



AFRL-AFOSR-VA-TR-2022-0054

Models and Protocols for Quantum Distributed Computation

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**09/03/2021
Final Technical Report**

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Air Force Research Laboratory
Air Force Office of Scientific Research
Arlington, Virginia 22203
Air Force Materiel Command

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

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1. REPORT DATE (DD-MM-YYYY) 03-09-2021	2. REPORT TYPE Final	3. DATES COVERED (From - To) 15 Jun 2016 - 14 Jun 2021
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4. TITLE AND SUBTITLE Models and Protocols for Quantum Distributed Computation	5a. CONTRACT NUMBER
	5b. GRANT NUMBER FA9550-16-1-0300
	5c. PROGRAM ELEMENT NUMBER 61102F

6. AUTHOR(S) Jason Morton	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) PENNSYLVANIA STATE UNIVERSITY 201 OLD MAIN UNIVERSITY PARK, PA US	8. PERFORMING ORGANIZATION REPORT NUMBER
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9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) AF Office of Scientific Research 875 N. Randolph St. Room 3112 Arlington, VA 22203	10. SPONSOR/MONITOR'S ACRONYM(S) AFRL/AFOSR RTA2
	11. SPONSOR/MONITOR'S REPORT NUMBER(S) AFRL-AFOSR-VA-TR-2022-0054

12. DISTRIBUTION/AVAILABILITY STATEMENT
A Distribution Unlimited: PB Public Release

13. SUPPLEMENTARY NOTES

14. ABSTRACT
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15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON TRISTAN NGUYEN
a. REPORT	b. ABSTRACT	c. THIS PAGE			
U	U	U	UU	4	19b. TELEPHONE NUMBER (Include area code) 426-7796

Standard Form 298 (Rev.8/98)
Prescribed by ANSI Std. Z39.18

Final Report
Agreement Number FA9550-16-1-0300
Jason Morton, The Pennsylvania State University
Models and Protocols for Quantum Distributed Computation
Period of Performance 06/15/2016 to 06/14/2021

Overview

The project resulted in peer reviewed publications and preprints, several talks and invited presentations, and three successful trainees. A research associate on the project, Jacob Biamonte, became an Associate Professor and head of laboratory at a major international research university. A graduate student William Wright did the majority of his Ph.D. work on the project and is now a data scientist at a Fortune 50 company. He was supported by the grant in the last year, and his dissertation “An Algebraic Perspective on Computing with Data” was composed of problems from the project and a problem from his AFRL internship. Another graduate student Sara Jamshidi who worked on the project is now an Assistant Professor of Computer Science and Mathematics. These graduate students had ongoing engagement with AFOSR and AFRL, including collaboration independent of the PI.

As is typical with fundamental mathematics research, a significant number of results and software products remain to be published and will continue to acknowledge the grant as they reach the dissemination stage. One of the most important insights to come out of the work is a possible solution to the problem of generalizing graphical models to the contextual settings and from simplicial complexes to more general topological models. This is a problem I have been working on, in some form, since 2006. The project provided what I believe to be the key element of the solution, but there are still many details to work out.

Summary of Research Results

The most successful product to come out of the grant bibliometrically was the “Quantum Machine Learning” paper by Biamonte et al., which appeared in Nature and has received approximately 1245 citations to date. This work is viewed as foundational in the field and acknowledges the grant.

The setting for the project begins with networked computational agents with speed-of-light communication constraints in an adversarial “fog of war” scenario. The agent’s shared state can use entanglement. The agent’s state can also exhibit inconsistencies similar to quantum phenomenon because of versioning, latency, and missing data, as well as due to actual entangled state. The setting is related to quantum machine learning as well. We initially developed a new model of this situation, including proving that classical distributed computing challenges can lead to situations formally identical to quantum entanglement.

The next phase of the project involved developing methods for estimation and action in this new statistical regime using tools from quantum information, statistics, algebraic geometry, and

algebraic topology. This was divided primarily into work on state verification/tomography and on inference in the contextual setting.

The invariants for state verification work done with Will Wright (and presented at IIT in 2019) provide a way to do model selection in this setting, and expand upon the quantum tomography ideas in the initial proposal. The method enables quantum or classical state verification without having access to all the coefficients. R code is available for this method, in addition to the technical reports. Wright, a Ph.D. student who graduated in 2019 with support from the grant, assisted in defining the sheaf-theoretic setting for our project and the invariant-based state-verification method that is akin to model selection. His dissertation “An Algebraic Perspective on Computing with Data” was composed of problems from the project and a related problem from his AFRL internship.

Inference in the contextual setting presents many new challenges. The most exciting results in my opinion are the discoveries made in the last two years of the grant which provide strong evidence that ideas from algebraic topology such as delta complexes and homology can be applied to generalize hierarchical models and other Generalized Linear Models (GLMs) to settings in which contextuality plays an important role. Getting to this point required a deep investment by the PI in learning and developing new techniques in algebraic topology. Beyond testing for contextuality using homology as explored by Abramsky, this line of research provides an avenue to quantify and estimate the degree of contextuality present in an observed contextual quantum or classical data set, and thereby enable models that allow small penalized departures from the ideal of a single true global state. I expect this will be useful for a broad range of problems including distributed quantum computation, information fusion, and optimistic distributed database design. Inference in the presence of contextuality is also important for work on statistical privacy, which by adding noise after constructing marginals introduces contextuality. The work exists as a draft book and research code which is available on request but not yet ready for publication.

PUBLICATIONS & DISCOVERIES

(Issued, in press, planned in peer-reviewed journals, and in non-peer-reviewed journals)

Biamonte, J., Wittek, P., Pancotti, N. et al. Quantum machine learning. *Nature* 549, 195–202 (2017). <https://doi.org/10.1038/nature23474> (1245 citations according to Google Scholar, acknowledges this grant).

W. Wright, *An Algebraic Perspective on Computing with Data*. Dissertation, <https://etda.libraries.psu.edu/catalog/16889wcv124>

J. Morton, Contextuality from missing and versioned data. <https://arxiv.org/abs/1708.03264>

J. Morton, *Contextual Generalized Linear Models*.

J. Biamonte and V. Bergholm, Quantum Tensor Networks in a Nutshell, <https://arxiv.org/abs/1708.00006> (71 citations according to Google Scholar, acknowledges this grant).

G. Verdon, M. Broughton, and J. Biamonte, A quantum algorithm to train neural networks using low-depth circuits, <https://arxiv.org/abs/1712.05304>

J. Biamonte, Quantum Machine Learning Matrix Product States, <https://arxiv.org/abs/1804.02398>

Dissemination: talks and workshops relating to the research supported

Morton, J.R., IIT Algebraic Statistics Seminar, "Goodness of fit from algebraic invariants." Abstract: Many statistical models for discrete data have a set of algebraic invariants that vanish on any probability distribution which is a member of the model. We discuss a new idea for performing goodness-of-fit tests on these models using such algebraic invariants. This approach may work better near singularities and boundaries, and in other situations where the usual approaches such as the likelihood ratio test have problems. (November 1, 2019)

Morton, J. R., Invited Talk at Functor Categories, Model Theory, and Constructive Category Theory, Northeastern University / University of Tartu, Pärnu Estonia (July 15, 2019 - July 17, 2019).

Morton, J. R., 2018 AFOSR Cybersecurity Program Review, AFOSR, Arlington, VA, "Models and Protocols for Quantum Distributed Computation," Oral Presentations, 30 in attendance, Invited. (May 21, 2018 - May 25, 2018).

Co-organizer of workshop on Boltzmann machines, American Institute of Mathematics, San Jose, CA, 30 in attendance. (September 17, 2018 - September 21, 2018). Devoted to the mathematics of restricted Boltzmann machines, including quantum aspects and quantum machine learning. Included representatives from industry working on commercial quantum computers.

Morton, J.R. Slow inconsistent statistics Workshop on local algorithms, MIT/Microsoft October 15, 2016

Morton, J.R. Slow inconsistent statistics Workshop on Probability, Uncertainty and Decision, Air Force Research Laboratory June 29, 2016

Trainees

Sara Jamshidi (Graduated 2017, Postdoc at Department of Applied Mathematics, Illinois Institute of Technology, now Assistant Professor of Computer Science and Mathematics at Lake Forest College)

William Wright (Graduated 2019, currently data scientist at Lowe's Companies, Inc.)

Jacob Biamonte (Research Associate, currently Associate Professor at Skolkovo Institute of Science and Technology)