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Design of a Wavelength Scaled Cylindrical Array (WSCA)

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Ultra-wide bandwidth (UWB) cylindrical arrays are important apertures for applications requiring full 360 degree coverage in azimuth as they offer many benefits over multi-faced planar arrays for such applications. To maximize their performance and to demonstrate their benefits over planar arrays, the cylindrical array's aperture must have elements spaced near half-wavelength both circumferentially and vertically (along the axis of the cylinder) at the high frequency of operation. For UWB operation, the height and/or radius of the array is often set by the desired aperture size for the low frequency of operation. In many instances, this results in an array that is far larger than needed at the high frequency with far more elements than required for the low frequency. The high-density of elements results in a large element count which drives up array cost, complexity, and weight. In this report, the proposed cylindrical array topology extends the concept of a wavelength scaled planar array to a cylindrical array by placing elements on different lattices in different portions of the array.

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DESIGN OF A WAVELENGTH SCALED CYLINDRICAL ARRAY (WSCA)

1. Introduction

Ultra-wide bandwidth (UWB) cylindrical arrays are important apertures for applications requiring full 360 degree coverage in azimuth, as they offer many benefits over multi-faced planar arrays for such applications. Unlike planar arrays, cylindrical arrays are capable of true omnidirectional coverage and maintain constant beamwidth, gain, polarization, and impedance match over a full 360 degree scan area in azimuth. In To maximize their performance and to demonstrate their benefits over planar arrays, the cylindrical array's aperture must have elements spaced near half-wavelength both circumferentially and vertically (along the axis of the cylinder) at the high frequency of operation. For UWB operation, the height and/or radius of the array is often set by the desired aperture size for the low frequency of operation. In many instances, this results in an array that is far larger than needed at the high frequency with far more elements than required for the low frequency. The high-density of elements results in a large element count, which drives up array cost, complexity, and weight.

The wavelength scaled concept from [1–4] aims to reduce this element count issue for linear and planar arrays. The proposed total band of operation can be sub-banded with scaled array elements replacing sections of the aperture. This concept allows for reduction in element count while the total band of operation is still maintained.

In this report, the proposed cylindrical array topology extends the concept of a wavelength scaled planar array to a cylindrical array by placing elements on different lattices in different portions of the array. All of these are sub-lattices of the high frequency lattice, which simplifies the geometrical layout of the elements while preserving the inherent symmetry and periodicity of a cylindrical array. This concept reduces the element count of the array, and reduces the variation in achievable elevation beamwidth across the operational UWB frequency band.

2. Wavelength Scaled Array Concept

UWB arrays enable multifunction use from a single aperture. However, such apertures often have a high element count with the element being sized and spaced for the highest frequency in the band of operation. At the low end of the band of operation, the array is oversampled vertically, and at the high end, the vertical aperture size is often much larger than needed. The wavelength scaled concept was developed to address these concerns for wideband arrays. Discussion of the wavelength scaled concept will start with linear and planar arrays as this was the initial implementation of the concept, and will be followed by the extension to cylindrical array architecture types.

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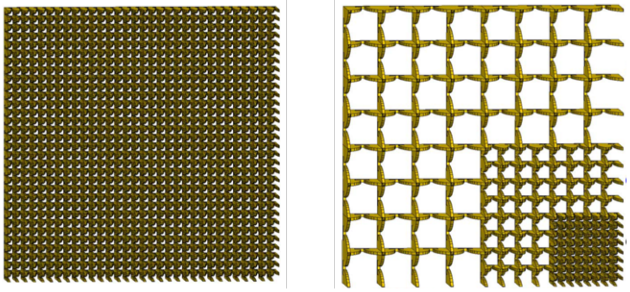


Fig. 1 — Example wavelength scaled concept for a planar array as presented in [5]. The array on the left is an example of a traditional wideband array with all elements spaced at half-wavelength at the high frequency. The array on the right demonstrates how the wavelength scaled concept reduces the element count by introducing scaled elements that cover sub-bands.

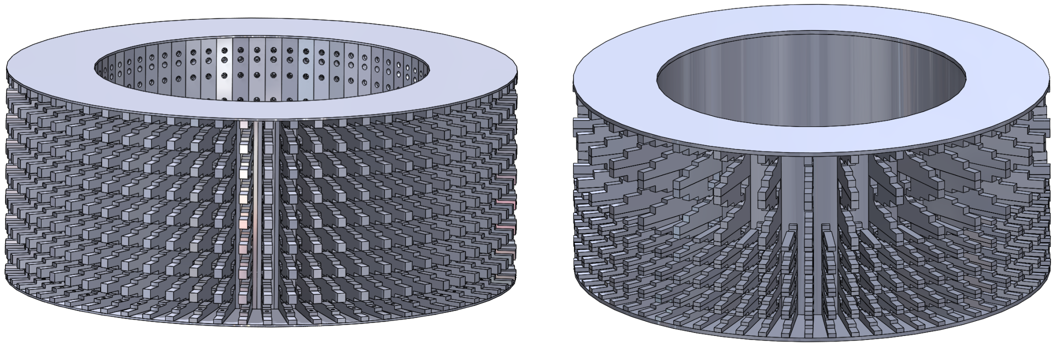
2.1 Linear and Planar Arrays

In a planar antenna array, the elements are arranged such that their apparent sources of radiation – commonly referred to as the phase centers – lie within a plane and have uniform spacing along a grid defined by a pair of basis vectors. With appropriate phase and amplitude excitation, the radiation pattern – including pointing direction – can be controlled without the need to manually rotate the antenna array. To avoid grating lobes, the elements should be spaced at – or near – a half-wavelength at the highest frequency of operation. The wavelength scaled concept was developed for planar arrays [1–4] proposing the total band of operation be sub-banded with scaled elements replacing sections of the aperture. In doing this, the element count can be reduced while the band of operation is maintained. An example of this architecture is shown in Fig. 1. The first array (left) is a traditional UWB planar array where the element is designed to operate over the full band of operation. In comparison, the second array (right) applied the wavelength scaled concept. The wavelength scaled array of Fig. 1 has three sub-bands. The high-band has wideband elements on the high-density lattice. The mid-band elements have a high frequency that is half that of the full operational band, while low-band elements have a high frequency that is a quarter of the overall high frequency. Each band has elements on a sub-lattice with inter-element separation of half-wavelength at the corresponding high frequency. This results in a 2:1 scaling between frequency bands.

2.2 Cylindrical Arrays

A cylindrical array is an antenna array where the element phase centers are arranged about a cylinder. These array geometries face similar difficulties to a planar array when operating over a wideband of operation. The height of the array dictates the achievable beamwidth in the elevation plane for the array. By sizing the array for low frequency gain and beamwidth requirements, the vertical beamwidth of the array is often significantly narrower than necessary at higher frequencies, and there are often more elements about the circumference of the array than needed at low frequencies. To account for this, portions of the array would typically go unused by systems using the array at high frequencies. Instead, we propose expanding the wavelength scaled concept demonstrated in [1–5] and applying it to a cylindrical array aperture, an example concept is shown in Fig. 2.

While lattice configurations are the focus in Chapter 3, initial discussion is presented here. The wavelength scaled cylindrical array (WSCA) concept focuses on wrapping multiple lattices - each a sub-lattice of the highest frequency lattice - about a central axis of a cylinder. Consequently, each sub-lattice



(a) Example Cylindrical Array Design (b) Example WSCA Design

Fig. 2 — Example wavelength scaled concept for a cylindrical array. The array on the left is an example of a traditional wideband cylindrical array with all elements spaced at half-wavelength at the high frequency. The array on the right demonstrates how the wavelength scaled concept reduces the element count with scaled elements to just a single sub-band for this example.

must be 2π-periodic to support wrapping about a cylinder. Each sub-lattice contains elements that cover a frequency band from the low frequency of the array (flow) to the high-frequency dictated by the lattice-spacing of a given sub-lattice (fn). The element spacing in each sub-lattice is a fraction (commonly, one half) of the highest-frequency wavelength λn = c/fn in the given sub-band. Since this is a two dimensional array, the ratio of the element densities in adjacent subl-attices is

$$\left(\frac{\lambda_n}{\lambda_{n-1}}\right)^2 = \left(\frac{f_{n-1}}{f_n}\right)^2 .$$

For operation at frequency f_{op}, elements in any sub-lattice that has a high frequency such that $f_o p < f_n$ will be used. Appropriate amplitude and phase calibration will be used to account for dissimilar elements. The sub-lattices reduce the sampling both circumferentially and vertically to reduce the element count of the array.

3. Lattice Configurations for WSCA

The wavelength-scaled concept is centered on nested sub-lattices, where the lower-density sub-lattices operate at lower frequency sub-bands dictated by lattice spacing relative to operational wavelength. The work in [1–5] shows various sub-lattice configurations for planar arrays. However, most of these focus on planar arrays with rectangular lattices having a 4:1 scaling in the density between sub-lattices. This is done for convenience in manufacturing and analysis. In this paper, sub-lattice configurations suitable for wavelength-scaled cylindrical arrays are presented where the sub-lattice must be 2π-periodic around the circumference of the cylinder. Manufacturing capabilities have improved in recent years because of technological advances and the growing capabilities of additive manufacturing. As a result, more complex lattice configurations are now possible. This flexibility in sub-lattice configuration allows greater control over the sub-band structure that gives the user the ability to tailor the array design to the desired operation of the array.

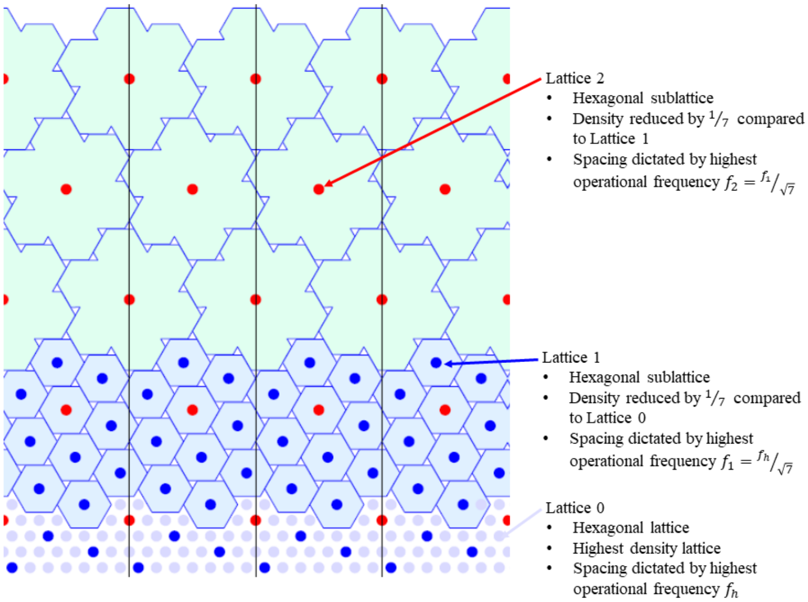
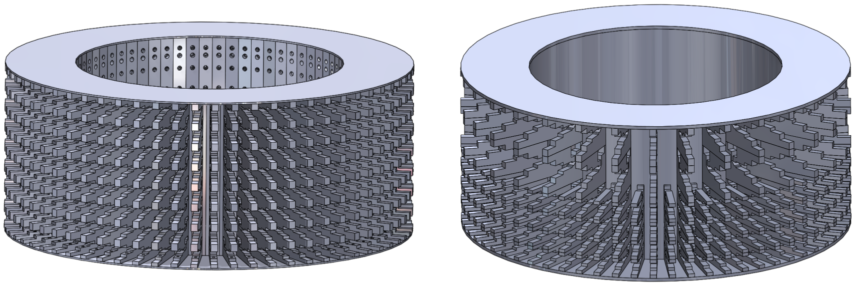


Fig. 3 — Illustration of unwrapped cylindrical lattice configuration for a WSCA. This example lattice configuration for an array has 1/√7 scaling between the upper frequency limits of the sub-bands, and each sub-band has elements lying on hexagonal sub-lattices. Element boundaries are also included for the upper two sub-bands.

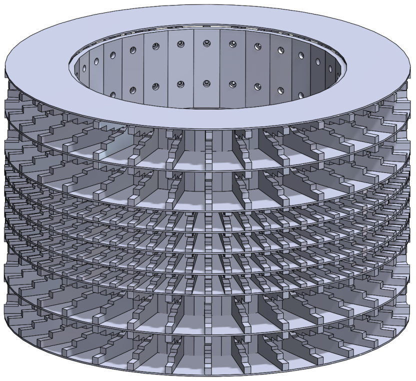
An example configuration is shown in Fig. 3, which is an unwrapped version of element locations in a cylindrical array. The horizontal dimension of this figure is the circumference of the array, and the vertical dimension is the array height. The critical feature here is that the lattice configuration is periodic so that it will be continuous when wrapped around the array. We see in Fig. 3 that the left and right edges of the array are identical, indicating a continuous cylindrical aperture. The dots indicate lattice points for three lattices of different densities, and the outlined polygons indicate unit cells for Lattice 1 and Lattice 2. The unit-cell is the geometry that is repeated about lattice points – the bounding geometry for an element that will be located on the given lattice. Lattice 0 is a standard hexagonal lattice. Its unit-cell is well-known and thus not shown on this figure to reduce complication in the figure. It should be noted, that this type of sub-lattice configuration is also applicable to planar wavelength scaled arrays.At the bottom of Fig. 3, light blue dots indicate the highest density lattice, referred to as Lattice 0. These dots indicate locations for elements operating up to the full desired operational frequency of the array, f_h. The dark blue dots indicate locations on a sub-lattice (Lattice 1) with a density equal to one-seventh the density of Lattice 0. The unit cells for these elements are also indicated on this figure. These elements operate at frequencies upper-bounded by f1 = fh$\sqrt{7}$. Finally, Lattice 2 has locations indicated by the red dots and represent another seven times reduction in element density. These elements operate at frequencies upper-bounded by f2 = f1$\sqrt{7}$.

4. Example WSCA Design

A traditional wideband cylindrical array, similar to the design presented in [6], designed to cover 2-10 GHz with vertical polarization is shown in Fig. 4(a). The elements are spaced to provide half-wavelength separation at 10 GHz both vertically and circumferentially. The array has a 12-inch diameter and a 4.8 inch height, resulting in 512 elements. The array of Fig. 4(b) implements the wavelength scaled concept where the top half of the array has elements arranged about a sub-lattice. The sub-lattice consists of antenna elements, which cover 2-5 GHz on a lower-density grid. The array of Fig. 4(b) has a total of 320 elements, a 25% reduction in element count compared to the array from Fig. 4(a). The array concept of Fig. 4(b) is adapted to the vertically symmetric configuration shown in Fig. 4(c).



(a) Example Cylindrical Array Design (b) Example WSCA Design 1



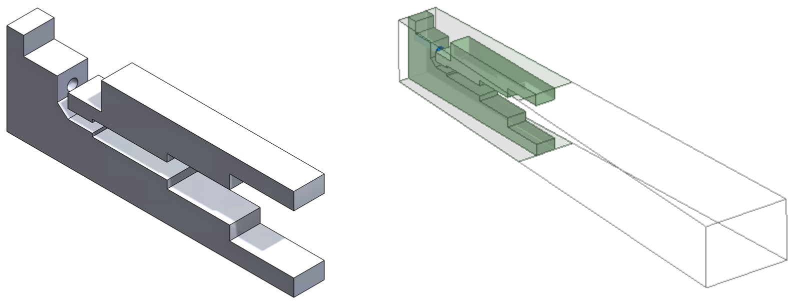
(c) Example WSCA Design 2

Fig. 4 — Example wavelength scaled cylindrical array designs. **(a)** Traditional wideband cylindrical array design **(b)** WSCA of (a) with 2 distinct sub-arrays arranged vertically in the array. The sub-bands use a 2:1 frequency scaling. **(c)** WSCA of (a) with 2 distinct sub-arrays arranged symmetrically about the vertical center of the array. The sub-bands use a 2:1 frequency scaling.

The designs shown in Fig. 4(b) and Fig. 4(c) consist of two sub-bands with band 1 covering the full desired bandwidth of 2-10GHz and band 2, the sub-band, with 2:1 frequency scaling of the high end of operation, 2-5GHz. The elements in band 1 and band 2 are step notch elements with the band 2 element appropriately scaled to operate with a high frequency half of the high frequency bandwidth of band 1. The element covering band 1 is named the full band element, whereas the element covering band 2 will be designated the *scaled element*. For a WSCA array with multiple sub-bands, there would be multiple scaled elements. The element designs and simulated results are presented in the following sections, with the active reflection coefficient simulations utilizing the phase mode simulation method as described in [7]. The embedded element patterns are simulated using a full array simulation technique to see any variations in pattern shape due to the wavelength scaled concept. For the design in Fig. 4(b), this means looking at a column where there are 4 full band elements and 2 scaled elements.

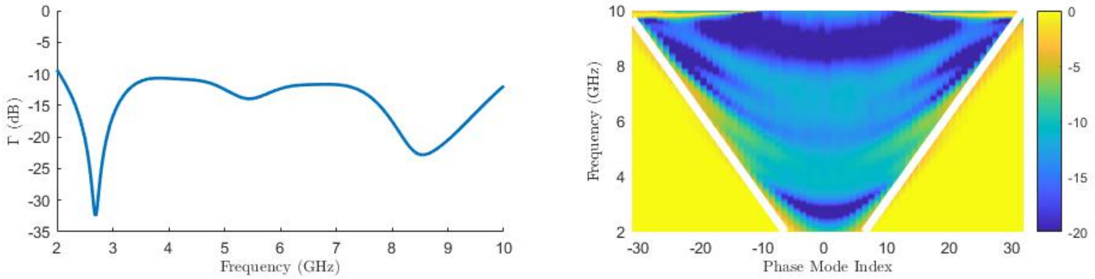
4.1 Full Band Element

The full band element and its unit cell setup are seen in Fig. 5. This element is spaced at 10GHz, band 1's high frequency, both vertically and circumferentially. This spacing is visualized with the wedge unit cell shown around the element in Figure 5(a), where the edges of the unit cell show the boundaries between the element and its neighbors. According to [7], the top and bottom walls of the unit cell are applied with periodic boundaries to simulate an infinite array in the vertical dimension. Similarly, periodic boundaries with a phase shift of $m\frac{2\pi}{N}$ are applied to the non-parallel, or wedge, walls on the side where m is the phase mode index (with $m = 0, 1, ..., N - 1$) and N the number of elements in the circumferential dimension. The simulated active reflection coefficient is seen in Fig. 6 where the element is seen to be well matched across the 2-10 GHz for mode 0. Mode 0 is the widest bandwidth mode and, therefore, sets the bandwidth of the element. Fig. 6(b) shows the active reflection coefficient, Γ, vs frequency and phase modes. The white lines define the theoretical limit for usable modes, defined as $m_{max} = \pm\frac{2\pi R}{\lambda}$ (where R is the radius), and shows good agreement with the simulated results. The embedded element patterns for the 4 elements in a column are seen in Fig. 7. The embedded element patterns start to have more variation in the pattern as the element gets closer to the scaled element (element 1 vs. element 4).



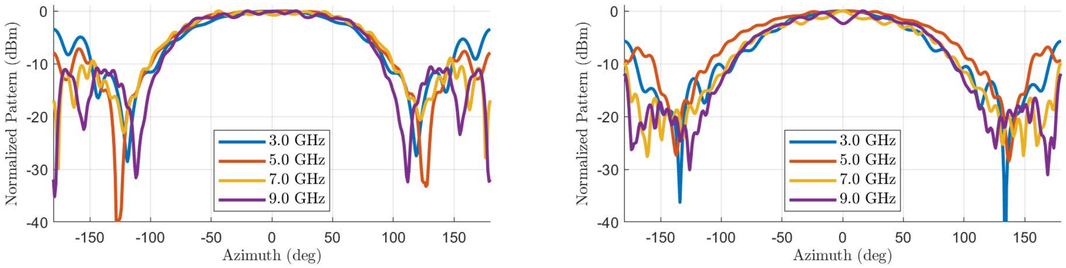
(a) Full Band Element (b) Unit Cell Simulation Setup

Fig. 5 — Full band element for the WSCA design presented in Fig. 4(b).

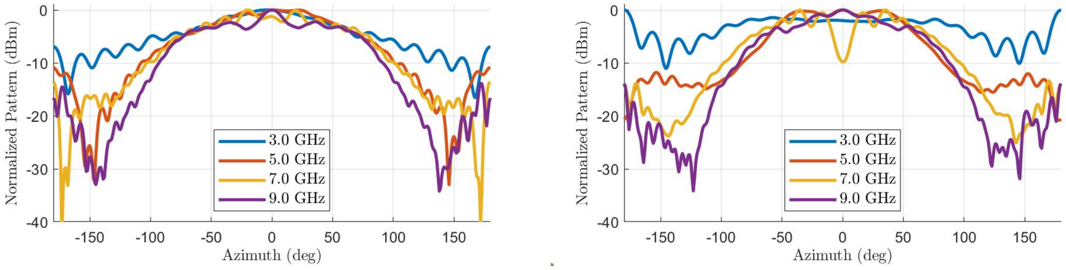


(a) Mode 0 Match (b) Active Reflection Coefficient

Fig. 6 — Match and active reflection coefficients for the full band element.



(a) Element 1 (b) Element 2

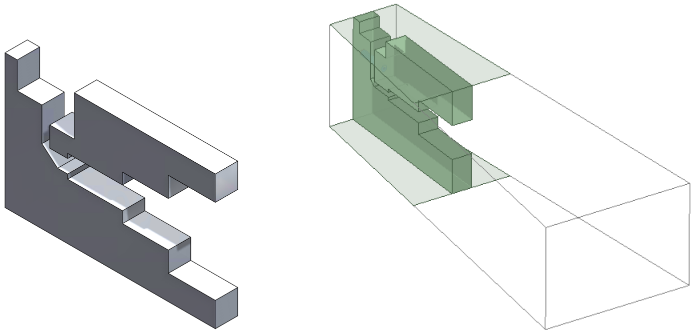


(c) Element 3 (d) Element 4

Fig. 7 — Embedded element patterns.

4.2 Scaled Element

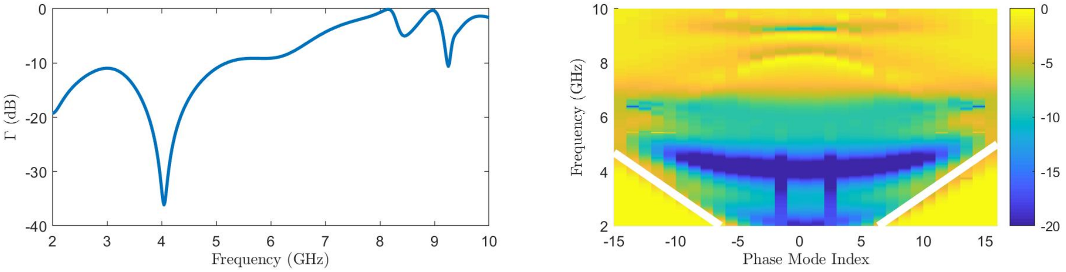
The scaled element and its unit cell setup are seen in Fig. 8. This element is spaced at 5GHz, band 2's high frequency, both vertically and circumferentially. Again, the spacing is visualized with the wedge unit cell shown around the element in Fig. 8(a). Boundary conditions are still applied according to [7]. The simulated active reflection coefficients are seen in Fig. 9 where the element is seen to be well matched across 2-5GHz for mode 0. Fig. 9(b) shows the active reflection coefficient, Γ, vs frequency and phase modes with the white lines still defining the theoretical limit for usable modes, well matched with simulated



(a) Full Band Element (b) Unit Cell Simulation Setup

Fig. 8 — Scaled element for the WSCA design presented in Fig. 4(b).

results. The embedded element patterns for the 2 elements in a column are seen in Fig. 10.

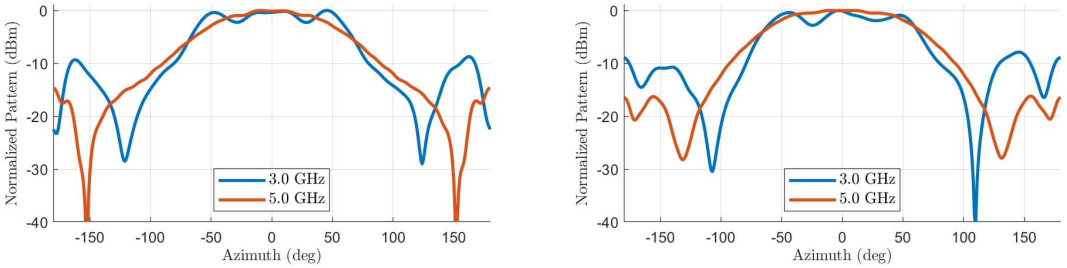


(a) Mode 0 Match (b) Active Reflection Coefficients

Fig. 9 — Match and active reflection coefficients for the full band element.

4.3 Performance Comparison

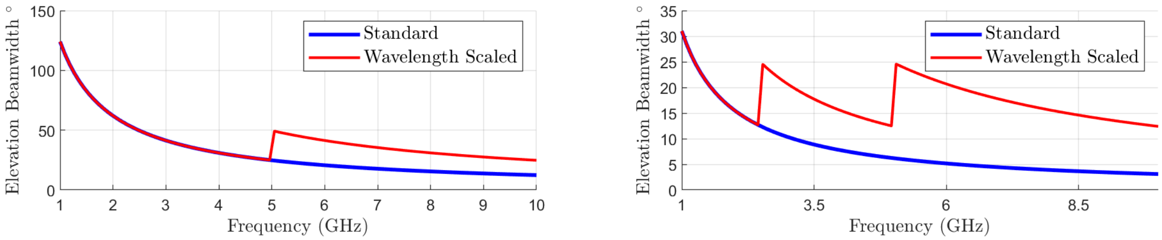
While the wavelength scaled concept reduced element count, it also offers performance benefits for certain applications, namely more consistent beamwidths across frequency. The elevation beamwidths as a function of frequency for the arrays of Fig. 4(a) and Fig. 4(b) are compared in Fig. 11(a). Application of the wavelength scaled concept has reduced the variation across the frequency band. This becomes more apparent in designs with more sub-bands as shown in Fig. 11(b). This plot shows elevation beamwidths for an array that has a 12 inch diameter and a 19.2 inch height. The array configuration has 3 sub-bands. Band 1 covers 2 GHz to 10 GHz, band 2 covers 2 GHz to 5 GHz, and band 3 covers 2 GHz to 2.5 GHz. The elevation beamwidths are flattening out across the operational frequency band of the array, which is desirable for certain applications.



(a) Element 1

(b) Element 2

Fig. 10 — Embedded element patterns.



(a)

(b)

Fig. 11 — Comparison of elevation beamwidths as a function of frequency. **(a)** Comparison of elevation beamwidth as a function of frequency for cylindrical arrays that are 12” in diameter and 19.2” in height. One array uses elements on a 10GHz half-wavelength lattice, while the other uses the wavelength scaled concept with 2:1 frequency scaling between 2 sub-bands. **(b)** Comparison of elevation beamwidth as a function of frequency for cylindrical arrays that are 12” in diameter and 19.2” in height. One array uses elements on a 10GHz half-wavelength lattice, while the other uses the wavelength scaled concept with 2:1 frequency scaling between 3 sub-bands.

5. Future Work

Traditional cylindrical arrays can be difficult to manufacture due to the rotational relationship between array elements as discussed in [6]. However, manufacturing advancements, specifically the advancement of additive manufacturing (AM), also known as 3D printing, has simplified the manufacturing of complex cylindrical array designs as demonstrated in [8] [9]. AM can fabricate complex geometries that are difficult or unable to be produced using traditional manufacturing. This ability to fabricate complex geometries with AM led the interest in extending the wavelength scaled concept to cylindrical arrays. WSCA designs are more complex than traditional cylindrical array designs; especially for the non-standard lattice structures presented in Section 3; and would be difficult to manufacture using traditional manufacturing. WSCA designs, such as those seen in Section 4, are easily fabricated with AM as demonstrated in Fig. 12.

Future work would look at fabricating and testing WSCA designs, such as those presented in Section 4. The fabrication process would follow a similar procedure to that described in [6] where the array is printed

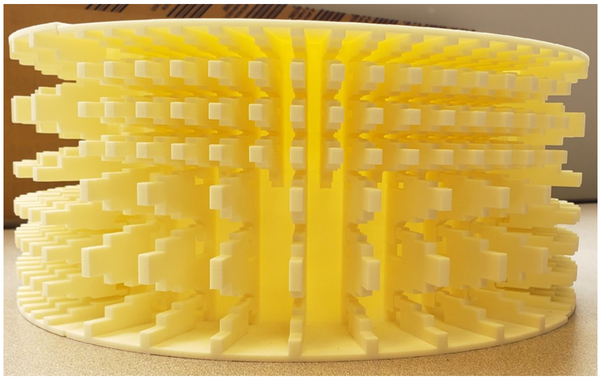


Fig. 12 — AM fabrication of WSCA Design 1 from Section 4. Printed using a selective laser sintering (SLS) printer.

using AM, metallized, and connectorized. Once the fabrication process was complete, array characterization measurements would be completed.

6. Conclusions

In this report, the wavelength scaled concept was proposed and designed for use with cylindrical array apertures. This work extends the wavelength scaled concept first utilized for linear and planar arrays. The wavelength scaled cylindrical array concept can greatly reduce the number of elements needed in the array, which can reduce array cost, complexity, and weight. Lattice configurations tailored for a WSCA are presented along with an example hexagonal lattice. Additionally, example WSCA designs are presented showing the potential element reduction for a 2-10 GHz array. Simulated results are shown for both the full band and scaled elements that are utilized in the example designs.

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