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Statistical Fatigue Failure Time of Unidirectional CFRP under Compression Loading

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14. ABSTRACT

The developed accelerated testing methodology (ATM) based on the matrix resin viscoelasticity for the creep and fatigue failure life prediction of fiber reinforced polymers (FRP) was applied to the statistical prediction of long-term creep and fatigue failure life for the longitudinal bending of unidirectional CFRP laminates which is an important basic item for the durability design of CFRP structures used for aircraft and others. The validity of our developed ATM for this creep and fatigue life prediction under the longitudinal bending of unidirectional CFRP was discussed herein. From that discussion, the following results were obtained.

1. Formulation of the statistical creep and fatigue failure time under bending load for the longitudinal direction of unidirectional CFRP based on the viscoelasticity of matrix resin was established assuming micro-buckling failure near the loading point.

2. The master curve of relaxation modulus of matrix resin in the very wide range of time was constructed by application of the time–temperature superposition to the data of DMA and creep tests measured at various temperatures.

3. Statistical creep failure times under bending loads at an arbitrary temperature were predicted by substituting the statistical flexural static strengths of unidirectional CFRP measured at several temperatures and the relaxation modulus of matrix resin into the formulation. Predicted statistical creep failure times agreed well with the measured ones. Then, the validity of our developed ATM for creep life prediction under the longitudinal bending of unidirectional CFRP was established.

4. Statistical fatigue failure times under bending loads at various temperatures were measured and it was confirmed that the master curve of flexural fatigue strength as the function of number of cycles to failure can be constructed based on the concept of ATM.

5. The failures in all cases of static, creep and fatigue bending loads at various conditions are due to the triggers of micro-buckling of carbon fibers in the matrix resin.

6. The long-term flexural creep and fatigue failure lives for the longitudinal direction of unidirectional CFRP laminates under bending loads could be predicted by substituting the measured parameters into the formulated equations of ATM. As results, it was cleared that the flexural creep strength decreases drastically with increasing time with similar behavior of the relaxation modulus of matrix resin, and that the flexural fatigue strength decreases scarcely with increasing of time and temperature as well as the number of cycles to failure.

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STATISTICAL FATIGUE FAILURE TIME OF UNIDIRECTIONAL CFRP UNDER COMPRESSION LOADING

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1.0 Summary

Our developed accelerated testing methodology (ATM) based on the matrix resin viscoelasticity for the failure life prediction of fiber reinforced polymers (FRP) was applied to the statistical prediction of long-term compressive creep and fatigue failure life under bending load for the longitudinal direction of unidirectional CFRP laminates which is an important basic item for the durability design of CFRP structures used for aircraft and others.

First, the master curve of relaxation modulus of matrix resin in the wide range of time at a reference temperature was obtained by applying the time-temperature superposition to the results of DMA and creep tests measured at various temperatures.

Second, the shape and scale parameters of compressive static strength at a reference time and temperature and the viscoelastic parameter were experimentally determined through the static bending tests for CFRP laminates. The statistical compressive creep failure times for CFRP laminates under bending load were predicted by substituting the parameters determined by the static bending tests into the formulation of ATM, and the validity of predicted results was confirmed by the creep bending tests for CFRP laminates at various load levels at a temperature.

Third, the master curve of non-dimensional compressive fatigue strength-number of cycles to failure (S-N master curve) was constructed by the fatigue bending testes for CFRP laminates at various temperatures and frequencies based on the viscoelasticity of matrix resin.

Finally, the long-term compressive creep and fatigue strengths under bending load for CFRP laminates were statistically and reliably predicted.

2.0 Introduction

Carbon fiber reinforced plastics (CFRPs) have been used for the primary structures of airplanes, ships, automobiles, and other vehicles, for which high reliability must be maintained during long-term operation. An accelerated testing methodology is strongly anticipated for predicting the long-term life of CFRP structures exposed to actual environmental temperatures, water, and other influences.

The mechanical behavior of CFRP matrix resin exhibits time-dependence and temperature-dependence, so-called viscoelastic behavior, not only above the glass transition temperature T_g, but also below T_g. Consequently, the mechanical behavior of CFRP presumably depends strongly on time and temperature [1–4]. In fact, the durability of CFRP is highly dependent on the environmental effects of temperature, water absorption, and so on. Many research papers have described CFRP durability under actual environmental conditions [5–9].

The author's earlier reports presented an accelerated testing methodology (ATM) for predicting the life of fiber-reinforced polymers (FRP) from the test data measured by the short time tests under elevated temperatures based on the time-temperature superposition principle holds for the matrix resin viscoelasticity [10–20]. Statistical formulations for scattered time-dependent and temperature-dependent static, creep, and fatigue strengths of CFRP were done based on Christensen's viscoelastic crack kinetics [21–25]. The overall results have been described in an earlier report [26].

The tensile strength along the longitudinal direction of unidirectional CFRP constitutes important and basic data for the reliable design of CFRP structures. The authors developed a testing method for tensile static, creep, and fatigue strengths in the longitudinal direction of unidirectional CFRP at elevated temperatures using resin-impregnated carbon fiber strands (CFRP strands) as the test specimen. Their strengths of CFRP strands were measured at various temperatures. Also, their strengths were inferred statistically using the formulated equations. The predicted values were compared with experimentally obtained data measured using CFRP strands. It was found that the tensile creep strengths of CFRP strands can be predicted from the measured data of statistical static strength of CFRP strands under various temperatures and the time- and temperature-dependent viscoelastic moduli of matrix epoxy resin [27, 28]. Furthermore, the tensile fatigue strengths of CFRP strands can be also predicted from the measured data of fatigue strengths of CFRP strand at room temperature in addition of statistical static strength of CFRP strands under various temperatures and the time- and temperature- dependent viscoelastic moduli of matrix epoxy resin [29].

The compressive strength along the longitudinal direction of unidirectional CFRP also constitutes important and basic data for the reliable design of CFRP structures. In this paper, the authors discuss the validity of our developed ATM for creep and fatigue life prediction under longitudinal bending of unidirectional CFRP laminates where the compressive fracture easily realized. In this case, the fracture mode of unidirectional CFRP laminates is the micro-buckling of carbon fiber in the compression side of bending specimen [24]. First, the viscoelasticity of matrix resin is evaluated based on the time-temperature superposition principle, and the flexural static strengths of CFRP laminates are measured statistically at various constant temperatures under a constant strain rate. Second, the statistical creep failure times under constant bending loads for CFRP laminates are predicted at a constant temperature by substituting the matrix resin viscoelasticity and the statistical flexural static strengths of CFRP laminates into the formulated equations of ATM, and the validity of predicted results was clarified by comparison with the creep failure times measured statistically using bending creep tests for CFRP laminates. Third, the statistical fatigue times under cyclic bending loads at various frequencies and temperatures for CFRP laminates are measured and the master curve of non-dimensional fatigue strength to show the relation of fatigue strength against the number of cycles to failure is constructed by using measured data. Finally, the long-term creep and fatigue life under bending loads of unidirectional CFRP laminates are compared with each other based on the viscoelasticity of matrix resin.

3.0 Formulation for Statistical Prediction of Creep and Fatigue Strengths of CFRP

The statistical time and temperature dependent quasi-static strength of CFRP and structures is formulated based on the viscoelasticity of matrix resin. The most important condition for this formulation is the time-temperature superposition principle held for the viscoelasticity of matrix resin. The following two conditions are added for establishing the formulation.

Condition A: Failure probability is independent of temperature and load histories.

Condition B: The time-dependent and temperature-dependent strength is controlled by the matrix resin viscoelasticity.

Figure 1 shows a CFRP structure suppressed by loading of several forces. Assuming that the matrix resin deformation in CFRP structure during loading is constrained perfectly by carbon fiber rigidity, the time-dependent strength of CFRP structure is controlled by the relaxation modulus of matrix resin.

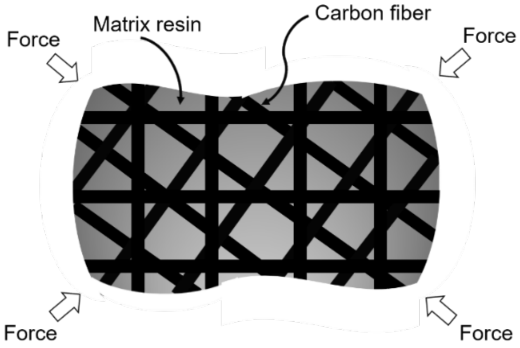


Figure 1. Mechanical model of CFRP

We have proposed the formulation of the statistical static strength σ_s of CFRP under constant strain rate loading based on the matrix resin viscoelasticity, as presented in the following equation, which was presented in our earlier work [30].

$$\log\sigma_{\mathrm{s}} = \log\sigma_0 + \frac{1}{\alpha}\log[-\ln(1-P_{\mathrm{f}})] + n_{\mathrm{R}}\log\left[\frac{E_{\mathrm{s}}^{*}(t,T)}{E_{\mathrm{r}}(t_0,T_0)}\right] \qquad (1)$$

In that equation, P_{f} signifies the failure probability, t denotes the failure time, t_0 represents the reference time, T stands for the temperature, T_0 stands for the reference temperature, and σ_0 and α respectively denote the scale parameter and the shape parameter on the Weibull distribution of static strength. In addition, n_{R} is the viscoelastic parameter; E_{r} and E_{s}^{*} respectively represent the relaxation and viscoelastic moduli of the matrix resin. The viscoelastic modulus E_s* for the static load with a constant strain rate is calculated as shown below.

$$E_{\mathrm{s}}^{*}(t,T)=E_{\mathrm{r}}(t/2\,,T) \tag{2}$$

The statistical creep strength σ_c can be ascertained by shifting the master curve of the static strength with log A based on Christensen's theory for the viscoelastic crack kinetics [21, 22]. Therefore, the master curve of the creep strength σ_c can be denoted by subscript "s" replaced with "c" in Equation (1) as shown below.

$$\log\sigma_{\mathrm{c}}=\log\sigma_{0}+\frac{1}{\alpha}\log[-\ln(1-P_{\mathrm{f}})]+n_{\mathrm{R}}\log\left[\frac{E_{\mathrm{c}}^{*}(t,T)}{E_{\mathrm{r}}(t_{0},T_{0})}\right] \tag{3}$$

In this equation, E_c* represents the viscoelastic modulus for a constant stress load.

$$E_{\mathrm{c}}^{*}(t,T)=E_{\mathrm{s}}^{*}(At,T)=E_{\mathrm{r}}(At/2\,,T) \tag{4}$$

The shifting amount log A, as ascertained from slope k_{R} of the logarithmic static strength against logarithmic failure time curve, is calculated as

$$\log A=\log\left(1+\frac{1}{k_{\mathrm{R}}}\right),\qquad k_{\mathrm{R}}=n_{\mathrm{R}}m_{\mathrm{R}} \tag{5}$$

where m_R represents the slope of the logarithmic relaxation modulus of the matrix resin against the logarithmic time curve. Parameter A shown in Equation (5) is obtainable from Christensen's viscoelastic crack kinetics. Parameter A for the case of bending load in the longitudinal direction of unidirectional CFRP is equal to 2 because the failure trigger mode is micro-buckling and is not crack propagation.

The above explanation for formulation indicates that the long-term creep failure time of CFRP can be statistically predicted by substituting the matrix resin relaxation modulus E_r and three parameters of CFRP static strength σ_0, α and n_{R} into Equation (3). These three parameters can be easily determined by the static tests of CFRP and the relaxation modulus of matrix resin.

We proposed the formulation of statistical fatigue strength of CFRP σ_f with fatigue degradation parameter F_f based on the matrix resin viscoelasticity, as shown below [30].

$$\log\sigma_f = \log\sigma_0 + \frac{1}{\alpha}\log[-\ln(1-P_f)] + n_R\log\left[\frac{E_f^*(t,T)}{E_r(t_0,T_0)}\right] - F_f(f,T,N_f) \tag{6}$$

The viscoelastic modulus E_f* is calculated using the following equation for the cyclic load for the case in which the stress ratio of the minimum stress / the maximum stress is zero. Fatigue degradation parameter F_f, as a function of frequency f, temperature T and the number of cycles to failure N_f, is obtainable by the following equation which is determined based on experimentation.

$$F_f(f,T,N_f) = F_f^*(f,T)\sum_{i=1}^{n} a_i \log(2N_f)^i, \qquad F_f^*(f_0,T_0) = 1 \tag{8}$$

The fatigue strength at $N_f = 1/2$ is equal to the static strength when failure time t is equal to 1/(2f). f_0 and T_0 are the reference frequency and temperature, respectively. The non-dimensional fatigue strength S_f can be defined as shown by the following equation.

$$\log S_f = \log\frac{\sigma_f}{\sigma_0} - n_R\log\left[\frac{E_f^*(t,T)}{E_r(t_0,T_0)}\right] = \frac{1}{\alpha}\log[-\ln(1-P_f)] - F_f(f,T,N_f) \tag{9}$$

Furthermore, the S_{f0} defined as the following equation is the master curve of non-dimensional fatigue strength when S_{f0} makes statistically one curve for various frequencies and temperatures.

$$\log S_{f0} = \frac{1}{\alpha}\log[-\ln(1-P_f)] - \frac{F_f(f,T,N_f)}{F_f^*(f,T)} \tag{10}$$

The time-temperature superposition for the viscoelasticity of matrix resin should hold for the frequency-temperature fatigue parameter $F_f^*(f,T)$.

4.0 Experiments

4.1 Specimens Employed

Unidirectional CFRP laminated plates were made by laminating and molding the prepreg using the autoclave method under the curing conditions presented in Table 1.

Table 1. Prepreg and curing condition

Fiber/Resin of prepreg (Torayca prepreg)	T800SC/2592
Thickness of prepreg	0.125 mm
Laminate configuration	$[0_8]$
Laminate thickness	2.0 mm
Plate size	200 mm × 200 mm
Curing method	Autoclave
Curing temperature	130 °C
Curing pressure	0.3 MPa
Curing time	2 hours

4.2 Determination Method of Relaxation Modulus of Matrix Resin

The relaxation moduli of matrix resin at various temperatures were determined by DMA and creep tests for the transverse direction of unidirectional CFRP laminates shown by Fig. 2. The details of determination process for the master curve of relaxation modulus of matrix resin are shown in our previous paper [31].

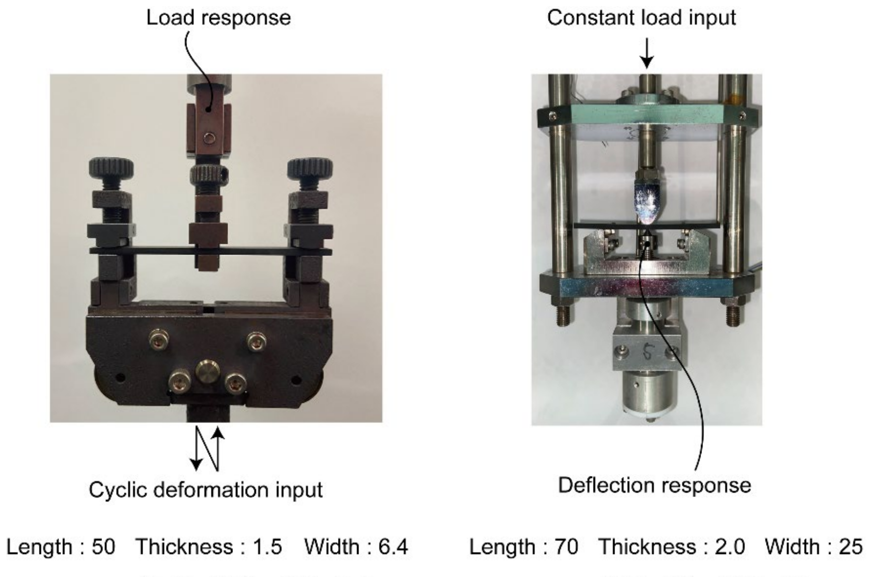
DMA tests were conducted at the cyclic deformation of maximum strain 0.06 %, various frequencies f from 0.01 Hz to 10 Hz and various temperatures T from 25 ºC to120 ºC, and the load responses were measured. The storage moduli $E'(f, T)$ for the transverse direction of unidirectional CFRP laminates obtained were converted to the storage moduli E' of matrix resin by using Chamis's role of mixture [32] and the mechanical properties of carbon fiber T800S [33], and the E' of matrix resin were converted to the relaxation moduli $E_r(t, T)$ based on the following linear viscoelasticity [34].

$$E_r(t,T) = E'(\ f,T\ ), \qquad t = 1/\pi^2 f \tag{11}$$

Creep tests were performed as the confirmation of reliability for the long-term relaxation modulus determined by the results of DMA tests measured in the short time range. Creep tests were conducted arbitrary constant loads from 5 N to 50 N at various temperatures T from room temperature to 90 °C and the deflection response were measured from 1 minute to 1000 minutes. The creep compliances $D_c(t, T)$ for the transverse direction of unidirectional CFRP laminates obtained were converted to the creep compliance D_c of matrix resin and the D_c of matrix resin were converted to the relaxation moduli $E_r(t, T)$ based on the following linear viscoelasticity [34].

$$E_r(t,T) = \frac{a}{D_c(t,T)}, \qquad a = \frac{1}{\Gamma(1+m)\Gamma(1-m)} \tag{12}$$

The long-term reliability for the relaxation moduli at various temperatures in the wide range of time determined from the results of DMA tests were confirmed by combining the results of creep tests.



Length : 50 Thickness : 1.5 Width : 6.4
Span : 37.5 Unit : mm

DMA test (Dual cantilever)

Length : 70 Thickness : 2.0 Width : 25
Span : 50 Unit : mm

Creep test (Three point bending)

Figure 2. DMA and creep tests for the transverse direction of unidirectional CFRP laminates.

4.3 Static, Creep and Fatigue Bending Tests of CFRP laminates

Beam specimens were cut from the laminated plates, as shown in Fig. 3. Static, creep and fatigue bending tests were performed with three- point bending in the fiber direction of the unidirectional CFRP laminates with aluminum foil as the cushion material at the loading point, as shown in Fig. 3. Flexural static strength was found by setting the test speed to 2 mm/min at six temperature levels in the range of room temperature to 120 °C. The number of specimens at each temperature is 20 specimens or 10 specimens, respectively.

The bending creep tests were conducted at a constant temperature of 80 °C and three load levels using the same testing machine for static test. The number of specimens at each load level is 20 specimens, respectively.

The bending fatigue tests were conducted at five temperature levels in the range of room temperature to 100 °C and frequency $f = 2$ Hz under zero stress ratio. These tests were also performed at $f = 0.2$ Hz at 85°C.

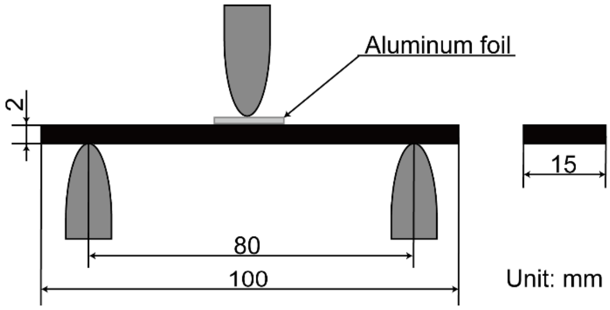


Figure 3. Three-point bending test for unidirectional CFRP laminates.

5.0 Results and Discussion

5.1 Relaxation Modulus of Matrix Resin

DMA and creep tests were conducted at various constant temperatures for the fiber transverse direction of unidirectional CFRP. The storage modulus *E*' versus the period (the inverse of frequency) *t* by DMA test and the creep compliance D_c versus the elapsed time *t* by creep test were measured at various temperatures *T* for the transverse direction of unidirectional CFRP. The relaxation moduli of matrix resin in the wide range of time over 7 decades from 10^{-4} min to 10^{3} min at various temperatures were obtained using the linear viscoelasticity and role of mixture, as shown in Fig. 4.

The master curve of relaxation modulus at reference temperature T_0 = 25 °C shown in Fig. 5 was obtained by horizontal and vertical shifting of the relaxation moduli at various temperatures shown in Fig. 5 with respect to the logarithmic time axis and the logarithmic relaxation modulus axis based on the modified time-temperature superposition principle by which the reliable long-term prediction of viscoelastic coefficient is available [35]. Figure 6 shows the time–temperature (horizontal) shift factor a_{T0} (*T*) and the temperature (vertical) shift factor b_{T0} (*T*), which are obtained by constructing the master curve of the relaxation modulus.

The loss tangent $\tan\delta$ versus frequency *f* at various temperature was measured by DMA tests and the master curve of $\tan\delta$ versus reduced frequency *f'* at reference temperature $T_0 =$ 25 °C obtained using the time-temperature (horizontal) shift factor shown in Fig. 6 is shown in Fig. 7.

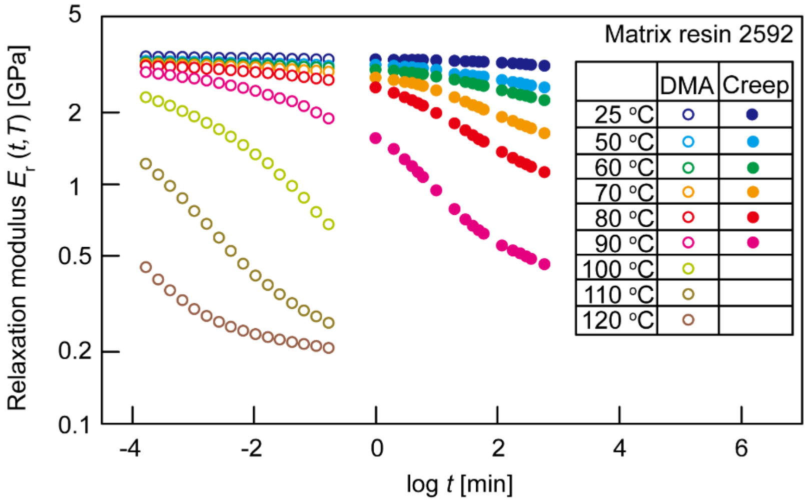


Figure 4. Relaxation moduli of matrix resin at various temperatures.

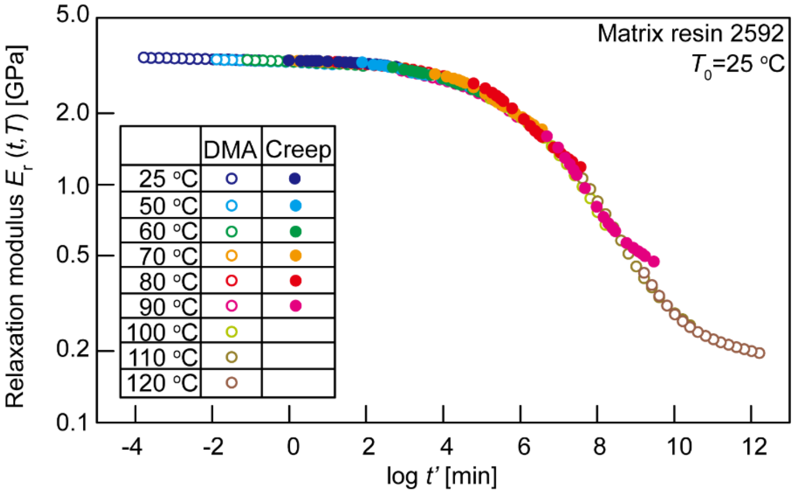


Figure 5. Master curve of relaxation modulus of matrix resin at T_0 = 25 °C.

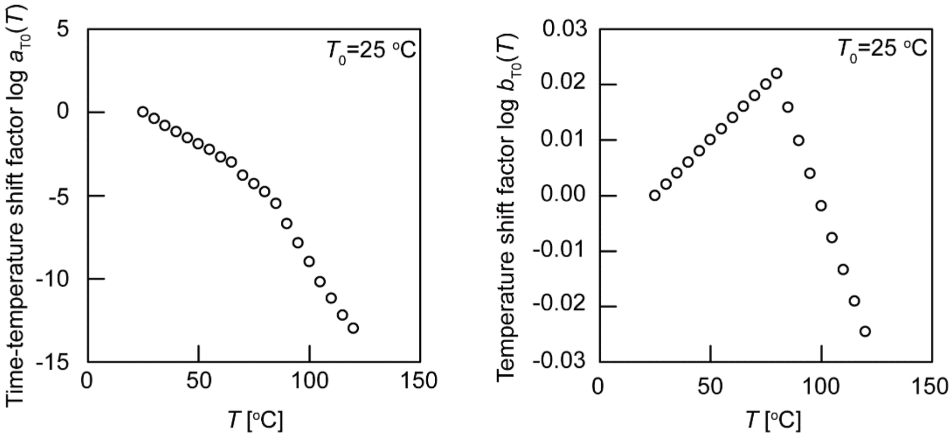


Figure 6. Horizontal and vertical shift factors for relaxation modulus of matrix resin

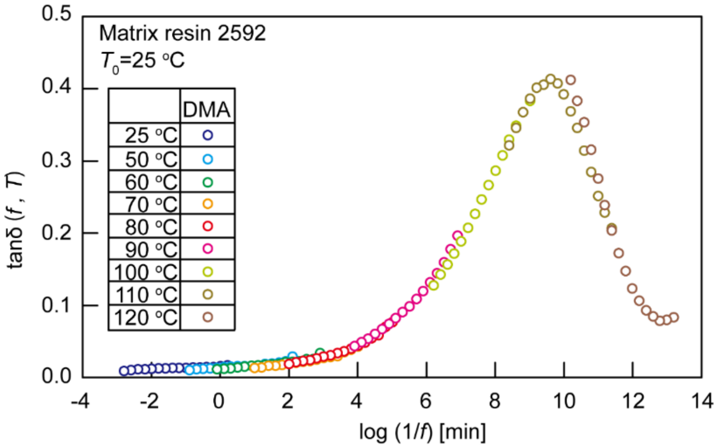


Figure 7. Master curve of loss tangent of matrix resin at T_0 = 25 °C.

5.2 Statistical Flexural Static Strength of CFRP laminates

Flexural static strengths of unidirectional CFRP laminates obtained from the maximum loads on the load-deflection diagrams are shown against temperature in Fig. 8.

Weibull distributions for static strength at various temperatures are depicted in Fig. 9 for the CFRP laminates. On the table in this figure, α_s is the shape parameter and β_s is the scale parameter of the CFRP laminates. The horizontal axis is the static strength σ_s /the scale parameter β_s. Although the scale parameter decreases according to the temperature rise, the shape parameter maintains an almost constant value for the CFRP laminates to the temperature rise. The average of shape parameter α_s at six temperatures and the scale parameter β_s at temperature T = 25 °C in this figure can be inferred as the shape parameter α and scale parameter σ_0 of the static strength at the reference temperature T_0 = 25 °C, and the reference failure time t_0 = 1 min used in Equation (1).

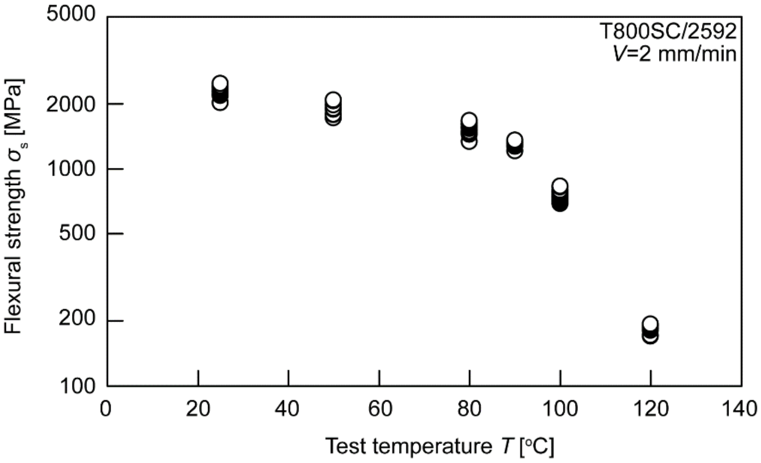
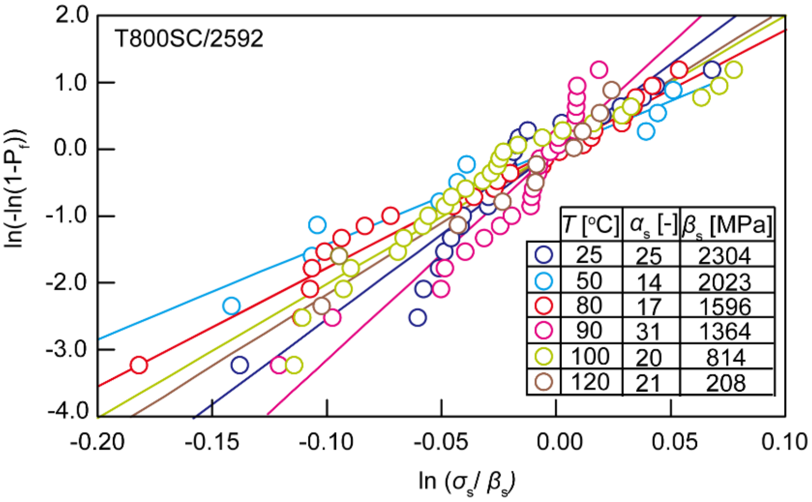


Figure 8. Flexural static strength versus temperature for unidirectional CFRP laminates.



	T [°C]	α_s [-]	β_s [MPa]
○	25	25	2304
○	50	14	2023
○	80	17	1596
○	90	31	1364
○	100	20	814
○	120	21	208

Figure 9. Weibull distributions of flexural static strengths at various temperatures for CFRP laminates.

5.3 Relation between Flexural Static Strength of CFRP Laminates and Viscoelastic Modulus of Matrix Resin

Figure 10 shows the relation between the flexural static strength of unidirectional CFRP laminates and the viscoelastic modulus of the matrix resin. Linear approximation was made based on data of 25 °C, 50 °C, and 80 °C, which are temperatures below the glass transition temperature of the matrix resin. The viscoelastic parameter $n_R = 0.97$ was obtained.

All parameters in Equations (1) and (3) for predicting statistical flexural static and creep strengths for unidirectional CFRP laminates were obtained as presented in Table 2

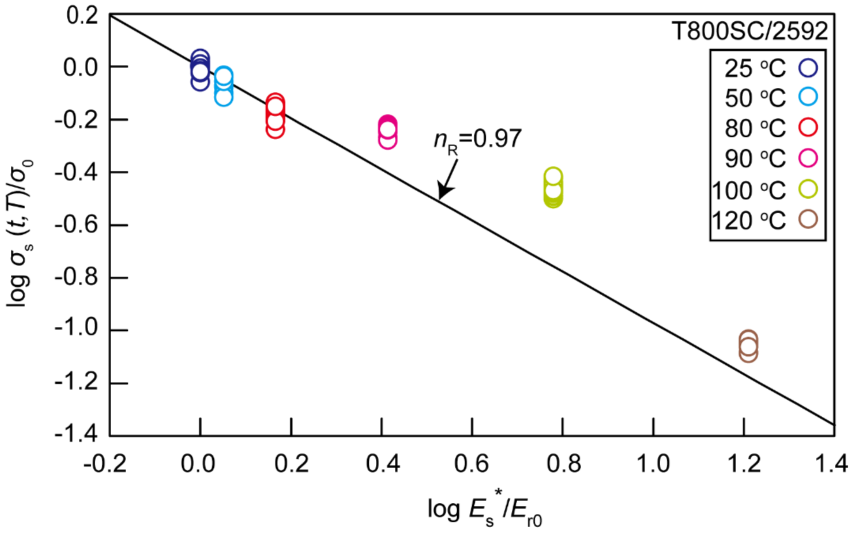


Figure 10. Flexural static strength of CFRP laminates versus viscoelastic modulus of matrix resin.

Table 2 Parameters of statistical flexural static and creep strengths for CFRP laminates

σ_0 [MPa]	α	n_R
2,304	21.0	0.97

5.4 Statistical Flexural Creep Strength versus Failure Time for CFRP laminates

Figure 11 presents prediction of the statistical flexural creep strength versus failure time at temperature $T = 80$ °C, as obtained by substituting the data of Table 2 and Figs. 5 and 6 into Equation (3). The flexural creep failure times measured experimentally at three levels of constant stress are presented in this figure. The measured creep failure times agree well with the predicted curve for which the failure trigger mode is the micro-buckling.

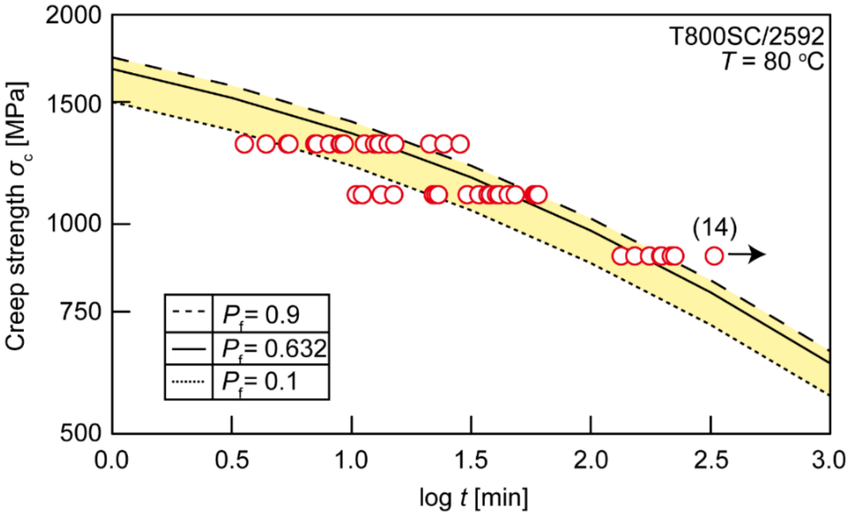


Figure 11. Predicted statistical flexural creep strengths versus failure time and measured ones.

5.5 Statistical Flexural Fatigue Strength against Number of Cycles to Failure for CFRP Laminates

Fatigue bending tests for unidirectional CFRP laminates were conducted at four temperatures with frequencies $f = 2$ Hz and the stress ratio of minimum stress/maximum stress $R = 0.05$, and additionally at temperature $T = 80$ °C with frequency $f = 0.2$ Hz. Flexural fatigue strength σ_{f} of CFRP laminates was measured against the number of cycles to failure N_{f} through testing. Figure 12 portrays the flexural fatigue strength versus the number of cycles to failure for CFRP laminates at various frequencies and temperatures. The flexural fatigue strengths of CFRP laminates decrease markedly with increase of the number of cycles to failure and temperature.

Figure 13 presents the non-dimensional fatigue strength S_{f} versus the number of cycles to failure N_{f} (S-N curves) at various frequencies and temperatures. These S-N curves show also strong frequency- and temperature- dependent behavior. These S-N curves make only one curve as shown in Fig. 14 if the frequency-temperature fatigue parameter $F_{\mathrm{f}}^{*}(f,T)$ defined in Equation (10) are selected to the inverse of reduced frequency $1/f'$at a reference temperature $T_0 = 25$ °C as shown in Fig. 15. This one curve is the master curve of fatigue strength at a reference condition of $T_0 = 25$ °C and $f_0 = 2$ Hz. The loss tangent tan δ versus $1/f'$at $T_0 = 25$ °C is shown in Fig. 14 by using Fig. 7. This figure indicates that the frequency-temperature fatigue parameter $F_{\mathrm{f}}^{*}(f,T)$ increases monotonically with the inverse of reduced frequency $1/f'$ in the similar manner as the loss tangent, which is one of expressions for the viscoelasticity of matrix resin. The relation between the frequency-temperature fatigue parameter $F_{\mathrm{f}}^{*}(f,T)$ and the loss tangent $\tan\delta\,(f,T)$ shown in Fig. 16 is approximately linear.

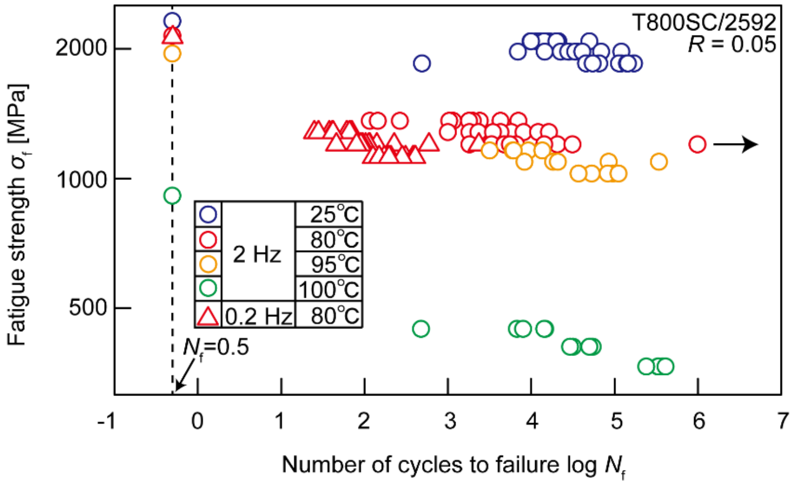


Figure 12. Fatigue strength versus the numbers of cycles to failure for CFRP laminates.

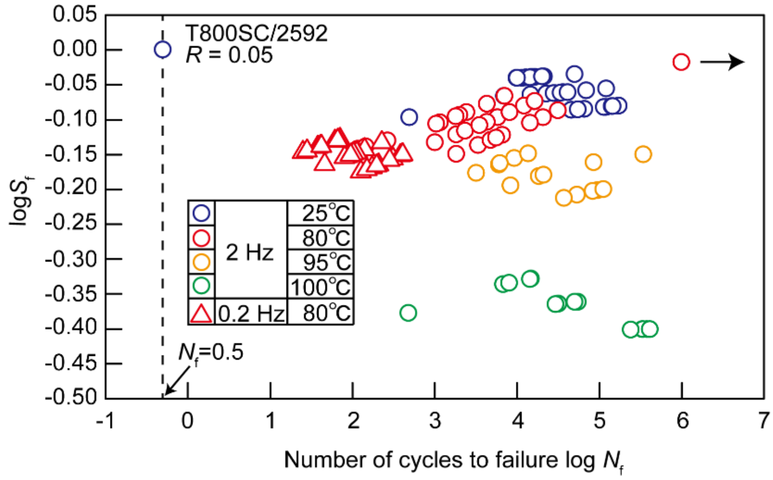


Figure 13. S-N curves at various conditions for CFRP laminates.

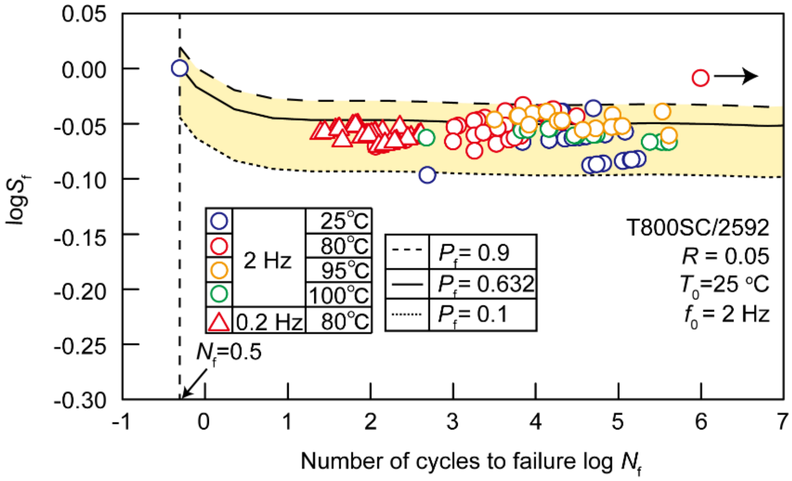


Figure 14. S-N master curve at reference condition for CFRP laminates.

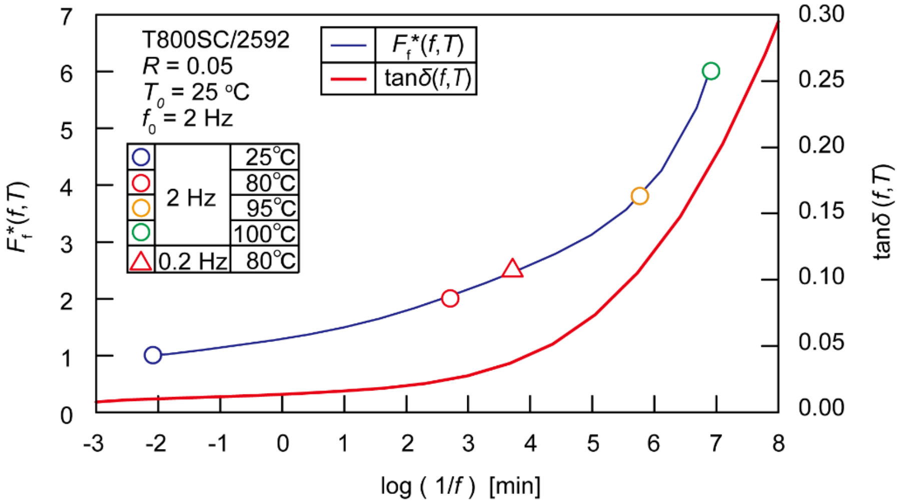


Figure 15. Frequency-temperature fatigue parameter and loss tangent versus inverse of frequency at a reference condition of T_0 = 25 °C and $f_0 = 2$ Hz.

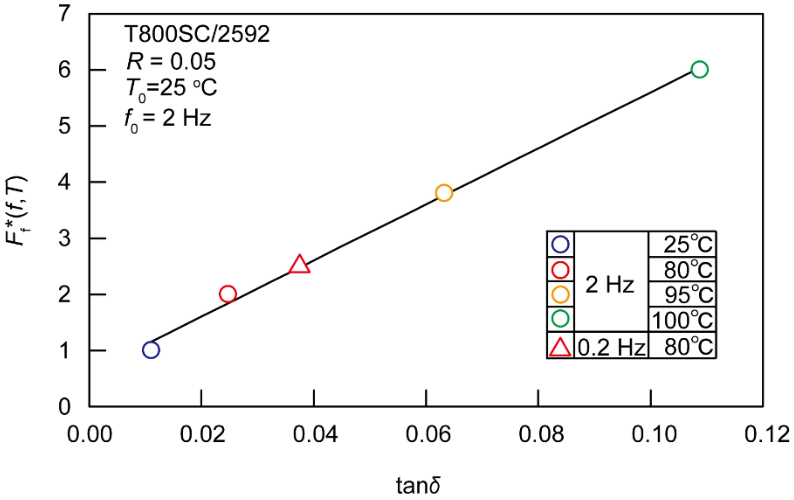
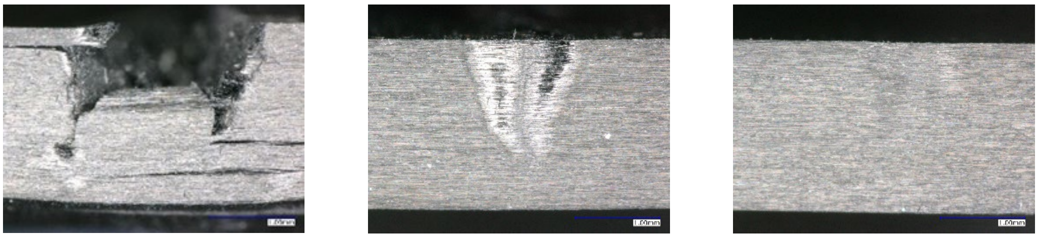


Figure 16. Frequency-temperature fatigue parameter versus loss tangent

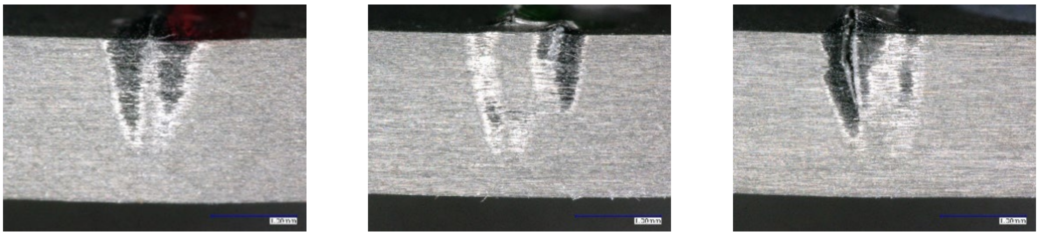
5.6 Fractgraphs of Static, Creep and Fatigue Failures under Bending Loads

The failures in all cases of static, creep and fatigue bending loads at various conditions are due to the triggers of micro-buckling of carbon fibers in the matrix resin at the compression side of specimen at the loading point as shown in Fig. 17.



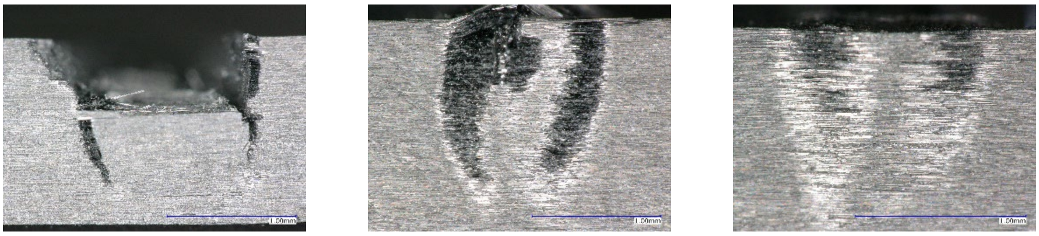
25 °C/2mm/min 100 °C/2mm/min 120 °C/2mm/min

Static



80°C/900 MPa 80°C/1100 MPa 80°C/1300 MPa

Creep



25 °C/2 Hz/2074 MPa 80 °C/2 Hz/1197 MPa 100 °C/2Hz/407 MPa

Fatigue

Figure 17. Side views of bending specimens of unidirectional CFRP laminates after loading

5.7 Long-term Prediction of Flexural Creep and Fatigue Strengths of Unidirectional CFRP Laminates

Figure 18 shows long-term predictions for the relaxation modulus of matrix resin and the flexural creep and fatigue strengths of unidirectional CFRP laminates. This figure clarifies that long-term flexural creep strength of unidirectional CFRP laminates are controlled strongly by the relaxation modulus of matrix resin. The flexural creep strength decreases drastically with increasing time with similar behavior of the relaxation modulus of matrix resin and this decreasing accelerates with increasing of temperature. On the other hand, the flexural fatigue strength decreases scarcely with increasing of time and temperature as well as the number of cycles to failure N_f. Especially, the fatigue strength keeps a constant with increasing N_f in the region of large number of cycles to failure, which is considered as the fatigue limit. This behavior is due to the fact that the trigger of failure is the micro-buckling of carbon fibers in the matrix resin.

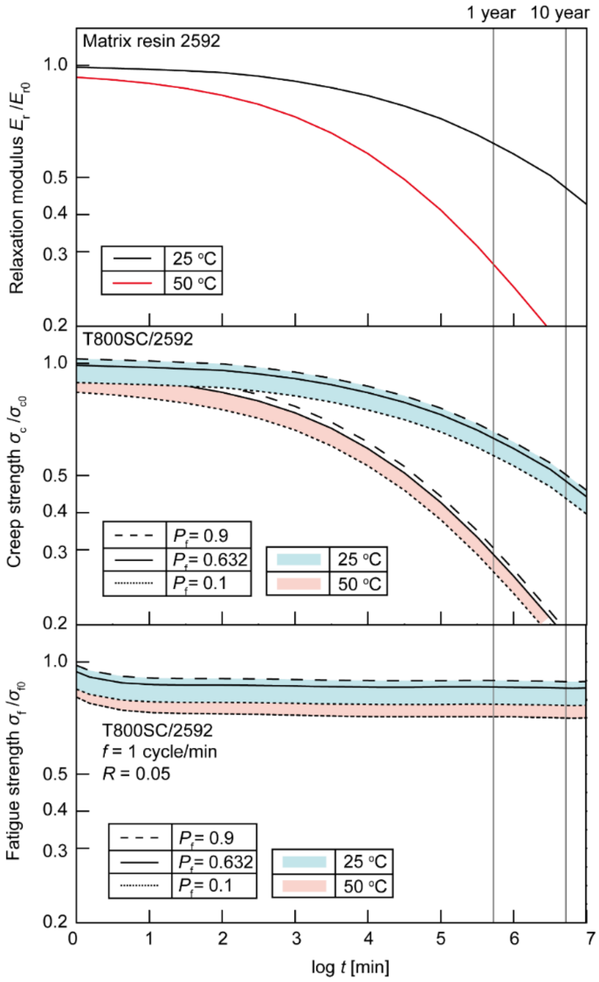


Figure 18. Long-term prediction of relaxation modulus of matrix resin and flexural creep and fatigue strengths of unidirectional CFRP laminates

6.0 Conclusion

Our developed accelerated testing methodology (ATM) based on the matrix resin viscoelasticity for the creep and fatigue failure life prediction of fiber reinforced polymers (FRP) was applied to the statistical prediction of long-term creep and fatigue failure life for the longitudinal bending of unidirectional CFRP laminates which is an important basic item for the durability design of CFRP structures used for aircraft and others. The validity of our developed ATM for this creep and fatigue life prediction under the longitudinal bending of unidirectional CFRP was discussed herein. From that discussion, the following results were obtained.

1. Formulation of the statistical creep and fatigue failure time under bending load for the longitudinal direction of unidirectional CFRP based on the viscoelasticity of matrix resin was established assuming micro-buckling failure near the loading point.

2. The master curve of relaxation modulus of matrix resin in the very wide range of time was constructed by application of the time–temperature superposition to the data of DMA and creep tests measured at various temperatures.

3. Statistical creep failure times under bending loads at an arbitrary temperature were predicted by substituting the statistical flexural static strengths of unidirectional CFRP measured at several temperatures and the relaxation modulus of matrix resin into the formulation. Predicted statistical creep failure times agreed well with the measured ones. Then, the validity of our developed ATM for creep life prediction under the longitudinal bending of unidirectional CFRP was established.

4. Statistical fatigue failure times under bending loads at various temperatures were measured and it was confirmed that the master curve of flexural fatigue strength as the function of number of cycles to failure can be constructed based on the concept of ATM.

5. The failures in all cases of static, creep and fatigue bending loads at various conditions are due to the triggers of micro-buckling of carbon fibers in the matrix resin.

6. The long-term flexural creep and fatigue failure lives for the longitudinal direction of unidirectional CFRP laminates under bending loads could be predicted by substituting the measured parameters into the formulated equations of ATM. As results, it was cleared

that the flexural creep strength decreases drastically with increasing time with similar behavior of the relaxation modulus of matrix resin, and that the flexural fatigue strength decreases scarcely with increasing of time and temperature as well as the number of cycles to failure.

7.0 Acknowledgements

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9.0 Appendix

9.1 Awards (2019~Present)

1. Sia Nemat-Nasser Medal from Society for Experimental Mechanics in 2019
2. SEM Fellow Award from Society foe Experimental Mechanics in 2021
3. B. J. Lazan Award from Society for Experimental Mechanics in 2022

Note: The formal presentation of 2021 SEM Award and 2022 B. J. Lazan Award will take place at the Awards Luncheon on June 15, Wednesday during the SEM Conference and Exposition on Experimental and Applied Mechanics that is scheduled for June 13-16, 2022 in Pittsburgh, Pennsylvania.

9.2 Publications (2019~Present)

Book Chapter

1. Miyano, Y. and Nakada, M. "Statistical Long-Term Creep Failure Time of Unidirectional CFRP", Chapter of "Advances in Thick Section Composite and Sandwich Structures", Editor Sung W. Lee, Springer (2020).

Journal papers

1. Miyano, Y. and Nakada, M., "Statistical Time and Temperature Dependent Static and Creep Strengths of Unidirectional CFRP under Tension and Bending Loads", *Materials System*, Vol.36, pp.11-15 (2019).
2. Nakada, M., Miyano, Y., Yamanaka, S. and Matsumoto, T., "OHT and OHC Fatigue Strengths of Interlaminar Toughened Quasi-isotropic CFRP Laminates Using Benzoxazine Resin as Matrix, *Materials System*, Vol.36, pp.31-36 (2019).
3. Nakada, M., Miyano, Y., Morisawa, Y., Nishida, H., Hayashi, Y. and Uzawa, K., "Prediction of statistical life time for unidirectional CFRTP under creep loading" *Journal of Reinforced Plastics and Composites*, Vol.38, pp.938-946 (2019).
4. Nakada, M. and Miyano, Y., "Temperature dependence of statistical fatigue strengths for unidirectional carbon fiber reinforced plastics under tension loading", *Journal of Composite Materials,* published online, November 2019, DOI:10.1177/0021998319886629.
5. Nakada, M., Miyano, Y., Morisawa, Y., Isaki, T., Hirano, T. and Uzawa, K., "Statistical life prediction of unidirectional carbon fiber/polypropylene tape under creep tension load", *Journal of Reinforced Plastics and Composites*, published online, January 2020, DOI:10.1177/0731684419900319.

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12. Nakada, M., Miyano, Y., Nakagawa, H., Kageta, S., Tajima, Y., Saito, H., Morimoto., Y. and Kageyama, K., “Cooling rate influence on the warp of a CFRP asymmetric laminated beam”, *Mechanics of Time-Dependent Materials* published on line: March 2021, doi.org/10.1007/s11043-021-09493-1.
13. Nakada, M., Miyano, Y., Kageta, S., Nishida, H., Hayashi, Y. and Uzawa, K., “Prediction of statistical life time for unidirectional CFRTP under cyclic loading”, *Journal of Reinforced Plastics and Composites*, published online April 2021, DOI: 10.1177/07316844211005542.
14. Miyano, Y., Nakada, M. and Kageta, S., “Statistical Assessment of Tensile Static, Creep and Fatigue Strengths for Unidirectional CFRP”, *Experimental Mechanics,* published on line 13 May 2021, doi.org/10.1007/s11340-021-00711-y.
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Conference Keynote Presentation

1. Miyano, Y. and Nakada, M., "Accelerated Testing Method for Durability of CFRP ~Recent Twenty Years of Progress~", *22nd International Conference on Composite Materials*, August 11~16, 2019, Melbourne Australia.

Conference Presentations

1. Miyano, Y., Nakada, M., "Temperature Dependence of Statistical Fatigue Strengths for Unidirectional CFRP under Tension Loading", *Annual Conference and Exposition on Experimental and Applied Mechanics*, June 3~6, 2019, Reno, Nevada, USA.
2. Nakada, M., Miyano, Y., Morisawa, Y., Isaki, T., Hirano, T. and Uzawa, K., "Statistical Life Prediction of Unidirectional CF/PP Tape under Creep Tension Load", *22nd International Conference on Composite Materials*, August 11~16, 2019, Melbourne Australia.
3. Miyano, Y., Nakada. M and Kageta, S., "Formulation for Statistical Tensile Static, Creep and Fatigue Strengths for Unidirectional CFRP", *Virtual SEM XIV International Congress*, Online, September 14~17, 2020.
4. Miyano, Y., Nakada, M. and Kageta, S., "Statistical Accelerated Testing Methodology for Long-term Life of Unidirectional CFRTP under Water Absorption", *2021 SEM Annual Conference and Exposition on Experimental and Applied Mechanics*, Online, June 14~17, 2021.
5. Nakada, M., Miyano, Y., Morisawa, Y., Nishida, H., Hayashi,Y. and Uzawa, K., "Prediction of long-term creep and fatigue strengths of unidirectional CFRP using thermoplastic epoxy resin as matrix", *17th Japan International SAMPE Symposium and Exhibition*, December 1~3, 2021, Tokyo, Japan.
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8. Nonaka, T., Nakada, M., Miyano, Y., Morisawa, Y., Isaki, T., Hirano, T. and Uzawa, K., "Effect of Water Absorption on the Tensile Strength of CF/PP UD-Tape", *17th Japan International SAMPE Symposium and Exhibition*, December 1~3, 2021, Tokyo, Japan.