REPORT DOCUMENTATION PAGE	*Form Approved* *OMB No. 0704-0188*

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1. REPORT DATE *(DD-MM-YYYY)*
03/01/2021

2. REPORT TYPE
Conference Paper

3. DATES COVERED *(From - To)*

4. TITLE AND SUBTITLE
Superposition Model for Achromatic Surface Lenses

5a. CONTRACT NUMBER
W56KGU-18-D-0004

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
Joseph N Mait

5d. PROJECT NUMBER
0721D070EX

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
The MITRE Corporation
7515 Colshire Drive
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8. PERFORMING ORGANIZATION REPORT NUMBER
PRS-21-0520

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
United States Army (ACC-APG)

10. SPONSOR/MONITOR'S ACRONYM(S)
ACC-APG

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
DISTRIBUTION STATEMENT A. Approved for public release: distribution unlimited.

13. SUPPLEMENTARY NOTES
Preprint to Flat Optics: Components to Systems 2021, Washington, DC, 27 June–1 July 2021.
https://www.osapublishing.org/abstract.cfm?URI=FLATOPTICS-2021-FTu2C.4

14. ABSTRACT

15. SUBJECT TERMS
Symbolic and Algebraic Manipulation; design algorithm

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON**
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**			Susan Carpenito
				2	**19b. TELEPHONE NUMBER** *(Include area code)* 781-271-7646

Superposition Model for Achromatic Surface Lenses

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Abstract: I present a model for an achromatic surface lens based on the superposition of lenses that focus discrete wavelengths independent of one another. The model potentially provides insight into the physical behavior of experimentally demonstrated lenses. It also suggests a simple design algorithm. © 2021 The MITRE Corporation ALL RIGHTS RESERVED.

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This research was developed with funding from the Defense Advanced Research Projects Agency (DARPA). The views, opinions and/or findings expressed are those of the author and should not be interpreted as representing the official views or policies of the Department of Defense or the U.S. Government.

1. Introduction

Structured surface elements offer the potential for optical functionality in a small volume but their dispersion is a severe drawback for broadband imaging. To overcome this problem, surface lenses with heights greater than a wavelength have been used to generate coincident foci for wavelengths related harmonically [1].

Recently, surface elements have been demonstrated that provide achromatic focusing over a band of wavelengths that are not harmonically related [2]. Although the experimental results are indisputable, a physical explanation for their behavior is lacking. Without question, the physical mechanism differs from the one that enables dispersion control by shaping subwavelength features. [3].

I propose a model for the demonstrated behavior based on lens superposition. This model potentially provides insight into the element's physical behavior and suggests a simpler design algorithm than that used in Ref. [2].

2. Lens Superposition Model

The model assumes achromatic performance is generated by a coherent summation of the optical phases necessary to focus each wavelength to f, i.e., a superposition of lenses,

$$\begin{aligned} S(r) &= \exp\left(j\phi_{R_0,0}\right) \\ &\times \left(\frac{1}{K}\right)\sum_{k=0}^{K-1}\exp\left[j\left(\phi_{R_0,k}-\phi_{R_0,0}\right)\right]\exp\left\{j\left(2\pi f\right)\left[\frac{n\left(\lambda_k\right)-1}{\lambda_k}\right]\left[\sqrt{1+\left(\frac{R_0}{f}\right)^2}-\sqrt{1+\left(\frac{r}{f}\right)^2}\right]\right\}, \\ &= \left|S(r)\right|\exp\left[j\psi(r)\right],\ r=[0,R]. \end{aligned} \tag{1}$$

Each lens shares the same quadratic structure defined by the aperture $D = 2R$ and the focal length f. The term R_0 is the radial location at which the geometry is zero for all lenses. This geometric structure is scaled by a wavelength dependent term.

The waveband over which achromaticity is desired is given by a discrete set of K wavelengths. Since the element response is temporally incoherent, each lens can have a different phase offset $\phi_{R_0,k}$.

Note that the superposition is dependent upon relative, not absolute, phase offset. Thus, there exist $K-1$ unknown phase values. Since the radial location R_0 is also unknown, the total unknowns is K.

To examine the impact of these unknowns on the superposition $S(r)$, I assumed a 5-mm focal length and 650-μm diameter. I considered 7 equally-spaced wavelength between 400 and 700 nm. The radial coordinate was quantized into 250 values separated by 1.30 μm. The element material is maP1200G photoresist.

In general, $S(r)$ is complex, i.e., it has both magnitude and phase. From Eq. (1), when the phase offset for all lenses is zero $S(r) = 1$ at $r = R_0$. This is represented in Figs. 1(a) and (b) for $R_0 = 0$. Note that the magnitude ranges between 0 and 1 and the phase in Fig. 1(c) is an aberrated quadratic phase.

When the phases are random, the deviation in $|S(r)|$ is reduced and the phase $\psi(r)$ is again an aberrated quadratic phase whose aberrations differ from those with zero phase. See Figs. 1(e)-(g).

Reducing deviations in magnitude increases light throughput and element efficiency. It stands to reason there exists at least one set of values R_0 and $\phi_{R_0,k}$ that yield a minimum deviation in $|S(r)|$.

3. Proposed Design Algorithm

If fabrication technology were capable of generating a complex transmission simply, evaluating Eq. (1) with the optimal values mentioned above yields an optimally designed element. Unfortunately, this is not the case. The photolithographic fabrication of surface elements yields structures that are phase-only. A simple way to generate a phase element from $S(r)$ is to set its magnitude to unity,

$$\begin{aligned} P_{ph}(r) &= \frac{S(r)}{|S(r)|}, \\ &= \exp\left[j\psi(r)\right]. \end{aligned} \tag{2}$$

If $|S(r)|$ is nearly constant, the error introduced by retaining only phase is minimal.

The fabricated structure $h(r)$ is generated by unwrapping the phase $\psi(r)$, converting phase to height, and wrapping the height using the maximum feature height h_{max}. I assumed h_{max} = 2.60 μm and used 650 nm as the reference wavelength to convert phase to height.

$$h(r) = \mathrm{mod}\left[\mathrm{unwrap}\left\{\left(\frac{f}{2\pi}\right)\left[\frac{\lambda_k}{n(\lambda_k)-1}\right]\psi(r)\right\}, h_{max}\right]. \tag{3}$$

The resulting structures for zero and random phase are represented in Figs. 1(d) and (h).

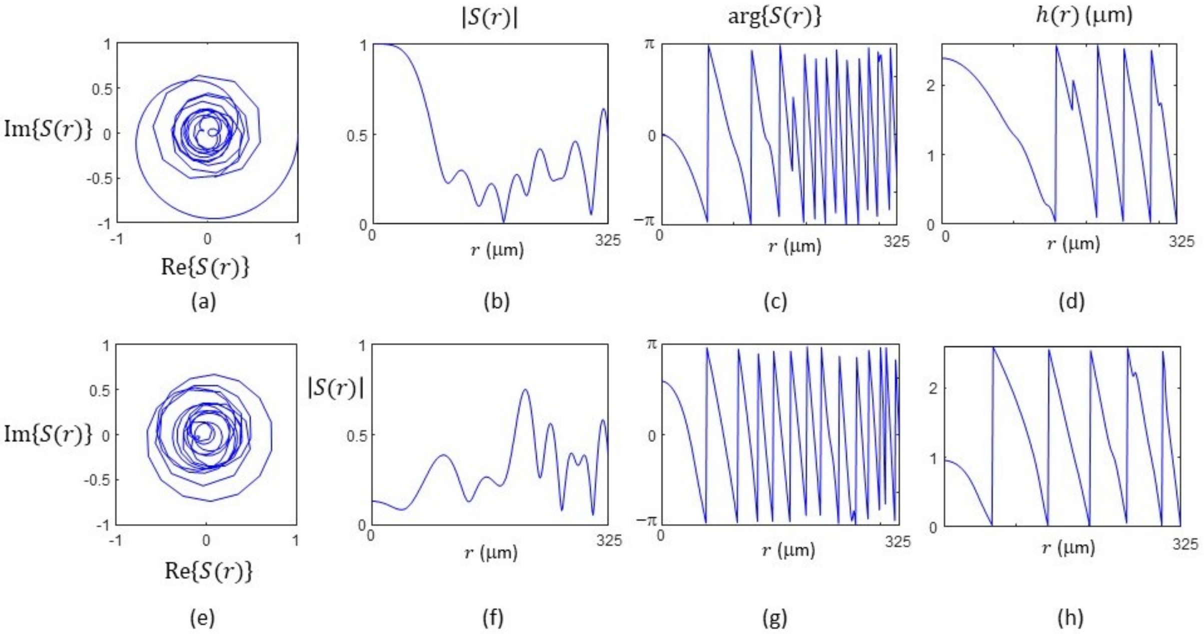

Fig. 1. Lens superposition. For zero phase offset, (a) complex-plane representation of $S(r)$, (b) magnitude $|S(r)|$, (c) phase of $S(r)$, and (d) designed profile $h(r)$. (e)-(h) Same as (a)-(d) except for random phase offset.

4. Concluding Comments

Preliminary results with random phase values indicate the viability of the design approach. However, the model and design algorithm have not been fully validated. Optimization of the parameters R_0 and $\phi_{R_0,k}$ to minimize deviations in $|S(r)|$ is still under development. We are also exploring the criteria for selecting the wavelength used to convert phase to height.

I acknowledge many useful discussions with Joe van der Gracht, Predrag Milojkovic, Jeremy Teichman, Rajesh Menon, Rohith Chandresekar, Augustine Urbas, and Markus Testorf while developing the model presented.

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