OPTIMAL AIRCRAFT MANEUVERING MODELS FOR CRUISE MISSILE ENGAGEMENT: A MODELING AND COMPUTATIONAL STUDY

THESIS

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AFIT-ENS-MS-22-M-142

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AFIT-ENS-MS-22-M-142

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THESIS

Presented to the Faculty
Department of Operational Sciences
Graduate School of Engineering and Management
Air Force Institute of Technology
Air University
Air Education and Training Command
in Partial Fulfillment of the Requirements for the
Degree of Master of Operations Research

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March 24, 2022

AFIT-ENS-MS-22-M-142

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Abstract

Given the increased threat and proliferation of adversary military capabilities, this research seeks to develop reasonably accurate and computationally tractable models to optimally maneuver aircraft to intercept cruise missile attacks. The research leveraged mathematical programming to model the problem, informed by constraints representing a system of (temporal) difference equations. The research began by comparing six models having alternative representations of velocity and acceleration constraints while analyzing situations with stationary targets. The Multiple Aircraft, Multiple Stationary Target Engagement Problem with Box Constraint Bounds (MAMSTEP-BC) Model yielded superior overall performance and was further analyzed through alternative mathematical programming model enhancements to create feasible flight profiles, in terms of leveraging a valid sequence of maneuvers. Lastly, the MAMSTEP-BC model was modified to maneuver aircraft to engage moving targets.

This model proved effective with multiple aircraft and multiple targets when optimizing the time needed to engage. MAMSTEP-BC was able to maintain a high-level of granularity by accounting for aircraft and pilot limitations while managing to generate optimal solutions quickly for both stationary and moving targets.

Acknowledgements

I would like to express my sincere thanks to my research advisor, Dr. Brian Lunday. His vast knowledge and dedication to helping guide me through this work is greatly appreciated.

Izaiah G. LaDuke

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I. Introduction

To introduce the underlying problem, Section 1.1 starts with the motivation for this study and presents background knowledge to provide the reader with its strategic context. Sections 1.2 and 1.3 respectively formalize the problem statement of the research and identify the three research questions addressed by the work. Lastly, Section 1.4 explains the organization of the remaining chapters of the research.

1.1 Motivation and Background

Since its creation in 1947, one of the core missions of the United States Air Force (USAF) has been that of Air Superiority. As defined by the Department of Defense (DoD), air superiority includes gaining enough dominance in the air, by means of an air battle, to allow the force to conduct its operations without being prohibited by air and missile threats (United States Joint Chiefs of Staff, 2015). While this mission is of high importance, the USAF acknowledges that, during a conflict with a near-peer adversary, air superiority may not be achieved at all places at all times (Lemay Center for Doctrine, 2019).

In recent years, the United States has become threatened by near-peer competitors in both industry and military advancements. The main country of interest has been China. Due to increased assertive, authoritarian threats by China, the United States must continue to have a robust presence in the Indo-Pacific. Part of this increased presence includes amplifying its power and ability to disrupt direct threats before they are able to reach American soil (Joseph R. Biden Jr., 2021). The main problem addressed by this research is to develop reasonably accurate and computationally tractable models to optimally maneuver multiple aircraft to engage multiple adversary cruise missiles. This problem is important because the United States (U.S.) and China are in tight competition to be the worlds most influential superpower, leading to strong military advancements by both countries (Office of the Director of National Intelligence, 2021). These advancements by China will require the U.S. to make both smart defense investments and decisions (United States Joint Chiefs of Staff, 2017). The scope of this project is to identify how to properly maneuver aircraft to neutralize positively identified long-range cruise missiles. The main goal of the research is to determine the level of model granularity to be used with respect to the airspace, aircraft maneuverability, time, and target capabilities. The known data requirements for this research will be the capabilities of U.S. fighters and weapons, in addition to Chinese cruise missile specifications. The outputs of this research will be the selection of the best model, the total time to intercept, computation time, and computational feasibility.

1.2 Problem Statement

Given positively identified long-range cruise missiles launched against a fixed site and multiple aircraft conducting defensive counter air patrols as part of a layered defense, efficiently identify how to maneuver the aircraft to effectively neutralize the cruise missiles.

1.3 Research Questions

The following questions will be addressed throughout the research process:

1. What are the established techniques, in terms of model fidelity and computa-

tional complexity, to model aircraft maneuvering-and-routing decisions for the underlying problem?

2. How can cruise missile neutralization be best integrated into an aircraft maneuvering-and-routing model?

3. What combined aircraft maneuvering-and-routing and threat neutralization modeling components yield a tractable, sufficiently granular representation for practical use?

1.4 Organization of the Thesis

The remainder of this thesis is organized as follows. Chapter II discusses literature related to national defense, vehicle routing problems, and mathematical programming formulations involving difference equations to maneuver or route entities. The following research was conducted in three different phases. Chapter III presents the work examined in Phase 1, which develops and tests alternative models to maneuver multiple aircraft to engage stationary targets. Presented in Chapter IV, Phase 2 of the research explores alternative mathematical programming model enhancements to create feasible flight profiles from Phase 1 of the research. In Chapter V, the work examined in Phase 3 is presented in which a final model is developed and tested to maneuver multiple aircraft to engage moving targets. Chapter VI concludes the thesis with the major outcomes of the work and introduces possible avenues for future research on the topic of temporal network routing models.

II. Literature Review

This chapter summarizes important literature relevant to modeling and solving defensive counter air problems. Section 2.1 highlights literature related to the topic of missile defense and aircraft maneuvering. Section 2.2 discusses selected literature related to the modeling methodology of vehicle routing problems. Each section is organized respectively with first the discussion of the broader topical areas, followed by highlighting literature exhibiting more specific modeling and analysis techniques relevant to this research.

2.1 Missile Defense and Aircraft Maneuvering

Karako (2019) examines the 2018 National Defense Strategy and 2019 Missile Defense Review's focus of "new era" missile defense. The author contends the United States' proposed actions within this review are not enough to meet current or emerging missile threats from our adversaries. The author argues that the U.S. must drastically change its policy to be able to successfully counter missile threats from major adversaries such as China and Russia. These threats have been changing in recent years, and policy cannot simply follow that of years past. This research informs the development of our model because it highlights some key considerations when defending against enemy missile threats. These considerations ensure the model in our work maintains a proper scope and represents a solution to an issue that is relevant and promotes national security. Although the author discusses defense layering with a higher level of complexity than the problem being modeled, defensive counter air is still one of the critical layers in that defense system, making this work relevant.

Patel and Goulart (2011) examine flight paths for counter hijack control systems. This model determines possible structures that could be targeted by a hijacked plane and uses a three degree-of-freedom nonlinear model to create a horizon that avoids trajectories resulting in hitting these objects. This paper successfully determines feasible trajectories allowing for aircraft avoidance in real-time applications. This paper informs development of our work because its model uses almost the exact opposite objective function to what is considered herein. This problem uses real-time data to divert from a specific trajectory whereas this thesis attempts to determine a trajectory resulting in a collision. Some of the ways the authors use KKT conditions in their constraints also may be of use in modeling the problem.

Pachter et al. (2019) examine a pursuit-evasion differential game. This game involves an aircraft being attacked by an enemy missile while simultaneously being protected by a defense missile. The goal is to use differential equations with kinematics assumptions to maximize the separation between the friendly aircraft and the enemy missile, while the enemy missile wants to minimize that same distance. This paper informs development of our model because of the authors' use of kinematics simplification and assumptions. These assumptions may be needed if the granularity in our proposed model needs to be decreased when trying to successfully solve the model.

Kumar and Shima (2017), the authors adopt a nonlinear approach to modeling protecting an aircraft from an attacking missile. This guidance strategy uses both sophisticated linear and nonlinear kinematics to formulate an engagement between a defender and an attacker as point mass vehicles in a Cartesian coordinate system. They use a technique known as sliding-mode control with the nonlinear methodology to help ensure mission effectiveness. Although this paper is very physics focused, it can still inform the formulation of our model by showing the differences between a linearized and nonlinear model when using the kinematics of flight. This can help with adjusting the granularity of the model in this work.

In the next work, Carr (2017) examines aircraft missile evasion by using optimal control theory. This dissertation focuses on using differential equations to model a cruise missile evasion problem by maximizing the miss distance of the attacking missile. The author then compares the performance of this specific event across multiple different scenarios each with different strategies. This paper informs development of our model because it describes an aircraft maneuvering problem without having to use any networks. This formulation is an example of how highly complex the methods in the thesis could become if a high level of granularity is preferred and the computing time and budget allow for it. The broad topic of avoiding missiles by a maneuvering aircraft is similar to that of targeting cruise missiles so the insights from this work may translate well to this thesis.

In the last piece of literature on missile defense, Wilson (2021) looks to address the problem of minimizing the risk of rival states successfully making multi-layered attacks against the United States. This research seeks to find the best way to position and maneuver a fixed number of friendly fighter aircraft to destroy enemy cruise missiles. This model leverages a network formulation with a series of nodes and arcs and discrete time periods. This paper directly informs development of our model because it is the previous work upon which this work builds. The insights in this work are at the lower end of the granularity scale, with time being discretized and aircraft only being allowed to travel along a finite number of arcs to a finite number of nodes. The goal of the current work is to remove some of the assumptions about aircraft maneuverability that were necessary in this work to create a better model with respect to *both* fidelity and tractability. This topic of intercepting Chinese cruise missiles is nearly identical, but the methodology used by Wilson should be much different than the intended approaches taken by this thesis.

2.2 Vehicle Routing Problem Methodology

Also relevant to the underlying problem is the literature on vehicle routing, much of which falls within the domain of Vehicle Routing Problem (VRP) research. VRP literature informed early explorations, model development, and preliminary testing regarding how to best address the problem examined herein. Although this thesis does not directly present the VRP-informed models developed, the conceptual motivation provided by these works remains relevant to review, as they influenced the modeling techniques considered and choices made when seeking highly granular aircraft routing models, as well as models that were computationally tractable.

In the first work by Braekers et al. (2016), the authors examine the topic of a vehicle routing problem with a wide lens and categorize different articles. This paper classifies 277 different VRP articles by analyzing trends apparent in the research. The level of detail in this taxonomy is the first of its kind and categorizes the works into very specified areas. The results of this paper allow readers to choose combinations of identifiers to find relevant VRP articles. This paper informs our research by pointing to relevant VRP articles based off identifiers of my choosing. This literature review allows for a more specific approach to a VRP rather than trying to adopt a weak structure that does not appropriately fit the model developed in this thesis.

Next, Laporte et al. (2013) examine many of the different influential contributors in recent years involving VRP modeling and analysis. This paper briefly describes the major threads of VRP research related to modeling and solution methodologies, and it references key papers contributing these developments. Some different methodologies mentioned include using heuristics, applying exact algorithms through set partitioning problems, and even directly solving linear mixed-integer programs with CPLEX to address synchronization problems of a realistic size. This paper contributes to our model by suggesting different ways that a VRP can be modeled as it relates to problem complexity. These insights portend the use of a VRP modeling structure for both low and high granularity missile defense problems. Some specifics of different methodologies referenced in this paper proved useful by directing us to VRP literature specific to military aircraft targeting.

Zhao et al. (2018) examine a slightly different type of problem known as a multi agent system. This system looks at directed graphs with a spanning tree to solve a specified-time consensus problem. This model uses a network (directed graph) to show the interaction between multiple agents. The agents in this work are satellites in orbit. This paper was useful in preliminary phases of model development because of its successful combination of multiple topics. This work combines a specified-time problem with linear multi agent systems on directed graphs. This work presents how to increase the complexity of a network problem with regards to time throughout the model run.

Choi et al. (2019) examine another VRP containing Unmanned Aerial Vehicles (UAVs) being used to deliver packages in urban areas. The goal of this work is to minimize operating costs, reduce street traffic, and successfully deliver packages on time without collisions between the UAVs and the urban environment. This VRP uses an optimization model with a two-layered urban flight network to create collision free paths for the UAVs to make their deliveries. This paper contributes to our model development because of the constraints within the authors' VRP. Specifically, constraints on the payload, flight time, and flight window provide insights on how to incorporate highly granular factors into a mathematical model. Also, it may be possible to consider targeting the obstacles that were meant to be avoided to be more closely aligned with a missile defense type problem. This paper reduces a lot of the kinematics to a low granularity problem to help with a model that could be solved quickly, but may not have sufficient results when trying to model cruise missile interception.

Radmanesh and Kumar (2016) utilize Mixed Integer Linear Programming (MILP) and path smoothing to plan UAV flight routes. This paper specifically examines the routes of aircraft in a formation that must avoid an incoming aircraft attempting to join the formation. The authors implement a novel fast-dynamic MILP and uses a cost function to minimize both the cost and energy of the UAVs. This paper informs development of our model because it offers an approach to solving an aircraft maneuvering problem without nodes and arcs while staying linear in nature. Some of the constraints in this formulation may be of use in our thesis as a middle of the road granularity solution to the overall problem. The topic of aircraft maneuvering adds to the paper's relevance because trying to seek a target is quite the opposite of avoiding an incoming object, allowing for possible changes to make the math very similar.

As referenced by Laporte et al. (2013), the work by Quttineh et al. (2013) examines military aircraft targeting and attacking a specified number of stationary ground targets while actively avoiding specific targets such as hospitals and civilians. This vehicle routing model uses both synchronization between a targeting aircraft and an attacking aircraft as well as precedence in targeting to create optimal flight paths with specified attack locations. This problem looks to maximize the outcome of the air-to-ground attack while minimizing the time required to conduct the mission. This paper contributes to our model development because of its ability to discretize the feasible attack space. This paper examines different feasible directions of targeting and attacking in a way that could be used in a VRP model in this thesis. The work presented in this paper describes a method of building a slightly higher level model in terms of granularity while maintaining a network with nodes and arcs. This would provide a modeling technique to improve upon the work by Wilson (2021) that leveraged a discrete time network but failed to recognize aircraft orientation.

III. Phase 1 - Maneuvering Multiple Aircraft to Engage Stationary Targets

This chapter presents the modeling, testing, and analysis related to Phase 1 of the research. Section 3.1 presents three alternative modeling approaches that leverage difference equations (DEs) for multiple aircraft to engage multiple stationary targets on a Cartesian plane. These models vary in terms of fidelity and expected computational tractability with respect to the manner in which bounds are imposed on aircraft velocities and accelerations. Section 3.2 sets forth and conducts testing related to the relative efficacy and efficiency of leading commercial solvers to address instances of the modeling variants, and Section 3.3 summarizes conclusions prior to examining Phase 2 of the research: improving upon Phase 1 flight profiles through multicriteria optimization and iterative modeling.

3.1 Phase 1 Difference Equation Model Formulation

The difference equation model consists of a combination of a control problem utilizing difference equations and a demand coverage problem, that is, a temporal set covering problem. The objective of the problem is to minimize the cumulative engagement times of the cruise missiles by each aircraft. Prior to formulating any optimization models, it is necessary to define the following sets, parameters, and decision variables.

Sets

- T: discrete time periods wherein aircraft maneuvering occurs, indexed on t, where $T = \{0, 1, ..., |T|\}$.
- K: set of aircraft used for engagements, indexed on k.
- I: set of stationary enemy targets to engage, indexed on i.

Parameters

- (x_i^{tgt}, y_i^{tgt}): x- and y-coordinates of the known stationary enemy target i
- (x_k^{init}, y_k^{init}): initial x- and y-coordinates of friendly aircraft k
- $(\dot{x}_k^{init}, \dot{y}_k^{init})$: initial x- and y-components of velocity for friendly aircraft k
- $(\ddot{x}_k^{init}, \ddot{y}_k^{init})$: initial x- and y-components of acceleration for friendly aircraft k
- (v_k^{min}, v_k^{max}): minimum and maximum allowed speed (i.e., magnitude of velocity) for aircraft k
- v_k^{man}: maneuvering velocity magnitude of aircraft k
- (a_k^{max-}, a_k^{max+}): maximum allowable *negative* and *positive* acceleration magnitude for aircraft k
- r_k: weapons range for friendly aircraft k
- M: scalar for the maximum possible distance to any target in the scenario
- T_{tot}: Total time for the engagement
- δ: Time increment for the difference equations, determined by $\frac{T_{tot}}{|T|-1}$

Decision Variables

- (x_{kt}, y_{kt}): x- and y-coordinates of friendly aircraft k at time period t
- $(\dot{x}_{kt}, \dot{y}_{kt})$: x- and y-components of velocity for friendly aircraft k at time period t
- $(\ddot{x}_{kt}, \ddot{y}_{kt})$: x- and y-components of acceleration for friendly aircraft k at time period t

- ϕ_{ikt}: binary variable equal to 1 if target i is engaged by aircraft k at time period t, and 0 otherwise
- ψ^x_{kt}: Boolean switch to be used on the lower bound for velocity in the x-direction for aircraft k at time period t
- ψ^y_{kt}: Boolean switch to be used on the lower bound for velocity in the y-direction for aircraft k at time period t

3.1.1 DE Model with Nonlinear Bounding Constraints for Acceleration and Velocity

The first model being considered adopts the most accurate representation of bounds on aircraft acceleration and velocity, albeit at the cost of a nonlinear representation that portends challenges to computational tractability. What follows is the problem formulation of the **Multiple Aircraft, Multiple Stationary Target Engagement Problem with Nonlinear Bounds** (**MAMSTEP-NLB**):

$$\min \sum_{i\in I}\sum_{k\in K}\sum_{t\in T} t\phi_{ikt} \tag{1}$$

$$\text{s.t. } x_{kt} = x_{k(t-1)} + \delta\dot{x}_{k(t-1)},\ \forall\, k \in K, t \in T \setminus \{0\}, \tag{2}$$

$$y_{kt} = y_{k(t-1)} + \delta\dot{y}_{k(t-1)},\ \forall\, k \in K, t \in T \setminus \{0\}, \tag{3}$$

$$\dot{x}_{kt} = \dot{x}_{k(t-1)} + \delta\ddot{x}_{k(t-1)},\ \forall\, k \in K, t \in T \setminus \{0\}, \tag{4}$$

$$\dot{y}_{kt} = \dot{y}_{k(t-1)} + \delta\ddot{y}_{k(t-1)},\ \forall\, k \in K, t \in T \setminus \{0\}, \tag{5}$$

$$(x_{k0}, y_{k0}) = (x_k^{init}, y_k^{init}),\ \forall\, k \in K, \tag{6}$$

$$(\dot{x}_{k0}, \dot{y}_{k0}) = (\dot{x}_k^{init}, \dot{y}_k^{init}),\ \forall\, k \in K, \tag{7}$$

$$(\ddot{x}_{k0}, \ddot{y}_{k0}) = (\ddot{x}_k^{init}, \ddot{y}_k^{init}),\ \forall\, k \in K, \tag{8}$$

$$(x_i^{tgt} - x_{kt})^2 + (y_i^{tgt} - y_{kt})^2 \leq r_k^2 \phi_{ikt} + M(1 - \phi_{ikt}),\ \forall\ i \in I, k \in K, t \in T, \quad (9)$$

$$\sum_{k \in K} \sum_{t \in T} \phi_{ikt} \geq 1,\ \forall\ i \in I, \quad (10)$$

$$\dot{x}_{kt}^2 + \dot{y}_{kt}^2 \leq (v_k^{max})^2,\ \forall\ k \in K, t \in T, \quad (11)$$

$$\dot{x}_{kt}^2 + \dot{y}_{kt}^2 \geq (v_k^{min})^2,\ \forall\ k \in K, t \in T, \quad (12)$$

$$\ddot{x}_{kt}^2 + \ddot{y}_{kt}^2 \leq (a_k^{max})^2,\ \forall\ k \in K, t \in T, \quad (13)$$

$$\sqrt{\ddot{x_{kt}}^2 + \ddot{y_{kt}}^2} \leq \frac{a_k^{max} - 1}{v_k^{man} - v_k^{min}} \sqrt{\dot{x_{kt}}^2 + \dot{y_{kt}}^2 - v_k^{min}} + 1,\ \forall\ k \in K, t \in T, \quad (14)$$

$$\phi_{ikt} \in \{0, 1\},\ \forall\ i \in I, k \in K, t \in T. \quad (15)$$

This formulation seeks to minimize via the objective function (1) the sum of the target engagement times. Constraints (2) and (3) update the position of each aircraft within the imposed Cartesian plane, for each time period, based upon their respective velocities during the previous time period, as well as the δ-parameter that discretizes the time horizon. Constraints (4) and (5) perform similar updates to each aircraft's velocity using their respective acceleration vectors. Constraints (6)-(8) respectively affix the initial position, velocity, and acceleration of each aircraft. Constraint (9) limits engagements to only occur when an aircraft is located within a distance of r_k to a target, and Constraint (10) requires each target be engaged at least once. (Via the objective function formulation, an optimal solution will not engage any target more than once.)

The aircraft velocity is constrained by circular upper and lower bounds, modeled by Constraints (11) and (12). Constraint (13) bounds the acceleration vector for each aircraft over the time horizon via a circular upper bound. The relationship between the magnitude of velocity and acceleration is bounded by Constraint (14). This bound restricts flight of the aircraft to a generalized, symmetric flight envelope, as depicted by a chart of speed versus load factor (otherwise known as a $v-N$ diagram). Without this bound, the aircraft may be allowed to maneuver in a fashion that would damage the structure of the aircraft or create impossible G-forces given the current velocity. Finally, Constraint (15) enforces a binary restriction on the relevant decision variable ϕ to determine the times of target engagements.

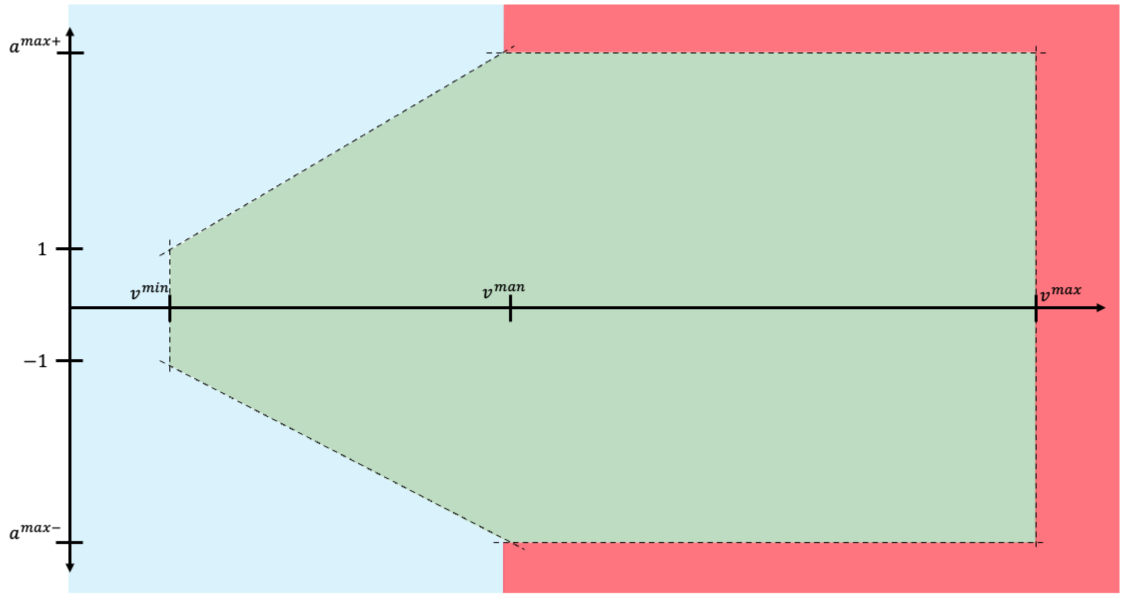

Figure 1. Example v-N Diagram Bounds on Aircraft Velocity and Acceleration

3.1.2 DE Model with Linear Approximating Constraints for Acceleration and Velocity

The next model being examined slightly reduces the accuracy of the acceleration and velocity bounds by using a linear approximation. This linear approximation is made in an attempt to increase the computational tractability of the model at the expense of model fidelity. This model is known as the **Multiple Aircraft, Multiple Stationary Target Engagement Problem with Linear Approximating Bounds** (**MAMSTEP-LAB**). Equations (11)–(13) from the MAMSTEP-NLB model will be replaced with the following constraints for velocity and acceleration bounds.

$$\dot{x}_{kt} + \dot{y}_{kt} \leq v_k^{max}, \ \forall\ k \in K, t \in T, \tag{16}$$

$$-\dot{x}_{kt} + \dot{y}_{kt} \leq v_k^{max}, \ \forall\ k \in K, t \in T, \tag{17}$$

$$-\dot{x}_{kt} - \dot{y}_{kt} \leq v_k^{max}, \ \forall\ k \in K, t \in T, \tag{18}$$

$$\dot{x}_{kt} - \dot{y}_{kt} \leq v_k^{max}, \ \forall\ k \in K, t \in T, \tag{19}$$

$$\dot{x}_{kt} + \dot{y}_{kt} \geq v_k^{min} - 2v_k^{max}\psi_{kt}^x - 2v_k^{max}\psi_{kt}^y\ \forall\ k \in K, t \in T, \tag{20}$$

$$\dot{x}_{kt} - \dot{y}_{kt} \geq v^{min} - 2v_k^{max}\psi_{kt}^x - 2v_k^{max}(1 - \psi_{kt}^y), \ \forall\ k \in K, t \in T, \tag{21}$$

$$-\dot{x}_{kt} + \dot{y}_{kt} \geq v_k^{min} - 2v_k^{max}(1 - \psi_{kt}^x) - 2v_k^{max}\psi_{kt}^y, \ \forall\ k \in K, t \in T, \tag{22}$$

$$-\dot{x}_{kt} - \dot{y}_{kt} \geq v_k^{min} - 2v_k^{max}(1 - \psi_{kt}^x) - 2v_k^{max}(1 - \psi_{kt}^y), \ \forall\ k \in K, t \in T, \tag{23}$$

$$\ddot{x}_{kt} + \ddot{y}_{kt} \leq a_k^{max+}, \ \forall\ k \in K, t \in T, \tag{24}$$

$$\frac{a_k^{max+}}{a_k^{max-}}\ddot{x}_{kt} + \ddot{y}_{kt} \leq a_k^{max+}, \ \forall\ k \in K, t \in T, \tag{25}$$

$$\ddot{x}_{kt} + \ddot{y}_{kt} \geq a_k^{max-}, \ \forall\ k \in K, t \in T, \tag{26}$$

$$\frac{a_k^{max-}}{a_k^{max+}}\ddot{x}_{kt} + \ddot{y}_{kt} \geq a_k^{max-}, \ \forall\ k \in K, t \in T \tag{27}$$

$$\psi_{kt}^x, \psi_{kt}^y \in \{0, 1\}, \ \forall\ k \in K, t \in T. \tag{28}$$

Constraints (16)-(19) impose rectilinear bounds on the maximum velocity of each aircraft. Constraints (20)-(23) impose rectilinear lower bounds on the minimum velocity of each aircraft, depending on the whether each of the $\dot{x}_{kt}$ and $\dot{y}_{kt}$ components are positive or negative, which are indicated by the binary ψ_{kt}^x and ψ_{kt}^y variables defined in Constraint (28). Lastly, Constraints (24)-(27) impose upper and lower bounds on the aircraft acceleration.

3.1.3 DE Model with Box Constraints for Acceleration and Velocity

The next model being examined slightly reduces the accuracy of the acceleration and velocity bounds once again by using simple box constraints. Again, this is done in an attempt to further increase the computational tractability of the model at the expense of model fidelity. This model is known as the **Multiple Aircraft, Multiple Stationary Target Engagement Problem with Box Constraint Bounds** (**MAMSTEP-BC**). Equations (11)–(13) will be replaced with the following constraints for velocity and acceleration bounds.

$$\dot{x}_{kt} \leq v_k^{max}, \; \forall \; k \in K, t \in T, \tag{29}$$

$$-\dot{x}_{kt} \leq v_k^{max}, \; \forall \; k \in K, t \in T, \tag{30}$$

$$\dot{y}_{kt} \leq v_k^{max}, \; \forall \; k \in K, t \in T, \tag{31}$$

$$-\dot{y}_{kt} \leq v_k^{max}, \; \forall \; k \in K, t \in T, \tag{32}$$

$$\dot{x}_{kt} \geq v_k^{min} - 2v_k^{max}\psi_{kt}^x, \; \forall \; k \in K, t \in T, \tag{33}$$

$$-\dot{x}_{kt} \geq v_k^{min} - 2v_k^{max}(1 - \psi_{kt}^x), \; \forall \; k \in K, t \in T, \tag{34}$$

$$\dot{y}_{kt} \geq v_k^{min} - 2v_k^{max}\psi_{kt}^y, \; \forall \; k \in K, t \in T, \tag{35}$$

$$-\dot{y}_{kt} \geq v_k^{min} - 2v_k^{max}(1 - \psi_{kt}^y), \; \forall \; k \in K, t \in T, \tag{36}$$

$$\ddot{x}_{kt} \leq a_k^{max+}, \; \forall \; k \in K, t \in T, \tag{37}$$

$$\ddot{y}_{kt} \leq a_k^{max+}, \; \forall \; k \in K, t \in T, \tag{38}$$

$$\ddot{x}_{kt} \geq a_k^{max-}, \; \forall \; k \in K, t \in T, \tag{39}$$

$$\ddot{y}_{kt} \geq a_k^{max-}, \; \forall \; k \in K, t \in T \tag{40}$$

$$\psi_{kt}^x, \psi_{kt}^y \in \{0, 1\}, \; \forall \; k \in K, t \in T. \tag{41}$$

Constraints (29)-(32) impose upper bounds on velocity in each component for all aircraft. Constraints (33)-(36) set lower bounds on aircraft velocity depending on the sign of $\dot{x}_{kt}$ and $\dot{y}_{kt}$, indicated once again by the binary variables ψ^x_{kt} and ψ^y_{kt}, determined via Constraint (41). Lastly, Constraints (37)-(40) impose both upper and lower bounds on acceleration for each aircraft in the model.

3.2 Phase 1 Testing, Results, and Analysis

The testing conducted in this research used six separate mathematical models with 30 randomized instances of each. Each of the three models listed above were tested two separate times: once with Constraint (14), and once instead with Constraint (42), a simplified form to reduce the number of roots within the constraint.

$$\ddot{x_{kt}}^2 + \ddot{y_{kt}}^2 \leq 2\left(\frac{a_k^{max}-1}{v_k^{man}-v_k^{min}}\right)\left((1-v_k^{min})\frac{a_k^{max}-1}{v_k^{man}-v_k^{min}}\right)\sqrt{\dot{x}_k^2+\dot{y}_k^2} + \left(\frac{a_k^{max}-1}{v_k^{man}-v_k^{min}}\right)^2\left(v_k^{min}\right)^2 + \dot{x}_k^2 + \dot{y}_k^2 - 2\left(\frac{a_k^{max}-1}{v_k^{man}-v_k^{min}}\right)v_k^{min} + 1 \quad (42)$$

Instances were randomly generated with two patrolling aircraft and two stationary enemy targets. These instances of aircraft and targets are generated using a rectangular grid as shown in Figure 2. The patrolling aircraft have uniform randomized locations and directions within the blue zone, whereas the enemy targets have uniform random stationary locations within the red zone. Common values imposed for the aircraft include initial velocity, maximum/minimum/maneuvering velocities, maximum/minimum accelerations, and targeting range, as reported in Table 1.

The testing for this research was done on a Lenovo Yoga Thinkpad equipped with an Intel Core i7-6500U processor with 2.50 GHz processing speed and 16 GB of RAM. Each instance was run within the General Algebraic Modeling System (GAMS) using version 33.2.0. The commercial solver scip was used for each instance of the optimiza-

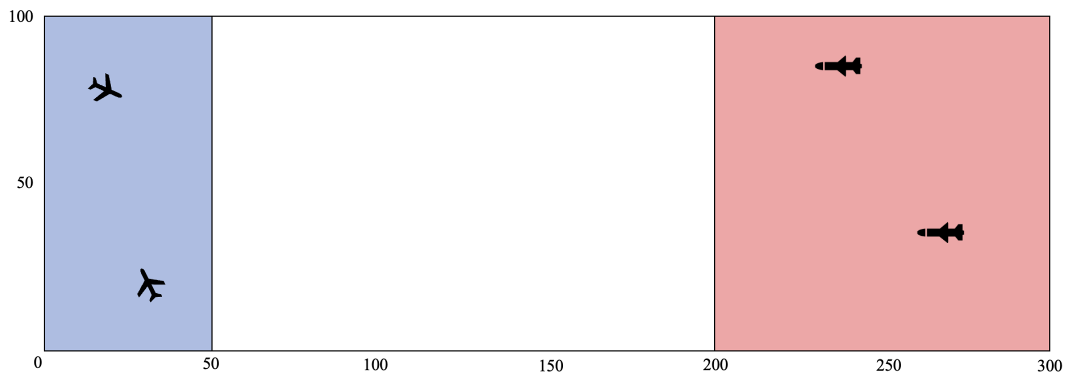

Figure 2. Simulated Airspace

Table 1. Aircraft Common Values used for Testing

Aircraft Parameter	Value
Initial Velocity	400 kts
Min Velocity	170 kts
Max Velocity	700 kts
Man Velocity	250 kts
Initial Acceleration	1 g
Min Acceleration	-7 g
Max Acceleration	7 g
Range	20 km

tion problem. This solver was selected because of its design for global optimization and superlative performance relative to other commercial solvers during preliminary testing with a rudimentary problem instance.

From this testing, it was found that all three models run with Constraint (14) resulted in scip (incorrectly) identifying the instance as being infeasible for all 30 iterations. When using Constraint (42), scip identified an optimal solution 15 times when using the MAMSTEP-NLB model. This model represented the most accurate constraints for the problem. Among the 15 instances for which an optimal solution was not identified, scip identified a feasible (i.e., integer-valued) solution for two of them, and it terminated due to a 300 second time limit for the remaining 13 instances. When relaxing the constraints to the MAMSTEP-LA model, scip identified the solution to be infeasible for all 30 instances. Lastly, when further relaxing the constraints to the MAMSTEP-BC model, the solver was able identify optimal solutions all 30 times. While this model is less computationally complex for modeling aircraft maneuvering, the model is able to return optimal results for 100% of the instances while reducing the objective value by 3.32% on average. This reduction is due to the relaxation of bounds on velocity and acceleration and allowing for a higher magnitude due to the nature of a simple box constraint. For the two models that returned feasible solutions for all 30 instances, output statistics for the time to solve the models within GAMS were recorded in Table 2.

Table 2. Phase 1 Results

Model	Avg. Time (sec)	SD (sec)
MAMSTEP-NLB	241	84.02
MAMSTEP-BC	35.03	12.77

3.3 Phase 1 Conclusions

From the results found during Phase 1 testing, it was determined that the only model to move forward to Phase 2 testing is the MAMSTEP-BC model. The MAMSTEP-LA model was set aside because it failed to find a feasible solution in every instance of the problem. When looking at the remaining two models, we lose some model fidelity when substituting MAMSTEP-BC for MAMSTEP-NLB, but the model results do not have a drastic change when doing so. The objective value is within 3.32% of the original on average while significantly reducing the computation time. Furthermore, the MAMSTEP-BC model was able to reach optimality twice as many times as the MAMSTEP-NLB model. As the research continues past Phase 1, the model will continue to add constraints and likely slow down the solver. Because of this, it was decided that the MAMSTEP-BC model would be the only one retained for further testing.

IV. Phase 2 - Improving Upon Phase 1 Flight Profiles through Multicriteria Optimization and Iterative Modeling

In this chapter, the work from Phase 1 is continued with the MAMSTEP-BC model. Upon examining the optimal solutions to instances from the MAMSTEP-BC model, it is noticed that they do not prescribe a flight path for the aircraft that comports with what a pilot would find reasonable to implement. This outcome results from the existence of alternative optimal solutions. Within this chapter, the model is modified alternatively by implementing either of two different multicriteria optimization techniques or by applying a feasible region reduction method to identify a valid flight profile for the aircraft. Section 4.1 discusses the work conducted in Phase 2 and the motivation behind it. Section 4.2 introduces the three models considered within this phase of research and describes their differences. Next, in Section 4.3, we test the models and analyze the results that come from each model. Finally, conclusions about Phase 2 of the research are made in Section 4.4.

4.1 Discussion and Motivation

The results found in Phase 1 of the research indicate to us an optimal solution to the problem exists. This problem yields an appropriate graph of the position of each aircraft, as depicted in Figure 3, wherein the dots are time-specific locations of each aircraft. This plot indicates one maneuver (turn) by each aircraft near their respective initial locations, after which each aircraft traverses a direct path towards their respective targets before maneuvering once again near the targets.

In this plot of the aircraft positions, we notice different spacing between positions at each maneuver, indicating a high variability in aircraft velocity over the engagement. This variability can be seen in Figure 4, which plots the magnitude of the respective aircraft velocities over time. Near the beginning of the solution each aircraft speeds up rapidly to approximately their maximum speeds to approach the targets. Once near the targets, they both abruptly decrease their velocity. Aircraft 2 seems to maintain this velocity once at the target, but Aircraft 1 exhibits three noticeable velocity decrements over the remaining model run time.

These inconsistent velocities raised some concerns about possible acceleration changes throughout the model instance. The respective aircraft acceleration magnitudes over the course of the engagement can be seen in Figure 5. This figure indicates a large number of acceleration changes throughout the entirety of the run, for both Aircraft 1 and Aircraft 2. These erratic accelerations would result in a flight path that is unacceptable for a pilot to implement, both in terms of their fatigue in maneuvering the aircraft and the fatigue imposed on their body via so much acceleration and deceleration. Due to the nature of this impractical flight path, we are motivated to find whether and how fast a modified model can not just obtain an optimal targeting solution but also to identify viable flight paths for the aircraft.

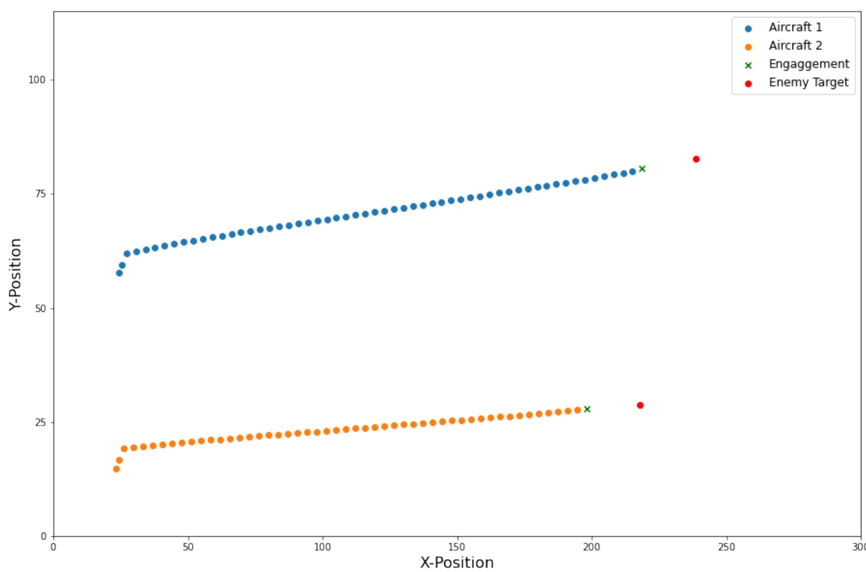

Figure 3. Phase 1 Position Example

Three different approaches are taken in this phase of the research to discriminate among alternative optimal solutions and create reasonable flight paths to allow for

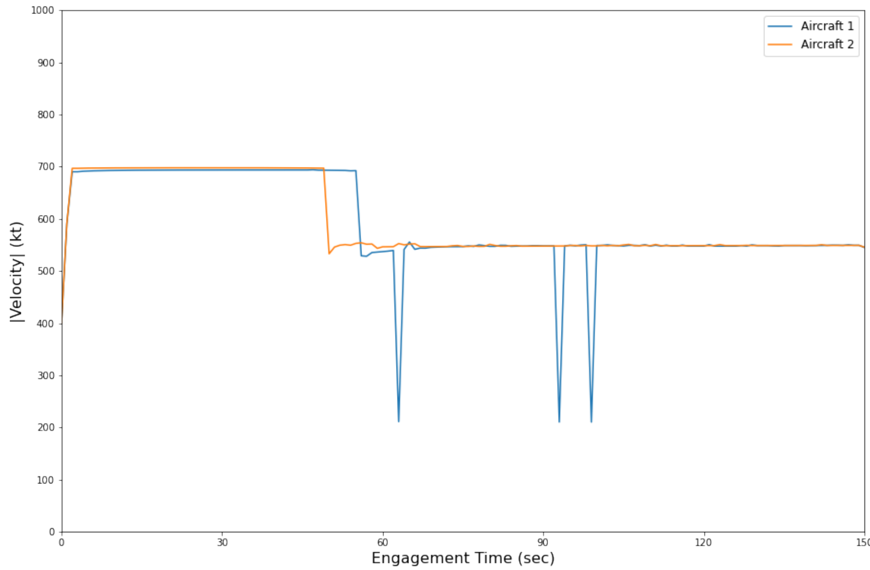

Figure 4. Phase 1 Velocity Example

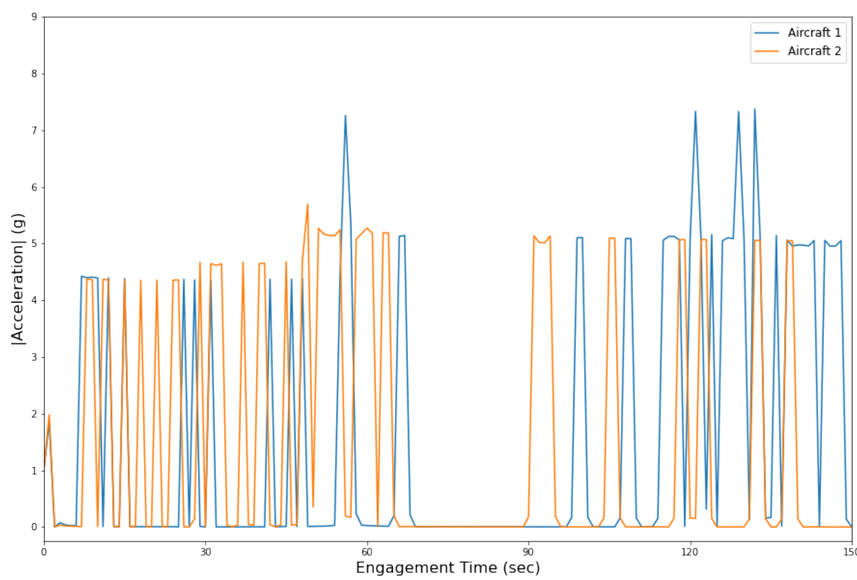

Figure 5. Phase 1 Acceleration Example

target engagement. These three approaches include two multiobjective optimization techniques: the Weighted Sum Method and the ϵ-Constraint Method. We also introduce a way of iteratively changing the initial MAMSTEP-BC model parameters that allows us to find alternative optimal solutions that have more reasonable flight paths. We denote this method as the Feasible Region Reduction Method, as it incrementally increases the size of the time domain until a feasible solution can be identified, and then repeats this process with the parameter bounding the maximum (and minimum) acceleration magnitude.

4.2 Phase 2 Difference Equation Model Formulations

4.2.1 Weighted Sum Method

As introduced by Ehrgott (2005), the Weighted Sum Method is a multiobjective optimization approach that assigns a weight to each of the different criteria within the objective function. For the MAMSTEP-BC model being analyzed as a result of Phase 1 testing, the original criteria only considered the overall time to engage each individual enemy target. The weighted sum method will be used to minimize the change in the acceleration, also known as jerk, throughout the flight profile.

To implement this approach, four new decision variables must be added to the model. These additional decision variables, all related to aircraft jerk in the x- and y-components are defined below.

Additional Decision Variables

- $\dddot{x}^{+}_{kt}$: Positive x-component of jerk for friendly aircraft k at time period t
- $\dddot{x}^{-}_{kt}$: Negative x-component of jerk for friendly aircraft k at time period t
- $\dddot{y}^{+}_{kt}$: Positive y-component of jerk for friendly aircraft k at time period t
- $\dddot{y}^{-}_{kt}$: Negative y-component of jerk for friendly aircraft k at time period t

The reasoning for having four different variables, one positive and one negative component for both the x and y directions has to do with avoiding non-linearities within the model when modeling the absolute values, a technique described by Bazaraa et al. (2010). The summation of these new variables is added to the original objective function, but given a small weight to prevent this new component from becoming the primary objective in the model. This new weighted sum objective function is shown in Equation (43).

$$\min \sum_{i\in I}\sum_{k\in K}\sum_{t\in T} t\phi_{ikt} + 0.001\sum_{k\in K}\sum_{t\in T\setminus\{0,1\}} \left(\dddot{x}^{+}_{kt} + \dddot{x}^{-}_{kt} + \dddot{y}^{+}_{kt} + \dddot{y}^{-}_{kt}\right) \tag{43}$$

The values of the added decision variables are determined by two additional constraints. Equation (44) determines the added x-components of aircraft jerk by calculating the change in acceleration from one time period to the next. Similarly, the y-components of aircraft jerk are calculated by Equation (45). Constraints (46)-(49) impose non-negative bounds on each of the components for aircraft jerk.

$$\dddot{x}^{+}_{kt} - \dddot{x}^{-}_{kt} = \frac{\ddot{x}_{kt} - \ddot{x}_{k(t-1)}}{\delta},\ \forall\ k \in K, t \in T \setminus \{0,1\}, \tag{44}$$

$$\dddot{y}^{+}_{kt} - \dddot{y}^{-}_{kt} = \frac{\ddot{y}_{kt} - \ddot{y}_{k(t-1)}}{\delta},\ \forall\ k \in K, t \in T \setminus \{0,1\} \tag{45}$$

$$\dddot{x}^{+}_{kt} \geq 0, \forall\ k \in K, t \in T \tag{46}$$

$$\dddot{x}^{-}_{kt} \geq 0, \forall\ k \in K, t \in T \tag{47}$$

$$\dddot{y}^{+}_{kt} \geq 0, \forall\ k \in K, t \in T \tag{48}$$

$$\dddot{y}^{-}_{kt} \geq 0, \forall\ k \in K, t \in T. \tag{49}$$

4.2.2 ϵ-Constraint Method

The second multicriteria optimization technique, the ϵ-Constraint Method also discussed by Ehrgott (2005), addresses an additional objective function via a constraint within the model. This additional constraint is represented via Equation (50).

$$\min \sum_{i \in I} \sum_{k \in K} \sum_{t \in T} t\phi_{ikt} \leq \epsilon \tag{50}$$

To utilize this method, one solves the original MAMSTEP-BC model (i.e., minimizing Equation (1)), augmented with Constraints (44)-(49). The parameter ϵ is set equal to the optimal objective function value resulting from this solution. Subsequently, a modification of MAMSTEP-BC is solved; it is augmented with Constraint (50), and it instead minimizes Equation (51). This outcome will discriminate among any alternative optimal solutions to the original MAMSTEP-BC problem in favor of the solution that minimizes the sum of the jerk on the aircraft.

$$\min \sum_{k \in K} \sum_{t \in T \setminus \{0,1\}} \left(\dddot{x}^{+}_{kt} + \dddot{x}^{-}_{kt} + \dddot{y}^{+}_{kt} + \dddot{y}^{-}_{kt} \right) \tag{51}$$

4.2.3 Feasible Region Reduction Method

The last technique used in this phase of research is a method of iteratively increasing the size of the feasible region from an initially infeasible instance until it becomes feasible, and an optimal solution is found. This process of iterating the model is automated within the GAMS modeling software. The approach taken begins by using the original parameters in MAMSTEP-BC for minimum and maximum acceleration and attempts to solve the model with a subset of only one time step (i.e., $T = \{0\}$). This process continually iterates the set (e.g., $T = \{0, 1\}$, $T = \{0, 1, 2\}$) until all targets can be engaged. Additionally, this iterative approach saves which of the aircraft engages with a target in the optimal solution. With this new subset of the original set T now being used, as well as only considering the aircraft that engage a target, we drop the minimum and maximum acceleration values down to -0.1 g and 0.1 g.

Solving this model is attempted and the acceleration values grow with a magnitude of 0.1 g upon each infeasible solution until an optimal solution is found. This technique of reducing the feasible region by limiting the time horizon and again reducing the space by minimizing the magnitude of the maximum acceleration finds an alternative optimal solution that, in turn, bounds the jerk if it does not directly minimize it. This method is implemented without the need to add any constraints or make changes to the original objective function.

4.3 Phase 2 Testing, Results, and Analysis

Each of the three methods mentioned above were run using the same system as Phase 1 of the research. One randomly generated instance using the same parameters and random seed from Phase 1 was used across all three methods. The run times for each of these instances are reported below in Table 3.

Table 3. Phase 2 Results

Model	**Run Time (min)**
Weighted Sum	128.70
ϵ-Constraint	872.98
Feasible Region Reduction	0.57

In the first model using the Weighted Sum Method, the acceleration plot, as seen in Figure 6, indicates fairly constant acceleration across the entire time horizon as compared to the reuslts of Phase 1 testing. Aircraft 1 exhibits one maneuver at the start of the run that aligns with increasing the velocity to begin the engagement, as seen in Figure 7. Acceleration changes are seen again near the 20 and 60 second marks. These changes align with the aircraft slowing down and once again increasing its velocity during a maneuver. Each of these maneuvers can be viewed as direction changes within the map of the engagement, as seen in Figure 8. Aircraft 2 does make an engagement during this iteration, and the resulting graphs indicate a constant acceleration as well as a smooth increase in velocity at the start and maintaining that velocity throughout the iteration run time. The position plot indicates the aircraft flew in one direction and held that direction throughout the engagement.

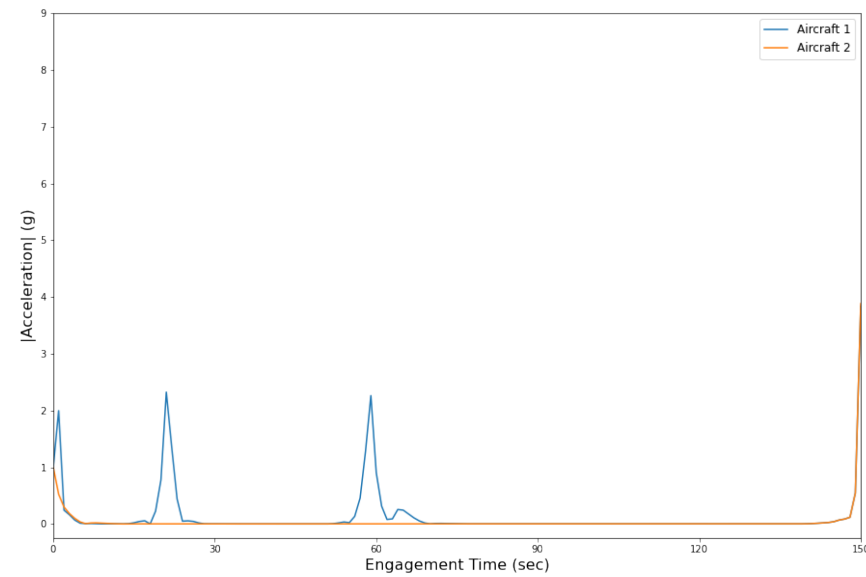

Figure 6. Acceleration: Weighted Sum Method

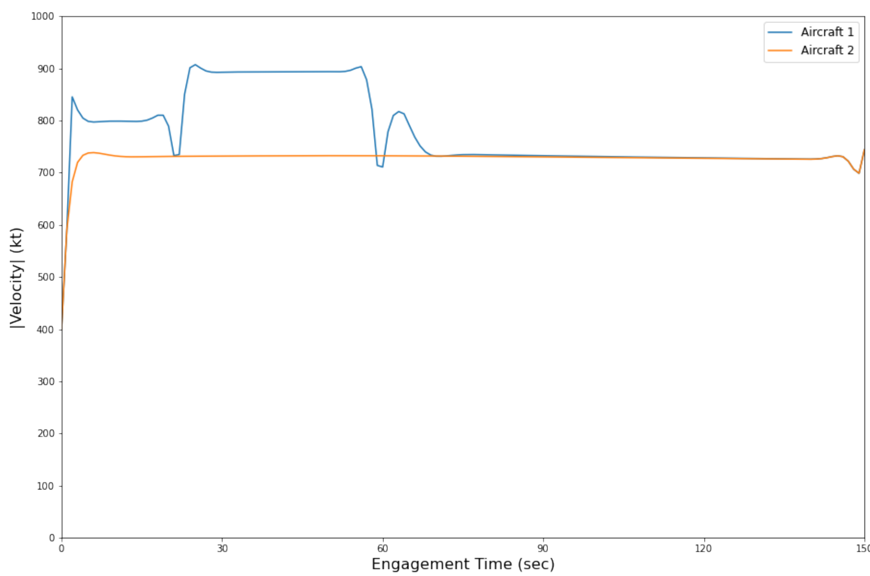

Figure 7. Velocity: Weighted Sum Method

Similar results can be found when considering the ϵ-Constraint Method, as depicted in Figures 9-11. When looking at the aircraft acceleration for Aircraft 1, there

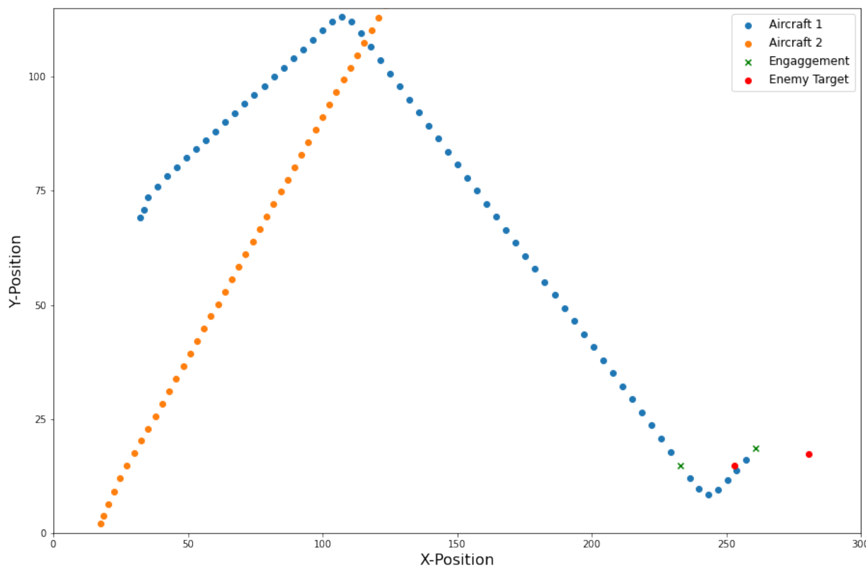

Figure 8. Position: Weighted Sum Method

exists three similar spikes in acceleration around the same times during the engagement, but with slightly less magnitude than in the Weighted Sum Method. Also, the small secondary peak after the last main engagement is no longer visible in the acceleration plot, Figure 9. Aircraft 2 exhibits a similar acceleration as in the Weighted Sum Method. An initial maneuver is made at the beginning of the run and the acceleration drops to near zero for the remainder of the engagement. These maneuvers are more visible in Figure 10 where we look at the velocity over the course of the engagement. Aircraft 1 has very smooth increases and decreases in velocity leading up to and at the the same time as the peaks in acceleration. Aircraft 2 exhibits a steady increase in velocity over the entirety of the run. This decrease in the velocity variation can be seen by the more even spread of points throughout the position plot in Figure 11. The maneuvers of the aircraft engaging the targets, Aircraft 1, are made at nearly the same coordinates, but the resulting plot has slightly less abrupt direction changes, as indicated by the small acceleration spikes and gradual velocity changes in Figures 9-10.

Lastly, the results of the Feasible Region Reduction Method are plotted in Figures

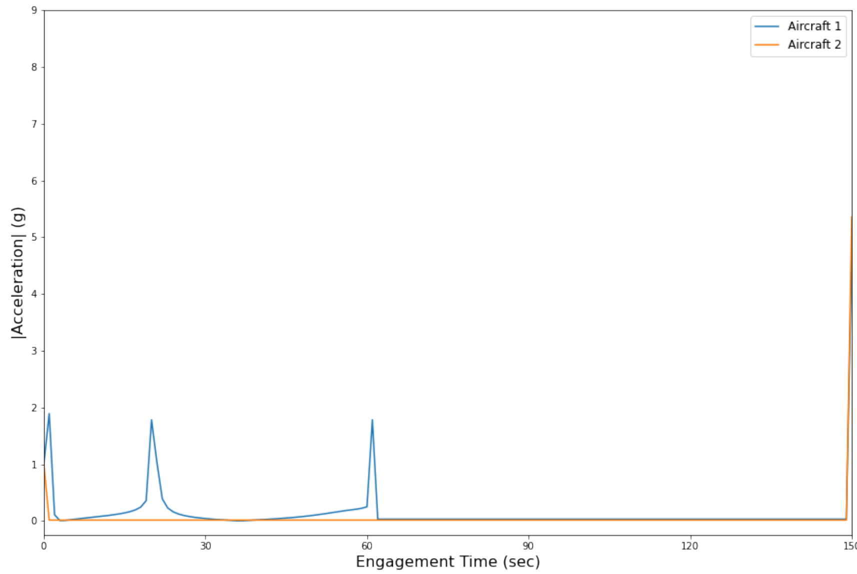

Figure 9. Acceleration: ϵ-Constraint Method

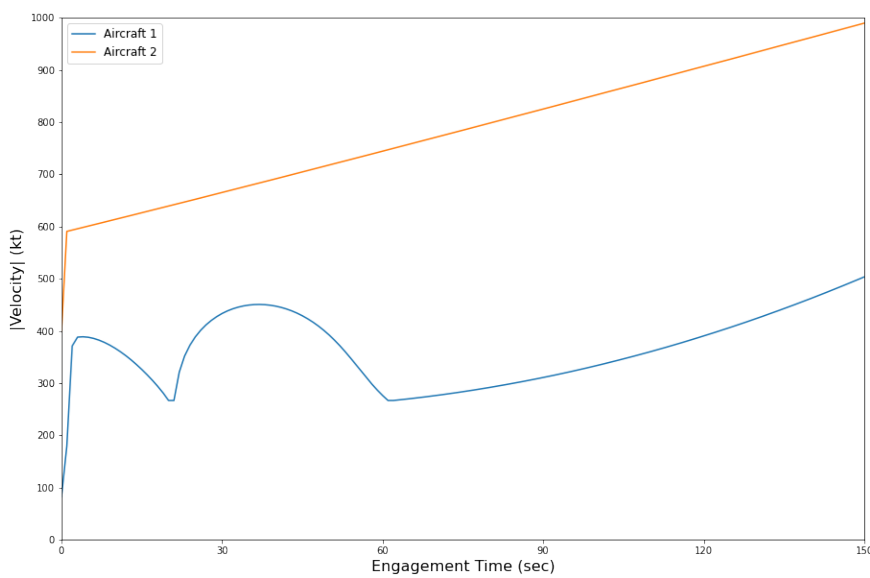

Figure 10. Velocity: ϵ-Constraint Method

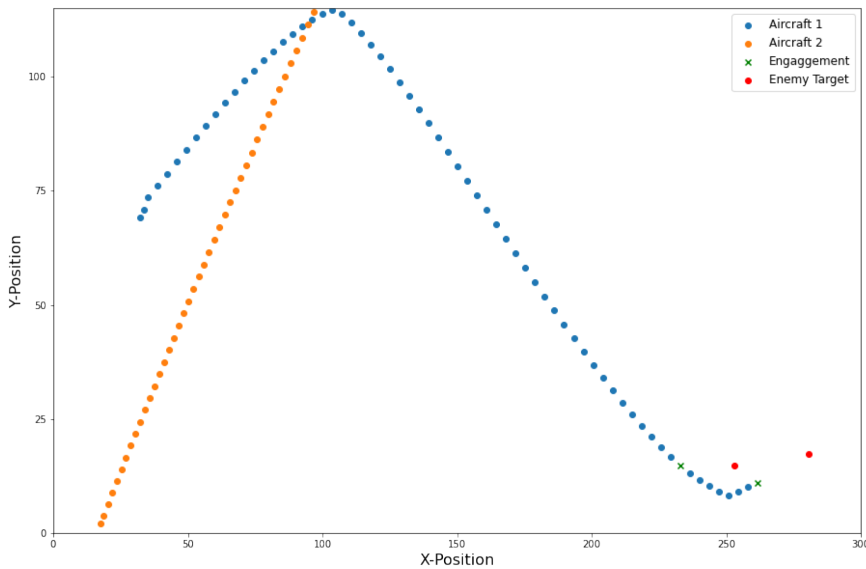

Figure 11. Position: ϵ-Constraint Method

12-14. In these results, we see that there continue to be alternative optimal solutions, as the maneuvers are once again different. With this method, the model is able to set aside aircraft that do not engage targets in the first feasible solution to the instance, thus removing Aircraft 2 from this scenario. It can be seen in Figure 12 that there are now four acceleration spikes. The first spike remains at the beginning of the run, but the second and third increases occur at slightly later times in the engagement. There is then one final jump at the end of the run. These accelerations align with three increases in velocity and one decrease during maneuvers, as seen in Figure 13. When looking at the resulting positions of Aircraft 1 over time, we observe a very familiar shape as previously seen in Figures 8 and 11, but with one additional direction change near the end of the engagement and slightly more variation in spacing between points, indicative of the changes in velocity.

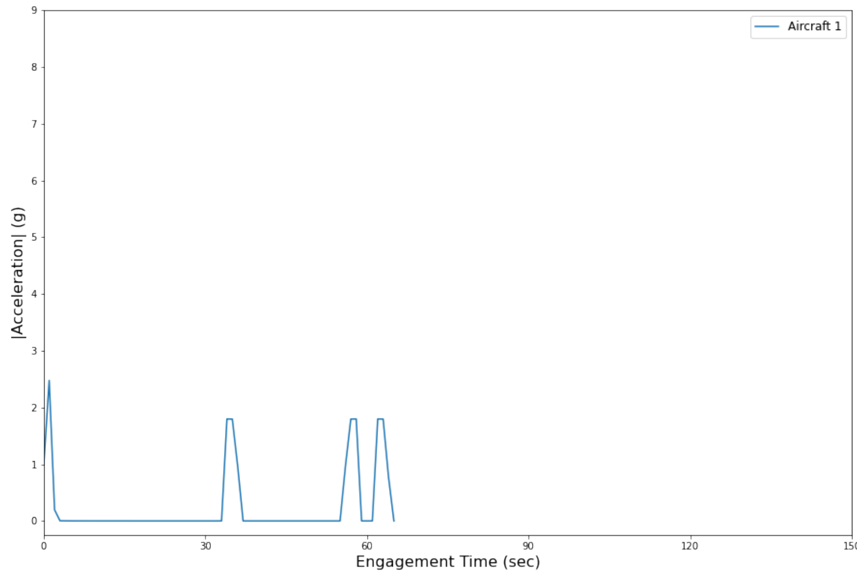

Figure 12. Acceleration: Feasible Region Reduction Method

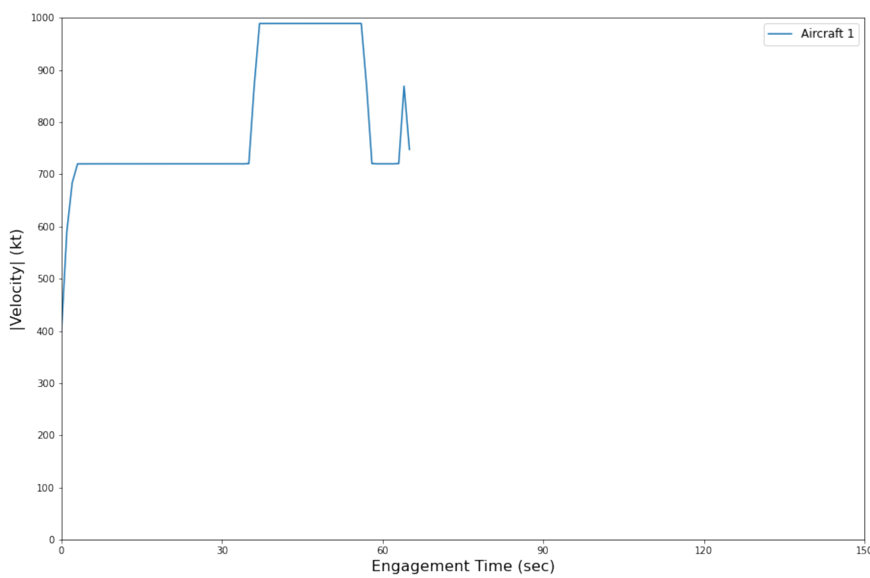

Figure 13. Velocity: Feasible Region Reduction Method

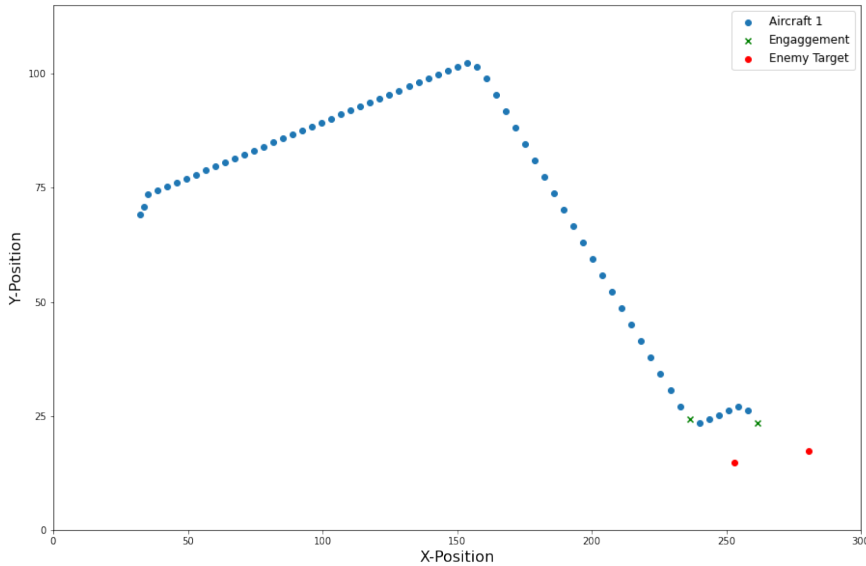

Figure 14. Position: Feasible Region Reduction Method

4.4 Phase 2 Conclusions

From the results found during Phase 2 testing, we can conclude that it is possible to quickly discriminate among the alternative optimal solutions identified in Phase 1 testing. Each of the three modeling approaches tested in Phase 2 yielded slightly different results, in terms of aircraft position, velocity, and acceleration. The feasibility of these models validates the idea of being able to smooth the flight of the aircraft over the course of the engagement. Whereas the results of the Weighted Sum Method and the ϵ-Constraint Method are slightly better in terms of requiring less maneuvering and inducing more gradual changes in velocity for the instance examined, it can be concluded that the Feasible Region Reduction Method is the only viable method due to its superlative run time. One of the primary objectives of this paper is to find a model that has a tractable, sufficiently granular solution, while remaining practical to use. When using the Feasible Reduction Method, we are able to validate that a tractable model is attainable without the necessity of long run times. For the remainder of the testing in this paper, the minimization of jerk will not be included, as the primary objective is to focus on implementing moving targets into an already highly granular model.

V. Phase 3 - Engaging Moving Targets with Multiple Aircraft

In this chapter, the work from Phases 1 and 2 are continued by further constraining the MAMSTEP-BC model to include moving targets. The chapter will present the modeling, testing and analysis related to Phase 3 of the research. Section 4.2 presents the appropriate changes to the MAMSTEP-BC model to allow for moving targets. Section 5.2 describes and reports the results testing with the higher fidelity model. Lastly, Section 5.3 discusses conclusions from Phase 3 of the research.

5.1 Phase 3 Difference Equation Model Formulation

As discussed in Chapter 3 of the research, the MAMSTEP-BC model was formulated utilizing Equations (1)-(10), (14)-(28), and (29)-(40). The first change needed to modify the previous MAMSTEP-BC model to allow for moving targets is to develop new parameters allowing us to calculate the different locations of moving targets. For this chapter, it is assumed that each target has a known initial location, direction, and fixed velocity. To calculate these values, we must first define a constant velocity for each target i. Next, the current parameters x_i^{tgt} and y_i^{tgt} must be modified to include an index for time as well. Next, parameters for the final location (aim point) of each target must be created for both the x- and y-components. The final parameters needed to modify this model include x- and y-components for the direction of travel for each target. These new parameters are represented in the new model as shown below.

Additional Parameters

- v_i^{tgt}: constant velocity of enemy target i.
- x_{it}^{tgt}: x-component of the location for enemy target i at each time step t.

- y_{it}^{tgt}: y-component of the location for enemy target i at each time step t.
- x_i^{aim}: location of enemy target i's aim point in the x-direction.
- y_i^{aim}: location of enemy target i's aim point in the y-direction.
- x_i^{dir}: unit vector x-component of the fixed direction of target i.
- y_i^{dir}: unit vector y-component of the fixed direction of target i.

The values for $(x_{i0}^{tgt}, y_{i0}^{tgt})$ are randomly generated to place the initial targets within the red enemy region of Figure 2. The aim point (x_i^{aim}, y_i^{aim}) for each target is randomly generated along the leftmost bound of the blue region in Figure 2. Leveraging these values, the values for (x_i^{dir}, y_i^{dir}) are calculated using the following unit vector equation:

$$x_i^{dir} = \frac{x_i^{aim} - x_{i0}^{tgt}}{\sqrt{(x_{i0}^{tgt} - x_i^{aim})^2 + (y_{i0}^{tgt} - y_i^{aim})^2}} \tag{52}$$

$$y_i^{dir} = \frac{y_i^{aim} - y_{i0}^{tgt}}{\sqrt{(x_{i0}^{tgt} - x_i^{aim})^2 + (y_{i0}^{tgt} - y_i^{aim})^2}}. \tag{53}$$

Once the direction of each target is determined, the locations for the remaining values of t can be calculated using the following equations:

$$x_{it}^{tgt} = x_{i0}^{tgt} + v_i^{tgt} \delta t x_i^{dir}, \ \forall \ i \in N, t \in T \setminus \{0\}, \tag{54}$$

$$y_{it}^{tgt} = y_{i0}^{tgt} + v_i^{tgt} \delta t y_i^{dir}, \ \forall \ i \in N, t \in T \setminus \{0\}. \tag{55}$$

Once the time indexed location of each target is calculated, Equation (9) from the original MAMSTEP-BC model can be modified to incorporate this new parameter utilizing the following additional constraint:

$$(x_{it}^{tgt} - x_{kt})^2 + (y_{it}^{tgt} - y_{kt})^2 \leq r_k^2 \phi_{ikt} + M(1 - \phi_{ikt}), \ \forall \, i \in I, k \in K, t \in T. \qquad (56)$$

5.2 Phase 3 Testing, Results, and Analysis

The finalized model within Phase 3 of this research was tested using the same system referenced in both Phase 1 and 2 of this work. In Phase 3, we only consider one model, but that model is tested with 30 randomly generated instances utilizing the same seed used in previous phases. Each instance includes two randomly generated aircraft and targets with random fixed directions. The aircraft used the same common value parameters as defined by Phase 1 of the research in Table 1.

The summary statistics for these 30 trials can be seen in Table 4. The model successfully found an optimal solution in all 30 iterations. Furthermore, this model ran approximately 5 seconds faster than the MAMSTEP-BC model with stationary targets in Phase 1 on average (see comparison in Table 4).

Table 4. Phase 3 Results

Model	**Avg. Time (sec)**	**SD (sec)**
MAMSTEP-BC (moving)	30.03	8.52
MAMSTEP-BC (stationary)	35.03	12.77

One of the randomized instances was plotted to show its positions, velocities, and accelerations to further analyze our results. Unlike the results of Phase 1, the example plotted from Phase 3 indicates a much smoother engagement by both aircraft, without the need for implementing an alternative approach as done in Phase 2. Figure 15 shows the movement by each aircraft as well as the target's movement until the time of engagement. Each aircraft begins their respective maneuvering by increasing their airspeed and decreasing it again to make turns to align towards the target. Each of the turns shown in Figure 15 align with a velocity decrease in Figure 16 as well as a jump in acceleration magnitude indicated by the spikes in Figure 17, with none of the spikes going over 2.5 g's of acceleration and each aircraft having a maximum of 5 maneuvers.

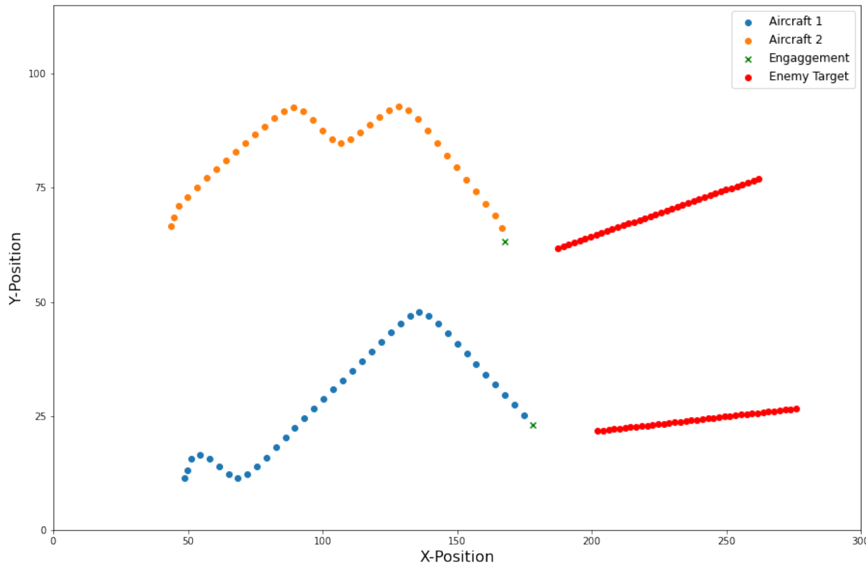

Figure 15. Phase 3 Position Example

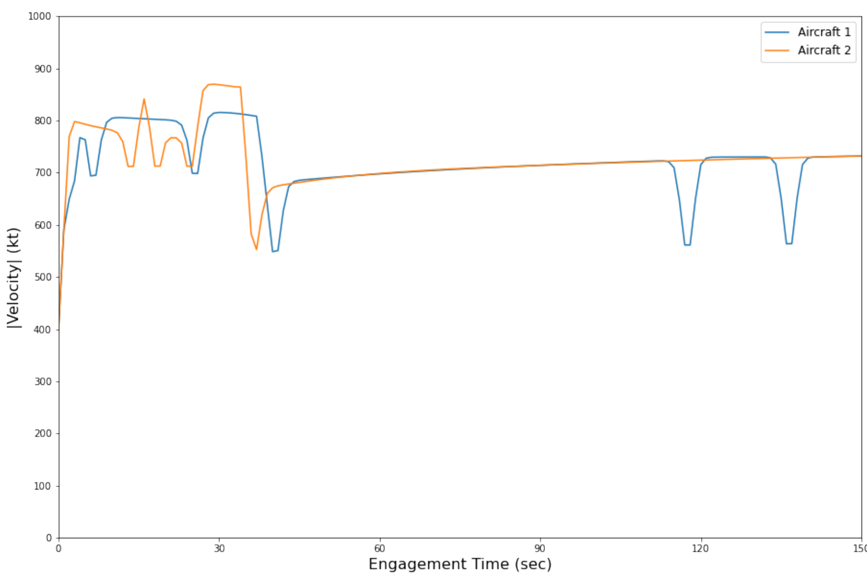

Figure 16. Phase 3 Velocity Example

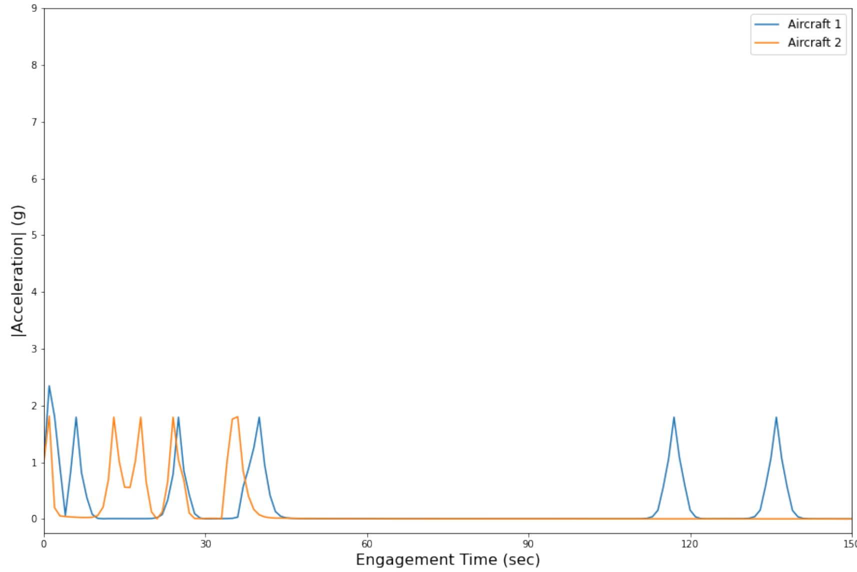

Figure 17. Phase 3 Acceleration Example

5.3 Phase 3 Conclusions

Overall, the results of Phase 3 signify the feasibility of the formulated model when testing multiple aircraft with multiple moving targets. The aircraft successfully engage each target optimally in all 30 trials, doing so in less time on average than the model designed for stationary targets. (This result comports with intuition because the moving cruise missiles are attacking targets in the general proximity of the aircrafts' original locations.) Additionally, the instance plotted in Figures 15-17 indicate the ability of the model to produce a feasible flight path that does not contain grossly erratic maneuvers that are infeasible for a human pilot to perform (or endure). Such a result does not preclude potential improvements to the prescribed flight paths; as is visible in Figure 15, both aircraft perform a sequence of two consecutive turns (right, then left) that are arguably unnecessary. In general, success of Phase 3 testing validates the ability to transform the model in Phase 1 to include moving targets without diminishing the fidelity of the model or increasing its computational complexity.

VI. Conclusion

This research created, validated, and tested alternative mathematical models to maneuver multiple aircraft to effectively neutralize positively identified long-range cruise missiles. As discussed in Chapter I, this research sought to address three key research questions by determining: (1) which modeling techniques to implement in terms of model fidelity and computational complexity to best represent the identified problem; (2) how neutralization of the enemy targets can best be integrated into the model; and (3) which components of the model best yield a sufficiently granular representation of aircraft maneuvering for practical use.

Phase 1 of the research formulated model variants through a series of difference equations and with alternative means of constraining velocity, acceleration, and their relationship with respect to an aircraft's $v-N$ flight envelope, minimizing an objective function calculating the combined time to neutralizing all targets within a specified range of an aircraft. In testing, simple box constraints on velocity and acceleration maintained high fidelity models while reducing the relative computational complexity, answering the first two research questions. The commercial optimization software solved this model with stationary targets quickly and provided meaningful results but sometimes provided an alternative optima with an infeasible flight path. These results motivated Phase 2 of the research, which found that smoothing the flight path through a feasible region reduction method validated the models' ability to quickly solve a the problem while providing an optimal flight path that is feasible to maneuver. In Phase 3, the model from Phase 1 was successfully adapted to consider moving targets with a fixed aimpoint and constant velocity. This combined model maintained a high level of granularity and was able to solve the model faster on average than the instances in Phase 1 of testing, successfully answering the third research question.

6.1 Additional Contributions

This research contributes to the field by demonstrating an innovative approach to optimization through difference equations that balances the need for a highly granular aircraft maneuvering model while maintaining quick computation times that allow for many iterations. This research can be extended by conducting more testing scenarios including more aircraft and targets. Such a study will inform limitations of the model as the instance size continues to grow. If the model fails to maintain reasonable run times for many aircraft with many targets, it may be necessary to decompose the problem into subsets of aircraft and solve the associated subproblems in parallel.

Lastly, the model could be further built upon to consider the orientation of the aircraft with respect to the target when engaging. This addition would allow for the model to maximize the probability of destroying a target based off where the target is approached from and where it can be hit.

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704–0188

1. REPORT DATE *(DD–MM–YYYY)*
24–03–2022

2. REPORT TYPE
Master's Thesis

3. DATES COVERED *(From — To)*
August 2020 — March 2022

4. TITLE AND SUBTITLE
Optimal Aircraft Maneuvering Models for Cruise Missile Engagement: a Modeling and Computational Study

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
LaDuke, Izaiah G., 2d Lt, USAF

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Air Force Institute of Technology
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WPAFB OH 45433-7765

8. PERFORMING ORGANIZATION REPORT NUMBER
AFIT-ENS-MS-22-M-142

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
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(937) 904-6539

10. SPONSOR/MONITOR'S ACRONYM(S)
SDPE

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION / AVAILABILITY STATEMENT
DISTRIBUTION STATEMENT A. APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.

13. SUPPLEMENTARY NOTES

14. ABSTRACT
Given the increased threat and proliferation of adversary military capabilities, this research seeks to develop reasonably accurate and computationally tractable models to optimally maneuver aircraft to intercept cruise missile attacks. The research leveraged mathematical programming to model the problem, informed by constraints representing a system of (temporal) difference equations. The research began by comparing six models having alternative representations of velocity and acceleration constraints while analyzing situations with stationary targets. The Multiple Aircraft, Multiple Stationary Target Engagement Problem with Box Constraint Bounds (MAMSTEP-BC) Model yielded superior overall performance and was further analyzed through alternative mathematical programming model enhancements to create feasible flight profiles, in terms of leveraging a valid sequence of maneuvers. Lastly, the MAMSTEP-BC model was modified to maneuver aircraft to engage moving targets.

This model proved effective with multiple aircraft and multiple targets when optimizing the time needed to engage. MAMSTEP-BC was able to maintain a high-level of granularity by accounting for aircraft and pilot limitations while managing to generate optimal solutions quickly for both stationary and moving targets.

15. SUBJECT TERMS
linear programming, vehicle routing problem, difference equations, temporal networks

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			Dr. Brian J. Lunday, AFIT/ENS
					19b. TELEPHONE NUMBER *(include area code)*
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