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Mitigating correlated noise in quantum machines

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14. ABSTRACT Noise is ever-present in quantum devices and has been studied in detail for decades. However, typical noise models make rather strong assumptions to make the problem tractable. In reality, noise in quantum computers is more complex and is a major obstacle standing in the path of developing functional quantum technologies. This project has worked to translate theoretical advances into experimental reality. Several achievements and efforts have advanced this work. First was to characterize non-Markovian noise in real quantum computers. Second was development of a complete toolkit for non-Markovian noise characterization. Third was showing that multi-time non-Markovian correlations can be used to reduce the noise of any quantum computer. And fourth was uncovering that just like in space, entanglement in time can also have a nontrivial manifestation.			
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The setting: quantum computers and complex noise

Noise is ever-present in quantum devices and has been studied in detail for decades. However, typical noise models make rather strong assumptions to make the problem tractable. Namely, the noise almost always is assumed to be uncorrelated in time or *Markovian*. Consider a sequence of gates, say G_1 , G_2 , G_3 , ... applied on a qubit. One usually assumes that the error in G_3 is independent of that in G_2 and G_1 , and the error in G_2 is independent of that in G_1 . This is the Markov assumption in practice.

In reality, noise in quantum computers is more complex. That is, the error in G_2 will depend on the errors in G_1 as well as the choice of gate G_1 (it could be the T gate or the phase gate, etc.). Similarly, G_3 may depend on both G_1 and G_2 , in a rather complicated manner. Such correlations in space-time, are often called *non-Markovian*, which are typically ignored because of the fundamental challenges in describing them.

Today, non-Markovian noise is a major obstacle standing in the path of developing functional quantum technologies. This is because, when such noise is not suppressed then its complexity grows with the size of the computation. That is, the noise becomes as complex as the computation. This then makes it impossible for a quantum computer to perform a useful computation that is non-trivial.

The reason why non-Markovian noise has not been studied in detail has to do with the invasive nature of quantum measurements. In the last five years, my group has developed a complete theoretical framework to overcome this foundational challenge. We are able to describe *any* non-Markovian quantum noise in straightforward manner [Pollock-2018-PRA, Pollock-2018-PRL, Milz-2020-Quantum, Milz-2021-PRX Quantum].

The present project is concerned with translating the theoretical advances made by my group into experimental reality. And in the past two years, we have made remarkable progress on this front. Last year we were able to successfully characterize non-Markovian noise on several IBM's commercial-grade devices. Our achievements are possible by combining the theoretical prowess of the Monash group with hands-on experience with real quantum computers of the University of Melbourne group, which has a remarkable track record on the practical side of quantum computing [Dang-2019-Quantum, White-2020-NatComm, Dang-2021-arXiv].

1 Noise characterization [White-2020-NatComm]

Our first achievement was to characterize non-Markovian noise in real quantum computers, published in the highly prestigious journal *Nature Communications*. There we report the following important facts:

- Unlike the previous experiments reporting non-Markovian phenomena, we did not engineer the complex noise. Rather we characterized the actual non-Markovian noise that plagues these devices.
- Our experiment reports multitime quantum correlations. This has not been achieved before as all prior studies average two-time correlations over a time duration. The averaging is especially problematic, as doing so makes it impossible to design better control over the device.
- Our characterization outperforms the state-of-the-art tools like the *gate set tomography*. This is not meant to be a comparison as GST makes the Markov assumption. However, it does mean that future characterization methods must integrate our tools designed specifically to address complex noise.

2 A toolkit for complex noise characterization [White-2021a-arXiv]

As noted above, there is a crucial difference between our noise characterization methods and previous studies of non-Markovianity. Our characterization can be directly used to better control the noise in a machine. To do this in practice, we have built on Ref. White-2020-NatComm] by developing a complete toolkit for non-Markovian noise characterization in Ref. [White-2021a-arXiv]. Here, we address important scaling issues, as well as estimation issues stemming from incomplete data and measurement errors (SPAM), and then navigate around the noise using this characterization information. The specific achievements are:

- We combine the non-Markovian noise characterization with a maximum likelihood estimation procedure. To do so, we must account for the causality conditions of the process, restricted control set of real quantum devices, as well as shot-noise and SPAM errors. The resultant toolkit reduces the

number of circuits required for characterization from $\mathcal{O}(24^k)$ to $\mathcal{O}(10^k)$, for one qubit where k is the number of timesteps.

- Even after implementing the maximum likelihood estimation procedure, the characterization still suffers from exponential scaling with the number of timesteps k . We address this by integrating above method with the theory of quantum Markov order theory [Taranto-PRL-2019, Taranto-PRA-2019, Taranto-npjQI-2021] and temper the exponential scaling to a linear scaling, i.e., $\mathcal{O}(k \cdot 10^\ell)$, where ℓ is the fixed Markov order. This is not only practical, it provides an accessible diagnostic to the complexity of device noise. It is important to note that without the maximum likelihood estimation a Markov order integration would not be possible.
- Finally, we show the direct utility of these tools. Namely, we characterize noise for Markov orders of $\ell = \{1, 2, 3\}$ and use this information to significantly increase the fidelity of several NISQ devices by using non-Markovian correlations as a resource. We show that the performance of the machine improves when the Markov order is chosen to be higher. This leads to a trade-off relation between characterization complexity and accuracy of the characterization. This is probably our most impactful achievement, establishing a trade-off relation between characterization complexity and its precision.

This work is under review in [PRX Quantum](#).

3 Resource extraction from noise [Berk-2021-arXiv]

In 2021, we developed a formal theory for quantifying the resources residing within a quantum process [Berk-2021-Quantum]. Building on this, we have now shown that multitime non-Markovian correlations can be used to reduce the noise of any quantum computer. Our new protocol, generalize the well-known dynamical decoupling protocol used for suppressing decoherence. As such, we show that our protocol is always as good as dynamical decoupling, but in practice outperforms dynamical decoupling by orders of magnitude.

In short, our protocol first characterizes noise and then finds an optimal decoupling sequence. This can be done on the go and incrementally to suppress the characterization complexity. In fact, Ref. [White-2021a-arXiv] experimentally demonstrates better control over a quantum computer. This is achieved by suppressing noise proportional to the degree of accuracy of the characterization.

This is a major achievement and as such this work is under review in [Nature Physics](#). (Getting past the editors is highly nontrivial).

4 A toolkit for many-time physics [White-2021b-arXiv]

It is well-known that entanglement, spooky correlation in space, plays a crucial role in the physics of interacting many bodies. This field, *many-body physics*, is a very mature topic that focuses on different phases of matter that are of immense interest, both for foundational and technological reasons. For instance, one can determine if a material will behave like a superfluid given some parameters.

We have recently uncovered that just like in space, entanglement in time can also have a nontrivial manifestation [Milz-2021-SciPost]. This suggests that entanglement, and other correlations, in time, will lead to a new class of dynamical phases, opening the door to a whole new branch of physics, which we call *many-time physics*. We anticipate many-time physics to be just as rich, but so far remains unexplored.

The characterization toolkit of §2 opens windows to new physics. In particular, we have developed a method in [White-2021b-arXiv] to acquire facets of many-time physics from an incomplete characterization of quantum noise. In particular, we can estimate entanglement, entropies, and other fundamental quantities. We are currently in the process of integrating *shadow tomography* [Huang-2020-Nature Physics] into the framework of many-time physics. This will allow understanding many features of quantum noise and quantum dynamics with only a few measurements. Quantum computers are fertile ground for this new branch of physics, and arguably better placed than studying spatial quantum features.

Our framework is naturally integrated with the language of tensor networks. We have shown that quantum processes are naturally cast as matrix product operators [Pollock-2018-PRA] for finite memory, as well as the likes of multiscale entanglement renormalization ansatz (MERA) for slowly decaying correlations. In 2018, at KITP in Santa Barbara, we integrated a newly developed machine learning protocol designed specifically for matrix product operators by Dario Poletti of Singapore University of Technology & Design [Guo-2020-PRA]. We are now finalizing an experiment that will report an efficient characterization multitime correlations, a central element of many-time physics, with machine learning [Huang-2022-In prep.].

5 Other progress

In the past year, we have also developed other theoretical and experimental tools to determine coarse features of a quantum process. In particular, we have extended the well-known randomized benchmarking protocol to the non-Markovian regime [[Figueroa-Romero-2021-PRX Quantum](#)]. We have also extended classical Markov monogamy inequalities to the quantum regime [[Capela-2021-arXiv](#)]. The former is published in PRX Quantum and the latter is under review in PRX Quantum.

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