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**Finite Set Statistics on Manifolds for Space Object Detection, Tracking,
Identification, and Characterization**

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14. ABSTRACT
New statistical methods have been developed to improve the treatment of uncertainty in space situational awareness. First, a deeper understanding of the previously developed adapted structural (AST) coordinates has been achieved, and the good performance of these coordinates has been demonstrated for the orbital tracking problem. Second, a new observation-centered nonlinear Kalman filter (OCKF) has been developed. Third, a new Normal:conditional-normal distribution has been developed to describe the uncertainty in the propagated angles-only position, especially after long-term propagation. Using this distribution can lead to improved performance in the association problem. Further, the OCKF and NCN contributions together provide an effective closed-form solution to the filtering problem under long-term propagation, especially with high eccentricity, a setting where standard methods struggle.

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Final Report

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Title: Finite Set Statistics on Manifolds for Space Object Detection, Tracking, Identification and Characterization

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Funded collaborators: Weston Faber (Applied Defense Solutions; now L3Harris), Islam Hussein (Applied Defense Solutions; now at Thornton Tomasetti); John Paffett (AppliedSpace)

Funded PhD student at Leeds: Shambo Bhattacharjee

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1 Summary

New statistical methods have been developed to improve the treatment of uncertainty in space situational awareness. First, a deeper understanding of the previously developed *adapted structural* (AST) coordinates has been achieved, and the good performance of these coordinates has been demonstrated for the orbital tracking problem. Second, a new *observation-centered* nonlinear Kalman filter (OCKF) has been developed. Third, a new *Normal:conditional-normal* distribution has been developed to describe the uncertainty in the propagated angles-only position, especially after long-term propagation. Using this distribution can lead to improved performance in the association problem. Further, the OCKF and NCN contributions together provide an effective closed-form solution to the filtering problem under long-term propagation, especially with high eccentricity, a setting where standard methods struggle.

2 Introduction

The project fits within the theme of finite set statistics (FISST) for space situational awareness (SSA). More specifically, the objective of the grant is to adapt ideas from directional statistics to improve the methods and algorithms for tracking and identification in SSA. The focus of the work is on improved tracking of space debris. That is, the resident space object (RSO) is regarded as non-cooperative and only occasional measurements are taken. Hence the uncertainties in the state can be nontrivial, and the time gap between successive measurements can be large (e.g. weeks or more).

This grant is a continuation of a previous AFOSR EOARD-funded project, FA9550-16-1-0099. The specific objectives of the current grant are to deepen the understanding of the methods developed under the previous grant and to produce several papers for journal publication.

The key consideration in our work is a suitable choice of coordinate system. Standard coordinate systems include Cartesian-ECI (earth centered inertial) coordinates, Keplerian elements and equinoctial coordinates. However, none of the standard coordinate systems is entirely satisfactory and some of them have severe limitations. Such limitations may not be important for certain states with small enough errors and short enough propagation times, but none of the standard coordinate systems works well for all initial states. Our approach is to represent uncertainty in terms of a notional point cloud about a “central” initial state. Then local coordinates are used to parameterize the uncertainty about this initial state. Provided the initial uncertainty is Gaussian in Cartesian-ECI coordinates, the uncertainty in these local coordinates will also be approximately Gaussian, whatever the choice of central initial state. This approach to representing uncertainty is closely related to the use of first-order Taylor expansions for certain representations of orbital elements.

The second step is to use these local coordinates to represent the propagated state, again in an approximately Gaussian form.

If the coordinate system is ill-chosen, then initial Gaussian perturbations give rise to extremely non-Gaussian distributions at a future time. However, with a good choice of coordinate system, approximate Gaussianity holds for much longer propagation times. As a result, AST coordinates lead to a statistically efficient and numerically tractable Bayesian filter for orbital tracking; they can also be used to develop effective methods for association problems.

3 Methods, Assumptions and Procedures

We have developed new statistical distributions and new coordinate systems to give improved methods in SSA to tackle problems such as association and tracking. For the most part Keplerian dynamics has been assumed, partly for simplicity, and partly so that underlying patterns can be seen more clearly. The methods have been assessed through computer simulation and mathematical expansions. More detail is in the next section.

4 Results and Discussion

- (a) AST coordinates were proposed under the previous grant. The current grant has given a deeper understanding and stronger evidence of their Gaussian behavior and their good statistical efficiency in the filtering problem. Recall one step of the filtering algorithm starts at an initial time with an estimated state and associated uncertainty; these are then propagated to a later time. An observation at the later time is followed by a Bayesian update for the estimated state and its associated uncertainty.
 - (i) A stronger intuition for AST coordinates has been given by presenting them as a local version of equinoctial coordinates.
 - (ii) The definition of AST coordinates has been slightly modified so that the propagation equations become exactly linear.

- (iii) It has been shown that at the initial time, AST coordinates are approximately a linear transformation of Cartesian-ECI coordinates under a wide range of initial conditions (low or high eccentricity, low or high uncertainty and any value of the inclination). Further, AST coordinates are much better behaved under propagation.
 - (iv) Using AST coordinates, the measurement or observation equation can be very non-linear under high eccentricity. For the treatment of this problem, see the next item, (b).
- (b) A new type of “observation centered” Kalman filter has been developed to deal with the problem in (a)(iv). The unscented Kalman filter (UKF) and extended Kalman filter (EKF) are commonly used in nonlinear situations, but they can perform very badly in the orbital tracking problem under long-term propagation with high eccentricity. The new filter comes in two closely related versions: unscented and extended, with abbreviations OCUKF and OCEKF. These new filters fix the problems with the UKF and EKF in this setting and they avoid the need for iteration in the iterated filters, IUKF and IEKF.
- (c) With respect to the “central” basis in three-dimensional Euclidean space, the two-dimensional angles-only part of the propagated state in Cartesian coordinates can be represented in terms of “longitude” and “latitude”, where the longitude is steadily increasing in time, and the latitude oscillates about the “equator”.

The behavior of the propagated angles-only distribution has been explored in detail. This behavior can be described using a novel “normal:conditional-normal (NCN)” distribution. Under long-term propagation, the NCN distribution cannot be adequately approximated by a bivariate Gaussian distribution. In particular, in the setting of a break-up event, the NCN distribution exhibits a distinctive pinching behavior when the propagation time is a near-multiple of the half orbital period. The term “pinched normal” (PN) distribution can be used to describe the special case of the NCN distribution in this setting.

- (d) By combining the ideas behind AST coordinates, the OCKF and the NCN distribution, it is possible to construct a closed-form filter for the orbital tracking problem that is highly efficient under long-term propagation, even in the case of high eccentricity, a setting where standard methods such as the EKF and UKF can go badly astray. The underlying principle is to carry out one step of the filter in two stages: first update the state distribution using the OCEKF for the observed longitude, and second update the state distribution using the NCN distribution for the observed latitude given the longitude.

The work has been reported in two arXiv papers [1, 2] and several conference papers. Two conference papers [3, 4] were presented by John Kent at the IAC in Washington, DC, 21–25 October 2019. One conference paper [5] was presented by John Kent at MFI 2020, 14-16 September 2020, held online. Another paper (“A closed form comprehensive solution to the orbital tracking problem”) pulling together the achievements of the grant is nearing completion. Our intention is to submit this and the two arXiv papers for journal publication imminently. In addition, the PhD student supported by the grant, Shambo Bhattacharjee, successfully completed his PhD in 2020.

5 Conclusions

The SSA community has shown considerable interest in this work, and the current grant has strengthened the arguments in favor of our AST coordinates and related ideas. Evidence is growing to demonstrate that our algorithms reliably provide improved speed and accuracy compared to existing methods under long-term propagation. Although most of the results have appeared in the public domain through conference proceedings and arXiv papers, an immediate objective is to submit a journal paper to publicize and explain our contributions more effectively, in order to facilitate the adoption of our algorithms by the community.

References

- [1] John T. Kent, Shambo Bhattacharjee, Weston R. Faber, and Islam I. Hussein. Observation-centered Kalman filters, 2019. arXiv:1907.13501.
- [2] John T. Kent, Shambo Bhattacharjee, Weston R. Faber, and Islam I. Hussein. Revisiting the orbital tracking problem, 2019. arXiv:1909.03793.
- [3] S. Bhattacharjee, J. T. Kent, W. R. Faber, and I. I. Hussein. Revisiting the filtering problem. In *70th International Astronautical Congress, Washington, D.C., 21–25 October, 2019*, 2019. Paper IAC-19,A6,7,6,X50344.
- [4] S. Bhattacharjee, J. T. Kent, W. R. Faber, and I. I. Hussein. Understanding the distribution of the propagated angles-only position vector. In *70th International Astronautical Congress, Washington, D.C., 21–25 October, 2019*, 2019. Paper IAC-19,A6,9,4,X50342.
- [5] J. T. Kent, S. Bhattacharjee, W. R. Faber, and I. I. Hussein. A unified approach to the orbital tracking problem. In *2020 IEEE International Conference on Multisensor Fusion and Integration for Intelligent Systems (MFI 2020), 14–16 September 2020, Karlsruhe, Germany*, pages 82–87, 2020.

6 List of Symbols, Abbreviations and Acronyms

AST: (Adapted ST) structural coordinates to describe the state of an orbiting object.

FISST: Finite Set Statistics

RSO: Resident Space Object

SSA: Space Situational Awareness

ECI: Earth Centered Inertial

UKF, EKF: Unscented and Extended Kalman Filters

IUKF, IEKF: Iterated Unscented and Extended Kalman Filters

OCUKF, OCEKF: Observation Centered Unscented and Extended Kalman Filters

NCN: Normal:conditional-normal distribution

PN: Pinched normal distribution