Final Report

Award Information

Award Number	N00014-16-1-2971
Title of Research	Unsteady Velocity Decomposition for Hydrodynamic Assessment of Advanced Naval Vessels
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Organization	University of Michigan
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Technical Section

Technical Objectives

There are several tools that can be used to aid the design of the next generation navy. The use of empirical-based performance equations are suitable for designs that are similar to previous ships. Unfortunately the accuracy of these methods precludes the utilization of revolutionary ideas for materials, ship type, etc. Model tests will continue to aid the design of future vessels, although experiments are expensive, time consuming, and theoretically limited due to scaling issues related to the need to test at scales that are much smaller than the prototype. Potential-flow-based methods have an important role in assessing design performance. The most important aspect of potential-flow based methods is the high-fidelity in which the wave field can be predicted. Unfortunately, if only potential flow is used, the designer may be lead to ship shapes that have poor performance with regard to the viscous drag, and hence potential flow alone is not sufficient as the basis for a high-fidelity design tool. Finally, Navier-Stokes-based methods hold the most promise for the highest fidelity tools since they naturally include both the wave and viscous aspects of the hydrodynamic solution. The principle drawback of such methods is the computational expense that makes them useless with regard to long-time simulation in a realistic seaway.

This work is focused on combining the potential-flow based methods together with field methods to achieve the best of both worlds. Previous work has been done using the framework of velocity decomposition to achieve domain reduction that is necessary for the field method, while computing a viscous potential solution that is valid outside of the rotational region. The theoretical work in this proposal will be useful for the analysis and discovery of the hydrodynamic processes that govern the performance of the broadest range of ship type. Furthermore the proposed work is applicable to all hydrodynamic performances, for example calm-water performance, design loads in extreme seas, coupled fluid-structure interaction in slamming, et cetera.

Technical Approach

0.1 Methodology

The proposed work is based on the past research into velocity decomposition methods by the PI. Early work developed the core idea of finding a viscous potential via a novel iterative algorithm [Edmund et al., 2011]. Then the work of [Rosemurgy et al., 2013, Rosemurgy IV, 2014, Rosemurgy et al., 2016] extended the method to lifting problems in two dimensions with and without a free-surface. In this work the methodology will be extended to treat three-dimensional flows with bodies near or piercing the water surface. The proposed work is organized to uncover and discover the interaction between the viscous potential and the complete flow for the realistic cases of unsteady wave, viscous, body problems.

Velocity decomposition requires the determination of a boundary value problem for a viscous-potential function. The PI has succeeded to derive new BVPs that govern a viscous potential for steady lifting and non-lifting flows in two and three-dimensions. Preliminary analysis has shown that for low-frequency motions a steady viscous potential can be found using a similar formulation. As part of the proposed activity, the specification of the BVP that governs an unsteady viscous potential will be developed. Then, the unsteady viscous potential will be calculated with the higher-order panel method aegir.

Mathematical Details of Velocity Decomposition Framework

For the problem considered in this proposal, viscous effects, such as the boundary layer and viscous wake, are known to be confined within regions near bodies (the reduced domain) and the fluid flow is irrotational for the rest of the domain.

The sketch of a general domain for the problems consider in this work is shown in Figure 1, where S_∞ represents the boundary of the flow domain infinitely distant from the body, V_R is the reduced domain that contains all vortical flow region, S_{OR} is the outlet boundary of the reduced domain, S_R is the sum of the rest of the boundaries of the reduced domain, S_B is the surface of the body, S_W is the wake surface behind the body.

The total flow velocity field $\mathbf{u}$, is expressed as the sum of an irrotational component $\nabla\varphi$ (the viscous-potential velocity) and a vortical component, $\mathbf{w}$, which goes to zero outside the vortical region of the flow field.

$$\mathbf{u}(\mathbf{x},t) = \nabla\varphi(\mathbf{x},t) + \mathbf{w}(\mathbf{x},t) \tag{1}$$

Therefore, this problem is decomposed into two sub-problems, one is a Navier-Stokes sub-problem in the reduced domain and the other is a viscous-potential sub-problem in the whole domain.

Navier-Stokes Sub-problem This sub-problem is governed by the incompressible version of the Navier-Stokes equations and the continuity equation, shown in 2 and 3, which are derived from

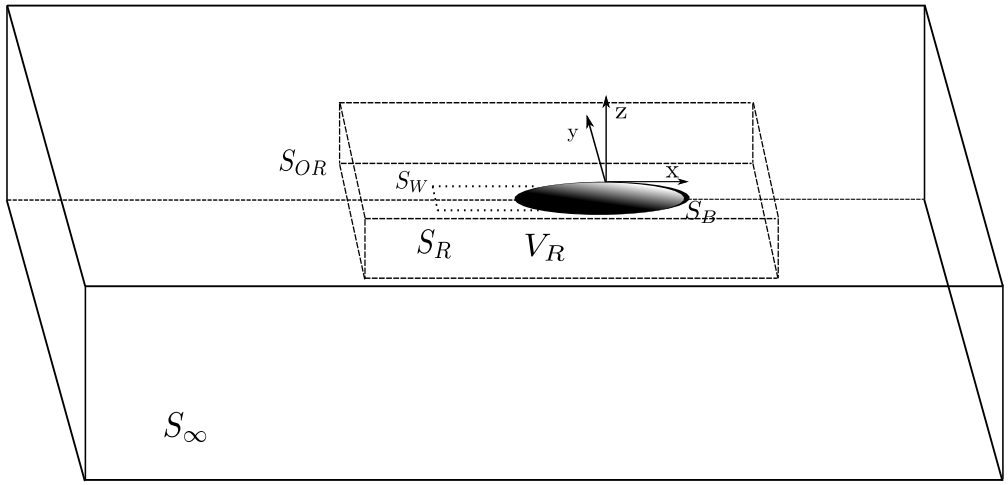

Figure 1: Sketch of the flow domain

conservation of mass and momentum with the assumptions of constant viscosity and density.

$$\nabla \cdot \mathbf{u} = 0 \tag{2}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u}) = -\frac{\nabla p}{\rho} + \mathbf{g} + \nabla \cdot [\nu(\nabla \mathbf{u} + \nabla \mathbf{u}^T)] \tag{3}$$

where $\mathbf{u} = \mathbf{u}(\mathbf{x}, t)$ is the total velocity field that varies in space and time, $\mathbf{u}\mathbf{u}$ in the second term means the tensor product, p is the pressure, ρ is the density of the fluid and ν is the kinematic viscosity, $()^T$ is the transpose, $\mathbf{g}$ is the gravitational acceleration vector.

The velocity of a viscous fluid on the body is subject to the no-slip boundary condition which requires the fluid velocity to be equal to the velocity of the body, (4).

$$\mathbf{u} = \mathbf{u}_B \qquad \text{on } S_B \tag{4}$$

where $\mathbf{u}_B$ is the velocity of the body. The velocity boundary condition at the boundary (excluding the outlet boundary) of the reduced domain (5) is provided by the solution from the viscous-potential sub-problem.

$$\mathbf{u} = \nabla \varphi \qquad \text{on } S_R \tag{5}$$

The velocity boundary condition at the outlet boundary S_{OR} of the reduced domain is set to a zero-normal gradient. The viscous-potential velocity can not be applied here, as the flow is rotational. The pressure outside of the reduced domain can be calculated through the Bernoulli equation using the viscous-potential velocity. The initial velocity field of the domain excluding boundaries is prescribed by the initial condition, (6).

$$\mathbf{u}(\mathbf{x}, 0) = \mathbf{u}_0(\mathbf{x}) \tag{6}$$

where $\mathbf{u}_0$ is the initial velocity field.

$$\mathbf{u}(\mathbf{x},t) = \nabla\varphi(\mathbf{x},t) \tag{7}$$

$$\nabla^2\varphi = 0 \tag{8}$$

$$\tag{9}$$

To include the viscous effects in the body boundary condition and generate the correct viscous potential, a viscous body boundary condition is used to replace the conventional non-penetration condition for potential. As derived in [Edmund, 2012, Rosemurgy IV, 2014, Rosemurgy et al., 2016], the viscous body boundary condition for viscous potential is stated as in (10), where $\hat{n}$ is the local normal vector. This viscous boundary condition is also enforced on the wake boundary S_W to model the viscous wake.

$$\frac{\partial\varphi(\mathbf{x},t)}{\partial n} = -\mathbf{w}(\mathbf{x},t)\cdot\hat{n} \tag{10}$$

Because the total velocity satisfies the continuity (2) and the irrotational part of the velocity is divergence-free, the vortical component of the velocity must also be divergence-free.

$$\nabla\cdot\mathbf{w} = 0 \tag{11}$$

Then (11) can be expressed in the local orthogonal coordinate system and integrated along the normal direction out to a distance δ. After rearranging, It can be simplified to equation (12).

$$w_n(0) = \int_0^\delta \left(\frac{\partial w_{t_1}}{\partial t_1} + \frac{\partial w_{t_2}}{\partial t_2}\right)\mathrm{d}n + w_n(\delta) \tag{12}$$

The upper limit of the integration δ is set to land outside the vortical region. Then the vortical component at δ is zero, and (12) can be further simplified to (13).

$$w_n(0) = \int_0^\delta \left(\frac{\partial w_{t_1}}{\partial t_1} + \frac{\partial w_{t_2}}{\partial t_2}\right)\mathrm{d}n \tag{13}$$

This can be substituted into the Neumann condition (10), and the viscous Neumann boundary condition becomes (14).

$$\begin{aligned}\frac{\partial\varphi}{\partial n} &= -\mathbf{w}\cdot\hat{n} = -w_n \\ &= -\int_0^\delta \left(\frac{\partial w_{t_1}}{\partial t_1} + \frac{\partial w_{t_2}}{\partial t_2}\right)\mathrm{d}n = \chi\end{aligned} \tag{14}$$

In an infinite fluid domain, the velocity is also subjected to the radiation condition, which means the disturbance due to the body decreases as the increase of the distance away from the body.

Even though the time variable is not explicitly present in the Laplace governing equation, both the viscous potential and the vortical velocity may be time dependent for unsteady flow. The unsteadiness can be implicitly included through the viscous boundary condition, (10). The time dependency in (10) may be due to the unsteadiness from body motion, boundary layer or wake of the flow. Based on the characteristics of the unsteadiness, different unsteady velocity decomposition algorithms can be applied [Chen et al., 2015].

Progress Statement Summary

This the final report, and all program objectives were achieved and surpassed. The project was able to support five PhD's, and a post-doc. The doctoral students were Yang Chen, Jose Mesa, Matthew Graham, Paul White, and Bradford Knight. Many publications have resulted from the work, and the support of the Office of Naval Research is gratefully appreciated.

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