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Quantum Information in Gravity

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EXECUTIVE SUMMARY

The purpose of this document is to provide an overview of the work done under the basic research project "Quantum information in gravity."

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QUANTUM INFORMATION IN GRAVITY

1. BACKGROUND

Quantum information systems are highly susceptible to environmental coupling. This coupling both causes systems to decohere as a detriment to quantum information processing, but also allows quantum systems to be used as precise sensors, able to outperform their classical counterparts. This deleterious effect of noise is well documented in the literature [e.g. 1–3], which, along with other foundational challenges, have limited the realizations of quantum information processing technologies. Noise sources, which can include thermal, vibrational, and electromagnetic radiation, have limited quantum computational processes to the 10's of qubits and only allow for shallow gate depths. In addition to the noise sources already mentioned, a gravitational field will couple with, and rotate the spin of, a qubit. This is due to the direct coupling of quantum information systems to gravitational fields described by Einstein's general theory of relatively [4]. So unlike traditional forms of classical information, quantum information gravitates. While small, the effect of gravity is cumulative. For example, on a large enough computational space, gravitational noise can erase the computational speed up of Grover's algorithm [5].

2. INTRODUCTION

Quantum particles will interact with a gravitational field, and slowly change over time. For massive spin-1/2 particles, this coupling induces a rotational change of the spin. For massless particles, this change presents in phase. Let us consider a massive spin-1/2 particle described by the Dirac equation; the induced rotation is called the Wigner rotation [4, 6]. That is, the unitary transformation δU that describes the infinitesimal Lorentz transformation $\delta\Lambda(x)$ in the local inertial frame at x, gives the net effect of a the gravitational field. On a spinor $\psi_{p,\sigma}$ of momentum p and spin σ, the differential effect of a gravitational field is

$$\delta U \psi_{p,\sigma} = N(p, \delta\Lambda p) \sum_{\alpha} \delta D_{\sigma,\alpha} \psi_{\delta\Lambda p,\alpha}, \tag{1}$$

where $N(p, \delta\Lambda p)$ is a normalization factor and $\delta D_{\alpha\beta}$ is the unitary operator that represents the infinitesimal Wigner rotation. In the case that the Wigner rotation occurs on a single direction, the operator D on a single qubit simplifies to

$$D = e^{i\sigma\Omega/2}, \tag{2}$$

where Ω is the total angle of the Wigner rotation integrated over the entire path of the particle, and $\sigma = \alpha\sigma_x + \beta\sigma_y + \gamma\sigma_z$ is a superposition of Pauli operators that describes the direction of the Wigner rotation. This Wigner rotation depends on the gravitational field and trajectory of the particle, and is ultimately a function of time. For a complete discussion of the Wigner rotation, including formulas for its calculation, see [4, 7–13] and the references therein.

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3. QUANTUM INFORMATION IN GRAVITY

3.1 Relativistic Information Processing

In this section, we will describe the general relativistic effects on massive spin-1/2 particles. The net effect of a Lorentz transformation can be described by the unitary transformation in eqn. 1. We shall mainly concern ourselves with one-particle (positive energy) states that are in a superposition of states with definite momentum and spin: $|p, 0\rangle$ and $|q, 1\rangle$; for convenience we omit the indices of the 4-momentum of the particle, and use the computational basis $\{|0\rangle, |1\rangle\}$ to label the spin states. We use a treatment similar to that of [14, 15] and note that the expression $|p, s\rangle$ denotes a state of definite spin projection s and momentum p. However, in contrast to massless particles, one can choose a single arbitrary quantization axis to describe the (potentially non-definite) spin states of massive particles with different momenta. Lastly, we will take the combined system to be $\mathcal{H}_p \otimes \mathcal{H}_s$, where $\mathcal{H}_p$ and $\mathcal{H}_s$ are the momentum and spin state Hilbert spaces, respectively.

At this point, we should discuss the meaning of a quantum state of definite momentum and spin projection in curved spacetime. It is not possible to directly express spinors or universal quantization axes within the context of curved spacetime. Thus we need to define local inertial frames at each point of spacetime. This is done through a tetrad field $e_a^{\ \mu}(x)$ that covers the entire spacetime (in this notation, Greek indices refer to general coordinates and Latin indices to the coordinates in the local inertial frames) [4]. Then, the spin projection and associated quantization axes of a quantum state in curved spacetime are described by the inertial observers at each point of the tetrad field.

Therefore, the unambiguous description of the state of a spin-1/2 particle has to be denoted by

$$|p^a(x), j, \sigma; x^\mu, e_a^{\ \mu}(x), g_{\mu\nu}(x)\rangle, \tag{3}$$

which represents a state of spin j, spin projection σ, and definite momentum $p^a(x)$ as observed from the local inertial frame, with coordinates $x^a = e^a_{\ \mu}(x)x^\mu$ at a point x^μ defined by the tetrad field $e_a^{\ \mu}(x)$ in a curved spacetime described by the metric tensor $g_{\mu\nu}(x)$. The description of the spin state and associated quantization axes can only be given with respect to the local inertial frames in each point of spacetime as defined by the tetrad field.

Let us consider the example of a spin-1/2 particle in circular orbit around a static isotropic gravitational field produced by a spherically symmetric black hole [8]. As shown in fig. 1, the particle moves from point A to point B traversing an angle θ. We can use a tetrad field to define the local inertial frame (t, x^1, x^2, x^3) at point A and the local inertial frame $(\tilde{t}, \tilde{x}^1, \tilde{x}^2, \tilde{x}^3)$ at point B (where the axes in the "1" direction are parallel to the radial direction and the axes in the "3" direction are parallel to the tangent direction). The difference between these two frames is a trivial rotation of θ. So, if the local observer in A defines the quantization axis as x^1, the local observer in B will have to rotate its axes by an angle θ to agree on the quantization axis.

Additionally, in general relativity, a vector has to undergo parallel-transport along the curved manifold in order to compare local inertial frames at different points in spacetime. In particular, the tetrad field is used to provide the rule to transform a spin vector between two points in spacetime. In the example described, it can be shown that the spin vector S_A is transformed into S_B by applying both the trivial rotation θ and the gravitationally induced Wigner rotation Ω.

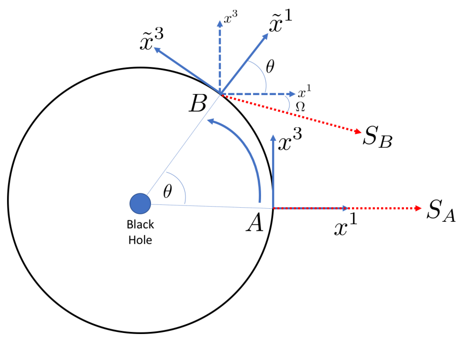

Fig. 1—Notional depiction of a massive spin-based qubit in circular orbital motion around a static spherically symmetric black hole. The spin projection is described with the local inertial frames defined by a tetrad field in each point of spacetime. The rotation by θ corresponds to a trivial rotation of the axes, while the angle Ω is the Wigner angle induced by the gravitational field.

Therefore, once we have used a tetrad field to define local inertial frames at all points of spacetime, an arbitrary quantization axis can be defined at some specific point (e.g., the x^1 direction in point A). Furthermore, the transformation of spin vectors between two different points in spacetime depends on the dynamics of the particle and the tetrad field (e.g., the Wigner rotation Ω with respect to the agreed quantization axis in the process of going from A to B).

With this in mind, and for the sake of simplicity, we rewrite the quantum states as

$$|p^a(x), j, \sigma; x^\mu, e_a^{\ \mu}(x), g_{\mu\nu}(x)\rangle \longrightarrow |p, \sigma\rangle. \tag{4}$$

Furthermore, the state given by

$$\alpha|p, \sigma_p\rangle + \beta|q, \sigma_q\rangle \tag{5}$$

denotes a superposition of two states of definite momentum p and q as observed in a local inertial frame defined by some tetrad field. Furthermore, some arbitrary axis in this local inertial frame has been used to quantize the spin projection.

In situations where the motion of the particle is confined to a plane, the Wigner rotation takes place along a single direction (e.g. around the y-axis), and the corresponding 2-level unitary transformation is given by the unitary matrix

$$D = e^{i\sigma_y \Omega_p/2} = \begin{pmatrix} \cos\left(\Omega_p/2\right) & \sin\left(\Omega_p/2\right) \\ -\sin\left(\Omega_p/2\right) & \cos\left(\Omega_p/2\right) \end{pmatrix}, \tag{6}$$

where Ω_p is the *Wigner angle*. In general, the Wigner angle depends on the momentum p and the Lorentz transformation Λ; for an explicit calculation of the Wigner angle, see [4].

Taking the quantization axis parallel to the z-direction (i.e. eigenvectors of the σ_z spin operator), the effect of the Lorentz transformation on the two positive energy Dirac states (spin up and spin down) is given by

$$\hat{U}|p,0\rangle = \cos\left(\frac{\Omega_p}{2}\right)|\Lambda p,0\rangle - \sin\left(\frac{\Omega_p}{2}\right)|\Lambda p,1\rangle, \tag{7}$$

$$\hat{U}|p,1\rangle = \sin\left(\frac{\Omega_p}{2}\right)|\Lambda p,0\rangle + \cos\left(\frac{\Omega_p}{2}\right)|\Lambda p,1\rangle. \tag{8}$$

Note that the concept of the Wigner rotation can be easily generalized for the study of Dirac states in the presence of classical gravitational fields described by Einstein's general relativity [4, 13].

To analyze this effect, we took a somewhat toy model and assume that we have a black box emitting the state

$$|\Psi\rangle = r|p,0\rangle + s|q,1\rangle, \tag{9}$$

with $|r|^2 > |s|^2$. We assume that momentum and spin were described by a fixed axis with respect to frame of reference, which all parties have knowledge of. Upon tracing out the momentum, i.e., just measuring the spin of the system, the reduced state is

$$\rho = \mathrm{tr}_p(|\Psi\rangle\langle\Psi|) = \begin{pmatrix} |r|^2 & 0 \\ 0 & |s|^2 \end{pmatrix}. \tag{10}$$

This state looks like an imperfectly prepared spin state, and it may be determined that for information processing purposes that it is an adequate approximation of the pure state.

There are two types of relativistic noise that we considered. One was kinematic, i.e., when the Lorentz transformation is induced from kinematic motion between two inertial frames of reference. The other is due to gravity from a massive body. As we showed in [16, 17], the states like those in eqn. 10 are becoming more pure and more distinguishable; however, this fact does not necessarily correlate to an increase in information transfer. This is due to the specific way the receiver and transmitter have agreed to encode information. Here we will use an example to illustrate this. Let us assume that an observer and particle are traveling at relativistic speeds with respect to one another, and further assume that the observer's motion is orthogonal to the particle's direction of travel. We also assume that the observer is moving with a constant velocity of $(9/10)c$. In fig. 2, we plot the entropy of the reduced state $\mathrm{tr}_p(|\Psi\rangle\langle\Psi|)$ pre- and post-evolution as a function of the momenta p and q. The change induced by kinematic motion is small; however, when not confining ourselves to kinematic motion, in the most general case the Wigner rotation can affect the entropy much more drastically. We have plotted this change in fig. 3. For example, in the general case (fig. 3), we obtain an interval of values that produce the minimum entropy state; however, in fig. 2, the entropy of the evolved state approaches 0 only when $v_p \to c$ and $v_q \to 0$, and when $v_q \to c$ and $v_p \to 0$. Since the entropy is decreasing toward a local minima in those cases, it would be reasonable to expect that those values maximize the capacity of the channel, as well. This intuition is incorrect for the former case. We have plotted the capacity as we range over velocity in fig. 4. From this plot, it is clear that that capacity does not take a local maximum when $v_p = c$ and $v_q = 0$, and additionally, we can see that the capacity never attains a value of 1, as it does under non-kinematic assumptions. The reason for this difference is that because the Wigner angle induced by kinematics can only range between 0 and $\pi/2$, and when $v_i = c$, then $\Omega_i = \pi/2$. In fig. 5, we have plotted the Wigner angle as a function of the particle's velocity. From this plot, the Wigner angle only

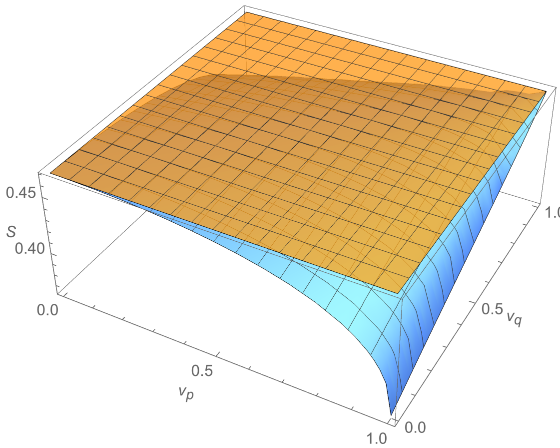

Fig. 2—Plot of the entropy pre- and post-evolution due to kinematic noise for the state in eqn. (10) with $r = \sqrt{9/10}$ and $s = \sqrt{1/10}$. The plane gives reference to the entropy pre-evolution, whereas the entropy post-evolution varies with velocities v_p and v_q, which are given as fractions of c.

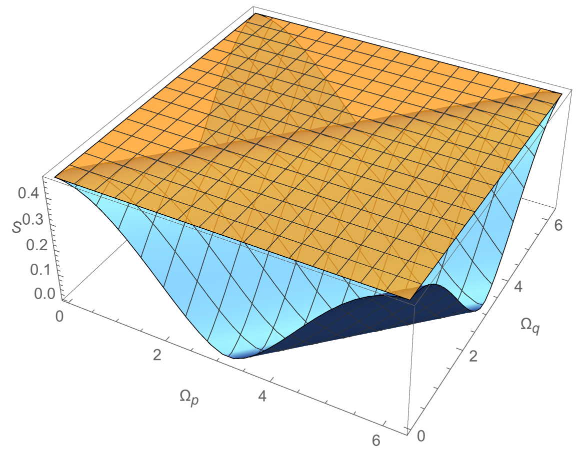

Fig. 3—Plot of the entropy pre- and post-evolution for the state in eqn. (10) with $r = \sqrt{9/10}$ and $s = \sqrt{1/10}$. The plane gives reference to the entropy pre-evolution, whereas the entropy post-evolution varies with Ω_p and Ω_q.

changes significantly at ultra relativistic velocities, at which point the change in the Wigner angle becomes highly non-linear. Using fig. 5, we are able to to reconcile the general capacity plot with the kinematic capacity plot. As a consequence, the Wigner angle never produces enough of an effect to greatly improve the capacity and, in this case, is more likely to be detrimental to information processing then beneficial.

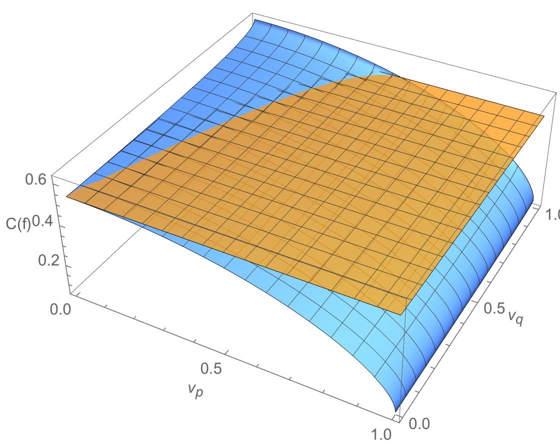

Fig. 4—The channel capacity when transmitting in the $\{\pm z\}$ basis (which varies over v_p and v_q) and the $\{\pm y\}$ basis (constant plane).

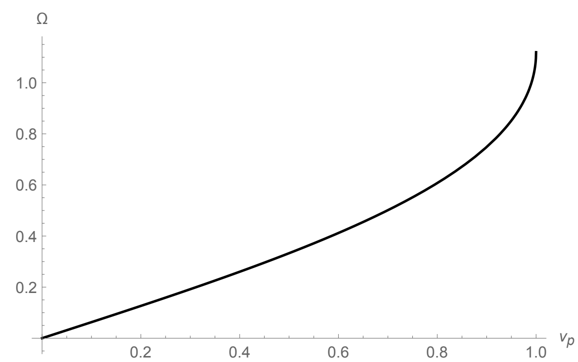

Fig. 5—Plot of the Wigner angle as a function the velocity of the particle p.

Gravitational effects, on the other hand, can produce Wigner angles in the full interval $[0, 2\pi]$. Let us consider an example of a static, stationary, neutrally charged, spherically symmetric black hole described in the Schwarzchild metric. In this example, the Wigner angle is completely dependent on the gravitational field generated by that black hole and the trajectory of the particle [4, 8, 10]. When the particle is following a free falling circular orbit of radius r_p around a black hole of mass M, the Wigner angle for a single orbital period is

$$\Omega_p = 2\pi\sqrt{\delta}\left(1 - \frac{Kr_s}{2r_p f}\frac{1}{K+\sqrt{\delta}}\right) - 2\pi, \tag{11}$$

where

$$K \equiv \frac{1-\frac{r_s}{r_p}}{\sqrt{1-\frac{3r_s}{2r_p}}}, \quad \delta \equiv 1 - \frac{r_s}{r_p}, \quad \text{and} \quad r_s \equiv 2M, \tag{12}$$

and all given in natural units ($G = 1, c = 1$); note though, from General Relativity, r_p must be larger than $\frac{3}{2}r_s$ [4]. Then, once a particle has completed an entire circular orbit in the Schwarzchild spacetime, Ω_p is the total rotation of the spin due solely to the presence of the gravitational field. Consequently, this effect is purely relativistic and caused by the interaction of a spin-$\frac{1}{2}$ quantum field with a classical gravitational field. In figs. 6 and 7, we can see the behavior of the entropy of the reduced state $\mathrm{tr}_p(|\Psi\rangle\langle\Psi|)$ after the completion

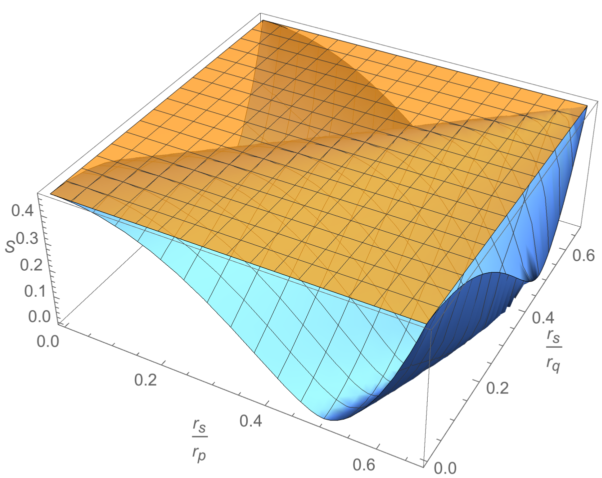

Fig. 6—Plot of the entropy pre- and post-evolution for the state in eqn. (10) with $r = \sqrt{9/10}$ and $s = \sqrt{1/10}$. The plane gives reference to pre-evolution, whereas the entropy varies after completion of a free falling circular orbit around a black hole of mass $r_s = 2M$ and orbital distances r_p and r_q.

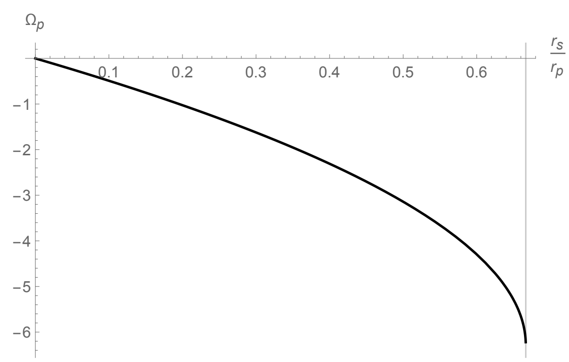

Fig. 7—Plot of the Wigner angle as a function the radial distance r_p of a particle from a massive gravitational body of mass $r_s = 2M$ after the completion of a circular orbit.

of a circular orbit at the orbital distances of r_p and r_q, and the the behavior of the Wigner angle as a function of orbital distance r_p. However, as we can see from fig. 8, the noise caused by gravity can induce a channel with unit capacity.

In summation, we determined that noise contributed by kinematic motion was minimal, and that it would take an extremely massive body to create a measurable amount of noise. We hazard the complete dismissal of smaller gravitational effects (like those encountered while orbiting Earth) as inconsequential though. The Wigner angle is complex to calculate [4], and it would be difficult, if not impossible, to calculate for a massive body that was not spherically symmetric. Therefore, if a quantum computation was executed on Earth's surface, correcting for the small Wigner rotation would most likely not be possible, and the effect would be cumulative.

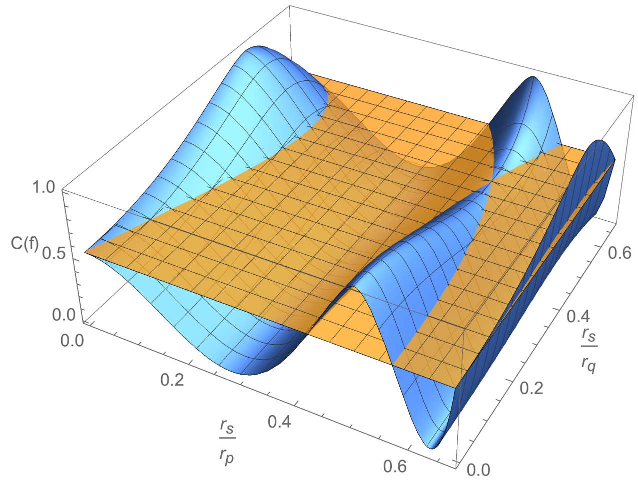

Fig. 8—The channel capacity when transmitting in the $\{\pm z\}$ basis (which varies over r_p and r_q) and the $\{\pm y\}$ basis (constant plane).

In addition, part of this work was inspired by wondering if a state state $|\Psi\rangle$ with

$$|r|^2 = 1/2 + \varepsilon \text{ and } |s|^2 = 1/2 - \varepsilon \tag{13}$$

and $0 < \varepsilon << 1$ could be used for transmitting information to another reference frame stegenographically. As long as the momenta and Wigner angles were chosen carefully, the information could be transferred reliably. Since in the preparer's frame of reference, the state is close to the completely mixed state, it would look like random noise to anyone trying to intercept the message in that frame, regardless of the measurement basis used. Thus we could send hidden information from one frame to another. However, we showed that the increase in capacity from kinematic noise is negligible. If one were to use a state with $r = \sqrt{.501}$ and $s = \sqrt{.499}$, the maximal capacity one could achieve from kinematic motion would be approximately 0.06. Therefore, this could only be used stenographically in the neighborhood of a black hole.

Finally, there have been discussions about the reduced density matrix, the appropriate measurement observables related to relativistic spin-1/2 particles [18–22], and whether or not the linear application of Wigner rotations creates a paradox. In [17], we address the issue of the amount of information ascribed to a reduced density matrix in relativistic settings, and in [23], we show that there is no paradox created by the linear application of Wigner rotations.

3.2 Computation

As we have previously mentioned, Earth's gravitational field effects the informatic content of a single qubit negligibly. However, we have shown that gravitational noise accumulates during computation, even with the most basic operations. Given a large enough space and a deep enough computation [5], relativistic noise sources are no longer negligible and must be accounted for. Here, we discuss ways to mitigate this noise using an approach similar to decoherence free subspaces. These methods rely on more passive error correction protocols than some of the more popular error correcting codes. In this effort, we improved upon the model provided in [4] to give an explicit formula for the noise operator. Originally this operator was written as a numerical approximation, but we showed that it can be simplified as follows: for a n-qubit system, assuming the Wigner rotation occurs along a single direction for each qubit, the operator is described by

$$D = e^{i\Omega \sum_j \Delta_j / 2}, \tag{14}$$

where

$$\Delta_j = \underbrace{I_2 \otimes \cdots \otimes I_2}_{j-1} \otimes \sigma_j \otimes \underbrace{I_2 \otimes \cdots \otimes I_2}_{n-j}, \tag{15}$$

Ω is the Wigner angle applied to each qubit, and σ_j has a potentially different direction for each qubit. While we are assuming the direction of the Wigner rotation is constant, we are not assuming they are equal across each qubit as they occupy different positions in space time. We will assume though that each Wigner angle affecting each qubit is equal, but will vary throughout the computation.

In [25], we showed that the gravitational noise operator does not commute with general single qubit operations, which are some of the fundamental building blocks of quantum algorithms. Therefore, for even the most basic computations, this noise will become cumulative. However, we did show that by embedding these computations in a slightly larger Hilbert space that the computations are unaffected by noise, similar to decoherence free subspaces. Additionally, as the computational space grows, the percentage of extra qubits needed for the embedding tends toward zero. We favored this approach since active error correcting codes have a high computational cost and require full communication across the quantum system [24]. The complete details of this work are currently under review for publication [25].

In work done prior to the author joining this project, the previous PI modeled expressions for the effect of gravity on adiabatic systems. For these systems, it is difficult to measure gravity's affect on runtime because there was no standardization for how to establish adiabatic evolution time. This is opposed to a gate model quantum computer, for which there is a well-defined metric for how many gates an operation utilizes, and therefore how many extra computational steps gravity induces. We came to the conclusion that there will be a change in computational time, and that gravity will impact the system, but quantifying this change in run time becomes a problem. This effort was done in conjunction for developing the theoretical foundation to detect gravitational effects using the D-Wave; however, implementing this in practice though proved to be extremely difficult because it would require a dedicated D-wave machine for approximately three weeks due to reboot times.

3.3 Sensing

The previous PI completed the technical development of these two gravity-based sensing efforts. One was to analyze a new quantum-based approach for gravimetry. This design uses information encoded qubits to measure gravitation, which can provide resolution beyond the de Broglie limit of atom-interferometric gravimeters. It was shown that this approach also offers an advantage over current state-of-the-art gravimeters in its ability to detect quadrupole field anomalies. These quadrupole anomalies allow for the possibility to search the ocean for objects of interest, having application in search and recovery, e.g., locating a submerged aircraft on the ocean floor, based on the difference between the specific quadrupole signature of the object of interest and that of other objects in the environment. The range of possible applications strongly motivates implementation and testing of the proposed technology. This work was published in [26] which contains the full technical details of this work.

In addition to the induced rotation on massive spin-1/2 particles, gravitational effects cause a phase change to photons. An analysis of quantum-based methods for improved gravimetric sensing was performed; this analysis demonstrated that photon entanglement can provide an additional source of target-state information beyond what is possible using purely classical sensing techniques. Additionally, we estimated the effect of gravity on a photon if it was passed between space-based nodes, such as satellites. While this work could be extended to an arbitrary number of satellites, we restricted ourselves to three. We showed that there is a measurable phase change in this instance, and the phase will depend on the path of the photon through the gravitational field. Ultimately, we showed that it is theoretically possible to measure the effect of gravity on this state using interferometry; such a measurement is helpful in quantifying the effect of gravity on quantum information systems. As an application, we proposed a quantum-based system for large-scale space-based detection of small near-earth objects (NEOs). The objective of the system is to measure extremely small deviations in the background gravitational field within a defined surveillance region to identify potentially dangerous NEO intrusions as early as possible. The proposed system is composed of a set of widely-separated line-of-sight emitter-receiver pairs that exchange entangled photons so that the signature of a moving object can be discerned from subtle gravitation-induced spin effects. The key advantage of the system is that the detection does not require direct illumination of the target. A potentially more important practical advantage is that the system can be implemented using relatively simple interferometric measurements. The work here was presented at [27].

Note: The author would like to note that they were the principle investigator on this project for only a year and a quarter. The original principle investigator was Dr. Marco Lanzagorta and was responsible for not only the conception of this project, but also a majority of the research. The author has made their best effort to capture the research done under this project.

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First and foremost, we would like to thank Dr. Marco Lanzagorta for the excellent work he did on this project while he served as PI. We would also like to thank Dr. Keye Martin and Dr. Daniel Bonior for their curious questions, insightful discussions, and helpful comments throughout this project.

REFERENCES

1. S. Aaronson, “Read the fine print,” *Nature Physics* **11**(4), 291–293 (2015).

2. F. Leymann and J. Barzen, “The bitter truth about gate-based quantum algorithms in the NISQ era,” *Quantum Science and Technology* **5**(4), 044007 (2020).

3. J. Preskill, “Quantum Computing in the NISQ era and beyond,” *Quantum* **2**, 79 (2018), ISSN 2521-327X, doi:10.22331/q-2018-08-06-79. URL `http://dx.doi.org/10.22331/q-2018-08-06-79`.

4. M. Lanzagorta, *Quantum information in gravitational fields* (Morgan & Claypool, 2014).

5. M. Lanzagorta and J. Uhlmann, “Quantum computational complexity in curved spacetime,” in *Information and complexity*, pp. 227–248 (World Scientific, 2017).

6. S. Weinber, *The Quantum Theory of Fields* (Cambridge University Press, New York, 1995).

7. D. Boulware, "Spin-1/2 quantum field theory in Schwarzschild space," *Physical Review D* **12**(2), 350 (1975).

8. H. Terashima and M. Ueda, "Einstein-Podolsky-Rosen correlation in a gravitational field," *Phys. Rev. A* **69**(3), 032113 (2004).

9. A. Peres and D. Terno, "Quantum information and relativity theory," *Rev. of Mod. Phys.* **76**(1), 93 (2004).

10. P. Alsing, G. Stephenson Jr, and P. Kilian, "Spin-induced non-geodesic motion, gyroscopic precession, Wigner rotation and EPR correlations of massive spin 1/2 particles in a gravitational field," *arXiv preprint arXiv:0902.1396* (2009).

11. L. Parker and D. Toms, *Quantum field theory in curved spacetime: quantized fields and gravity* (Cambridge University Press, 2009).

12. P. Alsing and G. Stephenson Jr, "The Wigner rotation for photons in an arbitrary gravitational field," *arXiv preprint arXiv:0902.1399* (2009).

13. M. Lanzagorta and M. Salgado, "Detection of gravitational frame dragging using orbiting qubits," *Class. Quantum Gravity* **33**(10), 105013 (2016).

14. A. Peres, P. F. Scudo, and D. R. Terno, "Quantum entropy and special relativity," *Physical review letters* **88**(23), 230402 (2002).

15. R. M. Gingrich and C. Adami, "Quantum entanglement of moving bodies," *Physical Review Letters* **89**(27), 270402 (2002).

16. T. Crowder and M. Lanzagorta, "The scope of a relativistic quantum process with spin-momentum entanglement," Proceedings of the International Conference on Unconventional Computation and Natural Computation (Springer), 2018, pp. 46–58.

17. T. Crowder and M. Lanzagorta, "Quantum information processing in the neighborhood of a black hole," *Nat. Comp.* **18**(3), 549–561 (2019).

18. L. C. Céleri, V. Kiosses, and D. R. Terno, "Spin and localization of relativistic fermions and uncertainty relations," *Physical Review A* **94**(6), 062115 (2016).

19. P. L. Saldanha and V. Vedral, "Physical interpretation of the Wigner rotations and its implications for relativistic quantum information," *New Journal of Physics* **14**(2), 023041 (2012).

20. E. Taillebois and A. Avelar, "Spin-reduced density matrices for relativistic particles," *Physical Review A* **88**(6), 060302 (2013).

21. T. Choi, "Relativistic spin operator and Lorentz transformation of the spin state of a massive Dirac particle," *Journal of the Korean Physical Society* **62**(8), 1085–1092 (2013).

22. H. Bauke, S. Ahrens, C. H. Keitel, and R. Grobe, "Relativistic spin operators in various electromagnetic environments," *Physical Review A* **89**(5), 052101 (2014).

23. M. Lanzagorta and T. Crowder, "Comment on "Wigner rotations and an apparent paradox in relativistic quantum information"," *Phys. Rev. A* **96**(2), 026101 (2017).

24. M. Nielsen and I. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, 2010).

25. T. Crowder and M. Lanzagorta, "Gravitationally Invariant Subspaces in Quantum Computing," *Under Review* (2021).

26. M. Lanzagorta and J. Uhlmann, "Theoretical foundations for design of a quantum Wigner interferometer," *IEEE Journal of Quantum Electronics* **55**(1), 1–7 (2018).

27. M. O. Lanzagorta, J. Uhlmann, S. Venegas-Andraca, and O. Jitrik, "Opto-gravimetric quantum sensing (Conference Presentation)," Proceedings of the Radar Sensor Technology XXIV, volume 11408 (International Society for Optics and Photonics), 2020, p. 114080P.