NAVAL POSTGRADUATE SCHOOL

MONTEREY, CALIFORNIA

THESIS

EXPANDING OPTIMIZATION OF ENERGY EFFICIENT UAV ROUTING IN SUPPORT OF MARINE CORPS EXPEDITIONARY ADVANCED BASE OPERATIONS WITH MULTIPLE SUPPLY DEPOTS

by

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September 2021

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REPORT DOCUMENTATION PAGE			*Form Approved OMB No. 0704-0188*
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instruction, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188) Washington, DC, 20503.			
1. AGENCY USE ONLY ***(Leave blank)***	**2. REPORT DATE** September 2021	**3. REPORT TYPE AND DATES COVERED** Master's thesis	
4. TITLE AND SUBTITLE EXPANDING OPTIMIZATION OF ENERGY EFFICIENT UAV ROUTING IN SUPPORT OF MARINE CORPS EXPEDITIONARY ADVANCED BASE OPERATIONS WITH MULTIPLE SUPPLY DEPOTS			**5. FUNDING NUMBERS**
6. AUTHOR(S) Jeffrey R. Haller			
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey, CA 93943-5000			**8. PERFORMING ORGANIZATION REPORT NUMBER**
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) N/A			**10. SPONSORING / MONITORING AGENCY REPORT NUMBER**
11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.			
12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release. Distribution is unlimited.			**12b. DISTRIBUTION CODE** A

13. ABSTRACT (maximum 200 words)

United States Marine Corps expeditionary units require last-mile resupply of essential items while maintaining a small operational footprint. The use of automated unmanned aerial vehicles (UAVs) allows for units to be resupplied upon request, while keeping a low footprint in the operational area. Jatho in 2020 described an optimization model that prescribes optimal UAV routes and flight trajectories while accounting for wind conditions and known obstacles between requested resupply units. Jatho's model allows only for UAVs to depart and return to one supply depot. This limits a UAV's ability to recharge and resupply, thereby limiting the number of units a UAV can visit. We expand the UAV routing model seen in Jatho to include multiple supply depots, eliminating the constraint for a supply UAV to use only one depot. Additional supply depots give UAVs the ability to recharge and resupply to fulfill further requests. We also describe a new depot selection model, allowing planners to choose the set of depots that can be expected to perform best in a given set of operational scenarios.

14. SUBJECT TERMS optimal routing, UAV, UAV routing, energy efficiency, logistics			**15. NUMBER OF PAGES** 67
			16. PRICE CODE
17. SECURITY CLASSIFICATION OF REPORT Unclassified	**18. SECURITY CLASSIFICATION OF THIS PAGE** Unclassified	**19. SECURITY CLASSIFICATION OF ABSTRACT** Unclassified	**20. LIMITATION OF ABSTRACT** UU

NSN 7540-01-280-5500 Standard Form 298 (Rev. 2-89) Prescribed by ANSI Std. 239-18

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
September 2021

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ABSTRACT

United States Marine Corps expeditionary units require last-mile resupply of essential items while maintaining a small operational footprint. The use of automated unmanned aerial vehicles (UAVs) allows for units to be resupplied upon request, while keeping a low footprint in the operational area. Jatho in 2020 described an optimization model that prescribes optimal UAV routes and flight trajectories while accounting for wind conditions and known obstacles between requested resupply units. Jatho's model allows only for UAVs to depart and return to one supply depot. This limits a UAV's ability to recharge and resupply, thereby limiting the number of units a UAV can visit. We expand the UAV routing model seen in Jatho to include multiple supply depots, eliminating the constraint for a supply UAV to use only one depot. Additional supply depots give UAVs the ability to recharge and resupply to fulfill further requests. We also describe a new depot selection model, allowing planners to choose the set of depots that can be expected to perform best in a given set of operational scenarios.

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List of Acronyms and Abbreviations

BVP	boundary value problem
C2	command and control
CBC	Computational Infrastructure for Operations Research branch and cut
COAMPS	Coupled Ocean/Atmosphere Mesoscale Prediction System
DoD	Department of Defense
DVRP	dynamic vehicle routing problem
EABO	expeditionary advanced base operations
GMTSP	generalized multiple depot, multiple travelling salesman problem
ILP	integer linear program
MDMTSP	multiple depot multiple traveling salesman problem
MDMTSFLP	multiple depot, multiple traveling salesman facility-location problem
MDVRP	multiple depot vehicle routing problem
MILP	mixed-integer linear program
mTSP	multiple-traveling salesman problem
PMP	Pontryagin's maximum principle
PSO	particle swarm optimization
PTSFLP	probabilistic traveling salesman facility location problem
SDVRP	split delivery vehicle routing problem
TSFLP	traveling salesman facility location problem
TSP	traveling salesman problem
UAV	unmanned aerial vehicle
USMC	United States Marine Corps
VRP	vehicle routing problem
Wh	watt-hours

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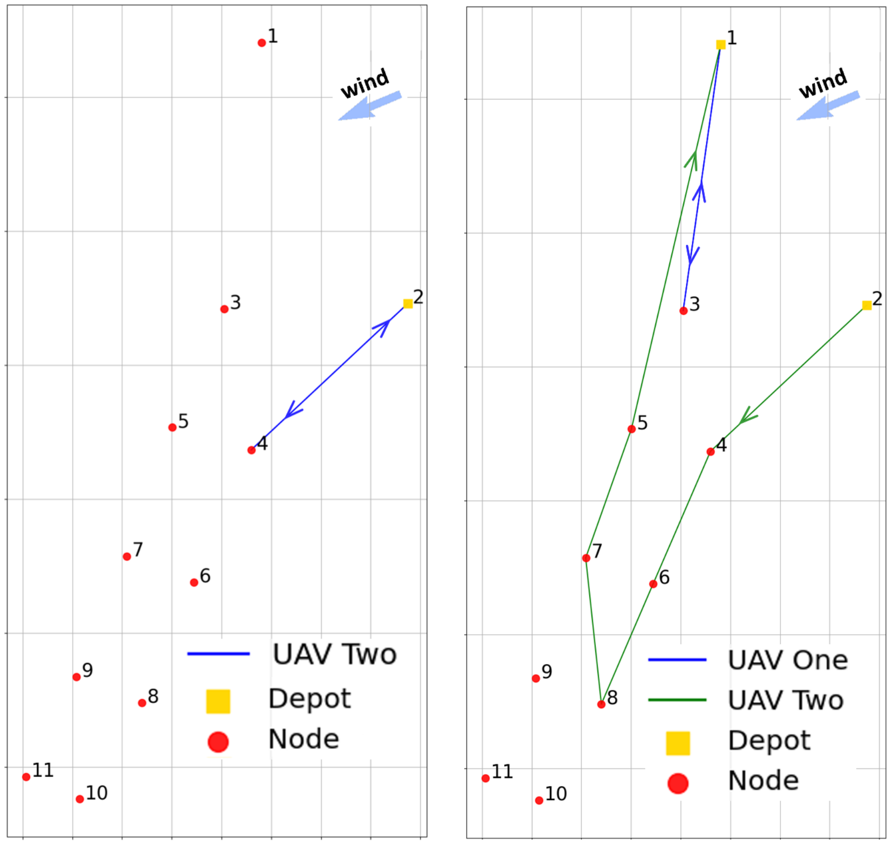
Executive Summary

The United States Marine Corps (USMC) expeditionary advanced base operations (EABO) concept relies on a forward force capable of dynamic adaptation to the operational environment. Now more than ever, our adversary is capitalizing on emerging technologies, capable of long-range fires targeting our critical centers of gravity. We must produce a force able to fight in a dynamic operational environment through reshaping legacy base infrastructure into smaller, more mobile units while maintaining a low operational footprint.

This thesis answers the EABO logistics challenge by developing an energy-aware optimal unmanned aerial vehicle (UAV) routing system providing Marines further sustainment in the operational environment. We expand on previous work from Jatho (2020), which presents single depot UAV routing model, by incorporating the use of multiple supply depots.

We show the advantage of using multiple depots in different network environments while considering actual wind conditions. Figure 1 illustrates the benefit of allowing a UAV to utilize additional depots. Figure 1b highlights a UAV's ability to service four extra demand nodes when we allow it to depart from one depot and return to another, vice restricting the UAV to using a single depot as shown in Figure 1a.



(a) Utilizing Single Depot: 2 (b) Utilizing Multiple Depots: 1 and 2

Figure 1. UAV Routing Using a Single Depot vs. Multiple Depots

Our second contribution is a depot selection model facilitating a comprehensive, and easily employable, methodology for EABO logisticians during operational planning. Our model uses data from probabilistic operational scenarios through a two-stage optimization problem to select the best depot locations among a set of potential locations, and provides respective UAV routing for each scenario using the selected depots. This model enhances the strategic planning process and lays the foundation for future research and development in last-mile EABO logistics.

Acknowledgments

Thank you to both of my advisors, Dr. Emily Craparo and Dr. Vladimir Dobrokhodov, as well as my second reader, CDR Nick Ulmer, for all of your overwhelming support, patience, and motivation throughout my thesis process. I would like to thank my friends and family who have been with me, cheering me on every step of the way. Finally, I would like to thank my best friend and number one fan, Jamie. Thank you for always being by my side and believing in me.

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CHAPTER 1:
Introduction

> For EABO to achieve its potential as a future operational concept that provides new and better options to future decision makers, there must be significant force development guided by a cogent operational vision for how EABO can be employed in support of future naval strategy. How we answer the question, "EABO to do what?" is critical to creating the compelling vision necessary to guide naval force development—both Navy and Marine Corps—towards a common operational purpose with distinct, but integrated, tactical capabilities. (United States Marine Corps [USMC] 2018, p. 69)

The United States Marine Corps (USMC) expeditionary advanced base operations (EABO) concept relies on a forward force capable of dynamic adaptation to the operational environment. Now more than ever, our adversary is capitalizing on emerging technologies, capable of long-range fires targeting our critical centers of gravity. We must produce a force able to fight in a dynamic operational environment through reshaping legacy base infrastructure into smaller, more mobile units while maintaining a low operational footprint.

Legacy supply logistics utilize large signature bases, which are easily targeted and exploited by the enemy: "A force dependent upon fixed forward-basing could not long persist or effectively partner if its most valuable military capabilities were based on readily targetable and highly vulnerable locations" (USMC 2018, p. 9). The EABO concept combines forward Marine Corps units as the inside force leveraging low-signature abilities, with command and control (C2) capabilities and long-term sustainment of the outside force. It is imperative to develop and expand upon emerging technologies to resupply and sustain widely dispersed Marine forces fighting in a contested environment.

> Logistical distribution centers and intermodal transportation hubs are potential single point of failure facilities that will operate best outside of the range of enemy long-range fires. Moving commodities from distribution hubs, be they afloat or ashore, across contested seas directly to [expeditionary advanced bases]

> supporting distributed naval forces, is the crux of the new logistics challenge. (USMC 2018, p. 62)

The new logistics challenge is answered through the utilization of unmanned aerial vehicle (UAV) technology. In general, a fundamental limitation of UAVs is on-board energy capacity, restricting time in flight and distance traveled. To address this limitation, we adopt an approach solving the energy-optimal guidance task that accounts for the limited energy onboard a UAV and enables "harvesting" energy from the environment.

An energy-aware optimal UAV routing system supports EABO strategic concepts by allowing Marines further sustainment to operate as the inside force, while maintaining a low operational signature. Last-mile logistics within the EABO construct would require Marines to be resupplied with mission-essential items (such as specialty medical supplies, lightweight ammunition, or technical support elements) in a time-efficient manner. UAVs offer a relatively inexpensive, easily deployable option to fulfill prioritized demand requests through energy-aware routing.

This thesis gives two solutions supporting EABO. Previous work by Jatho (2020) presents a vehicle routing model that utilizes weather-based flight trajectories to provide energy-optimal UAV routing from a single supply depot to resupply Marines in the operational environment. We first expand on previous work by allowing UAVs to operate from multiple supply depots. Combining the use of additional supply depots with energy-aware routing algorithms increases energy efficiency, and most importantly, satisfies more demand requests of USMC personnel in the field of operations.

The second element of this thesis provides a strategic planning tool for logisticians to best develop asset infrastructure within the operational environment. We present a model to select the best locations of supply depots, based on future probabilistic scenarios, allowing a logistician to "create a more dispersed, resilient and hard to target forward-basing infrastructure" (USMC 2018, p. 50).

Our work focuses on achieving one of the main goals of the Marine Corps EABO concept by rebalancing logistics capabilities, reducing the traditional logistics stockpile ashore and relying on a distribution system that delivers sea-based supplies to smaller, dispersed units ashore (USMC 2016, p. 23).

CHAPTER 2:
Background

Optimal UAV routing in our work builds upon a foundation of the work presented in Jatho (2020). We incorporate additional supply depots into our model, changing the vehicle routing problem (VRP) model seen in Jatho (2020) into a multiple depot vehicle routing problem (MDVRP) model. Then, we utilize the framework of our enhanced model to develop a depot selection model to help logisticians choose the best locations for supply depots based on probabilistic future demand scenarios of the operational environment.

2.1 Review of Jatho (2020)

Jatho (2020) describes a two-step approach combining a boundary value problem (BVP) providing optimal flight trajectories with a mixed-integer linear program (MILP) vehicle routing model. The result is an optimal energy-aware UAV routing system.

Jatho's work introduces two important concepts. The first focuses on optimal flight trajectories of UAVs based on terrain and time-varying weather conditions, specifically wind, obtained from the Coupled Ocean/Atmosphere Mesoscale Prediction System (COAMPS). The second creates a UAV routing system based on an energy associated cost matrix from optimal flight trajectories, that services demands according to hierarchical request prioritization.

2.1.1 Optimal Flight Trajectories with BVP

Jatho's first step solves for optimal flight trajectories within a network containing a supply depot and demand nodes. Within this work, each trajectory represents an energy-optimal segment of flight between two boundary points, while the optimal routing refers to the best ordered set of segments to fly to fulfil the logistics request. Jatho accounts for flight dynamics of a particular class of UAVs and required energy to fly between nodes by applying the Pontryagin's maximum principle (PMP) to solve the BVP, taking into account geographic "no-go" flight areas and time-varying wind data (Jatho 2020). The result is an asymmetric energy cost matrix for any UAV to fly from any node i to any node j.

Jatho describes aerodynamics of a UAV with an autopilot system, which simplifies the nonlinear nature of flight dynamics in a flight trajectory optimization problem, where the flight trajectory is defined as the path and the airspeed along that path (Jatho 2020). He derives optimal flight controls utilizing wind conditions, provided by COAMPS, interacting with a UAV's flight control inputs (pitch, roll, airspeed) (Jatho 2020). He utilizes a boundary value problem implementation from previous work by Dobrokhodov et al. (2020) to solve for optimal control inputs, given wind conditions and "no-go" areas, to produce an optimal flight trajectory from node i to node j and the overall cost matrix C_{ij} used in his routing model (Jatho 2020).

Figure 2.1 illustrates optimal flight trajectories of a UAV flying in the presence of time-varying wind conditions. Figure 2.1 is taken from Jatho (2020) illustrating an environment with one depot and 21 demand nodes. Great circle flight paths are shown in blue and the optimal flight paths are shown in red. Though it is difficult to visually distinguish individual flight paths in the figure, clearly there is a significant lack of overlap between red and blue trajectories, indicating that efficiency can be improved by accounting for wind when planning trajectories.

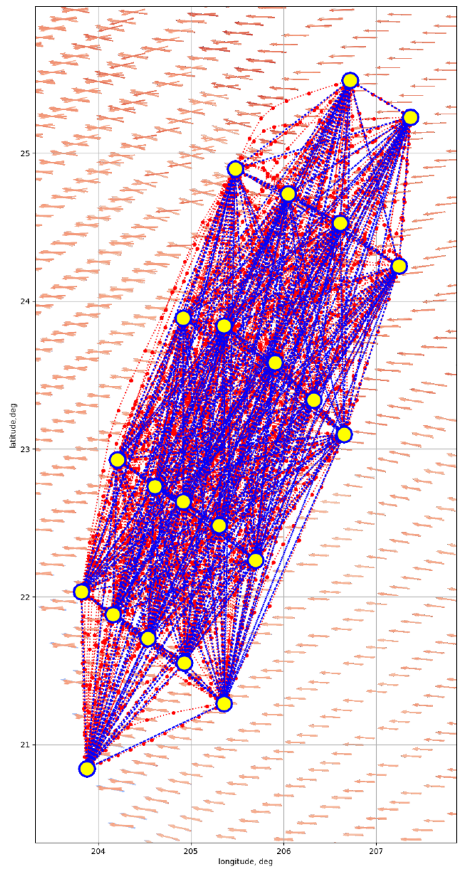


Figure 2.1. Optimal Flight Paths from Figure 5.8 in Jatho (2020).

2.1.2 Jatho's UAV Routing Model

The second piece of Jatho's work is a UAV routing model that uses the optimal flight trajectory energy cost matrix as input data. His model is based on the multiple-traveling salesman problem (mTSP) and builds a VRP model to develop optimal servicing routes for UAVs within the network of multiple destination nodes serviced from a single depot. (Jatho 2020).

The Traveling Salesman Problem

The traveling salesman problem (TSP) solves for the minimum-cost route for a salesman to depart from his starting node and visit every other node in the network. In a graph G, there are n nodes representing destinations for the salesman to visit, and X_{ij} arcs representing possible routes from node i to node j. The TSP uses one salesman departing from and returning to one starting node (or depot). Jatho uses the Miller et al. (1960) formulation of the TSP (Jatho 2020).

The Multiple Traveling Salesman Problem

The mTSP is an extension of the TSP to include multiple salesmen. Jatho uses a formulation from Kaempfer and Wolf (2019) which includes an additional index k to represent individual salesmen (UAVs) and modifies the main decision variable to X_{ijk}, representing the decision to send salesman k along arc X_{ij} (Jatho 2020). The conservation of flow within this mTSP formulation prevents multiple salesmen from visiting the same node; each node i is only allowed to be visited by a single salesman k.

The Vehicle Routing Problem

Jatho's UAV routing model is a VRP, which is a variation of the mTSP. His work cites Cheng et al. (2018)'s description of the VRP as a mTSP with additional constraints modeling vehicle capacities, demand nodes, demand time windows, and vehicle selection (Cheng et al. 2018; Jatho 2020). He describes previous work from Dorling et al. (2016) of a VRP focusing on minimizing delivery cost and delivery time. Additionally, Jatho uses fixed demands where demand node locations and requests are known before a UAV departs and do not change while the UAV is enroute (Jatho 2020). Work by Jaillet and Lu (2011) describes "online" and "offline" routing problems. An "online" problem describes dynamic servicing and rerouting where a UAV can receive demand requests enroute and can re-route to fulfill new demands. Jatho's model solves an "offline" problem.

Summary and Formulation

Jatho's model utilizes a central supply depot as a hub for multiple delivery UAVs to service demands within the network. The model allows demand nodes the ability to give a level of priority, or importance, to their respective demand requests. This ability gives service UAVs the flexibility to fulfill demand requests with higher level priority over the requests with a lower level priority. A UAV servicing a demand node receives a respective "reward" based on the priority level of the demand request. The objective in Jatho's model is to maximize the total reward by UAVs in the network (Jatho 2020). The model provides an output of the optimal routes for each UAV and gives the respective cost, in energy or time, for that route.

The complete formulation of Jatho (2020)'s UAV routing model is:

Sets and Indices:

$i, j \in N = \{1, ..., n\}$ Nodes

$k \in M = \{1, ..., m\}$ UAVs

Data:

C_{ijk} = Energy for UAV k to travel from node i and node j [Watt-Hours]

d_j = Demand at node j [Pounds]

s_k = Capacity of UAV k [Pounds]

$EnergyMax_k$ = Energy capacity of UAV k [Watt-Hours]

ϵ = Penalty weight

R_j = Reward for delivering supplies to node j, based on demand priority

Decision Variables:

X_{ijk} = Binary variable representing whether or not UAV k travels along arc X_{ij}

u_i = Dummy variable to prevent degenerate subtours

$$\max \sum_{i=1}^{n} \sum_{j:j>1, j\neq i} \sum_{k=1}^{m} X_{ijk} R_j - \epsilon \sum_{i=1}^{n} \sum_{j:j>1, j\neq i} \sum_{k=1}^{m} X_{ijk} C_{ijk} \tag{2.1}$$

$$\sum_{j=2}^{n} X_{1jk} \leq 1 \qquad \forall\, k \in \{1, ...m\} \tag{2.2}$$

$$\sum_{i=2}^{n} X_{i1k} \leq 1 \qquad \forall\, k \in \{1, ...m\} \tag{2.3}$$

$$\sum_{j=1}^{n} \sum_{k=1}^{m} X_{ijk} \leq 1 \qquad \forall\, i \in \{2, ...n\} \tag{2.4}$$

$$\sum_{i=1}^{n} \sum_{k=1}^{m} X_{ijk} \leq 1 \qquad \forall\, j \in \{2, ...n\} \tag{2.5}$$

$$\sum_{i=1}^{n} X_{irk} = \sum_{j=1}^{n} X_{rjk} \qquad \forall\, r \in \{2, ...n\} \quad \forall\, k \in \{1, ...m\} \tag{2.6}$$

$$u_i - u_j + (n-m) * \sum_{k=1}^{m} X_{ijk} \leq n - m - 1 \qquad \forall\, i, j \in \{2, ...n\} \quad \forall\, k \in \{1, ...m\} \tag{2.7}$$

$$\sum_{i=1}^{n} \sum_{j:j>1, j \neq i} d_j X_{ijk} \leq s_k \qquad \forall\, k \in \{1, ...m\} \tag{2.8}$$

$$\sum_{i=1}^{n} \sum_{j:j>1, j \neq i} C_{ijk} X_{ijk} \leq EnergyMax_k \qquad \forall\, k \in \{1, ...m\} \tag{2.9}$$

$$X_{ijk} \in \{0, 1\} \qquad \forall i, j \in \{1, ...n\} \tag{2.10}$$

$$u_i \in \{\mathbb{Z}\} \qquad \forall i \in \{2, ...n\} \tag{2.11}$$

Figure 2.2 shows the results of Jatho's UAV routing model from the same network environment, with one depot node and 21 demand nodes, shown in Figure 2.1 above. Jatho describes both the efficiency and effectiveness of these routing results by comparing them to two alternative routes that also exist within the network, but not chosen as optimal routes by the model. His comparison yields energy savings between 2.5 to 11 percent and highlights efficiency of a logistician utilizing a computer to implement the boundary value problem methodology and solve the UAV routing model for optimal results, rather than conducting by-hand calculations (Jatho 2020).

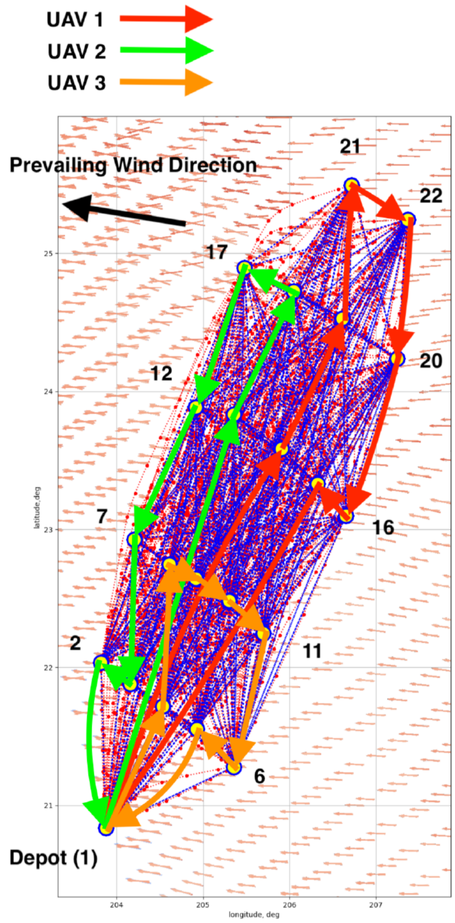


Figure 2.2. Optimal UAV Routing from Figure 5.9 in Jatho (2020).

2.2 Multiple Depots

Reviewing the UAV routing model in Jatho (2020) allows us to understand the model's foundation in both the mTSP and VRP. Our work focuses on enhancing Jatho's model to a MDVRP utilizing concepts of the multiple depot multiple traveling salesman problem (MDMTSP).

2.2.1 The MDMTSP

The MDMTSP is a variant of the mTSP to include multiple depots, vice a single depot, salesmen can utilize as starting and ending routing locations within a network. There is much literature describing exact algorithms and heuristics for the mTSP with a single depot, but less discussing the MDMTSP. The work in Malik et al. (2007) presents an approximation algorithm for the generalized multiple depot, multiple travelling salesman problem (GMTSP) using a degree constrained minimum spanning tree computed by a Lagrangian relaxation (Malik et al. 2007). Their algorithm assumes symmetric costs satisfying the triangle inequality and yields an approximation factor of 2 (Malik et al. 2007).

Oberlin et al. (2009) considers a transformation of the MDMTSP into a single asymmetric TSP. They apply the well-known Lin-Kernighan heuristic to their transformation, proving its effectiveness and showing that high quality solutions can be produced by standard algorithms in a relatively time-efficient manner (Oberlin et al. 2009).

Work in Benavent and Martinez (2013) presents a polynomial transformation of the MDMTSP to the TSP, an integer linear program (ILP) formulation for the MDMTSP, and what they claim to be the first polyhedral study of the MDMTSP (Benavent and Martinez 2013). Their ILP formulation uses inequalities from work seen in Laporte et al. (1986) and further developed by Belenguer et al. (2011) which prevent solutions containing salesmen service paths that start and end at different depots (Benavent and Martinez 2013). Our work does not consider this formulation because we give the salesmen (UAVs) in our model the ability to return to any available depot.

Perhaps the closest study in relation to our work comes from Kara and Bektas (2006) and their presentation of ILP formulations for both the single and multiple depot TSP. They consider two multi-depot cases: a fixed destination and a nonfixed destination MDMTSP. Their fixed destination problem determines a total of m tours such that the salesmen must return to their original depots, whereas the nonfixed destination problem does not restrict salesmen to a specific return depot, but the number of salesmen at each depot must be the same at the end of all tours as it was at the beginning (Kara and Bektas 2006). Additionally, their formulation requires that all customers are visited exactly once (Kara and Bektas 2006). In contrast to the work shown in Kara and Bektas (2006), we do not restrict salesmen to a particular return depot, nor do we require depots to retain a certain number of salesmen at the end of a tour. Additionally, our formulation does not require all customers to be serviced.

The MDVRP

The MDVRP is a special case of the MDMTSP where vehicles are used as salesmen, having limited capacities in fuel and/or payload. A MDVRP problem is regarded as more practical and challenging than the single depot VRP because it involves multiple depots (Samsuddin et al. 2018). Difficulties in the MDVRP lie within the layers of decision making. A decision maker, in our case the USMC logistician, must identify appropriate depots that allow UAVs to service demands, without exceeding UAV energy capacities. MDVRP application and solving methods are presented very nicely in Samsuddin et al. (2018). Methods solving the MDVRP include exact, heuristic, and metaheuristic (Samsuddin et al. 2018). Samsuddin et al. (2018) focuses on single and population-based metaheuristic methods for solving the MDVRP. Their work concludes population-based metaheuristics provide better performance and recommend improvement by either combining both types of algorithms (single or population-based) or hybridizing with the same class (Samsuddin et al. 2018).

Yadlapalli et al. (2009) considers a Langrangian-based algorithm for the MDMTSP using UAVs as Dubins' vehicles, thus making it a MDVRP. Their work aims at motion planning of the vehicles through n points in a plane choosing the vehicles that provide minimum cost tours of all vehicles considered (Yadlapalli et al. 2009). They use Euclidean distance as cost and propose a generalized version of Held-Karp's method to solve the combinatorial problem providing a lower bound within five percent of the optimum (Yadlapalli et al. 2009).

Our work focuses on exact algorithms solving the MDVRP because we assume there will be no more than 50 demand nodes in a last-mile EABO logistics scenario (Jatho 2020).

2.2.2 Facility Location and Depot Selection

The traveling salesman facility location problem (TSFLP) focuses on selecting the best supply depot locations to minimize the overall cost of fulfilling demand requests within a network.

The multiple depot, multiple traveling salesman facility-location problem (MDMTSFLP) seen in Chan and Baker (2005) addresses methods for depot selection based on demands in the network, where depot locations are pre-staged and fixed. Chan and Baker use a heuristic based approach combining minimum spanning forests (MSF) and a modified Clarke-Wright

(CW) procedure to choose which depots to use and create optimal delivery routing (Chan and Baker 2005).

Shen and Chen (2017) present “a two-tier particle swarm optimization (PSO) framework” determining optimal depot locations and routing in the MDVRP (Shen and Chen 2017). Their work solves for optimal depot locations through external PSO, then uses internal PSO to solve for optimal vehicle routing. Shen and Chen (2017) shows the external swarm technique reduces the routing cost produced by the internal swarm stage by about 13.16 percent (Shen and Chen 2017). Their work requires each vehicle to use a single depot by constructing the MDVRP as a clustering problem, assigning one vehicle to one cluster (Shen and Chen 2017).

Laporte et al. (1986) uses ILP formulation to propose what they believe is the first exact algorithm optimally solving the facility location problem on networks containing up to around twenty nodes (Laporte et al. 1986). Our work uses similar concepts seen in Laporte et al. (1986), including upper bounds on the number of depots in the network and number of vehicles at each depot, as well as simultaneous solutions for location and routing (Laporte et al. 1986). In contrast to our focus, Laporte et al. (1986) assigns an associated cost of using a particular depot location, requires each vehicle route to start and end at the same depot, and allows demand nodes to be visited more than once.

The work in Bertsimas (1989) studies the TSFLP and the probabilistic traveling salesman facility location problem (PTSFLP). Their focus on the TSFLP improves heuristics to show optimal depot locations are at nodes which always require a visit (Bertsimas 1989). The TSFLP approach seen in Bertsimas (1989) assigns a probability of requiring service to each node in a network, then selects the best locations among the nodes as depots such that “the expected distance traveled over all possible instances of the problem is minimized” (Bertsimas 1989). This method increases complexity of the problem due to re-optimization of vehicle tours in each instance. Bertsimas (1989) address this issue by using an a priori tour technique developing heuristics for the PTSFLP, yielding solutions within 25 percent of the optimal TSFLP (Bertsimas 1989).

CHAPTER 3:
Expanding the UAV Routing Model with Multiple Supply Depots

3.1 Importance of Multiple Supply Depots

Jatho (2020) provides a UAV routing model that yields an optimal solution for resupplying Marine units in a contested environment. However, this model is constrained by the use of only one supply depot. This requires all UAVs to resupply, recharge, depart, and return to a single location within the network.

We now describe an extension of Jatho's model to include multiple supply depots. We allow each UAV to depart and return to one of a set of supply depots. This, in turn, allows a UAV to possibly service additional resupply requests while decreasing the overall energy expenditure of the service route.

3.1.1 Sets and Indices

The addition of multiple depots requires identifying distinct subsets of N to represent a set of depot nodes and demand nodes. Sets P and Q partition set N into depot nodes and demand nodes, respectively.

3.1.2 Data and Parameters

The primary source of data is a cost matrix consisting of all enumerated flight routes in the network, given as the solution of the boundary value problem. C_{ijk} represents the cost (in watt-hours (Wh)) for UAV k to travel from node i to node j. The cost matrix is asymmetric due to the influence of weather conditions. For example, due to wind speed and direction, the cost for UAV 1 to travel from node 1 to node 2 (C_{121}) may not necessarily equal the cost for UAV 1 to travel from node 2 to node 1 (C_{211}). The demand at any node j is given by d_j (pounds). The payload capacity of UAV k is given by s_k (pounds). The starting depot of UAV k is b_k. The energy capacity of UAV k is $EnergyMax_k$ (Wh).

The model's primary objective is to maximize the reward-weighted deliveries completed. R_j is the reward a UAV is given by servicing a demand at demand node j. This reward is based on the given demand node's importance level. Like Jatho, we designate one of three importance levels for each demand node: routine, priority, or urgent (Jatho 2020). We set the reward for a priority level request to five times that of a routine request, and that of an urgent level request to 10 times that of a routine request. In the event that there are multiple optimal solutions with respect to the primary objective, the model uses total energy expenditure as a tie-breaker. The small parameter ϵ is a scaling factor which transforms the energy cost to units of reward. It multiplies the energy expenditure term in the objective function (shown in Equation 3.3) ensuring its value is small relative to that of the primary objective value.

3.1.3 Decision Variables

This model uses the same decision variables described in Jatho (2020). Variable X_{ijk}, as shown in Equation 3.1, is a binary variable that reflects whether UAV k travels from node i to node j (Jatho 2020). Variable u_i is used to prevent subtours.

$$X_{ijk} = \begin{cases} 1, & \text{if UAV } k \text{ travels from node } i \text{ to node } j \\ 0, & \text{otherwise} \end{cases} \tag{3.1}$$

$$u_i \in \mathbb{Z} \qquad\qquad \forall i \in Q \tag{3.2}$$

3.1.4 Objective Function

The primary objective of the model is to maximize the total reward given to the fleet of UAVs servicing of demand requests in the network. Equation 3.3 shows the objective function in its entirety, including both the reward servicing demand nodes and the small tie-breaker penalty for energy consumption.

$$\max \sum_{i=1}^{n} \sum_{j:j\neq i} \sum_{k=1}^{m} X_{ijk} R_j - \epsilon \sum_{i=1}^{n} \sum_{j:j\neq i} \sum_{k=1}^{m} X_{ijk} C_{ijk} \tag{3.3}$$

3.1.5 Constraints

The main families of constraints are built from the mTSP and the UAV routing model used in Jatho (2020). These constraints are augmented to support the addition of multiple depots, representing a MDVRP.

Multiple Traveling Salesman Constraints

$$\sum_{j \in Q} X_{b_k jk} \leq 1 \qquad \forall\, k \in \{1, ...m\} \tag{3.4}$$

$$\sum_{i \in Q} \sum_{j \in P} X_{ijk} \leq 1 \qquad \forall\, k \in \{1, ...m\} \tag{3.5}$$

$$\sum_{i=1}^{n} \sum_{k=1}^{m} X_{ijk} \leq 1 \qquad \forall\, j \in Q \tag{3.6}$$

$$\sum_{j=1}^{n} \sum_{k=1}^{m} X_{ijk} \leq 1 \qquad \forall\, i \in Q \tag{3.7}$$

$$\sum_{i \in N} X_{irk} = \sum_{j \in N} X_{rjk} \qquad \forall\, r \in Q; k \in \{1, ...m\} \tag{3.8}$$

$$u_i - u_j + (n - m) * \sum_{k=1}^{m} X_{ijk} \leq n - m - 1 \qquad \forall\, i, j \in Q \tag{3.9}$$

Constraint 3.4 ensures that each UAV departs from its starting depot b_k at most once, and constraint 3.5 ensures each UAV returns to a depot node at most once. These constraints are modified from Jatho (2020) Equations 2.2 and 2.3, to control a UAV's starting depot, b_k, and allow for a UAV to return to any depot within the set P.

Constraints 3.6 and 3.7 ensure at most one entry and one exit, respectively, per demand node. Constraint 3.8 is a conservation of flow constraint for all demand nodes and UAVs. It ensures the summation of all entrances equals the summation of all exits for each demand nodes and UAV. This constraint guarantees if a UAV enters a demand node, it will also exit that node.

Constraint 3.9 is a subtour elimination constraint that ensures degenerate subtours of demand nodes do not exist. This constraint is taken and modified from Miller et al. (1960).

Constraints from Jatho (2020)

$$\sum_{i=1}^{n} \sum_{j:j\neq i} d_j X_{ijk} \leq s_k \qquad \forall\, k \in \{1, \ldots m\} \tag{3.10}$$

$$\sum_{i=1}^{n} \sum_{j:j\neq i} C_{ijk} X_{ijk} \leq EnergyMax_k \qquad \forall\, k \in \{1, \ldots m\} \tag{3.11}$$

Constraint 3.10 is a UAV supply capacity constraint that ensures the total supplies delivered on a UAV's route is not greater than the UAV's carrying capacity, s_k.

Constraint 3.11 is an energy constraint that limits the cost of a UAV's route to be less than or equal to the maximum energy of the UAV.

Multiple Depot Constraints

$$\sum_{j\in Q} X_{b_k jk} = \sum_{i\in Q} \sum_{j\in P} X_{ijk} \qquad \forall\, k \in \{1, \ldots m\} \tag{3.12}$$

Constraint 3.12 controls depot flow. It ensures every UAV that leaves its starting depot returns to a depot.

3.1.6 Complete Formulation

Sets and Indices:

$i, j \in N = \{1, \ldots, n\}$	Nodes
$k \in M = \{1, \ldots, m\}$	UAVs
$P \subseteq N$	Depot nodes
$Q \subseteq N$	Demand nodes

Data:

C_{ijk} = Energy for UAV k to travel from node i and node j [Wh]

d_j = Demand at node j [Pounds]

s_k = Capacity of UAV k [Pounds]

b_k = Starting depot of UAV k

$EnergyMax_k$ = Energy capacity of UAV k [Wh]
ϵ = Penalty weight
R_j = Reward for delivering supplies to node j, based on demand priority

Decision Variables:
X_{ijk} = Binary variable representing whether or not UAV k travels from node i to node j
u_i = Dummy variable to prevent degenerate subtours

$$\max \sum_{i=1}^{n} \sum_{j:j\neq i} \sum_{k=1}^{m} X_{ijk} R_j - \epsilon \sum_{i=1}^{n} \sum_{j:j\neq i} \sum_{k=1}^{m} X_{ijk} C_{ijk} \tag{3.13}$$

$$\sum_{j\in Q} X_{b_k jk} \leq 1 \qquad \forall\, k \in \{1, ...m\} \tag{3.14}$$

$$\sum_{i\in Q} \sum_{j\in P} X_{ijk} \leq 1 \qquad \forall\, k \in \{1, ...m\} \tag{3.15}$$

$$\sum_{i=1}^{n} \sum_{k=1}^{m} X_{ijk} \leq 1 \qquad \forall\, j \in Q \tag{3.16}$$

$$\sum_{j=1}^{n} \sum_{k=1}^{m} X_{ijk} \leq 1 \qquad \forall\, i \in Q \tag{3.17}$$

$$\sum_{i\in N} X_{irk} = \sum_{j\in N} X_{rjk} \qquad \forall\, r \in Q; k \in \{1, ...m\} \tag{3.18}$$

$$\sum_{j\in Q} X_{b_k jk} = \sum_{i\in Q} \sum_{j\in P} X_{ijk} \qquad \forall\, k \in \{1, ...m\} \tag{3.19}$$

$$u_i - u_j + (n-m) * \sum_{k=1}^{m} X_{ijk} \leq n - m - 1 \qquad \forall\, i, j \in Q \tag{3.20}$$

$$\sum_{i=1}^{n} \sum_{j:j\neq i} d_j X_{ijk} \leq s_k \qquad \forall\, k \in \{1, ...m\} \tag{3.21}$$

$$\sum_{i=1}^{n} \sum_{j:j\neq i} C_{ijk} X_{ijk} \leq EnergyMax_k \qquad \forall\, k \in \{1, ...m\} \tag{3.22}$$

$$X_{ijk} \in \{0, 1\} \qquad \forall i, j \in \{1, ...n\} \tag{3.23}$$

$$u_i \in \mathbb{Z} \qquad \forall i \in Q \tag{3.24}$$

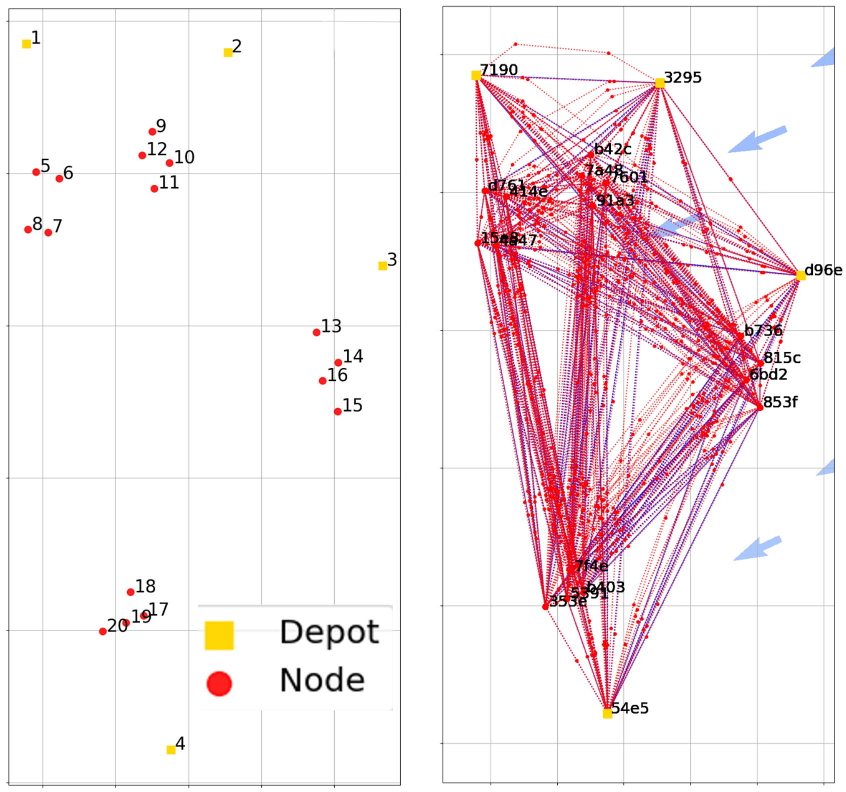
3.2 Results

We now exercise the MDVRP model to gain insight on optimal UAV routing and energy savings from using multiple depots versus a single depot. Similar to Jatho (2020), we solve for the optimal solution for the UAV routing model using Computational Infrastructure for Operations Research branch and cut (CBC) solver and Pyomo software (version 5.7.1), on a computer with 16 GB RAM and a 1.8 GHz CPU (Hart et al. 2011) and (Hart et al. 2017). Each network environment described in this section contains about 119-357 constraints, 229-1536 decision variables, and can be solved in approximately 42-229 seconds. We examine two networks, or operational environments, of potential last-mile logistics scenarios on the island of Maui, HI. Of note, the choice of Maui for the operational environment is due to actual obtained weather data from COAMPS. The first network depicts a "clustered" demand node logistics scenario illustrating depots as the parent nodes to their respective clusters of demands. A clustered network is used for simplicity of model validation and does not restrict UAV servicing. The second network shows demand nodes shared by two depots. In this environment, it is less clear what the optimal UAV servicing routes are. For each network, we compare the results from utilizing a single depot versus multiple depots.

3.2.1 Clustered Demand Nodes

Network Environment

We first consider the network environment shown in Figure 3.1. This network features a northeasterly wind as depicted by the blue arrows in Figure 3.1b.



(a) Operational Environment Network

(b) Enumerated Optimal Flight Paths

Figure 3.1. Operational Environment Network of a Clustered Last-Mile Resupply Logistics Scenario

Input Data and Assumptions

The data for this network includes requests from demand nodes and UAV parameters. All demand nodes have a routine importance level and request three pounds of supplies each. We use a homogeneous fleet of four UAVs each with a supply capacity of 15 pounds and a maximum energy capacity of 500,000 Wh; all data is notional and does not necessarily represent a specific UAV. We assume all demand requests are entered and received at the same time and UAVs depart a supply depot at the same time.

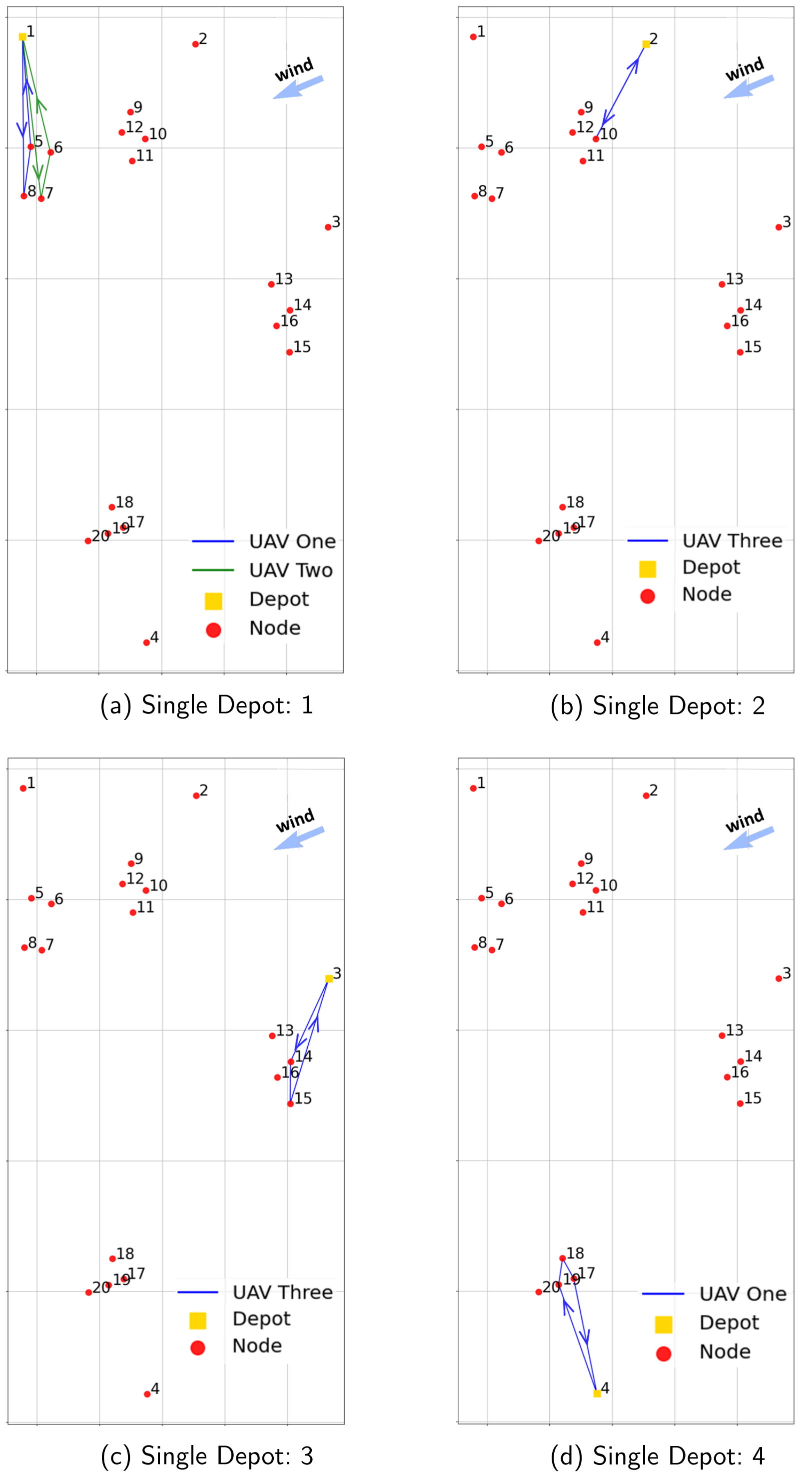
Single Depot Routing

Figure 3.2 shows a series of routing scenarios restricting four resupply UAVs to using one depot. Figure 3.2a depicts node 1 as the resupply depot. Here, the model uses two of four available UAVs to service node 1's respective demand node cluster. The remaining UAVs are unused due to the energy required to service the other demand nodes.

Figure 3.2b uses only one UAV and is only able to service one demand request. The costs of traveling to and from unserviced nodes 9, 11, and 12 exceed UAV energy constraints, thus the remaining three UAVs are not utilized. A roundtrip cost from node 2 to node 9 ($C_{2,9}$ + $C_{9,2}$) is about 663,707 Wh, to node 11 is 603,915 Wh, and to node 12 is roughly 724,680 Wh. The remaining UAVs in this scenario are unable to service the other demand nodes because of their energy capacity of 500,000 Wh.

Figure 3.2c uses one UAV to service nodes 14 and 15. Nodes 13 and 16 are left unserviced due to UAV energy constraints. Figure 3.2d uses one UAV to service demand nodes 17, 18, and 19, leaving demand node 20 unserviced.

In summary, Figure 3.2 highlights the limitations of using a single depot in this network. In each instance mentioned above, UAVs can only service the open depot's respective cluster of demands, leaving the remaining demand requests unfulfilled. Additionally, UAVs are left unused due to the energy required to service the remaining demands from a single depot.



(a) Single Depot: 1

(b) Single Depot: 2

(c) Single Depot: 3

(d) Single Depot: 4

Figure 3.2. UAV Routing for a Resupply Clustered Demand Environment Highlighting Routes of Four UAVs Using One Depot

Multiple Depot Routing

Figure 3.3 utilizes depots 1, 2, 3, and 4. UAVs One, Two, Three, and Four start at depots 1, 2, 3, and 4 respectively but are allowed to return to any depot. The results from this model highlight the importance of using multiple depots, as well as the ability of a UAV to utilize a tailwind.

The serviced demand nodes in the respective clusters of depot nodes 1, 2, 3, and 4 remain unchanged from Figure 3.2. However, what is most notable is the servicing route of UAV Two. Originally shown in Figure 3.2b, UAV Two was only able to service node 10 because it was constrained to return to depot node 2. It was able to complete that out-and-back tour with a total energy expenditure of 482,031 Wh. By relaxing the constraint of returning to a single depot, UAV Two in Figure 3.3 can use the environmental energy from a tailwind to service additional demand nodes 9, 11, and 12 then return to depot node 1 with a total energy expenditure of 487,321 Wh.

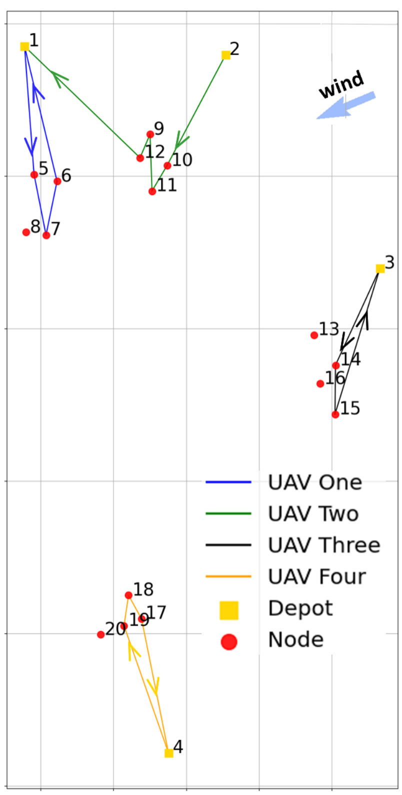
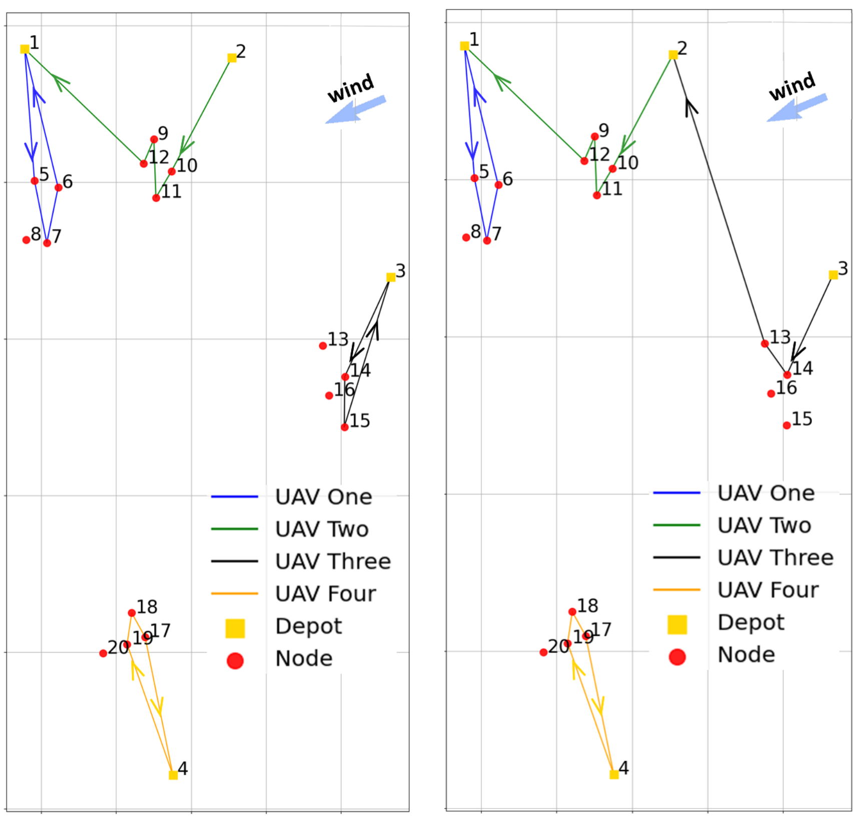


Figure 3.3. UAV Routing for a Resupply Clustered Demand Environment Highlighting Routes of Four UAVs Using Multiple Depots

Importance Level

Figure 3.4 shows the model's ability to prioritize an urgent demand request. In Figure 3.4a, all demand nodes have an importance level of routine, the lowest level of importance in the model. Figure 3.4b shows a logistics scenario with all demand nodes requesting supplies with importance level routine, except for node 13 which has a request at importance level urgent.

Figure 3.4b illustrates UAV Three's ability to fulfill the routine request of demand node 14 and the urgent request of demand node 13, then utilize energy-saving properties of a tailwind to return to depot node 2.



(a) All Demands routine Request Importance Level

(b) Demand Node 13 urgent Request Importance Level

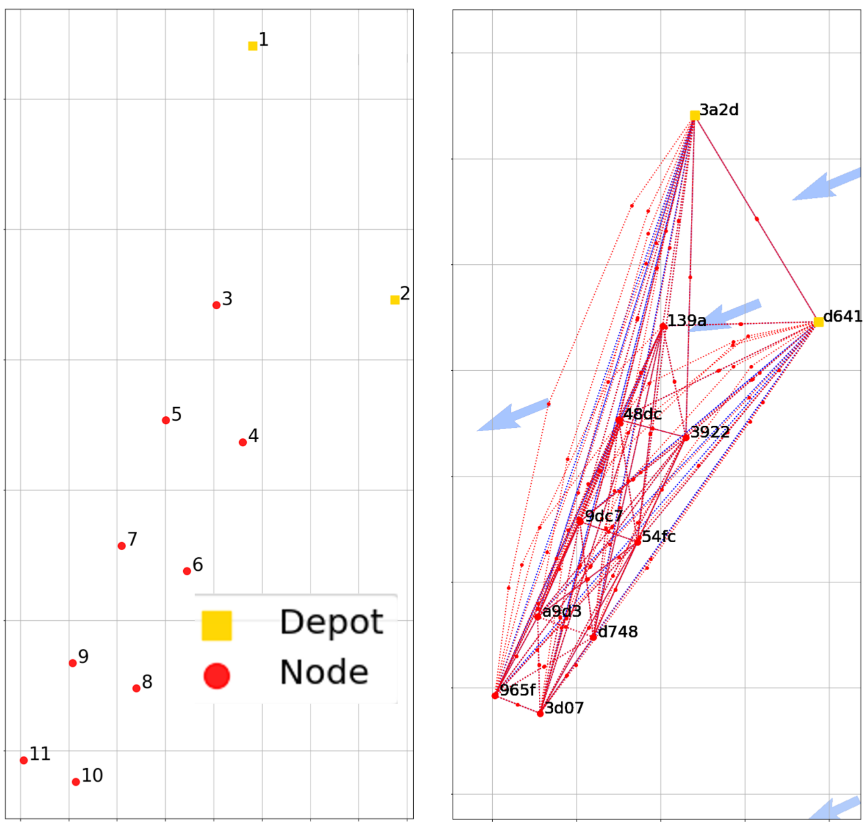
Figure 3.4. UAV Routing Illustrating Model Results of a Routine Request versus an Urgent Request

3.2.2 Demand Nodes Between Two Depots

The clustered nature of the demand nodes in the previous network environment serves to build intuition and validate that the model is functioning as expected. When demand nodes are more or less evenly distributed between depot nodes, it is less obvious what optimal UAV routes should look like.

Network Environment

Figure 3.5 depicts an “ambiguous” network, with nine demand nodes almost evenly distributed between two depot nodes. This network features a northeasterly wind as illustrated by the blue arrows in Figure 3.5b.



(a) Operational Environment Network

(b) Enumerated Optimal Flight Paths

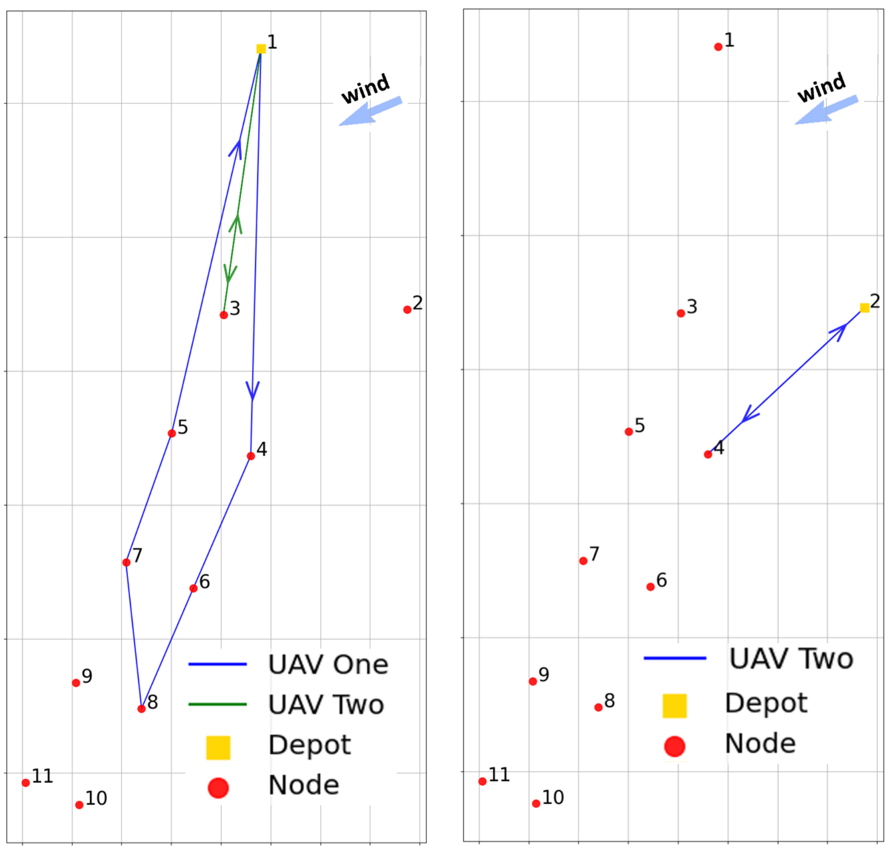
Figure 3.5. Operational Environment Network of Demand Nodes between Two Depot Nodes in a Last-Mile Resupply Logistics Scenario

Input Data and Assumptions

For this scenario, all demand nodes have a routine importance level and request three pounds of supplies each. We use a homogeneous fleet of four UAVs, each with a supply capacity of 15 pounds and a maximum energy capacity of 1,200,000 Wh.

Single Depot Routing

Figure 3.6 shows UAV routing results using only a single depot in the network. Figure 3.6a depicts two UAVs and their respective routes departing from and returning to depot node 1. Figure 3.6b shows UAV routing for the use of depot node 2. Figure 3.6b's result uses one UAV and is only able to service demand node 4.



(a) Single Depot: 1 (b) Single Depot: 2

Figure 3.6. Single Depot UAV Routing for Demand Nodes Between Two Depot Nodes

Multiple Depot Routing

Figure 3.7 depicts the routes of two UAVs using two depots. UAVs One and Two start their servicing route at depots 1 and 2 respectively. The same nodes are serviced as were shown in Figure 3.6a. However, using multiple depots decreases the overall energy expenditure of a UAV servicing demand nodes 4, 5, 6, 7, and 8. The routes of UAV One shown in Figure 3.6a, and UAV Two shown in Figure 3.7 service the same demand nodes. UAV One's route in Figure 3.6a is $1 \rightarrow 4 \rightarrow 6 \rightarrow 8 \rightarrow 7 \rightarrow 5 \rightarrow 1$ and has a total energy expenditure of 1,150,543 Wh. UAV Two's route in Figure 3.7 is $2 \rightarrow 4 \rightarrow 6 \rightarrow 8 \rightarrow 7 \rightarrow 5 \rightarrow 1$ with a total energy expenditure of 1,084,528 Wh. This is approximately 6% savings of energy to service the same demand nodes with one UAV.

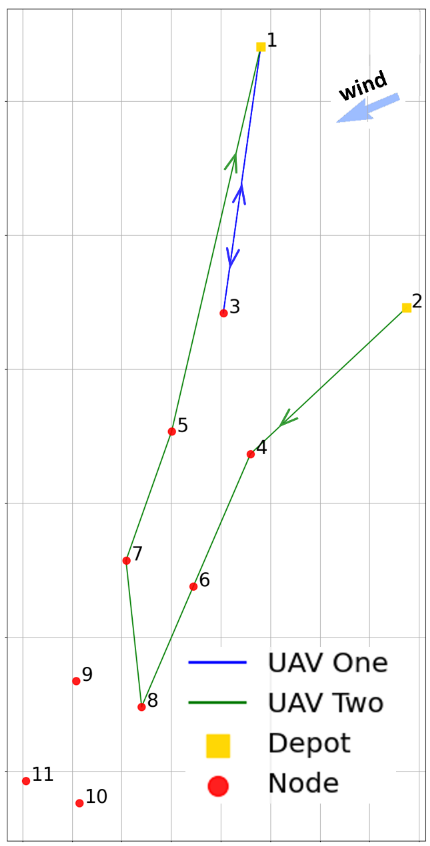


Figure 3.7. Multiple Depot UAV Routing for Demand Nodes Between Two Depot Nodes

CHAPTER 4:
Depot Selection

4.1 Introduction

The second main contribution of this thesis is a depot selection model. This model enhances the strategic planning process of EABO and lays the foundation for future research and development. The overall goal of the planning process is a force structure meeting the demands of the operational environment that provides a feasible and affordable capabilities mix (Mazarr et al. 2019).

In a 2018 RAND study, Mazarr et al. (2019) conduct research and analysis as part of a three-phase project titled *Defense Planning for a New Era* (Mazarr et al. 2019). Their research contributes to Phase Two of the project describing the current defense planning process used by the Department of Defense (DoD), focusing on how scenarios are developed and employed to support defense planning (Mazarr et al. 2019). The study by Mazarr et al. (2019) highlights the importance of robust planning methods and emphasizes, "No matter what the starting point for planning, the methods that assess a proposed future force's performance in a variety of future scenarios are what ultimately provide valuable information to decisionmakers" (Mazarr et al. 2019).

The motivation for our depot selection model is built on facilitating a comprehensive, and easily employable, methodology for EABO logisticians during operational planning.

4.2 The Depot Selection Model

4.2.1 Description

The depot selection model can be thought of as a two-stage optimization problem. First, the model chooses depots among a set of pre-planned possible depot locations. Second, the model finds the optimal UAV delivery routes with the chosen depot locations. During the logistics planning phase of a military operation, logisticians will first select possible locations for a supply depot. The selection of possible locations could depend on a variety of factors such as terrain, proximity to the adversary, line-of-sight propagation, weather forecasts, and operational footprint. This planning phase provides the depot selection model with a set of possible locations to choose from. The model uses a scenario-based approach, giving each scenario a weighted probability of occurrence. Each scenario is specified by demand location and importance level of resupply requests. For each scenario, the model constructs a valid MDVRP solution. The goal of the model is to maximize the expected performance across a set of potential future scenarios by selecting the best depot locations to use among the set of all possible locations. The results are chosen depot locations and the optimal UAV routes for each scenario.

4.2.2 Sets and Indices

This model uses the same sets and indices as the MDVRP model described in Chapter 3 and also includes an additional set for possible scenarios, represented by Ω and indexed by ω.

4.2.3 Data and Parameters

The data for this model is similar to what was described in Chapter 3. The difference now is that each scenario, ω, holds its own unique data and parameters. This is indicated by a superscript ω on the cost, demand, and reward parameters. For example, the demand at node j in scenario ω is defined by d_j^ω. New data and parameters added to the model include p^ω, $maxdep$, and $maxUAV_i$. The probability that a scenario ω will occur is represented by p^ω and the maximum number of depot nodes for the model to select is represented by $maxdep$. During an operation, it may only be feasible to provide a certain number of supply depots due to various constraints. Additionally, each depot may only be able to house a certain number of UAVs. The maximum number of UAVs to house at depot node i is represented by $maxUAV_i$.

4.2.4 Decision Variables

The main addition of decision variables is given by Equation 4.3, where Y_i is a binary variable that represents if a depot location is selected by the model. Equations 4.1 and 4.2 are modified from the previous MDVRP Equations 3.1 and 3.2 for distinction between scenarios.

$$X_{ijk}^{\omega} = \begin{cases} 1, & \text{if UAV } k \text{ travels from node } i \text{ to node } j \text{ in scenario } \omega \\ 0, & \text{otherwise} \end{cases} \tag{4.1}$$

$$u_i^{\omega} \in \mathbb{Z} \qquad\qquad \forall i \in Q, \omega \in \Omega \tag{4.2}$$

$$Y_i = \begin{cases} 1, & \text{if depot } i \text{ is selected} \\ 0, & \text{otherwise} \end{cases} \tag{4.3}$$

4.2.5 Objective Function

The primary objective of the depot selection model is to maximize expected performance across all scenarios by choosing which depot locations to select. This model solves the same objective as the MDVRP model to obtain optimal UAV routing, but includes scenarios and their respective weighted probability of occurrence. Based on the parameters of each scenario, the model will choose the best depots to select which will maximize the overall reward in the objective function shown in Equation 4.4.

$$\max \sum_{i=1}^{n} \sum_{j:j\neq i} \sum_{k=1}^{m} \sum_{\omega\in\Omega} p^{\omega} X_{i,j,k}^{\omega} (R_j^{\omega} - \epsilon C_{i,j,k}^{\omega}) \tag{4.4}$$

4.2.6 Constraints

The constraints in this model are composed of modified MDVRP model constraints outlined in Chapter 3, and additional depot selection constraints.

MDVRP Model Constraints

The constraints given below are taken from the MDVRP model and modified to include distinct scenarios ω. Additionally, the parameter of UAV starting location, b_k, is removed from the constraints to allow the model to choose which depots UAVs should be housed in and depart from. This relaxes the constraint on where each UAV k starts its route.

$$\sum_{i\in P}\sum_{j\in Q} X^{\omega}_{i,j,k} \leq 1 \qquad \forall\, k \in K, \omega \in \Omega \tag{4.5}$$

$$\sum_{i\in Q}\sum_{j\in P} X^{\omega}_{i,j,k} \leq 1 \qquad \forall\, k \in K, \omega \in \Omega \tag{4.6}$$

$$\sum_{i=1}^{n}\sum_{k=1}^{m} X^{\omega}_{i,j,k} \leq 1 \qquad \forall\, j \in Q, \omega \in \Omega \tag{4.7}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{m} X^{\omega}_{i,j,k} \leq 1 \qquad \forall\, i \in Q, \omega \in \Omega \tag{4.8}$$

$$\sum_{i\in N} X^{\omega}_{i,r,k} = \sum_{j\in N} X^{\omega}_{r,j,k} \qquad \forall\, r \in Q, k \in K, \omega \in \Omega \tag{4.9}$$

$$\sum_{i\in P}\sum_{j\in Q} X^{\omega}_{i,j,k} = \sum_{i\in Q}\sum_{j\in P} X^{\omega}_{i,j,k} \qquad \forall\, k \in K, \omega \in \Omega \tag{4.10}$$

$$u^{\omega}_{i} - u^{\omega}_{j} + (n-m) * \sum_{k=1}^{m} X^{\omega}_{i,j,k} \leq n-m-1 \qquad \forall\, i, j \in Q, \omega \in \Omega \tag{4.11}$$

$$\sum_{i=1}^{n}\sum_{j:j\neq i} d^{\omega}_{j} X^{\omega}_{i,j,k} \leq s_k \qquad \forall\, k \in K, \omega \in \Omega \tag{4.12}$$

$$\sum_{i=1}^{n}\sum_{j:j\neq i} C^{\omega}_{i,j,k} X^{\omega}_{i,j,k} \leq EnergyMax_k \qquad \forall\, k \in K, \omega \in \Omega \tag{4.13}$$

Constraints 4.5 and 4.6 ensure that each UAV, in each scenario, departs and returns to a depot node only once. Constraints 4.7 and 4.8 limit one entry and one exit, respectively, per demand node in each scenario.

Constraint 4.9 is a conservation of flow constraint for all demand nodes and all UAVs, in each scenario. It ensures the summation of all entrances of demand nodes equals the summation of all exits of demand nodes for all UAVs. This constraint guarantees if a UAV enters a demand node, it will also exit that node.

Constraint 4.10 controls depot flow. It ensures each UAV that departs a depot returns to a depot for each scenario.

Constraint 4.11 is a subtour elimination constraint that ensures degenerate subtours of demand nodes do not exist in each scenario. This constraint is taken and modified from Miller et al. (1960). Constraint 4.12 is a UAV supply capacity constraint for each scenario that ensures the total supplies delivered on a UAV's route is not greater than the UAV's carrying capacity, s_k. Constraint 4.13 is an energy constraint for each scenario that limits the cost of a UAV's route to be less than or equal to the maximum energy of the UAV.

Depot Selection Constraints

Constraints 4.14, 4.15, and 4.16 are all added to the MDVRP model to create the depot selection model.

$$\sum_{j=1}^{n}\sum_{k=1}^{m} X_{i,j,k}^{\omega} \leq maxUAV_i \cdot Y_i \qquad \forall\, i \in P, \omega \in \Omega \tag{4.14}$$

$$\sum_{i=1}^{n}\sum_{k=1}^{m} X_{i,j,k}^{\omega} \leq maxUAV_j \cdot Y_j \qquad \forall\, j \in P, \omega \in \Omega \tag{4.15}$$

$$\sum_{i\in P} Y_i \leq maxdep \tag{4.16}$$

Constraints 4.14 and 4.15 control the number of UAVs allowed to depart from, and return to a depot. The number of UAVs that can depart on a resupply route from a depot cannot be greater than the total number of UAVs currently at that depot. Likewise, the number of UAVs allowed to return to a particular depot cannot be greater than the allowable number of UAVs that depot can house, represented by $maxUAV_j$. These constraints use the binary variable X_{ijk}^{ω} to represent the control of maximum UAVs departing and entering depots. Constraint 4.14 ensures that summation of arcs out of depot j is less than or equal to the maximum number of UAVs allowed to be housed at that depot. Constraint 4.15 ensures that summation of arcs into depot j is less than or equal to the maximum number of UAVs allowed to be housed at that depot. These constraints are applied for each depot, in each scenario. Constraint 4.16 ensures the total number of depots selected by the model is less than or equal to the maximum number of depots able to be provided for the operation.

4.2.7 Complete Formulation

Sets and Indices:

$i, j, r \in N = \{1, ..., n\}$	Nodes
$k \in K = \{1, ..., m\}$	UAVs
$P \subseteq N$	Potential depot nodes
$Q = N \setminus P$	Demand nodes
$\omega \in \Omega = \{1, ..., W\}$	Scenarios

Data:

$C^{\omega}_{i,j,k}$ = Energy for UAV k to travel from node i and node j in scenario ω [Watt-Hours]

d^{ω}_{j} = Demand at node j in scenario ω [Pounds]

s_k = Capacity of UAV k [Pounds]

$EnergyMax_k$ = Energy capacity of UAV k [Watt-Hours]

ϵ = Penalty weight

R^{ω}_{j} = Reward for delivering supplies to node j in scenario ω, based on demand priority

p^{ω} = Probability that scenario ω will occur

$maxdep$ = Maximum number of depot nodes to select

$maxUAV_i$ = Maximum number of UAVs to house at node i

Decision Variables:

$X^{\omega}_{i,j,k}$ = Binary variable representing whether or not UAV k travels along arc (i, j) in scenario ω

u^{ω}_{i} = Dummy variable to prevent degenerate subtours

Y_i = Binary variable representing whether or not depot i is selected

$$\max \sum_{i=1}^{n} \sum_{j:j\neq i} \sum_{k=1}^{m} \sum_{\omega\in\Omega} p^{\omega} X^{\omega}_{i,j,k} (R^{\omega}_{j} - \epsilon C^{\omega}_{i,j,k}) \tag{4.17}$$

$$\sum_{i\in P}\sum_{j\in Q} X^{\omega}_{i,j,k} \leq 1 \qquad \forall\, k \in\ K, \omega \in \Omega \tag{4.18}$$

$$\sum_{i\in Q}\sum_{j\in P} X^{\omega}_{i,j,k} \leq 1 \qquad \forall\, k \in\ K, \omega \in \Omega \tag{4.19}$$

$$\sum_{i=1}^{n}\sum_{k=1}^{m} X^{\omega}_{i,j,k} \leq 1 \qquad \forall\, j \in Q, \omega \in \Omega \tag{4.20}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{m} X^{\omega}_{i,j,k} \leq 1 \qquad \forall\, i \in Q, \omega \in \Omega \tag{4.21}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{m} X^{\omega}_{i,j,k} \leq maxUAV_i Y_i \qquad \forall\, i \in P, \omega \in \Omega \tag{4.22}$$

$$\sum_{i=1}^{n}\sum_{k=1}^{m} X^{\omega}_{i,j,k} \leq maxUAV_j Y_j \qquad \forall\, j \in P, \omega \in \Omega \tag{4.23}$$

$$\sum_{i\in P} Y_i \leq maxdep \tag{4.24}$$

$$\sum_{i\in N} X^{\omega}_{i,r,k} = \sum_{j\in N} X^{\omega}_{r,j,k} \qquad \forall\, r \in Q, k \in\ K, \omega \in \Omega \tag{4.25}$$

$$\sum_{i\in P}\sum_{j\in Q} X^{\omega}_{i,j,k} = \sum_{i\in Q}\sum_{j\in P} X^{\omega}_{i,j,k} \qquad \forall\, k \in\ K, \omega \in \Omega \tag{4.26}$$

$$u^{\omega}_i - u^{\omega}_j + (n-m) * \sum_{k=1}^{m} X^{\omega}_{i,j,k} \leq n - m - 1 \qquad \forall\, i, j \in Q, \omega \in \Omega \tag{4.27}$$

$$\sum_{i=1}^{n}\sum_{j:j\neq i} d^{\omega}_j X^{\omega}_{i,j,k} \leq s_k \qquad \forall\, k \in\ K, \omega \in \Omega \tag{4.28}$$

$$\sum_{i=1}^{n}\sum_{j:j\neq i} C^{\omega}_{i,j,k} X^{\omega}_{i,j,k} \leq EnergyMax_k \qquad \forall\, k \in\ K, \omega \in \Omega \tag{4.29}$$

$$X^{\omega}_{i,j,k} \in \{0,1\} \qquad \forall i, j \in\ N, \omega \in \Omega \tag{4.30}$$

$$Y_i \in \{0,1\} \qquad \forall i \in\ P \tag{4.31}$$

$$u^{\omega}_i \in \mathbb{Z} \qquad \forall i \in Q, \omega \in \Omega \tag{4.32}$$

4.3 Results

We now run the depot selection model using two network environments with potential scenarios depicting different locations and parameters of expeditionary units requesting supplies. The results tell which depots to select and give optimal UAV routing for each scenario using the chosen depots. Each environment described in this section contains about 366-754 constraints, 1540-3466 decision variables, and can be solved in approximately 14-269 seconds subject to a 50,000 "maxNode" evaluation parameter.

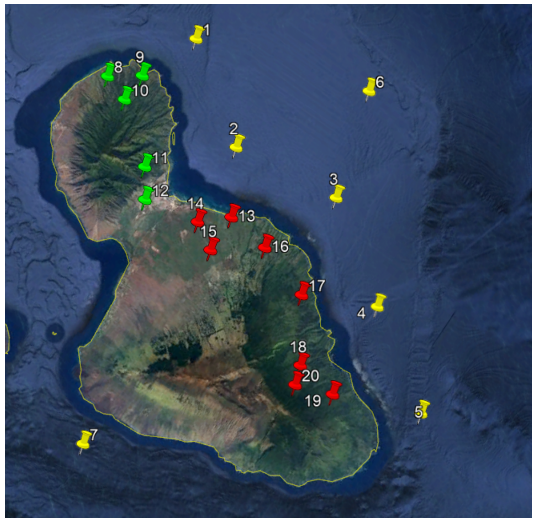
We obtain results from a no-wind environment with three scenarios, and the "clustered" network outlined in Chapter 3 using wind data obtained from COAMPS.

4.3.1 No-Wind Environment with Three Scenarios

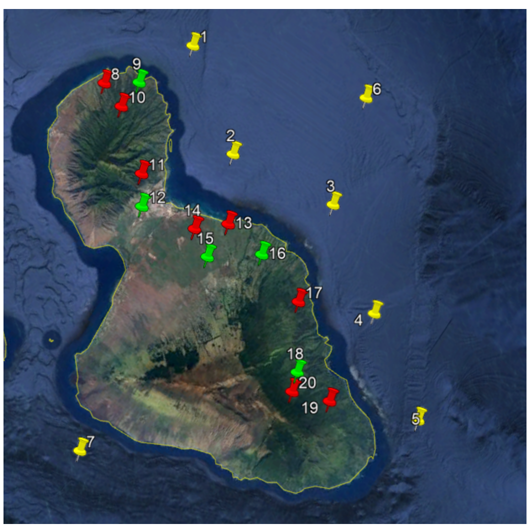
The first environment we consider uses three scenarios and is a hypothetical representation of five expeditionary units conducting operations on the island of Maui, HI. The motivation for this approach is a logistician's planning of last-mile resupply operations using probabilistic locations of five units requesting supplies, at a specific point in time. During the planning phase, an operation may require the five units to meet at a rendezvous point at a certain time, and it is the logistician's job to determine the best locations for two supply depot ships. Planners will use historical data for the probabilities of each scenario and a potential set of depot locations. The model is then run to determine which depot locations to select from the set of all possible locations.

Network Environment

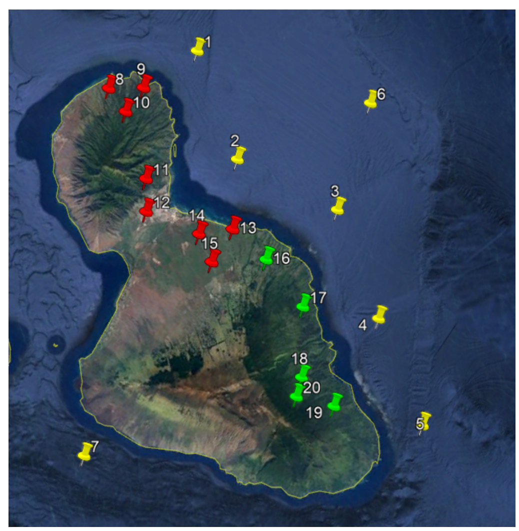
Figure 4.1 illustrates the first network we consider with three possible scenarios a logistician may encounter during a last-mile resupply operation. The figure depicts potential depot node locations (yellow pins) and a series of possible locations for five expeditionary units requesting resupplies, or demand nodes (green and red pins). Green pins in the figure represent "active" demand nodes and red pins represent "inactive" demand nodes for the scenario. Potential depot locations are fixed throughout all scenarios, but demand nodes change in each scenario as indicated by active or inactive. A logistician may also change other demand node and UAV parameters across scenarios. These parameters include demand importance level, requested supplies in pounds, UAV carrying capacity, and UAV energy capacity. The locations of demand nodes, variability of these parameters, and probability of occurrence make each scenario unique.



(a) Scenario 1



(b) Scenario 2



(c) Scenario 3

Figure 4.1. Three Possible Scenarios of Five Expeditionary Units Requesting Supplies

Data and Assumptions

The model uses a single cost matrix of distance (in miles) throughout all scenarios, regardless of demand node activity status. We use a no-wind, symmetric cost matrix in this network for simplicity and validation of the model, though the model can also obtain time-varying wind data from an asymmetric cost matrix as described in Chapter 3. The ultimate rendezvous point for the operation is the northwestern point of the island. Scenario and UAV data are shown in Figure 4.2.

	Supplies Requested (lbs)	Importance Level
Scenario 1		
Node 8	3	routine
Node 9	3	routine
Node 10	3	routine
Node 11	3	priority
Node 12	3	urgent
Scenario 2		
Node 9	3	routine
Node 12	3	priority
Node 15	3	urgent
Node 16	3	urgent
Node 18	3	urgent
Scenario 3		
Node 16	3	priority
Node 17	3	urgent
Node 18	3	urgent
Node 19	3	urgent
Node 20	3	urgent

(a) Scenario Data

	Supply Capcity (lbs)	Energy Capacity (miles)
Scenario 1		
UAV 1	15	40
UAV 2	15	40
UAV 3	15	40
Scenario 2		
UAV 1	15	40
UAV 2	15	40
UAV 3	15	40
Scenario 3		
UAV 1	15	40
UAV 2	15	40
UAV 3	15	40

(b) UAV Data

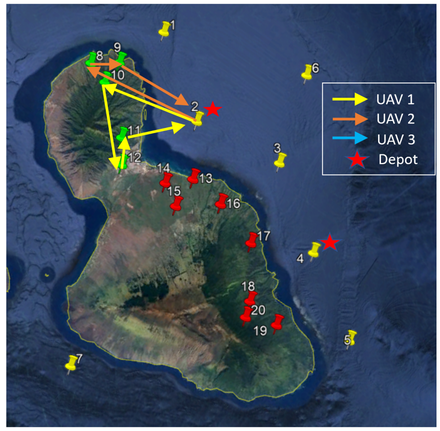
Figure 4.2. Three Possible Scenarios of Five Expeditionary Units Requesting Supplies

For planning purposes, we assume that the further away from the rendezvous point a unit (demand node) is, the higher level of importance its demand request will be. We can compare the importance level of demand requests shown in Figure 4.2a with the locations of the demand nodes shown in Figure 4.1. Scenarios 2 and 3 have more higher importance level requests because the respective demand nodes in those scenarios are further from the rendezvous point (Northwestern tip of Maui).

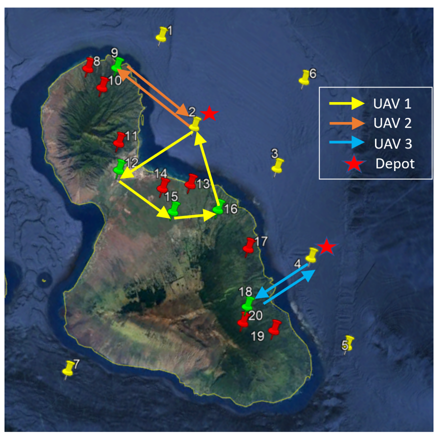
Scenarios 1, 2, and 3 have probabilities of occurrence 0.5, 0.1, and 0.4 respectively. For this simulation, the five units have higher probabilities of either being close in proximity to the rendezvous point or very far away from the rendezvous point the time when supplies are needed. These scenarios are for illustrative purposes only; in true operational planning, scenario probabilities will be obtained through proper analytical methods. The model chooses two depots from the set of depot nodes $\{1, 2, 3, 4, 5, 6, 7\}$. Each depot node can house three UAVs.

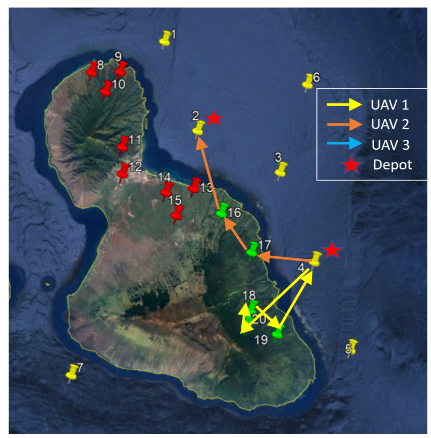
Model Results

The model chooses depots 2 and 4. Figure 4.3 shows the model output for each scenario and the respective optimal UAV routes. By choosing depots 2 and 4, the model is able to provide optimal UAV routing servicing all demand requests in each scenario.



(a) Scenario 1 Results





(b) Scenario 2 Results (c) Scenario 3 Results

Figure 4.3. UAV Routes of Three Possible Scenarios of Five Expeditionary Units Requesting Supplies Utilizing Chosen Depots 2 and 4

4.3.2 Cluster Environment Depot Selection

Figure 4.4 shows the results from the depot selection model run on the cluster demand environment outlined in Chapter 3. This network uses an asymmetric cost matrix of energy (in Wh), as well as the same data and assumptions mentioned previously in 3.2.1. The model is tasked to choose two depots from the set of depots $\{1, 2, 3, 4\}$. Each depot can house four UAVs. These results show it is more efficient and cost effective to use depot locations 1 and 2 when we expect all USMC units may have a routine demand request and dispersed in the operational environment as shown in Figure 4.4.

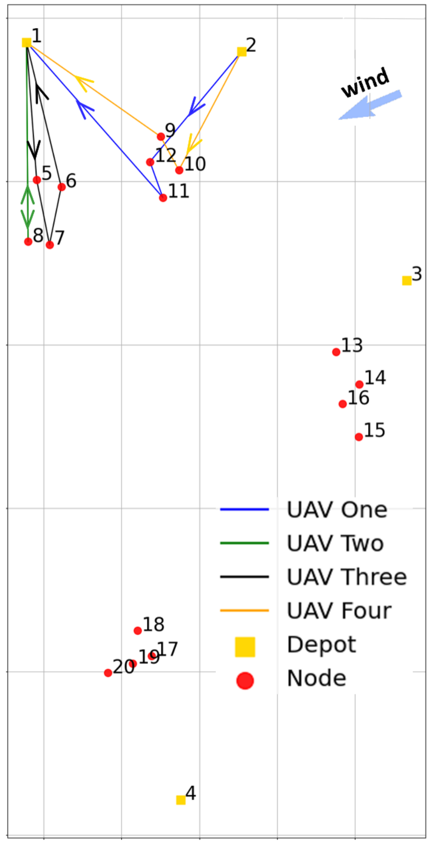


Figure 4.4. Depot Selection Model Choosing Two Depots in a Clustered Demand Network

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CHAPTER 5: Conclusion and Future Work

5.1 Conclusion

Using wind data and energy-aware flight path algorithms, we developed two models providing optimal UAV routing for Marines requesting supplies during last-mile EABO logistics. Through expansion of the vehicle routing model described in Jatho (2020), we have highlighted how multiple depots allow UAVs to fulfill more demand requests and improve energy efficient routing. Additionally, the depot selection model can help strategic planners choose the best locations for supply depots based on probable future operational scenarios. Our work continues to improve the performance and energy efficiency of UAV routing in last-mile resupply logistics.

5.2 Future Work

Our models can be improved through future work focusing on respective limitations. The multiple depot UAV routing model is limited in its ability to provide item-specific demand requests, timestamped UAV delivery deadlines, partial demand servicing, and dynamic UAV re-routing. We believe improving these areas will yield a better representation of real-world EABO logistics, and ultimately lead to better servicing of USMC units in the field of operations. Additionally, future work should include emphasis on expanding the depot selection model for large scale simulation.

5.2.1 The UAV Routing Model

Demand Specificity

The vehicle routing model currently only supports demand requests in weight. The demand nodes are not able to request specific items to be delivered by a resupply UAV. In real life last-mile logistics, Marines need specific supplies rather than a generic package. Our current model represents supplies delivered as a generic package because the model only supports demands based on weight. Future work should include line item demand requests.

This could include items such as specialty medical supplies, nuts and bolts, and other lightweight supplies requested by units in a last-mile scenario. A UAV might then be limited on load capacity in both weight and available space. It is more realistic to deliver specific items to Marines requesting supplies.

Delivery Deadlines

Our model assumes simultaneous demand requests and prioritizes deliveries based on a tiered importance level (routine, priority, urgent). It is more realistic to create a UAV routing model with scheduling based on timestamped demand requests. Future work should allow a UAV's route to prioritize both importance level and the time at which the request was sent. We recommend using queuing theory to determine the proper methods for creating a UAV routing model with scheduled servicing.

Partial Servicing

A partial servicing, or split delivery, UAV routing model could potentially increase energy efficiency of UAV routes. Currently, a demand request can only be serviced by a single UAV. This constrains the model in creating an all or nothing fulfillment of requests. For example, a UAV that is carrying five pounds of supplies is not able to service a demand node requesting seven pounds of supplies. The demand is left unfulfilled. A split delivery model would allow a UAV to fulfill what it can of a request. In the aforementioned example, that UAV could service five pounds of the seven requested, and the demand node would still require a service of the remaining two pounds of supplies. The work in Dror and Trudeau (1989) describes a split delivery vehicle routing problem (SDVRP) which showed savings in both the distance traveled by vehicles as well as the total number of vehicles used to service demand nodes (Dror and Trudeau 1989).

Dynamic UAV Re-routing

To further increase efficiency, a delivery routing model should also consider new demand requests that may be sent while a UAV is en route. Currently, the UAVs in our model do not have the ability to re-route and potentially fulfill additional requests that come in while that UAV is servicing other demand nodes. Our model provides a static optimal UAV routing system. The work seen in Kucharska (2019) describes dynamic vehicle routing problem (DVRP) modeling techniques which dynamically design a set of routes for vehicles to serve customers, while new customer requests arrive in the system (Kucharska 2019). In a real-world EABO last-mile logistics scenario, demand requests are sent into the system spontaneously whenever units in the field need supplies. Allowing UAV re-routing in flight would give the model the ability to adapt to incoming demand requests without forcing the UAV to return to a supply depot for routing instructions. This would potentially improve overall effectiveness of the model by fulfilling additional prioritized demands dynamically. This principle could also be applied to account for supply depot ship relocation while a UAV is conducting a servicing route.

5.2.2 Depot Selection

Expansion for Large-Scale Simulation

The depot selection model is a strategic planning tool used to help best design operational infrastructure and place command assets. Strategic planning aims at preparing a command for problems that could occur during an operation. The depot selection model can help logisticians choose the best depot locations by simulating future last-mile demand scenarios. The examples in our work illustrating the model's use were arbitrarily chosen and developed from personal military experience. However, expansion of the model for large-scale simulation could give a better representation of future demand scenarios, producing improved selection of supply depot locations.

5.3 Final Thoughts

We must continue building on our work to improve conditions for resupplying Marines in a contested environment. Today's fight against our adversary requires units to keep a low operational footprint while maintaining combat readiness. The future of the United States Marine Corps relies on technological advancements further developing its EABO concept. Deployment of UAVs used in resupply logistics provides further sustainment and protection of our most valuable asset—personnel.

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