

**ADVANCED FINITE ELEMENT TECHNIQUES FOR
COMPUTING TRANSVERSE MODES OF OPTICAL FIBERS:
OPTICAL FIBER MODE SOLVER COMPUTATIONAL
MODEL PROJECT**

Jay Gopalakrishnan

**Portland State University
1600 SW 4th Ave
Portland, OR 97201-5522**

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DR. JACOB GROSEK, DR-II
Work Unit Manager

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DR. IMELDA ATENCIO, DR-IV
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1.0 SUMMARY

This report describes the progress and achievements associated with the development of an advanced computer model for accurately finding optical fiber mode data; improving upon past techniques already found in the scientific literature. This effort has been conducted with support from collaborators at the Oden Institute of the University of Texas at Austin and the Air Force Research Laboratory (AFRL), Directed Energy Directorate, Laser Division (RDL). The products of this project, namely the simulation code and improved understanding of how to find optical fiber mode data, has been transferred to the AFRL/RDL team in support of their applications.

2.0 INTRODUCTION

The transverse modes in optical fibers form a basis for the justification and implementation of AFRL's Coupled Mode Theory (CMT) fiber amplifier models. This project focuses on sharpening the toolset for accurate computation of transverse modes and their confinement losses. This includes offering the AFRL the ability to model fibers with embedded microstructures that impose sophisticated boundaries that can be captured by curvilinear finite elements.

While the project began by focusing on the further development of the discontinuous Petrov Galerkin (DPG) Finite Element Method (FEM) for the purpose of modeling optical fiber amplifier guided mode data, all methodological options were investigated. An uncertainty quantification analysis of numerically calculated modes, first considering the Generalized Polynomial Chaos (gPC) approach [1, 2] was originally planned, but initial research revealed that this task is better handled after more fundamental questions on mode computation have been answered. The main thrust of the project evolved into developing the mathematical theory behind, and building a robust and versatile computer model for finding, optical fiber guided modes (and associated mode data) using any suitable finite element discretization in conjunction with the FEAST algorithm (a numerical eigensolver technique; FEAST = "fast eigensolver"). Furthermore, the project aimed to resolve issues with finding the correct mode loss values when fibers are coiled.

The most current version of our advanced finite element eigensolver (optical mode solver) tool for the AFRL captures the mode loss data in any arbitrary, yet manufacturable, straight fiber under any suitable discretization (including finite element size), polynomial order (of the FEM discretization), and position of the outer radial boundary condition in the appropriate fiber region, which is a type of perfectly matched layer (PML) – an absorbing boundary layer. Based on our literature review and to the best of our knowledge, we are the first to show a proper mathematical convergence of optical mode solver model.

Our efforts also revealed that the existing methodologies for finding the mode confinement losses under fiber coiling conditions have been making unquantified approximations that certainly affect the loss values. We have reported to our AFRL collaborators what these approximations are, and have begun to investigate numerical approaches that will address these approximations and will, hopefully, provide provable convergence to the correct confinement losses. In fact, one of our graduate students, Pieter Vandeberge, is currently a summer 2021 AFRL Scholars Program intern at the AFRL/RDL working on this topic.

3.0 METHODS, ASSUMPTIONS, AND PROCEDURES

The main assumption is that the propagating optical wave can be decomposed into individual transverse modes, each with corresponding amplitudes that can grow or attenuate as they propagate along the length of the fiber waveguide. Guided modes are solved separately in the two dimensional (2D) transverse cross-sectional domain of the given optical fiber, and are primarily determined by the distribution and magnitude of the refractive index throughout this domain. Computationally, mode solving is formulated as a Helmholtz

eigenvalue problem, where the modes are the eigenvectors and the square of the mode propagation constants are the eigenvalues. The mode confinement losses are calculated as twice the imaginary component of the propagation constant. We found that the FEAST algorithm [3–6] is a versatile and efficient numerical method for solving this eigenproblem, allowing us to apply desirable finite element (FE) discretizations as needed.

Of greatest concern is the implementation of the outer radial boundary condition, and its effect on the calculated mode data. Indeed, much of our effort has focused on ensuring that this boundary condition has been resolved, leading to a mathematically convergent simulation tool. However, issues with the implementation of a suitable outer radial boundary condition soon resurfaced as we then considered the stress effects due to fiber bending. As with the rest of the optical fiber community, we assume that, at least locally to a given position along the fiber length, the fiber is circularly coiled. The geometry of this circular coiling leads to a non-constant parameter space that grows out to infinity, which makes the implementation of any finite outer radial boundary condition an approximation that is expected to affect the calculated mode confinement losses.

Work on the closely related task of propagation models benefited from the seamless integration of the output from the mode solver code as input to the CMT propagation code that we built in-house. Since these two codebases require different tools, they are implemented as two different git repositories, with the propagation code depending on the mode solver code. All team members have password access to these secure repositories, including those at the Oden Institute and at the AFRL/RDL.

Note that the details of our modeling efforts and procedures are thoroughly delineated in submitted and published journal articles, which will be referenced in the next section.

4.0 RESULTS AND DISCUSSION

Our *python*¹ code [7], created at the inception of the project, has now matured. It is being used in other projects of interest to the AFRL involving studies on modeling the *transverse mode instability*², efficiency, and expected thermal loads in high-power fiber laser amplifiers. Computer modeling facilities for the latter are built into another *Bitbucket*³ repository [8], which utilizes both *python* and *c++* coding languages.

The first theoretical investigations of the convergence properties of the FEAST eigensolver, using the DPG discretization, found that it does not generate spurious modes and is guaranteed to achieve convergence under certain conditions. This result is reported in the journal publication [5].

Proofs of convergence when other discretizations are used were established in an abstract setting (that covers many discretizations beyond the DPG discretization). These results are published in one of the top mathematical journals in the field [6].

As the project progressed, further results were obtained using the both codebases [7, 8]. Significant results included an equivalent short fiber for gain computations [9, 10] and extensive first-of-a-kind-simulation results for a two-tone thulium-doped fiber [11]. Both the publications [10] and [11] acknowledge this contract in recognition of the partial support it provided to the students in the team.

To make further progress on the central goals of the project, we then proceeded to extend the codebase in [7] to cover non-self adjoint eigenproblems. This was a necessary step in preparation to compute leaky modes; i.e. when mode loss is present/expected. We then proceeded to study various options for computing confinement

¹*Python* is an open source programming language.

²The *transverse mode instability* is a thermally-induced detrimental nonlinearity that reduces output beam quality in high-power fiber laser amplifiers.

³*Bitbucket* is a type of git repository for sharing and collaborating on files, documents, and codes, with options for password protection and other security protocols.

losses of leaky modes accurately. Months of literature review found that loss values are often reported in the optics literature without any convergence studies and are difficult or impossible to reproduce. This then led us to examine the mathematical literature for cutting-edge advances in PML techniques used for truncation of the infinite-domain eigenproblem to a computable finite-domain eigenproblem. We found that a frequency-dependent PML suggested in [12] appeared to produce the best results. However, this PML has not yet been widely adopted because it requires one to solve a nonlinear eigenproblem.

In view of this situation, we embarked on developing an innovative solver for that nonlinear eigenproblem, after reducing the nonlinearity to a polynomial nonlinearity. We developed a novel and efficient adaptation of the FEAST algorithm for solving such polynomial eigenproblems together with a mathematical guarantee that the algorithm will recover the correct modes. When combining it with the frequency-dependent PML of [12], additional advantages surfaced. For example, any spurious mode supported only in the PML region becomes part of the eigenspace of an infinite eigenvalue and is automatically filtered out. The new algorithm can also use elliptical, and/or other, more complex closed contours, to effectively probe the eigenvalue space while staying away from the deformed essential spectrum. This is crucial for avoiding myriads of spurious modes that otherwise would get calculated and presented as potential solutions to the given problem. All these new results are presented in [13], where a hollow-core fiber (a type of microstructure fiber) was used to exemplify the success of our approach.

Work on integration of the mode solver with propagation code was central to the dissertation [14] of Tathagatha Goswami, who successfully defended his PhD in 2021, and was supported, in part, by this contract. Another student, Dow Drake, also supported partially by this contract, completed his Masters degree and contributed substantially to the codebase [8] and to the initial research [10]. The collaborators at the Oden Institute consulted and advised both the AFRL team and our group in the development of this 2D FE-FEAST eigensolver. In fact, our collaboration has been conducted through both formal and informal meetings, events, and workshops with these teams [15]. For example, the progress and achievements of this project have been presented at the annual AFRL mini-workshops on fiber amplifier model development, and at the annual reviews of the Air Force Office of Scientific Research (AFOSR) Computational Mathematics Portfolio.

5.0 CONCLUSIONS

This project has contributed to the theory, design, and implementation of accurate and efficient computational transverse mode solvers for general optical fibers, including microstructured fibers. The DPG methodology has proved to provide a solid foundation for both targeted applications: computation of transverse eigenmodes and direct numerical simulations of optical fibers. More generally, eigensolver methodologies applicable to any discretization were also developed. AFRL partners have continual access to the git repositories containing the numerical tools developed in the course of this project. A major accomplishment of this effort is the fact that a provably convergent mathematical approach for finding mode loss values has been discovered and implemented in our new optical mode solver computational tool. Once the remaining issues with the outer radial boundary condition under fiber coiling conditions have been resolved, we concur with the Oden Institute's assessment that "If more time and funding were available for this project, then returning to the original goal of investigating a uncertainty quantification strategy for the optical mode solver would be recommended" [16].

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LIST OF ACRONYMS AND ABBREVIATIONS

2D	two dimensional
AFRL	Air Force Research Laboratory
AFOSR	Air Force Office of Scientific Research (AFRL directorate)
CMT	coupled mode theory (optical fiber modeling approach)
DPG	discontinuous Petrov-Galerkin (finite element discretization approach)
FE	finite element (used to discretize a problem domain)
FEAST	fast eigensolver numerical technique
FEM	finite element method (discretization of a problem domain)
gPC	generalized polynomial chaos (uncertainty quantification approach)
i.e.	(Latin) id est meaning “that is”, “specifically”, “namely”
PML	perfectly matched layer (absorbing boundary condition/region within the computational domain)
RDL	Laser Division with AFRL, Directed Energy Directorate

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