AFRL-AFOSR-UK-TR-2022-0037

ATHENA: Aero-Thermo-Elastic Nonlinear reduced order modeling for hypersonic Airframes

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03/30/2022
Final Technical Report

DISTRIBUTION A: Distribution approved for public release.

Air Force Research Laboratory
Air Force Office of Scientific Research
European Office of Aerospace Research and Development
Unit 4515 Box 14, APO AE 09421

REPORT DOCUMENTATION PAGE

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1. REPORT DATE 20220330	**2. REPORT TYPE** Final	**3. DATES COVERED**	
		START DATE 20180930	**END DATE** 20210929

4. TITLE AND SUBTITLE
ATHENA: Aero-Thermo-Elastic Nonlinear reduced order modeling for hypersonic Airframes

5a. CONTRACT NUMBER	**5b. GRANT NUMBER** FA9550-18-1-0508	**5c. PROGRAM ELEMENT NUMBER**
5d. PROJECT NUMBER	**5e. TASK NUMBER**	**5f. WORK UNIT NUMBER**

6. AUTHOR(S)
Paolo Tiso

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Eidgenössische Technische Hochschule ETH Rämistrasse 101 Zurich 8092 CH	**8. PERFORMING ORGANIZATION REPORT NUMBER**

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) EOARD UNIT 4515 APO AE 09421-4515	**10. SPONSOR/MONITOR'S ACRONYM(S)** AFRL/AFOSR IOE	**11. SPONSOR/MONITOR'S REPORT NUMBER(S)** AFRL-AFOSR-UK-TR-2022-0037

12. DISTRIBUTION/AVAILABILITY STATEMENT
A Distribution Unlimited: PB Public Release

13. SUPPLEMENTARY NOTES

14. ABSTRACT
See final report for details.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT** SAR	**18. NUMBER OF PAGES** 19
a. REPORT U	**b. ABSTRACT** U	**c. THIS PAGE** U		

19a. NAME OF RESPONSIBLE PERSON DAVID SWANSON	**19b. PHONE NUMBER** *(Include area code)* 785-6565

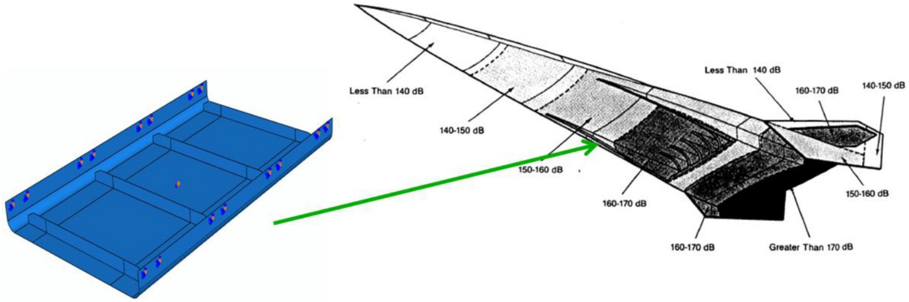
ETH zürich

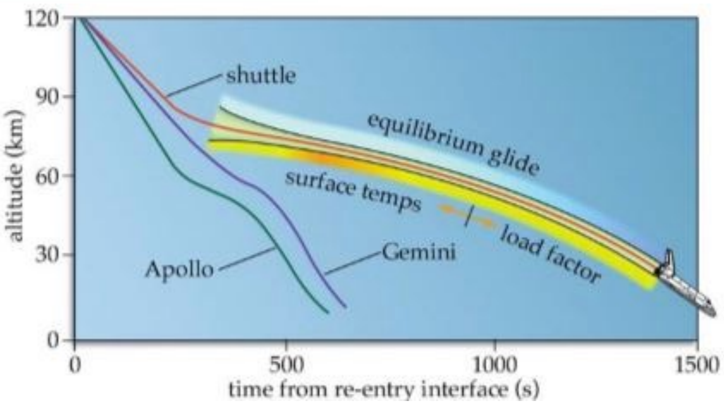
Aero-**TH**ermo-**E**lastic **N**onlinear reduced order modeling for hypersonic **A**irframes: **ATHENA**

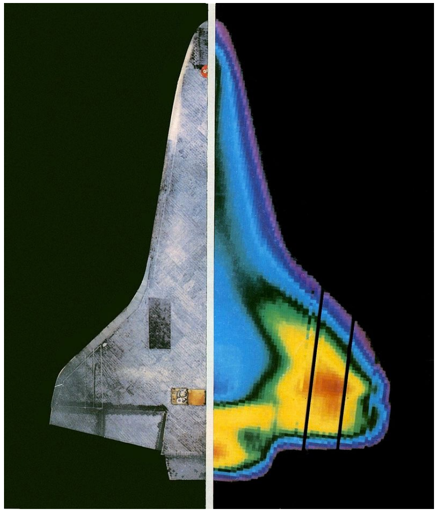
Morteza Karamooz Mahdiabadi, Paolo Tiso

Final Report

Motivation







Assess the **structural dynamics** in presence of **time-varying thermal and aerodynamic loads**, for **long times**: **fatigue analysis**

AFOSR – Grant No. 1191404 – *Reduced Order Modeling for Hypersonic Aeroelasticity:* ***ROMA*** | Jan 2016-Dec 2018

AFOSR – Grant No. FA9550-18-1-0508 - *Aero-THermo-Elastic Nonlinear reduced order modeling for hypersonic Airframes:* ***ATHENA*** | Sept 2019-Sept 2021

Motivation – what model features we need?

- Arbitrary geometries: **large FEM**
- Temperature field **space and time dependent**
- **Geometric nonlinearities** to account for
  - bending stiffness reduction due to thermally induced compression loads
  - Buckling/snapthrough
  - Large deflections
  - Limit cycle oscillations
- Temperature dependent material properties **– degradation**
- Aero loads via **piston-theory** + **fluctuations** due to turbulence
- Aero-thermo-mechanical **coupling**

Motivation – where is the bottleneck?

- Design/optimization require **lots of analyses**
- **High Fidelity Models** (HFMs) are **unfeasible** do to time and memory resources required (1 iteration might require hours/days)
- Current practice: **neglect coupling/transient response**, or resort **to lumped models** to trade for speed

☞ Need for **Reduced Order Models** (ROMs)

Outcome: ROMs that enable design iterations at reasonable times (order of magnitude faster than HFMs), while **keeping all the essential features of the HFMs**, without requiring abstraction, simplification or lumping.

One-way thermo-elastic coupling

1. S. Jain, P. Tiso, **Journal of Sound and Vibration**, (2020) [PDF]

Thermal problem

$$\mathbf{M}_T\mathbf{T}' + \mathbf{f}_T\left(\mathbf{T}\right) = \mathbf{h}(\tau)$$

$$\mathbf{T}(\tau)$$

Structural problem

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{f}\left(\mathbf{u}, \mathbf{T}\right) = \mathbf{g}\left(t\right)$$

Solve the thermal problem first, then pass the $\mathbf{T}(\mathbf{x},\tau)$ to the structural problem.

Non-existence of

1. ***equilibrium***
2. ***invariant spaces for reduction***

Standard techniques (hot modes, cold modes, etc),not well grounded

Temperature evolves slowly when compared to structural periods:
time scale dichotomy.

"Thermal" time

"mechanical" time

$$\tau = \epsilon t, \quad \epsilon \ll 1$$

$$\frac{d\star}{dt} = \epsilon \frac{d\star}{d\tau}$$

During a characteristic time interval for thermo problem, not much is happening to the structure

Method of Multiple Scales

Solution depends **on two different time scales**

$$\mathbf{u} = \mathbf{u}(t, \tau)$$

Expansion in ϵ

$$\mathbf{u}(t, \tau) = \mathbf{u}_0(t, \tau) + \epsilon \mathbf{u}_1(t, \tau) + \epsilon^2 \mathbf{u}_2(t, \tau) + \ldots$$

Leading order: nonlinear problem

$$\mathcal{O}(1): \qquad \mathbf{M}\frac{\partial^2 \mathbf{u}_0}{\partial t^2} + \mathbf{C}\frac{\partial \mathbf{u}_0}{\partial t} + \mathbf{f}\left(\mathbf{u}_0, \mathbf{T}(\tau)\right) = \mathbf{p}\left(t, 0\right)$$

Linear system with slow temperature variation: τ is a parameter!

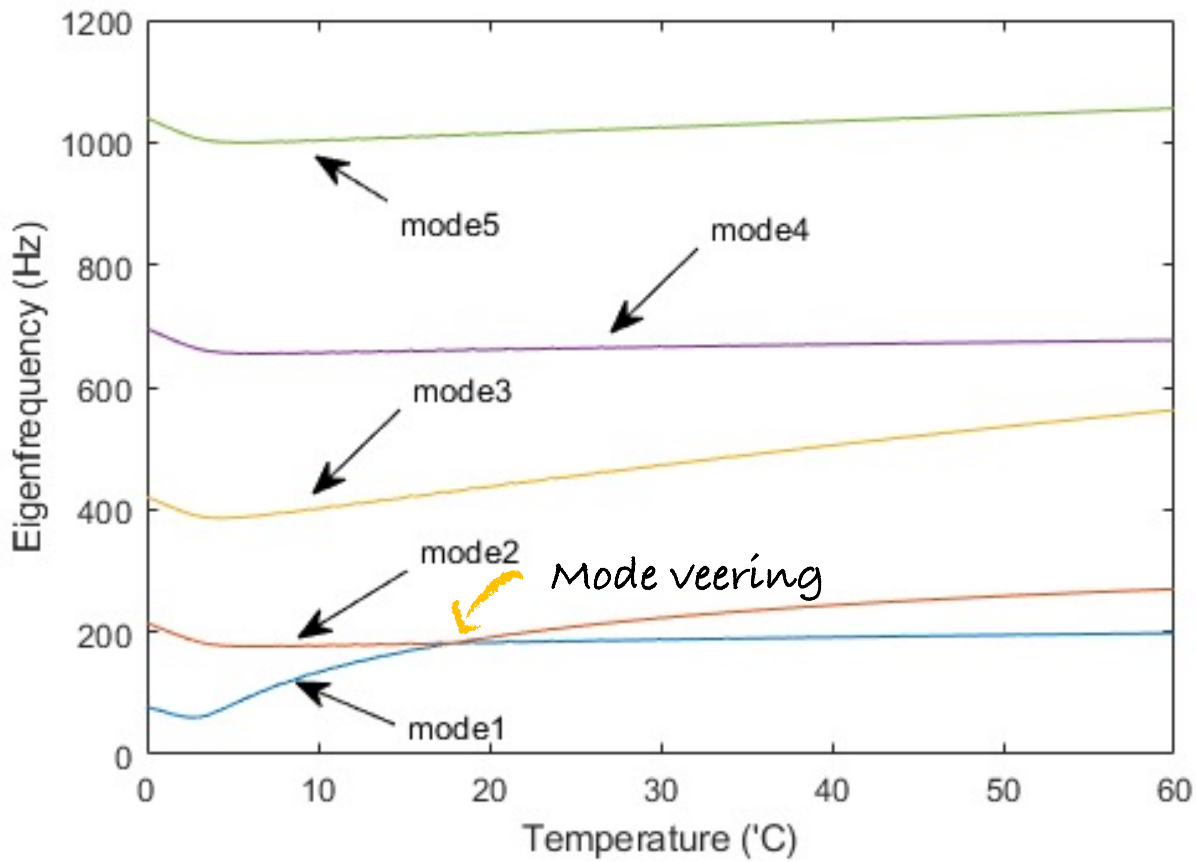
Parameter

$$\mathbf{M}\frac{\partial^2 \mathbf{u}_0}{\partial t^2} + \mathbf{C}\frac{\partial \mathbf{u}_0}{\partial t} + \mathbf{K}\left(\mathbf{T}(\tau)\right)\mathbf{u} + \mathbf{b}(\mathbf{T}(\tau)) = \mathbf{p}\left(t, 0\right)$$

This justifies parametric equilibrium… …and reduction basis.

$$\mathbf{u}_{eq}(\tau) = -\mathbf{K}\left(\mathbf{T}(\tau)\right)^{-1}\mathbf{b}(\mathbf{T}(\tau)) \qquad \left[\mathbf{K}(\mathbf{T}(\tau)) - \omega_i^2(\tau)\mathbf{M}\right]\phi_i(\tau) = \mathbf{0}$$

Modal basis variation



Modes might experience ***veering*** as temperature changes. This can be taken into account by the algorithm discussed in [4].

Galerkin projection

$$\boldsymbol{\Phi}(\tau)^T\mathbf{M}\boldsymbol{\Phi}(\tau)\frac{\partial^2\mathbf{q}_0}{\partial t^2}+\boldsymbol{\Phi}(\tau)^T\mathbf{C}\boldsymbol{\Phi}(\tau)^T\frac{\partial\mathbf{q}_0}{\partial t}+\boldsymbol{\Phi}(\tau)^T\mathbf{f}\left(\mathbf{q}_{eq}(\tau)+\boldsymbol{\Phi}(\tau)\mathbf{q}_0,\mathbf{T}(\tau)\right)=\boldsymbol{\Phi}(\tau)^T\mathbf{p}\left(t,0\right)$$

✓ The ROM adapts to the underlying, slowly changing equilibrium;

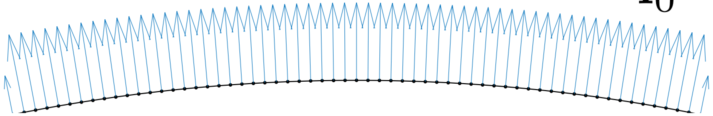
✓ No basis time derivatives present;

Example:

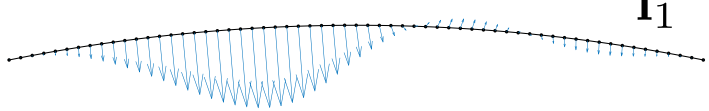
turbulence

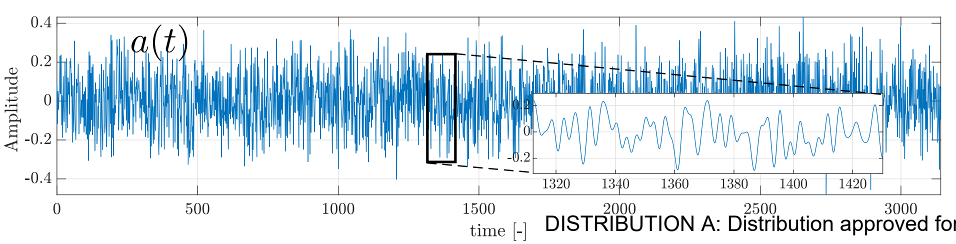
$$\mathbf{p}(t,\epsilon)=\mathbf{l}_0\sin\omega_c t+\epsilon\mathbf{l}_1 a(t)$$

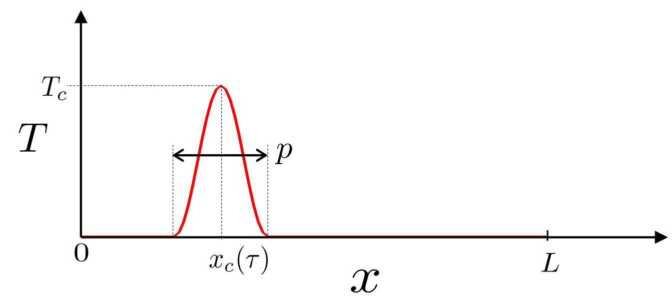
Uniform load shape $\mathbf{l}_0$

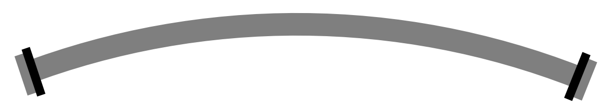


Perturbation shape $\mathbf{l}_1$



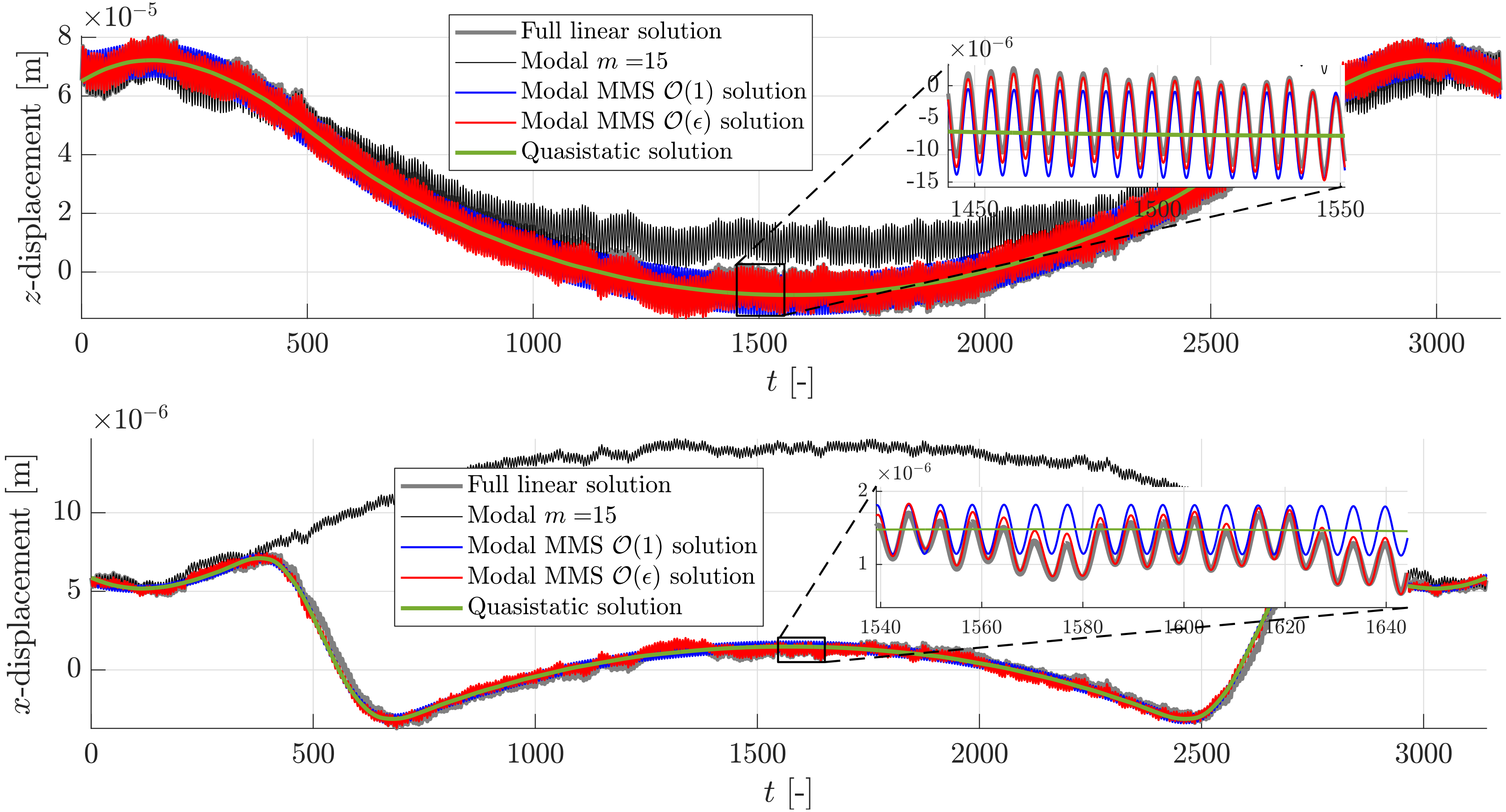






- Temp. profile traveling over the arch
- Mechanical load excites first 3 modes
- ROM: 5 modes, interpolated between 19 temperature configurations

Example: Curved Arch



Parametric ROM

$$\boldsymbol{\Phi}(\tau)^T\mathbf{M}\boldsymbol{\Phi}(\tau)\frac{\partial^2\mathbf{q}_0}{\partial t^2}+\boldsymbol{\Phi}(\tau)^T\mathbf{C}\boldsymbol{\Phi}(\tau)^T\frac{\partial\mathbf{q}_0}{\partial t}+\boldsymbol{\Phi}(\tau)^T\mathbf{f}\left(\mathbf{q}_{eq}(\tau)+\boldsymbol{\Phi}(\tau)\mathbf{q}_0,\mathbf{T}(\tau)\right)=\boldsymbol{\Phi}(\tau)^T\mathbf{p}\left(t,0\right)$$

The highlighted terms need to be efficiently computed.

How to make the ROM efficient *online*?

1. *Non-intrusive computation of ROM terms*
2. *Efficient interpolation*

Non-intrusive methods

$$\tilde{\mathbf{M}}\ddot{\mathbf{q}} + \tilde{\mathbf{C}}\dot{\mathbf{q}} + \tilde{\mathbf{K}}\mathbf{q} + \tilde{\mathbf{f}}_{nl}(\mathbf{V}\mathbf{q}) = \tilde{\mathbf{g}}$$

Function of modal coordinates only!

$$\mathbf{V}^T \mathbf{f}_{nl}(\mathbf{V}\mathbf{q}) = (\tilde{\mathbf{f}}_{nl})_I = \sum_i^m \sum_j^m \alpha_{ijI} q_i q_j + \sum_i^m \sum_j^m \sum_k^m \beta_{ijkI} q_i q_j q_k$$

Coefficients of **assumed nonlinear force** obtained by **fitting** with nonlin. forces necessary to hold structure into the shape of dominant modes, **at chosen amplitudes.**

Done conveniently with **off-the shelf FE programs** (Abaqus, Ansys, …): "**non-intrusive**".

Implicit Condensation and Expansion (ICE)

M. I. McEwan, J. R. Wright, J. E. Cooper, and A. Y. T. Leung, "***A combined modal/finite element analysis technique for the dynamic response of a non-linear beam to harmonic excitation***," Journal of Sound and Vibration, vol. 243, pp. 601-624, 2001.

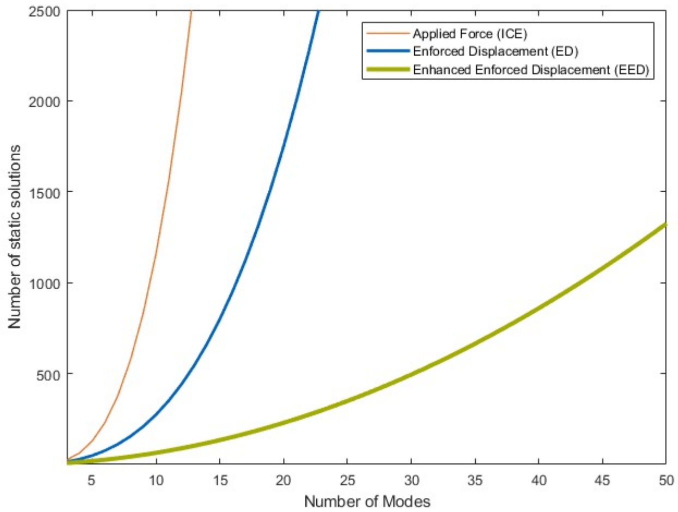
Enforced Displacements (ED)

A. A. Muravyov and S. A. Rizzi, "***Determination of nonlinear stiffness with application to random vibration of geometrically nonlinear structures***," Computers & Structures, vol. 81, pp. 1513-1523, 2003

Non-intrusive methods

State of the art:

1. They require lots of static solutions/evaluations
2. The modes necessary to capture the geometric nonlinearity (***Dual modes***) in the reduction basis also require extensive computations



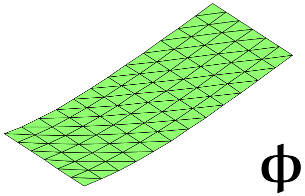
Our contribution:

Non-intrusive Modal derivatives appended to the linear modes basis

M.K. Mahdiabadi, P. Tiso, A. Brandt, DJ Rixen, **MSSP** (2020) [PDF]

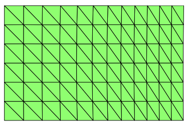
$$\mathbf{K}_t\left(\mathbf{u}_{eq}(\tau),\mathbf{T}(\tau)\right)\boldsymbol{\theta}_{ij} = -\left[\frac{\partial^2\mathbf{f}}{\partial\mathbf{u}\partial\mathbf{u}}\left(\mathbf{u}_{eq},\mathbf{T}(\tau)\right)\cdot\boldsymbol{\phi}_j\right]\boldsymbol{\phi}_i$$

Modal derivatives automatically account for geometrically nonlinear deformation induced by vibration modes. Their computation *is systematic*, as compared to Dual Modes, and lead to generally more consistent results.



$\boldsymbol{\phi}_1$

Vibration Mode



$\boldsymbol{\theta}_{11}$

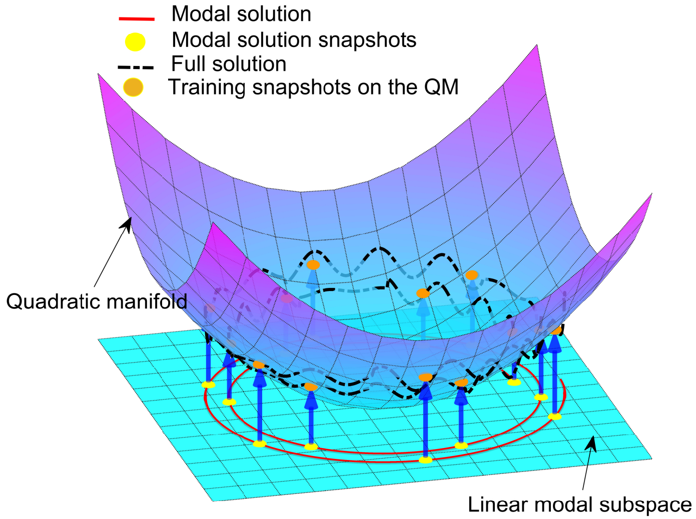
Modal Derivative

Non-intrusive methods

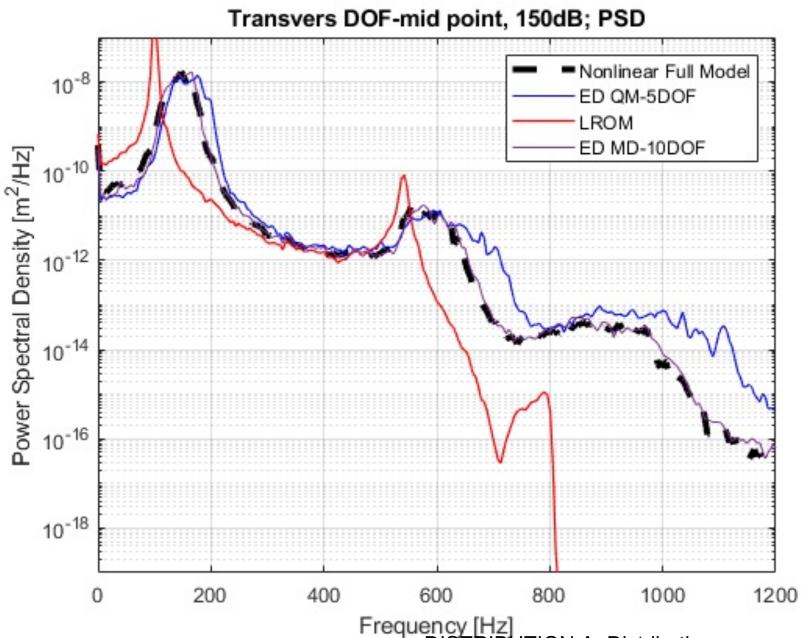
Our contribution (cont'd):

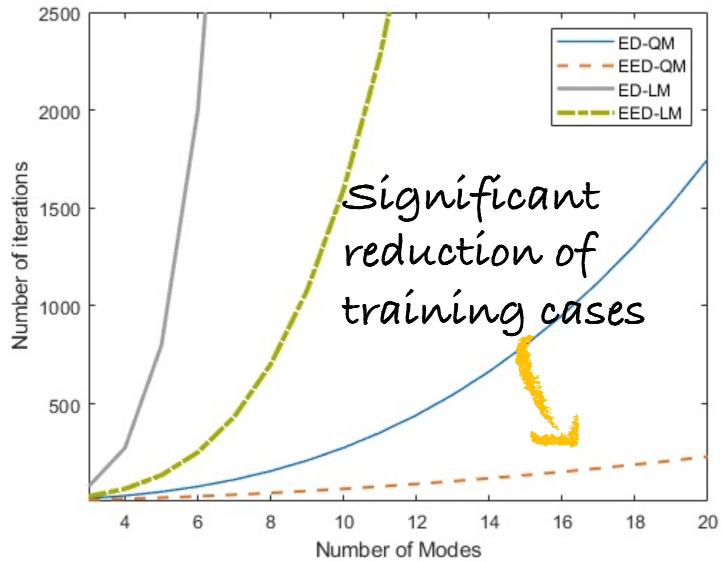
$$\mathbf{u} = \mathbf{\Xi}(\mathbf{q}) = \mathbf{\Phi q} + \frac{1}{2}\mathbf{\Theta qq}, \ \ \mathbf{\Xi} : \mathbb{R}^m \rightarrow \mathbb{R}^n$$

Usage of **quadratic manifold** (built with VMs and MDs) to significantly reduce the offline cost of training cases.

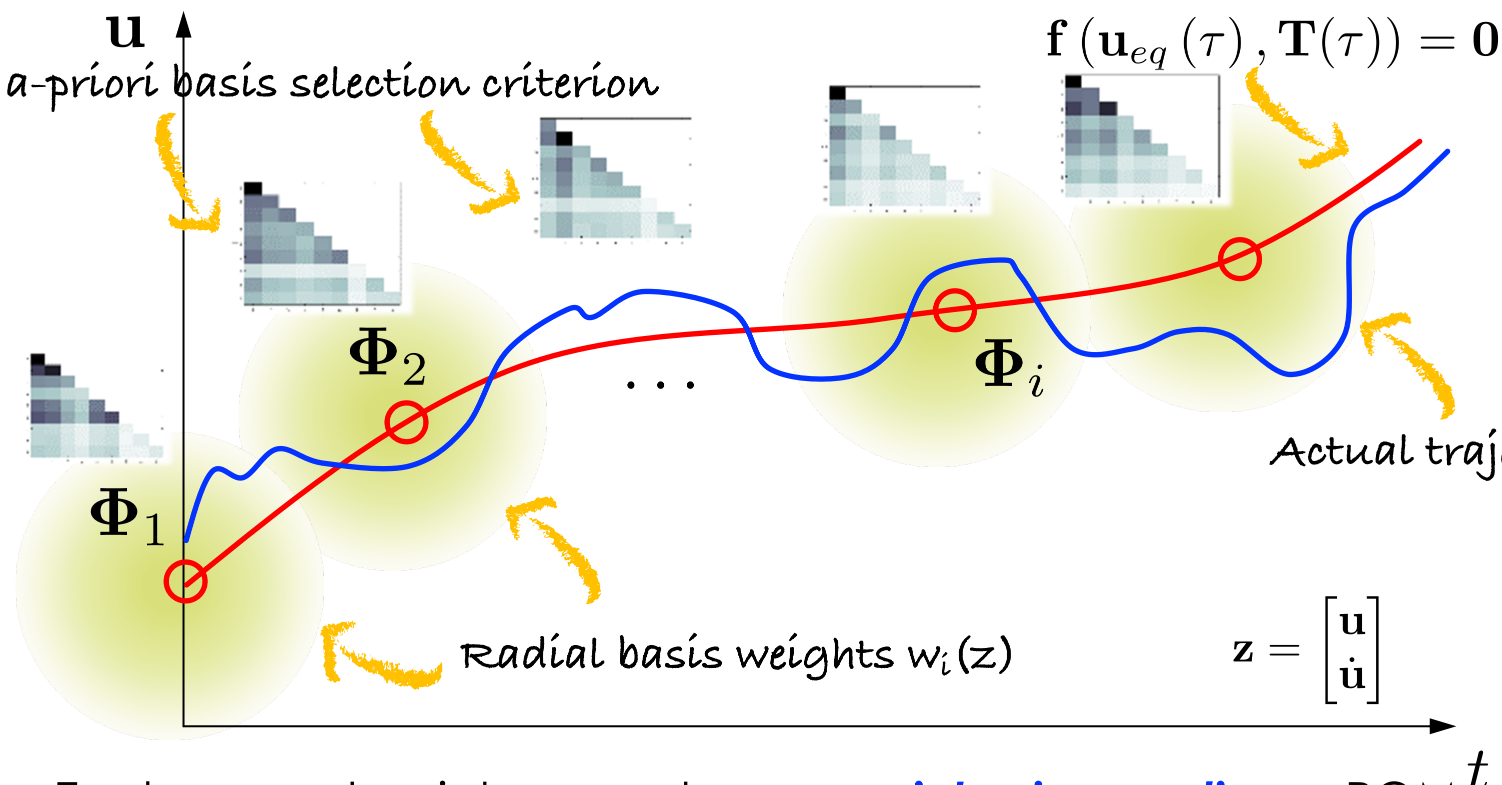


Example: flat beam

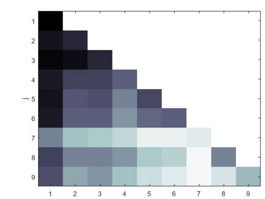




Interpolated ROM



Max. modal interaction



$$\hat{\mathbf{M}}\ddot{\mathbf{q}}(t) + \hat{\mathbf{K}}^{(1)}\mathbf{q}(t) = \hat{\mathbf{f}}(t)$$

$$W_{ij} = \int_0^T \left| q_i(t)\, q_j(t) \right| dt.$$

Each support point generates a ***non-intrusive nonlinear*** ROM

$$\dot{\tilde{\mathbf{z}}} = \mathbf{A}_i(\tilde{\mathbf{z}} - \tilde{\mathbf{z}}_i) + \tilde{\mathbf{f}}_i(\tilde{\mathbf{z}} - \tilde{\mathbf{z}}_i) + \tilde{\mathbf{p}}$$

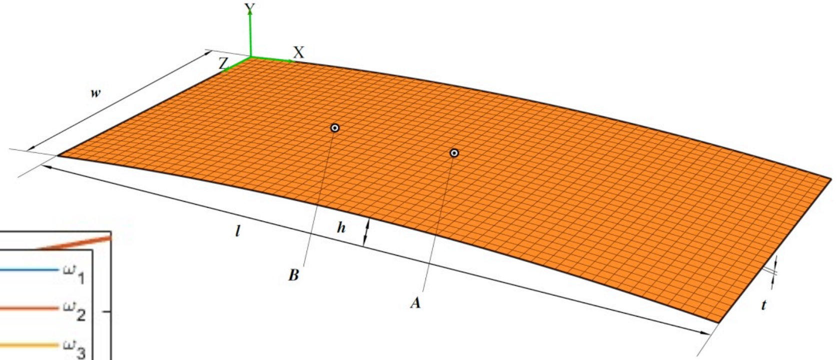
ROMs are weighted online by radial basis functions

$$\dot{\tilde{\mathbf{z}}} = \sum_i^N w_i(\tilde{\mathbf{z}} - \tilde{\mathbf{z}}_i) \left[\mathbf{A}_i(\tilde{\mathbf{z}} - \tilde{\mathbf{z}}_i) + \tilde{\mathbf{f}}_i(\tilde{\mathbf{z}} - \tilde{\mathbf{z}}_i) + \tilde{\mathbf{p}} \right]$$

Example: curved shell

Temperature distribution: uniform in space, increasing in time

- ROM: 9 modes, interpolated between 25 temperature configurations



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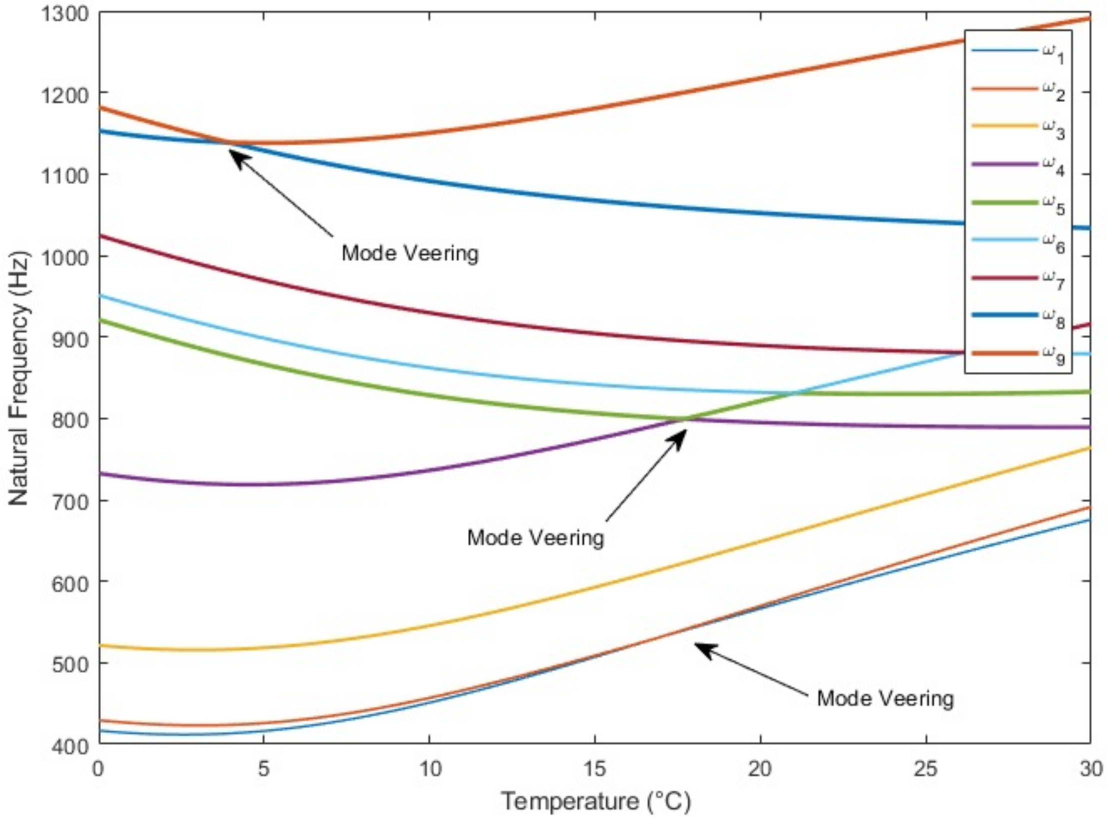
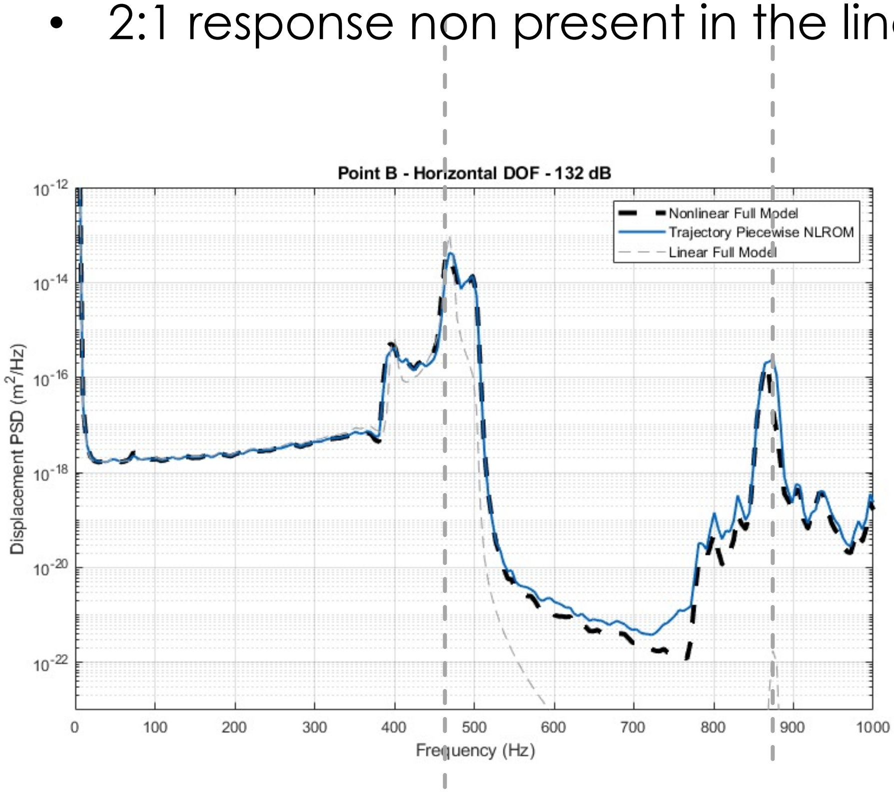
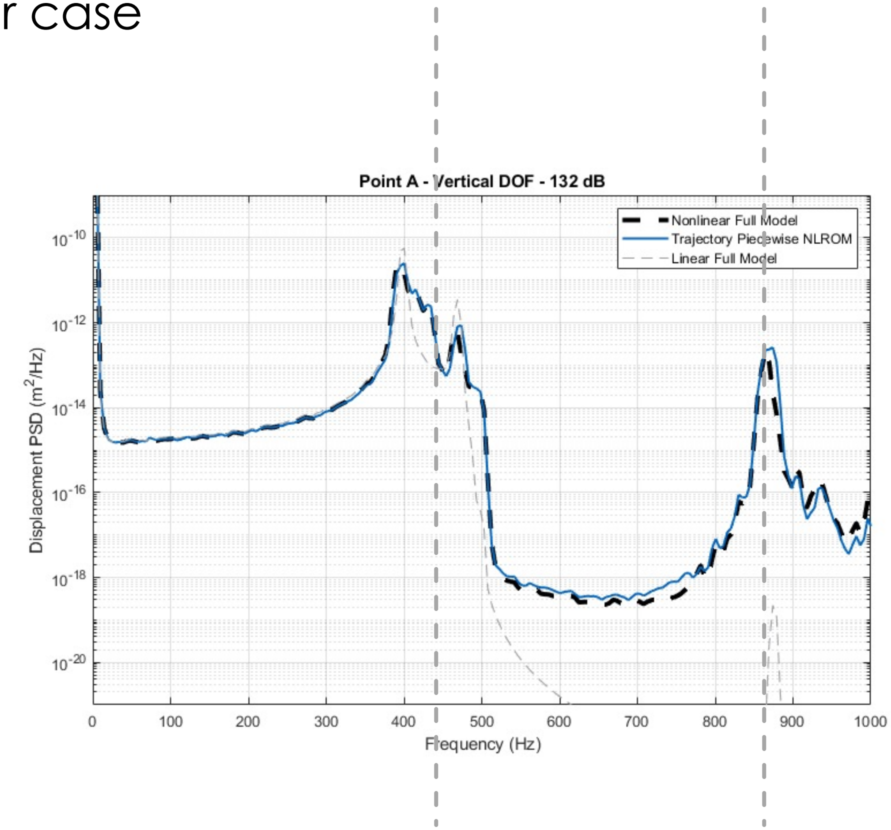


Example: curved shell

1. MK Mahdiabadi, P. Tiso, **under preparation**

- 2:1 response non present in the linear case





16 cores
25 equilibrium points
31 seconds real time
ROM: 5.1 hours
Full order model: about 11 days

Summary

- **Slow variation** in temperature is not only **physically relevant** but also essential to **justify** model reduction using an **adaptive reduction basis.**
- Use of **multiple scales method** to exploit the time scale dichotomy.
- Order epsilon **correction important** in presence of **small mechanical loads.**
- In the **geometrically nonlinear** setting, the basis is enriched using the **modal derivatives** corresponding to significant **vibration modes.**
- **Non-intrusive** construction of ROM using **modal derivatives** and **quadratic manifolds** for increased offline efficiency
- For **hyper-reduction** of nonlinear terms, it is possible to avoid HFM based training by **lifting modal solutions on quadratic manifolds**.
- **Online ROM interpolation** for efficiency once precomputed equilibria and basis at support points are available.

Remaining Technical Challenges

- Inclusion of aerodynamic and turbulence
  - Prof. Dimitris Drikakis (un. Cyprus) will develop an **acoustic model** which will be validated against high-fidelity numerical simulations and experiments, and **will offer the model to ETH** for inclusion into the **acoustic-thermo-elastic analysis** framework for hypersonic airframes.
  - Thermal buckling/snap-through?
- Parametrization
  - Recently developed a ROM that includes geometric uncertanties (J. Marconi, P. Tiso, F. Braghin, CMAME, 2019)
  - An idea for slightly curved structures?
- Efficient online interpolation for reduced operators
  - Trajectory Piecewise weighted Linearization

Publications

1. MK Mahdiabadi, P. Tiso, **under preparation**
2. MK Mahdiabadi, P. Tiso, A. Brandt, DJ Rixen, **MSSP** (2020)
3. MK Mahdiabadi, A Bartl, D Xu, P Tiso, DJ Rixen, **Journal of Sound and Vibration** (2019)
4. *S. Jain*, P. Tiso, **Journal of Sound and Vibration,** (2020)
5. *S. Jain*, P. Tiso, **ASME Journal of Computational and Nonlinear Dynamics** (2019)
6. *S. Jain*, P. Tiso, G. Haller, **Journal of Sound and Vibration** (2018)
7. *S. Jain*, P. Tiso, **ASME Journal of Computational and Nonlinear Dynamics** (2018)
8. *J.B. Rutzmoser*, D.J. Rixen, *S. Jain*, P. Tiso, **Computers & Structures** (2017)
9. *S. Jain*, P. Tiso, J.B. Rutzmoser, D.J. Rixen, **Computers & Structures** (2017)