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Ensuring Equitable Access to Liver Transplant Using Linear Programming Duality, Network Flow, and Simulation

by

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14. ABSTRACT

Donor livers are allocated via the 'Model for End-Stage Liver Disease (MELD)' score. Women are consistently 4.8% less likely to receive a liver transplant than men. Women are disadvantaged by their smaller abdominal cavities which cannot accommodate larger donated livers. They are also disadvantaged by lower natural creatinine levels, a waste product correlated with liver disease. We proposed increasing female MELD scores to reach an equitable allocation of livers. To decide how many points should be added, we created a linear program to model liver allocation. This linear program used transplant data to test how total MELD points and total number of lives saved changes when a new constraint to enforce equity is introduced. Next, we used the linear programming duality theorem to calculate how many points should be added to female MELD scores. We then designed a network flow model to capture the size incompatibilities that women face when they are only able to accept small livers. We used data analysis to decide how to restrict the flow of livers between small donors and large patients and created new allocation rules reserving smaller livers for smaller patients to equalize the rate of transplant between all size groups.

15. SUBJECT TERMS

women, duality theorem, network flow, liver transplant, Model for End Stage Liver Disease (MELD)

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ENSURING EQUITABLE ACCESS TO LIVER TRANSPLANTATION USING LINEAR PROGRAMMING DUALITY, NETWORK FLOW, AND SIMULATION

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Abstract

Donor livers to transplant patients are allocated based on medical urgency and liver disease severity, via the ‘Model for End-Stage Liver Disease (MELD)’ score. The goal of allocation is to prevent candidates from dying while waiting for a liver, yet many candidates die on the list. Women are consistently 4.8% less likely to receive a liver transplant than men. Women are disadvantaged by their generally smaller abdominal cavities which cannot accommodate larger donated livers. They are also disadvantaged by lower creatinine levels; creatinine is a waste product made by the liver which signifies renal dysfunction at high levels. Since women generally have less muscle mass than men, they produce less creatinine which makes their liver disease appear less severe in the MELD score.

We propose increasing MELD scores for women, to increase their access to donated livers. To decide how many points should be added to women’s MELD scores, we created an ideal linear program for liver allocation. This linear program uses real transplant data from 2016 to test how the total MELD points and number of lives saved by an allocation of livers changes when a new constraint to enforce equity across sexes is introduced. Next, we used the duality theorem of linear programming to calculate how many points should be added to womens’ MELD scores to achieve fairness. The transplant community relies heavily on scoring mechanisms, rather than optimization, so designing an allocation rule based on a score makes it more likely to be accepted and implemented.

Next, we designed a network flow model to capture the size incompatibilities that women face when they are only able to accept small livers because of the size of their abdominal cavity. We use the Max Flow Min Cut theorem to restrict the flow of livers between small donors and large patients and create new allocation rules reserving smaller livers for smaller patients to equalize the rate of transplant between all size groups.

Finally, we tested my proposed score boost using the Liver Simulated Allocation Model (LSAM) to ensure these changes work in practice when medical details like diagnosis and physical characteristics like age, race, blood type, height, and weight are included. The simulation tested our model results by matching organs to patients one at a time, whereas the linear program and network flow model assumed future knowledge of all donated livers over the next year.

We have calculated a score boost for women to correct the bias they receive for lower natural creatinine levels and tested it with the Liver Simulated Allocation Model. We also created size restrictions using a network flow algorithm to correct for the disadvantage women have due to their overall smaller stature. Finally, we tested some of these policy changes in the Liver Simulated Allocation Model.

Keywords: women, duality theorem, network flow, liver transplant, Model for End Stage Liver Disease (MELD)

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Background

Organ transplantation, the “surgical removal of a healthy organ from one deceased or living individual and its placement into another person”, has been a lifesaving treatment since the first successful transplant was performed in 1954 (Kizer et al., 2022). Each transplantable organ has its own allocation system balancing efficiency (lives saved) with equity among transplant candidates. Livers are currently prioritized using the Model for End-Stage Liver Disease (MELD) as well as a few other factors like blood type, body size, and geographic location. The MELD score is based on four main components: international normalized ratio, total bilirubin, creatinine, and serum sodium levels. Each patient’s MELD score ranges from 6 to 40; a higher score is a lower estimate of their chances of surviving their liver disease over the next three months (Darden et al., 2020). When a liver becomes available, it is offered to the patient with the highest MELD score first to maximize lives saved.

The Organ Procurement and Transplantation Network (OPTN) sets and implements organ allocation policies, maintains the wait list, and collects data on transplantation. In March of 2000, they published “the OPTN Final Rule”, which states that allocation:

1. Shall seek to achieve the best use of donated organs, and avoid organ waste;
2. Shall set priority rankings based on sound medical judgement;
3. Shall balance medical efficiency (extra life years) and equity (waiting time), without discriminating patients based on their race, age, and blood type;
4. Shall be reviewed periodically and revised as appropriate (The Organ Procurement and Transplant Network, 2000)

We focus on the third point to correct womens’ disadvantage under the MELD system, while maintaining efficiency and equity for other demographics. Using these guidelines, the OPTN designed a different point system for each organ that prioritizes candidates on a waitlist for an available organ. The components of the point system vary by organ. For example, kidney transplant candidates are prioritized mainly by the time they have been on the waitlist. This is because candidates with kidney failure can receive dialysis, a procedure that serves as an 'artificial kidney' to regulate body function while waiting for a transplant. Liver transplant candidates cannot be prioritized by wait time because no procedure like dialysis exists for end-stage liver failure. Instead, liver candidates are prioritized by the severity of a candidate's disease and the predicted length of time that they can survive without a transplant.

Many studies have shown a gender disparity that disadvantages women waiting for a liver transplant. According to the data collected by the United Network for Organ Sharing from 1995 to 2012, women were consistently 4.8% less likely to receive a transplant, graphed in Figure 1. Even though data shows that women have a longer life expectancy after liver transplant then men, they are still less likely to receive one (Darden et al., 2020).

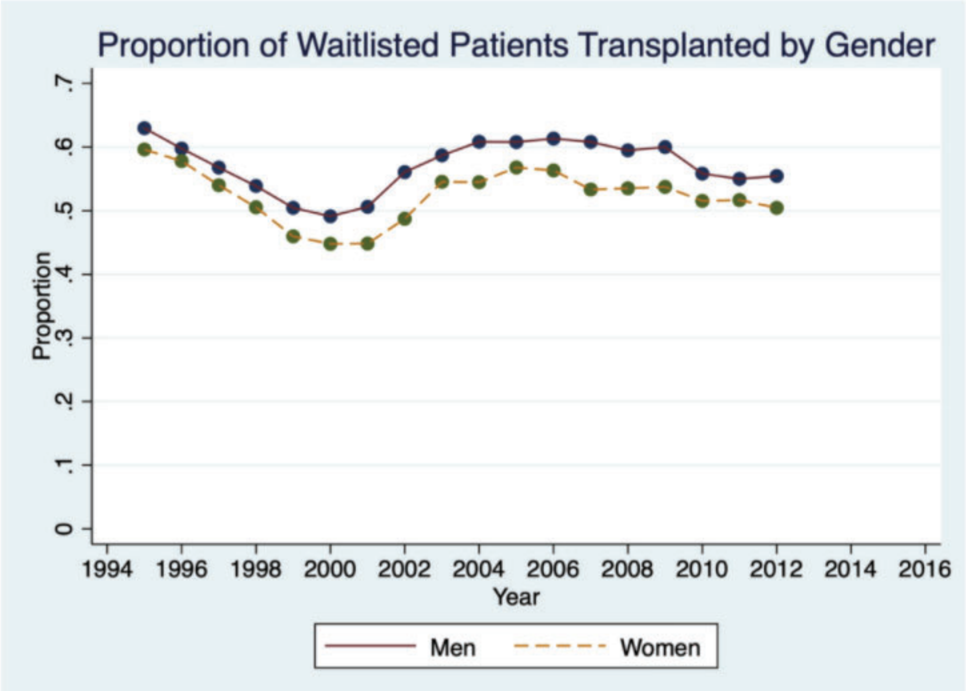


Figure 1: Gender Disparity in Liver Transplantation (Darden et al., 2020)

There are two explanations for this observed disparity between men and women. First, the natural creatinine level is much lower in women than in men. Creatinine is a medical waste product of creatine, an amino acid made by the liver as a result of normal muscle metabolism. A high level of creatinine signifies renal dysfunction and is a large component of a patient's MELD score (Cleveland Clinic, 2021). Because men naturally have more muscle tissue, they have a natural creatinine level that is 30% higher and therefore a physiologic advantage in their likelihood to receive a liver transplant.

The other possible explanation for the lower transplant rates among women is a patient's physical size. A recipient with a large abdominal cavity can accept small or large livers, while patients with a smaller abdominal cavity can only accept small livers. Livers are offered to transplant candidates according to MELD, regardless of the size of the donor or candidate. Women are more likely to be forced to decline livers when they are too large. Because men have larger abdominal cavities on average, their range of acceptable donors is larger (Darden et al., 2020).

I was motivated to study this when Professor Gentry first taught me about the scarcity of livers for transplantation. The demand for organs far outstrips the supply, and patients often die while waiting for a transplant. In 2012, over 15,000 patients were put on the waiting list, nearly 6,000 transplants were performed, but more than 2,000 patients died while awaiting a liver (Thompson et al., 2004). It is imperative that the system for allocating livers is as equitable as possible and does not discriminate against any patients of differing sex, race, blood type, et cetera. Correcting this issue could have a huge impact on thousands of women each year by providing them life-saving liver transplants.

Previous Work

The article “Persistent Sex Disparity in Liver Transplantation Rates” was published in 2020 and explains MELD scoring in detail, including the stages of revision it has gone through and the current problems with the system. The authors discussed the policy reform completed in 2002 that corrected the racial discrimination of the MELD system, but worsened the gender disparity in liver transplantation. Women are now 4.9% less likely to receive a transplant than men, compared to a less drastic 2.1% before the revision in 2002. They agreed with most other medical professionals that the three main reasons for this issue is a lower amount of natural creatinine, a smaller abdominal cavity, and possible regional socioeconomic differences (Darden et al., 2020).

The paper “Height Contributes to the Gender Difference in Wait-List Mortality Under the MELD-Based Liver Allocation System” studied the relationship between a woman’s height and weight and her probability of receiving a liver transplant. They found that womens’ MELD scores do not correctly represent their true medical condition - women are at 19% higher risk of wait-list mortality then men. They concluded that a possible explanation for a high wait-list mortality is that women are typically shorter and need smaller organs, which are preferentially offered to pediatric recipients (Lai et al., 2010).

We frequently referenced a paper published in 2004 titled “Simulating the Allocation of Organs for Transplantation.” The authors make it very clear how important it is to constantly reevaluate the standards we use to allocate organs because it is a complex and dynamic problem. Simulation allows allocation policies to be tested with valid, up-to-date, medically detailed data before the policies are implemented. The authors explain the timeline of candidate listing, organ procurement, transplant surgery, and finally post-graft success very clearly. This is shown in Figure 2:

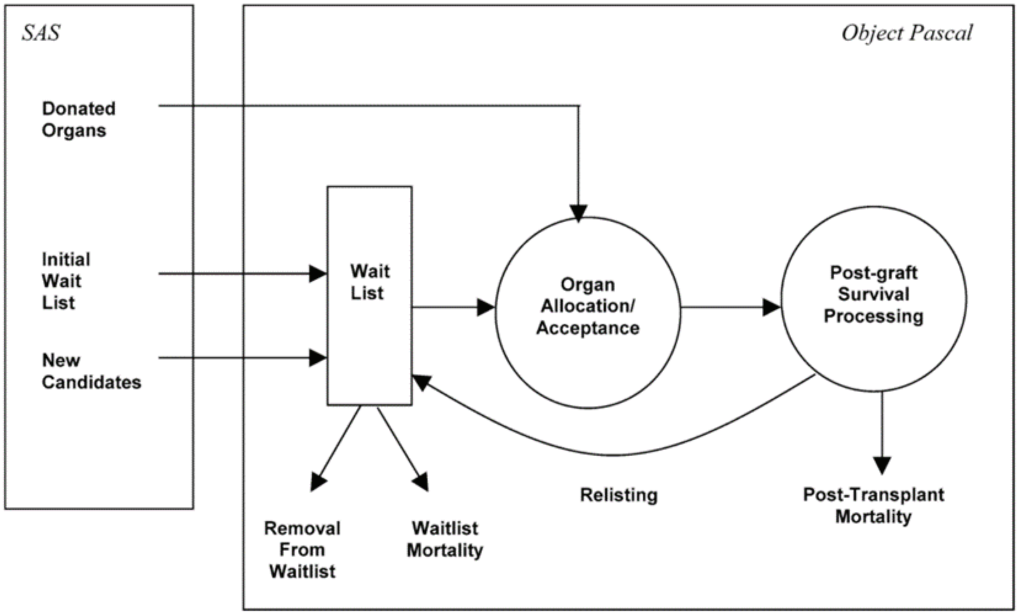


Figure 2: Diagram of the Candidate Listing and Liver Allocation Process (Thompson et al., 2004)

One of the major steps of my project tests the effects of adding points to womens' MELD scores in the Liver Simulated Allocation Model to show the transplant community how it will affect the efficiency and equity of the system (Thompson et al., 2004). A recommended expansion of this project would be to test the allocation rules we develop in the network flow model in LSAM as well. A more in-depth description of the development, operation, and results of the Liver Simulated Allocation Model can be found under 'Simulation to Test MELD Score Boost for Women'.

Our linear program for designing a MELD score boost for women was inspired by Bertsimas et al.'s "Fairness, Efficiency, and Flexibility in Organ Allocation for Kidney Transplant." In this paper, the authors design a scoring mechanism for kidney allocation to maximize the Life Years Gained from Transplantation (LYFT), subject to equity constraints. The authors tested adding or removing ‘fairness’ constraints based on race, gender, body mass index, and blood type, and observed the changes in the transplant system’s total life years gained from transplantation. This idealized linear program assumes foresight of every available organ and patient over the next year, before any matches are made. In reality, organs are sequentially matched when they are recovered to available patients without utilizing any future information. Even though this idealized problem is not realistic, it is still very useful in understanding the effect that a new equity constraint will have on the overall efficiency, or LYFT, of the transplant system. Bertsimas et al. then use this idealized linear program to design a point system, based on the organ’s allocation criteria, that maximizes LYFT while simultaneously enforcing the selected equity constraints (Bertsimas et al., 2013). In our linear program, we maximize the sum of MELD scores to maximize lives saved while enforcing an equity constraint for womens’ transplants.

To interpret the results of our ideal linear program, we used data from a paper titled “Correcting the Sex Disparity in MELD-Na.” In this paper, the authors define the precise relationship between a patient’s MELD score and their likelihood of survival without a liver transplant. Using data collected from 2016 to 2019, they estimated the mean 90-day survival rate without a transplant as a function of the MELD score assigned to the patient (Figure 3). They found that the mortality rate for women is typically slightly higher than the mortality rate for men at the same MELD score, showing that female patients have worse outcomes regardless of the allocation system. For example, at a MELD score of 30, women have a survival rate of 0.49 compared to a male survival rate of 0.53. Using the Kaplan-Meier method, they calculated an appropriate score boost for womens’ MELD scores to correct the difference in mortality rates to be 1-3 points (Wood et al. 2021). The authors of this paper used a different statistical method to solve the same issue of womens' MELD scores and designed a very similar-sized score boost, showing that the result we found was significant.

MELD score	Male survival rate	Female survival rate	MELD score	Male survival rate	Female survival rate
15	0.9639	0.9623	28	0.631	0.5931
16	0.9596	0.9517	29	0.5735	0.5499
17	0.9491	0.9423	30	0.5334	0.4993
18	0.9334	0.9244	31	0.4818	0.4673
19	0.9177	0.9094	32	0.4573	0.4282
20	0.911	0.8812	33	0.3988	0.365
21	0.8868	0.8673	34	0.3464	0.3477
22	0.8671	0.8308	35	0.31	0.2827
23	0.8386	0.8039	36	0.2714	0.2702
24	0.8075	0.7657	37	0.1755	0.2046
25	0.7657	0.74	38	0.1803	0.1997
26	0.7249	0.6955	39	0.1482	0.1074
27	0.6686	0.6527	40	0.0924	0.1036

Figure 3: 90-day Survival Rate for Men and Women at Same MELD Score (Wood et al., 2021)

There is much published research detailing the disparity in the number of transplants that go to men and women, making it a widely recognized issue among the transplant community.

Designing a MELD Score Boost for Women Using Linear Program Duality

We created an ideal linear program to maximize the number of MELD points of patients that received a liver transplant. Since higher-MELD patients are more likely to die if they do not get a transplant, this objective function maximizes the number of lives saved through transplant. This ideal linear program has a set of constraints: (1) that only one liver is assigned to one candidate, (2) that only one patient can accept a liver and no partial livers can be allocated, and (3) a gender equity constraint. This fairness constraint sets a lower bound on the percentage of total transplants that go to women, because the ideal distribution of livers would allocate the same percentage of transplants to the percentages of men and women on the list. The model we constructed is:

$$
\begin{aligned}
\text{maximize:}\quad & \textstyle\sum_{p\in P} MELD(p)x_p \\
\text{subject to:}\quad & \textstyle\sum_{p\in P} x_p \le 7409 \\
& \sum_{p\in P} f_p\, x_p \ \ge 0.4 * 7409 \\
& x_p \in \{0,1\} \text{ for all } p \in P
\end{aligned}
$$

To evaluate our model, we used real transplant data collected by the Organ Procurement and Transplant Network (OPTN) from January 1st 2016 to January 1st 2017. This dataset includes a unique identifier for each candidate (patient id), the sex of the candidate, the MELD score of the candidate at the time they were listed, the BMI, height and weight of the candidate, and the maximum and minimum donor weight the candidate is willing to consider. We found that 40% of the patients on the list during this period were women out of a total 12,295 total candidates. For equity, the gender equity constraint must then ensure that at least 40% of the available 7,409 livers go to a female candidate. The only set in this model is one of all of the patients, $p \in P$, listed on the transplant list in 2016. The only decision variable in this model, x_p, is a binary variable that takes the value of one if a patient is assigned a donor organ. This preliminary model also includes only two parameters: the MELD score of each patient and a gender indicator. The parameter indicating MELD score, referred to as *MELD(p),* was assigned to each patient in the set P to be optimized in the objective function. The gender indicator, f_p, is one if the patient is female and zero if the patient is male. This parameter is used in the gender equity constraint to ensure that 40% of the available organs are assigned are to a female candidate.

The results of this optimization show how much efficiency might be lost by adding a gender equity constraint. Running the model without the gender equity constraint shows how the current transplant system would allocate livers, and the objective function value shows the number of MELD points transplanted without enforcing equity that year.

This model assumes an unrealistic foresight of all organs and patients at the beginning of the analysis period. In reality, both organs and patients flow into the system randomly and organs must be sequentially assigned to patients using no future information. This complication is shown in Figure 4. On the left is a representation of our linear program, where organs are being matched with a patient pair simultaneously as if all transplant patients and all donor organs are available at the same time. On the right, how allocation rules actually work is shown. Only one organ is available at a time and the donor organ will be matched to the transplant candidate ranked at the top of the list to receive a transplant first, here a male patient with a MELD score of 39.

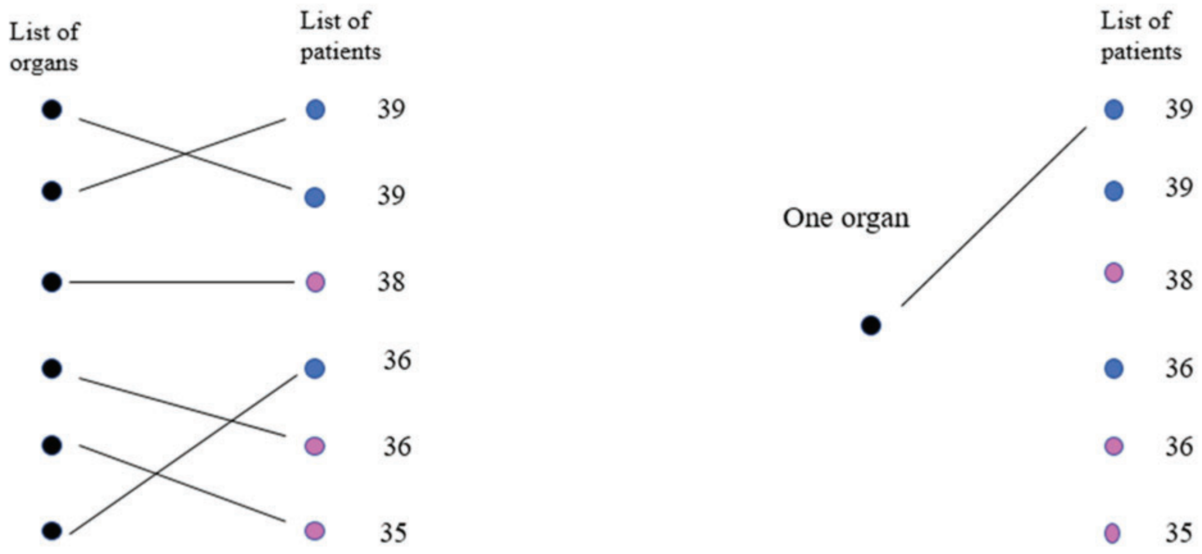


Figure 4: Idealistic vs. Realistic Liver Allocation

Even though the MELD system ranks the male patient at the top of the list, it is possible that the female patient shown to be third on the list with a MELD score of 38 is actually in more dire need of a transplant due to their physiological disadvantages. In Figure 5, we demonstrate how the transplant waiting list would be ordered with MELD score boosts for women. Allocation under the current MELD system is shown on the left, with the possible score bonus added to womens' MELD scores in red. On the right, the transplant waiting list is resorted after incorporating the score boost for women. Now, the first donor organ that becomes available will be offered to the female patient with a MELD score of 39.2 as she has the most urgent need for a liver transplant. This patient can then choose to accept or decline the donor liver; it would then go to the male patient with a MELD score of 39 and so on.

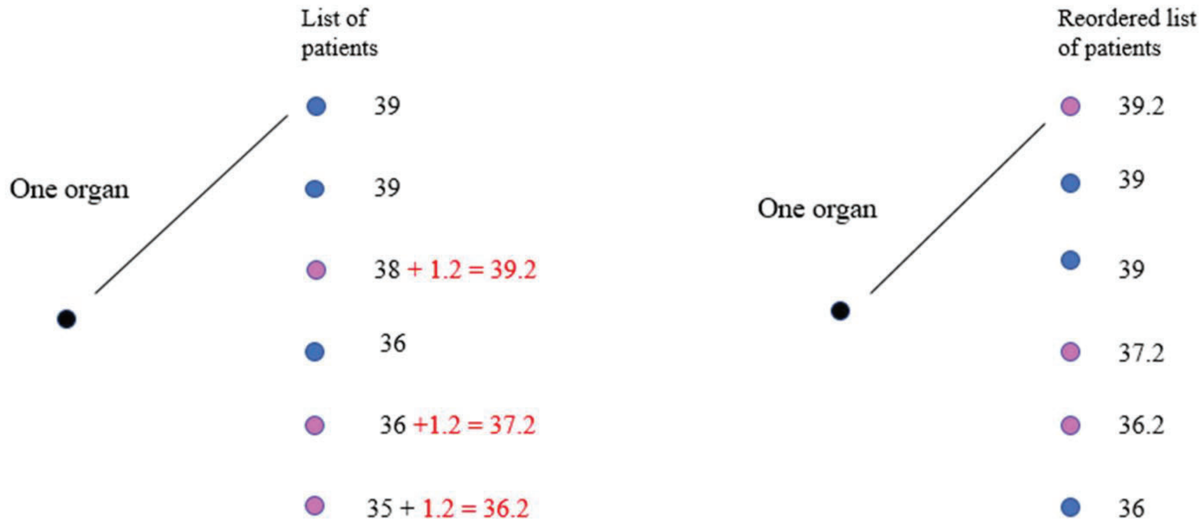


Figure 5: Realistic Liver Allocation with Added Score Boost for Women

In this diagram, we chose 1.2 arbitrarily. Using linear programming duality allows us to mathematically derive this score boost by converting the optimization function into a dual model whose decision variable value can be added to women's MELD scores to account for their disadvantage, without knowing which organs will become available and when. In the dual of the original integer program, the gender equity constraint can be converted into the dual multipliers shown in the model below. Duality claims that optimization problems can be viewed from two perspectives, the primal problem or the dual problem. The dual problem provides a lower bound to the solution of the primal problem (Rader, 2010).

To convert the primal into the dual, all constraints must be typical linear constraints. In the original model, the x_p decision variable is a binary variable and therefore must be converted into two linear constraints before constructing the dual. This is known as a 'linear relaxation', shown below:

$$\text{maximize: } \sum_{p \in P} MELD(p) x_p$$

$$\text{subject to: } \sum_{p \in P} x_p \leq 7409$$

$$\sum_{p \in P} f_p\, x_p \geq 0.4 * 7409$$

$$x_p \leq 1 \text{ for all } p \in P$$

$$x_p \geq 0 \text{ for all } p \in P$$

After performing a linear relaxation, we create the dual model. In the dual of a linear program, each constraint is converted into a dual multiplier in the objective function based on the sign of the variable and inequality. Duality claims that each optimization can be viewed from two perspectives: the primal and the dual (Rader, 2010). In a classic business analytics linear program, the objective function of the primal model would calculate the maximum profit of the business. You then would convert this model into the dual to solve for the minimum cost. In this case, the objective function of the primal model maximizes the number of MELD points transplanted. When converted to the dual, the objective function now minimizes the penalty on the maximum MELD points transplanted caused by enforcing 40% equity.

The decision variables in a dual program are known as ***'shadow prices'*** and show the marginal utility of changing the allocation of a single organ. After solving this model, the shadow price y_t will indicate the additional efficiency, in total MELD points, if one additional liver was available for transplant. The shadow prices y_1 through y_{12295} are a result of the linear relaxation and simply take on the value of the MELD score for that specific patient. Finally, the shadow price y_f is the additional number of MELD points transplanted if one fewer woman received a transplant. The most important result of the project so far comes from this step: the negative value of y_f indicates the number of MELD points that we are willing to sacrifice for one additional female transplant recipient. This value is the score boost that we will add to women's MELD scores in order to reach equity. Using duality ensures that we are not arbitrarily adding points to the MELD score that are not needed. For example, if the gender equity constraint is 'loose', easy to satisfy and women are receiving enough transplants, y_f will be assigned a value of zero and no points will be added. If the gender equity constraint is tight and not enough women are receiving transplants, y_f will be nonzero and will take the negative value of the number of points needed to reach equity. The full python code used to evaluate the linear program is located in the Appendix. The final dual model developed in step two is shown below:

$$\begin{aligned}
\text{minimize:} \quad & 7409y_t + (0.40 * 7409)y_f + (y_1 + y_2 + \cdots + y_{12295}) \\
\text{subject to:} \quad & y_t - f_p y_f + y_1 \geq MELD(1) \\
& y_t - f_p y_f + y_2 \geq MELD(2) \\
& \quad \vdots \\
& y_t - f_p y_f + y_{12295} \geq MELD(12295) \\
& y_t \geq 0,\ y_f \leq 0,\ y_1,\ y_2, \ldots, y_{12295} \geq 0
\end{aligned}$$

Ideal Linear Program and Dual Program Results

Even though the ideal model does not account for different size donor organs and patients or the sequential nature of organ allocation, it is still a useful tool to demonstrate the tradeoff between maximizing MELD points, the expected number of lives saved, and enforcing equity for women. The results of the ideal linear model and dual model using data from 2016, enforcing different percentages of donor organs given to female patients are shown the table and figures below. Our ideal linear model uses a binary decision variable x_p to model the fact that patients cannot be given partial organ transplants. In order to solve the dual model for the shadow price y_f, we had to use a linear relaxation to make the binary constraint into two linear constraints which is why the number of women transplanted and total MELD points at each percent equity is no longer an integer value.

We evaluated the primal model using python, the pyomo linear programming package, and the gurobi optimization solver, and ran it using USNA's Mazu Remote Desktop server. With no constraint enforcing equity for women, women receive 39.18% of the donor organs with 2,903 female patients being transplanted. The maximized total MELD points of the transplanted patients was 18,3697 points, and we estimated the number of lives saved to be about 2198.39 using the mortality rates from Figure 3. When enforcing equity at 40% since 40% of the waiting list in 2016 was female, the maximized total MELD points decreased to 18,3636, the number of women receiving a liver transplant increased to 2,963, and the expected number of lived saved decreased slightly to 2,197. We systematically varied the percent of transplants for women in the equity constraint to document the tradeoff requiring higher transplant rates for women, and maximizing MELD (Figure 7), or expected number of lives saved through transplant (Figure 8).

% organs given women	total MELD points	Number of women transplanted	# lives saved	Dual model shadow price
30.00%	183697	2903	2198.39	0
37.00%	183697	2903	2198.39	0
37.50%	183697	2903	2198.39	0
38.00%	183697	2903	2198.39	0
38.50%	183697	2903	2198.39	0
39.00%	183697	2903	2198.39	0
40.00%	183636.4	2963.6	2297.05	1
40.50%	183578.71	3000.64	2196.14	2
41.00%	183504.6	3037.68	2195.17	2
41.50%	183421.79	3074.73	2194.18	3
42.00%	183310.6	3111.78	2193.13	3
42.50%	183199.52	3148.83	2192.09	3
43.00%	183088.39	3185.87	2191.05	3
43.50%	182971.34	3222.915	2190.01	4
44.00%	182823.2	3259.96	2188.97	4
44.50%	182659.98	3297	2187.77	5
45.00%	182474.75	3334.05	2186.34	5
45.50%	182262.43	3371.1	2184.94	6
46.00%	182040.16	3408.14	2183.55	6
48.00%	181149.76	3556.32	2177.97	7
50.00%	180032	3704.5	2170.69	8
53.00%	178062.53	3926.77	2157.71	11
57.00%	174652.31	4223.13	2136.72	13
60.00%	171614	4445.4	2118.64	15

Figure 6: Ideal Model Results at Varied Percent Equity

As shown in Figure 8, there is an inverse relationship between the total MELD points of the transplant recipients and the percentage of donor livers that must be given to women. The trend line is flat until 40% because, up to that point, the constraint enforcing equity for women is not tight and therefore has no effect on the objective function. While this negative relationship makes it seem like we are not allocating organs to the patients with high MELD scores that need transplants the most, it is not concerning because we recognize that womens' MELD scores are lower than they should be because the current MELD system discriminates against them. If we add points to womens' MELD scores like we deem necessary, the curve illustrating the tradeoff between equity and efficiency would not be as steep.

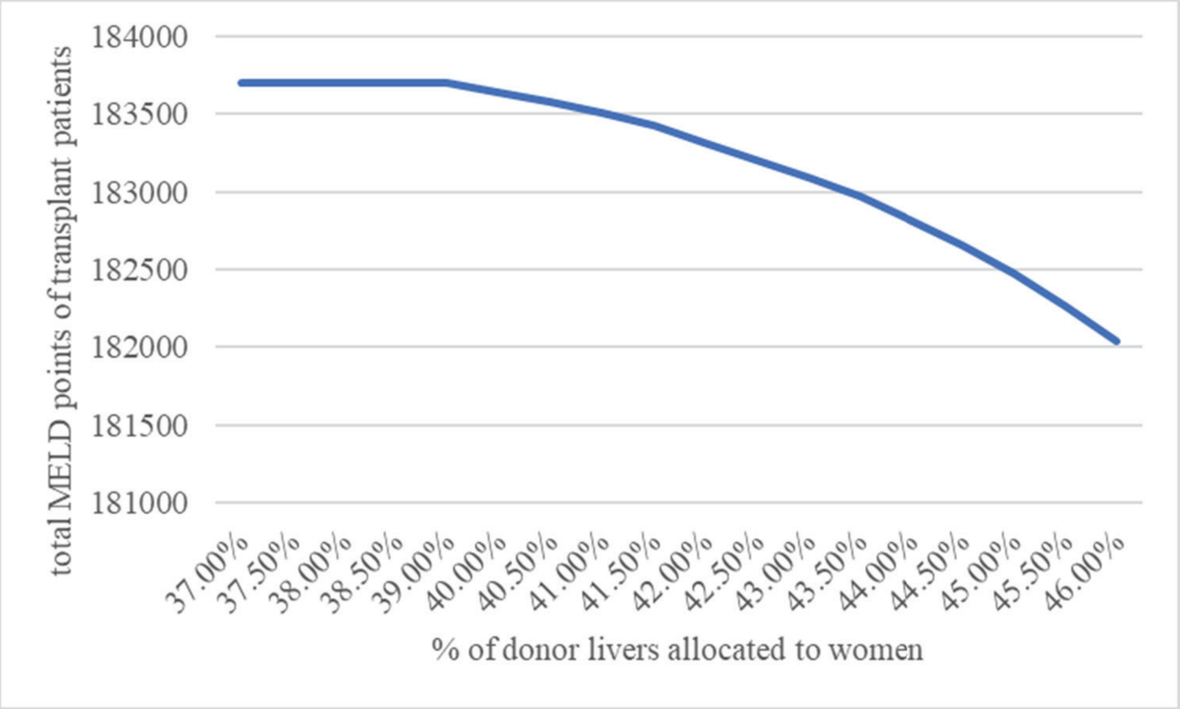


Figure 7: Tradeoff Between Equity and Maximum MELD Points

Figure 8 illustrates the tradeoff between equity, as the percentage of donor organs given to women, and the estimated total number of lives saved. Even though livers are currently allocated by the MELD system, this relationship most accurately reflects the tradeoff described in the third point of the "OPTN Final Rule." It directly compares efficiency as the estimated number of lives saved through transplantation with equity, the fair allocation of organs to both men and women. The trend lines are very similar in Figure 7 and Figure 8 because the mortality rate of each transplant patient is directly derived from the patient's MELD score. They are not completely identical however because, as shown in Figure 3, the mortality rates are different for men and women at the same MELD score. If the mortality rates were identical and MELD scores did not disadvantage women, this curve would also be less steep.

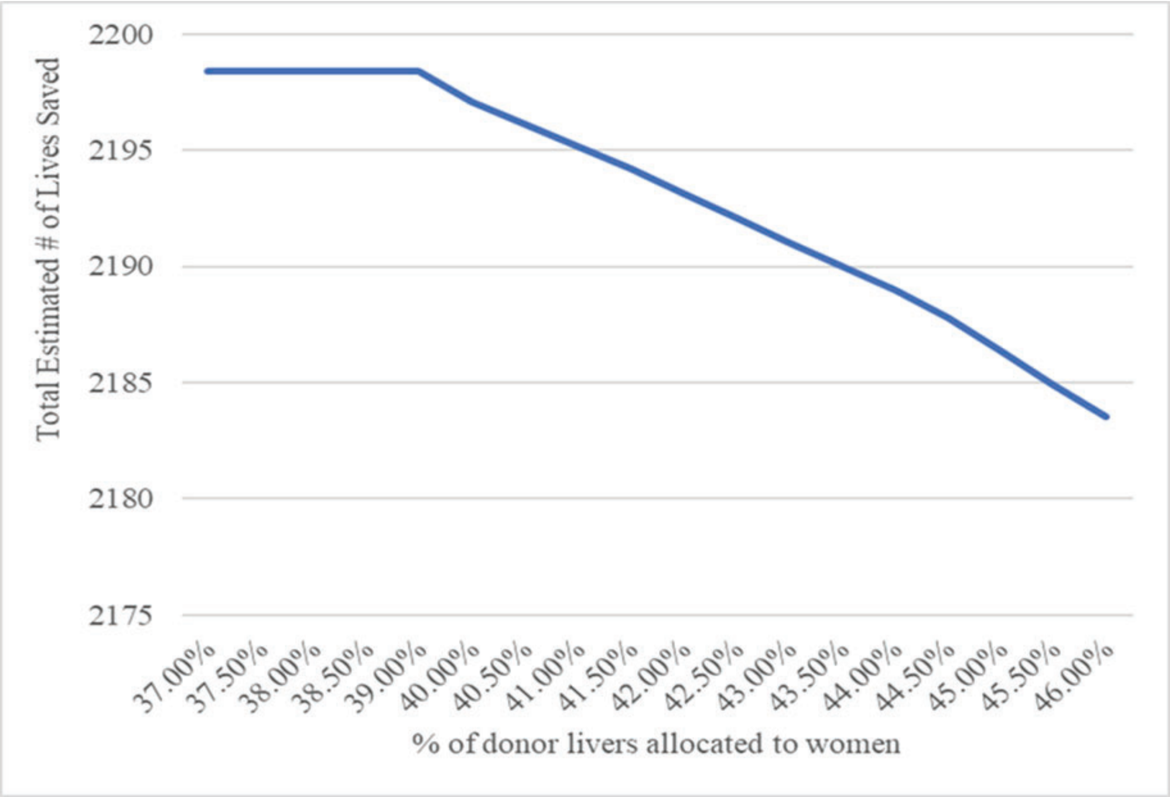


Figure 8: Tradeoff Between Equity and Estimated Number of Lives Saved

We solved the dual model for various percentages of donor livers that must be allocated to women. From the dual model, we found the shadow-price of the gender equity constraint, y_f, the number of MELD points that are sacrificed by allocating an additional donor organ to a female patient until reaching that equity level. As the percentage of organs required to go to women increases, it becomes more difficult to reach and requires us to sacrifice more MELD points to get to that level. The number of MELD points that should be added to womens' MELD scores to enforce equity is $-y_f$. The values of y_f are all negative, resulting in a general positive trend in the number of MELD points added as the percentage of donor organs allocated to women increases (Figure 9).

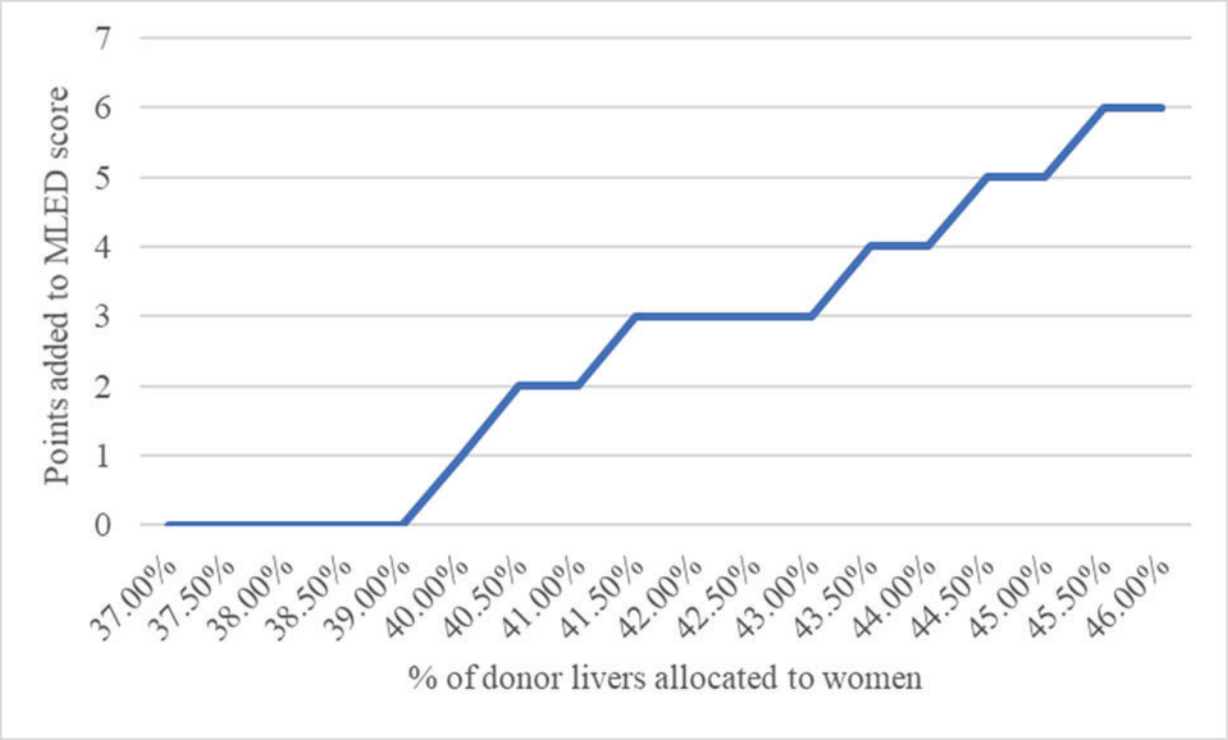


Figure 9: Number of Points to Add to Female MELD Scores to Enforce Various Levels of Equity

The shadow-price of the gender equity constraint indicates the number of points that should be added to womens' MELD scores will be non-zero only if the gender equity constraint is 'tight'. With this specific dataset, women receive 39.18% of the donor organs without enforcing any minimum percentage. The table of data collected from the ideal and dual model indicates that zero MELD points are necessary until you enforce equity at 40% because until that point, the gender equity constraint is not needed. The value of interest in Figure 9 is the MELD score boost corresponding to allocating 40% of the donor organs to women. We concluded that one point must be added to the MELD scores of all female patients in order to reach equity. Upon finding these results, we concluded that they were significant because they are consistent with the results of the paper "Correcting the Sex Disparity in MELD-Na", which found that about 1 to 3 points should be added to the MELD scores of women (Wood et al., 2021). Finding consistent results shows that our ideal linear program and dual program is another suitable method to solve the issue that lower creatinine levels pose for women.

Simulation to Test MELD Score Boost

We test the effect that adding MELD score boosts will have on womens' transplant rates in a realistic manner using a discrete event simulation through the previously developed Liver Simulated Allocation Model (LSAM). The original version of the Liver Simulated Allocation Model was finished in September of 2001 by the Scientific Registry of Transplant Recipients and is used to study likely effects of liver allocation policy changes on organ offers acceptance, waitlist survival, and posttransplant survival. It implements the concrete rules we develop and random components to reflect the uncertainty of acceptance decisions and transplant success when organs are offered to potential recipients. We can adjust the allocation rules and model parameters to simulate the MELD score boost that we have designed. We defined a new rule applied to only women on the transplant list that added the point value determined from the negative value of the shadow price y_f in step two to their prior MELD score. The LSAM then uses a random number generator to simulate the varying characteristics of both donors and organs, and the acceptance decisions of patients (Scientific Registry of Transplant Recipients, 2019).

After finding that we should add one point to womens' MELD scores from my dual model, we compared this value with the results of running the same equity percentage requirements in the Liver Simulated Allocation Model (LSAM). Instead of enforcing a specific percentage of women to receive transplants, we incrementally added MELD points to the scores of women and observed the change in equity. A side-by-side comparison of the percentage of transplants given to women in both models, with MELD boosts of different size, is shown below in Figure 10.

Female MELD Boost Given	Percent of Transplants to Women (Dual)	Percentage of Transplants to Women (LSAM)
0	39.18	36.37
1	40.00	38.46
2	40.50	40.34
3	41.50	42.41
4	43.50	44.83
5	44.50	46.31
6	45.50	48.49
7	48.00	50.20
8	50.00	51.91
9	-	54.02
10	-	55.33
11	53.00	57.58
12	-	59.34
13	57.00	61.05
14	-	62.52
15	60.00	64.36

Figure 10: Comparison in MELD Boost Results from the LSAM and Dual Models

While the results are not identical, they are similar enough to show consistent significant results. There are a few possible explanations to the different results we see in our dual model versus the Liver Simulated Allocation Model. First, the LSAM is a much more elaborate simulation model and therefore includes the possibility that women can decline organ offers. This could explain why the LSAM percentages are lower for the MELD boosts between zero and two points. The variation could also be due to one important difference in the data used. The 2016 dataset we used for my ideal and dual model only includes the 7,409 patients that were added to the list in 2016. The dataset used in LSAM is more extensive and includes the transplant candidates added to the list in 2016 in addition to the patients that were already on the list in the previous year, which could skew the result away from what we calculated in the dual model.

Another explanation for the different results from the LSAM and dual models is the fact that MELD scores are required to be integer values. It is possible that the percentage of women receiving transplants would be more similar between models for MELD scores with decimal values, as the integer MELD boosts encompass a large range of equity percentages. Because the difference between a MELD boost of one point and two points is already so substantial, it is not realistic to try and fix both the size restriction and underestimation of illness due to creatinine levels with only a MELD score boost. Adding any additional points to account for the size restriction that women face in accepting organs could overcorrect the inequity and does not really target the real issue. While a MELD boost would put women higher on the list and therefore get them more organ offers that they can now afford to decline, ‘saving’ smaller organs for smaller patients like women would be a more precise and accurate way to correct this disadvantage. These results led us to develop the network flow algorithm that will be my next step in continuing this project.

Designing Size Restrictions Using Network Flow

After completing analysis of the ideal linear program and dual program, we have found that adding one point to womens' MELD scores might correct the negative impact of lower creatinine. After evaluating the issue further and reading the paper "Sex Does Matter in Liver Allocation – Time to Address Existing Sex-Based Disparities" by Dr. Willscott Naugler and Dr. Susan Orloff from the Oregon Health and Science University, we found that a MELD score boost alone might not solve the size compatibility problem. Naugler and Orloff argue that a simple correction of the MELD score, like we proposed, could fix the underestimation of renal function by creatinine measurement. However, a change in MELD score is unlikely to fix the lack of available donor livers due to size incompatibilities that women face (Naugler et al., 2020). It is crucial to account for this size disadvantage in our combined model because it can drastically increase the amount of time that a woman spends waiting for a transplant. Because of size incompatibility, there may be instances where it is prudent to give a smaller patient with a lower MELD score a donor liver before a larger patient with a higher MELD score. We aimed to model this dynamic optimization problem using a method that is both static and linear, but still accurate.

Fortunately, a very recently available (June 2021) paper offered an alternative solution to the size issue. The paper "Eliminating Transplant Waiting Time Inequities – With an Application to Kidney Allocation in the USA" represented a similar issue for kidney allocation using a network flow model (Figure 11). In kidney allocation, a major concern is sensitization by blood type. Some patients are heavily restricted on the donor kidneys they can accept. For example, patients with an O blood type can only accept organs from an O blood type donor. On the other hand, patients of an AB blood type can accept donor kidneys from any blood type (van de

Klundert et al., 2021). This is similar to the restrictions that women face on donor liver size, as small donors can offer their liver to any patient, but small patients can only accept small livers. This work used the Max Flow Min Cut Theorem to restrict the flow of organs between specific donor supply and patient demand nodes (van de Klundert et al., 2021).

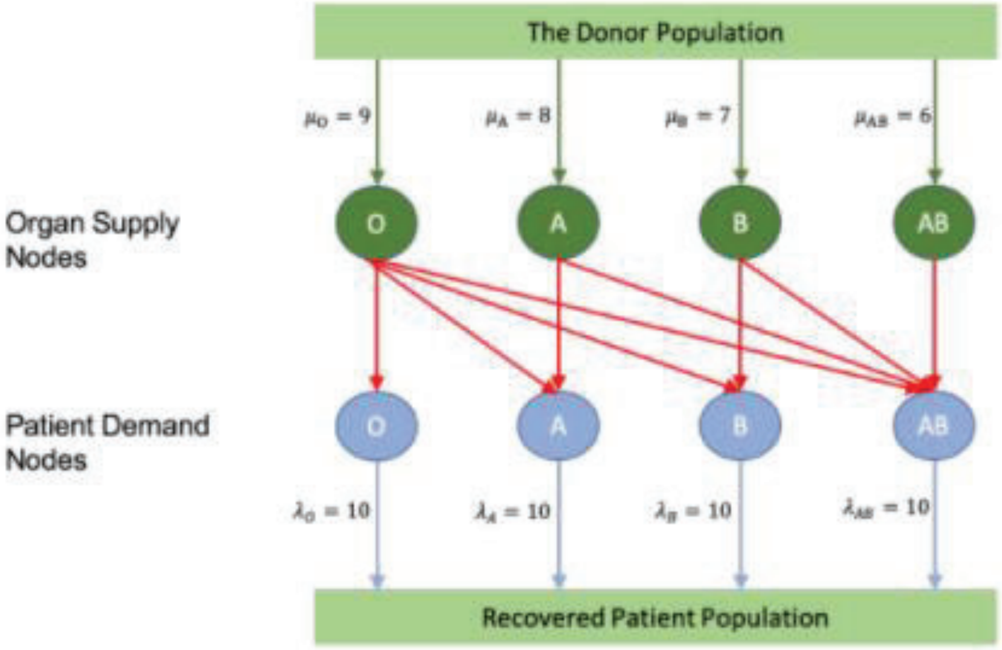


Figure 11: Network Flow Model of Kidney Allocation by Blood Type (van de Klundert et al., 2021)

For simplicity, the original linear program assumes that all donors and patients are the same size and can accept all livers offered to them. To make the model more accurate, we had to account for the fact that when people of smaller stature like women are offered large organs, they often have to decline the offer because the organ will not fit inside their body cavity. To capture this size incompatibility, we developed a network flow model to detail the flow of livers from donors of all sizes to transplant patients of all sizes, shown in Figure 12 below.

In this diagram, there is an inflow of donor livers from the donor population, of varying sizes, into donor supply nodes that range from the smallest organs to the largest organs. From the supply nodes, the donor livers flow to the patient demand nodes. As shown by the arcs connecting the supply nodes to the demand nodes, some patients can only accept organs from certain donor supply nodes. The different number of sources for each patient demand node makes the flow of donor organs to each demand node extremely uneven. For example, the smallest patients can only accept the smallest organs while the largest patients can accept organs from any donor size category. Large patients, usually men, will always have a higher rate of liver transplant simply because they have more sources of donor supply and smaller patients, like women and children, have very few sources.

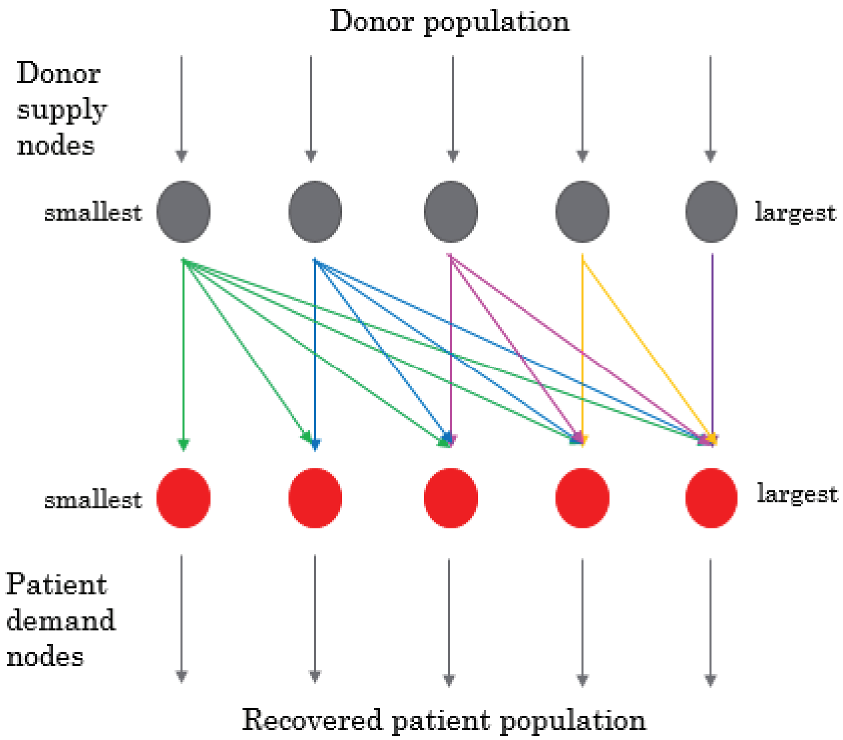


Figure 12: Example Network Flow Model of Donor Livers

The diagram shown in Figure 12 is a simple example. To develop a more realistic and data-based model, we matched the patient and donor listing data from 2016 by matching the patient identification numbers. In this combined dataset, there were 4181 available donor livers. 4181 of the 11,598 transplant patients received a liver transplant. We divided the range of sizes for patients and donors into 10 equal groups by height, as height is generally a better indicator of liver size compatibility than weight or body mass index. The donor groups are denoted by the labels D1, D2, D3, et cetera through D10. D1 is the smallest donor group by height, while D10 is the largest donor group by height. The middle donor groups like D5 and D6 are average height and have livers that are acceptable by every size patient. Similarly, the patient groups are labeled P1 through P10. P1 is the smallest patient group by height, while P10 is the largest patient group by height. The middle patient groups like P5 and P6 are average height and can typically accept any size donor organ. The groupings, with the upper and lower height boundary, are shown below in Figure 13.

Donor Size Grouping	Lower Boundary (Min Height)	Upper Boundary (Max Height)	Patient Size Grouping	Lower Boundary (Min Height)	Upper Boundary (Max Height)
D1	144.78 cm	154.94 cm	P1	130.48 cm	154.9 cm
D2	154.95 cm	160 cm	P2	154.91 cm	160.02 cm
D3	160.01 cm	165 cm	P3	160.03 cm	165.1 cm
D4	165.01 cm	167.64 cm	P4	165.11 cm	167.64 cm
D5	167.65 cm	172.72 cm	P5	167.65 cm	172.72 cm
D6	172.73 cm	177 cm	P6	172.73 cm	177 cm
D7	177.01 cm	180 cm	P7	177.01 cm	180.3 cm
D8	180.01 cm	183 cm	P8	180.31 cm	182.88 cm
D9	183.01 cm	187.96 cm	P9	182.89 cm	187.96 cm
D10	187.97 cm	203.2 cm	P10	187.97 cm	205.74 cm

Figure 13: Donor and Patient Size Groupings for Network Flow Model

For the flow of organs from donors to patients to be perfectly equitable, each patient group should receive the same proportion of total transplants as their proportion of the total transplant list. We calculated these proportions and then compared them to observe which patient groups are at a size disadvantage. However, we see that in reality, the smallest five patient groups receive less transplants than is equitable, and the largest five patient groups receive more transplants than is equitable. This tells us that patients smaller than 172.72 centimeters in height generally have a harder time finding compatible organ matches. The comparison is shown in Figure 14 below.

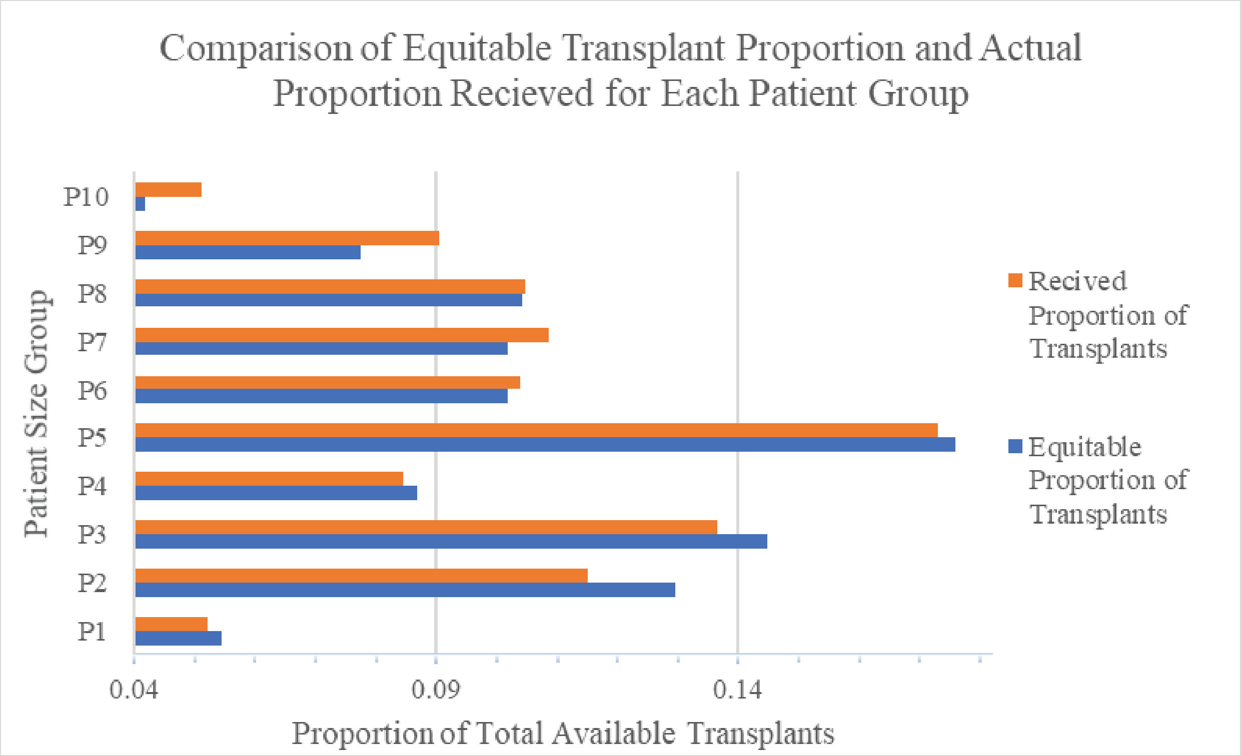


Figure 14: Comparison of Equitable Transplant Proportion and Actual Proportion Received for Each Patient Group

After observing these trends, we simulated this phenomenon using a network flow model. A network flow model, specifically a max flow, is a type of linear program that maximizes flow from a source node to a sink node while it travels through intermediate nodes and edges. In this model, we ensure that all organs are effectively used by setting the flow out of the source as 4181 organs and the flow into the sink as 4181 organs. Between the ten donor groups and ten patient groups, we have 100 available donor-patient pairs, which serve as the edges connecting donor supply and patient demand nodes in the original network flow model. If we created no rules restricting flow and all organs were compatible for all patients with no regard for size, this would be an accurate way to model the flow of livers between donors and patients on the transplant list. The intermediate nodes of the full, unrestricted network flow model are shown below in Figure 15.

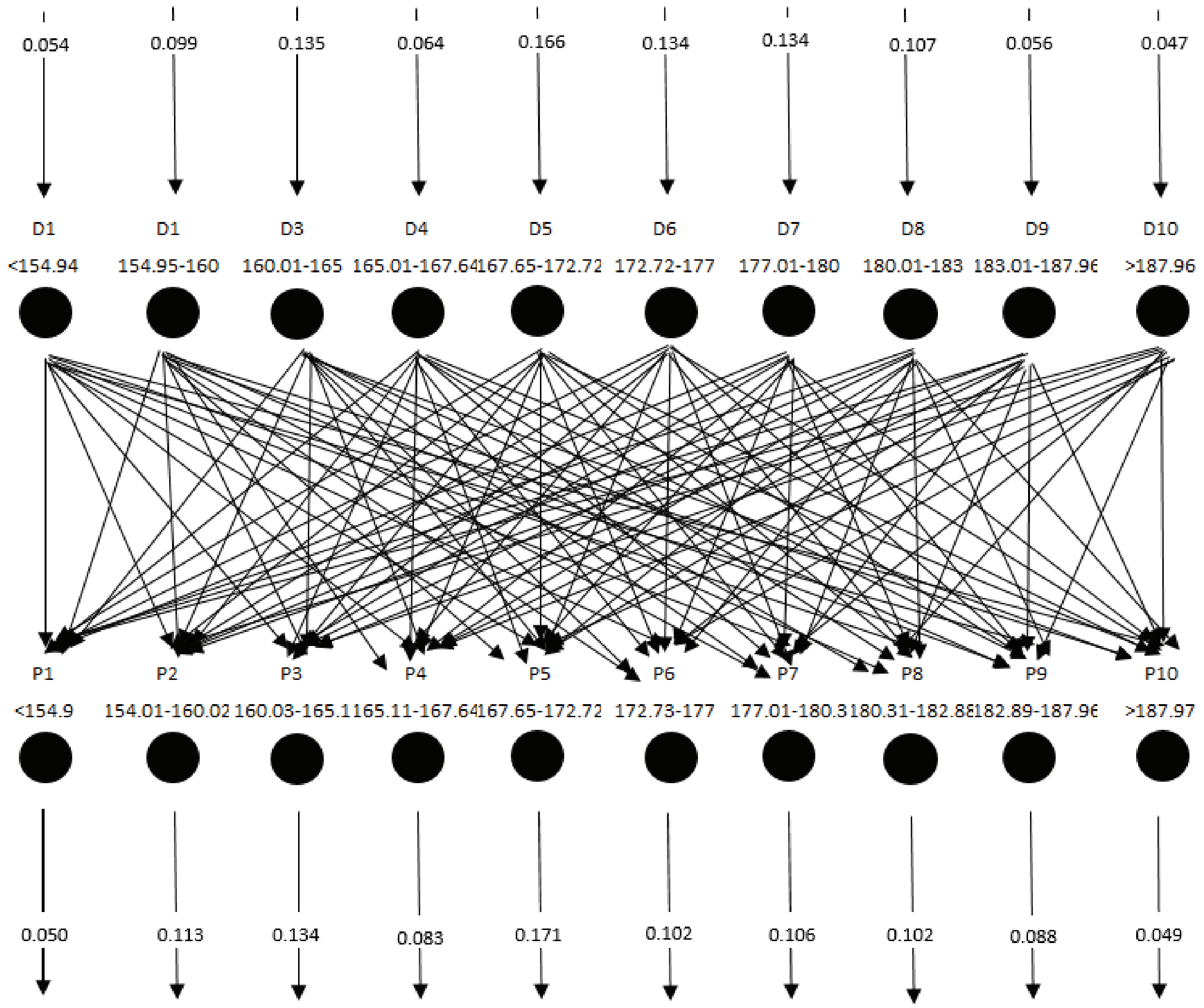
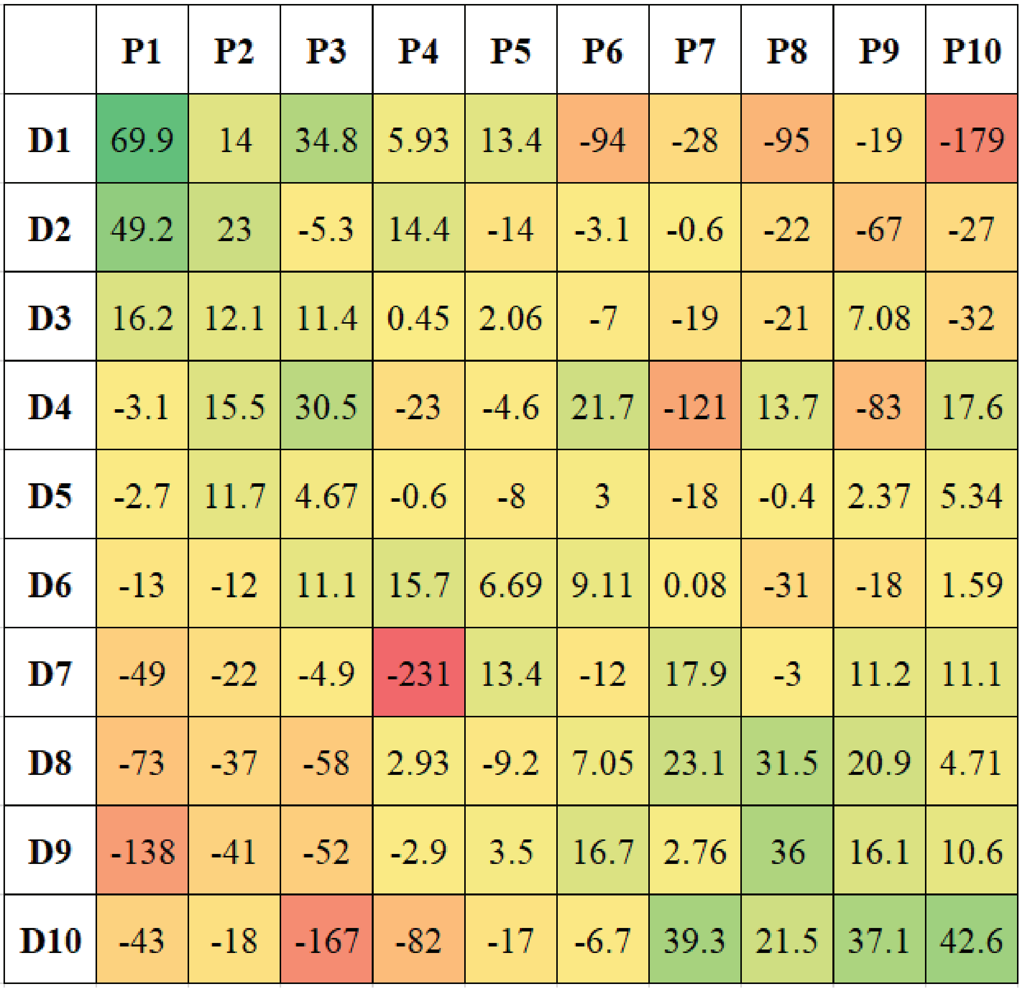


Figure 15: Full Network Flow Model Using Analysis of 2016 Transplant Data

To accurately model the flow of organs between donors and patients to account for the disadvantage women face due to size, we have to restrict the edges that symbolize the donor-patient transplant pairs. We cut the edges along which donor livers can flow for two reasons: (1) infeasible pairs due to incompatible size matches, and (2) the rules we create to prioritize smaller patients. First, to determine which pairs were infeasible, we calculated the percentage of donor livers from each donor size group that went to each patient size group. We then compared this percentage to the percentage of donor organs this patient group should have received if each patient group got the amount of donor organs equal to their proportion of the complete list of transplant candidates. This percentage is what we consider to be the ‘fair’ distribution of donor organs. A drastically lower percentage shows that this donor-patient pair is likely unfeasible as the size difference may be too large. The density chart below shows the percentage difference between the fair distribution of organs and the actual proportion that flowed between the specified donor and patient group.



	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**	**P10**
D1	69.9	14	34.8	5.93	13.4	-94	-28	-95	-19	-179
D2	49.2	23	-5.3	14.4	-14	-3.1	-0.6	-22	-67	-27
D3	16.2	12.1	11.4	0.45	2.06	-7	-19	-21	7.08	-32
D4	-3.1	15.5	30.5	-23	-4.6	21.7	-121	13.7	-83	17.6
D5	-2.7	11.7	4.67	-0.6	-8	3	-18	-0.4	2.37	5.34
D6	-13	-12	11.1	15.7	6.69	9.11	0.08	-31	-18	1.59
D7	-49	-22	-4.9	-231	13.4	-12	17.9	-3	11.2	11.1
D8	-73	-37	-58	2.93	-9.2	7.05	23.1	31.5	20.9	4.71
D9	-138	-41	-52	-2.9	3.5	16.7	2.76	36	16.1	10.6
D10	-43	-18	-167	-82	-17	-6.7	39.3	21.5	37.1	42.6

Figure 16: Density Chart of the Severity of Size Incompatibility

Any box with a red color of varying intensity indicates that the donor-patient pair group received less organs than equitable. We assumed that the darkest red pairs received such low numbers of organs because the pair was generally infeasible due to size. If the first percentage was more than approximately 30% smaller than the second percentage, we considered this donor-patient pair to be incompatible and removed this 'edge' from the overall network flow model. Because size incompatibility is not a strict cutoff, we removed some edges that had less than a 30% difference to maintain consistency.

After repeating this for each of the 100 donor-patient pair groups, we decided to remove the following edges: (D1, P6), (D1, P7), (D1, P8), (D1, P9), (D1, P10), (D2, P9), (D2, P10), (D7, P1), (D7, P2), (D7, P3), (D7, P4), (D8, P1), (D8, P2), (D8, P3), (D9, P1), (D9, P2), (D9, P3), (D10, P1), (D10, P2), (D10, P3), and (D10, P4). The infeasible edges are shown in red in the flow model in Figure 17.

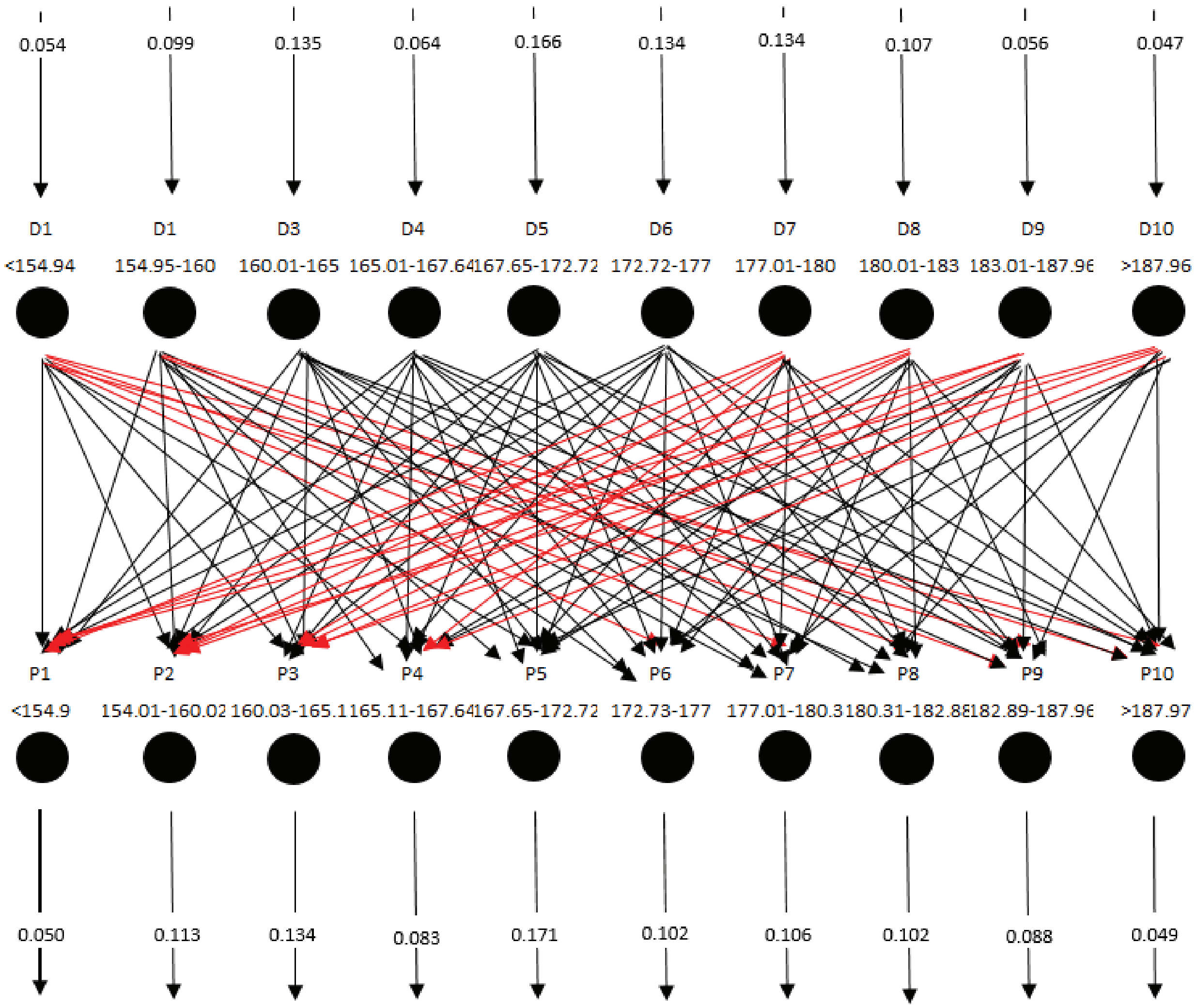


Figure 17: Network Flow Model with Infeasible Edges Shown in Red

The network flow model with the infeasible edges removed is shown in Figure 18.

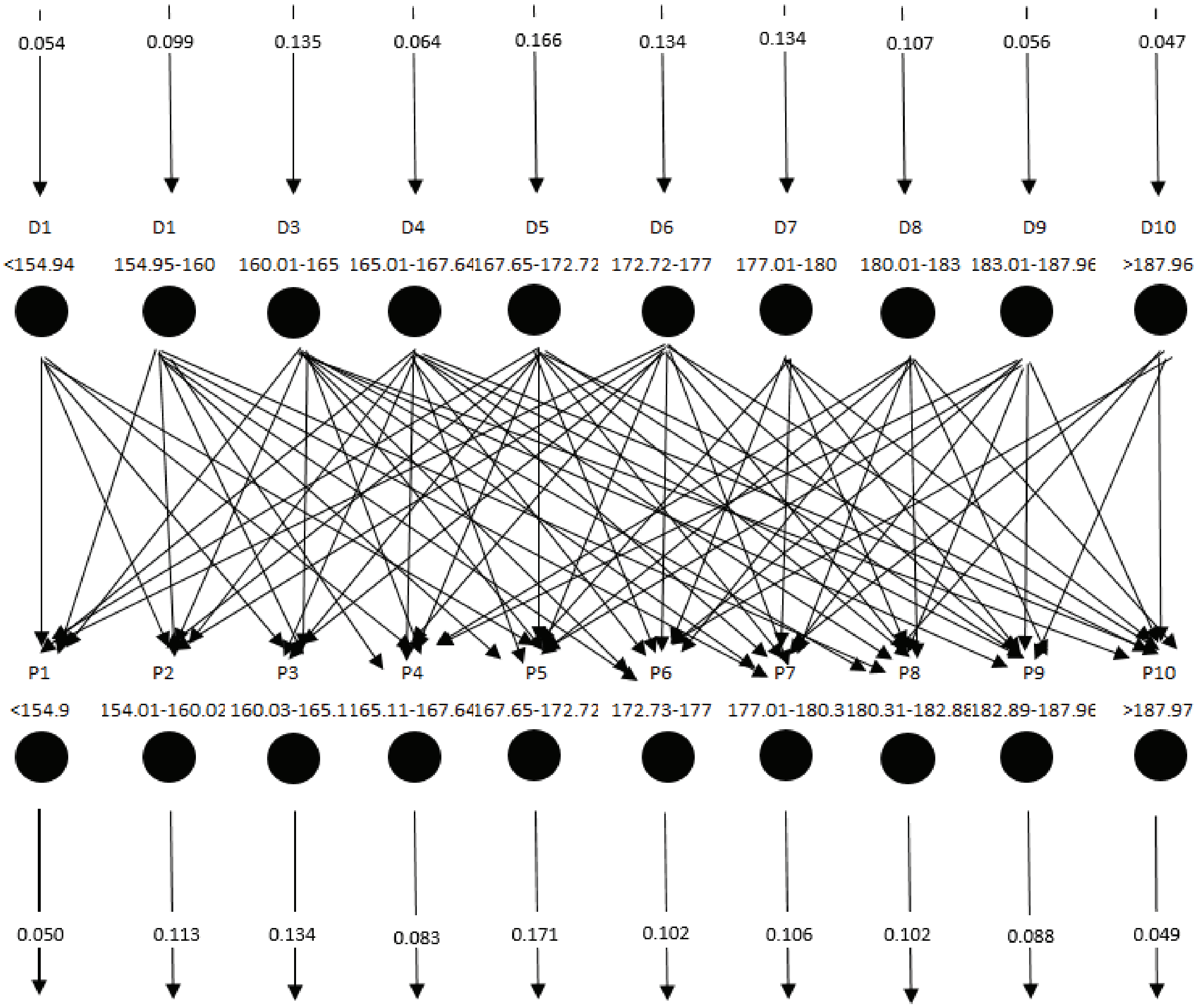


Figure 18: Network Flow Model with Infeasible Edges Removed

We next coded this network flow model into python, again using the linear programming module Pyomo and the optimization solver Gurobi. In this model, we have two sets. *NF.nodes* is the set of all of the nodes in the model, including all 20 patient and demand nodes and a dummy source and supply node to simplify the execution of the model. *NF.edges* is the set of all of the edges in the model, including all of the edges shown in Figure 18 above in addition to 20 edges that think the donor and patient group nodes to the source and sink. There are three parameters. The first two parameters, *NF.source* and *NF.sink*, simply designate which of the nodes are the dummy source and sink node as there are different inflow and outflow expectations for those two nodes. *NF.capacity* is a parameter that designates the max flow allowed along each edge in *NF.edges*. For the edges connecting donor supply nodes to patient demand nodes, the value is fixed at zero if the pair is infeasible or 4181 if the pair is feasible. For the edges connecting the source node to the donor supply nodes, the capacity is the number of donor organs we have of that size. Finally, for the edges connecting the patient demand nodes to the sink node, the capacity is the number of organs each patient group should get if they were fairly distributed based on the proportion of the total candidate list that the patient group occupies.

The decision variable *NF.x* is the flow of organs along each edge in the model, which is maximized in the objective function with the goal to use the available organs as efficiently as possible. The first constraint makes sure that the flow along each edge does not exceed the capacity of that edge. The second constraint, starting with the if statement, requires that the flow in and out of each intermediate node are equal. The abstract model design is shown below and the full python code is shown in Appendix B.

```
maximize: $\sum_{(i,j)\in E} x[i,j]$ for all $(i,j) \in edges$ if $j = sink$
subject to: $x[i,j] \leq capacity[i,j]$ for all $(i,j) \in edges$
    if $k = source$ or $k = sink$:
        skip $Constraint$
    $inflow = \sum_{(i,j)\in E} x[i,j]$ for all $(i,j) \in edges$ if $j = k$
    $outflow = \sum_{(i,j)\in E} x[i,j]$ for all $(i,j) \in edges$ if $i = k$
    $inflow = outflow$
```

This model is similar to the first ideal linear program we solved: it assumes an unrealistic foresight of all donors and transplant patients at the beginning. It shows how donor livers will flow between donor and patient size groups to satisfy the equity requirements defined in the capacity of the patient groups, but in an unrealistic, simplistic manner. The results are still useful for a comparison with the real results from the 2016 transplant data even though the model is ideal. Because the model shows the results of a perfectly equitable allocation of donor livers, the comparison shows us which patient groups are overserved and underserved in reality. This comparison is shown below in Figure 19. We found that in 2016 the five smallest patient groups received too few organs and the five largest patient groups received too many organs, with the exception of patient group eight receiving the exact equitable amount.

Patient Group	Model Results	Results from 2016 Data	Difference
P1	220	210	-10
P2	534	473	-61
P3	597	562	-35
P4	355	344	-11
P5	728	715	-13
P6	418	426	8
P7	418	446	28
P8	428	428	0
P9	316	370	54
P10	167	205	38

Figure 19: Comparison of 2016 Real Data and Network Flow Model

One method to correct this disadvantage that women, and smaller patients in general, face due to their smaller abdominal cavities, shown in the comparison above, is to create new allocation rules that restrict the flow along some of the paths connecting donor supply nodes and patient demand nodes. This enforces similar transplant rates for each patient size and reduce the disparity in transplant rates between men and women. In this diagram, the authors of the paper divided the network flow of donor kidneys into two separate subnetworks. They cut the red path connecting the O blood type and A blood type to the B blood type and AB blood type and no longer let donor organs flow between them (van de Klundert et al., 2021).

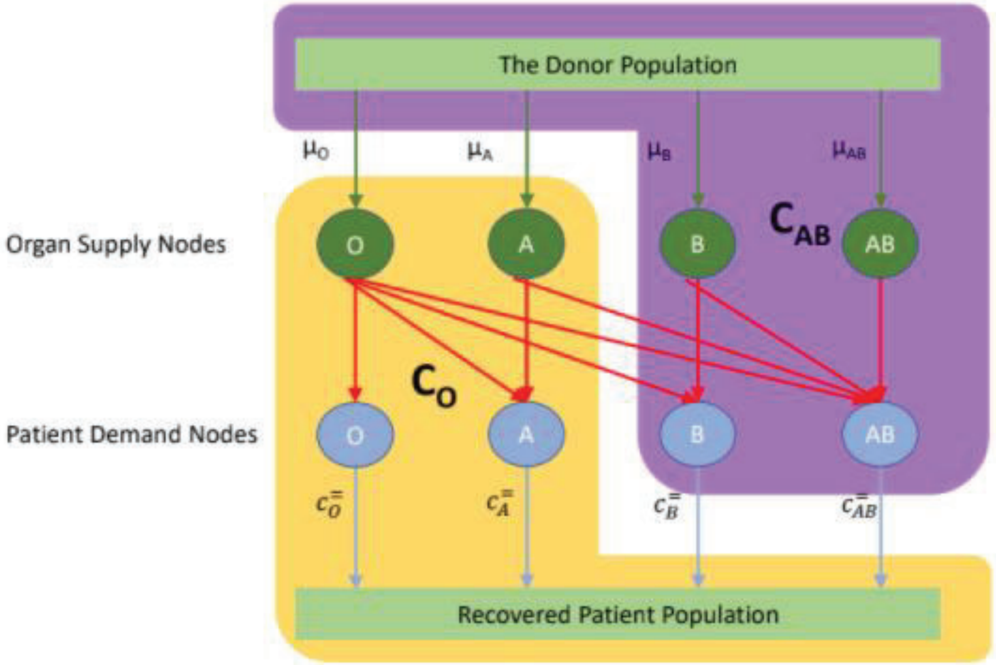


Figure 20: Max Flow Min Cut Theorem Applied to Kidney Allocation (van de Klundert et al., 2021)

We followed a similar process and cut some of the paths connecting smaller donors to larger patients in order to reserve the smaller livers for smaller patients who are in more need of donor organs. After comparing the model results to the 2016 results, we were able to identify which exact patient group were not reaching the equitable transplant number naturally. To help smaller patients receive more organs, we cut all of the edges connecting the smallest five donor supply nodes to the largest five patient supply nodes to reserve those organs for the smaller patients. Specifically, we cut the following edges: (P2, P6), (P2, P7), (P2, P8), (P3, P6), (P3, P7), (P3, P8), (P3, P9), (P3, P10), (P4, P6), (P4, P7), (P4, P8), (P4, P9), (P4, P10), (P5, P6), (P5, P7), (P5, P8), (P5, P9), and (P5, P10). We then changed these capacities in the data file and re-executed the python code to see if there was still a feasible solution. If there was no longer a feasible solution, we would consider our implemented rules too harsh as the network would no longer be able to effectively use all of the donor organs. When executing the model after implementing the new rules, we found that we were still able to transplant all of the available organs while prioritizing an equitable flow to the smaller patients. The edges that we cut to promote flow to the smaller patients are shown in blue in Figure 21.

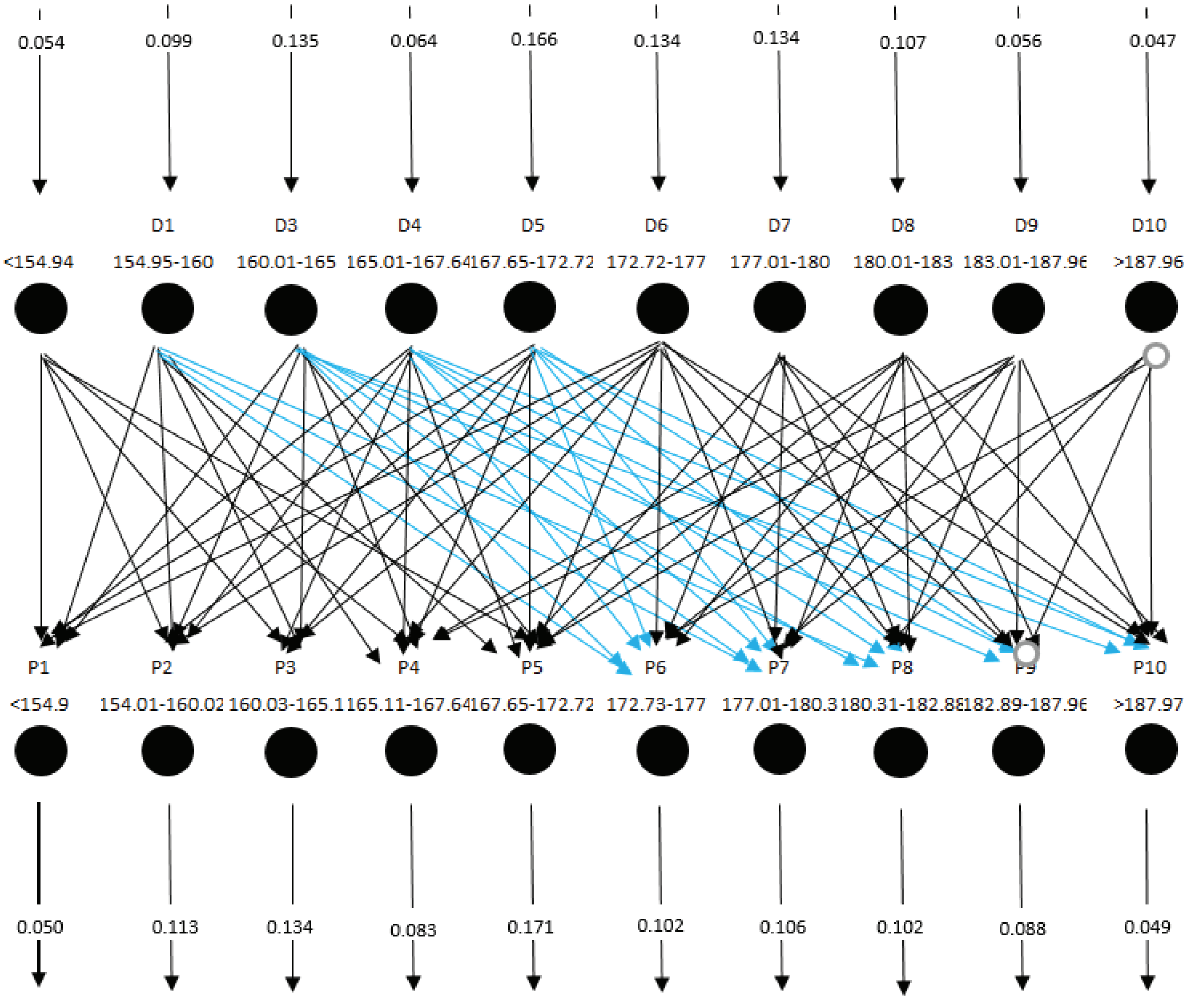


Figure 21: Network Flow Model with Restricted Edges to Promote Equity Shown in Blue

The network flow model with the edges we removed to promote equity under the new flow rules is shown in Figure 22.

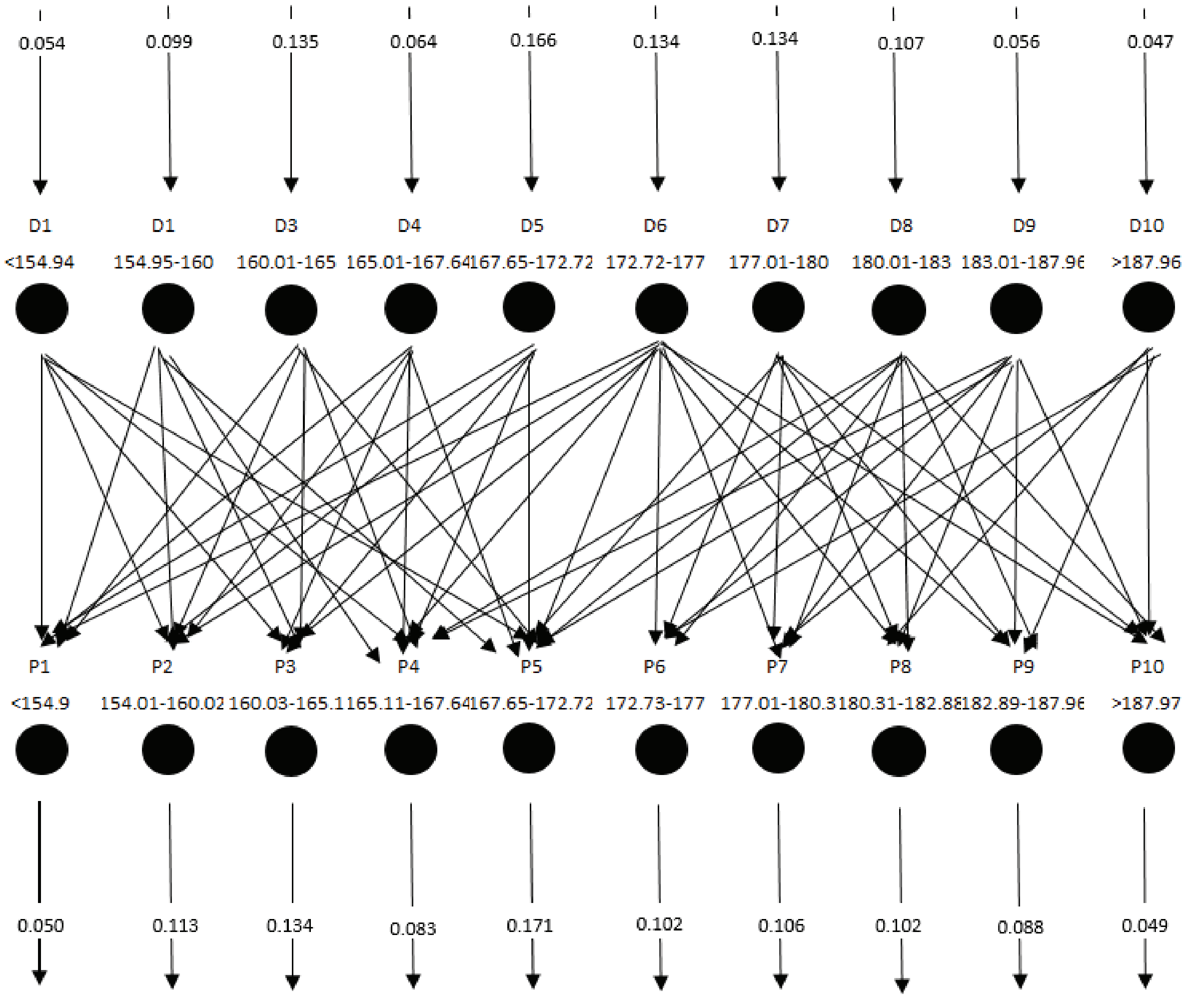


Figure 22: Network Flow Model with Restricted Edges Removed

At this point, we decided not to use the Liver Simulated Allocation Model (LSAM) to test our allocation rules that cut off the smallest donors from the largest patients in the network flow model. The fact that the network flow model was still able to allocate all 4181 organs successfully with 18 additional edges cut was enough proof for us that these rules were not too stringent to the point that larger patients would be disadvantaged. If pyomo could not execute the model, which would occur if not all 4181 organs could be transplanted with the additional edges removed, we would know that our rules were too strict.

Future Work and Conclusion

The ideal linear program and dual program and the network flow model separately address the two main issues that women face in receiving a fair proportion of liver transplants. To address the discrimination of female patients as a whole, the combination of the two models is needed. After executing the linear program and its dual, we found that one point needs to be added to the MELD score of female patients to account for their lower natural creatinine levels. This modification changes the actual organization of the waiting list before any donor organs are offered to transplant patients.

After executing the network flow model, we recommend that the OPTN change rules to not allow organs to flow from the smallest five donor groups to the largest five patient groups. Specifically, we cut the following edges: (P2, P6), (P2, P7), (P2, P8), (P3, P6), (P3, P7), (P3, P8), (P3, P9), (P3, P10), (P4, P6), (P4, P7), (P4, P8), (P4, P9), (P4, P10), (P5, P6), (P5, P7), (P5, P8), (P5, P9), and (P5, P10). This modification would still allow all donated organs to flow to some patient, but result in equitable proportions of each patient group getting transplanted.

In the future, we would like to use the Liver Simulated Allocation Model (LSAM) to test our allocation rules that cut off the smallest donors from the largest patients in the network flow model. Because the final network flow allocated all 4181 organs successfully with 18 additional edges cut, we were able to assume that our rules were accurate. However, both these flow restriction rules and the additional point we added to womens' MELD scores should be tested more thoroughly in LSAM before implementation. This would be a big undertaking, however necessary, as the Liver Simulated Allocation Model can give us information that is not shown in simpler mathematical modeling like the quality of donor going to each patient group, a cost estimate of new policies due to a factor like increased travel distance to a hospital, or any unintentional consequences that the new policy will have on other demographics not addressed by the new rules (Scientific Registry of Transplant Recipients, 2019). This research could also be refined further by running the same dual model and network flow model using additional years of transplant data other than 2016 to ensure that the trends are constant across time.

In addition to these offered expansions of this specific project, it should also be recognized that the methods described throughout this report do not only apply to the inequality that women face in transplantation. Similar models can also be used to attack problems that other demographics like patients of rare blood type, racial minorities, patients of lower socioeconomic status, pediatric patients, et cetera also face in transplantation. They can even be applied to the methods used to allocate other organs like lungs, hearts, kidneys, and intestines that are also inherently inequitable. While the specific results of this project are summarized above, the dual program and network flow model designed to achieve those results are also extremely valuable as they can be easily adopted for future research.

The number of successful organ transplants increased by 5.9% in 2021, even with the COVID-19 pandemic, and will likely continue to increase each year (Kizer et al., 2022). As the number of patients seeking an organ transplant increases while the supply of organs is still scarce, it becomes increasingly important that the methods the medical community uses to allocate available donor organs is equitable and fair to all patients. It is well researched and documented that inequities exist in all aspects of organ transplantation. As stated by Boulware and Mohottige in the recently published report by the National Academy of Science and Medicine, “it is time for [the medical and scientific community] our actions to move beyond describing and acknowledging inequities. Rather, we must commit to enacting solutions that rectify inequity through multidimensional approaches that address fundamental causes." This completed trident research is part of the effort to move away from simply noting that inequities exist in this medical practice and start truly attacking the problem in an effort to make life-saving organ transplants more accessible to all.

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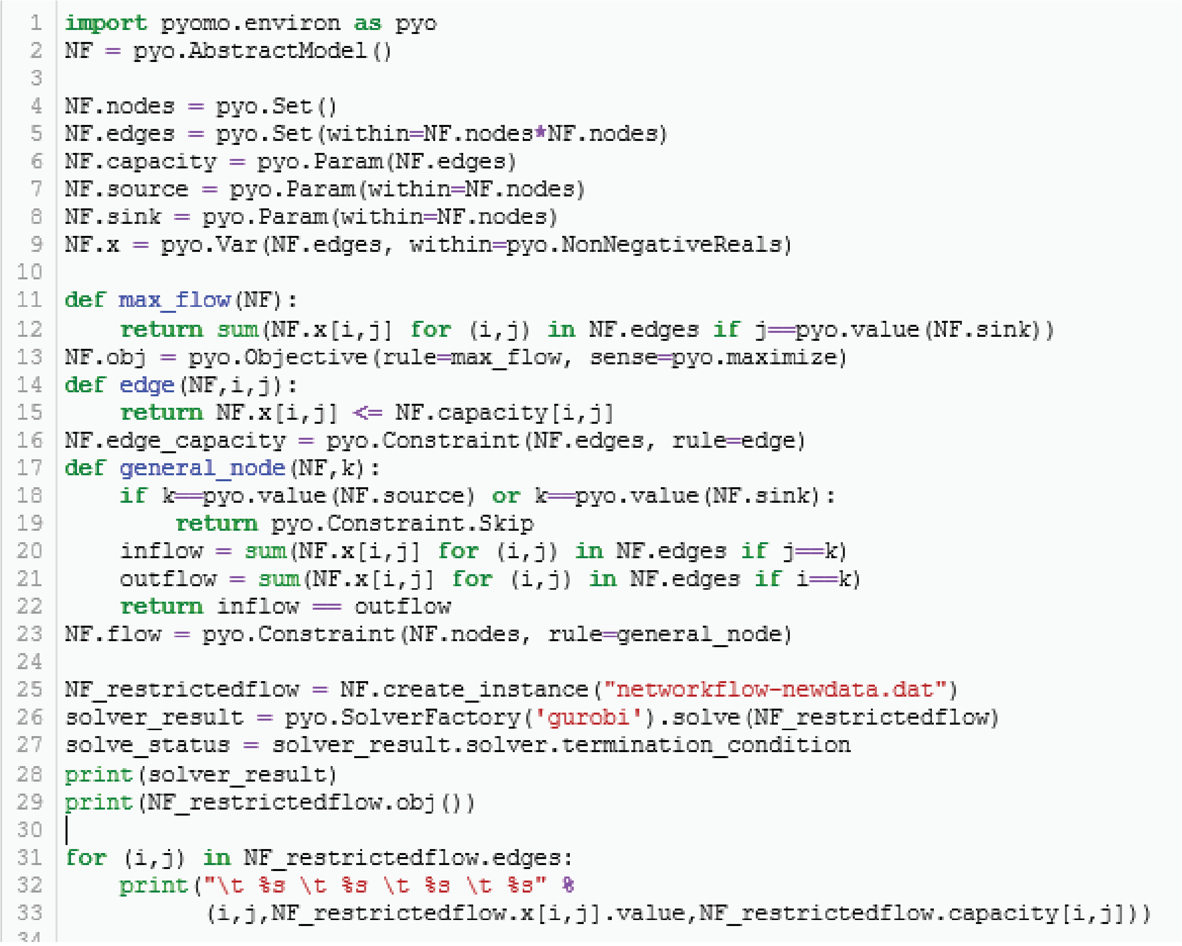
Appendix A

```
 1 import pyomo.environ as pyo
 2 LS = pyo.AbstractModel()
 3 LS.dual = pyo.Suffix(direction=pyo.Suffix.IMPORT)
 4 
 5 LS.px_id = pyo.Set()
 6 LS.is_female = pyo.Param(LS.px_id)
 7 LS.meld = pyo.Param(LS.px_id)
 8 LS.mort = pyo.Param(LS.px_id)
 9 LS.y = pyo.Var(LS.px_id, domain=pyo.NonNegativeReals)
10 
11 def MELD(LS):
12     return sum(LS.meld[i]*LS.y[i] for i in LS.px_id)
13 LS.obj = pyo.Objective(rule=MELD, sense=pyo.maximize)
14 def one_patient(LS):
15         return sum(LS.y[i] for i in LS.px_id) <= 7409
16 LS.one_patient_rule = pyo.Constraint(rule=one_patient)
17 def woman_fairness(LS):
18     return sum(LS.is_female[i]*LS.y[i] for i in LS.px_id) >= (0.40*7409)
19 LS.woman_rule = pyo.Constraint(rule=woman_fairness)
20 def linear_relaxation(LS, i):
21     return (LS.y[i] <= 1)
22 LS.lr = pyo.Constraint(LS.px_id, rule=linear_relaxation)
23 
24 LSequity = LS.create_instance("20162017_nodonor.dat")
25 solver_result = pyo.SolverFactory('gurobi').solve(LSequity)
26 solve_status = solver_result.solver.termination_condition
27 print(solver_result)
28 print(LSequity.obj())
29 
30 print(f'y_t = {LSequity.dual[LSequity.one_patient_rule]}')
31 print(f'y_f = {LSequity.dual[LSequity.woman_rule]}')
32 
33 women=0
34 for i in LSequity.px_id:
35     x = LSequity.is_female[i]*LSequity.y[i].value
36     women = women + x
37 print(f'{women} women recieved a transplant in 2016.')
38 print(f'{(women/7409)*100}% of the donor organs went to a female recipient.')
39 lives_saved = 0
40 for i in LSequity.px_id:
41     y = (1-LSequity.mort[i])*LSequity.y[i].value
42     lives_saved = lives_saved + y
43 print(f'The total number of lives saved through liver transplant in 2016 is {lives_saved}.')
```

Figure 23: Pyomo Python Code of Creatinine Model

This example enforces 40% of the donor organs to go to female transplant patients, equivalent to the proportion of the transplant list that was female in 2016. To evaluate the current MELD system with no gender equity constraint, remove line 17 and 18.

Appendix B



```python
import pyomo.environ as pyo
NF = pyo.AbstractModel()

NF.nodes = pyo.Set()
NF.edges = pyo.Set(within=NF.nodes*NF.nodes)
NF.capacity = pyo.Param(NF.edges)
NF.source = pyo.Param(within=NF.nodes)
NF.sink = pyo.Param(within=NF.nodes)
NF.x = pyo.Var(NF.edges, within=pyo.NonNegativeReals)

def max_flow(NF):
    return sum(NF.x[i,j] for (i,j) in NF.edges if j==pyo.value(NF.sink))
NF.obj = pyo.Objective(rule=max_flow, sense=pyo.maximize)
def edge(NF,i,j):
    return NF.x[i,j] <= NF.capacity[i,j]
NF.edge_capacity = pyo.Constraint(NF.edges, rule=edge)
def general_node(NF,k):
    if k==pyo.value(NF.source) or k==pyo.value(NF.sink):
        return pyo.Constraint.Skip
    inflow = sum(NF.x[i,j] for (i,j) in NF.edges if j==k)
    outflow = sum(NF.x[i,j] for (i,j) in NF.edges if i==k)
    return inflow == outflow
NF.flow = pyo.Constraint(NF.nodes, rule=general_node)

NF_restrictedflow = NF.create_instance("networkflow-newdata.dat")
solver_result = pyo.SolverFactory('gurobi').solve(NF_restrictedflow)
solve_status = solver_result.solver.termination_condition
print(solver_result)
print(NF_restrictedflow.obj())

for (i,j) in NF_restrictedflow.edges:
    print("\t %s \t %s \t %s \t %s" %
          (i,j,NF_restrictedflow.x[i,j].value,NF_restrictedflow.capacity[i,j]))
```

Figure 24: Pyomo Python Code of Size Incompatibility Model

To apply the cuts made between the smallest five donor groups and the largest five patient groups, the model code was not changed. This change was made in the dataset, “networkflow-newdata.dat”, by changing the capacity for those edges to zero.