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Beating shot noise using multi-port interferometers

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“Correlated Boson Sampling Metrology (CoBoSaME)”

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Abstract:

We have developed a theoretical framework to assess the metrological power of a reconfigurable multiport interferometer. This framework is a new formalism that can describe arbitrary linear setups using the Gaussian state technology, combined with the classical Fisher information studies. This framework revealed that earlier proposals based on boson sampling did not achieve any advantage with respect to conventional Gaussian interferometry, thereby shaking the foundations of this field and of our project’s working hypothesis. In parallel to this work, motivated by the need to simulate large non-Gaussian problems, we have found a new toolkit of quantum-inspired algorithms and quantum computing algorithms for performing multivariate numerical analysis—i.e. from solving partial differential equations to constructing and analyzing large multivariate probability distributions. This research activity has led to two scientific contributions (see items a) and b) in the List of Publications). Additionally, the CoBoSaME project has permitted to address other questions beyond the scope of its research program which has been collected in several peprints as well (see items c), d) and e)).

Introduction:

CoBoSaME aims at **probing the sensing capabilities of multiport interferometers** using techniques inspired by boson sampling. As explained in the proposal, there were initial works that showed a measurable advantage of such devices when compared with other classical or Gaussian interferometry schemes. CoBoSaME’s theoretical efforts were devoted to **computing the phase resolution of a reconfigurable photonic circuits** under a variety of input states. This requires the study of the Fisher information for the whole set of Gaussian resources (e.g. input coherent states or homodyne detection) endowed with an arbitrary number of modes (i.e. multivariate Gaussian distributions). With such a framework we could provide detailed information about the potential quantum advantages to be reached in multiport interferometers, such as the ones available at Air Force Research Laboratories.

Results, Discussion:

I) Multiport interferometry

Instead of directly using the Symmetric Logarithmic Derivative (SLD), we developed a new algebraic treatment to compute the full Fisher information of the optical field in the multiport interferometer device. This represents one of the major achievements of the research work, which renders several advantages compared to the earlier and more difficult phase-estimation frameworks. In particular, we successfully recovered the vast majority of preceding results for small interferometric circuits exploiting Gaussian resources. Furthermore, it also provides useful insight on the interplay between the resources and experimental imperfections, such as losses and non-ideal detectors, which could open new ways for harnessing the input resources in order to mitigate degrading effects.

Most importantly, this technique allowed us to address the central question in CoBoSaME, showing that ***the original boson-sampling-inspired metrology strategies don’t provide a real metrological advantage*** with respect to the best classical strategy, for any number of modes. In other words, ***the literature on top of which CoBoSaME was designed was proven wrong***, disrupting our original research program.

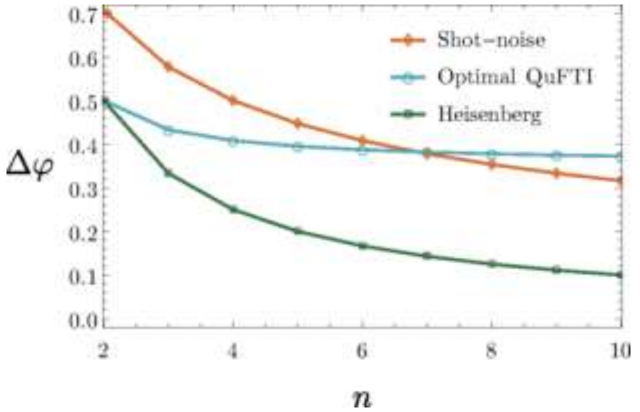


Fig. 1. In the work by J. P. Olson et al, Phys. Rev. A 96, 013810 (2017), this plot showed the potential advantage of Boson-Sampling schemes (QuFTI) for interferometry. However, our new calculation schemes show that the red line underestimates the capability of interferometry with coherent states. An updated calculation places the red line below the blue one so that QuFTI is actually worse than trivial interferometry with coherent states.

II) Quantum computer and quantum-inspired description of multivariate functions

Despite this setback, our project has had some important spin-offs. One of them is the development of a **quantum computing toolbox for solving complex multivariate analysis problems**. The original motivation for this research was to find an efficient computational description of the multivariate probability distributions that appear when studying continuous variable states ---e.g. the states of light passing through multiport interferometers--. These states are smooth Gaussian functions or deformation thereof, but they are too "large" for conventional processing with traditional algorithms.

To cope with this problem, we developed efficient encodings of such distributions that work in a Near-term Intermediate Scale Quantum (NISQ) computer. The encoding assumes a uniform discretization of each spatial variable "x" using s=0,1,…2^n-1 points

$$x_s = -\frac{L_x}{2} + s\Delta x$$

where "n" is the number of qubits, Lx the length of the interval and Δx the spacing between lattice points. Once this discretization is constructed, we encode the integer "s" in a set of "n" qubits, so that √f(x) or f(x) (different choices lead to different algorithms) becomes the actual wavefunction in the quantum computer.

We **proved analytically** and confirmed **numerically that smooth differentiable functions** have a simple encoding in quantum registers with **moderate amounts of entanglement**. Simulations with Gaussian states, such as the one of our multiport interferometer studies, required entanglement of N-ebits for an N-variable function, which is an affordable limit near-term intermediate scale quantum computers.

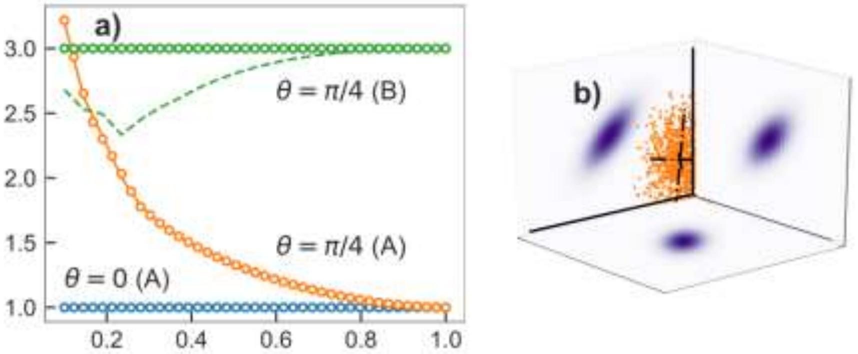
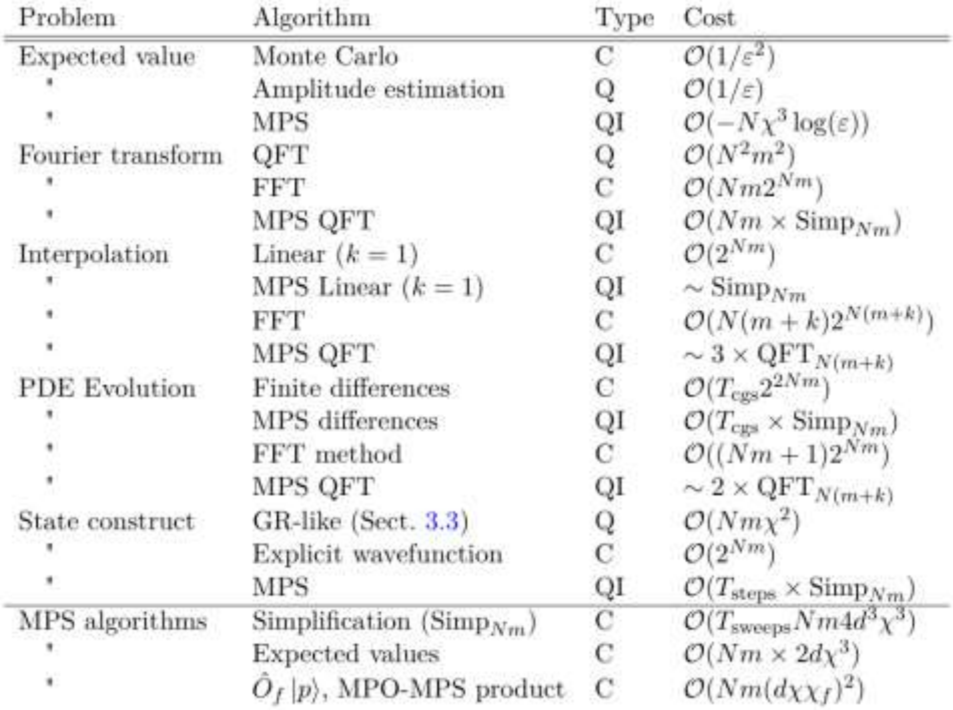


Fig. 2. A three-variable squeezed and rotated probability distribution is encoded using a quantum register. With a trivial ordering of qubits (A), the amount of entanglement diverges, and the state is

hard to build. However, if we reorder the qubits by significance (B), the amount of entanglement is bounded to 3. This bound becomes 1 and 2 for one-dimensional and two-dimensional distributions.

This encoding suddenly opens the door to solving problems outside our initial scope: it can be used to solve differential equations, perform signal analysis, interpolate functions and do all sorts of quantum numerical analysis tasks, with provable computational advantages. Moreover, for the type of functions that usually appear in our sensing scenarios---i.e. smooth differentiable distributions with finite bandwithds---, we found that those algorithms admit an efficient representation in terms of matrix product states (MPS), which can be efficiently simulated in classical computers, thus opening us the door to explore arbitrarily complex sensing scenarios.

The table below summarizes **the algorithms that can be applied to this encoding**, both **in quantum computers and using tensor network states**. Each algorithm has an associated cost, measured in time or number of operations. In some cases, such as Fourier transforms and interpolation, there are exponential gains over classical algorithms that need to explore the exponentially large amount of data required to encode a function.



Problem	Algorithm	Type	Cost
Expected value	Monte Carlo	C	$\mathcal{O}(1/\varepsilon^2)$
"	Amplitude estimation	Q	$\mathcal{O}(1/\varepsilon)$
"	MPS	QI	$\mathcal{O}(-N\chi^3\log(\varepsilon))$
Fourier transform	QFT	Q	$\mathcal{O}(N^2m^2)$
"	FFT	C	$\mathcal{O}(Nm2^{Nm})$
"	MPS QFT	QI	$\mathcal{O}(Nm \times \mathrm{Simp}_{Nm})$
Interpolation	Linear ($k=1$)	C	$\mathcal{O}(2^{Nm})$
"	MPS Linear ($k=1$)	QI	$\sim \mathrm{Simp}_{Nm}$
"	FFT	C	$\mathcal{O}(N(m+k)2^{N(m+k)})$
"	MPS QFT	QI	$\sim 3 \times \mathrm{QFT}_{N(m+k)}$
PDE Evolution	Finite differences	C	$\mathcal{O}(T_{\mathrm{cgs}}2^{2Nm})$
"	MPS differences	QI	$\mathcal{O}(T_{\mathrm{cgs}} \times \mathrm{Simp}_{Nm})$
"	FFT method	C	$\mathcal{O}((Nm+1)2^{Nm})$
"	MPS QFT	QI	$\sim 2 \times \mathrm{QFT}_{N(m+k)}$
State construct	GR-like (Sect. 3.3)	Q	$\mathcal{O}(Nm\chi^2)$
"	Explicit wavefunction	C	$\mathcal{O}(2^{Nm})$
"	MPS	QI	$\mathcal{O}(T_{\mathrm{steps}} \times \mathrm{Simp}_{Nm})$
MPS algorithms	Simplification (Simp_{Nm})	C	$\mathcal{O}(T_{\mathrm{sweeps}}Nm4d^3\chi^3)$
"	Expected values	C	$\mathcal{O}(Nm \times 2d\chi^3)$
"	$\hat{O}_f\lvert p\rangle$, MPO-MPS product	C	$\mathcal{O}(Nm(d\chi\chi_f)^2)$

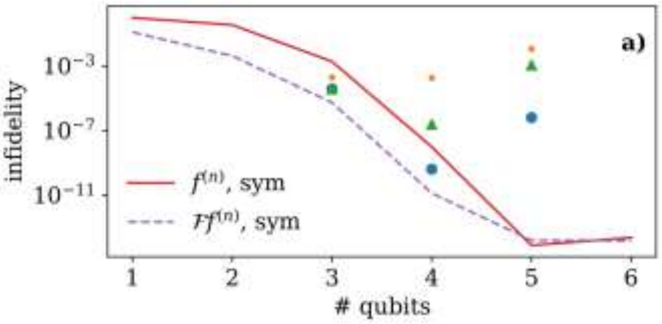
This work and the aboved mentioned algorithsm is available as e-print in the arXiv, under https://arxiv.org/abs/1909.06619 and is under referral by the journal Quantum

III) Quantum simulation of PDE’s in real quantum computers

Building on the ideas from Part II) we have implemented some proof-of-concept quantum simulations where we use an IBM-Q quantum computer to solve the partial differential equation of a (i) quantum harmonic oscillator and (ii) a superconducting qubit, using the quantum computer itself. For instance, the equation of the quantum harmonic oscillator is

$$\left[\frac{1}{2}\frac{d^2}{dx^2} + \frac{1}{2}x^2\right] f(x) = Ef(x)$$

Our first idea is to use the quantum Fourier transform to efficiently interpolate the function over dense grids, while simultaneously obtaining good estimates of the derivate and position operator. This means that, given a function f(x) encoded in a quantum computer, we can estimate the “energy” E associated to it. The second idea is to restrict our search for functions f(x) to a family of variational quantum states that are built with a finite set of parameterized gates.

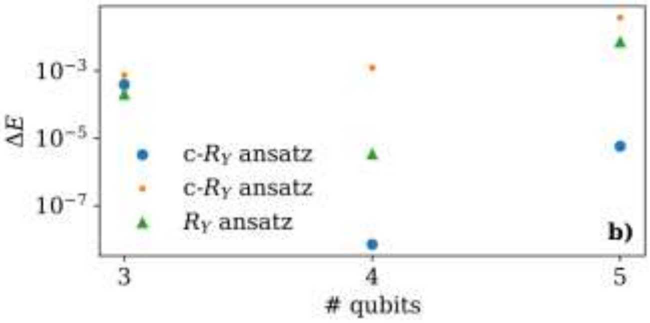


Our **quantum variational PDE solver** therefore optimizes the variational parameters to find the best approximation to the solution of the equation. We are currently tuning this algorithm, testing different optimization methods and different variational ansatz.

This first plot studies the fidelity that can be achieved by simulating the probability distribution of a harmonic oscillator using different algorithms. The red and dashed lines assume a perfect encoding of the function in N qubits (N=1-6), combined with interpolation to 2^17 points. As shown in the plot, an encoding with 4 qubits can almost perfectly reconstruct the ground state of the problem, perfectly interpolating to any other resolution, both in Fourier and position space.

The dots and markers in the same plot assume that we use a variational ansatz to describe the function. The blue circles use a controlled-rotation ansatz of our invention that encodes the inversion symmetry of the problem. The green triangles use the so called RY ansatz from IBM's Qiskit library. Both have been optimized for fidelity under ideal conditions and reflect the potential accuracy of the ansatz in solving the problem.

Once we state the accuracy of the ansatz, we can apply them to the energy optimization itself, using a simulated quantum computer, measurements (1000 shots per optimization step) and a stochastic optimization algorithm (SPSA) which is compatible with the error fluctuations of the quantum computer. Those results are shown as orange dots in the plot.



Naturally, the fidelity worsens and increases to around 10^-3, due to the statistical uncertainty introduced by quantum measurements. This increase in the infidelity comes also with a similar deviation in the estimate of the energy. However, note that the actual accuracy in the energy (0.001) lays well below the uncertainty of the measurements (0.03), which is a promising result.

Summary

1. The working hypothesis of this project have been disrupted, finding that key manuscripts were incorrect in the presentation of their results. This means that our original proposals for boson-sampling inspired metrology are not feasible and need to be redesigned.

2. We have found that there is a promising line of research in developing quantum algorithms for numerical analysis, and applying them to various problems, from studying non-Gaussian interferometry protocols, to as solving differential equations for fluid dynamics and engineering. [eprint available]

3. We have demonstrated that these ideas can be used to develop quantum inspired algorithms: i.e., classical programs that have an advantage when solving partial differential equations, interpolating functions, integrating, etc. These algorithms have immediate application in the simulation of stochastic processes, such as in risk analysis and finance. [eprint available]

4. We have started a third work that combines those quantum analysis methods with variational quantum algorithms in the IBM-Q quantum computers. Our work shows that it is possible to

solve useful equations, such as the harmonic oscillator and the equations for a charge and flux qubit using the quantum computer itself [manuscript in preparation].

List of Publications and any Significant Collaborations that resulted from your AOARD supported project: In standard format showing authors, title, journal, issue, pages, and date, for each category list the following:

a) papers published in peer reviewed journals

- Antonio A. Valido, *Quantum dissipation of planar harmonic systems: Maxwell-Chern-Simons theory*, Phys. Rev. D **99**, 016003 (2019).

b) papers published in peer-reviewed conference proceedings,

c) papers published in non-peer-reviewed journals and conference proceedings,

d) conference presentations without papers,

We do not have contributions of types b)-d)

e) manuscripts submitted but not yet published

- Antonio A. Valido and Juan José García-Ripoll, *Gaussian phase sensitivity of boson-sampling-inspired strategies*, to be submitted.
- Juan José García-Ripoll, *Quantum-inspired algorithms for multivariate analysis: from interpolation to partial differential equations*, arXiv:1909.06619. Submitted to Quantum.
- Antonio A. Valido, *Statistical physics of flux-carrying Brownian particles*, arXiv:1905.03323.
- Al-Tarazi Assuabay, Alejandro J. Castro, and Antonio A. Valido, *Wigner instability of the damped Hirota equation*, arXiv:1910.11045.

f) provide a list any interactions with industry or with Air Force Research Laboratory scientists or significant collaborations that resulted from this work.

Unfortunately, due to our cancelation of travel arrangements for APS March Meeting, we have not been able to arrange a visit to the Air Force Research Laboratory that were originally planned. We welcome further interaction by online means if the researchers in the lab find that this work merits some collaboration, or to answer any questions that arise from the report.

DD882: As a separate document, please complete and sign the inventions disclosure form.

Neither the researchers (Juan José García Ripoll, Antonio Alejandro Valido) nor CSIC claim any patentable invention arising from these works.

Important Note: If the work has been adequately described in refereed publications, submit an abstract as described above and refer the reader to your above List of Publications for details. If a full report needs to be written, then submission of a final report that is very similar to a full length journal article will be sufficient in most cases. This document may be as long or as short as needed to give a fair account of the work performed during the period of performance. There will be variations depending on the scope of the work.