



AFRL-AFOSR-VA-TR-2022-0006

High resolution coherent interferometric imaging in random media

**LILIANA BORCEA
REGENTS OF THE UNIVERSITY OF MICHIGAN
503 THOMPSON ST
ANN ARBOR, MI, 48109
USA**

**10/19/2021
Final Technical Report**

DISTRIBUTION A: Distribution approved for public release.

Air Force Research Laboratory
Air Force Office of Scientific Research
Arlington, Virginia 22203
Air Force Materiel Command

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing the burden, to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.
PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.

1. REPORT DATE (DD-MM-YYYY) 19-10-2021		2. REPORT TYPE Final		3. DATES COVERED (From - To) 01 Apr 2021 - 30 Sep 2021	
4. TITLE AND SUBTITLE High resolution coherent interferometric imaging in random media				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER FA9550-21-1-0166	
				5c. PROGRAM ELEMENT NUMBER 61102F	
6. AUTHOR(S) Liliana Borcea				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) REGENTS OF THE UNIVERSITY OF MICHIGAN 503 THOMPSON ST ANN ARBOR, MI 48109 USA				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) AF Office of Scientific Research 875 N. Randolph St. Room 3112 Arlington, VA 22203				10. SPONSOR/MONITOR'S ACRONYM(S) AFRL/AFOSR RTB1	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S) AFRL-AFOSR-VA-TR-2022-0006	
12. DISTRIBUTION/AVAILABILITY STATEMENT A Distribution Unlimited: PB Public Release					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT We introduced a novel methodology for high resolution synthetic aperture radar (SAR) imaging through heterogeneous (random) media. We developed three different imaging algorithms, justified them from first principles and studied with analysis and numerical simulations their resolution and robustness to noise and to the uncertainty of the fluctuations of the wave speed. The algorithms enhance the existing imaging technology because on one hand they mitigate random multiple scattering effects and on the other hand they give resolution that is comparable to that in homogeneous (non-scattering) media.					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			ARJE NACHMAN
U	U	U	UU	6	19b. TELEPHONE NUMBER (Include area code) 426-8427

Standard Form 298 (Rev.8/98)
Prescribed by ANSI Std. Z39.18

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing the burden, to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.
PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.

1. REPORT DATE (DD-MM-YYYY) 10-12-2021		2. REPORT TYPE Final technical report		3. DATES COVERED (From - To) 04-01-2021 to 09-30-2021	
4. TITLE AND SUBTITLE High Resolution Coherent Interferometric Imaging in Random Media				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER FA9550-21-1-0166	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S) Liliana Borcea				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) University of Michigan Office of Research and Sponsored Projects 503 Thompson St Ann Arbor, MI 48109-1340				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) USAF, AFRL DUNS 143574726 AF OFFICE OF SCIENTIFIC RESEARCH 875 NORTH RANDOLPH STREET, RM 3112 ARLINGTON VA 22203-1954 JULIA A. CARRICO 7036966564 julia.carrico@us.af.mil				10. SPONSOR/MONITOR'S ACRONYM(S) AFOSR	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT Approved for Public Release; distribution is Unlimited.					
13. SUPPLEMENTARY NOTES Program Director: Dr. Arje Nachman, AFOSR/RTB1, (703) 696-8427, arje.nachman@us.af.mil					
14. ABSTRACT We introduced a novel methodology for high resolution synthetic aperture radar (SAR) imaging through heterogeneous (random) media. We developed three different imaging algorithms, justified them from first principles and studied with analysis and numerical simulations their resolution and robustness to noise and to the uncertainty of the fluctuations of the wave speed. The algorithms enhance the existing imaging technology because on one hand they mitigate random multiple scattering effects and on the other hand they give resolution that is comparable to that in homogeneous (non-scattering) media.					
15. SUBJECT TERMS Imaging; Wave scattering in random media; High resolution imaging; Spectral analysis.					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT UU	18. NUMBER OF PAGES 11	19a. NAME OF RESPONSIBLE PERSON Liliana Borcea
a. REPORT UU	b. ABSTRACT UU	c. THIS PAGE UU			19b. TELEPHONE NUMBER (Include area code) 734-913-5342

Final report for AFOSR Grant FA9550-21-1-0166
High Resolution Coherent Interferometric Imaging in Random Media

Liliana Borcea
Department of Mathematics
University of Michigan, Ann Arbor

1 Objectives

The objective of this short term grant was to develop a novel synthetic aperture radar (SAR) methodology for imaging remote targets through a random medium, which is a model of atmospheric turbulence. The attributes of the methodology are:

1. It has a rigorous mathematical justification that is rooted in the theory of wave propagation in random media.
2. It is robust with respect to the uncertainty of the small scale features (random inhomogeneities) of the medium through which the waves propagate from the SAR platform to the targets and back.
3. It is robust to ambient (additive) noise.
4. It gives a better resolution than the existing imaging methodology in random media.

2 Accomplishments

We have completed the work outlined in the proposal and the results are in the following publication:

L. Borcea, J. Garnier, “Imaging in Random Media by Two-Point Coherent Interferometry”, accepted for publication at SIAM J. Imaging Sciences. Currently in press.

Abstract: *This paper considers wave-based imaging through a heterogeneous (random) scattering medium. The goal is to estimate the support of the reflectivity function of a remote scene from measurements of the backscattered wave field. The proposed imaging methodology is based on the coherent interferometric (CINT) approach that exploits the local empirical cross correlations of the measurements of the wave field. The standard CINT images are known to be robust (statistically stable) with respect to the random medium, but the stability comes at the expense of a loss of resolution. This paper shows that a two-point CINT function contains the information needed to obtain statistically stable and high-resolution images. Different methods to build such images are presented, theoretically analyzed, and compared with the standard imaging approaches using numerical simulations. The first method involves a phase retrieval step to extract the reflectivity function from the modulus of its Fourier transform. The second method involves the evaluation of the leading eigenvector of the two-point CINT imaging function seen as the kernel of a linear operator. The third method uses an optimization step to extract the reflectivity function from some cross products of its Fourier transform. The presentation is for the synthetic aperture radar data acquisition setup, where a moving sensor probes the scene with signals emitted periodically and records the resulting backscattered wave. The generalization to other imaging setups, with passive or active arrays of sensors, is discussed briefly.*

Motivation and summary of the results: It has been known for at least twenty years that cumulative scattering effects in random media can be mitigated in imaging by working with cross-correlations of measurements collected at sensors. This insight comes from the theory of wave propagation in random media, which quantifies the net scattering effect on the components (modes) of the wave field as follows:

- Scattering causes loss of coherence of the wave. This manifests in the exponential decay of the expectation of the mode amplitudes with the range of propagation. For high frequency SAR imaging through the turbulent atmosphere and through clouds, this means that beyond a few kilometers, the mean (coherent) wave is small. Because classic image formation relies on this coherent wave, it is useless at such ranges.
- The incoherent wave i.e., the random fluctuations gain strength at increasing range and eventually become dominant. These fluctuations are not noise. They carry information about the targets that we wish to image.
- The statistical de-correlation of the wave fluctuations plays an important role in the signal processing needed to extract the target information. The de-correlation is quantified by two intrinsic scales: the decoherence frequency Ω_d and the decoherence length X_d . These scales depend on: the central frequency, the range, the amplitude and the length scale of the fluctuations of the wave speed. In practice, X_d and Ω_d can be determined from the SAR data using statistical signal processing or by optimization, during the process of imaging.

The Coherent Interferometric (CINT) methodology developed by the PI in collaboration with Josselin Garnier from Ecole Polytechnique France, George Papanicolaou from Stanford and Chrysoula Tsogka from UC Merced is rooted in this theory. The CINT imaging function $I^{\text{CINT}}(\mathbf{y})$ is formed at imaging points \mathbf{y} in the range and cross-range plane by superposing over the antenna locations and the bandwidth the empirical cross-correlations of the data, synchronized relative to \mathbf{y} . There are two key steps:

1. The calculation of the empirical cross-correlations. Due to the statistical de-correlation of the incoherent wave, the cross-correlations must be carried out over antenna offsets $\leq X_d$ and in time windows of duration $\geq 1/\Omega_d$. The windowing introduces a smoothing in the signal processing that combined with the superposition over the antenna locations and the bandwidth gives a statistically stable imaging function $I^{\text{CINT}}(\mathbf{y})$.
2. The synchronization, which CINT does by migration to the imaging point \mathbf{y} . In the frequency domain, migration amounts to the removal of the deterministic phase given by the product of the frequency with the round trip travel time from the SAR antenna to \mathbf{y} .

The resulting CINT imaging function $I^{\text{CINT}}(\mathbf{y})$ is peaked near the targets and enjoys the important property of statistical stability (robustness) with respect to the realization of the random medium. This means that $I^{\text{CINT}}(\mathbf{y}) \approx \mathbb{E} [I^{\text{CINT}}(\mathbf{y})]$, where \mathbb{E} is the expectation with respect to the law of the random process that models the fluctuations of the wave speed. The trouble is that this robustness comes at the expense of resolution. We no longer have the resolution limits of imaging in a homogeneous medium with wave speed c , given by $h_{\perp} = \lambda L/a$ in cross-range and $h = c/B$ in range, where λ is the central wavelength, L is the range, a is the synthetic aperture length and B is the bandwidth. We get instead $H_{\perp} = \lambda L/X_d$ and $H = c/\Omega_d$. Since $X_d \ll a$ and $\Omega_d \ll B$ in regimes with strong scattering effects, the loss of resolution is significant: $H_{\perp} \gg h_{\perp}$ and $H \gg h$.

Our research shows that it is possible to improve the resolution while keeping the statistical stability of imaging. To do so, we work with a “two point CINT function” $I(\mathbf{y}, \mathbf{y}')$. Like $I^{\text{CINT}}(\mathbf{y})$, this is obtained via superposition of the empirical cross-correlations of the measured wave. The difference is in the synchronization i.e., the migration step. Because the cross-correlations are quadratic in the data, we realized that migration to a single point \mathbf{y} , like in CINT, is good only if the imaging scene consists of point targets that

are further apart than the CINT resolution limits. Otherwise, it is better to migrate to a pair of points $(\mathbf{y}, \mathbf{y}')$ in the imaging region, because a careful analysis shows the following: If \mathbf{y} and \mathbf{y}' are two points in the support of the targets, then $I(\mathbf{y}, \mathbf{y}')$ has a peak at the center point $(\mathbf{y} + \mathbf{y}')/2$ with the poor resolution (H_{\perp}, H) and at the offsets $\pm(\mathbf{y} - \mathbf{y}')$ with the high resolution (h_{\perp}, h) .

The difficulty of working with the two point CINT function is that it is not an image. It is not easy to relate the peaks of $I(\mathbf{y}, \mathbf{y}')$ to target locations in complicated scenes and, moreover, it is difficult to search in a four dimensional space. Additional processing is needed to extract an image from $I(\mathbf{y}, \mathbf{y}')$. CINT is the simplest possible processing, and it amounts to evaluating $I(\mathbf{y}, \mathbf{y}')$ on the diagonal:

$$I^{\text{CINT}}(\mathbf{y}) = I(\mathbf{y}, \mathbf{y}).$$

As explained above, this gives blurry images. Our research supported by this grant resulted in three different methods of extracting a better image from $I(\mathbf{y}, \mathbf{y}')$:

Method 1: This method is based on the fact that

$$\int_{\mathbb{R}^2} d\mathbf{y} \int_{\mathbb{R}^2} d\tilde{\mathbf{y}} e^{-i\mathbf{K}\cdot\tilde{\mathbf{y}}} I\left(\mathbf{y} + \frac{\tilde{\mathbf{y}}}{2}, \mathbf{y} - \frac{\tilde{\mathbf{y}}}{2}\right) \approx C|\hat{\rho}(\mathbf{K})|^2 e^{-\frac{\kappa_{\perp}^2 h_{\perp}^2}{2} - \frac{\kappa^2 h^2}{2}},$$

where $C > 0$ is a constant and $\hat{\rho}(\mathbf{K})$ is the Fourier transform of the unknown reflectivity function $\rho(\mathbf{y})$ of the targets, evaluated at the ‘‘spatial frequency’’ vector $\mathbf{K} = (K_{\perp}, K)$. The left hand side in this equation can be calculated from the SAR data, so we can estimate the modulus of the Fourier transform $|\hat{\rho}(\mathbf{K})|$ in the rectangular domain $\mathbf{K} \in (-3/h_{\perp}, 3/h_{\perp}) \times (-3/h, 3/h)$. From this estimate we can reconstruct the reflectivity $\rho(\mathbf{y})$ via phase retrieval. The advantages of this approach are:

- The reconstruction of $\rho(\mathbf{y})$ has the high resolution (h_{\perp}, h) expected from SAR imaging in homogeneous media. This means that the random medium effects are mitigated without losing resolution.
- The imaging is straightforward and one can use one of the many available phase retrieval algorithms.
- The algorithm works for extended targets, not just constellations of point targets.

The disadvantages are:

- Phase retrieval is not robust to noise unless we have strong prior information about the reflectivity, such as $\rho(\mathbf{y}) \geq 0$.
- Even with such prior, since the modulus $|\hat{\rho}(\mathbf{K})|$ of the Fourier transform of the reflectivity is not affected by an overall phase, the reconstruction is good up to a rigid body translation of the imaging scene and a reflection with respect to the origin.

Method 2: The two point CINT function $I(\mathbf{y}, \mathbf{y}')$ can be viewed as the kernel of a linear integral operator, whose leading eigenfunction can be used to locate the targets. We carried out a detailed spectral analysis of this operator in the case of a scene with point targets, and showed that if they are sufficiently well separated, or if there are well separated clusters of targets, then the leading eigenfunction is an imaging function. That is, it peaks in the vicinity of the targets. The advantages of this approach are:

- Reflectivities of any sign (not just positive) can be recovered.
- There is no translation or reflection ambiguity as in the case of method 1.

- The imaging is simple and non-iterative.
- The imaging is very robust with respect to noise.

The disadvantage is: The targets have to be separated at distance $\geq H_{\perp}$ in cross-range and $\geq H$ in range. If that is the case, then the leading eigenfunction gives an image with resolution $(\sqrt{h_{\perp}H_{\perp}}, \sqrt{hH})$ that is better than the (H_{\perp}, H) resolution of CINT but worse than the ideal resolution (h_{\perp}, h) . Note that CINT cannot discern between positive and negative reflectivities, while this method does. If the targets are not well separated, then this method does not give a resolution improvement.

Method 3: This method is based on the fact that

$$\int_{\mathbb{R}^2} d\mathbf{y} e^{-i\tilde{\mathbf{K}}\cdot\mathbf{y}} \int_{\mathbb{R}^2} d\tilde{\mathbf{y}} e^{-i\mathbf{K}\cdot\tilde{\mathbf{y}}} I\left(\mathbf{y} + \frac{\tilde{\mathbf{y}}}{2}, \mathbf{y} - \frac{\tilde{\mathbf{y}}}{2}\right) \approx C \hat{\rho}\left(\mathbf{K} + \frac{\tilde{\mathbf{K}}}{2}\right) \overline{\hat{\rho}\left(\mathbf{K} - \frac{\tilde{\mathbf{K}}}{2}\right)} e^{-\frac{\kappa_{\perp}^2 h_{\perp}^2}{2} - \frac{\kappa_{\perp}^2 h^2}{2} - \frac{\tilde{\kappa}_{\perp}^2 H_{\perp}^2}{2} - \frac{\tilde{\kappa}_{\perp}^2 H^2}{2}},$$

where the bar denotes complex conjugate and C is a positive constant. The left hand side in this equation can be computed from the SAR data, so the problem reduces to reconstructing the reflectivity $\rho(\mathbf{y})$ from $\hat{\rho}\left(\mathbf{K} + \frac{\tilde{\mathbf{K}}}{2}\right) \overline{\hat{\rho}\left(\mathbf{K} - \frac{\tilde{\mathbf{K}}}{2}\right)}$ estimated at $\mathbf{K} \in (-3/h_{\perp}, 3/h_{\perp}) \times (-3/h, 3/h)$ and at $\tilde{\mathbf{K}} \in (-3/H_{\perp}, 3/H_{\perp}) \times (-3/H, 3/H)$. This we can do efficiently with optimization. The advantages of this approach are:

- If the additive noise is not too high, then we can reconstruct $\rho(\mathbf{y})$ with the high resolution (h_{\perp}, h) expected from SAR imaging in homogeneous media.
- The reconstruction does not rely on a strong prior, such as having point targets or a positive reflectivity. Unlike method 1, it gives not only the support of the targets but the sign of the reflectivity as well.
- There is no translation or reflection ambiguity as in the case of method 1.

The disadvantage is that the reconstruction is more sensitive to additive noise than method 2. This can be mitigated via some filtering process, at the expense of resolution. Thus, we have a robustness vs. resolution trade-off.

While carrying out this project, we realized that it is possible to use empirical cross-correlations to image moving targets. This is a nontrivial extension of the work described above, which we hope to do under AFOSR auspices in the near future.

3 Other scientific activities:

- **Editorial boards:**

1. Editor in chief: SIAM Journal on Multiscale Modeling and Simulations
2. Communications of the American Mathematical Society
3. SIAM Journal on Uncertainty Quantification
4. Journal of Computational Physics
5. Inverse Problems
6. Inverse Problems and Imaging

- **Scientific boards**

1. Elected to the SIAM Board of Trustees, for a 3 year term, starting January 1, 2021.
2. Scientific advisory board, ICERM Institute at Brown University, 2018-2021.
3. Scientific Advisory Board of the Johann Radon Institute for Computational and Applied Mathematics, Linz Austria, 5 year term starting 2018.

- **Plenary lectures and colloquia** (a few plenary talks were postponed for 2022 due to COVID)

1. Power exchange and onset of energy equipartition among surface and body waves in random media, SIAM Annual meeting minisymposium talk, 2021.
2. Reduced Order Model approach for inverse scattering, plenary talk via zoom, Verification, Validation, and Uncertainty Quantification conference at University of Chicago, May 10-14, 2021.
3. Reduced Order Model approach for inverse scattering, plenary talk via zoom, Thematic Programme: Computational Uncertainty Quantification: Mathematical Foundations, Methodology & Data, University of Vienna, May 2 - June 24, 2022.
4. Reduced Order Model approach for inverse scattering, Applied Mathematics Seminar via zoom, Stanford University, May 19, 2021.
5. Power exchange and onset of energy equipartition among surface and body waves in random media, University of Michigan PDE seminar, April 15, 2021.

- **Organizer of conferences**

1. SIAM Annual Meeting, 2021. Organizing committee.
2. SIAM PDEs meeting, 2022 (postponed due to COVID). Scientific committee.
3. BIRS (Banff) Workshop 21w5035: Women in Inverse Problems, via zoom, December 5-10, 2021. Co-organizer with Chrysoula Tsogka.