S-1803

DECLASSIFIED

6 November [redacted]

NRL Report No. S-1803

FR-1803

NAVY DEPARTMENT

A Theoretical Study of The Speed of Fall of Depth Charges in Water.

NAVAL RESEARCH LABORATORY
ANACOSTIA STATION
WASHINGTON, D. C.

Number of Pages: Text - 13 Tables - 5 Plates - 5

Prepared by:

Dr. John M. Ide, Physicist

M. F. M. Osborne, Junior Physicist

Reviewed by:

Dr. H. C. Hayes, Principal Physicist
Superintendent, Sound Division

Approved by:

H. G. Bowen, Rear Admiral, USN., Director

Distribution:

BuShips (3)
NDRC (5)
BuOrd (2)
OpNav (Fleet Training) (1)
OpNav (DNC) (1)
Coord. Research (2)
British Central Scientific Mission (5)
Radio & Sound Laboratory, San Diego (1)
" " " " New London (1)
East Coast Sound School, Key West (1)
West Coast Sound School, San Diego (1)
Files (27)

ect

TABLE OF CONTENTS

TABLES

PLATES

[redacted]

[redacted]

ABSTRACT

The present report estimates from theoretical considerations the probable effectiveness of five methods of increasing the velocity of descent of depth charges. These are (1) streamlining the bombs, (2) adding extra weight, (3) increasing their stability, (4) applying self-propulsion, and (5) giving the bombs an initial velocity at the moment of striking the water. Two sizes of charge, and four shapes, are discussed.

It is concluded that stability is prerequisite to any improvement in falling speed, and that streamlining, internal propulsion or a combination thereof will produce several hundred percent increase in the average falling speed to target depth. Bombs striking the water with appreciable speed will reach target depth one or two seconds sooner than those falling from rest, but this improvement is not sufficient to justify elaborate ejection equipment. Eccentric weighting of the depth charges will considerably increase their speed, largely because of improvement in stability.

I. INTRODUCTION:

1. The rate of descent of the charges is a most important factor in the effectiveness of anti-submarine depth charge attack. In NRL Report 1776 of 27 August 1941, it was shown that doubling or tripling the speed of fall of the present "ash cans" would greatly increase the probability of successful attack on submerged submarines.

2. The purpose of this report is to estimate from theoretical considerations the probable effectiveness of five methods of increasing the velocity of descent of depth charges. These are (1) streamlining the bomb, (2) adding extra weight, (3) increasing the stability of the bomb, (4) applying self-propulsion, and (5) giving the bomb an initial velocity at the moment of striking the water. Two sizes of charge are considered, — a bomb containing 300 lbs of explosive, as in the present USN depth charge, and a relatively light contact bomb containing 30 lbs of explosive. For each bomb weight, four different shapes are discussed: (1) the cylindrical shape of the present "ash can", in which the length (L) is 1.57 times the diameter (d); (2) a spherical shape; (3) the shape of an ovarian ellipsoid, with the length equal to 3 times the diameter; and (4) a "dirigible" streamlined shape with a ratio of length to largest diameter of five to one. These shapes illustrate increasing degrees of streamlining. The effects of weighting and of internal propulsion upon terminal speed are estimated for special cases. For each of the four special shapes, curves of speed and depth as functions of time are computed and plotted for fall through water from rest, and for fall from an initial speed (S_o) equal to one-half the terminal speed (S_t). The report concludes with a discussion of the tabulated and plotted results.

II. MATHEMATICAL ANALYSIS:

3. The speed of fall of depth charges in water may be computed from Newton's second law of motion, written in a form which includes the resistance of the water. The desired results are the terminal speeds of bombs of various sizes and shapes, and their speed and depth as a function of time. The simple equations for falling bodies, derived in elementary mechanics, require elaboration when applied to depth charges. In water the pull of gravity is reduced by the buoyancy, in accordance with Archimedes' principle, and in addition, the drag of the water on the body introduces a resistance which is proportional to the square of the velocity. If internal propulsion is employed, an additional term must be added to the pull of gravity. In mathematical terms:

$$M\, dS/dt = a - bS^2 \qquad (1)$$

[redacted]

Where S = the speed of fall of the bomb at time t.
M = the effective mass of the bomb, including the hydrodynamic "carried mass" arising from the kinetic energy of the moving liquid.
a = an effective gravity constant, including the effect of buoyancy and internal propulsion.
bS^2 = the drag of the water on the bomb.

4. The speed of fall increases until it reaches a definite terminal speed, S_t, at which the effective downward pull on the bomb is balanced by the drag. After the acceleration reaches zero the bomb continues to fall at constant speed S_t. The terminal speed is obtained from equation (1) by setting the right hand side equal to zero. Thus:

$$S_t^2 = \frac{a}{b} \qquad (2)$$

This is a special case of equation (1) when $dS/dt = 0$. Numerical solutions of equation (2) for terminal speeds (S_t) are easily obtained for desired conditions. Table II gives (S_t) as a function of the degree of streamlining, and Table III gives values of (S_t) for various sizes, shapes, weightings, and degrees of internal propulsion. Terminal speeds are shown in graphical form on Plate I. It will be shown later that the size of the bomb determines (a), and its shape determines (b).

5. Going back to equation (1), a first integration, by separation of variables, yields an equation for speed of fall (S) against time (t):

$$S = S_t \tanh (k + qt) \qquad (3)$$

Where $k = \frac{1}{2} \log \left(\frac{S_t + S_o}{S_t - S_o} \right)$, a term arising from the constant of integration.

S_o = the speed of the bomb at the instant of striking the surface of the water ($t = 0$). This may be called the initial velocity.

q = $\sqrt{ab}/M$, the constants (a), (b), and (M) being defined as in equation (1).

6. Solutions of equation (3), for four shapes and two sizes of bombs, are plotted in the form of curves on Plates II and III.

7. Further integration of equation (3) yields an expression for depth of fall (h) as a function of time (t):

$$h = \frac{M}{b} \log \left(\frac{\cosh (k + qt)}{\cosh k} \right) \qquad (4)$$

[redacted]

Where the constants retain their previous significance.

8. Curves of depth as a function of time, computed from this equation for four shapes and two sizes of bombs, are plotted on Plates IV and V. The effect of initial velocity is shown by the dashed curves on Plates II, III, IV, and V.

9. The four equations outlined above determine the theoretical behavior of depth charges falling in water, when the proper numerical substitutions are made.

III. EVALUATION OF CONSTANTS

10. In order to compute terminal velocities, speed-time curves, and depth-time curves for depth charges, it is necessary to evaluate the constants k, q, M, a and b, which occur in the four equations of the preceding section. The integration constant (k) is expressed in terms of initial and terminal speeds (S_o) and (S_t) respectively. (q) is a composite of (M), (a) and (b). It will therefore be sufficient to evaluate the last three constants.

A. Effective Mass (M).

11. The effective dynamic mass of the bomb (M) will be considered first. It is shown in treatises on hydrodynamics (e.g. Lamb's "Hydrodynamics", § 68 and § 92) that the kinetic energy of the liquid set into motion by a falling body may be taken into account by adding an apparent mass (M') to the actual mass of the body (M_b), to obtain the effective mass (M) which is accelerated. The hydrodynamic "carried mass" (M') is a fraction of the mass of fluid displaced by the moving body. For a cylinder falling in a direction perpendicular to its length, $M' = \rho V$; for a sphere $M' = \frac{1}{2}\rho V$; and for moderately streamlined shapes $M' = \frac{1}{5}\rho V$, where (ρ) is the density of the liquid, and (V) the volume of the falling body. The theory has been experimentally verified for spheres, by R. G. Lunnon, Proc. Roy. Soc., Series A, Vol. 118, p. 680 (1928). Thus in equation (1), $M = M_b + M'$, where (M_b) is the actual mass of the bomb, and (M') the "carried mass" computed as above for each shape of bomb. Table II gives values of (M) computed for four shapes and two sizes of bomb.

B. Effective Gravity Force (a).

12. The second numerical factor in the equation of motion of the bomb, equation (1), is the effective gravity force (a). If the bomb falls without internal propulsion the gravity force, ($Vg\sigma$), will be opposed by the buoyancy ($Vg\rho$). Thus:

$$a = Vg(\sigma - \rho) \qquad (5)$$

where the quantities not yet named are (g), the acceleration of gravity, and (σ) the mean density of the bomb. In order to study the effects of weighting and of internal propulsion on the terminal speeds of bombs, displayed in Table III and Plate I, it is assumed that the ratio of the mass of the explosive to the total mass of the bomb is successively 0.3, 0.5, 0.7, and 0.9. In order to compute speed-time and depth-time curves,

[redacted]

exhibited in Plates II, III, IV and V, the assumption is made that this ratio is 0.7, as in the present "ash can". It is assumed that 16% of the volume of the bomb is air, and the density of iron is taken as 490 lbs. per cubic foot, the density of TNT as 100 lbs. per cubic foot, and of sea water as 64 lbs. per cubic foot. The acceleration of gravity as usual is 32.2 feet per second per second.

13. For example, a bomb containing 300 lbs. of explosive has a volume $V = 3.8$ cubic feet, a mean density $\sigma = 113$ lbs. per cubic foot, and hence an effective gravity constant (a) of 6000 poundals. For the light bomb containing 30 lbs. of explosive, (a) is 600 poundals.

14. If a propulsive force of the rocket type is added to the pull of gravity on the bomb, the constant (a) is increased:

$$a = Vg\,(\sigma - \rho) + Tg \qquad (6)$$

where (T) is the thrust of the propellant in pounds. For the bomb containing 300 lbs. of explosive, the thrust produced by the discharge of 75 lbs. of rocket explosive in 10 seconds, at an average velocity of 6400 feet per second, will be:

$$T = \frac{75 \times 6400}{10g} = 1500 \text{ lbs.}$$

Similarly the discharge of 30 lbs. of rocket explosive will produce a thrust of 600 lbs. during the ten second interval. Modern rocket performance suggests that such thrusts are practical, using ordinary explosives and well designed nozzles. Experimental investigation of rocket propulsion under water would be required to confirm this.

15. In Table III and Plate I, values of terminal speed are shown for two degrees of internal propulsion, corresponding to weights of rocket explosive equal to 10% and 25% of the entire weight of explosive.

16. Of course the rocket principle is not the only means of obtaining internal propulsion. For example, a propeller drive similar to that used for torpedoes could produce the required thrust. Since a self-propelled depth charge will reach target depth in a few seconds, and requires no steering or balancing devices, a relatively simple driving mechanism could be used, powered by a small battery short-circuited through the motor as the bomb enters the water. Table III is computed for the special case of rocket drive, but the results are applicable to any type of self-propulsion.

C. <u>Drag Factor (b).</u>

17. The shape of the bomb, and with it the effect of streamlining,

[redacted]

enters into the equation of motion through the final term (bS^2). This represents a force known as drag, which acts on the falling body in the direction opposite to its motion. The drag is formulated in terms of the momentum imparted by the body to the fluid in unit time. This is expressed in the literature of applied hydrodynamics as $(\frac{1}{2} \rho C_d A S^2)$. Here (ρ) is the density of the fluid, (S) the speed of the moving body, (A) the area of the body projected on a plane normal to the direction of motion, and (C_d) a constant of proportionality known as the drag coefficient, or specific resistance. Comparing this formula for drag with the final term of equation (1), it is apparent that (b) must be defined as $(\frac{1}{2} \rho C_d A)$ in order to have the proper physical significance. The density (ρ) is known, and (A) is easily computed for any assumed shape and volume of bomb. Evaluation of (C_d) will complete the data required for computation of the drag factor, (b).

18. The drag coefficient (C_d) involves complex phenomena which will be briefly outlined. The reader is referred to standard texts, such as Prandtl and Tietjens "Applied Hydro- and Aero-mechanics", or Dodge and Thompson's "Fluid Mechanics", for full discussions of the resistance of immersed bodies.

19. On the basis of the Newtonian or momentum conception of fluid resistance, it was long thought that for a given shape and orientation of the body, the drag coefficient was independent of its size and velocity. Experience has shown, however, that viscosity effects in the medium, neglected in the Newtonian theory, cause the drag coefficient to depend on size and velocity, except in the few cases where the geometry of flow is determined by sharp edges (e.g. for a plate moving in a direction normal to its plane). Thus the drag coefficient for a falling body is, in general, a function of its velocity, size, and shape. Fortunately the principle of dynamic similarity may be employed to combine the influence of the first two variables into a single parameter, known as the Reynolds' number. The Reynolds' number can be derived from dimensional considerations, or from the experimental conditions which ensure that bodies of the same shape but different size and velocity possess the same value of drag coefficient. It was discovered by Reynolds that two geometrically similar bodies in the same medium have the same drag coefficient (specific resistance) when their velocities are inversely proportional to their corresponding linear dimensions. A comparison of drag coefficients in different media shows that the dependence upon velocity is essentially the same in all cases where the kinematic viscosity has the same value. These relations may be combined. Let (L) be the length of some corresponding linear dimension for any one of a number of geometrically similar bodies, (S) the velocity of the body, and (ν) the kinematic viscosity (ratio of coefficient of viscosity to the density of the fluid). Then the drag coefficient (C_d) is the same for all combinations of size and velocity for which $R_n = LS/\nu$ has the same value. The ratio (R_n) is called the Reynolds' number of the flow system. In the case of geometrically similar bodies the liquid currents

[redacted]

are similar, and the flow lines have the same form, when they correspond to equal Reynolds' numbers.

20. Experimental curves of drag coefficients versus Reynolds' numbers are available from wind tunnel tests, for various shapes of bodies. Values of (C_d), relevant for depth charges falling in water, may be obtained from published curves for bodies in air, once the proper Reynolds' numbers are known. For evaluation of the Reynolds' numbers for motion in sea water, the kinematic viscosity is taken as 1.23×10^{-5} square feet per second, and the principal dimension of the bomb is from one to three feet depending on its size. When the depth charge attains the modest falling speed of one foot per second, $R_n = 200{,}000$ for the large bomb (300 lbs. of explosive), and $R_n = 80{,}000$ for the small bomb (30 lbs. of explosive). After a few seconds (R_n) will increase toward values of one to ten million at the terminal speeds of the bombs. In Table I values of drag coefficients taken from a standard text, are tabulated for the four shapes under study.

Table I

Drag Coefficients for Falling Bodies

Shape	C_d for $R_n > 500{,}000$
Cylinder	0.40 to 1.0*
Sphere	0.22
Ellipsoid (L/d = 3)	0.06
Dirigible (L/d = 5)	0.04

*For $R_n < 500{,}000$

21. The reduction in drag coefficient obtainable by streamlining the depth charge is apparent from the table, which shows that the cylindrical "ash can" has a drag coefficient ten times greater than that for a truly streamlined shape. The difference may be even greater, since the drag coefficient at low speeds ($R_n = 500{,}000$) is larger than shown in the table. The increase in (C_d) at low Reynolds' numbers is most marked for cylinders, where $C_d = 1$ is a probable value for the "ash can" shape, until it reaches a speed greater than three feet per second. Bombs of more streamlined shape retain the low drag coefficients of Table I to lower Reynolds' numbers than the "ash can", and will reach the critical (R_n) more quickly since they fall faster. Hence their actual drag coefficients will be close to the tabulated values.

[redacted]

22. The factors entering into the computation of the constants (M), (a) and (b), occurring in the equations of motion of the bomb, have now been discussed. Actual computations of these constants, as well as of (K), (q), $(\frac{M}{b})$ and (S_t), are tabulated in Table II, for two sizes and four shapes of bombs.

IV. DISCUSSION OF RESULTS

A. Terminal Speeds

22. The terminal speeds, computed from equation (2), are displayed as a function of weighting and internal propulsion, for the present cylindrical shape and for a streamlined "dirigible" shape, and for two sizes of bombs, in Table III, and are plotted on Plate I. Table II shows terminal speeds as a function of increasing streamlining, for bombs without internal propulsion.

23. The effect of weighting is shown by varying the ratio of the mass of explosive to the mass of the bomb from 0.3 to 0.9 in steps. Weighting alone, in the practical range, increases terminal speed in the same proportion as it does the weight of the bomb. Thus the addition of 160 lbs. of lead to the present "ash can", would increase both the weight and the terminal speed 30%. Weighting increases the terminal speed of the streamlined bomb in the same proportion, but since (S_t) is much larger in this case, the increase is also larger. Regardless of the shape or size of bomb, weighting alone increases (S_t) and (M_b) by the same percentage.

24. If the bomb is self propelled by means of a rocket explosive, the terminal speed is increased in the ratio 1 : 2 : 3, as the charge available for rocket propulsion increases from 0 to 10% and then to 25% of the whole amount of explosive. The more heavily weighted the bomb, the less effective is the addition of self-propulsion, since gravity becomes more nearly comparable to the internal thrust. The same ratio of increase due to propulsion holds for the streamlined as for the "ash can" shape, but complete streamlining increases all values of terminal speed by a factor of almost 6. Terminal speeds greater than 200 feet per second are theoretically attainable by internal propulsion of the fully streamlined bomb.

25. Self-propulsion alone doubles or triples the terminal speed of the bomb. Complete streamlining to dirigible shape increases the terminal speed by a factor of six. Weighting alone, in the practical range, increases terminal speed in the same proportion as it does the weight. Combined weighting and propulsion adds nothing beyond the effect of propulsion alone, since the propulsion thrust is so large compared with gravity that it dominates the force function. Combined streamlining and propulsion increases terminal speed from 12 to 205 feet per second, by the tremendous factor of 17, but the further addition of weighting adds no more.

[redacted]

26. The above discussion holds in detail for the small bomb, containing 30 lbs. of explosive, with the difference that each value of terminal speed for the small bomb is 0.68 times its corresponding value for the large bomb. The terminal speed is proportional to the sixth root of the mass of the bomb.

27. It is apparent from the above that internal propulsion and streamlining, or a combination thereof, are the most effective means of increasing the terminal speed of depth charges in water. Weighting must be regarded as incidental, and valuable chiefly as it may improve stability.

B. Streamlining

28. The effect of increasing degrees of streamlining on the terminal speed for large and small bombs is shown in Table II. With the progression from cylinder, to sphere, to ellipsoid (L/d = 3), to dirigible shape (L/d = 5), the increase in terminal speed is proportional to 1: 1.5 : 4 : 6, respectively. The same proportional increases are obtained for the small bombs, but the actual values are two thirds of those for the large bomb. This sequence was adopted for this study, not because it is proposed to use these exact shapes, but to indicate the relative increases to be expected as streamlining is improved. It is clear from the results that the ellipsoidal shape with length equal to three times the diameter gives effective streamlining. Recourse to the comparatively awkward "dirigible" shape is hardly necessary. In fact a reasonable compromise with practical conditions suggests the adoption of a ratio of length to diameter of three, with a streamlined head, and a "boat tail" form at the rear of the bomb to partially streamline the wake. Such a shape, similar to that of an artillery projectile with streamlined rather than ogival head, could be adopted without drastic revision of the means of stowage and ejection of the depth charges. Speed curves similar to those shown for the hypothetical ovarian ellipsoid, of L/d = 3, would be expected for such a shape. The "boat tail" shape at the rear is actually more stable against "yaw" or tumbling than the completely streamlined dirigible form.

C. Stability

29. In this study it is assumed that the bombs will fall in a direction parallel to the longest dimension, without "yaw", spin, or other departure from a simple trajectory. If they are not stable in this sense, the drag will be greater, and the observed falling speeds will be smaller than the computed values. The factor of stability is most important, and probably accounts for most of the difference between computed and observed terminal speeds for the present USN depth charge. Theoretically the "ash can" should quickly reach a falling speed of 12 feet per second, actually it does not fall faster than 8 or 9 feet per second.

[redacted]

30. The present charge, with its center of gravity close to the geometric center, can roll and tumble, and fall either endways (parallel to its axis), or sideways (perpendicular to its axis). The drag ($\frac{1}{2}\rho C_d A$) is slightly larger for endways fall, but there is nothing to prevent either mode of fall, or end over end, and the course may not be straight downward, especially if the cylinder has an initial spin and falls sideways. In addition to uncertainty about the mode of fall, the Reynolds' number of the motion may be less than 500,000 for a few seconds at the start, and this will more than double the drag and reduce the speed by 40%. The lack of stability and the low Reynolds' number at the start of the fall are sufficient to explain the discrepancy between computed and observed falling speeds. The unstable motion may also cause the bomb to fall in an erratic curve, so that it reaches target depth at a point some distance from the position at which it was dropped. This is in addition to uncertainties introduced by turbulent water in the destroyer's wake.

31. Instability may be remedied by constructing the bomb, or weighting the present "ash can", so that the center of gravity is offset from the center of buoyancy. As the charge falls, the center of gravity must lead the center of buoyancy, if the bomb is to maintain the desired orientation as it descends. The addition of internal propulsion, initial velocity, streamlining, or a combination of these, will increase the Reynolds' number of the motion by increasing the speed of fall. Under these improved conditions, the values computed in this study should agree fairly well with experiment, probably within 20%.

32. Whatever the shape of the bomb, stability should be assured by offsetting the center of gravity in the direction of the bomb head. All the conclusions of this study presuppose this type of stability.

D. Speed-time and Depth-time Curves

33. The factors which determine terminal speed have been discussed, and the conclusion drawn that streamlining and self-propulsion are the most effective means of increasing this quantity. Although the terminal speed is valuable as an upper limit to the actual speed, it is the average speed of the bomb on its way to target depth which is important in making an attack. It is of value to know how rapidly the terminal speed is approached, and to what depths the bombs will fall in various times.

34. Curves showing speed versus time and depth versus time, for the two sizes and four shapes of bombs, are computed from the equations of motion and plotted on Plates II, III, IV and V. Plates II and III show speed of fall plotted against time for large and for small bombs, respectively. Plates IV and V show the depth of fall plotted against time for large and small bombs, respectively. The full line curves on

each plate show the relations when the bomb enters the water from rest, and the dashed curves the relations when the bomb strikes the water with initial speed equal to one half its terminal speed. On the right hand side of Plate II the bombs are sketched to scale, showing the relative shapes in the falling position.

35. It may be seen from the speed-time curves on Plates II and III that the cylinder and sphere reach nine tenths of their terminal speeds in 1 to 2 seconds after striking the water, the ellipsoid in 3 to 5 seconds, and the dirigible shape in 5 to 8 seconds. The difference between large and small bombs, and the presence or absence of initial velocity, is included in the ranges given. The small bombs take 10% longer to reach terminal speed than the large ones of similar shape, and the presence of initial velocity $S_o = 1/2\ S_t$ decreases the time required to reach terminal speed by 1 to 3 seconds, the amount increasing with streamlining.

36. The actual "ash can" may require several additional seconds to reach terminal speed, owing to turbulence of the water, instability of fall, and the low Reynolds' number of the motion at the start of the descent.

37. From the curves on Plates IV and V it may be seen that the time required for the bomb to reach a target at moderate depth, say 200 feet, varies greatly with the degree of steamlining and slightly with the initial speed. Thus Plate III gives data for the following table:

Table IV

Effect of Streamlining on Speed and Time to Target Depth

Description	Time for Bomb to Reach 200' Depth	Average Speed to Depth of 200 Feet
Present "ash can", unstable fall.	25 seconds	8 ft. per sec.
Cylinder, stable fall from rest	18 seconds	11 ft. per sec.
Cylinder, from $S_o = 1/2\ S_t$	17 seconds	12 ft. per sec.
Sphere, from rest	13 seconds	15 ft. per sec.
Ellipsoid, from rest	7 seconds	29 ft. per sec.
Dirigible, from rest	6 seconds	33 ft. per sec.
Dirigible, from $S_o = 1/2\ S_t$	4 seconds	50 ft. per sec.

[redacted]

38. The table shows that the time required for the bomb to reach target depth is decreased 7 seconds by stabilizing, an additional 12 seconds by streamlining, and a further 1 to 2 seconds by ejecting the bomb with an appreciable initial velocity. The table also shows that the average theoretical speed of the bomb on its way to a depth of 200 feet, is about 12 feet per second for the cylinder, not much affected by initial velocity; and 30 to 50 feet per second for the streamlined shapes, whose average speed is definitely greater if they strike the water at $S_o = 1/2\ S_t$.

E. Internal Propulsion of Streamlined Bombs:

39. The curves and tables of this report show that surprising bomb speeds should be attainable by the combination of streamlining and self-propulsion. Table V gives the average falling speeds theoretically attainable for fully streamlined self-propelled depth charges of two sizes and two degrees of internal propulsion. These speeds range from 55 to 100 feet per second, or from 33 to 59 knots, respectively. The table also shows the time in seconds required for the bomb to reach nine tenths of its terminal speed, and the depth corresponding to this speed. The average speeds are computed for target depths of 200 feet. Since streamlined self-propelled bombs should reach this depth in 2 to 4 seconds after striking the water, their detonation would endanger the attacking destroyer. The danger may be minimized by using a depth charge thrower, or by employing relatively light bombs adjusted to explode on contact with the target.

F. Illustrative Special Cases:

40. The following special cases illustrate the modifications in depth charge design suggested by this discussion. A target depth of 200 feet is assumed, and a "pay load" of 300 pounds of explosive.

Case I. Present USN "ash can", cylindrical depth charge. The fall may be unstable, particularly in a turbulent wake. Observed terminal speed is 8 or 9 feet per second. Time to target depth, 25 seconds or more.

Case II. The "ash can", modified by advancing the center of gravity by 30% weighting at one end. This should assure stable fall, attaining a terminal speed of 12 to 15 feet per second, and requiring 16 to 18 seconds to reach target depth.

Case III. A newly designed bomb, with streamlined head, "boat tailed" rear like an artillery projectile, and length equal to three times the diameter. The data for the ellipsoid of the same fineness ratio should apply to this case. This bomb will probably attain an average speed of 30 ft. per second, a terminal speed of 50 feet per second, and will reach target depth 7 or 8 seconds after striking the water.

[redacted]

Case IV. A completely streamlined bomb of dirigible shape, with length five times the diameter, and the center of gravity advanced for stability. The terminal speed should be 68 feet per second, and the bomb should reach target depth in 4 to 6 seconds.

41. The addition of internal propulsion using rocket explosives or driving propellers will increase all terminal speeds 100% to 200%, and correspondingly shorten the time to reach target depth.

V. CONCLUSIONS

42. A theoretical study of the falling of bombs through water leads to the following concusions:

(1) Stability of fall is pre-requisite to any improvement in falling speed. Improved stability alone, from offsetting the center of gravity, would give the present "ash can" 50% greater terminal speed, increasing it from 8 feet per second to almost 12.

(2) The falling speeds of depth charges may be increased by a factor of 4 by the moderate streamlining illustrated in Special Case III, and by a factor of 6 by complete streamlining. An average speed of 30 feet per second to a depth of 200 feet, should be attainable by moderate streamlining, compared with approximately 8 feet per second for the unstable and unstreamlined "ash can" now used by the USN.

(3) Weighting the bomb increases the falling speed by the same percentage that it increases the weight, but may produce an inefficient ratio of explosive to iron. Weighting is not recommended, except as a means of improving stability by shifting the center of gravity.

(4) Internal propulsion would further increase the falling speeds by 100 to 200%, but would require further development of propulsion technique and drastic modification of bomb design. An average speed to target depth amounting to 100 feet per second (59 knots) is theoretically possible from internal propulsion of a fully streamlined bomb. Hence it is clear that any bomb speed required by tactical considerations can be reached or even exceeded by proper design of the depth charge.

(5) The ejection of the charge with an initial velocity increases the speed for the first few seconds of fall, and the bomb should reach target depth one or two seconds sooner than without it. The improvement does not seem great enough to justify elaborate ejection equipment.

VI. SYMBOLS AND ABBREVIATIONS

S	Speed of fall of the depth charge (feet per second).
S_o	Speed of depth charge at instant of striking water (feet per second).
S_t	Terminal speed of depth charge (feet per second).
M_b	Weight of the depth charge (pounds).
M'	Hydrodynamic "carried mass" of water set in motion by bomb (pounds).
V	Volume of depth charge (cubic feet).
ρ	density of sea water (lbs. per cuib feet).
$\sigma = M_b/V$	The mean density of the depth charge (lbs. per cubic feet).
T	Thrust of internal propulsion (pounds).
A	Area of cross section of bomb in plane at right angles to its direction of motion (square feet).
L	Length of depth charge in the direction of its motion (feet).
d	Maximum diameter of depth charge (feet).
g	Acceleration of gravity, 32.2 feet per second per second.
h	Depth of fall of bomb below surface of the sea (feet).
C_d	Drag coefficient of the falling depth charge (numeric).
$R_n = LS/\nu$	Reynolds' number of the flow system (numeric).
ν	Kinematic viscosity of the fluid in which the bomb falls (sq. ft. per second).

Abbreviations — Composites of Above Symbols

$$a = Vg(\sigma - \rho) + Tg$$

$$b = \frac{1}{2} \rho C_d A$$

$$M = M_b + M'$$

$$q = \sqrt{ab}/M$$

$$k = \frac{1}{2} \log \frac{S_t + S_o}{S_t - S_o}$$

[redacted]

Table I

<u>Drag Coefficients for Falling Bodies</u>

R_n = Reynolds' Number of the Motion

C_d = Drag coefficient

Shape	C_d for $R_n \geq 500{,}000$
Cylinder	0.40 to 1.0*
Sphere	0.22
Ellipsoid (L/d = 3)	0.06
Dirigible (L/d = 5)	0.04

*For $R_n < 500{,}000$

[redacted]

Table II

Constants in the Equations of Motion, and Terminal Speeds for Bodies of Various Shapes

For an initial velocity equal to half the terminal velocity

$K = 0.549$ for $S_o = \frac{1}{2} S_t$

$K = 0$ for $S_o = 0$

For the large bomb, containing 300 lbs. of explosive:

M_b = 430 lbs. σ = 113 lbs. per cu. ft.

V = 3.8 cu. ft. $a = Vg(\sigma - \rho)$ = 6000 poundals.

Shape	$M_{lbs.}$	$L_{ft.}$	$d_{ft.}$	$A_{sq.ft.}$	b	q	M/b	S_t ft/sec
Cylinder	674	2.29	1.46	3.34	42.9	.753	15.8	11.8
Sphere	552	1.94	1.94	2.95	20.8	.640	26.6	16.9
Ellipsoid	480	4.02	1.34	1.41	2.71	.266	177	46.9
Dirigible	480	5.67	1.134	1.01	1.29	.184	372	67.9

For the small bomb, containing 30 lbs. of explosive:

M_b = 43 lbs. σ = 113 lbs/cu.ft.

V = 0.38 cu.ft. a = 600 poundals

Shape	$M_{lbs.}$	$L_{ft.}$	$d_{ft.}$	$A_{sq.ft.}$	b	q	M/b	S_t ft/sec
Cylinder	67.4	1.06	.676	.721	9.28	1.11	7.26	8.01
Sphere	55.2	0.90	.900	.636	4.49	0.94	12.3	11.5
Ellipsoid	48	1.87	.622	.304	.586	0.391	81.9	31.9
Dirigible	48	2.64	.528	.218	.278	0.270	172	46.2

[redacted]

Table III

Terminal Speed as a Function of Weighting and Self-Propulsion

(1) Ratio of Mass of Explosive to Mass of Bomb.
(2) No Internal Propulsion.
(4) Rocket Explosive = 25% of Total Explosive.
(3) Rocket Explosive = 10% of Total Explosive.

		Cylindrical Shape			Streamlined "Dirigible" Shape (L/d=5)		
	(1)	(2)	(3)	(4)	(2)	(3)	(4)
		ft/sec.	ft/sec.	ft/sec.	ft/sec.	ft/sec.	ft/sec.
300 lbs. Explosive	0.3	20.3	27.9	36.6	116.7	160	210
	0.5	15.4	25.7	36.0	88.7	148	207
	0.7	11.8	24.3	35.6	68.2	140	205
	0.9	9.2	23.6	35.6	52.2	135	204
30 lbs. Explosive	0.3	13.8	19.0	24.9	79.5	109	143
	0.5	10.5	17.5	24.5	60.2	101	141
	0.7	8.1	16.5	24.2	46.5	95	139
	0.9	6.2	16.1	24.2	35.6	92	139

[redacted]

Table IV

Effect of Streamlining on Speed and Time to Target Depth.

Description	Time for Bomb to Reach 200 ft. Depth	Average Speed to Depth of 200 Feet.
	Seconds	ft. per second
Present "ash can", unstable fall	25	8
Cylinder, from rest stable fall	18	11
Cylinder, from $S_o = \frac{1}{2} S_t$	17	12
Sphere, from rest	13	15
Ellipsoid, from rest	7	29
Dirigible, from rest	6	33
Dirigible, from $S_o = \frac{1}{2} S_t$	4	50

Table V

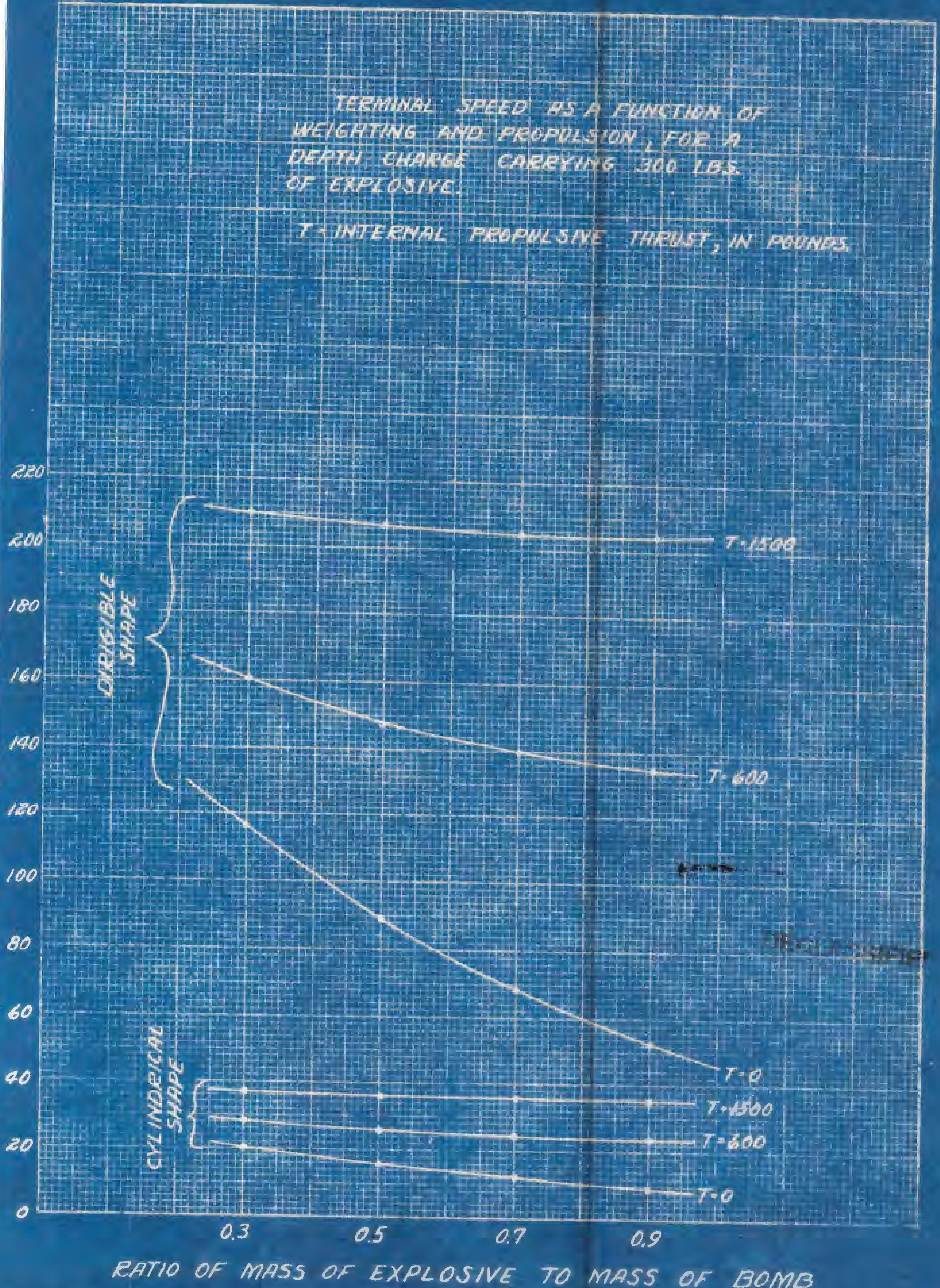
Computed Speeds of Streamlined Self-Propelled Depth Charges.

Description	Terminal Speed S_t	Seconds to reach 0.9 S_t	Depth where $S = 0.9\ S_t$	Average Speed to 200' Depth	Average Speed in Knots, to 200' Depth
	ft/sec.	seconds	feet	ft/sec.	Knots
Large Bomb 300 lbs. Explosive T = 1500 lbs.	205	2.9	316	100	59
Large Bomb 300 lbs. Explosive T = 600 lbs.	140	4.6	316	65	38
Small Bomb 30 lbs. Explosive T = 150 lbs.	139	2	146	82	49
Small Bomb 30 lbs. Explosive T = 60 lbs.	95	3	146	55	33

[redacted]

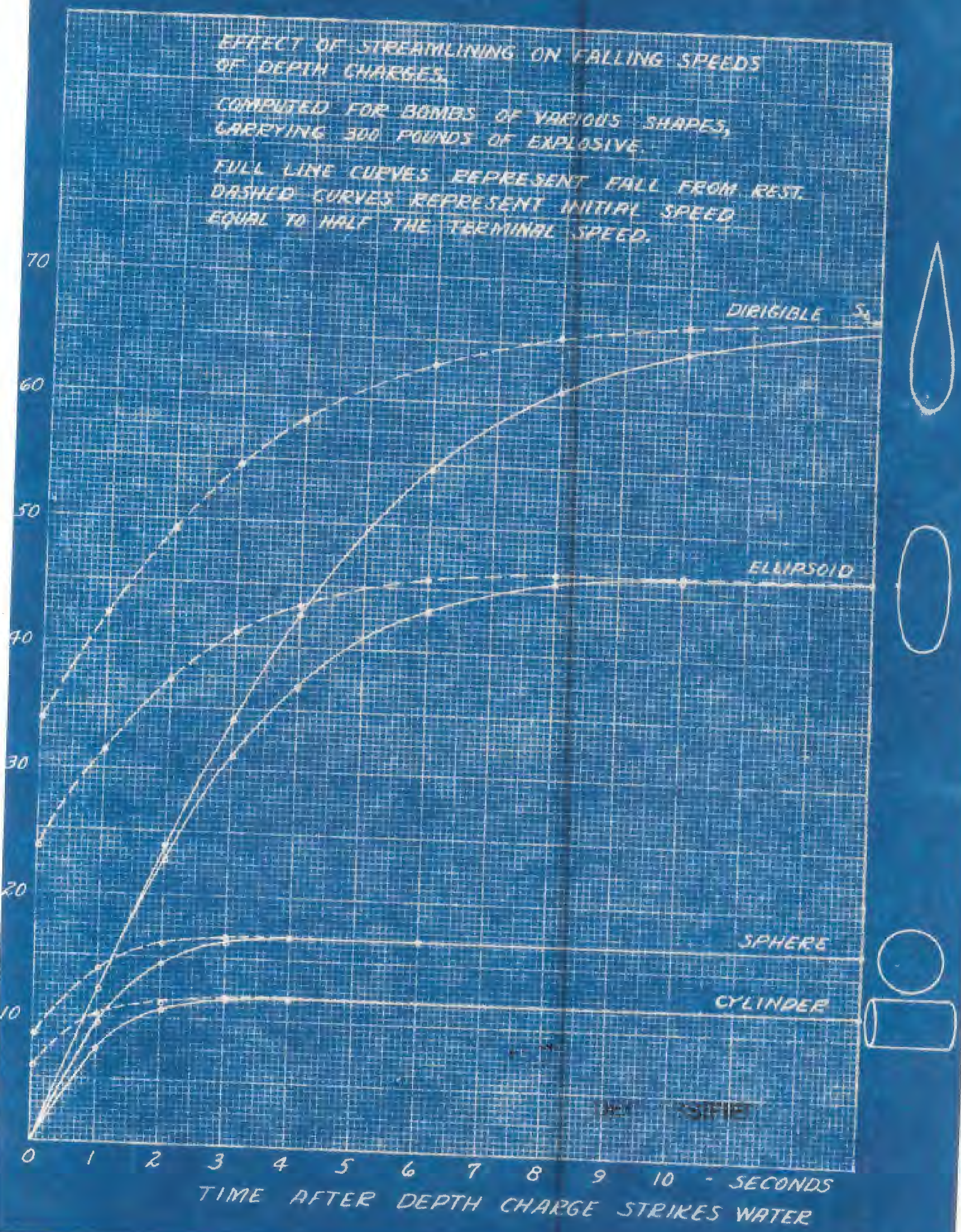
U. S. NAVAL RESEARCH LABORATORY
BELLEVUE, D. C.

DATE



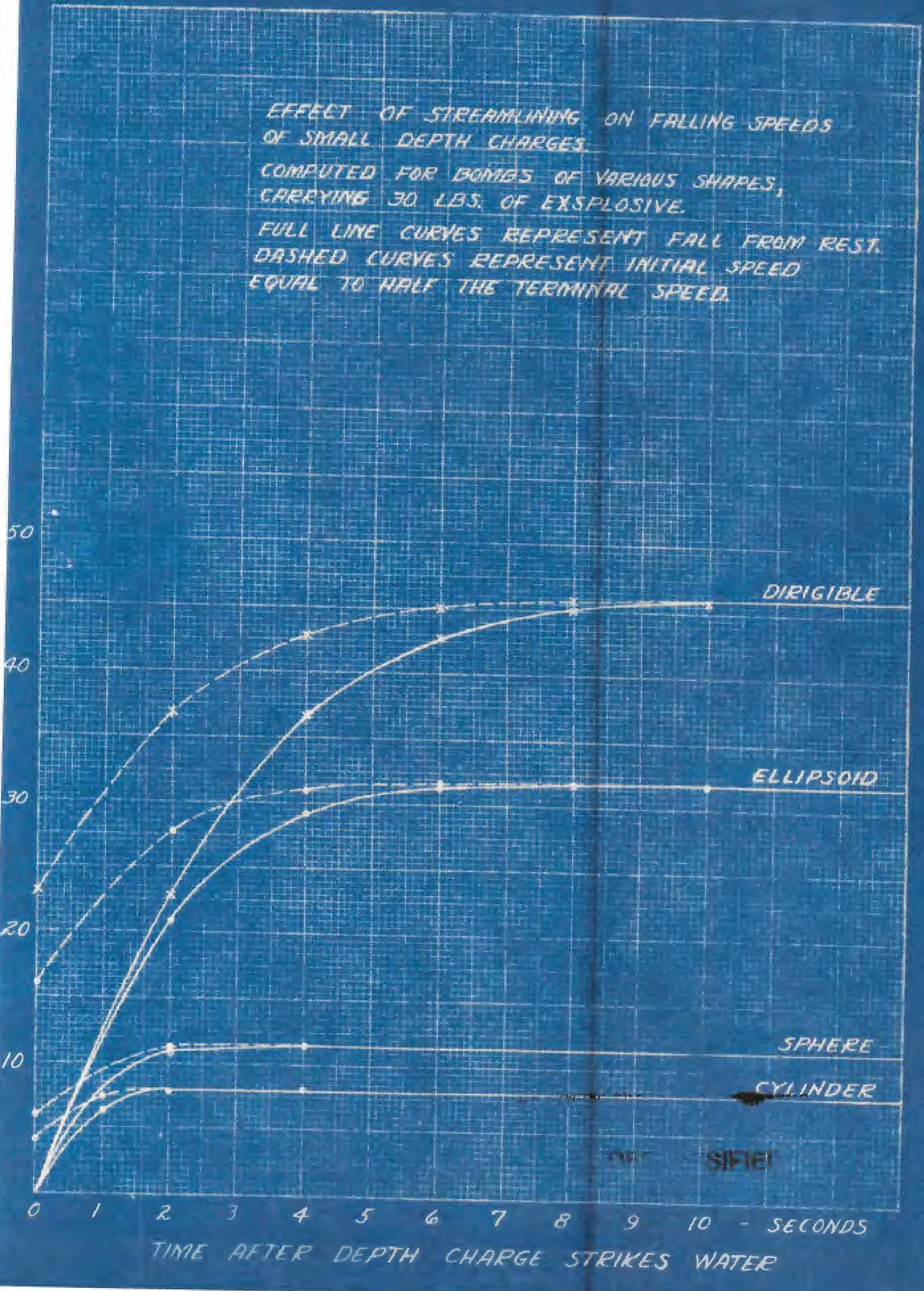
U. S. NAVAL RESEARCH LABORATORY
BELLEVUE, D. C.

DATE



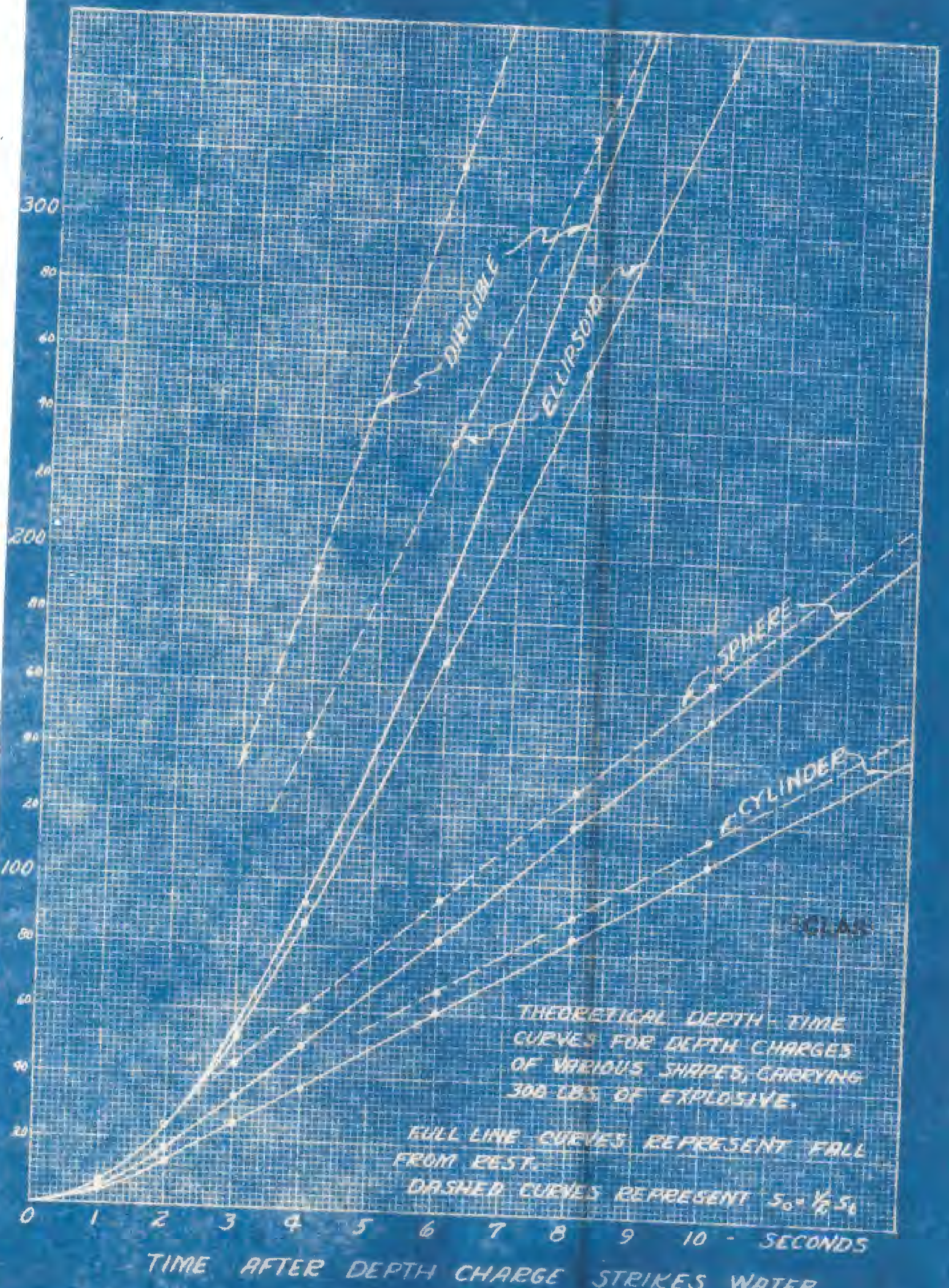
U. S. NAVAL RESEARCH LABORATORY
BELLEVUE, D. C.

DATE



U. S. NAVAL RESEARCH LABORATORY
BELLEVUE, D. C.

DATE



U. S. NAVAL RESEARCH LABORATORY
BELLEVUE, D. C.

DATE