

Case-Study Non-Equilibrium Particle-Dynamics Modeling of Swarm- on-Swarm Engagements

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14. ABSTRACT This report describes simulations demonstrating the modeling of swarm engagements using particle-dynamics models and artificial-potential based control algorithms. Such control algorithms are based on near-neighbor communication and near-neighbor tracking of noncooperative agents. These laws cause the swarms to mimic the actions of particles whose dynamics are defined by potential functions. The general approach of using such models is similar to that of nonequilibrium molecular-dynamics modeling of mixing dissimilar particulate materials. Swarmengagement scenarios can be complex because of small time-periods of engagement, multiple types of blue-red force interactions, and the requirement of near-neighbor target tracking. With respect to particle-dynamics representation of swarm-engagements, fundamental quantities that can represent characteristics of particles interactions are the defined potential functions, which can be functions of particle-particle separation, the types of particles interacting, and type of the interaction. These potential functions can provide formal representation of both deterministic and non-deterministic particle-particle interaction scenarios. The complexity of swarm interactions suggests that such a modeling tool is necessary, and can be used in creating potential-theory based control algorithms for swarm-on-swarm interactions						
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Introduction

Unmanned Aerial Vehicles have increasingly widespread proliferation throughout the world. This trend is due to the increasing interest in UAV use, driven by new and improving drone capabilities, coupled with decreasing manufacturing costs. With this increase in drone activity across civilian and military markets [1-3], there is a growing desire to develop control algorithms for managing drone-swarm interactions with different environments, and Counter Unmanned Aerial Vehicle Systems (C-UASSs) to stop drones from being used for adversarial purposes. This leads to two somewhat interconnected difficulties for the systems and control engineer. One of these is the need to develop control schemes that are capable of handling dozens, hundreds, and possibly thousands of collaborating agents, which may have vastly different characteristics and capabilities. Following from that is the problem of evaluating these schemes, which will require simulations of hundreds or thousands of agents and their interactions as the engagements evolve.

The sheer size and potential complexity of drone-swarm engagements suggests adopting particle-dynamics models and near-neighbor tracking algorithms as foundation for modeling and simulation, as well as for control algorithms. Specifically, the use of particle dynamics models and near-neighbors tracking algorithms should permit more control and convenience for assignment of object-interaction properties, which may be unique to drone-swarm engagements. In addition, it is conceivable that deterministic simulations of drone-engagement scenarios may be of interest for investigating specific situations. That is to say, generation of scenario ensembles for calculation of drone-drone interaction probabilities that are based on restricted classes of a priori assumptions concerning engagements, and thus modeled explicitly. However, particle dynamics simulations can be written for both deterministic and non-deterministic swarm-engagement scenario generation.

The general approach of using particle-dynamics models and near-neighbors tracking algorithms [4-14] for modeling swarm engagements is similar to that of nonequilibrium molecular-dynamics modeling of mixing dissimilar fluids or particulate materials, e.g., see simulations of reference [15] concerning mixing. With respect to particle-dynamics representation of swarm-engagements, the fundamental quantities that will represent characteristics of drone interactions are interparticle potential functions. These potentials can be functions of drone-drone separation, the types of drones, drone allegiance (red, blue, or white) and the nature of the interaction, e.g., avoidance, fraternal cooperation, or mutual-annihilation interactions. These potential functions provide formal representation of both deterministic and non-deterministic drone-drone interaction scenarios.

In the case of drone swarms, these potentials will not be physical. Instead, they are the result of control logic that attempts to mimic the action of particles following the dynamics that would result from motion in a potential field (hence the term “artificial potential”). It is not feasible to design a standard full-information control scheme to direct the actions of each agent in a swarm of hundreds or thousands. The information is not available, and the communication burdens could not be managed, even were a coherent strategy devised. Instead, a potential function is defined based on local conditions and some overarching goal. The controller on the individual agent then estimates the actions needed to reduce the function, and the control commands are based on this estimate.

The concept of using potential functions for representing swarms of unmanned aerial vehicles (UAVs) was inspired by observations of insect swarms and schools of fish [16], and has been a topic of study in collaborative control for some time. The utility of the approach has been demonstrated both in extensive theoretical work and in physical demonstration [17], especially as

it relates to construction of control algorithms. The potential functions used for these studies were global or far-field model formulations, and were primarily for coordination of homogeneous swarms, that is, swarms of one-color, identical UAVs. Such approaches require idealized communication systems and assume knowledge of all UAVs by all agents in the swarm [18]. This is infeasible in now-realistic large swarms, and work using limited-range or near-neighbor information has been pursued. The use of limited information leads inevitably to locally computed gradients that are only approximations of those of the overall potential. Some limited proofs of stability have however been derived [19, 20].

A significant result of these investigations was that, in principle, potential-function modeling of UAV swarms, with respect to control algorithm implementations, enables control behaviors that respond quickly to changing events for swarms consisting of many members. The potential functions used for the proposed study, based on particle dynamics models and near-neighbors tracking, are interparticle or local-field model formulations, and will allow coordination of heterogeneous drone swarms in the presence of opposing swarms. A characteristic of these potential functions will be the inclusion of the non-cooperative swarms in the function definition.

In order to achieve efficient particle-dynamics modeling, where interactions among multiple types of particles are near-field, and reduced computational complexity is required for feasibility of models used for control algorithms, optimal near neighbor tracking should be essential. Born out of efforts concerning battle management and simulations associated with the SDI program, are near-neighbor tracking methods based on adaptive Lagrangian grids [4-14], which have been shown to be highly efficient. Recently these methods were applied to Air-Traffic control algorithms [14].

This report suggests development of non-equilibrium particle-dynamics models of layered systems, which are structured for fusing sensor and model data to predict drone-drone interaction probabilities, based on current and anticipated future UAS capabilities. Associated with development of particle-dynamics models should be specialized computer programs for anticipated controller algorithm adaptability. Accordingly, model formulations of drone-swarm engagements should be with respect to two complementary categories, designated as “general,” formulations that potentially can require large-scale computational resources, and “elegant,” formulations of reduced computational complexity, adaptable for control algorithms. Controllers built around these elegant formulations should be based on simulation results of large-scale simulations using the general formulations.

General particle-dynamics models of drone-swarm engagements can use available particle dynamics software, e.g., LAMMPS, having encoded various particle-particle correlation metrics, where drone-drone interactions are specified by potential functions. This type of implementation can provide for general proof-of-concepts and levels of feasibility concerning the general approach of “non-equilibrium particle-dynamics” modeling of drone-swarm engagements. Elegant modeling of drone-swarm engagements will entail the development of specialized computer programs, whose general structure is in terms of particle-particle potential functions and near-neighbor-tracking algorithms, that are formally robust, and thus conveniently adaptable for control-algorithm utilization.

Preliminary assessment of particle dynamics software suggests that the LAMMPS software [15] should provide potential flexibility for general modeling of blue-red swarm interactions that are not conveniently representable using specialized agent-engagement software. Simulations using LAMMPS can be used for generation of scenario ensembles based on deterministic and non-deterministic a priori assumptions concerning types of red-blue engagements.

Anticipated development of particle-dynamics models for controller algorithms should be based on data fusion of model predictions and sensor measurements, at each engagement scenario. Models used for controller algorithms should be formulated in terms of optimized drone-drone potential functions, near-neighbor tracking algorithms, and scenario-dependent probability density distributions, which are associated with particle-dynamics state spaces. Controllers derived using these artificial potential functions have been studied for collaborative action in swarms, sometimes in the presence of non-cooperative agents [21], and it has been shown that such controllers are stable. This analysis must however be extended to cases of actively hostile opposing (red) swarms, and the associated possibility of losing members of the blue swarm.

Additionally, simulation of blue-red engagements should not be expected to be based purely on global potential functions. As the number of agents grows, the ability to compute the values of the functions, or even maintain the information necessary, rapidly exceeds the capacity of the agents and the communications systems. Instead, the controllers must rely on potentials and gradients computed from near-neighbor agents, sensing of nearby opposing agents, and estimates of the states of the larger swarm of opposing agents. A further complication is that the controllers must allow for swarms to divide to deal with multiple opposing agents, with no overarching commander to assign targets.

In principle, particle-dynamics simulations can generate statistical data that characterize blue-red engagements, using as input drone characteristics, including weapons and communication systems. For example, a drone would have attributes such as aerodynamic data, weight, and thrust, needed to model its ability to move through the air in a simulation. Drones and associated systems can be placed in a simulation environment, where various effects can be added such as clutter models. In the case of drone-swarm warfare, each engagement scenario, whose outcome is analyzed statistically, can be scripted, thus providing red and blue force UASs with objectives and movements.

Overall, particle-dynamics models should be able to evaluate drone-swarm interactions using particle-particle correlation metrics used in particle-dynamics simulations. Specifically, these models should output the location of red force and blue force drones, the current time step, and the various sensor-perceived locations of the red force and blue force drones. This data can be used to generate probability distributions, at each time step, of where the sensors think the drones are located. The probability distributions can be evaluated at various time steps for construction of mappings into a particle-dynamics model space. In addition to statistical data, in principle, one can include in a drone-swarm-engagement data space, deterministic information based on existing systems and drone engagements. This can provide for transition of models to actual controller-algorithm utilization concerning specific field applications.

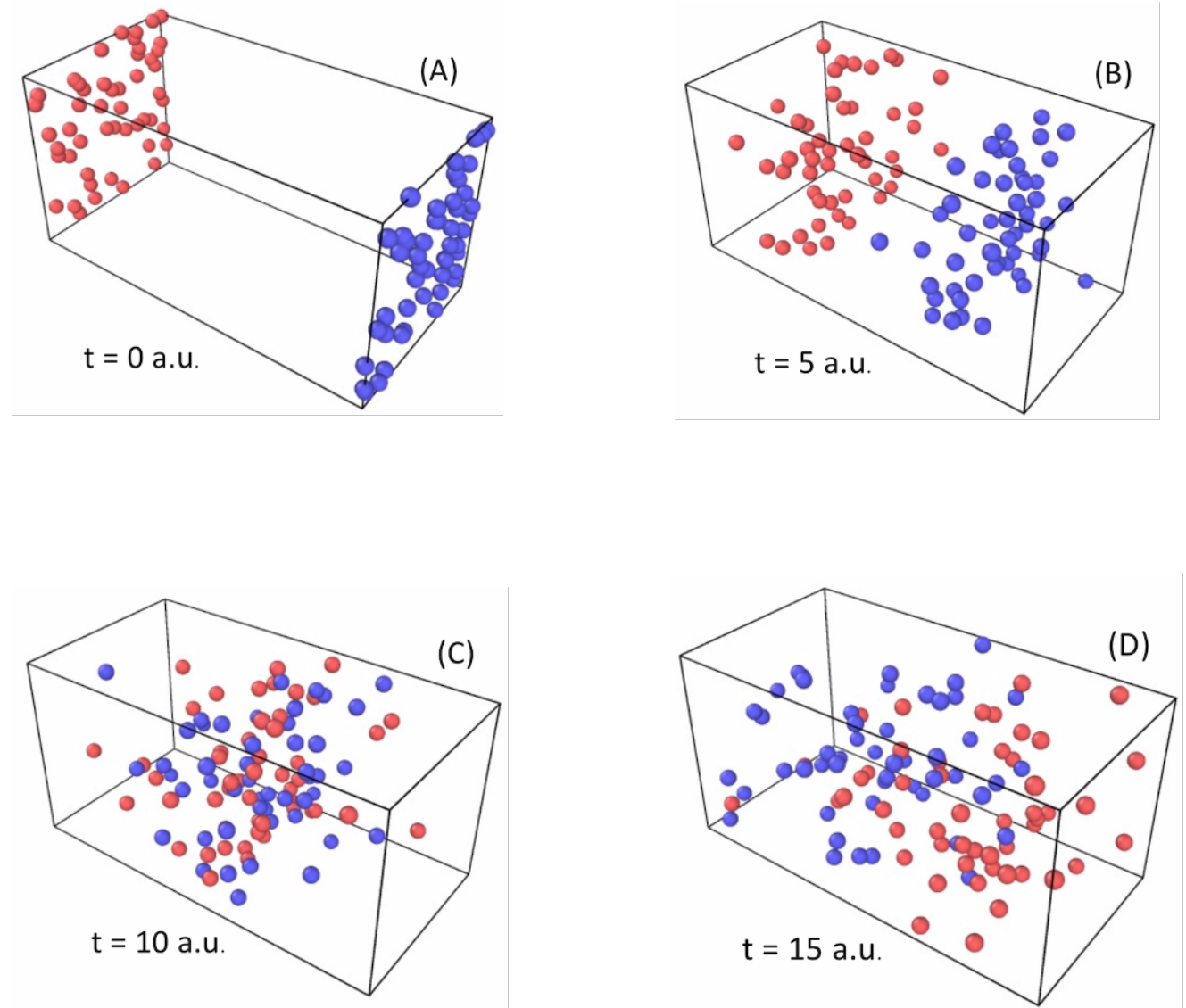
In what follows, prototype particle-dynamics simulations of blue-red engagements using the LAMMPS software [15] are described. The potential functions used for these simulations are described in reference [22] and briefly in the appendix.

Prototype Simulations

This section presents prototype particle-dynamics simulations of blue-red engagements. The design of these simulations, was not to demonstrate optimal blue-red engagement tactics, but rather general characteristics of using particle dynamics to simulate such engagements. Note that space and time scales for these simulations are arbitrary. For realistic engagement scenarios, space and time scales would be adjusted, i.e., scaled, consistently.

Our first prototype simulation is that of 50 blue particles engaging with 50 red particles, where blue seeks to “avoid” red. Accordingly, the assigned interaction potential is short-range repulsion, between all particles. The results of this simulation are shown in Figure 1.

The simulation shows the complex results of simple rules that are characteristic of these sorts of problems. The repulsive forces drive the two swarms to expand away from their initial positions at the ends of the control volume. As they reach and mix, the particles swirl about each other, avoiding all other particles.



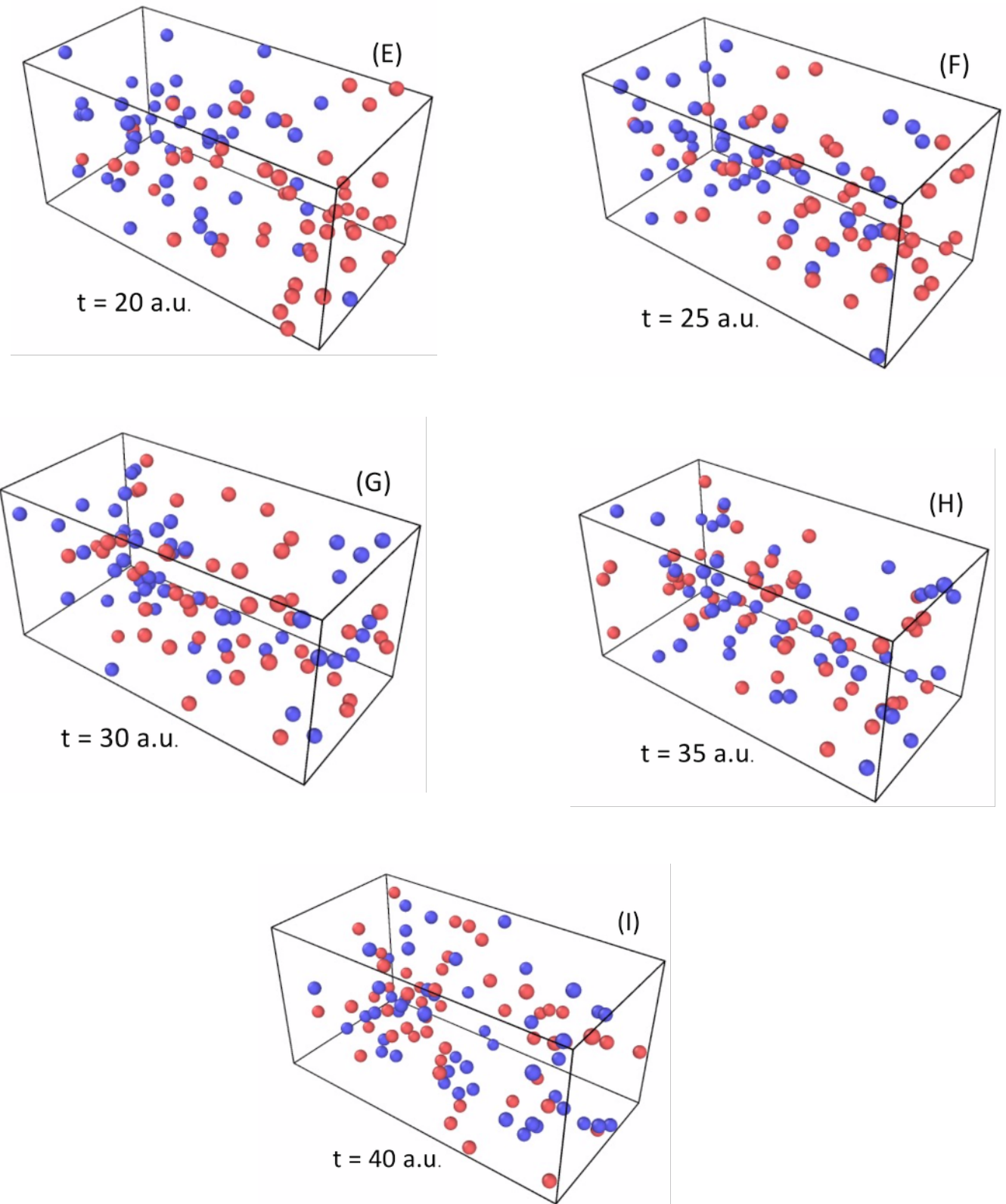
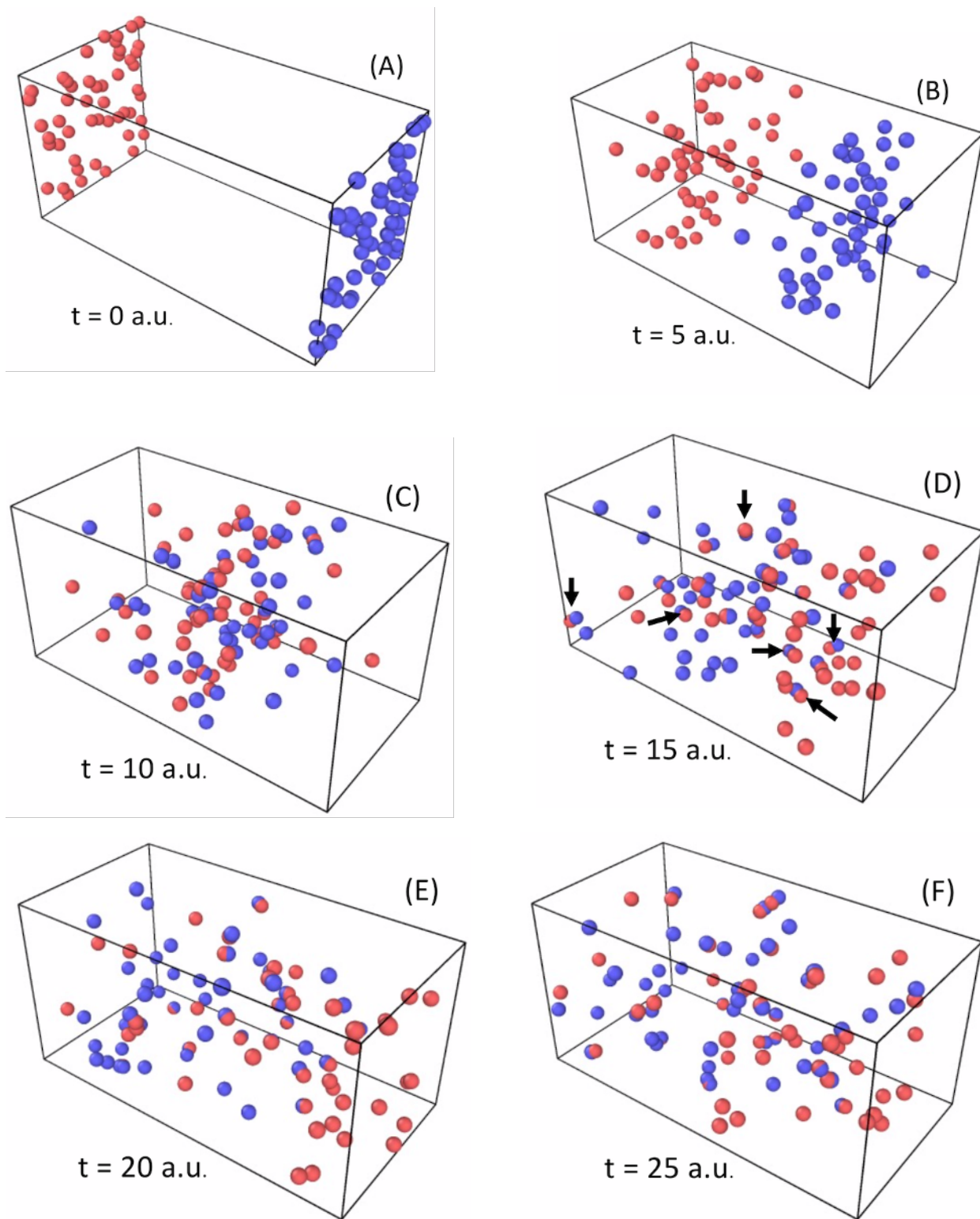


Figure 1. Swarm-engagement where blue avoids red.

Our second prototype simulation is that of 50 blue particles engaging with 50 red particles, where blue and red actively engage. Accordingly, the assigned interaction potential is short-range repulsion between particles of same color, but short-range attraction between blue and red. The results of this simulation are shown in Figure 2.



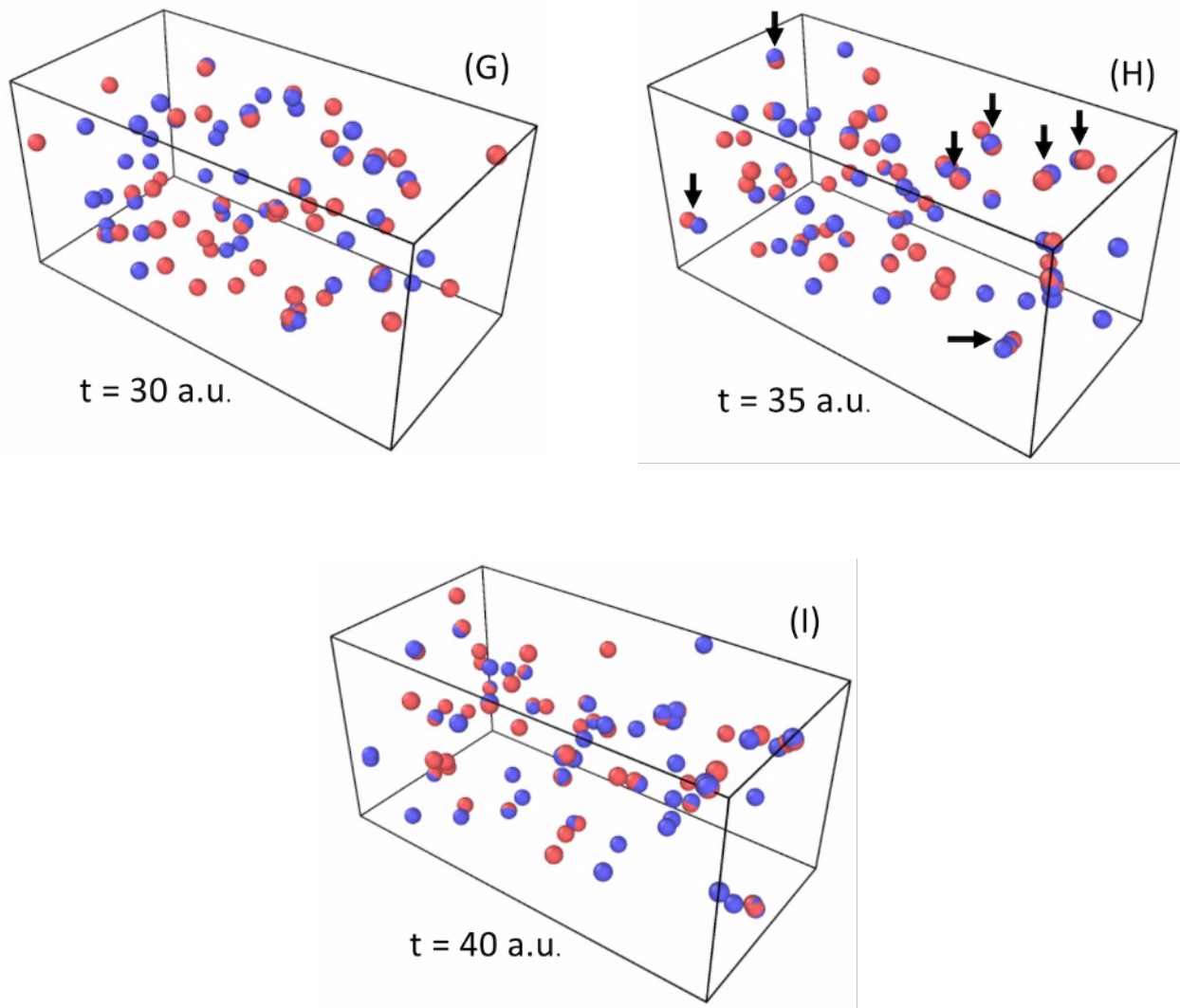
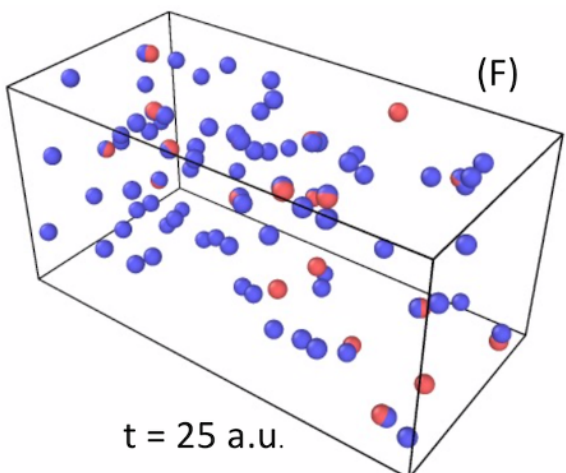
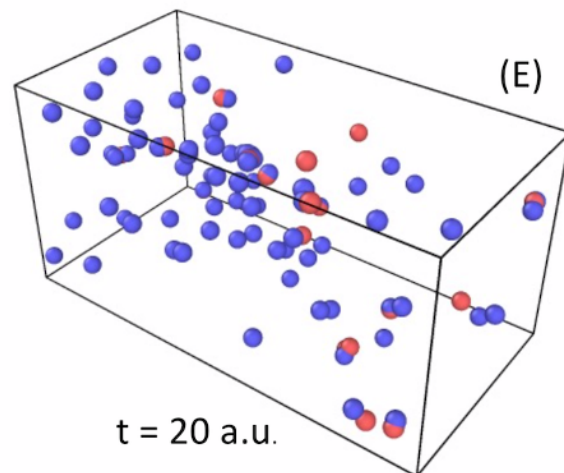
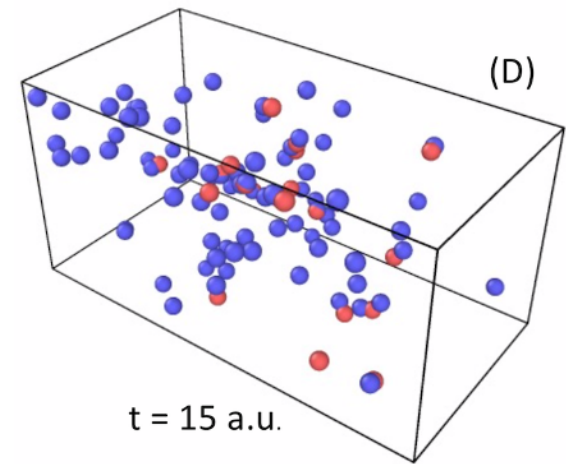
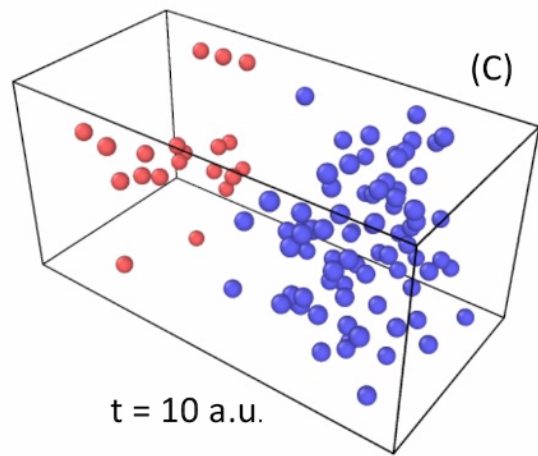
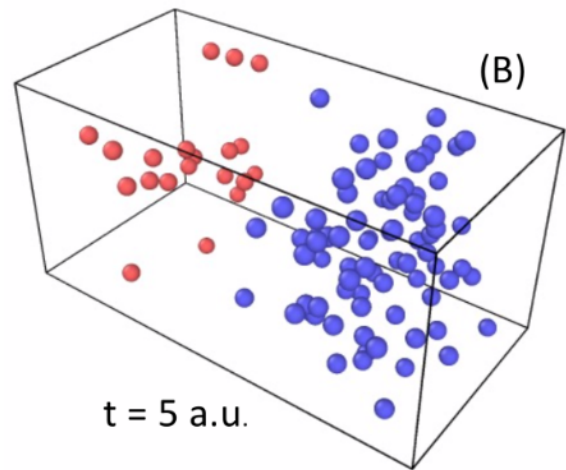
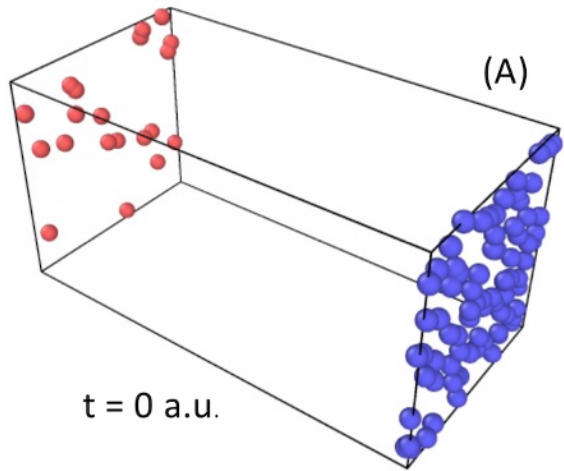


Figure 2. Swarm-engagement where 50 blue attract 50 red.

In this engagement, the two swarms again expand from their initial positions. The agents engage and couple, but the same-color repulsion prevents a simple evolution in which all particles drive directly to the nearest particle of the other team.

Our third prototype simulation is that of 80 blue particles engaging with 20 red particles, where blue seeks to overwhelm red. Again, the assigned interaction potential is short-range repulsion between particles of same color, but short-range attraction between blue and red. The results of this simulation are shown in Figure 3.

As this simulation evolves, some of the red particles are bound to one or more blue particles. The other blue particles are then repelled to seek other red particles.



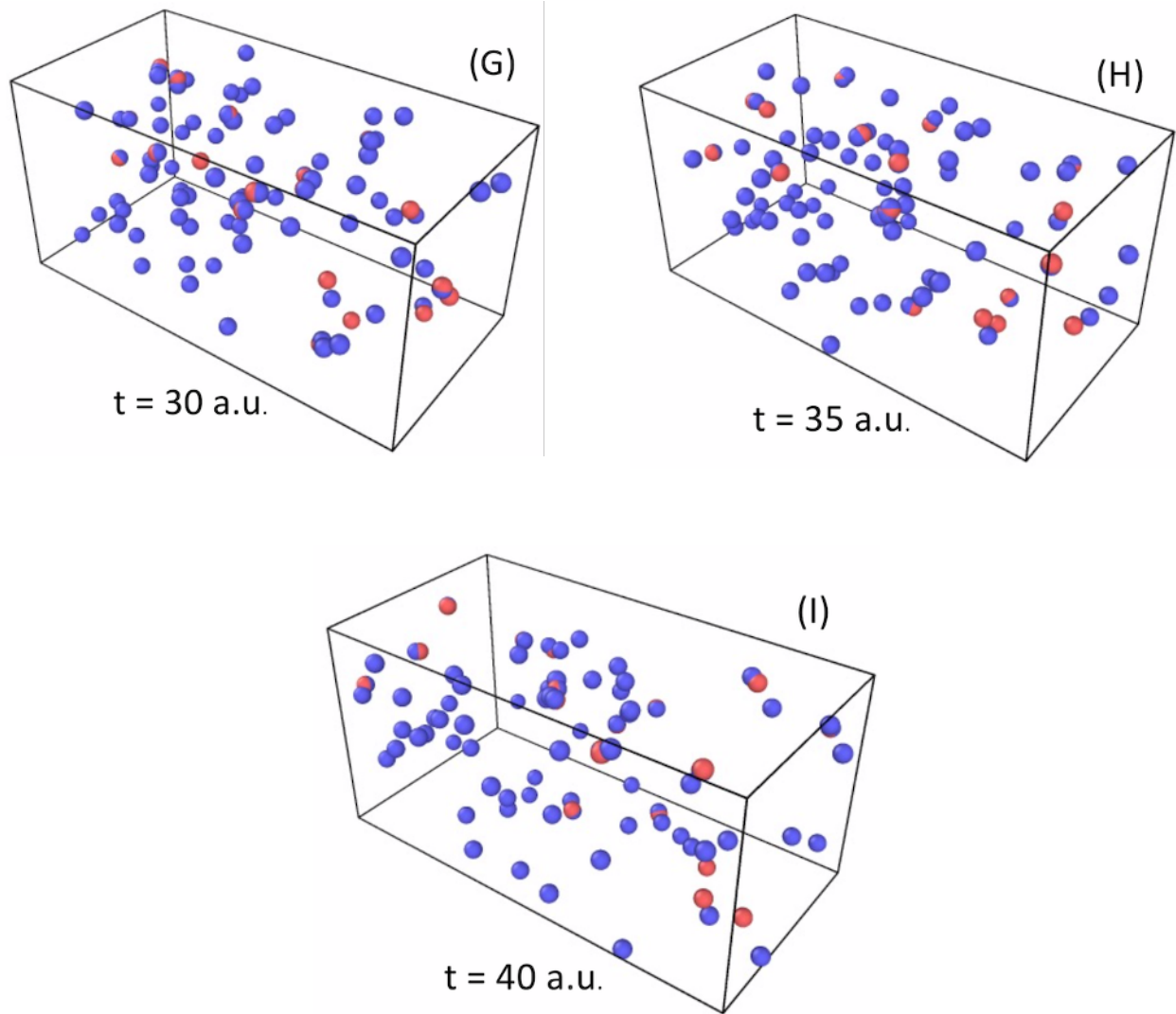


Figure 3. Swarm-engagement where 80 blue and 20 red actively engage.

Discussion

The prototype particle-dynamics simulations described here provide initial demonstration of using potential functions to represent different types of drone-swarm engagements. These simulations motivate examination and posing of various problems for application of this approach, specifically its adaptation for simulating many different types of swarm-engagement scenarios and for control-algorithm formalisms.

Consideration of swarm engagements involving thousands upon thousands of interacting model drones is well posed for simulations using existing particle-dynamics software, e.g., LAMMPS, which have evolved and are optimized for large-scale simulations. This existing capability should motivate simulation of swarm engagement scenarios involving large numbers of interacting drones.

The problem of simulating swarm engagements, associated with large numbers of interacting drones, by means of reduced-complexity particle dynamics models, in contrast to those structured for large-scale simulations, poses an open problem. This problem may be addressed, in principle,

using existing coarse-graining methods used in particle dynamics [22], which entail “lumping” of particle clusters into larger-particle representations. These types of representations should be necessary for adaptation to control-algorithm formalisms, which are inherently of reduced model complexity.

The potential function used in simulations described here is based on physical characteristics of atomic and molecular processes. There exists a large variety of different types of potential functions, having a wide range of different forms, which have been constructed for modeling different types of atomic scale processes. It is expected that these available potential functions may be applied for simulation of certain classes of drone-swarm engagement scenarios. The availability of potential functions, and their software implementation, poses the problem of investigating to what extent these functions can be adapted for simulation of drone-swarm engagements. For example, simulation of certain types of drone-swarm engagements, where there exist particle-particle annihilation interactions, may require potential functions effecting particle removal as a function of particle-particle interaction state.

It can be assumed, however, that the general characteristics of drone-swarm engagements are inherently different from those of atomistic interactions, associated with microscale physical processes. It is therefore anticipated that particle-dynamics modeling of drone-swarm engagements will require construction of new types of potential functions, which can more appropriately represent the potentially-large variety of drone-swarm interactions, and which can evolve with use of more advanced battle tactics. It should be noted that potentials used to direct behaviors and derive controllers need not represent physical quantities, only that their gradients produce behaviors that are within the capabilities of the physical agents.

Particle-dynamics modeling entails near-neighbors tracking of particles relative to each other. Existing near-neighbors tracking algorithms used for particle-dynamics simulations, as in the case of potential functions, have been developed according to characteristics of atomistic-scale physical processes. It is anticipated that particle-dynamics simulation of drone-swarm engagements will require near-neighbor tracking algorithms that are more appropriate.

Conclusion

This report describes non-equilibrium particle dynamics modeling and simulation of swarm engagement scenarios, and poses various problems motivating further investigation.

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Appendix: DPD Formalism

The swarm particles interact via the dissipative particle dynamics (DPD) potential as described by reference [22]. In this potential, a force between particle i and particle j is described by:

- 1) $\vec{F} = (F^C + F^D + F^R)\hat{r}_{ij} \quad r < r_c$
- 2) $F^C = Aw(r)$
- 3) $F^D = -\gamma w(r)^2(\hat{r}_{ij} \cdot \vec{v}_{ij})$
- 4) $F^R = \sigma w(r)\alpha\Delta t^{-\frac{1}{2}}$
- 5) $w(r) = 1 - \frac{r}{r_c}$

where

- 1) F^C is a conservative force
- 2) F^D is a dissipative force
- 3) F^R is a random force
- 4) \hat{r}_{ij} is a unit vector in the direction $\vec{r}_i - \vec{r}_j$
- 5) \vec{v}_{ij} is the vector difference in velocities of the two particles $\vec{v}_i - \vec{v}_j$
- 6) α is a Gaussian random number with zero mean and unit variance
- 7) Δt is the timestep size
- 8) $w(r)$ is a weighting factor that varies between 0 and 1
- 9) r_c is the cutoff radius
- 10) σ is set equal to $\sqrt{2k_B T \gamma}$, where k_B is the Boltzmann constant, T is the temperature parameter and
- 11) A and γ are arbitrary coefficients

The interaction between particles i and j , and like particles can be tuned by the coefficients A and γ . These coefficients determine how strongly the particles attract or repel one another.

The simulations were completed using the LAMMPS software with all quantities being unitless. Simulations were completed over a 50 x 50 x 100 block region with fixed boundaries (i.e., walls). The fixed boundaries means that particles do not interact across the boundary. Further the boundaries of the block region were set to reflect particles when they attempt to move through them. Reflection means that if a particle moves outside the wall on a timestep by a distance delta, then it is put back inside the wall by the same delta, and the sign of the corresponding component of its velocity is reversed.

For the prototype simulations described in this paper, the DPD formalism was applied to simulate the engagements. The coefficients A and γ determines the forces between the particles. When the coefficient A is less than zero, the forces tend to be attractive while for A greater than zero the forces tend to be repulsive. The γ coefficient can be used to add additional tuning to the forces between particles in an effort to modify their behavior. In our prototype simulations for red and blue particles, the DPD forces between particles are defined for like particles (e.g., blue and blue) and unlike particles (i.e., red and blue), with the DPD potentials having a cutoff of 5 distance units. For the simulation where blue seeks to “avoid” red we set the coefficients A and γ to 50 and 10 respectively. For the simulation where blue seeks to “attack” red we set the coefficients A and γ to -50 and 10 respectively. In both cases, for like particles, A was set to 40 and γ to 0. Both blue

and red particles are modeled as spheres with mass set to 4 mass units and diameter set to 4 distance units.