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Report on
Scale Model Tests for High-Pressure High-Temperature Steam Piping
First Partial Report

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ABSTRACT

Apparatus, capable of displacing the ends of a model pipe a known amount with respect to each other and of measuring the resultant reaction forces due to the pipe deformation, has been constructed. Experiments have been performed using model pipes of different shapes. End reactions for these pipes have been measured and in many cases compared to corresponding values obtained by calculation. The validity of scaling forces from a model pipe to the full scale pipe has been verified. The equivalence of the change of reaction forces caused by a temperature change and the corresponding change of reaction forces at constant temperature due to a displacement of the pipe ends equal to the temperature expansion have been demonstrated. The change of the modulus of elasticity with temperature must be considered to obtain this equivalence. It is found that the effects of pressure changes within the pipe on the end reactions are small and may be neglected. It is concluded that the determination of end reactions of piping systems by model tests is practical and that the accuracy is dependent mainly on the rigidity of the testing apparatus.

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APPENDICES

INTRODUCTION

(a) Authorization

1. This problem was authorized by Bureau of Engineering Project Order #69/40 of 15 September 1939.

(b) Statement of Problem

2. The determination of end reactions and stresses because of temperature and pressure changes in three dimensional pipe lines is a difficult procedure and is subject to error because of the long calculations involved or to necessary approximations. Small-scale models of any given pipe system can be constructed, and the end reactions can be measured directly. The purpose of this investigation is to construct small-scale pipe models and to develop methods for measuring the end reaction forces due to a pipe and the stresses within the pipe. The practicability and accuracy of these methods will be determined. The general object is to simplify and improve the reliability of the design of high temperature, high-pressure steam pipe lines by the use of suitable models.

3. The full scope of this problem is too broad for a single report. As the experimental and theoretical work progresses, partial reports will be prepared. This first report will contain a comparison of measured and calculated end reactions for pipe systems with no branch connections and will have for its purpose the determination of the accuracy and practicability of the use of model pipe systems.

(c) Known Facts Bearing on the Problem

4. In the design of piping allowance must be made for the stresses and end reactions due to combined pressure and expansion effects. Where the temperature and pressure changes are large, these effects become especially important. Pipe systems should be designed so that the expansion that takes place can be forced back into the pipe system with the least resulting stresses and reaction forces. The pipe system will be deformed slightly to take care of the change of pipe length. Restraints that hinder the free deformation of a pipe will in general cause greater stresses and reaction forces. Lack of rigidity in constraints and end anchorages will relieve pipe stresses and reaction forces. In pipe installations it is generally assumed that the ends of a pipe are held fixed with respect to both location and direction. An increase of pipe wall thickness will result in greater end reaction forces, making it undesirable to increase this thickness beyond a necessary value.

5. The theoretical base upon which the calculations of stresses and end reactions are built has been worked out mainly by William Hovgaard(1) and Thomas Karman(2). These calculations apply to pipe bends and take into account the tendency of a circular pipe section to become oval due to bending stresses. The change of moment of inertia is negligibly small for the deformations considered. The result of this de-

formation of the pipe is that the material furthest from the neutral axis, is brought closer to the neutral axis, thus relieving the stress in these more remote sections and adding stress at some intermediate point. The pipe is more flexible, therefore, than it would be if the stress varied directly as the distance from the neutral axis, such as it does in the case of bars or solid materials. The increase of flexibility of a pipe section, over what would be calculated from theory as applied to solid materials, is a function of the shape, diameter, and wall thickness of the pipe. The less the wall thickness of a pipe or radius of curvature of a pipe bend, the greater will be this increase of flexibility. The increase in flexibility is only for bending moments in the plane of the pipe bend.

6. It is assumed throughout this report, unless stated specifically to the contrary, that all transverse pipe sections are essentially circular and that no stresses are applied beyond the elastic limit of the material. Thus stresses and strains are always in direct proportion.

7. Hovgaard(1) has checked his theoretically-derived equations with experimental results obtained from full scale pipes and finds satisfactory agreement. It can be concluded that for pipes bent to radii of not less than four or five pipe diameters good agreement with theoretical and experimental values will be obtained. In three-dimensional pipe lines with no branches, end reactions and stresses can be laboriously arrived at by theoretical means (3) (4) (5) (6) and, if approximate methods are avoided, the answers obtained should be almost rigorously correct. The principal approximation in the calculation is that the radius of the pipe bend be at least four or five pipe diameters.

8. In order that much of the labor and opportunity for error involved in these calculations be eliminated, H. W. Semar(7) at Westinghouse has constructed apparatus so that end reactions of scale model pipe systems can be measured. Scale factors have been developed so that the forces and torques to be expected from the full-scale system can be calculated from the forces and torques measured on the model. Lately a descriptive article(8) has shown that the scale model method of testing has been extended to branch pipe systems, for which accurate mathematical solutions are impractical.

METHODS AND APPARATUS

(a) Principle of Operation

9. When a pipe, with ends fixed in rigid anchorage, expands because of an increase in temperature, the forces exerted by the ends of the pipe on the anchorage are the same as those produced by freeing one end of the pipe, allowing it to expand, and then restoring it to its original position. These reaction forces for small displacements are linearly proportional to the displacement of the free end of the pipe. The expansion forces can therefore be duplicated by a displacement of the free end of the pipe equal in magnitude to the expansion, but opposite in direction. Correction must be made for any change in modulus of elasticity that will occur at different temperatures.

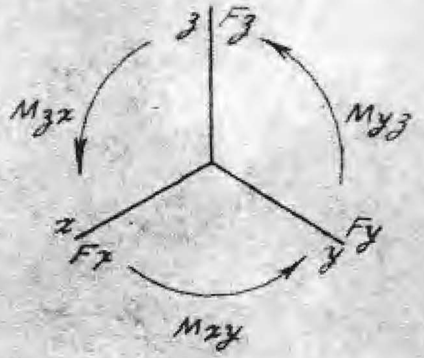

FIGURE 1- COORDINATE FORCES & MOMENTS.

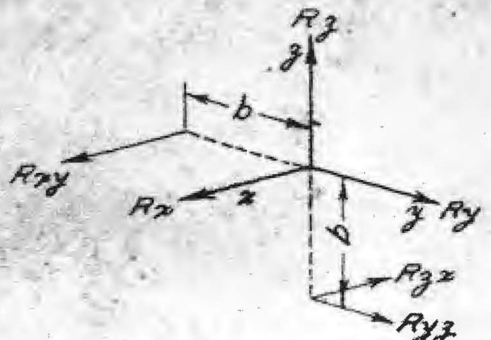

FIGURE 2- REACTION FORCES ARE USUALLY TAKEN AS THE FORCES EXERTED ON THE BRACKET OR PIPE.

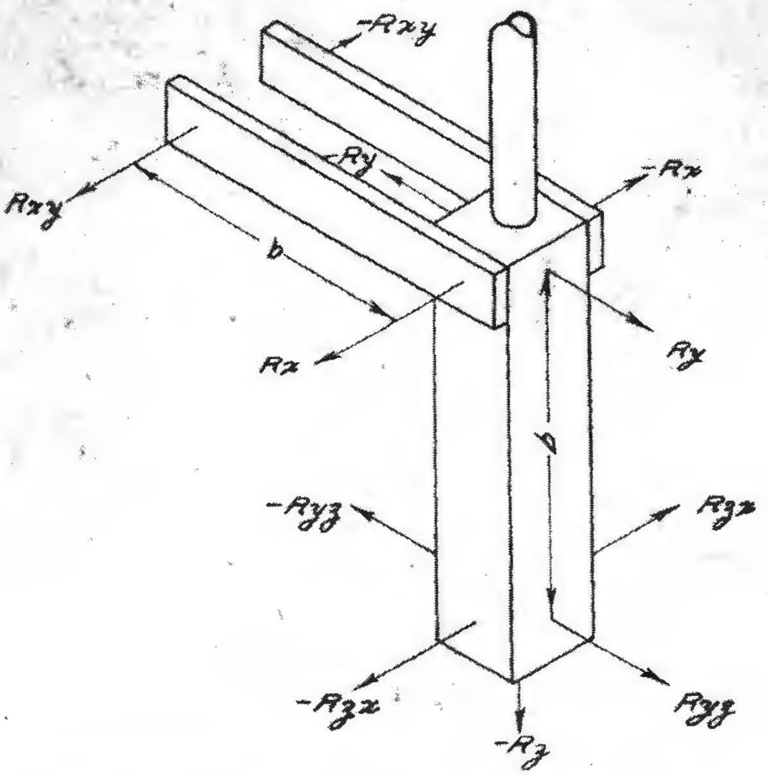

FIGURE 3- BRACKET FOR HOLDING FIXED END OF MODEL PIPE. THE BRACKET IS HELD BY PIN SUPPORTS AT THE INDICATED REACTION POINTS.

which is calibrated in pounds. A dial indicator is fastened to the opposite sleeve (See Plate 3) to determine when the slack, which is usually adjusted to about 0.002 inch, is taken up, at which point the force indicator just balances (or measures) the reaction force.

14. It will be noted that the pipe is mounted at the position where $-R_z$ should be located. This is no disadvantage, for the apparatus can be oriented so that R_z is always positive.

(d) Scale Factors

15. If the model were constructed exactly to scale and made of the same material as the full-size pipe, the forces on the full-size pipe would be the product of the corresponding forces on the model and the scale of the model.

16. In order to use standard sizes of steel tubing it is necessary to depart slightly from the true scale model. This can be done if the scale factor, s, satisfies the following condition:

$$s = \frac{R_m}{R_a} = \left(\frac{r_m}{r_a} \right)^2 \frac{t_a}{t_m}$$

17. Then for the same deflection the forces in the full-size pipe and the model will be in the ratio

$$\frac{E_a I_a}{E_m I_m} s^3 = K$$

and the moments in the ratio

$$\frac{E_a I_a}{E_m I_m} s^2 = \frac{K}{S}$$

where

K = model-to-pipe conversion factor

E = modulus of elasticity

I = moment of inertia

s = scale of model

R = radius of pipe bend

r = mean radius of pipe cross section

t = pipe wall thickness.

The subscripts m and a refer to the model and full-scale pipe respectively.

18. Let e represent the displacement of the ends of the full scale pipe with respect to each other. If there is no relative motion of the pipe anchorages to be considered, e will represent the expansion due to a temperature change (to a slight extent pressure change). If only the displacement due to temperature (and pressure) change is to be considered, the direction of displacement will be along a line connecting the ends of the pipe, and e can easily be calculated from thermal expansion data. The displacement of the ends of the model pipe, by the testing apparatus, should be along the same direction as e.

19. The reaction forces of the model pipe, in terms of force per unit length or pounds per inch displacement of the pipe ends relative to each other are

$$R'_x = \frac{\Delta Rx}{\Delta L} \quad \text{etc.,}$$

where ΔRx is the change in reaction force, and ΔL is the change of the relative position of the ends of the model pipe. This is the slope of the end reaction plotted as a function of the displacement. It should be remembered that the direction of e determines the direction of ΔL.

20. The actual forces and moments exerted by the full size pipe, due to an expansion e, are:

$$Fx = (Rx' + R'xy - R'zx)\ Ke$$

$$Fy = (R'y + R'yz)\ Ke$$

$$Fz = R'z\ Ke$$

$$Mxy = (b/s)\ R'xy\ Ke$$

$$Myz = (b/s)\ R'yz\ Ke$$

$$Mzx = (b/s)\ R'zx\ Ke$$

(e) <u>Apparatus</u>

21. The apparatus for testing end reactions of model pipe systems with no branches consists of the following essential parts:

(1) A rigid beam and right angle support (Plate 1 parts a and b) so constructed that the right angle support can be fixed at any position along the length of the beam. The face of the right angle support is perpendicular to the axis of the beam.

(2) A fixed head (Plate 3 and Plate 1 part c) by means of which the reaction forces are measured. This head is mounted solidly to one end of the beam.

(3) A movable head (Plate 2 and Plate 1 part d) that can be mounted at any height on the right angle support and that can be adjusted to move one end of the model pipe in any direction contained in the plane of the beam and right angle support. The movable head consists of a milling machine attachment with added reenforcements and a clamp support in which the movable end of the pipe can be held in any desired position.

22. In order that pipes with branch systems may be studied, a suitable bed plate must be substituted for the beam, and a sufficient number of right angle supports with movable heads must be provided. It is desirable, in this case, to be able to mount reaction measuring devices on the movable heads.

(f) Theoretical Ends of Model Pipe

23. The ends of a model pipe must be held rigidly in position and direction. At the fixed head a pipe end is placed in a bracket (Plate 4), and the theoretical end of the pipe is assumed at point E. Above this point the pipe is free to bend. At this point and at several points below the pipe is held by a total of 6 set screws and a centering spacer. The inside of the pipe, up to point E, is plugged with a tight fitting steel rod. Any pipe from 1/4" O.D. to 1" O.D. can be held in this bracket. The pipe is terminated in a similar manner in the clamp support at the movable head.

(g) Temperature and Pressure Variations

24. An electric furnace was constructed around a pipe by first covering the pipe with a uniform layer of asbestos tape. Thermocouple junctions (Chromel-alumel) had first been spot welded along various points on the pipe. On the insulating layer of tape was wound a continuous heating coil of #26 nichrome wire. At each end of the pipe was a short section of winding about two or three inches long, represented as a and d in figure 4, that was closely spaced, having 1/16" between turns. Taps were brought out at the ends of these sections. The remainder of the coil was wound with 1/8" between turns and a center tap obtained. The coil was then covered with uniform layers of asbestos and asbestos tape. The coils were connected as shown in Figure 4. Plate 1 shows such a pipe mounted in the testing apparatus. The box, e, is a junction for thermo - couples.

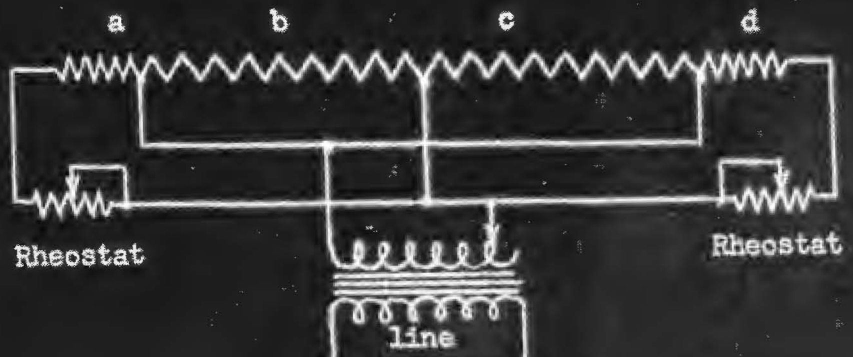

Figure 4.
Electric Furnace and Furnace Connections.

25. For high internal pipe pressures the steel plugs contained in the ends of the pipe were silver soldered into position. A small hole was drilled through the plug at the movable bend along the axis of the plug. The hole at the external end of the plug was enlarged, for a short distance, and tapped for a 1/8" pipe fitting. Pressure was obtained from a nitrogen tank with a suitable pressure reducer and transmitted through copper tubing to this hole. The pressure connection to the model pipe had no effect on any measured end reaction. This connection is illustrated in Plate 2.

(h) Rigidity of Apparatus

26. Marchant[9] calculated the end reactions for the pipe used by Semar[7] in his experimental measurement of its reaction forces. For the major reactions the calculated values agree reasonably well (within 20 percent) with the experimental values. However, the pipe used by Semar was of very flexible construction and provided no serious test of the rigidity of the apparatus used in testing. In order that the apparatus may be used with confidence it is necessary that the calculated and measured values of reaction agree reasonably well for pipes requiring fifty or one hundred times the reaction force per unit of deflection. Some advantage can be had by using smaller diameter model pipes, but as the length, wall thickness, and diameter of the pipe are not all independent variables, the decrease in stiffness cannot be reduced greatly. Appendix #1, which discusses the use of rods rather than tubes for models, should be noted in this respect.

27. It was found necessary to use dial indicators to measure the location of the top of the bracket and the clamp support with respect to the supporting beam and to compensate for undesired displacements of these parts because of insufficient rigidity. Plate 1, 2, and 3 show how this is accomplished. To compensate for any displacement of the fixed head a force is applied at Ry (see figure 3) to keep the top of the bracket in the correct position with respect to the supporting beam. To make the displacement of the pipe at the movable head agree with that given by the movable head scale (the milling machine attachment scale) a force is applied to the clamp support, perpendicular to the right angle support face (Plate 2). This not only corrects for the loss of translation of this part, but also relieves bending stresses in the movable head and its supports and so corrects for much of the rotation.

(i) Bending of Model Pipes

28. Seamless mild steel tubing was used for all pipe models except those discussed in Appendix #1. For the thick-walled models, bent to a large radius of curvature, sand or a low melting alloy may be used to prevent flattening. For thin walled models bent to a small radius of curvature, such as a 3/4" O.D. tube with 0.035" wall thickness bent to a 3" radius of curvature, a half and half mixture of wood's metal and lead was found to be a good filler. Wood's metal alone is too hard and causes the tubing to break, while lead alone is too soft and causes the tubing to wrinkle at the inside of the bend. The tubing is bent on specially constructed forms in a standard bending machine.

EXPERIMENTS AND DATA OBTAINED

(a) Classes of Experiments

29. Experiments performed fall into 4 general classes which are:

(A) The determination of end reactions and the comparison of the values found with those calculated from the dimensions and geometry of the model pipe.

(B) The determination of end reactions for pipe systems of different dimensions, but scale models of each other, and the comparison of results by calculating the forces to that of a common size pipe.

(C) The determination of the equivalence of reaction forces calculated from data obtained from a cold pipe and the experimental reaction forces obtained by the actual change of temperature and expansion of a pipe.

(D) The determination of the effect of internal pressure on the end reactions of both hot and cold pipes.

(b) Data Obtained

Experiment #1 - Class A

30. In this experiment five pipes, illustrated in figure 5, were studied. The calculated (see appendix 2) and measured values of end reactions are given in table 1. Due to symmetry the coordinate

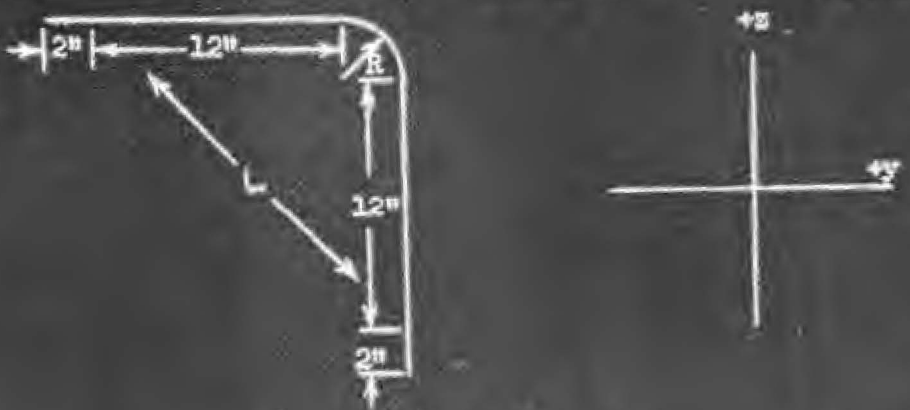

Figure 5.
Plane right-angle bends.

Pipes numbered 10, 12, 13, 14, and 15 are of this type and have wall-thickness and outside diameter of 0.035" and 0.75" respectively. The displacement, ΔL, is along a line, L, connecting the theoretical ends of the pipe. A 2" section at each end of the pipe is inactive and is clamped in the testing apparatus.

forces F_y and F_z are numerically equal and therefore (see equations paragraph 11)

$$-R_y = R_z = R_{yz}$$

The checks obtained in this manner give an indication of the accuracy with which the end reactions are measured. They do not, however, provide a true check on the accuracy of the experiment as the systematic errors due to bending of apparatus may be independent of this check.

31. The numerical values of the measured forces are in general less than the calculated values. This should be so, for all flexibility and bending of the apparatus take place in a manner that relieves the strain of the pipe, and, therefore, the actual measured forces will be less than calculated. The calculated forces assume perfect rigidity of the apparatus holding the pipe. Table I shows the measured forces to be less than the calculated forces in all cases except for pipe #15. It is probable that a variation of wall thickness accounts for this exception. For pipes of this type, with maximum end reaction less than 900 lbs. per inch of displacement, errors less than 10 per cent are obtained. These errors are in a direction away from safety.

Experiment #2. Class A

32. The type of pipe used in the second experiment is shown in Fig. 6 and Plate 6. This is a scale model of the three-dimensional pipe used as an illustration in the Piping Handbook[3]. The theoretical values of end reactions of this model pipe have been obtained by direct calculation, by a grapho-analytical method as described in the above handbook, and by scaling the values calculated in the handbook as described by Semar[7]. (See paragraph 15 to 18 and appendix #3). The values obtained by these two theoretical methods agree within a few percent. This can be regarded as a verification of the scale model theory.

R = 2.85"
O.D. = 0.75"
I.D. = 0.68"

Fig. 6.

Pipe #16. The three section of pipe are mutually perpendicular Reactions are measured according to the primed coordinate system. Forces and moments are calculated with reference to the unprimed coordinate system the axis of which are parallel to the three straight sections of pipe.

TABLE 1

Measured and Calculated Values of Reaction Forces for Plane Pipes Containing One Right Angle Bend. See Figure 5 for pipe dimensions and appendix 2 for calculations.

Reaction Force — Pounds per inch.

	Pipe #10 R=2.25"			Pipe #12 R=3.0"			Pipe #13 R=3.75"			Pipe #14 R=4.5"			Pipe #15 R=6.0"		
	Rz	Ryz	Ry	Rz	Ryz	Ry	Rz	Ryz	Ry	Rz	Ryz	Ry	Rz	Ryz	Ry
Measured	332	556	-880	320	520	-820	308	506	-808	288	500	-800	268	468	-744
Calculated	356	560	-916	344	544	-887	329	526	-855	308	505	-813	270	443	-713
Per cent dif. between abs. magnitudes of Meas. & Calc. values	-7.3	-0.7	-3.3	-7.0	-4.4	-7.0	-6.4	-3.8	-5.5	-6.5	-1.0	-1.0	-0.7	+5.6	+4.3
Check	Rz + Ryz = 888 Ry =-880			Rz + Ryz = 840 Ry =-820			Rz + Ryz = 814 Ry =-808			Rz + Ryz =788 Ry =-808			Rz + Ryz = 736 Ry =-744		

TABLE II

Comparison of the numerical values
obtained experimentally and theoretically
for pipe #16.

Force or Moment Component	Numerical Value of Force or Moment		Per Cent Difference
	Theoretical	Experimental	
Fx	505 lb/in	448 lb/in.	-11.4
Fy	2100	1850	-11.9
Fz	390	423	+ 8.5
Mxy	1560 in-lb/in	1450 in.-lb/in	- 7.0
Myz	7000	5912	-15.5
Mzx	1980	1858	- 6.3

33. A comparison of the measured forces and moments and the corresponding calculated values are given in Table II. Again it is shown that the measured forces are, in general, less than the calculated. Pipe #16 is a short, stiff pipe and provides a severe test of the model pipe apparatus. If errors are to be kept less than 15 percent, with the apparatus as used in this experiment, then the forces and moments per inch deflection, given in Table II, describe the maximum stiffness permissible for a model pipe to be tested.

34. The reactions due to pipe #16 as measured in one of the first arrangements of the testing apparatus were approximately one half of the calculated values. It would appear necessary, therefore, that all apparatus of this kind be tested with model pipes having known end reactions and having a stiffness comparable to that of the least flexible models to be encountered.

Experiment #3 Class B

35. In order to determine experimentally the validity of the scale model theory pipes were constructed of the shape illustrated in Fig. 7, the dimensions of which are given in Table III.

Figure 7.

Pipes #21, 22, 23, 24. These pipes contain three mutually perpendicular sections.

TABLE III

Pipe #	O. D.	W. T.	$\overline{ab}$ & $\overline{ef}$	$\overline{bc}$ & $\overline{de}$	$\overline{cd}$	R
21	.252 in	0.028 in	3.13 in	4.6 in	1.64 in.	1.05 in
22	.504	0.051	6.95	10.2	3.64	2.32
23	.752	0.071	11.14	16.4	5.84	3.72
24	1.004	0.095	15.0	22.0	7.85	5.0

36. These pipes are scale models of each other and satisfy the relations given in paragraph 16. The theoretical end reactions have not been determined for these pipes, but the average forces and moments for the four pipes, when scaled to a common diameter, have been determined. These are shown in Table IV. All of the reactions of the 1/4 inch diameter pipe could not be measured because of lack of clearance. The sizes of pipes used include the smallest and largest that can be placed in the testing apparatus. The dimensions of the smaller sizes are such that large percentage errors may occur. The percent variations from average values cannot be considered excessive. They indicate again the probable accuracy under extreme conditions of operation and roughly verify the scale model theory.

TABLE IV.

Comparison of reaction for pipes of different sizes which are scale models of each other. Units are in pounds per inch displacement or inch-pounds per inch displacement for forces and moments respectively. Absolute magnitudes are given for the reactions.

Force or Moment	Pipe #24 1" O.D.			Pipe #23 3/4" O.D.			Pipe #22 1/2" O.D.			Pipe #21 1/4" O.D.		
	Measured	Average of values of all Pipes Scaled to 1 inch Dia.	Per Cent Dif.	Measured Force or Moment	Average of Values of all Pipes Scaled to 3/4 inch Dia.	Per Cent Dif.	Measured Force or Moment	Average of Values of All Pipes Scaled to 1/2 inch Dia.	Per Cent Dif.	Measured Force or Moment	Average of Values of All Pipes Scaled to 1/4 inch Dia.	Per Cent Dif.
Fx	64	62.8	+ 2	44	48.6	- 9	45	41.2	+ 7	29.7	29.8	0
Fy	240	274	-12	218	212	+ 3	197	180	+10		130	-
Fz	88	96	- 4	68	71	- 5	62.5	60.4	+ 3	46.6	43.6	+ 6
Fxy	560	600	- 7	340	346	- 2	187	182	+ 3	61.7	59.4	+ 4
Fyz	3570	3540	+ 1	1880	2030	- 7	1125	1174	+ 6		350	-
Fzx	440	500	-12	260	288	-10	163	152	+ 7	50.7	49.6	+13

Experiment #4 Class C

37. A pipe similar to that shown in Fig. 5, except that the wall thickness was 0.049 inches, was used to determine the relations between end reactions and temperature. The radius of the pipe bend was 3.75 inches. An electric furnace was constructed about the pipe, as described in paragraph 24.

38. Plate 7 shows the relation between the reaction forces and the displacement of the ends of the pipe relative to each other. The displacement was along a line (L of Figure 5) connecting the ends of the pipe. The values of the end reactions due to any given displacement and at any temperature can be calculated if the end reactions are known for a given displacement at a given temperature. The modulus of elasticity must be known over the temperature range, as these reactions are linear functions of this modulus. (See Plates 8 and 9).

39. Plate 10 illustrates the relation between reaction forces and temperature. The solid circles represent measured forces due to the temperature change. The open circles represent reactions resulting from displacements of the movable end of the pipe at room temperature and corrected for the change of elastic modulus. The equation used in this calculation is given in paragraph 44. The displacement was equal to the free thermal expansion between the theoretical ends of pipe that would be caused by the indicated temperature change.

40. In Plate 11, on the right half of the graph, is plotted in heavy lines the end reactions versus displacement at a temperature of 75° F. The forces are due to a calculated expansion of 0.106 inches. The pipe was then heated, and the movable head was adjusted so that the reaction forces were kept zero. The total motion of the movable head, 0.109 inches, was the measured expansion. The movable end of the pipe was then displaced towards its original position, and the reaction forces were measured. These are plotted on the left half of the graph. The values of end reactions, due to this temperature change with the ends of the pipe in the positions occupied at room temperature, are indicated at the zero displacement location. The corresponding values calculated from room temperature data are also shown. The values check within the experimental accuracy of about 5 per cent.

Experiment #5 Class C.

41. Pipe #32, with the same dimensions as #23 (See Fig. 7 and Table III), except for a wall thickness of 0.049 inches and I.D. of 0.656 inches, had an electric furnace constructed about it. The same procedure was then carried out with this pipe as was done in the preceding experiment.

42. Plate 12 is similar to Plate 11 except that the measured displacement of the ends of the pipe due to the temperature change, rather than the calculated displacement, was used throughout. As there were 6 reaction forces to be considered, these were tabulated, as shown in Table V, rather than plotted.

TABLE V.

Comparison of end reactions obtained due to temperature change and the corresponding reactions calculated from cold pipe date.

Pipe #32

Reaction	Measured	Calculated	Per cent Difference
Rx	20.1 lbs.	20.9 lbs	4
Ry	69.4	67.2	3
Rz	7.3	7.2	1
Rxy	8.2 in lbs	8.6 in lbs	5
Ryz	43.7	44.0	1
Rzx	8.8	7.0	25

43. The expansion due to the temperature change from 77° F to 726° F was measured as 0.172 inches.

44. The measured reaction forces necessary to move the ends of the hot pipe back to the positions occupied when cold are given in the second column of Table V. The calculated forces were obtained from the equation below and are shown in the third column of Table V.

$$R_t = \frac{E_t}{E_{to}} \quad R_{to}$$

R_t refers to a reaction caused by a temperature change from to to t with the pipe ends held in rigid anchorage.

R_{to} refers to a reaction caused by a displacement of the ends of the pipe by an equal magnitude but in an opposite direction to that which would be caused by a free thermal expansion due to a temperature change from to to t. The reaction R_{to} is measured at room temperature. E_t and E_{to} refer to Young's modulus at the two temperatures.

45. The differences between measured and calculated values for all major reactions are less than 5 per cent. The large percentage difference between the Rzx reactions is not significant, as the numerical difference is small compared with the major forces.

46. Temperature variations among 9 thermocouples along the length of the pipe were not more than 20° F at the high temperatures. The temperature change during measurements at a given temperature point was not more than a degree. The temperature recorded was the average of the thermocouple readings.

Experiment #6 Class D

47. Effects of changes of internal pressure on the end reactions were determined for pipes #31 and #32 with temperature as a parameter. Some results are tabulated in Table VI. Only results due to pipe #32 are presented.

TABLE VI.

Effect on the end reactions of a change of internal pressure of 800 lbs/sq. in. Pipe #32. Reactions are given in pounds.

	△ Rx	△ Ry	△ Rz	△ Rxy	△ Ryz	△ Rzx
Calculated from Room Temp. Data	+.43	1.38	.14	.18	.9	.14
Calculated from 726° F data.	+.39	1.40	.15	.17	.9	.18
Measured at room temp.	0	1.6	.20	0	.8	0
Measured at 725° F.	-.4	2.0	.40	0	.9	-.1

48. The determination of the calculated values of end reaction change due to a pressure change is performed by first finding the change of longitudinal pipe stress due to the change of pressure. The change of length of the pipe due to this stress change is then calculated, and from end reaction versus displacement data, the reactions necessary to move the ends of the pipe back to their original positions are determined. It is interesting to note that under these assumptions the change of reaction forces due to a pressure change is independent of the modulus of elasticity (or temperature).

49. As the forces involved are very small, it must be expected that such factors as bends and non-circular transverse cross sections of the pipe, which are neglected in the calculated values, will greatly effect the result. The agreement of the results, as shown in Table VI, is therefore surprisingly good.

50. The end reaction forces due to an internal increase of pipe pressure are small. In the case illustrated they are less than 2 per cent of the reaction forces due to temperature change (see Table V). It may be considered permissible to neglect this effect.

CONCLUSIONS AND RECOMMENDATIONS

(a) Probable Errors

51. The principal source of error is, as has been emphasized before, not in the instruments used in measuring the reaction forces, but in the bending of the apparatus holding the model pipe. Apparatus can be constructed that with proper operating technique will give measured results agreeing within 15 per cent of those calculated.

52. The magnitude of the probable error is a function of the "stiffness" of the model pipe being tested. When the forces and moments, per inch of end displacement, are large (over 2000 pounds per inch or 7000 inch pounds per inch) the errors may be greater than 15 per cent.

53. Identical pipe models will give corresponding reaction forces that agree within a few percent. This spread of measurements is due to inaccuracies of the measuring instruments and variation of the pipe wall thickness. These errors are small.

54. The theoretical end of the pipe is accurately determined and correct in its usage for all measurements taken at room temperature. For high temperature work it is difficult to determine the displacement of the pipe ends due to the temperature change by means other than direct measurement, as the inactive ends of the pipe and the clamp supports are raised to quite high unknown temperatures. Thus, while the bending of the pipe takes place only between the theoretical ends of the pipe, there is a temperature gradient near these ends that disturbs expansion calculation.

55. The probable errors are to be expected in a direction away from safety - i.e. the measured reaction forces are in general less than those calculated.

(b) Facts Established

56. The experiments performed have established the following conclusions for model pipe systems with no branch connections:

1. Model pipe systems and testing apparatus are practical to construct and use.
2. End reactions can be measured within an accuracy of 15 per cent for pipes having moderate stiffness. (Forces and moments of 1000 lb/in. and 4000 in-lb/in. respectively).
3. The accuracy of model pipe testing apparatus should be tested by means of a standard model pipe having known end reactions and having a stiffness at least as great as other models to be tested.

4. Scale factors, as determined by Semar[7], offer no difficulties either theoretically or practically and can be applied with confidence.

5. It is impractical to work under conditions of high temperature for other than research conditions. All information can be obtained under room temperature conditions with simple reliable calculations. Hooke's law is assumed to hold in all cases.

6. End reactions due to internal pressure changes are small (about 2 per cent of the reaction due to temperature changes for high temperature systems) and may be neglected.

SUMMARY

57. If values of end reactions calculated for a full scale pipe and for its small scale model are determined and if the reactions are compared by the application of the suitable scale factors, the reactions agree.

58. Experimentally measured end reactions agree with corresponding calculated values within the limitations of the testing apparatus and the accuracies of measurement.

59. Measurements under high temperature conditions can be performed satisfactorily under good laboratory conditions, but it is considered impractical and unnecessary to work under these conditions, as all information within the limitation of Hooke's law can be obtained from experiments carried out at room temperature.

60. The effects of changes of internal pipe pressure on the end reactions are small. The change of end reaction forces due to pressure changes are linear functions of the pressure change.

61. For stiff pipes, where measurements are difficult when tubular models are employed, the use of rods for model construction for the determination of end reactions is believed justified. (See appendix 1).

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APPENDIX #1

The Use of Rods as a Substitute for Tubes in Model Pipe Systems. A discussion of a Paper by Fred M. Hill[12].

The advantages of using rods rather than tubes for the construction of model pipes are:

(1) Little difficulty is encountered in making suitable bends.

(2) The model can be made as long and flexible as is desired, for the ratio of length of model to pipe can be chosen independently of the diameter of the model. This is an important advantage for little trouble need be experienced with lack of rigidity of the model pipe testing apparatus.

The principal disadvantage in using rods rather than tubes for the construction of model pipes is the uncertainty of the meaning of the measured end reaction. The principal reactions must, of course, be larger than those for tubes, but the magnitude of this difference is a function of the shape and wall thickness of the tube and cannot be easily determined. The measurements of Hill indicate that in general this difference is not large.

When the flattening theory is disregarded, as is the case when rods are used for pipe model construction, the ratio of forces of pipe to model is

$$\frac{F_p}{F_m} = \frac{E_p I_p \quad \Delta L_p S^3}{E_m I_m \quad \Delta L_m}$$

and the ratio of moments of pipe to model is

$$\frac{M_p}{M_m} = \frac{E_p I_p \quad \Delta L_p S^2}{E_m I_m \quad \Delta L_m}$$

where

F = coordinate reaction force
M = Reaction moment
E = modulus of elasticity
I = moment of inertia
ΔL = deflection
S = ratio of lengths of model to pipe
p = subscript indicating pipe
m = subscript indicating model.

Mr. Hill[12] has shown that measured end reactions, using 1/4 inch diameter rods as scale models of pipes whose end reactions were known, when multiplied by the proper scale factor gave results in quite good agreement with the values determined for the full scale pipe. Mr. Hill has included in his report a study of the pipe used as an example in the Piping Handbook[3].

Pipe #16 was used in this laboratory as a model of this same pipe. End reactions were also determined in this laboratory for the Handbook pipe by means of a model constructed from a 3/8" diameter rod and neglecting the flexibility factor.

The first two tables below are from Mr. Hill's report in which rods were used as models. Table III gives the reactions as determined by use of pipe #16, and Table IV gives the results of this laboratory's calculations and experimental results using the 3/8 inch rod as a model. It will be noticed that the mathematical results obtained at this laboratory are different from those obtained by Hill* (Table II column 2). The source of error has not yet been determined. The moments tabulated in Tables III and IV should not be compared with those of Tables I and II as they are determined for opposite ends of the pipe.

The results signify that for stiff pipes, where measurements would be difficult using tubular models, the use of rods for model construction for the determination of end reactions is justified.

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* A check of the theoretical work at this Laboratory indicates that the mathematical results of Hill, as shown in Table II, are too high and should be equal to those given in Table IV. It is emphasized again that the moments as given in the first two tables and the last two tables are not comparable as they are determined for opposite ends of the pipe.

TABLE I.

Three-Dimensional Pipe Run Illustrated in the Piping Handbook. (Data from Hill). (Forces are given in pounds and moments in inch-pounds).

End Reaction	Mathematical by Tubular Theory	Solid Model	Difference	Per Cent Difference
Fx	5,010	5,510	500	+9.98
Fy	20,870	21,610	740	+3.55
Fz	3,855	4,019	164	+4.25
Mzy	220,652	270,000	49,348	+22.36
Mxy	1,786,070	1,727,000	49,070	-3.31
Mxz	381,548	350,000	31,548	-8.27

TABLE II

Three-Dimensional Pipe Run Illustrated in the Piping Handbook. Flattening Factor Omitted (Data from Hill) (Forces are given in pounds and moments in inch-pounds).

End Reaction	Mathematical by Beam Theory	Solid Model	Difference	Per Cent Difference
Fx	8,009	5,510	2,499	-31.20
Fy	30,918	21,610	9,308	-30.11
Fz	5,647	4,019	1,628	-28.82
Mzy	247,249	270,000	22,751	+ 9.20
Mxy	2,455,969	1,727,000	728,969	-29.68
Mxz	556,663	350,000	206,663	-37.12

TABLE III

Three-Dimensional Pipe Run Illustrated in the Piping Handbook. (Model Pipe #16, NRL Data) (Forces are given in pounds and moments in inch-pounds).

End Reaction	Mathematical by Tubular Theory	Tubular Model	Difference	Per Cent Difference
Fx	5,010	4,540	-470	- 9.4
Fy	20,870	18,700	-2,170	-10.4
Fz	3,855	4,290	+445	+11.6
Mxy	324,000	317,000	-7,000	- 2.2
Myz	1,480,000	1,250,000	-230,000	-15.6
Mzx	412,800	395,000	-17,800	- 4.3

TABLE IV

Three-Dimensional Pipe Run Illustrated in the Piping Handbook. Flattening factor Omitted (Rod used as Model, NRL Data and Calculations).

End Reaction	Mathematical by Beam Theory	Solid Model	Difference	Per Cent Difference
Fx	6,390	7,050	+660	+10.3
Fy	24,060	23,700	-360	- 1.5
Fz	4,800	4,370	-430	- 9.0
Mxy	358,000	365,000	+7000	+ 2.0
Myz	1,602,000	1,520,000	-82,000	- 5.1
Mzx	530,000	549,000	+19,000	+ 3.6

APPENDIX 2

Calculations for Plane Pipes Consisting of a 90 degree Bend and Tangents. Pipes #10 through #15.

For simple pipe systems in one plane general mathematical equations can be set up for any given type of bend or bends. These equations can give the end rractions as a function of the end displacement and their solutions, in graphical form, provide a quick and accurate means of determining their results for any particular type of pipe that is represented. Such a method is given by E.A. Wert, S. Smith and E.T. Cope[10].

The analytical method given below, for a quarter bend with tangents, follows this procedure. As the application will only be for a few pipes the equations will be solved for the particular cases rather than plotted.

The equations given in the above reference contain numerous errors and should not be applied without check. The nomenclature and coordinate system used below corresponds to that of Wert [10] and must be transformed to correspond to that employed in the body of this report.

The pipe structure is shown in the figure below. R is the radius of the pipe bend and m and n are the lengths of the tangents.

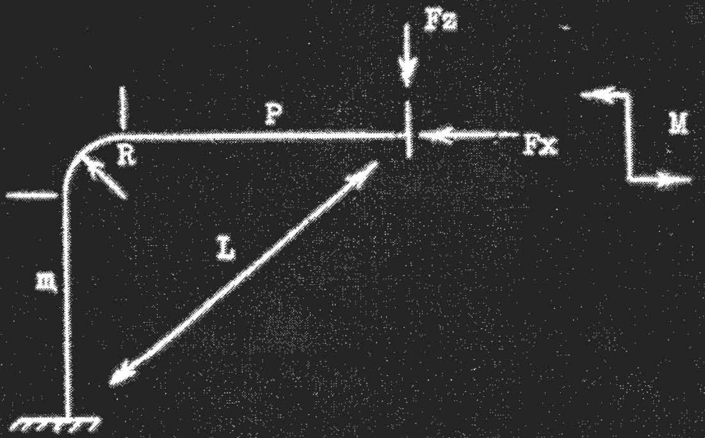


Figure 1

The reaction-displacement equation as solved by the graphs-analytical method[3] gives

$$EI\,\triangle x = jFx + kFz$$

$$EI\,\triangle y = fFx + hFz$$

where

$$j = \left[\frac{m^3 + 3m^2R + 3mR^2 + 0.681sR^2}{3} - \frac{(m^2 + 2mR + 0.726sR)^2}{4(m+s+p)}\right]$$

$$f = [(\frac{0.5m^2 + mR + 0.363sR}{m + s + p}) (mR + 0.637sR-0.5p^2) -R(mR +0.5m^2+0.318sR)]$$

$$h = [mR(p+R)+sR(0.5R+0.637p)-0.166p^3-(\frac{mp+mR+0.637sR+sp+0.5p^2}{m + s + p})(mR+0.637sR-0.5p^2$$

$$k = f$$

$$M = [\frac{mR+mp+0.637sR+sp+0.5p^2}{m + s + p}] F_z - [\frac{0.5m^2 + mR + 0.363sR}{m + s + p}] Fx$$

where $S = \frac{\pi R}{2K}$

$K = \frac{1 + 12h^2}{10 + 12h^2}$

$h = \frac{tR}{r^2}$

For the pipes of this type considered in these experiments

$m = p =$ 12 inches = length of tangents

O.D. = 0.75"

t = wall thickness = 0.035"

r = mean radius = 0.3575"

I = moment of inertia = 0.00505 in.^{4}

E = modulus of elasticity = 29×10^6 lbs/in^2

The displacement, ΔL, is along a line, L, connecting the ends of the pipe.

$$\Delta x = \Delta z = \frac{\Delta L}{\sqrt{2}}$$

and $Fx = Fz = F$

substitute values to get

$$F = \frac{1.037 \times 10^5 \Delta L}{j + f} \text{ pounds}$$

For different radii of pipe bends the following solutions are tabulated.

Pipe #	R Inches	j	f	F lbs/ΔL	M	Pipe End Reactions (lbs)* R^1_x	R^1_{zx}	R^1_z
[illegible]	1.5	553	-271	368	7.80F	368 ΔL	575 ΔL	943 ΔL
10	2.25	630	-339	356	7.85F	356 ΔL	560 ΔL	915 ΔL
12	3.00	716	-415	344	7.90F	344 ΔL	544 ΔL	888 ΔL
13	3.75	811	-496	329	8.00F	329 ΔL	526 ΔL	855 ΔL
14	4.5	920	-585	308	8.20F	308 ΔL	505 ΔL	813 ΔL
15	6.0	1170	-785	270	8.20F	270 ΔL	443 ΔL	713 ΔL

* The end reaction as tabulated in Table I of the body of this report are related to the above values by the relation

$$R_y = R^1_z$$

$$R_{yz} = -R^1_{zx}$$

$$R_z = -R^1_x$$

ΔL is unity

Reaction Calculations and Scaling Example for Pipe #16.

A full scale pipe of which pipe #16 is a scale model is given below. This pipe is similar to that given in the Piping Handbook[3] and the reactions will be numerically the same as those given in the handbook. The three straight sections of pipe are mutually perpendicular.

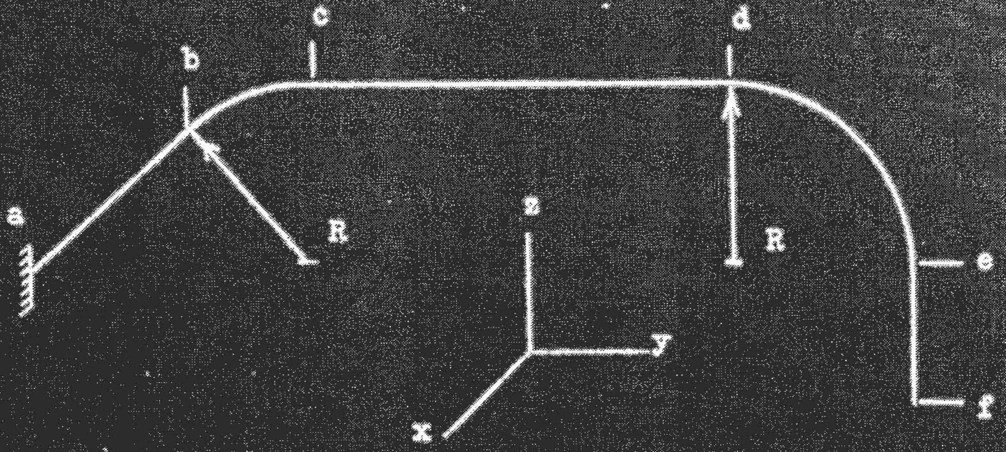


Figure 1

Full scale pipe similar to that in the Piping Handbook or model pipe #16.

Dimensions of full scale pipe

O.D. = 12.75" Wall thickness = 1/2"

I.D. = 11.75" Mean diameter = 12.25"

ab = 72"

cd = 138" $I = 360 \text{ in.}^4$

ef = 48" $E = 25 \times 10^6 \text{ lb/in}^2$

R = 60"

Steel tubing having an O.D. of 0.75" and wall thickness of 0.035" was used for the scale model. The ratio of diameters and the ratio of radii of the full scale and model pipe should be as near the same magnitude as convenient.

The scale is

$$s = \left(\frac{rm}{ra}\right)^2 \frac{ta}{tm} = \left(\frac{0.358}{6.125}\right)^2 \frac{0.5}{0.035} = \frac{1}{21}$$

See paragraph 17 of report for definition of symbols.

The dimensions for the model pipe are therefore:

O.D. = 0.75"
I.D. = 0.68"
ab = 3.43"
cd = 6.57
ef = 2.29
R = 2.86"

Wall thickness = 0.035"

$I = 5.02 \times 10^{-3}$ in.^{4}
$E = 29 \times 10^6$ lb. per in.^{2}

The calculations of reaction forces and moments were performed in a similar manner as was done for the full scale pipe[3], and the results are given below.

$$Fx = EI\,(2.04\,\Delta x - 2.86\,\Delta y + .45\,\Delta z)\,10^{-3}$$
$$Fy = EI\,(2.86\,\Delta x - 14.65\,\Delta y + 2.19\,\Delta z)\,10^{-3}$$
$$Fz = EI\,(0.45\,\Delta x - 2.20\,\Delta y + 1.72\,\Delta z)\,10^{-3}$$

$$Mxy = +6.90\,Fx + 0.92\,Fy$$
$$Myz = -4.46\,Fy - 5.91\,Fz$$
$$Mzx = -0.81\,Fz + 4.55\,Fx$$

Forces are positive when directed along the positive direction of the coordinate axes. The displacements Δx, Δy, and Δz, of the free end of the pipe are positive when in the positive direction of the coordinate axes. See figure below.

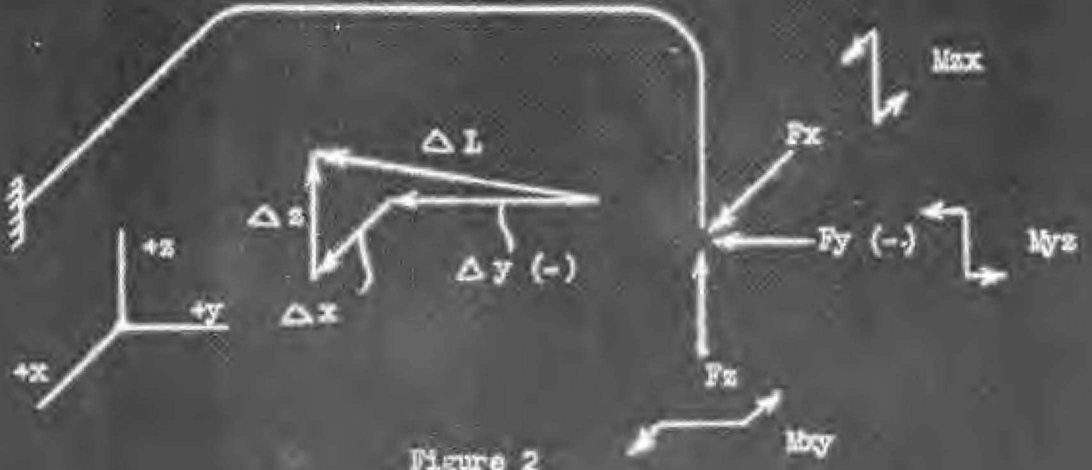


Figure 2

Reaction forces and Moments.

The change of length of the pipe due to temperature change will be such that (see figure 1 of this appendix).

$$\Delta x : \Delta y : \Delta z = \overline{ab} + R : \overline{cd} + 2R : \overline{ef} + r$$

This will make the displacement of the pipe ends take place along a line connect the ends of the pipe. The change in length between the pipe ends is

$$\Delta L = \sqrt{\Delta x^2 + \ y^2 + \ z^2}$$

and

$$\Delta x = 0.425 \Delta L$$
$$\Delta y = -0.830 \Delta L$$
$$\Delta z = 0.353 \Delta L$$

Substitute the values of Δx, Δy, Δz and EI in the equation for the coordinate forces and moments to get

$$F_x = 505 \Delta L \text{ (pounds)}$$
$$F_y = -2100 \Delta L$$
$$F_z = 390 \Delta L$$
$$M_{yx} = 1560 \Delta L \text{ (inch-pounds)}$$
$$M_{yz} = 7000 \Delta L$$
$$M_{zx} = 1980 \Delta L$$

In order that the reaction forces of the full scale pipe may be obtained from those determined for its scale model the model-to-pipe conversion factor, K, is determined (See paragraphs 15 to 20 of report body).

$$K = \frac{E_a I_a}{E_m I_m} s^3 = \frac{25 \times 10^6 \times 360}{29 \times 10^6 \times 0.00502} \times \left(\frac{1}{21}\right)^3 = 6.67$$

and

$$K/s = 6.67 \times 21 = 140$$

The forces and moments exerted on the full scale pipe are:

$$F_{ax} = F_{mx} K = -505 \times 6.67 \Delta L = -3370 \Delta L$$
$$F_{ay} = 14000 \Delta L$$
$$F_{az} = -2600 \Delta L$$
$$M_{axy} = M_{mxy} K \Delta L/s = 218000 \Delta L$$
$$M_{ayz} = 980000 \Delta L$$
$$M_{azx} = 277000 \Delta L$$

where the subscripts "a" and "m" refer to the full scale and to the model pipe respectively.

The expansion, ΔL, as determined for the full scale pipe corresponding to a temperature range of 60° F to 725° V, is 1.52 inches. If this value of ΔL is substituted in the last set of equation the forces and moments corresponding to this displacement are obtained. The absolute values of these are tabulated below together with the corresponding values obtained by the Handbook[3].

TABLE 1.

A comparison of End Reactions obtained by Calculation for Model Pipe #16 and scaled to the full scale pipe, and the end reactions calculated for the full scale (Handbook) pipe.

	Calc. from scale model pipe date	Handbook
Fax	5110 lbs.	5010 lbs
Fay	21300	20870
Faz	3950	3855
Maxy	331000 in-lbs	324000 in-lbs
Mayz	1490000	1480000
Maxz	420000	412800

The differences of corresponding reactions can be regarded as small numerical errors in the calculations.

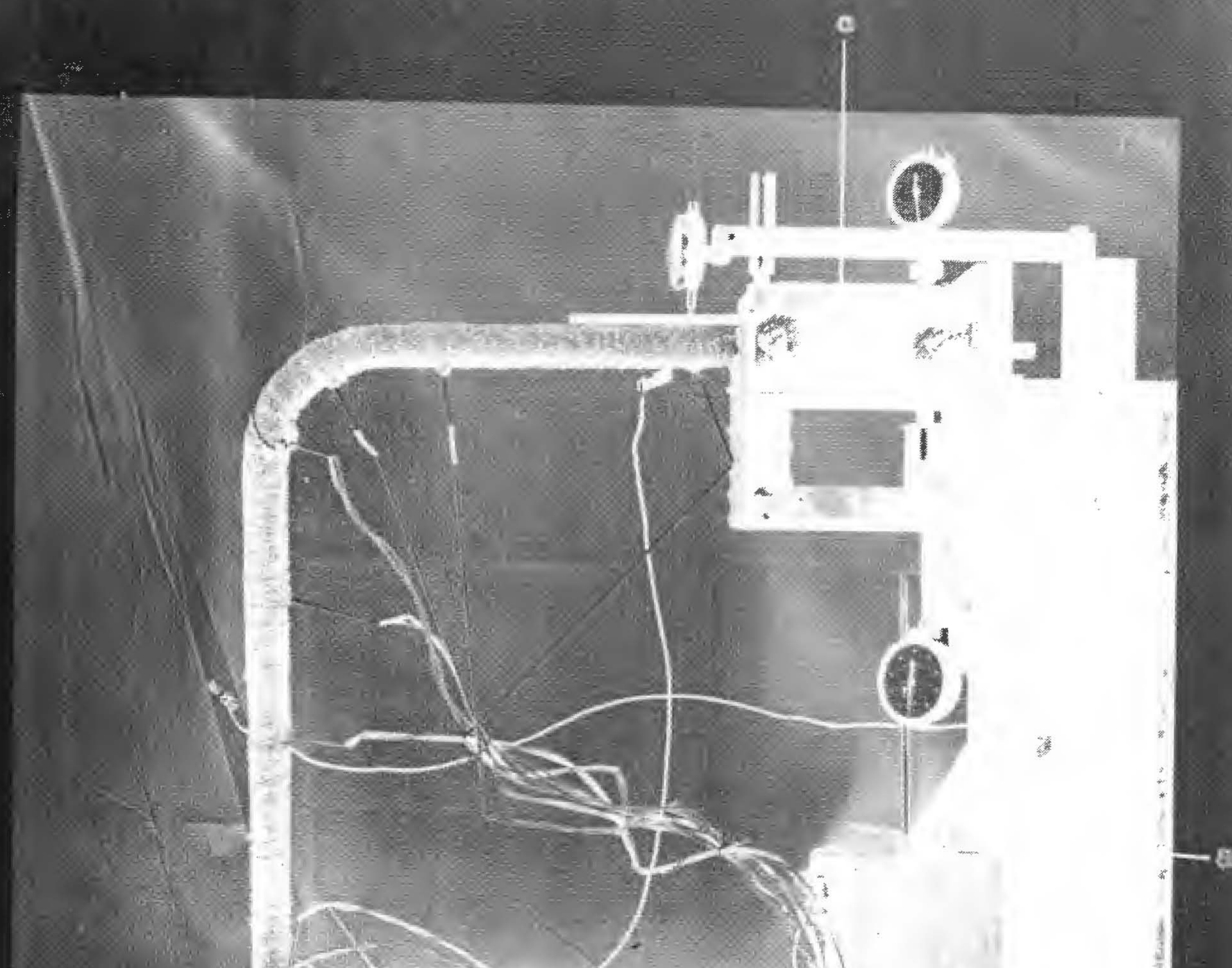


Plate 1

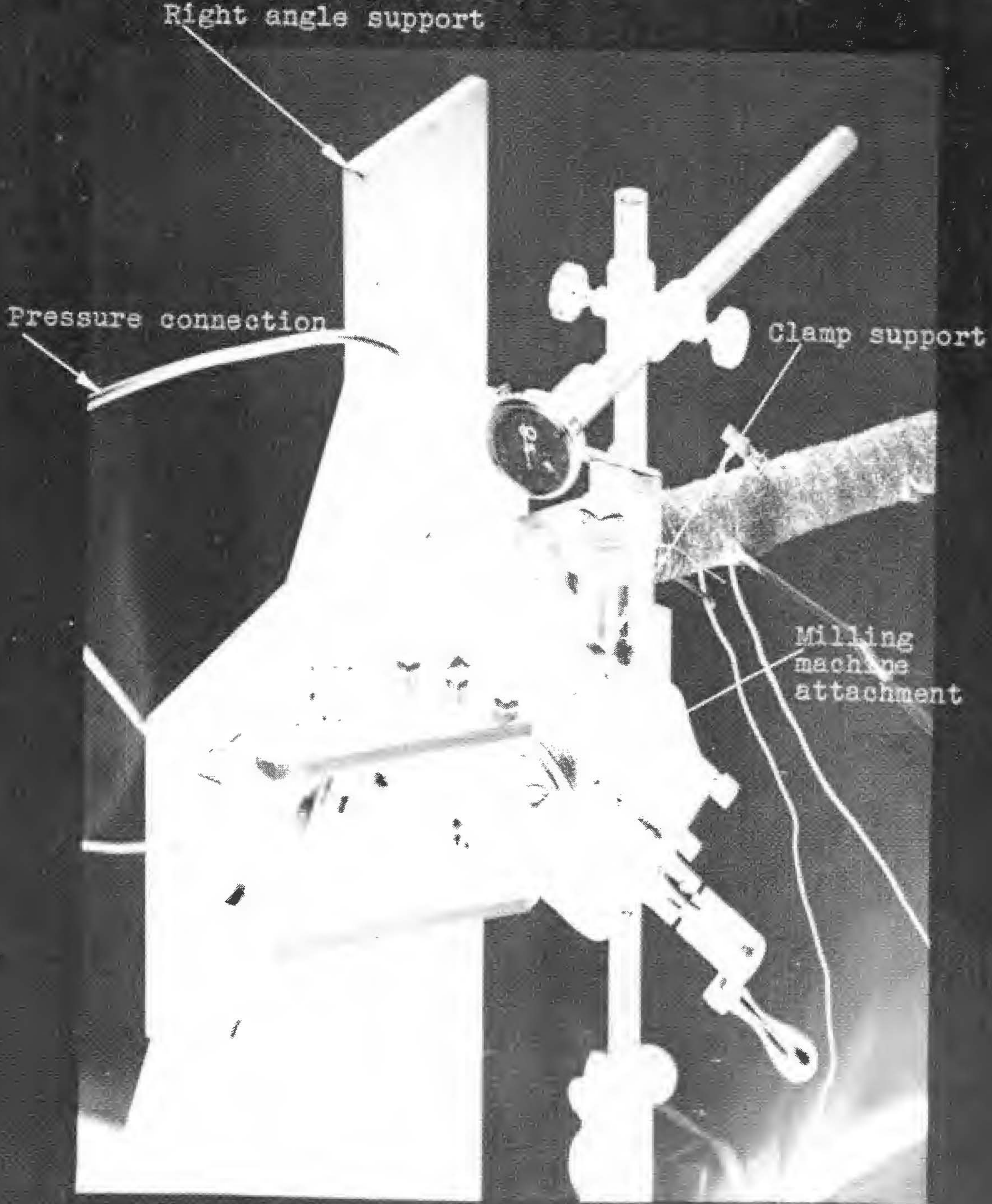

Right angle support
Pressure connection
Clamp support
Milling machine attachment

Plate 2

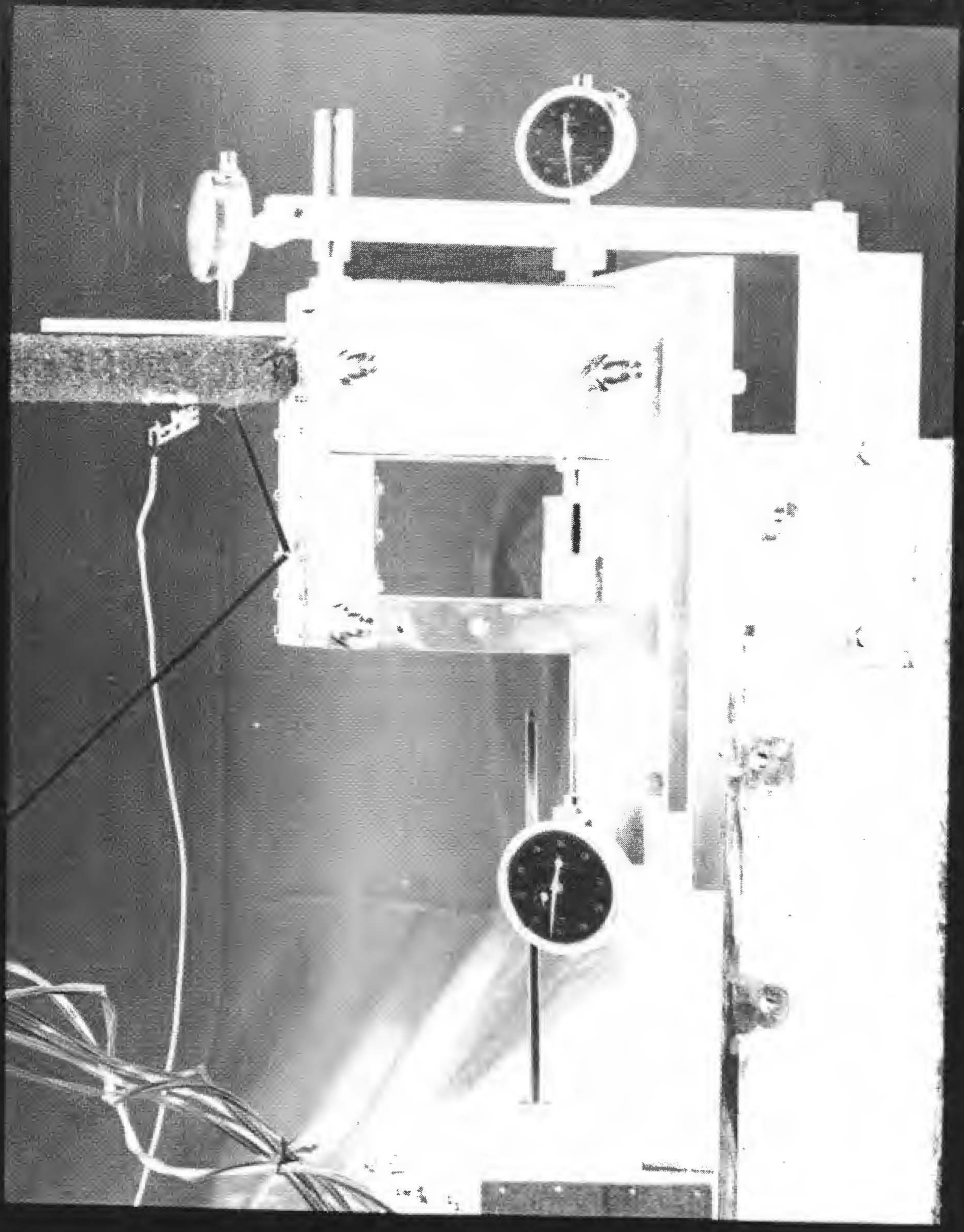
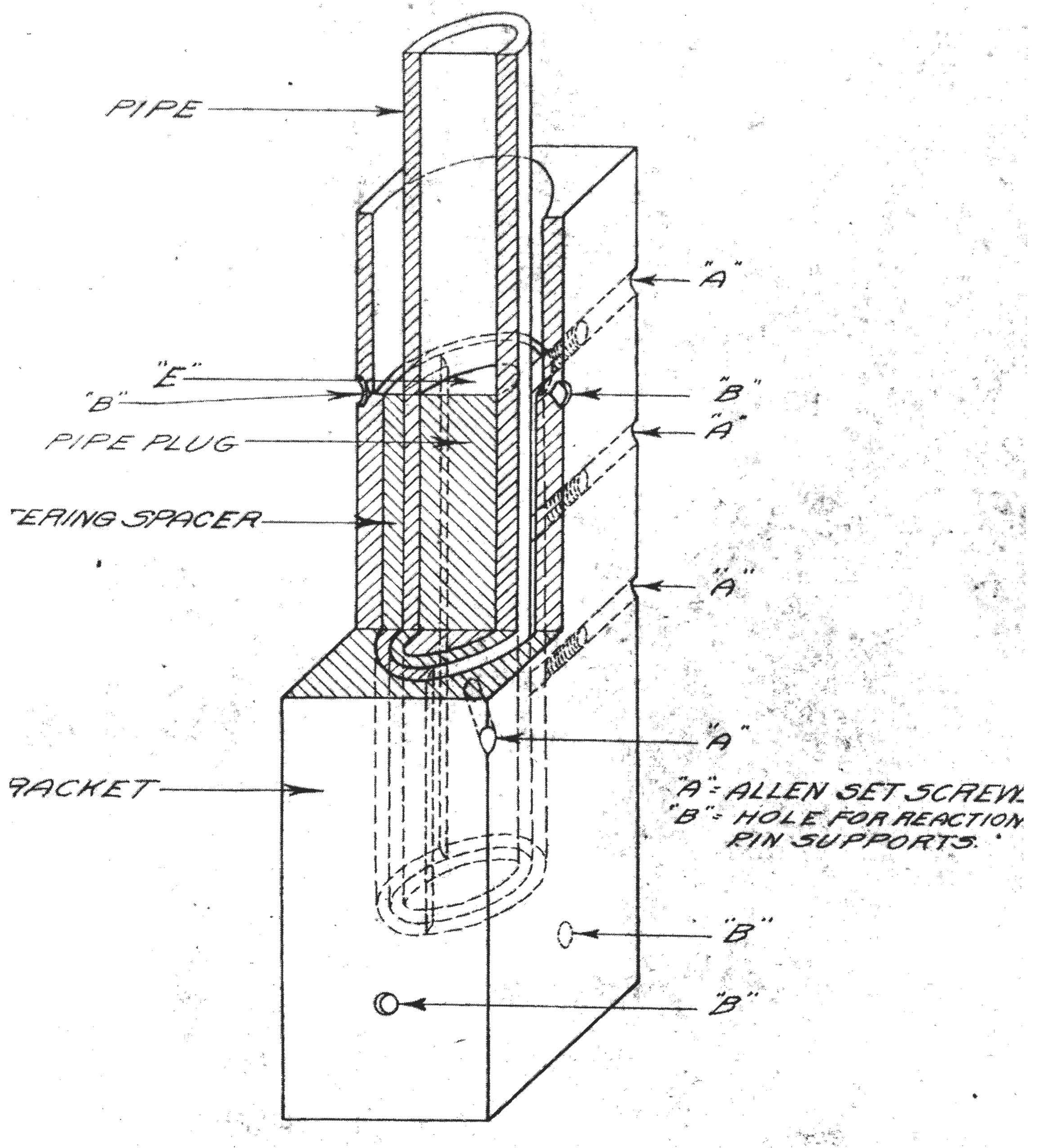
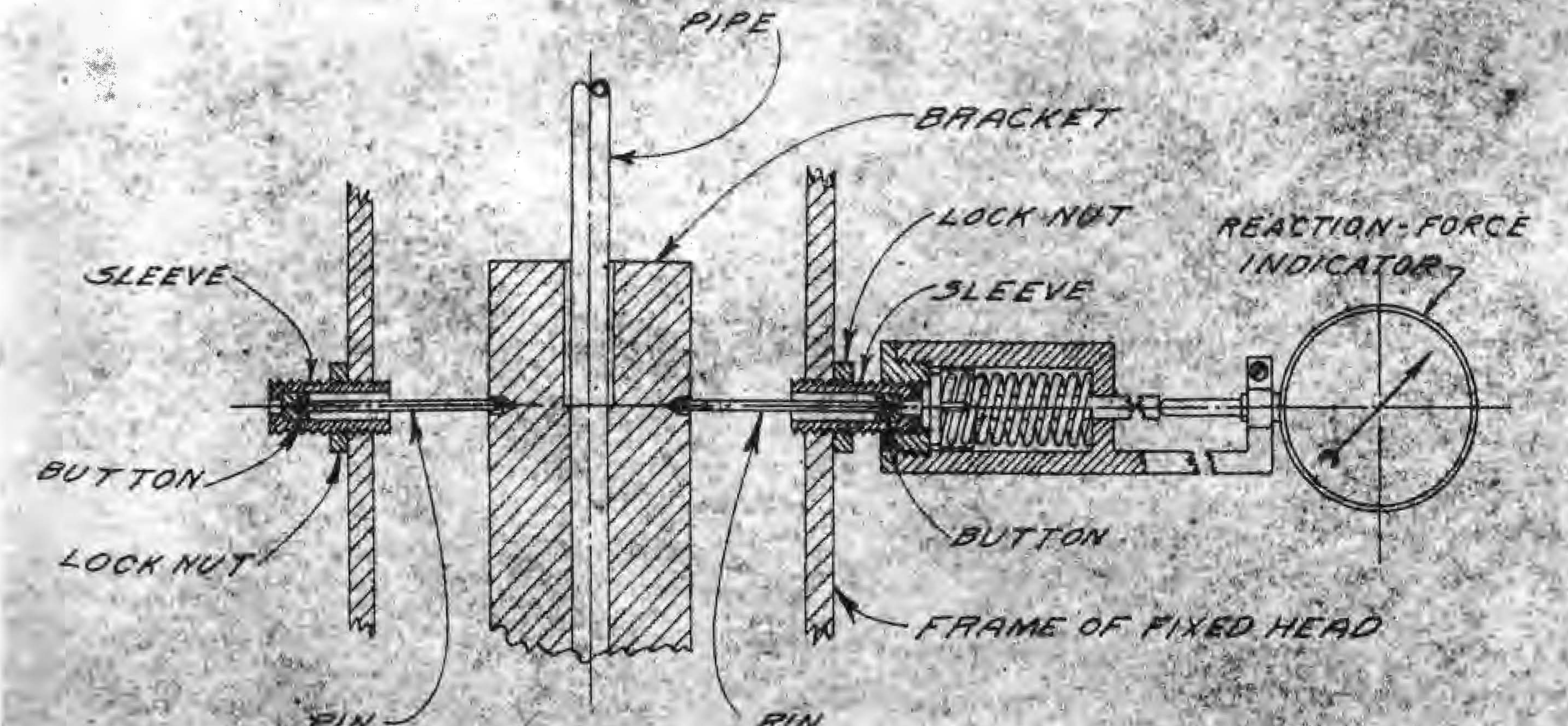


Plate 3



METHOD OF FIXING PIPE END IN BRACKET



METHOD OF SUPPORTING FIXED END OF MODEL PIPE AND REACTION INDICATOR

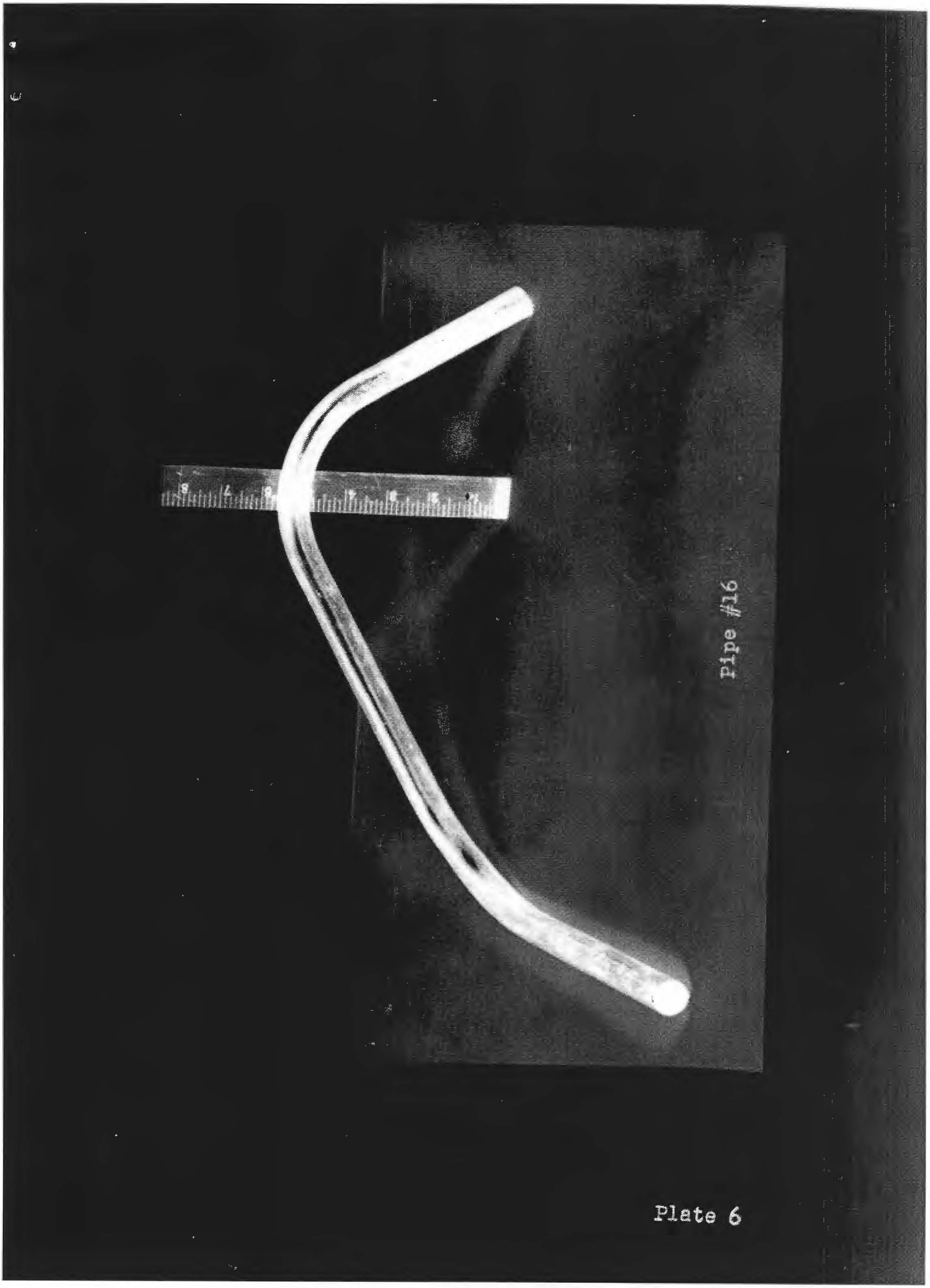


Plate 6

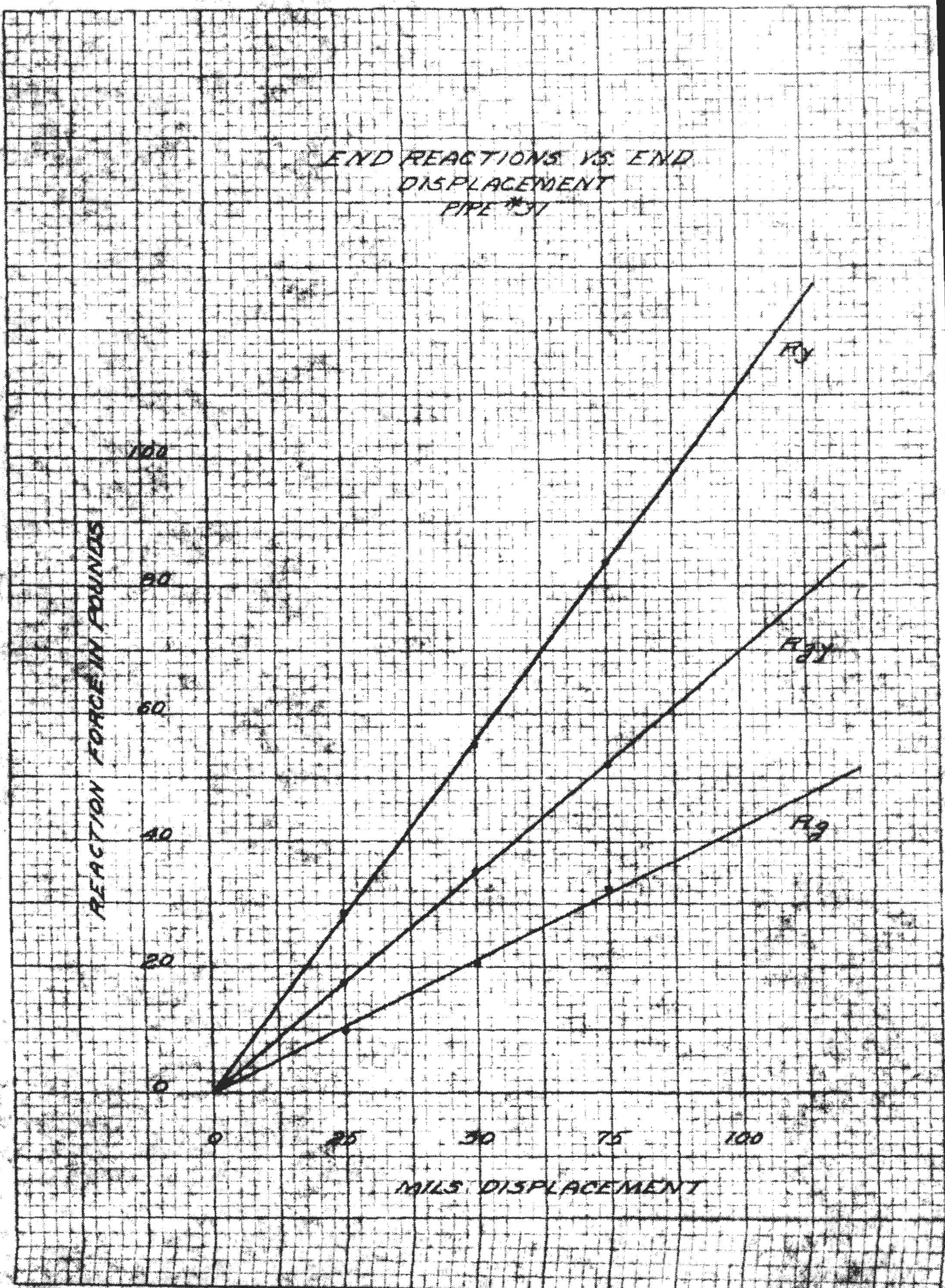
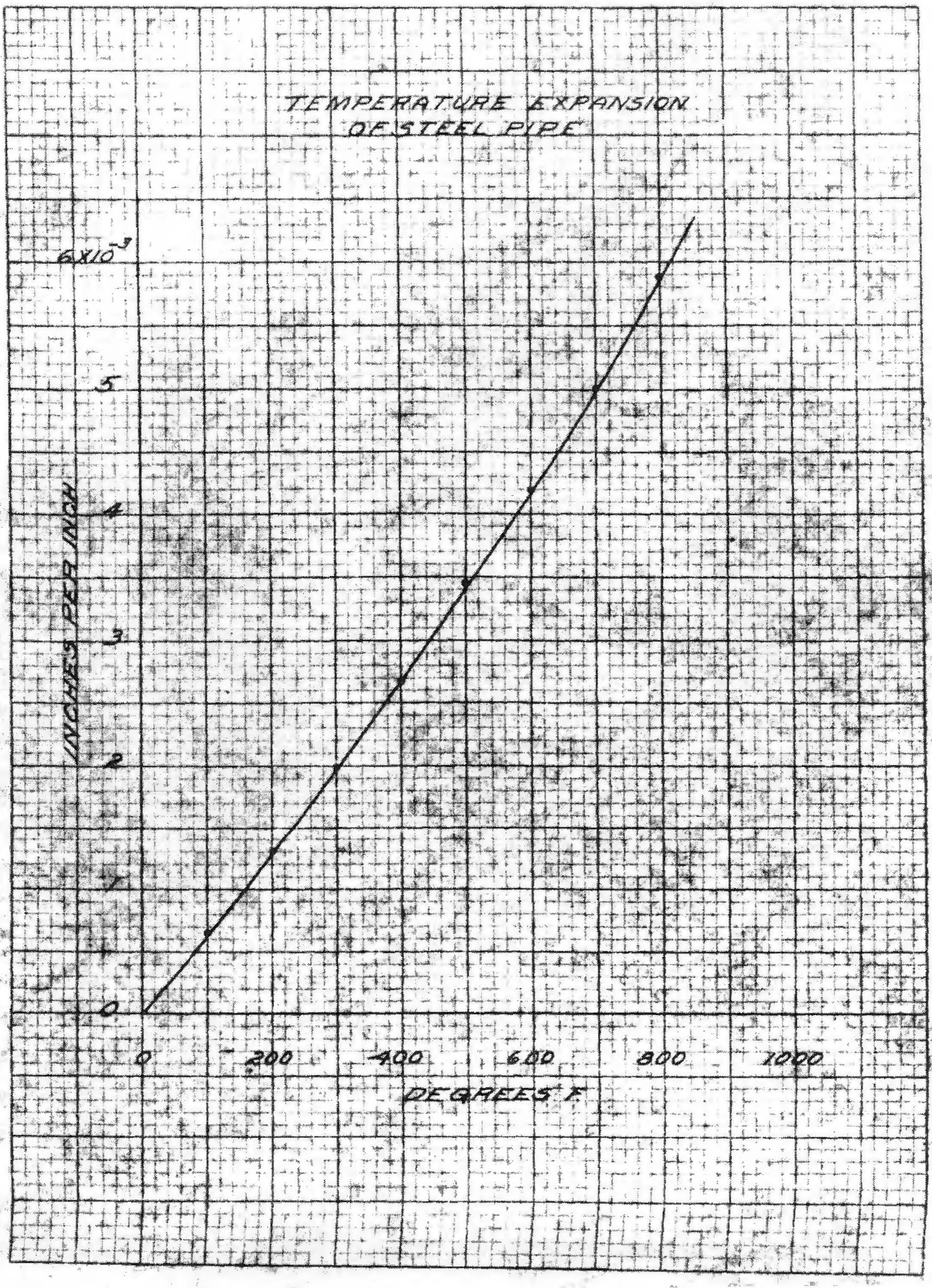

END REACTIONS VS. END
DISPLACEMENT
PIPE #51
REACTION FORCE IN POUNDS
100
80
60
40
20
0
0
25
50
75
100
MILS DISPLACEMENT
Ry
Rzy
Rz

PLATE 7


TEMPERATURE EXPANSION OF STEEL PIPE

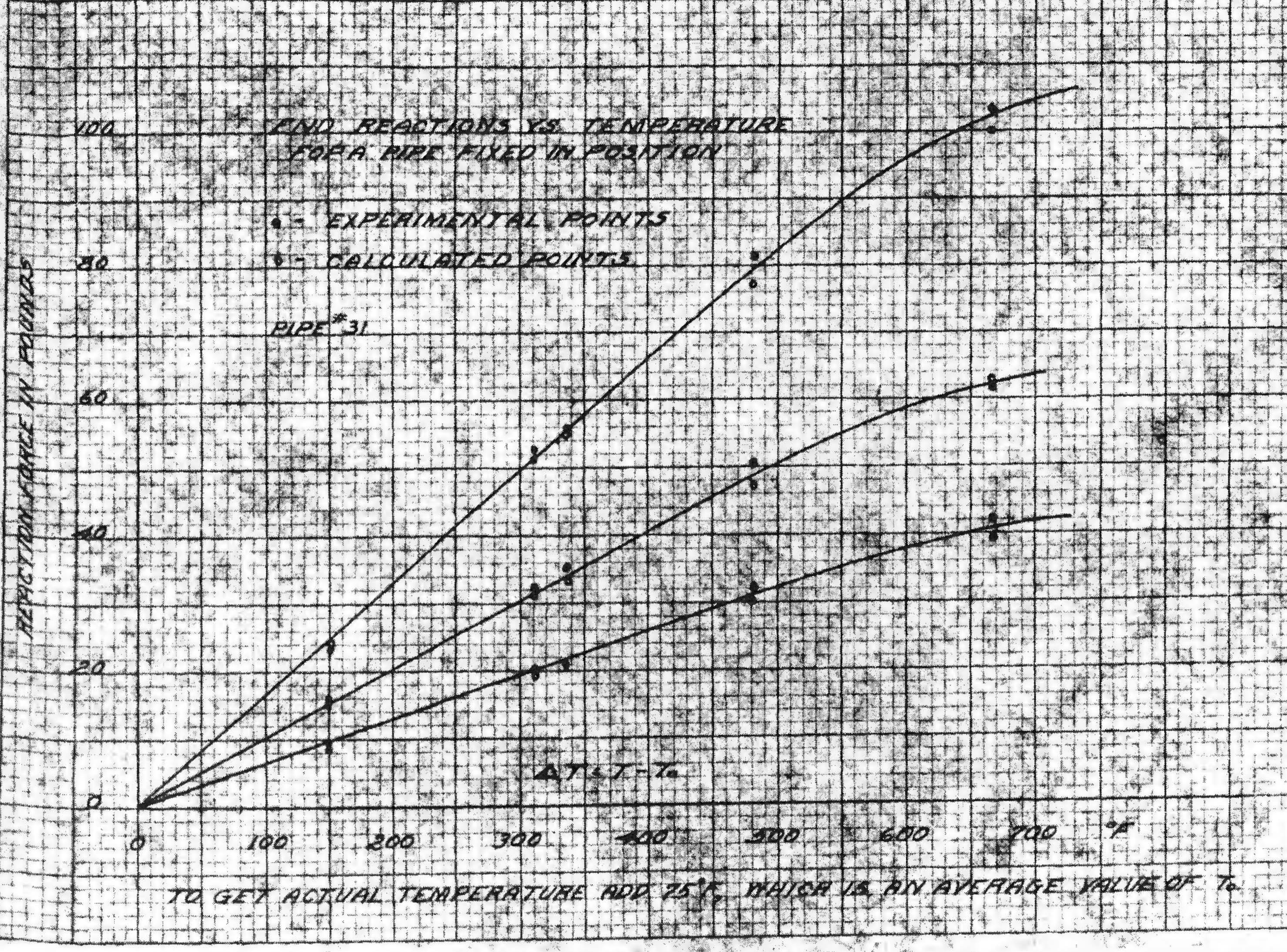
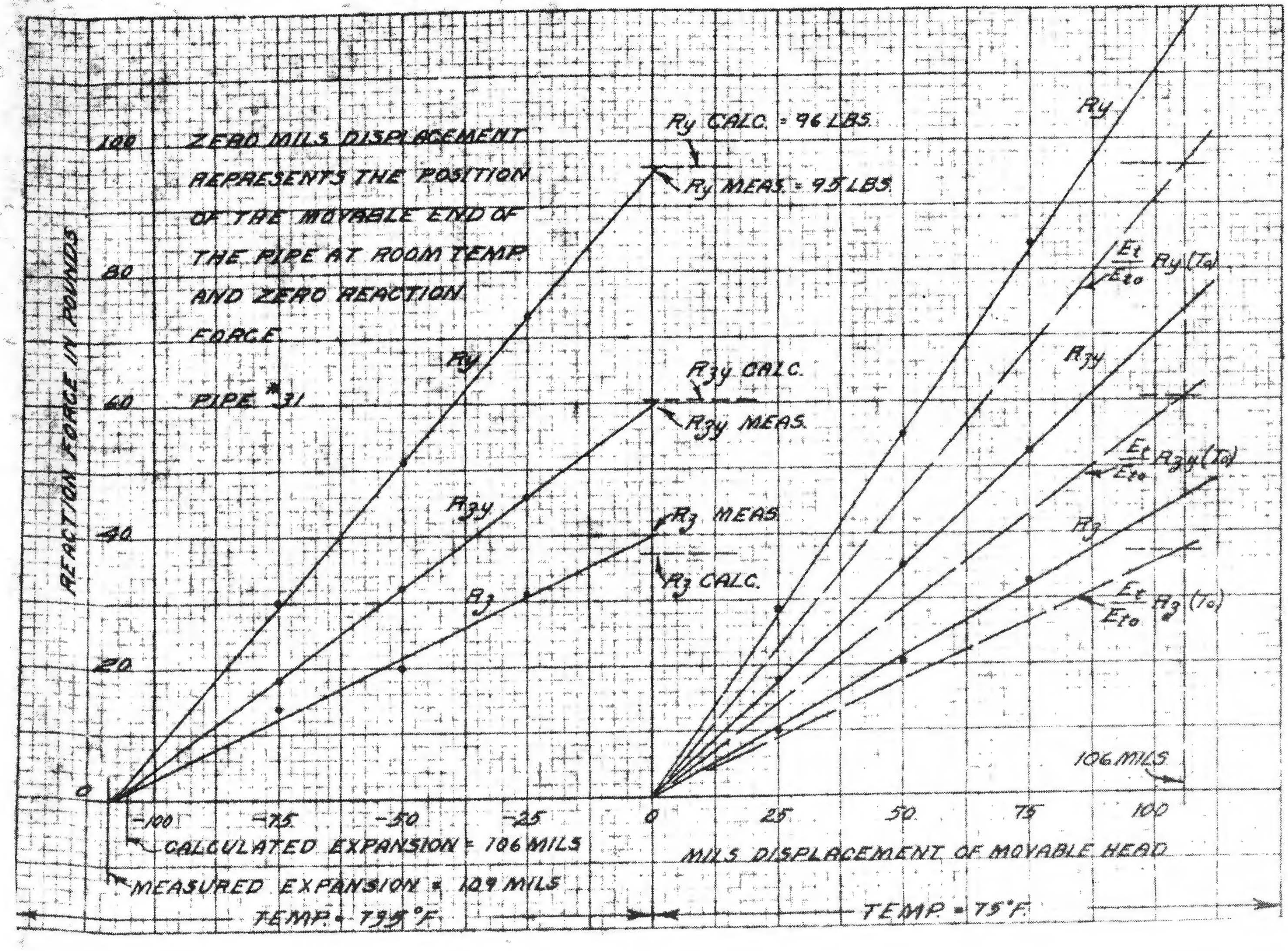

END REACTIONS VS. TEMPERATURE
FOR A PIPE FIXED IN POSITION
● - EXPERIMENTAL POINTS
○ - CALCULATED POINTS
PIPE # 31
REACTION FORCE IN POUNDS
100
80
60
40
20
0
0
100
200
300
400
500
600
700
°F
ΔT = T - To
TO GET ACTUAL TEMPERATURE ADD 75°F, WHICH IS AN AVERAGE VALUE OF To

PLATE 10


REACTION FORCE IN POUNDS
0
20
40
60
80
100
ZERO MILS DISPLACEMENT REPRESENTS THE POSITION OF THE MOVABLE END OF THE PIPE AT ROOM TEMP. AND ZERO REACTION FORCE
PIPE #31
Ry
Rzy
Rz
Ry CALC. = 96 LBS.
Ry MEAS. = 95 LBS.
Rzy CALC.
Rzy MEAS.
Rz MEAS.
Rz CALC.
Ry
Et/Eto Ry (To)
Rzy
Et/Eto Rzy (To)
Rz
Et/Eto Rz (To)
106 MILS
-100
-75
-50
-25
0
25
50
75
100
CALCULATED EXPANSION = 106 MILS
MEASURED EXPANSION = 109 MILS
TEMP. = 795°F
MILS DISPLACEMENT OF MOVABLE HEAD
TEMP. = 75°F

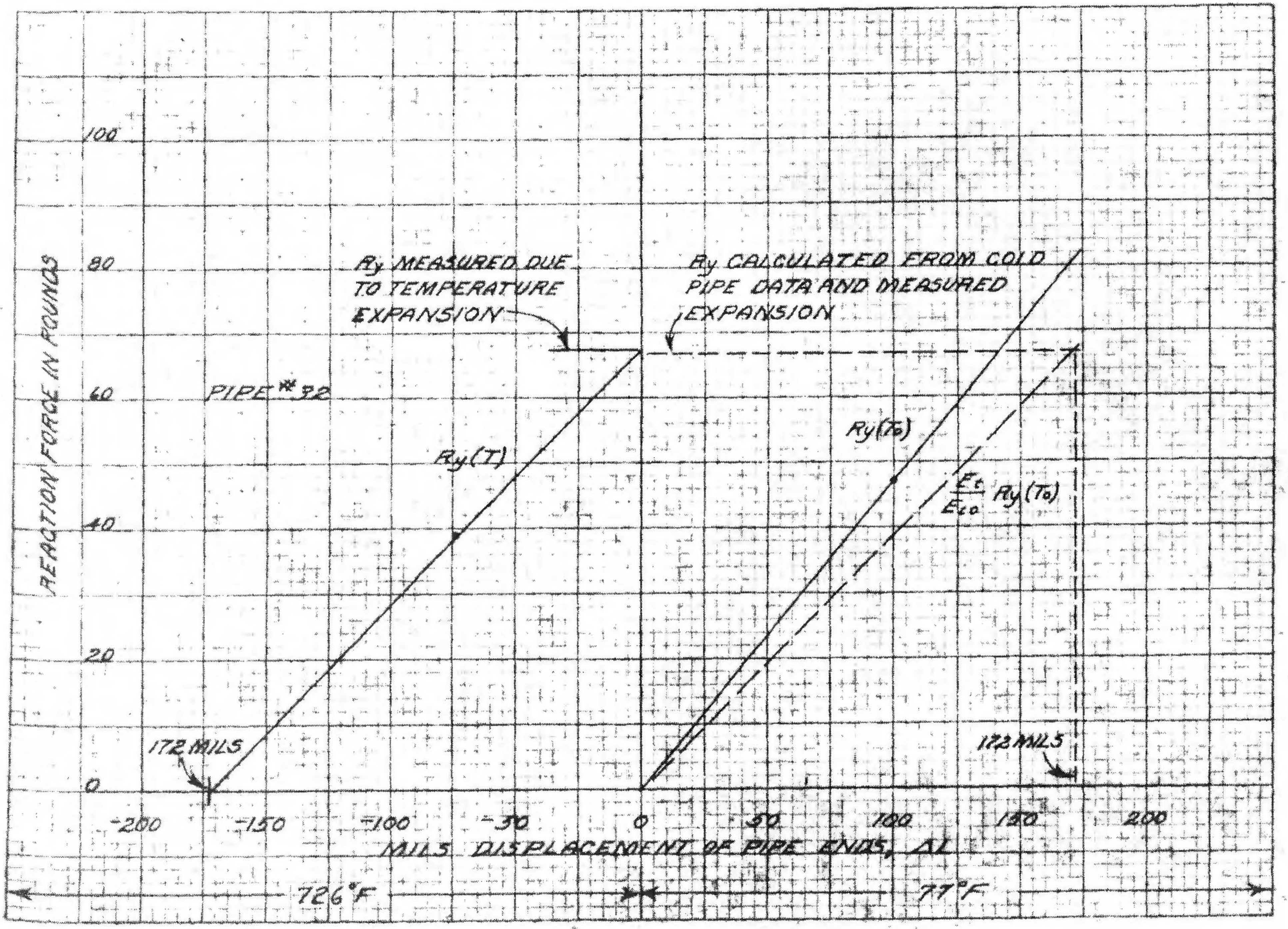

REACTION FORCE IN POUNDS
100
80
60
40
20
0
-200
-150
-100
-30
0
50
100
150
200
MILS DISPLACEMENT OF PIPE ENDS, ΔL
726°F
77°F
PIPE # 32
Ry MEASURED DUE TO TEMPERATURE EXPANSION
Ry CALCULATED FROM COLD PIPE DATA AND MEASURED EXPANSION
Ry(T)
Ry(To)
Et/Eto Ry(To)
172 MILS
172 MILS

PLATE 12