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ANALYTICAL MODELS FOR COUNTERFIRE WARFARE OF FIELD ARTILLERY

by

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**ANALYTICAL MODELS FOR COUNTERFIRE WARFARE OF FIELD
ARTILLERY**

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Submitted in partial fulfillment of the
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ABSTRACT

On the Korean Peninsula, many field-artillery units of the Republic of Korea and the Democratic People's Republic of Korea confront each other on opposite sides of the border. An artillery counter-fire engagement between the two sides is a realistic scenario. A commander of a field artillery unit must decide quite often when to move and/or when and where to fire. Typically, such decisions depend on the commander's intuition and experience without any formal analysis. This may lead to wrong tactical decisions. Thus, there is need for reliable battlefield analysis based on data such as unit size, the capability of intelligence-gathering and targeting assets, and the precision and lethality of fire. The main purpose of this thesis is to develop analytical models for counterfire warfare of field artillery using stochastic duels. We develop discrete-time Markov chain and continuous-time Markov chain models and apply them to study the effect of defensive and offensive scooting on the outcome of the exchange under various operational assumptions. The models, and the analyses they facilitate, can aid artillery field-commanders in making better, more informed, tactical decisions on the battlefield, in accordance with the given scenario. Our analytical models can be one component in the process in which the artillery develops its capability and effectiveness operationally and tactically.

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EXECUTIVE SUMMARY

In a combat situation, the commander of a field artillery unit has to decide when to move and/or when and where to fire. Even though field artillery tactics like ‘shoot-and-scoot’ were developed decades ago, such decisions depend quite often on the commander’s intuition, which could sometimes work, but it might also be unreasonable in many cases. A commander needs to operate rationally, according to and following an analysis of the battlefield situation. Such analysis needs to rely on data regarding Blue and Red, such as unit size, the capability of surveillance assets and counter-battery radar, and the precision and lethality of fire.

The main purpose of this thesis is to develop analytical models for counterfire warfare of field artillery using stochastic duels. The models are intended to analyze critical factors in artillery operations such as intelligence-gathering and targeting, firing, and scooting tactics to enhance survivability.

The models, and the analyses they facilitate, can aid artillery field-commanders in making better, more informed, tactical decisions on the battlefield, in accordance with the given scenario. The models may provide data-based tactical insights that will enable decision capabilities beyond the qualitative analysis that currently exists. Our analytical models can be one component in the process in which the artillery develops its capability and effectiveness operationally and tactically.

To this end, we develop three Markov chain models and apply them to study defensive- and offensive-scooting tactics under various operational assumptions.

The first model is a one-on-one model using a discrete-time Markov chain. Through this model, we can see how lethality, detection and target acquisition capabilities, and scooting tactics relate to and influence each other. This basic model provides some initial insights. For example, if one side has a high detection and lethality capabilities, it is to its advantage to scoot frequently. Frequent scooting will enhance its win probability. The on-on-one model is the basis for the next two models: Two-on-one and many-on-many models.

The two-on-one model is also a discrete-time Markov chain. This model is derived from the one-on-one model, and gives insights into the battle situation when the Blue force is numerically superior. The analysis for this model is similar to that of the one-on-one model. Of course, a slight difference exists in analysis results because there is a difference in the number of units. The main takeaways from the two-on-one model are (1) the advantage of Blue's offensive scooting is reduced compared to the one-on-one model; (2) the maximum decrease in win probability for the two-on-one model is larger than that for the one-on-one model; and (3) the advantage of Blue's defensive scooting is not as great as in the one-on-one model.

The third model is the most realistic as it captures a many-on-many combat situation. This model is a continuous-time Markov chain. Therefore, it also captures the element of the time, and simplifies the way we define transitions. This model provides a tool to analyze the combat situation for a larger number of units, different from the one-on-one model and the two-on-one model. The currently developed model shows several different results from the previous two models: (1) the Blue's win probability always decreases as the rate of Blue's offensive scooting increases. This means that the more frequently Blue scoots following a firing, the less Blue is likely to win. This surprising result is quite robust to additional, more extreme, choices of parameters, and should be subject to further explorations in future research; (2) When it comes to Blue's defensive scooting, the results are insensitive to Red's scooting strategy. That is, the conditions of detection capability and lethality are satisfied, Blue's win probability decreases regardless of the Red's scooting policy. This also requires further research.

These three models can be used as analysis tools to support field-artillery commanders in making tactical decisions on the battlefield, which are based on data about the combat capabilities and tactics of friendly and enemy forces.

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I. INTRODUCTION

A. BACKGROUND

On the Korean Peninsula, many field-artillery troops of the Republic of Korea (ROK) and the Democratic People's Republic of Korea (DPRK) confront each other on opposite sides of the border. The ROK Army considers the artillery force of the DPRK, including long-range artillery, a severe threat, and the ROK has continuously increased its forces to respond to that perceived threat. Besides standard artillery pieces, these forces also include assets such as unmanned aerial vehicles (UAV) for reconnaissance, counter-battery radar, multiple launch rocket system (MLRS), etc. Against that backdrop, an artillery counter-fire engagement between units of the two countries is therefore a realistic scenario.

The main task of field artillery units is to support maneuver units with indirect firepower. Before the maneuver forces are employed, the supporting artillery units must neutralize the enemy's artillery. To achieve this, the artillery units must defeat the enemy's artillery while maintaining their combat power. In this thesis, we use artillery units of the ROK as Blue and DPRK as Red.

In a real combat situation, the commander of a field artillery unit has to decide when to move and/or when and where to fire. Even though field artillery tactics like 'shoot-and-scoot' were developed decades ago, such decisions depend quite often on the commander's intuition, which could sometimes work, but it might also be unreasonable in many cases. Therefore, a commander needs to operate rationally, according to, and following an analysis of the battlefield situation. Such analysis needs to rely on data, regarding Blue and Red, including the number of troops, the capability of surveillance assets and counter-battery radar, and the precision and lethality of fire.

The goal of this thesis is to develop analytical models that will aid artillery field-commanders in making better tactical decisions on the battlefield, in accordance with the given scenario. These models provide tactical insights that enable decision capabilities beyond the qualitative analysis that currently exists. Also, better insights about the

dynamics of artillery exchanges can help the ROK better plan and execute such operations. Our analytical model can be one component in the process in which the ROK artillery develops its capability and effectiveness operationally and tactically.

After we build the model, we change several parameters, which allows us to see how the battle results vary. Also, through these models, commanders will be able to gain insights into whether more surveillance assets are needed, whether counter-battery radar's detection rates need to be higher, or whether they need to maneuver more often.

B. GENERAL SCENARIO

Two artillery forces—Blue and Red—are engaged in a fire exchange. The artillery pieces are self-propelled howitzers, so they have high mobility and firing ability. In addition, they can detect each other using intelligence-gathering and targeting assets.

We separate the intelligence-gathering and targeting assets into two groups. The first group comprises surveillance assets such as UAVs or ground reconnaissance forces that search for targets. The second group includes counter-battery radars, which can only target the enemy when the enemy fires. The target information from the surveillance assets has some errors and noise; if either force shoots using this information, its fire will be rather inaccurate. On the other hand, information from counter-battery radars is more precise, and so, fire based on radar information is more accurate than fire based on surveillance. The difference between *accurate fire* and *inaccurate fire* affects the level of lethality: the lethality of accurate fire is higher than that of inaccurate fire.

Furthermore, the detection probabilities for the two types of intelligence-gathering and targeting assets are different. Surveillance assets like UAVs or reconnaissance forces take time to locate the enemy; therefore, their detection rate is low, and at any given time, the detection probability is relatively low as well. The detection probability of counter-battery radar is almost instantaneous and is much higher than that of surveillance assets; however, a counter-battery radar works only when the enemy fires. Therefore, firing reveals the existence of the shooting location with some probability because both sides have counter-battery radars. We assume that intelligence-gathering and targeting assets are not affected by the artillery fire because they belong to a higher level of command.

Therefore, the surveillance assets search the enemy's location continuously, and the counter-battery radar units are always available.

When the enemy's location is detected by intelligence-gathering and targeting assets, this information (such as coordinates of the target location) is sent directly to the artillery unit. Blue and Red only can fire based on this information; they cannot fire absent such information.

Also, Blue and Red can scoot when they decide to. Scooting means Blue or Red leaves their current locations, and this action has pros and cons. If they scoot, the enemy loses information about their location. Therefore, the enemy cannot shoot, but they lose the time needed for shooting. We assume that Blue and Red can only scoot after they shoot or are being shot at. They do not scoot without shooting or being shot at.

C. SCOPE AND OBJECTIVES

The primary purpose of this thesis is to develop analytical models for counterfire warfare of field artillery using stochastic duels. Specifically, our main point is to create models that analyze the battle, focusing on critical factors in artillery operations such as intelligence-gathering and targeting, firing, and scooting to ensure survivability.

When it comes to the capability of intelligence-gathering and targeting assets and lethality of firing, the commander's decision is hardly reflected because these are set for each side to some extent. Therefore, after developing the model, we analyze the scooting policies according to the capability of intelligence-gathering and targeting assets and lethality of firing. Scooting policies can vary according to the commander's decision, and the results of these analyses can give the artillery commander insights into the battle situation.

Chapter II presents a one-on-one model using a discrete-time Markov chain with the scenario and assumptions. Chapter III discusses analyses for the one-on-one model. In the chapter, we conduct sensitivity analysis to Blue's scooting policies and find common features for the change to a win probability of Blue. We develop a two-on-one model in Chapter IV and conduct analyses for that model in Chapter V. The two-on-one model gives

insight into scooting policies when the Blue force is numerically superior. Chapter VI presents a many-on-many model using a continuous-time Markov chain, and analyses for the many-on-many model are shown in Chapter VII. We conclude our work in Chapter VIII with a summary and recommendations for potential future work.

D. LITERATURE REVIEW

Guzik (1988) analyzes the optimal moving strategy of artillery to support maneuver troops using continuous-time Markov chains. To determine the optimal moving strategy, Guzik derives a long run proportion of the time that artillery units remain states which are capable of supporting maneuver units, and the expected time that artillery remains available to support maneuver troops before being attacked. Guzik says that the number of shots fired from a position should not be the reason for determining a displacement, but that the imminent attack, time in position, and the need to maintain support for the supported unit should be the basis for the decision. Therefore, he argues that the moving tactic according to the number of shots needs to be changed.

Kress (1991) presents a two-on-one stochastic duel model. Kress takes into account the situation where two Blue units attack one Red unit which do not move. Blue units' mission is to occupy Red's position on or before a specific time. Kress discusses the best engagement tactics for Blue and Red, respectively. Also, Kress (1992) proposes a many-on-many stochastic duel model in which the situation is set to a mountain battle. In this situation, the attacking units are forced to approach the enemy in one column as they move along a mountain road. Therefore, only the front unit can attack and be exposed to the enemy's fire. On the other hand, the defending units are static during the battle and they can fire simultaneously at the exposed attacking unit in defensive positions. Finally, Kress analyzes Red's win probability according to kill rate ratio and force size.

Duke (1996) proposes an analytical model to conduct sensitivity analysis of effectiveness for Crusaders, which were required as new artillery systems at the time of his study, by changing the parameters using discrete-time Markov chains. Duke uses the time until the enemy kills Crusaders and the number of fire missions executed and lost by Crusaders as measures of effectiveness.

Shim (2017) analyzes the shoot-and-scoot tactics, especially the moving policy of one of the two sides, using continuous-time Markov chains. Shim focuses on one scenario: Blue knows Red's position, and Blue can move. However, Red is stationary and does not know Blue's position at first. As time goes by, the lethality of Red increases unless Blue does not move. Shim tries to get the best moving policy to maximize the benefit and minimize the risk. He recommends moving every 15 minutes on average and says that frequent moving during the early stages of the battle is the best option for Blue.

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II. ONE-ON-ONE MODEL

In this chapter, we present a discrete-time Markov model for a combat situation in which two artillery pieces are engaged in a one-on-one artillery counterfire battle. This simple model sets the stage for the more realistic engagements presented later.

A. THE SCENARIO

There are only two self-propelled artillery pieces: Blue and Red. They have high mobility and firing capabilities. They are engaged in a fire exchange.

Both Blue and Red have two kinds of intelligence-gathering and targeting assets; one is surveillance assets like UAVs and ground reconnaissance forces that search for targets. The other targeting assets are counter-battery radars that can detect enemy fire. Surveillance assets can detect the enemy regardless of whether the enemy fires or not, but these assets have a low detection probability. Although counter-battery radars have a high detection probability, they can only detect the enemy's position when the enemy fires. Surveillance assets and counter-battery radars belong to higher level of command; therefore, they are not affected by the fire of the artillery pieces and can observe the enemy continuously. Also, information from surveillance assets and counter-battery radars is sent to artillery pieces directly. Both artillery pieces can only fire when either surveillance or radar information regarding targeting becomes available. They cannot shoot without such information.

There are some errors and noise in the target information obtained from surveillance assets. Therefore, fire using this information may result in inaccurate fire. On the other hand, the information from counter-battery radars is more accurate, and so firing based on this information is more accurate than firing based on the information from surveillance assets. Obviously, the lethality of accurate fire is higher than that of inaccurate fire.

If commanders of either Blue or Red want to avoid being detected and attacked by the enemy, they can evade it by *scooting* from their current positions. When one side scoots, its enemy loses information, and it cannot fire. However, the scooting artillery pieces also

lose time for shooting. We assume that Blue and Red can scoot only when they shoot or are being shot at. If a piece scoots after firing at the enemy, we call it *offensive scooting*. If it scoots after being shot at, we call it *defensive scooting*.

B. DISCRETE-TIME (ABSORBING) MARKOV CHAIN

Before we define the states for the discrete-time Markov chain (DTMC), we define the possible states in which Blue and/or Red can be. We list these states in Table 1.

Table 1. Possible states for Blue and/or Red

States		Definitions
I	Idle	- Neither side is shooting. - The artillery piece could be 'scooting or just stationary' in some positions. - If Blue or Red scoots, they become idle.
F_1	Inaccurate Fire	- Firing at targets acquired by surveillance assets.
F_2	Accurate Fire	- Firing at targets acquired by counter-battery radars.
K	Killed	- Blue or Red is killed by enemy's inaccurate or accurate fire.

Now, we define the states of the DTMC for the one-on-one model:

$$\{(I, I), (I, F_1), (F_1, I), (F_1, F_1), (F_1, F_2), (F_2, F_1), (F_2, F_2), (K, *), (*, K), (K, K)\}$$

The first state in the parenthesis is the state of Blue and the second state in the parenthesis is the state of Red. Table 2 gives detailed descriptions of each state. States $(K, *)$, $(*, K)$, and (K, K) are absorbing states. States (I, F_2) and (F_2, I) do not exist because a counter-battery radar can only detect targets that fire. Therefore, it is impossible to transit to these states.

Table 2. Descriptions of each state of DTMC for the one-on-one model

States (B, R)	Descriptions
(I, I)	- Both Blue and Red are idle.
(I, F_1)	- Blue is idle, but Red shoots inaccurate fire at Blue.
(F_1, I)	- Blue shoots inaccurate fire at Red, but Red is idle.
(F_1, F_1)	- Both Blue and Red shoot inaccurate fire at each other.
(F_1, F_2)	- Blue shoots inaccurate fire at Red. - Red shoots accurate fire at Blue.
(F_2, F_1)	- Blue shoots accurate fire at Red. - Red shoots inaccurate fire at Blue.
(F_2, F_2)	- Both Blue and Red shoot accurate fire at each other.
$(K, *)$	- Blue is killed by Red. - Absorbing probability of this state is win probability of Red.
$(*, K)$	- Red is killed by Blue. - Absorbing probability of this state is win probability of Blue.
(K, K)	- Both Blue and Red are killed simultaneously. - Absorbing probability of this state is the probability of ending in a tie.

1. Parameters

In Table 3, we define the probability parameters for the model.

Table 3. Definitions and notations of probability parameters

Parameters		Definitions
Lethality	β_1	Blue killing Red using inaccurate fire
	β_2	Blue killing Red using accurate fire
	ρ_1	Red killing Blue using inaccurate fire
	ρ_2	Red killing Blue using accurate fire
Detection	δ_1	Blue detecting Red using surveillance
	δ_2	Blue detecting Red using counter-battery radars
	ε_1	Red detecting Blue using surveillance

Parameters		Definitions
	ϵ_2	Red detecting Blue using counter-battery radars
Scooting	b	Blue executes offensive scooting
	c	Blue executes defensive scooting
	α	Red executes offensive scooting
	γ	Red executes defensive scooting

2. General Assumptions

These are the assumptions that apply to the one-on-one model.

- Blue and Red start the duel in state (I, I) .
- Blue and Red have unlimited shells and fuel. Therefore, they can scoot and fire until one side or both sides are killed. The case of mutual annihilation exists because Blue and Red can fire at each other in the same time period, and thus, they can be killed simultaneously.
- Intelligence-gathering and targeting assets (surveillance assets and counter-battery radars) are always operating in their operation areas and they are not affected by the fire of the artillery pieces. In other words, they try to detect the enemy continuously.
- Firing can only occur following a detection by a surveillance assets or counter-battery radar.
- All shots are independent.
- Detecting by counter-battery radar and detecting by surveillance assets are independent. In case the target is detected by both surveillance assets and counter-battery radar at the same time, firing will be accurate.

- When Blue or Red detects the enemy, they must shoot. However, if they detect the enemy and are being shot at simultaneously, they can fire back or scoot.
- The probabilities of kill, detection, and scooting are independent of one another.
- The *measure of effectiveness (MOE)* is the win probability of Blue. The win probability of Blue is the ‘Probability of absorbing in state $(*, K)$ ’.

The detailed situations and assumptions for transitions are shown in Appendix A.

3. Transition Matrix for the Model

Based on the situations and assumptions for transitions just described, a transition matrix is shown in Figure 1.

	$(K, *)$	$(*, K)$	(K, K)	(I, I)	(I, F_1)	(F_1, I)	(F_1, F_1)	(F_1, F_2)	(F_2, F_1)	(F_2, F_2)
$(K, *)$	1	0	0	0	0	0	0	0	0	0
$(*, K)$	0	1	0	0	0	0	0	0	0	0
(K, K)	0	0	1	0	0	0	0	0	0	0
(I, I)	0	0	0	$\frac{(1-\varepsilon_1)}{(1-\delta_1)}$	$\varepsilon_1(1-\delta_1)$	$(1-\varepsilon_1)\delta_1$	$\varepsilon_1\delta_1$	0	0	0
(I, F_1)	ρ_1	0	0	$\frac{(1-\rho_1)}{(1-\delta_1)}(ac + (1-\alpha)c + \alpha(1-c))$	$\frac{(1-\rho_1)}{(1-\delta_1)}(1-\delta_1)$ $\frac{(1-\delta_2)}{(1-\alpha)(1-c)}$	0	$\frac{(1-\rho_1)}{(1-\alpha)(1-c)}\delta_1(1-\delta_2)$	0	$\frac{(1-\rho_1)\delta_2}{(1-\alpha)(1-c)}$	0
(F_1, I)	0	β_1	0	$\frac{(1-\beta_1)}{(1-\varepsilon_1)}(by + (1-\gamma)b + \gamma(1-b))$	0	$\frac{(1-\beta_1)}{(1-\varepsilon_1)}\varepsilon_1(1-\varepsilon_2)$ $\frac{(1-\delta_2)}{(1-\gamma)(1-b)}$	$\frac{(1-\beta_1)}{(1-\gamma)(1-b)}\varepsilon_1(1-\varepsilon_2)$	$\frac{(1-\beta_1)\varepsilon_2}{(1-\gamma)(1-b)}$	0	0
(F_1, F_1)	$\rho_1(1-\beta_1)$	$\beta_1(1-\rho_1)$	$\rho_1\beta_1$	$\frac{(1-\beta_1)}{(1-\rho_1)}(yc + (1-\gamma)c + \gamma(1-c))$	0	0	$\frac{(1-\beta_1)}{(1-\rho_1)}(1-\varepsilon_2)$ $\frac{(1-\delta_2)}{(1-\gamma)(1-c)}$	$\frac{(1-\beta_1)}{(1-\gamma)(1-c)}\varepsilon_2(1-\delta_2)$	$\frac{(1-\beta_1)}{(1-\gamma)(1-c)}(1-\varepsilon_2)\delta_2$	$\frac{(1-\beta_1)}{(1-\gamma)(1-c)}\varepsilon_2\delta_2$
(F_1, F_2)	$\rho_2(1-\beta_1)$	$\beta_1(1-\rho_2)$	$\rho_2\beta_1$	$\frac{(1-\beta_1)}{(1-\rho_2)}(yc + (1-\gamma)c + \gamma(1-c))$	0	0	0	$\frac{(1-\beta_1)}{(1-\gamma)(1-c)}(1-\delta_2)$	0	$\frac{(1-\beta_1)}{(1-\gamma)(1-c)}\delta_2$
(F_2, F_1)	$\rho_1(1-\beta_2)$	$\beta_2(1-\rho_1)$	$\rho_1\beta_2$	$\frac{(1-\beta_2)}{(1-\rho_1)}(yc + (1-\gamma)c + \gamma(1-c))$	0	0	0	0	$\frac{(1-\beta_2)}{(1-\gamma)(1-c)}(1-\varepsilon_2)$	$\frac{(1-\beta_2)}{(1-\gamma)(1-c)}\varepsilon_2$
(F_2, F_2)	$\rho_2(1-\beta_2)$	$\beta_2(1-\rho_2)$	$\rho_2\beta_2$	$\frac{(1-\beta_2)}{(1-\rho_2)}(yc + (1-\gamma)c + \gamma(1-c))$	0	0	0	0	0	$\frac{(1-\beta_2)}{(1-\gamma)(1-c)}(1-\rho_2)$

Figure 1. Transition matrix for the one-on-one model

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III. ANALYSES FOR THE ONE-ON-ONE MODEL

In this chapter, we implement the model developed in Chapter II.

A. SENSITIVITY ANALYSIS TO BLUE'S SCOOTING POLICY

For a given set of parameters values for lethality, detection, and Red's scooting policy, we study the sensitivity of win probability of Blue to changes in Blue's scooting policies.

1. Fixed Parameters

We fix the parameters for the lethality of inaccurate fire (β_1, ρ_1) and accurate fire (β_2, ρ_2), and for detection by surveillance assets (δ_1, ε_1) and counter-battery radars (δ_2, ε_2). We also fix the values for Red's scooting parameters (α, γ). Altogether, we analyze 100 combinations of parameter values (henceforth referred to as cases) that reflect various effectiveness scenarios for the two sides, as shown in Figure 2.

We select five combinations of lethality parameters, five combinations of detection parameters, and four combinations of Red's scooting policy – a total of 100 cases. For each case, Blue's win probability is computed as a function of Blue's scooting policy.

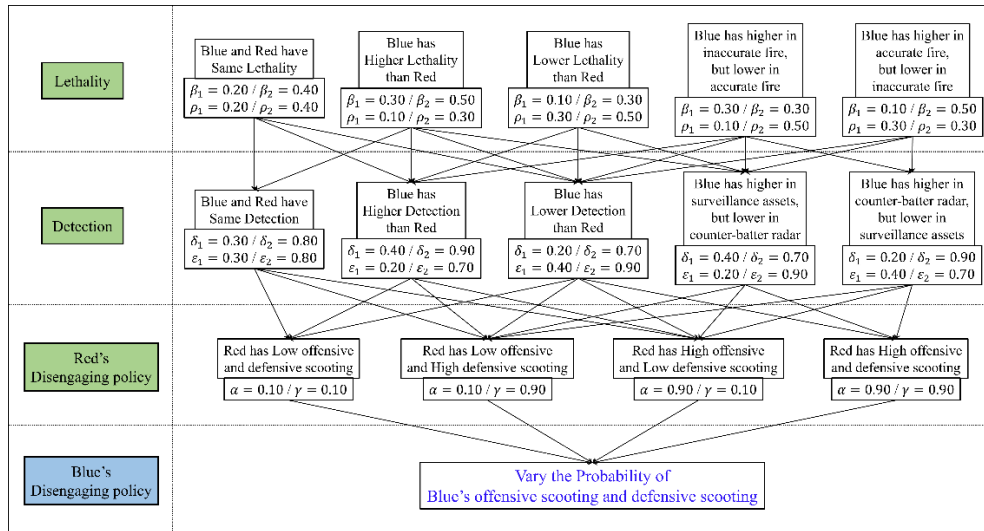


Figure 2. The parameter values for the one-on-one analysis

2. Sensitivity to the Probability of Blue's *offensive scooting* (b)

We vary the probability of Blue's offensive scooting (b) from 0 (never executing offensive scooting) to 1 (always executing offensive scooting after firing). For most of the cases, as the probability of Blue's offensive scooting increases, the probability that Blue wins increases too in almost linear fashion (91 out of the 100 cases). We focus on two types of special cases: One is when the win probability of Blue actually decreases as the probability of Blue's offensive scooting increases, and the other is when the win probability of Blue significantly increases as the probability of Blue's offensive scooting increases.

a. Cases when the Win Probability of Blue Decreases as the Probability of Blue's offensive scooting (b) Increases

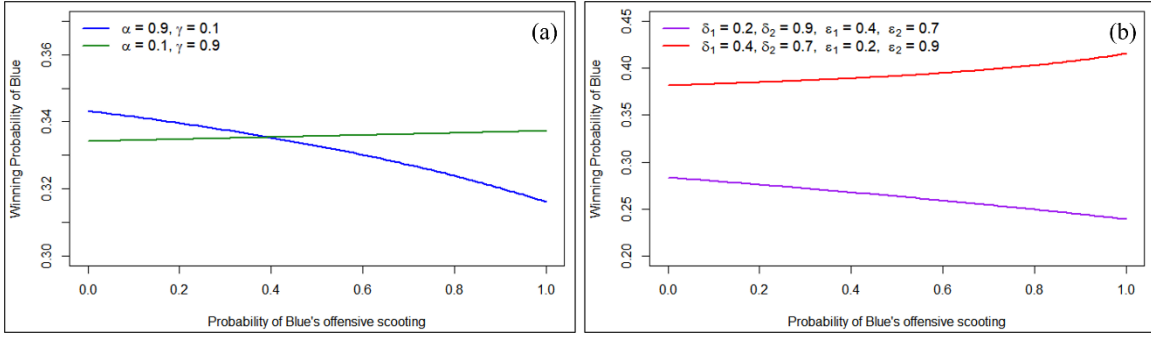
Only in nine of the 100 cases does Blue's win probability decrease as Blue's offensive scooting probability increases. The decrease of the win probability between no scooting and always scooting is small: 0.03 to 0.04. The common features in these cases are as follows.

- The killing probability of Blue's inaccurate fire (β_1) is lower than that of Red's inaccurate fire (ρ_1) and the killing probability of Blue's accurate fire (β_2) is higher than that of Red's accurate fire (ρ_2).
- The detection probability of Blue's surveillance assets (δ_1) is lower than that of Red's surveillance assets (ε_1).
- Red has a high probability of offensive scooting (α) and low probability of defensive scooting (γ).

Because the capability of Blue's surveillance assets is low, it is difficult to detect the location of Red unless it fires. Since the lethality of Blue's inaccurate fire is low, if Blue scoots frequently after shooting, it is hard for it to destroy Red even if Blue's surveillance assets identify the enemy. Furthermore, Blue cannot fully utilize its radar capabilities and the lethality of accurate fire because Red scoots frequently after shooting. Blue's increased offensive scooting seems to reinforce this. In addition, even when Red's defensive scooting is infrequent, if Blue scoots frequently after shooting, Blue misses

opportunities to fire at the enemy. In such cases, it is difficult to annihilate Red even through accurate fire.

The effect of Red's scouting intensities on Blue's win probability is demonstrated in Figure 3a. If offensive scouting probability (α) is high and defensive scouting probability (γ) is small, then the win probability of Blue decreases as Blue's offensive scouting intensity increases. If the reverse is true, then the win probability of Blue increases as Blue's offensive scouting intensity increases. In Figure 3b, we see the win probability of Blue as a function of detection probability. If detection probability of Blue's surveillance assets (δ_1) is lower than that of Red's (ε_1), then the win probability of Blue decreases as Blue's offensive scouting intensity increases. If the reverse is true, then the win probability of Blue increases as Blue's offensive scouting intensity increases.



(a) $\beta_1 = 0.1, \beta_2 = 0.5, \rho_1 = 0.3, \rho_2 = 0.3, \delta_1 = 0.3, \delta_2 = 0.8, \varepsilon_1 = 0.3, \varepsilon_2 = 0.8, c = 0.1$

(b) $\beta_1 = 0.1, \beta_2 = 0.5, \rho_1 = 0.3, \rho_2 = 0.3, \alpha = 0.9, \gamma = 0.1, c = 0.1$

Figure 3. Blue's win probability for two scenarios of scouting policies of Red (a) and two scenarios of detection probabilities (b)

Interestingly, if the win probability is already high, the Blue's win probability increases as the offensive scouting probability increases. Thus, this can be interpreted as the win probability of Blue is possibly increased by ensuring survival through frequent movement after shooting if Blue's overall combat effectiveness is high.

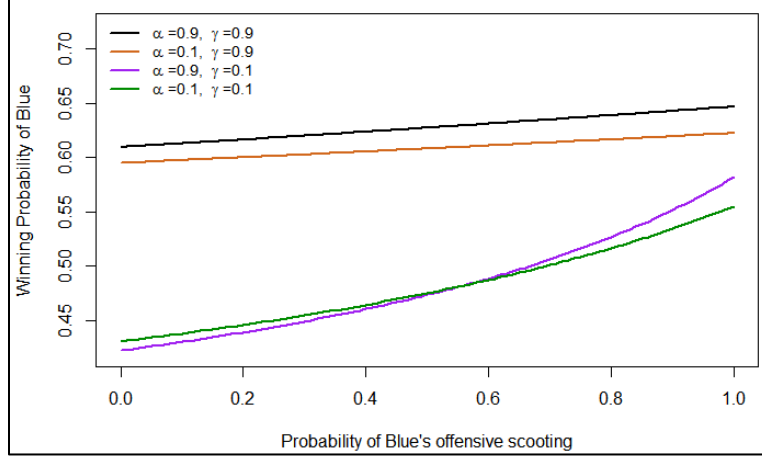
b. Cases when the Win Probability of Blue Significantly Increases as the Probability of Blue's offensive scooting (b) Increases

In 18 cases among the 100, Blue's win probability increases significantly as the probability of Blue's offensive scooting increases. In these cases, the difference in win probability between "no-scoot" and "always scoot" ranged between 0.1 and 0.25. The common features of these cases are as follows.

- The detection probability of Blue's surveillance assets (δ_1) is higher than that of Red's surveillance assets (ε_1).
- Red has low probability of defensive scooting (γ).

If the detection probability of Blue's surveillance assets is high, then Red's location can be identified quickly, even if the enemy does not fire, and this information can be used to fire at Red. Also, since the probability of Red's defensive scooting is low, there is a high probability that it will stay in its position even if it is under attack. In such a situation, if Blue's offensive scooting probability increases, it avoids Red's counter-battery radar detection and uses its surveillance assets to continuously find the enemy's location and fire, thus increasing Blue's victory probability.

In Figure 4, we see the change in Blue's win probability for four scenarios of Red's scooting policies. If the detection probability of Blue's surveillance assets is high and Red's defensive scooting probability is low, Blue's win probability increases significantly as Blue's offensive scooting probability increases, regardless of the probability of Red's offensive scooting.



$$\beta_1 = 0.2, \beta_2 = 0.4, \rho_1 = 0.2, \rho_2 = 0.4, \delta_1 = 0.4, \delta_2 = 0.7, \varepsilon_1 = 0.2, \varepsilon_2 = 0.9, c = 0.1$$

Figure 4. Blue's win probability for four scenarios of Red's scouting policies

3. Sensitivity to the Probability of Blue's *defensive scouting* (c)

We vary the probability of Blue's defensive scouting (c) from 0 (never executing defensive scouting) to 1.0 (always executing defensive scouting). Analyzing the 100 cases, we observe that as Blue's defensive scouting probability increases, Blue's win probability decreases in 57 cases and increases in 43 cases. This means that neither a decrease nor an increase in the probability of win is dominant for defensive scouting. In the following subsections, we focus on two situations: One is when the win probability of Blue significantly increases as Blue's defensive scouting probability increases, and the other is the case when the win probability decreases significantly. A change is considered significant when Blue's win probability changes by 0.05 or more when we move the probability of Blue's defensive scouting from 0 to 1.

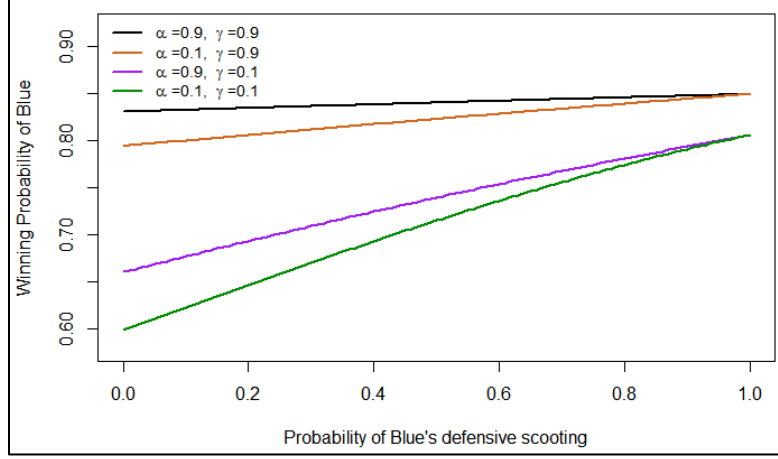
a. Cases when the Win Probability of Blue Significantly Increases as the Probability of Blue's defensive scouting (c) Increases

There are 18 cases in which the win probability of Blue significantly increases as the probability of Blue's defensive scouting increases. In these cases, the increase in win probability between "no-scoot" and "always scoot" ranges between 0.05 and 0.2. The common features of these cases are as follows.

- The killing probability of Blue's inaccurate fire (β_1) is higher than that of Red's inaccurate fire (ρ_1).
- The detection probability of Blue's surveillance assets (δ_1) is greater than that of Red's surveillance assets (ε_1).
- Red has a low probability of defensive scooting (γ).

We can easily imagine this situation. Blue's surveillance assets have a high probability of detecting the enemy, and the information from surveillance assets and precision of Blue are high, so it has high lethality in inaccurate fire. Also, in a situation where Red does not move frequently even when attacked, the more frequently Blue moves when attacked, the more Blue can detect and destroy Red before Red detects Blue again.

Figure 5 displays Blue's win probability for four scenarios of Red's scooting policy. If the detection probability of Blue's surveillance assets and the lethality of Blue's inaccurate fire are high, Blue's win probability increases significantly as Blue's defensive scooting probability increases. In addition, regardless of Red's scooting policy, as Blue's defensive scooting probability increases, Blue's win probability also increases. However, the lower the Red's defensive scooting probability, the greater the advantage Blue gains from frequent defensive scooting.



$$\beta_1 = 0.3, \beta_2 = 0.3, \rho_1 = 0.1, \rho_2 = 0.5, \delta_1 = 0.4, \delta_2 = 0.7, \varepsilon_1 = 0.2, \varepsilon_2 = 0.9, b = 0.9$$

Figure 5. Blue's win probability for four scenarios of Red's scouting policies

b. Cases when the Win Probability of Blue Significantly Decreases as the Probability of Blue's defensive scouting (c) Increases

There are 25 cases in which the win probability of Blue significantly decreases as the probability of Blue's defensive scouting increases. In these cases, the decrease in win probability between “no-scoot” and “always scoot” ranges between 0.05 and 0.25. The common features of these cases are as follows.

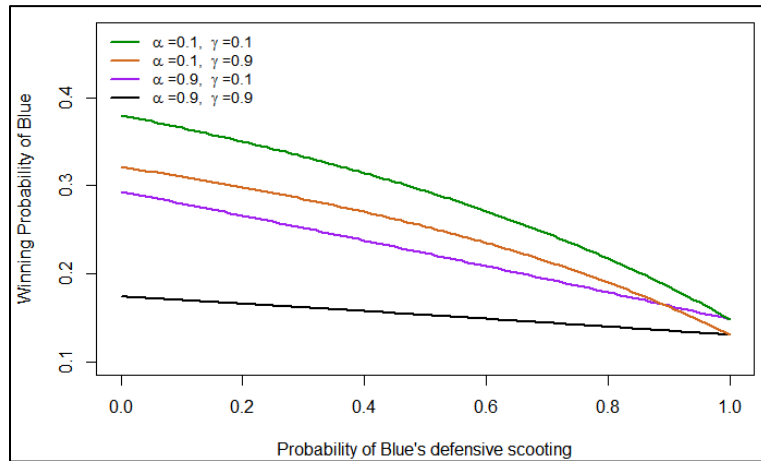
- The killing probability of Blue's inaccurate fire (β_1) is lower than that of Red's inaccurate fire (ρ_1).
- The detection probability of Blue's surveillance assets (δ_1) is less than or equal to that of Red's surveillance assets (ε_1).
- Red has a low probability of offensive scouting (α).

The low killing probability of Blue's inaccurate fire and the low detection probability of Blue's surveillance assets mean that it is difficult for Blue to locate Red unless Red attacks. In this situation, if there is a high probability of Blue scouting after being attacked, Blue has to find the enemy again more often. While Blue searches for the enemy, it could be attacked by Red because Red's surveillance assets have a higher

detection probability than those of Blue. Also, since Red's offensive scouting probability is low, Red seldom moves after firing and has a high probability of continuing to fire at Blue's position. Therefore, as Blue's defensive scouting probability increases, Blue's win probability decreases.

This result differs from our initial intuitive thought that the more often Blue moves when attacked, the higher the probability that Blue wins the battle. This is due to the limitation of the killing probability of this model. In this model, damage or reduction of combat power is not caused by enemy fire. In other words, the killing probability is limited to either dead or alive. If the concept of reducing combat power is reflected, the results may be different.

Figure 6 displays Blue's win probability for four scenarios of Red's scouting policies. If the detection probability of Blue's surveillance assets and the lethality of Blue's inaccurate fire are low, Blue's win probability decreases significantly as Blue's defensive scouting probability increases, regardless of Red's scouting policy. However, the lower the Red's offensive scouting probability, the greater the disadvantage of frequent defensive scouting by Blue.



$$\beta_1 = 0.1, \beta_2 = 0.5, \rho_1 = 0.3, \rho_2 = 0.3, \delta_1 = 0.2, \delta_2 = 0.9, \varepsilon_1 = 0.4, \varepsilon_2 = 0.7, b = 0.1$$

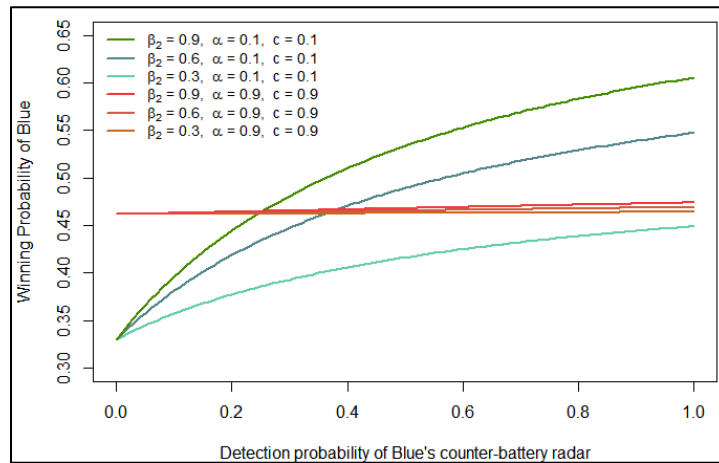
Figure 6. Blue's win probability for four scenarios of Red's scouting policies

B. SENSITIVITY OF BLUE'S WIN PROBABILITY TO THE PERFORMANCE OF COUNTER-BATTERY RADAR

In the sensitivity analysis to Blue's scooting policy, the killing probability of inaccurate fire (β_1, ρ_1) and the detection probability of surveillance assets (δ_1, ε_1) have a great influence on the results. On the other hand, the killing probability of accurate fire (β_2, ρ_2) and the detection probability of counter-battery radar (δ_2, ε_2) does not seem to have a significant effect in this one-on-one model.

In order to find the effect of counter-battery radar and accurate fire on Blue's win probability, we change the killing probability of Blue's accurate fire (β_2) and the detection probability of Blue's counter-battery radar (δ_2) while fixing the other lethality and detection parameters ($\beta_1, \rho_1, \rho_2, \delta_1, \varepsilon_1, \varepsilon_2$), and check the win probability of Blue.

Figure 7 shows an example that demonstrates the effect of counter-battery radar. Only when the probability of Blue's defensive scooting (c) and Red's offensive scooting (α) are low, does the win probability of Blue significantly increase as the detection probability of Blue's counter-battery radar increases. Otherwise, Blue's win probability only slightly increases. If Blue does not move frequently after being shot and Red does not move frequently after shooting, Blue can obtain location information about Red from the counter-battery radar and fire based on this information. Therefore, this is a natural result.



$$\beta_1 = 0.2, \rho_1 = 0.2, \rho_2 = 0.4, \delta_1 = 0.1, \varepsilon_1 = 0.1, \varepsilon_2 = 0.5, \gamma = 0.1, b = 0.1$$

Figure 7. Graph of sensitivity analysis to β_2 and δ_2

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IV. TWO-ON-ONE MODEL

In this chapter, we present a discrete-time Markov model for a combat situation in which two artillery pieces are engaged in two-on-one artillery counterfire battle. This model is expanded from the one-on-one model described in Chapter II.

A. THE SCENARIO

The scenario for the two-on-one model is similar to the one-on-one model. Three self-propelled artillery pieces—two for Blue and one for Red—are engaged in a fire exchange. The conditions of intelligence-gathering and targeting assets and scooting policies are the same as in the one-on-one model. Hence, the two-on-one model gives insights into the battle situation when the Blue force is numerically superior.

B. DISCRETE-TIME MARKOV CHAIN FOR THE TWO-ON-ONE MODEL

The possible states for Blue and Red and the detailed descriptions of each state are provided in Table 4. Each state is defined by three entries. There are 25 possible states (triplets) in the resulting state space. The first and second entries in a state are related to Blue, and the third entry relates to Red. The absorbing states are $(K, K, *)$, $(*, *, K)$, and (K, K, K) .

Red's shot is applied to the first Blue in parenthesis. That is, state (I, I, F_1) means Red fires at the first Blue in state I. Also, state (F_1, I, F_1) means Red fires at the first Blue, which is in state F_1 . The first Blue and the second Blue in parenthesis are interchangeable. In other words, if the current state is state (F_1, F_1, F_1) and attacked Blue (the first one) decides to scoot but the second one does not scoot, then the next state is (F_1, I, F_1) .

We assume that both Blue shooters share information from surveillance assets and counter-battery radars, so they always shoot the same type of fire when both Blue shoot. Therefore, states $(F_1, F_2, *)$ and $(F_2, F_1, *)$ do not exist.

In addition, states (I, I, F_2) and (K, I, F_2) do not exist because a counter-battery radar can only detect targets that fire. Therefore, it is impossible to transit to these states.

Table 4. Descriptions of each state of DTMC for two-on-one model

States (B, B, R)	Descriptions
(I, I, I)	- All three artillery pieces are idle.
(I, I, F_1)	- Two Blue are idle, but Red shoots inaccurate fire at one of Blue.
(F_1, I, I)	- One Blue shoots inaccurate fire at Red - The other Blue and Red are idle.
(F_1, I, F_1)	- One Blue and one Red shoot inaccurate fire at each other. - The other Blue is idle.
(F_1, I, F_2)	- One Blue shoots inaccurate fire at Red, but the other Blue is idle. - Red shoots accurate fire at one Blue.
(F_1, F_1, I)	- Two Blue shoot inaccurate fire at Red, but Red is idle.
(F_1, F_1, F_1)	- Two Blue and one Red shoot inaccurate fire at each other.
(F_1, F_1, F_2)	- Two Blue shoot inaccurate fire at Red. - Red shoots accurate fire at one Blue.
(F_2, I, I)	- One Blue shoots accurate fire at Red. - The other Blue and Red are idle.
(F_2, I, F_1)	- One Blue shoots accurate fire at Red, but the other Blue is idle. - Red shoots inaccurate fire at one of Blue.
(F_2, I, F_2)	- One Blue shoots accurate fire at Red, but the other Blue is idle. - Red shoots accurate fire at one Blue.
(F_2, F_2, I)	- Two Blue shoot accurate fire at Red, but Red is idle.
(F_2, F_2, F_1)	- Two Blue shoot accurate fire at Red. - Red shoots inaccurate fire at one Blue.
(F_2, F_2, F_2)	- Two Blue and one Red shoot accurate fire at each other.
(K, I, I)	- One Blue is killed. - Remaining Blue and Red are idle.
(K, I, F_1)	- One Blue is killed. - Remaining Blue is idle, but Red shoots inaccurate fire at Blue.
(K, F_1, I)	- One Blue is killed. - Remaining Blue shoots inaccurate fire at Red, but Red is idle.
(K, F_1, F_1)	- One Blue is killed. - Remaining Blue and Red shoot inaccurate fire at each other.
(K, F_1, F_2)	- One Blue is killed. - Remaining Blue shoots inaccurate fire at Red - Red shoots accurate fire at Blue.
(K, F_2, I)	- One Blue is killed. - Remaining Blue shoots accurate fire at Red, but Red is idle.
(K, F_2, F_1)	- One Blue is killed. - Remaining Blue shoots accurate fire at Red. - Red shoots inaccurate fire at Blue.
(K, F_2, F_2)	- One Blue is killed. - Remaining Blue and Red shoot accurate fire at each other.
$(K, K, *)$	- Two Blue are killed by Red. - Absorbing probability of this state is win probability of Red.
$(*, *, K)$	- Red is killed by Blue. - Absorbing probability of this state is win probability of Blue.
(K, K, K)	- Both Blue and Red are killed simultaneously. - Absorbing probability of this state is the probability of ending in a tie.

1. General Assumptions

The assumptions that apply to the two-on-one model are shown below.

- Blue and Red start the battle in state (I, I, I) .
- Blue and Red have unlimited shells and fuel. Therefore, they can scoot and fire until one side is killed or both sides are killed. The case of mutual annihilation exists because Blue and Red can fire at each other in the same time period, so they are able to be killed simultaneously.
- Intelligence-gathering and targeting assets (surveillance assets and counter-battery radars) are always operating in their operation areas and they are not affected by the fire of the artillery pieces. In other words, they try to detect the enemy continuously.
- Firing can only occur following a detection by surveillance assets or counter-battery radar.
- Red can only engage one at the time and a scooting Blue is not radar detectable.
- All shots are independent.
- Each Blue decides to execute offensive scooting independently when both Blue fire simultaneously.
- Only the Blue piece that is being shot at can execute defensive scooting.
- When a piece is both shooting and being shot at, scooting is derived from defensive scooting.
- Both Blue shooters share information from surveillance assets and counter-battery radars. Therefore, they always shoot the same type of fire when both Blue shoot.

- Detecting by counter-battery radar and detecting by surveillance assets' are independent of one another. In case the target is detected by both surveillance assets and counter-battery radar simultaneously, firing will be accurate.
- When Blue or Red detects the enemy, they must shoot. However, if they detect the enemy and are being shot at simultaneously, they can either fire back or scoot.
- The probability of kill, detection, and scooting are independent.
- The *measure of effectiveness (MOE)* is the win probability of Blue. Win probability of Blue is 'Probability of absorbing in state $(*,*, K)$ '.

2. Transition Matrix for the Two-on-One Model

(A, B, R)	$(K,K,*)$	$(*,*,K)$	(K,K,K)	(L,L)	$(L,F1)$	$(F1,L)$	$(F1,F1)$	$(F1,F2)$	$(F1,F1,F1)$	$(F1,F1,F2)$	$(F2,L)$	$(F2,F1)$	$(F2,F2)$	$(F2,F1,F1)$	$(F2,F1,F2)$	(K,L)	$(K,L,F1)$	$(K,F1,L)$	$(K,F1,F1)$	$(K,F1,F2)$	$(K,F2,L)$	$(K,F2,F1)$	$(K,F2,F2)$
$(K,K,*)$	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
$(*,*,K)$	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
(K,K,K)	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
(L,L)	0	0	0			0	0				0	0	0	0	0	0	0	0	0	0	0	0	
$(L,F1)$	0	0	0						0	0					0	0						0	
$(F1,L)$	0		0		0						0	0	0	0	0	0		0	0	0	0	0	
$(F1,F1)$	0		0																				
$(F1,F2)$	0		0					0	0	0					0								
$(F1,F1,F1)$	0		0																				
$(F1,F1,F2)$	0		0																				
$(F2,L)$	0		0		0	0	0				0	0	0	0	0	0	0	0	0	0	0	0	
$(F2,F1)$	0		0																				
$(F2,F2)$	0		0																				
$(F2,F1,F1)$	0		0																				
$(F2,F1,F2)$	0		0												0								
$(F2,L,F1)$	0		0		0	0	0																
$(F2,L,F2)$	0		0																				
$(F2,F2,F1)$	0		0		0	0	0	0	0	0					0								
$(F2,F2,F2)$	0		0		0	0	0	0	0	0					0								
(K,L)	0	0	0		0	0	0	0	0	0	0	0	0	0	0	0				0	0	0	
$(K,L,F1)$	0		0		0	0	0	0	0	0	0	0	0	0	0	0				0	0	0	
$(K,F1,L)$	0				0	0	0	0	0	0	0	0	0	0	0	0					0	0	
$(K,F1,F1)$	0				0	0	0	0	0	0	0	0	0	0	0	0							
$(K,F1,F2)$	0				0	0	0	0	0	0	0	0	0	0	0	0					0	0	
$(K,F2,L)$	0		0			0	0	0	0	0	0	0	0	0	0	0				0	0	0	
$(K,F2,F1)$	0				0	0	0	0	0	0	0	0	0	0	0	0				0	0	0	
$(K,F2,F2)$	0				0	0	0	0	0	0	0	0	0	0	0	0				0	0	0	

Figure 8. Transition matrix for the two-on-one model

V. ANALYSES FOR THE TWO-ON-ONE MODEL

In this chapter, we study the sensitivity of the win probability of Blue to changes to the parameters in the two-on-one model.

A. SENSITIVITY ANALYSIS TO BLUE’S SCOOTING POLICY

For a given set of parameters values for lethality, detection, and Red’s scouting policy, we study the sensitivity of the win probability of Blue to the changes in Blue’s scouting policies.

1. Fixed Parameters

We fix the parameters for the lethality of inaccurate fire (β_1, ρ_1) and accurate fire (β_2, ρ_2), and for detection by surveillance assets (δ_1, ε_1) and counter-battery radars (δ_2, ε_2). We also fix the values for Red’s scouting parameters (α, γ). As shown in Figure 2, we analyze 100 cases that reflect various effectiveness scenarios for the two sides. For each case, Blue’s win probability is computed as a function of Blue’s scouting policy.

2. Sensitivity to the Probability of Blue’s *offensive scouting* (b)

We vary the probability of Blue’s offensive scouting (b) from 0 (never executing offensive scouting) to 1 (always executing offensive scouting after firing). For most of the cases, as the probability of Blue’s offensive scouting increases, the Blue’s win probability increases (75 out of the 100 cases). Compared to the one-on-one model, the advantage of offensive scouting is reduced (from 91 to 75 out of 100 cases), but there are still more cases in which the Blue’s win probability increases than in cases where it decreases.

We focus on two types of special cases: One is when the win probability of Blue significantly increases as the probability of Blue’s offensive scouting increases, and the other is when the win probability of Blue significantly decreases as the probability of Blue’s offensive scouting increases. A change is considered significant when Blue’s win probability changes by 0.05 or more when we move the probability of Blue’s offensive scouting from 0 to 1.

a. Cases when the Win Probability of Blue Significantly Increases as the Probability of Blue's offensive scouting (b) Increases

There are 18 cases in which the win probability of Blue increases significantly as the probability of Blue's offensive scouting increases. In these cases, the increase in win probability between "no-scoot" and "always scoot" ranges between 0.05 and 0.15. The common features of these cases are as follows.

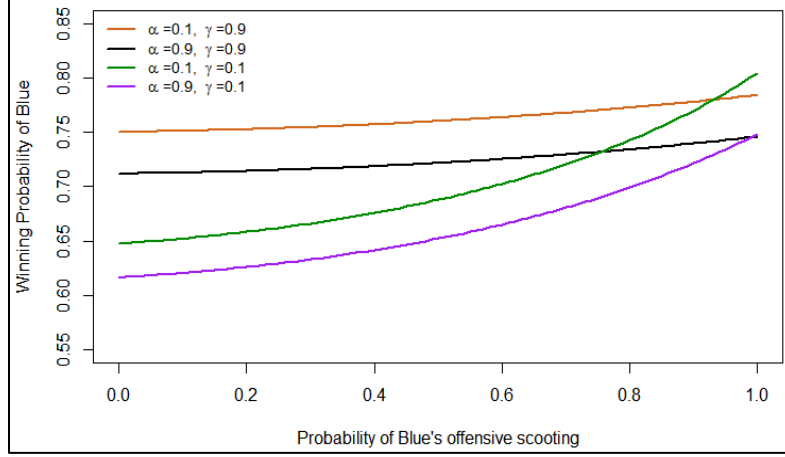
- The killing probability of Blue's inaccurate fire (β_1) and accurate fire (β_2) is less than or equal to that of Red's inaccurate fire (ρ_1) and accurate fire (ρ_2), respectively.
- The detection probability of Blue's surveillance assets (δ_1) is higher than that of Red's surveillance assets (ε_1).
- Red has a low probability of defensive scouting (γ).

Blue can quickly locate Red without the enemy firing if Blue's surveillance assets have a high detection probability. Also, Red is likely to stay in its position even if attacked because the probability of Red's defensive scouting is low. In this situation, if the probability of Blue's offensive scouting increases, it avoids Red's counter-battery radar detection and uses its own surveillance assets to continuously find the enemy's position. Therefore, the win probability of Blue increases.

However, the first condition (the lethality of Blue is lower than that of Red) is not easy to understand at first glance. The reason why is that if the lethality of Blue is high, the increase is not significant since the win probability of Blue is already over 0.9, regardless of the values of other parameters.

Figure 9 shows how the win probability of Blue increases, as a function of offensive scouting, with respect to the four scenarios of Red's scouting policy: (1) low offensive scouting (α), high defensive scouting (γ), (2) high offensive scouting (α), low defensive scouting (γ), (3) low offensive scouting (α), low defensive scouting (γ), and (4) high offensive scouting (α), low defensive scouting (γ). If the detection probability of Blue's surveillance assets is high, and the probability of Red's defensive scouting is low, then

Blue's win probability increases significantly as the probability of Blue's offensive scouting increases, regardless of the probability of Red's offensive scouting.



$$\beta_1 = 0.1, \beta_2 = 0.3, \rho_1 = 0.3, \rho_2 = 0.5, \delta_1 = 0.4, \delta_2 = 0.7, \varepsilon_1 = 0.2, \varepsilon_2 = 0.9, c = 0.9$$

Figure 9. Blue's win probability for four scenarios of Red's scouting policies

b. Cases when the Win Probability of Blue Significantly Decreases as the Probability of Blue's offensive scouting (b) Increases

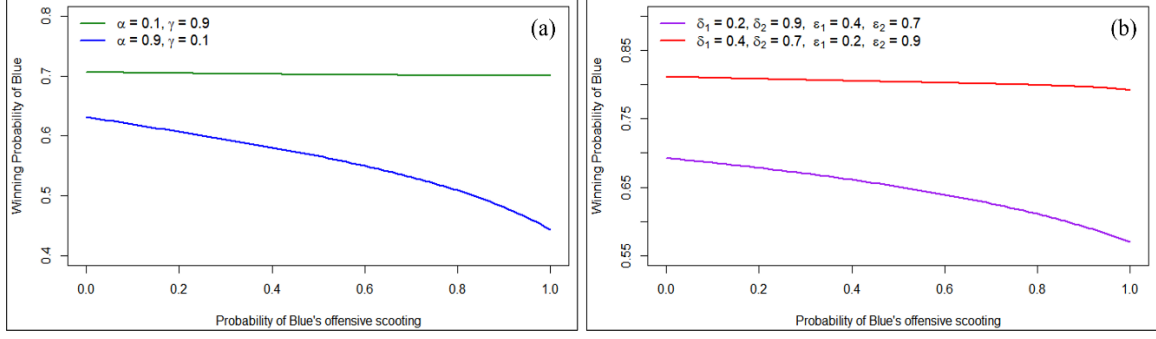
Compared to the one-on-one model, as the probability of Blue's offensive scouting increases, the win probability of Blue decreases more often in the two-on-one model, but most decrease cases are below 0.01, which is not significant. However, in terms of maximum decrease, it is 0.04 in the one-on-one model, whereas it is 0.19 in this model. We check the cases when the decrease in win probability of Blue is 0.05 or more. The common features in these cases are as follows.

- The killing probability of Blue's inaccurate fire (β_1) is lower than that of Red's inaccurate fire (ρ_1).
- The detection probability of Blue's surveillance assets (δ_1) is lower than that of Red's surveillance assets (ε_1).

- Red has high probability of offensive scooting (α) and low probability of defensive scooting (γ).

Due to the low detection probability of Blue's surveillance assets, it is difficult to locate Red without the enemy's firing. If Blue frequently executes offensive scooting, it is hard to annihilate the enemy even if it detects Red because of the low killing probability of its inaccurate fire. In addition, Blue does not take full advantage of its radar capabilities and accurate fire if it scoots frequently after shooting, and it seems that the high probability of Red's offensive scooting intensifies this. Also, even if the probability of Red's defensive scooting is low, Blue misses the opportunity to fire at the enemy because it executes offensive scooting after firing.

The effect of Red's scooting intensities is demonstrated in Figure 10a. If offensive scooting probability (α) is high and defensive scooting probability (γ) is small, then the win probability of Blue significantly decreases as Blue's offensive scooting intensity increases. If the reverse is true, then the win probability of Blue slightly decreases as Blue's offensive scooting intensity increases. In Figure 10b, we see the change in Blue's win probability according to detection probability. If the detection probability of Blue's surveillance assets (δ_1) is lower than that of Red's (ε_1), then the win probability of Blue significantly decreases as Blue's offensive scooting intensity increases. If reverse is true, then the win probability of Blue decreases very little as Blue's offensive scooting intensity increases.



(a) $\beta_1 = 0.1, \beta_2 = 0.5, \rho_1 = 0.3, \rho_2 = 0.3, \delta_1 = 0.2, \delta_2 = 0.9, \varepsilon_1 = 0.4, \varepsilon_2 = 0.7, c = 0.9$

(b) $\beta_1 = 0.1, \beta_2 = 0.5, \rho_1 = 0.3, \rho_2 = 0.3, \alpha = 0.9, \gamma = 0.1, c = 0.1$

Figure 10. Blue's win probability as a function of scouting policies of Red (a) and detection probability (b)

The difference between the one-on-one model and the two-on-one model in these cases is the win probability of Blue. In the one-on-one model, when Blue's win probability decreases, the highest win probability is 0.28. However, in the two-on-one model, the lowest win probability in these cases is 0.40, and the highest win probability is 0.77. In the two-on-one model, the decrease in Blue's win probability according to the probability of Blue's offensive scouting is large because there is more room for a decrease.

3. Sensitivity to the Probability of Blue's *defensive scouting* (c)

We vary the probability of Blue's defensive scouting (c) from 0 (never executing defensive scouting) to 1.0 (always executing defensive scouting). Analyzing the 100 cases, we observe that as Blue's defensive scouting probability increases, Blue's win probability decreases in 57 cases and increases in 43 cases, the same as in the one-on-one model.

However, in all cases where the win probability increases, the increase is never higher than 0.05. That is, the advantage of Blue's defensive scouting is not as great as in the one-on-one model. This is because the win probability of Blue is close to 0.9 or higher when the conditions that the win probability significantly increases in the one-on-one model are met since Blue has one more unit than Red. That is, there is no room for a significant increase.

Therefore, we focus on one situation: Where the win probability of Blue significantly decreases as Blue's defensive scooting probability increases. A change is considered significant when Blue's win probability changes by 0.05 or more when the probability of Blue's defensive scooting is changed from 0 to 1.

a. Cases when the Win Probability of Blue Significantly Decreases as the Probability of Blue's defensive scooting (c) Increases

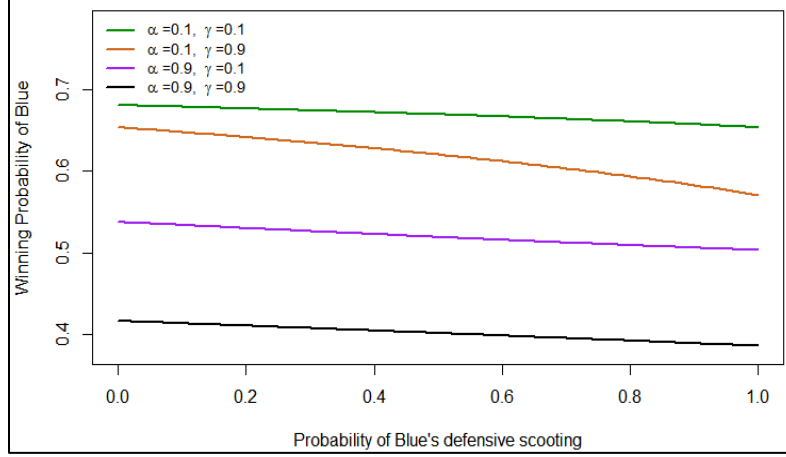
The common features of the cases in which the win probability of Blue decreases by 0.05 to 0.1 as the probability of Blue's defensive scooting increases are:

- The killing probability of Blue's inaccurate fire (β_1) is lower than that of Red's inaccurate fire (ρ_1).
- The detection probability of Blue's surveillance assets (δ_1) is less than or equal to that of Red's surveillance assets (ε_1).
- Red has low probability of offensive scooting (α) and high probability of defensive scooting (γ).

There is one different condition between the two-on-one model and the one-on-one model with respect to the conditions for the cases when the win probability of Blue significantly decreases as the probability of Blue's defensive scooting increases. In the two-on-one model, the condition of the high probability of Red's defensive scooting (γ) is added. This means that in the one-on-one model, Red's defensive scooting does not significantly affect the decrease in Blue's win probability as a function of Blue's defensive scooting, while it significantly affects the two-on-one model. The reason Red's defensive scooting affects the decrease in the two-on-one model seems to be that Red can dodge the concentrated fire of the two Blue units through defensive scooting.

Figure 11 shows the change in Blue's win probability for various scenarios of Red's scooting policies. If the detection probability of Blue's surveillance assets and the lethality of Blue's inaccurate fire are low, Blue's win probability decreases as Blue's defensive scooting probability increases, regardless of Red's scooting policy. However, when Red's

offensive scooting probability is low and defensive scooting probability is high, Blue's win probability decreases the most as the probability of Blue's defensive scooting increases.



$$\beta_1 = 0.1, \beta_2 = 0.3, \rho_1 = 0.3, \rho_2 = 0.5, \delta_1 = 0.2, \delta_2 = 0.9, \varepsilon_1 = 0.4, \varepsilon_2 = 0.7, b = 0.1$$

Figure 11. Blue's win probability for four scenarios of Red's scooting policies

B. SENSITIVITY OF BLUE'S WIN PROBABILITY TO THE PERFORMANCE OF COUNTER-BATTERY RADAR

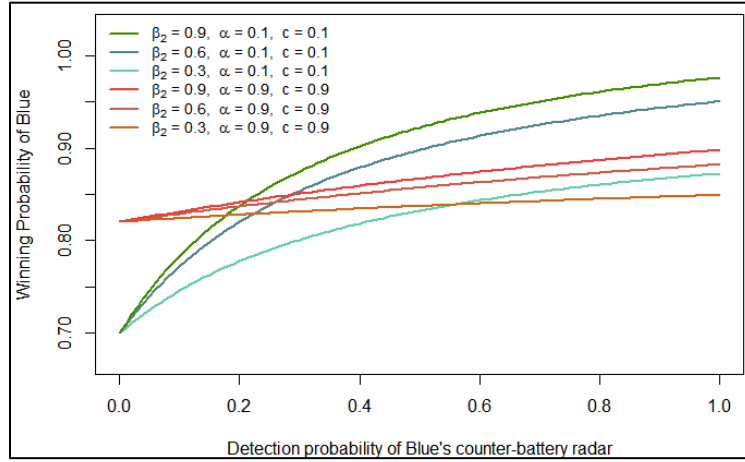
As with the sensitivity analysis result of the one-on-one model, the killing probability of accurate fire (β_2, ρ_2) and the detection probability of counter-battery radar (δ_2, ε_2) do not seem to have a significant effect on the two-on-one model.

In order to find the effect of counter-battery radar and accurate fire on Blue's win probability, we change the killing probability of Blue's accurate fire (β_2) and the detection probability of Blue's counter-battery radar (δ_2) while fixing the other lethality and detection parameters ($\beta_1, \rho_1, \rho_2, \delta_1, \varepsilon_1, \varepsilon_2$), and check the win probability of Blue.

Figure 12 displays an example that demonstrates the effect of counter-battery radar. As with the one-on-one model, only when the probability of Blue's defensive scooting (c) and Red's offensive scooting (α) are low does the win probability of Blue significantly increase as the detection probability of Blue's counter-battery radar increases. Otherwise, Blue's win probability only slightly increases. If Blue does not move frequently after being

shot at and Red does not move frequently after shooting, Blue can obtain location information about Red from the counter-battery radar and fire based on this information. Therefore, this is also a natural result like in the one-on-one model.

In the one-on-one model and two-on-one model, the reason why the killing probability of Blue's accurate fire (β_2) and the detection probability of Blue's counter-battery radar (δ_2) does not significantly affect Blue's win probability is the difference in characteristics of surveillance assets and counter-battery radar. That is, surveillance assets continue to find the enemy with a certain probability regardless of whether the enemy fires or not, whereas the counter-battery radar finds the location through enemy fire. That is, the counter-battery radar is effective only when these conditions are met: (1) the enemy fires, (2) counter-battery radar detects the enemy's fire, and (3) the enemy does not scoot after shooting. Therefore, the detection probability of the surveillance assets has a greater influence on Blue's win probability than that of counter-battery radar.



$$\beta_1 = 0.2, \rho_1 = 0.2, \rho_2 = 0.4, \delta_1 = 0.1, \varepsilon_1 = 0.1, \varepsilon_2 = 0.5, \gamma = 0.1, b = 0.1$$

Figure 12. Graph of sensitivity analysis to β_2 and δ_2

VI. MANY-ON-MANY MODEL

In this chapter, we present a continuous-time Markov model for a combat situation in which multiple artillery pieces, on each side, are engaged in artillery counterfire battle.

A. THE SCENARIO

There are initially $m > 1$ artillery pieces for Blue and $n > 1$ artillery pieces for Red. The two sides are engaged in counter-artillery battle. At any given time, each piece on a certain side is engaged in the same activity: firing, scooting or idling. For example, when Blue fires, all live pieces of Blue fire, and when Red scoots, all live pieces of Red scoot.

The battle ends when one side is annihilated, and the surviving side is the victor. The flow diagram of the states in the many-on-many battle is shown in Figure 13.

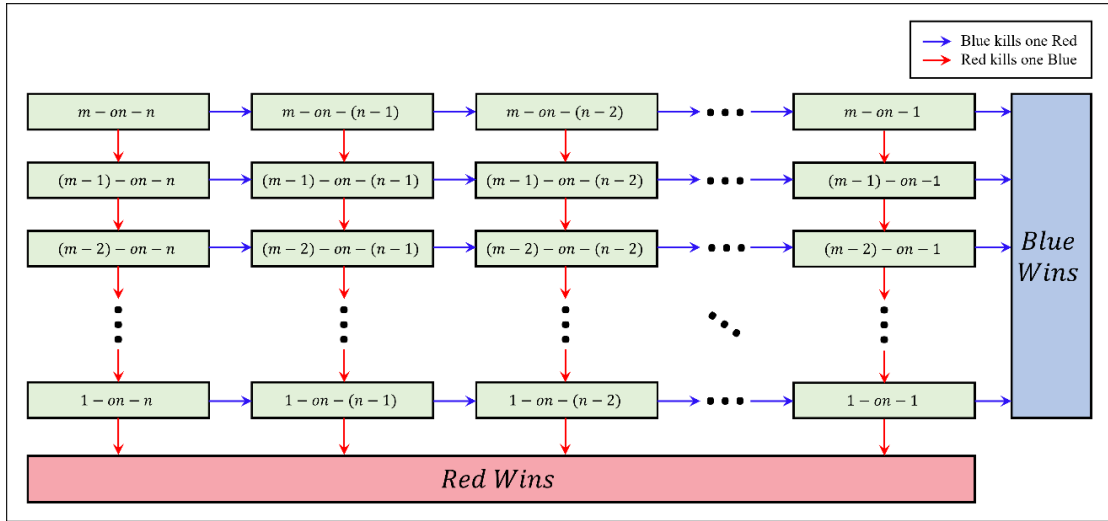


Figure 13. Flow diagram for the many-on-many battle

The targeting conditions (accurate and inaccurate fire) and targeting assets and scooting policies (offensive- and defensive scooting) are the same as in the one-on-one model.

B. CONTINUOUS-TIME (ABSORBING) MARKOV CHAIN

The possible states for Blue and/or Red are the same as in the one-on-one model, as shown in Table 1.

For the many-on-many model, we define a state $(i, x; j, y)$ where m, n are the number of live pieces on Blue and Red, respectively and x, y are the state of the Blue and Red units (I, F_1 , or F_2), respectively. The detailed descriptions of each state are given in Table 5.

Table 5. Descriptions of each state of Continuous-Time Markov Chain (CTMC) for many-on-many model

States	Descriptions
$(i, I; j, I)$	- Both i pieces of Blue and j pieces of Red are idle.
$(i, I; j, F_1)$	- i pieces of Blue are idle, but j pieces of Red shoot inaccurate fire at Blue.
$(i, F_1; j, I)$	- i pieces of Blue shoot inaccurate fire at Red, but j pieces of Red are idle.
$(i, F_1; j, F_1)$	- Both i pieces of Blue and j pieces of Red shoot inaccurate fire at each other.
$(i, F_1; j, F_2)$	- i pieces of Blue shoot inaccurate fire at Red. - j pieces of Red shoot accurate fire at Blue.
$(i, F_2; j, F_1)$	- i pieces of Blue shoot accurate fire at Red. - j pieces of Red shoot inaccurate fire at Blue.
$(i, F_2; j, F_2)$	- Both i pieces of Blue and j pieces of Red shoot accurate fire at each other.
$(K; *)$	- All Blue pieces are killed by Red. - Absorbing probability of this state is win probability of Red.
$(*; K)$	- All Red pieces are killed by Blue. - Absorbing probability of this state is win probability of Blue.

1. Parameters

We use the same parameters' notation as in the one-on-one model as shown in Table 3. However, in the CTMC model, these parameters are rates rather than probabilities.

Each Blue piece kills a Red piece at an exponential rate β_1, β_2 , for inaccurate- and accurate fire, respectively. A Red piece kills a Blue piece at an exponential rate ρ_1, ρ_2 , for inaccurate- and accurate fire, respectively.

Time to target Red by Blue is exponentially distributed with parameters δ_1, δ_2 , for surveillance assets and counter-battery radar, respectively. Time to target Blue by Red is exponentially distributed with parameters $\varepsilon_1, \varepsilon_2$, for surveillance assets and counter-battery radar, respectively.

Scooting is executed at exponential rates as well. The rates of Blue are b, c , for offensive scooting and defensive scooting, respectively, and the rates of Red are α, γ , for offensive scooting and defensive scooting, respectively. If at least one side scoots, both sides need to acquire their targets afresh.

2. General Assumptions

These are the assumptions that apply to the many-on-many model.

- Blue and Red start the battle in state $(m, I; n, I)$.
- Blue and Red have unlimited shells and fuel. Therefore, they can scoot and fire until all pieces of one side are killed.
- Intelligence-gathering and targeting assets (surveillance assets and counter-battery radars) are always operating in their operation areas and they are not affected by the fire of the artillery pieces. In other words, they try to detect the enemy continuously.
- Firing can occur only following a detection, and detection is always followed by firing.
- All inter-temporal times in the engagement (firing, detection, scooting) are independent of each other.
- The *measure of effectiveness (MOE)* is the win probability of Blue. Win probability of Blue is ‘Probability of absorbing in state $(*; K)$ ’.

3. Transitions Diagram for the Model

The transitions diagram for the many-on-many model is shown in Figure 14.

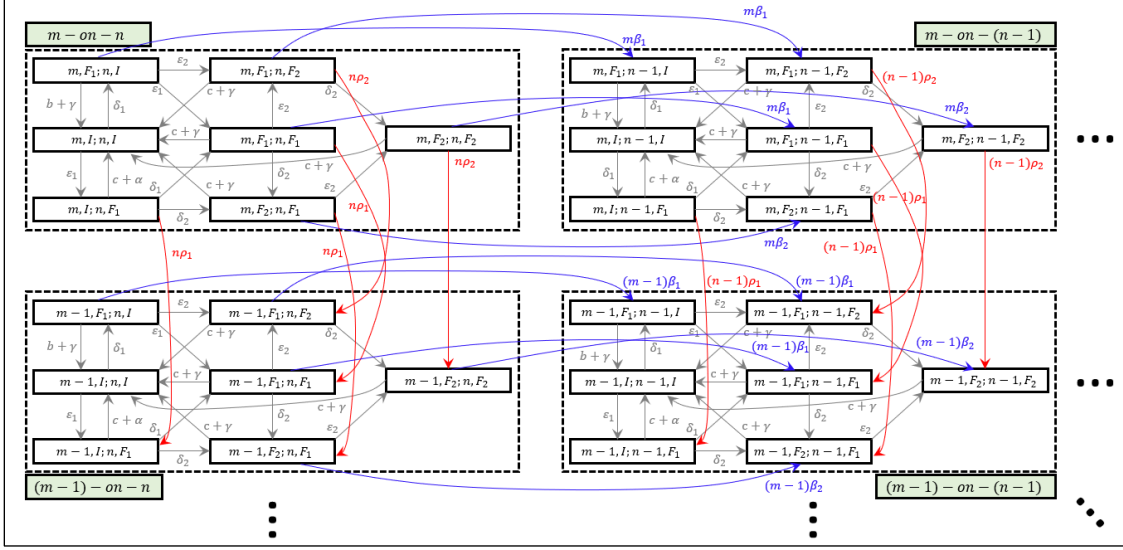


Figure 14. Transitions diagram for the many-on-many model

4. Sample Transitions

A sample of possible transitions, along with the transition probabilities for the many-on-many model are shown in Table 6.

Table 6. Sample transitions for the many-on-many model

Present State	Future State & Transition Probability
$(m, I; n, I)$	$(m, F_1; n, I) \text{ wp } \frac{\delta_1}{\delta_1 + \epsilon_1}$
	$(m, I; n, F_1) \text{ wp } \frac{\epsilon_1}{\delta_1 + \epsilon_1}$
$(m, F_1; n, I)$	$(m, F_1; n-1, I) \text{ wp } \frac{m\beta_1}{m\beta_1 + \epsilon_1 + \epsilon_2 + b + \gamma}$
	$(m, F_1; n, F_1) \text{ wp } \frac{\epsilon_1}{m\beta_1 + \epsilon_1 + \epsilon_2 + b + \gamma}$
	$(m, F_1; n, F_2) \text{ wp } \frac{\epsilon_2}{m\beta_1 + \epsilon_1 + \epsilon_2 + b + \gamma}$
	$(m, I; n, I) \text{ wp } \frac{b + \gamma}{m\beta_1 + \epsilon_1 + \epsilon_2 + b + \gamma}$
$(m, F_1; n, F_1)$	$(m, F_1; n-1, F_1) \text{ wp } \frac{m\beta_1}{m\beta_1 + n\rho_1 + \delta_2 + \epsilon_2 + c + \gamma}$
	$(m-1, F_1; n, F_1) \text{ wp } \frac{n\rho_1}{m\beta_1 + n\rho_1 + \delta_2 + \epsilon_2 + c + \gamma}$

	$(m, F_2; n, F_1) \text{ wp } \frac{\delta_2}{m\beta_1+n\rho_1+\delta_2+\varepsilon_2+c+\gamma}$
	$(m, F_1; n, F_2) \text{ wp } \frac{\varepsilon_2}{m\beta_1+n\rho_1+\delta_2+\varepsilon_2+c+\gamma}$
	$(m, I; n, I) \text{ wp } \frac{c+\gamma}{m\beta_1+n\rho_1+\delta_2+\varepsilon_2+c+\gamma}$

a. Transitions from $(m, I; n, I)$

From the state $(m, I; n, I)$, for $m, n > 1$, it is possible to transit to the states $(m, F_1; n, I)$ and $(m, I; n, F_1)$. When Blue and Red units are in state I , they should detect the enemy using the surveillance assets. If Blue's surveillance assets detect Red first, it transits to the state $(m, F_1; n, I)$. If Red's surveillance assets detect Blue first, it transits to the state $(m, I; n, F_1)$.

b. Transitions from $(m, F_1; n, I)$

From the state $(m, F_1; n, I)$, for $m, n > 1$, it is possible to transit to the states $(m, F_1; n - 1, I)$, $(m, F_1; n, F_1)$, $(m, F_1; n, F_2)$, and $(m, I; n, I)$. If Blue kills one Red using inaccurate fire, it transits to the state $(m, F_1; n - 1, I)$. If Red's surveillance assets detect Blue, it transits to the state $(m, F_1; n, F_1)$. If Red's counter-battery radar detects Blue, it transits to the state $(m, F_1; n, F_2)$. If at least one of the two sides decides to scoot, it transits to the state $(m, I; n, I)$.

c. Transitions from $(m, F_1; n, F_1)$

From the state $(m, F_1; n, F_1)$, for $m, n > 1$, it is possible to transit to the states $(m, F_1; n - 1, F_1)$, $(m - 1, F_1; n, F_1)$, $(m, F_2; n, F_1)$, $(m, F_1; n, F_2)$, and $(m, I; n, I)$. If Blue kills one Red using inaccurate fire, it transits to the state $(m, F_1; n - 1, F_1)$. If Red kills one Blue using inaccurate fire, it transits to the state $(m - 1, F_1; n, F_1)$. If Blue's counter-battery radar detects Red, it transits to the state $(m, F_2; n, F_1)$. If Red's counter-battery radar detects Blue, it transits to the state $(m, F_1; n, F_2)$. If at least one of the two sides decides to scoot, it transits to the state $(m, I; n, I)$.

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VII. ANALYSES FOR THE MANY-ON-MANY MODEL

In this chapter, we give a few numerical examples applying the many-on-many model. For a given set of parameters values for lethality, detection, and Red's scooting policy, we study the sensitivity of the win probability of Blue to changes in Blue's scooting policies.

A. FIXED PARAMETERS FOR SENSITIVITY ANALYSIS

We fix the parameters for the number of units on both sides (m, n), for the lethality of inaccurate fire (β_1, ρ_1) and accurate fire (β_2, ρ_2), and for detection by surveillance assets (δ_1, ε_1) and counter-battery radars (δ_2, ε_2). We also fix the values for Red's scooting parameters (α, γ). We select three combinations of the number of units, five combinations of lethality parameters, five combinations of detection parameters, and four combinations of Red's scooting policy. Altogether, we analyze 300 cases that reflect various effectiveness scenarios for the two sides, as shown in Figure 15. For each case, the win probability of Blue is computed as a function of Blue's scooting policy.

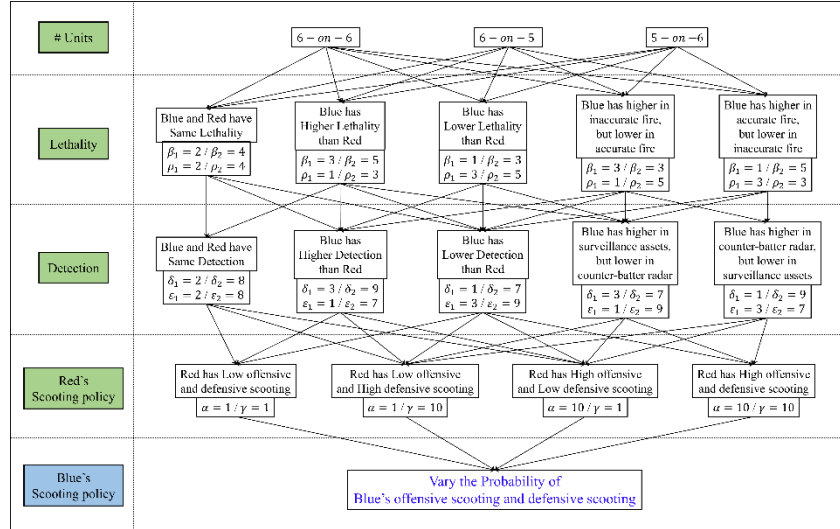


Figure 15. The parameter values of sensitivity analysis for the many-on-many model

B. SENSITIVITY TO THE RATE OF BLUE’S OFFENSIVE SCOOTING

We vary the rate of Blue’s offensive scouting (b) from 1 to 10 for each one of the 300 cases described in Figure 15, and produce a plot of Blue’s win probability as a function of the probability for offensive scouting. In all 300 cases, Blue’s win probability decreases as the rate of Blue’s offensive scouting increases. This means that the more frequently Blue scoots following a firing, the less likely Blue is to win. This trend is common to all the choices for number of units, capability of detection, or lethality of fire. The results differ from the results we obtained for the one-on-one model (91 out of 100 cases resulted in increase in win probability) and the two-on-one model (75 out of 100 cases resulted in an increase).

This surprising result is quite robust for additional, more extreme, choices of parameters, and should be subject to further explorations in future research.

C. SENSITIVITY TO THE RATE OF BLUE’S DEFENSIVE SCOOTING

We vary the rate of Blue’s defensive scouting (c) from 1 to 10. Analyzing all 300 cases, we observe that there are common combinations of parameters for which Blue’s win probability decreases as the rate of Blue’s defensive scouting increases. The common features of these cases are as follows.

- The killing probability of Blue’s inaccurate fire (β_1) is lower than that of Red’s inaccurate fire (ρ_1).
- The detection probability of Blue’s surveillance assets (δ_1) is less than or equal to that of Red’s surveillance assets (ε_1).

If the cases do not have the above characteristics, Blue’s win probability either decreases negligibly (by less than 0.01) or increases, as the rate of Blue’s defensive scouting increases.

The difference between the many-on-many model and the previous two models (the one-on-one model and the two-on-one model) is the condition of Red’s scouting policy. In the one-on-one model, we need a condition that Red has low probability of offensive

scouting (α). Also, in the two-on-one model, we have a condition that Red has low probability of offensive scouting (α) and high probability of defensive scouting (γ). However, in the many-on-many model, regardless of the Red's scouting policy, if the two conditions just mentioned are satisfied, Blue's win probability decreases.

Figure 16 shows the dependency of Blue's win probability on its defensive scouting probability for several Red's scouting policies. The first graph presents the six-on-six battle, the second one shows the six-on-five battle, and the third one represents the five-on-six battle. Blue's win probability depends on the number of units and Red's scouting policy. However, it can be seen that the decrease rate is not significantly different.

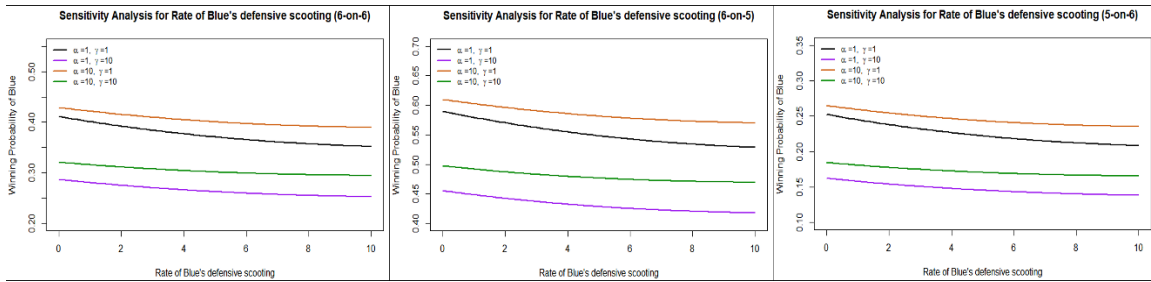


Figure 16. Blue's win probability for four scenarios of Red's scouting policies in the many-on-many model

This is also the difference between the many-on-many model and the previous two models. In the previous models, both the win probability of Blue and the change rate of Blue's win probability are different according to the Red's scouting policy. However, in the many-on-many model, the decrease rate of Blue's win probability according to the Red's scouting policy is similar. Intuitively, it is expected that both the win probability and the sensitivity of the win probability according to the Red's scouting policy would be different. As a result of the analysis, the win probability of Blue is different, but the sensitivity of the win probability is similar. This also requires further research.

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VIII. CONCLUSIONS

A. SUMMARY

The primary purpose of this thesis is to develop analytical models with stochastic duels for counterfire warfare of field artillery. In particular, our main thrust is to create models that analyze the battle, focusing on critical factors in artillery operations such as intelligence-gathering and targeting, firing, and scooting to ensure survivability. These models are intended to aid artillery field-commanders in making better tactical decisions on the battlefield, in accordance with the given scenario.

We develop three Markov chain models. The first model is the one-on-one model using a discrete-time Markov chain. Through this model, we can see how lethality, detection, and scooting relate to and influence each other. It also provides some basic tactical insights regarding which scooting policies can increase the win probability, depending on lethality and detection capability. For example, if one side has a high detection probability and lethality, it is to the advantage of this side to scoot frequently.

The second model is a two-on-one model using a discrete-time Markov chain. This model gives insights into the battle situation when the Blue force is numerically superior. The analysis for this model is similar to the one-on-one model. Of course, a slight difference exists in analysis results because there is a difference in the number of units. The main takeaways from this model are (1) the advantage of Blue's offensive scooting is reduced compared to the one-on-one model; (2) the maximum decrease in the win probability for the two-on-one model is larger than that for the one-on-one model; and (3) the advantage of Blue's defensive scooting is not as great as in the one-on-one model.

The third model is a many-on-many model that uses a continuous-time Markov chain. This model provides a tool to analyze the combat situation involving a larger number of units, unlike the one-on-one model and the two-on-one model. The currently developed model shows several results that differ from those in the previous two models, and we are not able to find exact reasons for those differences.

These three models can be used as analysis tools for commanders of field-artillery units who typically rely on their intuition to judge the battlefield situation. The developed models can give insights into implementing effective tactics based on data on the combat power and tactics of friendly and enemy forces.

B. FUTURE WORK

The main thrust of our work is the modeling. Therefore, we do not conduct full-scale design of experiments. Our analyses and examples only demonstrate the usefulness of the models as analytic tools. Based on our limited analysis, we present simple insights related to battlefield situations. We can get more information and insights through well-structured experiments.

Also, to make a more realistic model, future research can take into account the situation that parameter values may change according to the enemy's states. In our models, we use the same parameter values no matter what the enemy state is. In a real battle situation, if one side is attacked, the lethality of its fire could decrease, which affects the outcome of the battle. In addition, in the many-on-many model, the parameter value of offensive- and defensive scooting can be changed depending on the number of remaining live units. Through these enhancements, meaningful results can be derived by checking the change in the win probability according to the change in the scooting rate as the number of units decreases.

In the many-on-many model, each piece on a certain side is engaged in the same activity. That is, this model does not reflect distributed employment among artillery pieces, which is often used by self-propelled artillery units. The battery performs the operation together, but for survivability, the operation can be performed by dividing it into two or three sub-units. Developing a model considering this employment tactic is complicated because the number of states increases rapidly. For that reason, with the limited time and resources available it could not be reflected in our work. If this consideration is included in a model, it would be possible to develop a tool that analyzes the battlefield situation in a more realistic and detailed manner.

APPENDIX A. THE SITUATIONS AND ASSUMPTIONS FOR TRANSITIONS OF THE ONE-ON-ONE MODEL

Present State (<i>B, R</i>)	Future State (<i>B, R</i>)	Situations	Assumptions
<i>I, I</i>	<i>I, I</i>	- Both B's surveillance assets. and R's surveillance assets do not detect enemy.	- If enemy's location is not detected by surveillance assets, they cannot shoot.
	<i>F₁, I</i> (<i>I, F₁</i>)	- B's (R's) surveillance assets detect enemy. - R's (B's) surveillance assets do not detect enemy.	- If R's (B's) location is detected by B's (R's) surveillance assets, B (R) can shoot inaccurate fire.
	<i>F₁, F₁</i>	- Both B's surveillance assets. and R's surveillance assets detect enemy.	- If B and R detect the enemy by their surveillance assets, they can shoot inaccurate fire.
	* No transitions: <i>F₁, F₂</i> (<i>F₂, F₁</i>) / <i>F₂, F₂</i> / <i>K, *</i> / <i>*</i> , <i>K</i> / <i>K, K</i>		
<i>F₁, I</i> (<i>I, F₁</i>)	<i>I, I</i>	- No kill is recorded. - At least one of the two sides scoots.	- If at least one of the two sides decides to scoot, information from surveillance assets is lost.
	<i>F₁, I</i> (<i>I, F₁</i>)	- No kill is recorded. - Neither B nor R scoots. - R's (B's) surveillance assets and radar do not detect enemy.	- If B's (R's) location is not detected by R's (B's) surveillance assets and radar, R (B) cannot shoot.
	<i>F₁, F₁</i>	- No kill is recorded. - Neither B nor R scoots. - R's (B's) surveillance assets detect enemy. - R's (B's) radar do not detect enemy.	- If B and R do not scoot and R (B) detect enemy using surveillance assets, R (B) can shoot inaccurate fire. However, R (B) cannot shoot accurate fire because R's (B's) radar fails to detect enemy.
	<i>F₁, F₂</i> (<i>F₂, F₁</i>)	- No kill is recorded. - Neither B nor R scoots. - R's (B's) radar detect enemy.	- If B and R do not scoot and R (B) detect enemy using radar, R (B) can shoot accurate fire.
	<i>*, K</i> (<i>K, *</i>)	- B (R) kills R (B).	-
	* No transitions: <i>F₂, F₂</i> / <i>F₂, F₁</i> (<i>F₁, F₂</i>) / <i>I, F₁</i> (<i>F₁, I</i>) / <i>K, *</i> (<i>*, K</i>) / <i>K, K</i>		

Present State (<i>B, R</i>)	Future State (<i>B, R</i>)	Situations	Assumptions
<i>F₁, F₁</i>	<i>I, I</i>	- No kill is recorded. - At least one of the two sides scoots.	- If at least one of the two sides decides to scoot, information from surveillance assets is lost on both sides.
	<i>F₁, F₁</i>	- No kill is recorded. - Neither B nor R scoots. - Neither B's radar nor R's radar detects enemy.	- If B and R's position is detected by their surveillance assets, their position is kept track by surveillance assets until at least one of them decides to scoot. Therefore, they can shoot inaccurate fire repeatedly. - If B and R fail to detect enemy's location by their radar, they cannot shoot accurate fire.
	<i>F₂, F₁</i> (<i>F₁, F₂</i>)	- No kill is recorded. - Neither B nor R scoots. - B's (R's) radar detects enemy. - R's (B's) radar does not detect Enemy.	- If B and R's position is detected by their surveillance assets, their position is kept track by surveillance assets until at least one of them decides to scoot. Therefore, they can shoot inaccurate fire repeatedly. - B (R) can shoot accurate fire because its radar detects enemy. - If R's (B's) location is detected by B's (R's) radar while keeping track by B's (R's) surveillance assets, B (R) shoot accurate fire, not inaccurate fire.
	<i>F₂, F₂</i>	- No kill is recorded. - Neither B nor R scoots. - Both B's radar and R's radar detect enemy.	- If B and R's position is detected by their surveillance assets, their position is kept track by surveillance assets until at least one of them decides to scoot. Therefore, they can shoot inaccurate fire repeatedly. - If enemy's location is detected by radar while keeping track by surveillance assets, they shoot accurate fire, not inaccurate fire.
	<i>K, *</i>	- R kills B. - B does not kill R.	-
	<i>*, K</i>	- B kills R. - R does not kill B.	-

Present State (<i>B, R</i>)	Future State (<i>B, R</i>)	Situations	Assumptions
	<i>K, K</i>	- Simultaneous kill of both sides.	-
	* No transitions: <i>I, F₁</i> (<i>F₁, I</i>)		
<i>F₁, F₂</i> (<i>F₂, F₁</i>)	<i>I, I</i>	- No kill is recorded. - At least one of the two sides scoots.	- If at least one of the two sides decides to scoot, information from surveillance assets and radar is lost on both sides.
	<i>F₁, F₂</i> (<i>F₂, F₁</i>)	- No kill is recorded. - Neither B nor R scoots. - B's (R's) radar does not detect enemy.	- If B and R's position is detected by their surveillance assets or radar, their position is kept track until at least one of them decides to scoot. Therefore, they can shoot repeatedly. - If R's (B's) location is not detected by B's (R's) radar while keeping track by B's (R's) surveillance assets, B (R) cannot shoot accurate fire. Therefore, B (R) can shoot only inaccurate fire.
	<i>F₂, F₂</i>	- No kill is recorded. - Neither B nor R scoots. - B's (R's) radar detects enemy.	- If B and R's position is detected by their surveillance assets or radar, their position is kept track until at least one of them decides to scoot. Therefore, they can shoot repeatedly. - If R's (B's) location is detected by B's (R's) radar while keeping track by B's (R's) surveillance assets, B (R) shoot accurate fire, not inaccurate fire.
	<i>K, *</i>	- R kills B. - B does not kill R.	-
	<i>*, K</i>	- B kills R. - R does not kill B.	-
	<i>K, K</i>	- Simultaneous kill of both sides.	-
	* No transitions: <i>F₁, F₁</i> / <i>F₂, F₁</i> (<i>F₁, F₂</i>) / <i>I, F₁</i> (<i>F₁, I</i>) / <i>F₁, I</i> (<i>I, F₁</i>)		
<i>F₂, F₂</i>	<i>I, I</i>	- No kill is recorded. - At least one of the two sides scoots.	- If at least one of the two sides decides to scoot, information from radar is lost.
	<i>F₂, F₂</i>	- No kill is recorded. - Neither B nor R scoots.	- If B and R's position is detected by their radar, they can shoot accurate fire repeatedly until at least one of them decides to scoot.

Present State (B, R)	Future State (B, R)	Situations	Assumptions
	$K, *$	- R kills B. - B does not kill R.	-
	$*, K$	- B kills R. - R does not kill B.	-
	K, K	- Simultaneous kill of both sides.	-
	* No transitions: $I, F_1 (F_1, I) / F_1, I (I, F_1) / F_1, F_1 / F_1, F_2 (F_2, F_1) / F_2, F_1 (F_1, F_2)$		

APPENDIX B. THE SITUATIONS AND ASSUMPTIONS FOR TRANSITIONS AND TRANSITION PROBABILITIES OF THE TWO-ON-ONE MODEL

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
I, I, I	I, I, I	- Both B's surveillance assets and R's surveillance assets do not detect enemy.	- If enemy's location is not detected by surveillance assets, they cannot shoot.	$(1 - \delta_1)(1 - \varepsilon_1)$
	I, I, F_1	- B's surveillance assets do not detect enemy. - R's surveillance assets detect enemy.	- If B's location is detected by R's surveillance assets, R can shoot inaccurate fire. - If R's location is not detected by B's surveillance assets, B cannot shoot.	$(1 - \delta_1)\varepsilon_1$
	F_1, F_1, I	- B's surveillance assets detect enemy. - R's surveillance assets do not detect enemy.	- If R's location is detected by B's surveillance assets, both B can shoot inaccurate fire. - Both B shoot the same type of fire simultaneously because they share the surveillance assets and radar. - If B's location is not detected by R's surveillance assets, R cannot shoot.	$\delta_1(1 - \varepsilon_1)$
	F_1, F_1, F_1	- Both B's surveillance assets and R's surveillance assets detect enemy.	- If R's location is detected by B's surveillance assets, both B can shoot inaccurate fire. - Both B shoot the same type of fire simultaneously because they share the surveillance assets and radar. - If B's location is detected by R's surveillance assets, R can shoot inaccurate fire.	$\delta_1\varepsilon_1$
	* No transitions: $F_1, I, I / F_1, I, F_1 / F_1, I, F_2 / F_1, F_1, F_2 / F_2, I, I / F_2, I, F_1 / F_2, I, F_2 / F_2, F_2, I / F_2, F_2, F_1 / F_2, F_2, F_2 / K, I, I / K, I, F_1 / K, F_1, I / K, F_1, F_1 / K, F_1, F_2 / K, F_2, I / K, F_2, F_1 / K, F_2, F_2 / K, K, * / *, *, K / K, K, K$			

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
I, I, F_1	I, I, I	- No kill is recorded. - R scoots.	- If at least one of the two sides decides to scoot, information from surveillance assets is lost.	$\frac{(1 - \rho_1)(\alpha + (1 - \alpha)c)}{(1 - \delta_1)(1 - \delta_2)(1 - \varepsilon_1)}$
		- No kill is recorded. - Attacked B scoots, but R holds its position. - B's surveillance assets and radar do not detect enemy. - R's surveillance assets do not detect enemy.	- R can shoot at non-scooting B because R holds its position. However, R cannot fire because it does not detect non-scooting B. - If R's location is not detected by B's surveillance assets and radar, B cannot shoot.	
	I, I, F_1	- No kill is recorded. - Attacked B scoots, but R holds its position. - B's surveillance assets and radar do not detect enemy. - R's surveillance assets detect enemy.	- R can shoot at non-scooting B because R holds its position. - If non-scooting B is detected by R's surveillance assets, R can shoot inaccurate fire. - If R's location is not detected by B's surveillance assets and radar, B cannot shoot.	$\frac{(1 - \rho_1)((1 - \alpha)c + (1 - \delta_1)(1 - \delta_2)\varepsilon_1)}{(1 - \delta_1)(1 - \delta_2)}$
		- No kill is recorded. - Neither attacked B nor R scoots. - B's surveillance assets and radar do not detect enemy.	- If B's position is detected by R's surveillance assets, its position is kept track by surveillance assets until at least one of them decides to scoot. Therefore, R can shoot inaccurate fire repeatedly. - If R's location is not detected by B's surveillance assets and radar, B cannot shoot.	

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	F_1, I, I	- No kill is recorded. - Attacked B scoots, but R holds its position. - B's surveillance assets detect enemy. - B's radar do not detect enemy. - R's surveillance assets do not detect enemy.	- Even if attacked B scoots, non-scooting B can shoot inaccurate fire if B's surveillance assets detect enemy because R holds its position. However, B cannot shoot accurate fire because B's radar fails to detect enemy. - R can shoot at non-scooting B because R holds its position. However, R cannot fire because it does not detect non-scooting B.	$(1 - \rho_1)(1 - \alpha)c$ $\delta_1(1 - \delta_2)(1 - \varepsilon_1)$
	F_1, I, F_1	- No kill is recorded. - Attacked B scoots, but R holds its position. - B's surveillance assets detect enemy. - B's radar do not detect enemy. - R's surveillance assets detect enemy.	- R can shoot at non-scooting B because R holds its position. - If non-scooting B is detected by R's surveillance assets, R can shoot inaccurate fire. - Even if attacked B scoots, non-scooting B can shoot inaccurate fire if B's surveillance assets detect enemy because R holds its position. However, B cannot shoot accurate fire because B's radar fails to detect enemy.	$(1 - \rho_1)(1 - \alpha)c$ $\delta_1(1 - \delta_2)\varepsilon_1$
	F_1, F_1, F_1	- No kill is recorded. - Neither attacked B nor R scoots. - B's surveillance assets detect enemy. - B's radar do not detect enemy.	- If B and R do not scoot and B detects enemy using surveillance assets, B can shoot inaccurate fire. However, B cannot shoot accurate fire because B's radar fails to detect enemy. - Both B shoot the same type of fire simultaneously because they share the surveillance assets and radar. - If B's position is detected by R's surveillance assets, its position is kept track by surveillance assets until at least one of them decides to scoot. Therefore, R can shoot inaccurate fire repeatedly.	$(1 - \rho_1)$ $(1 - \alpha)(1 - c)$ $\delta_1(1 - \delta_2)$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	F_2, I, I	- No kill is recorded. - Attacked B scoots, but R holds its position. - B's radar detects enemy. - R's surveillance assets do not detect enemy.	- R can shoot at non-scooting B because R holds its position. However, R cannot fire because it does not detect non-scooting B. - Even if attacked B scoots, non-scooting B can shoot accurate fire if B's radar detects enemy because R holds its position.	$(1 - \rho_1)(1 - \alpha)c \delta_2(1 - \varepsilon_1)$
	F_2, I, F_1	- No kill is recorded. - Attacked B scoots, but R holds its position. - B's radar detects enemy. - R's surveillance assets detect enemy.	- R can shoot at non-scooting B because R holds its position. - If non-scooting B is detected by R's surveillance assets, R can shoot inaccurate fire. - Even if attacked B scoots, non-scooting B can shoot accurate fire if B's radar detects enemy because R holds its position.	$(1 - \rho_1)(1 - \alpha)c \delta_2 \varepsilon_1$
	F_2, F_2, F_1	- No kill is recorded. - Neither attacked B nor R scoots. - B's radar detects enemy.	- If B and R do not scoot and B detects enemy using radar, B can shoot accurate fire. - Both B shoot the same type of fire simultaneously because they share the surveillance assets and radar.	$(1 - \rho_1)(1 - \alpha)(1 - c)\delta_2$
	K, I, I	- R kills one of B. - R scoots.	- If at least one of the two sides decides to scoot, information from surveillance assets is lost on both sides.	$\rho_1(\alpha + (1 - \alpha)(1 - \delta_1)(1 - \delta_2)(1 - \varepsilon_1))$
		- R kills one of B. - R does not scoot. - B's surveillance assets and radar do not detect enemy. - R's surveillance assets do not detect enemy.	- R can shoot at alive B because R holds its position. However, R cannot fire because it does not detect alive B. - If R's location is not detected by B's surveillance assets and radar, B cannot shoot.	

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	K, I, F_1	- R kills one of B. - R does not scoot. - B's surveillance assets and radar do not detect enemy. - R's surveillance assets detect enemy.	- R can shoot at alive B because R holds its position. - If alive B is detected by R's surveillance assets, R can shoot inaccurate fire. - If R's location is not detected by B's surveillance assets and radar, B cannot shoot.	$\rho_1(1 - \alpha)(1 - \delta_1)(1 - \delta_2)\varepsilon_1$
	K, F_1, I	- R kills one of B. - R does not scoot. - B's surveillance assets detect enemy. - B's radar do not detect enemy. - R's surveillance assets do not detect enemy.	- R can shoot at alive B because R holds its position. However, R cannot fire because it does not detect alive B. - Alive B can shoot inaccurate fire if B's surveillance assets detect enemy because R holds its position. However, B cannot shoot accurate fire because B's radar fails to detect enemy.	$\rho_1(1 - \alpha)\delta_1(1 - \delta_2)(1 - \varepsilon_1)$
	K, F_1, F_1	- R kills one of B. - R does not scoot. - B's surveillance assets detect enemy. - B's radar do not detect enemy. - R's surveillance assets detect enemy.	- R can shoot at alive B because R holds its position. - If alive B is detected by R's surveillance assets, R can shoot inaccurate fire. - Alive B can shoot inaccurate fire if B's surveillance assets detect enemy because R holds its position. However, B cannot shoot accurate fire because B's radar fails to detect enemy.	$\rho_1(1 - \alpha)\delta_1(1 - \delta_2)\varepsilon_1$
	K, F_2, I	- R kills one of B. - R does not scoot. - B's radar detects enemy. - R's surveillance assets do not detect enemy.	- R can shoot at alive B because R holds its position. However, R cannot fire because it does not detect alive B. - Alive B can shoot accurate fire if B's radar detects enemy because R holds its position.	$\rho_1(1 - \alpha)\delta_2(1 - \varepsilon_1)$
	K, F_2, F_1	- R kills one of B. - R does not scoot. - B's radar detects Enemy. - R's surveillance assets detect enemy.	- R can shoot at alive B because R holds its position. - If alive B is detected by R's surveillance assets, R can shoot inaccurate fire. - Alive B can shoot accurate fire if B's radar detects enemy because R holds its position.	$\rho_1(1 - \alpha)\delta_2\varepsilon_1$
	* No transitions: $F_1, I, F_2 / F_1, F_1, I / F_1, F_1, F_2 / F_2, I, F_2 / F_2, F_2, I / F_2, F_2, F_2 / K, F_1, F_2 /$			

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	$K, F_2, F_2 / K, K, * / *, *, K / K, K, K$			
F_1, I, I	I, I, I	- No kill is recorded. - R scoots.	- If at least one of the two sides decides to scoot, information from surveillance assets is lost.	$(1 - \beta_1)\gamma$
	F_1, I, I	- No kill is recorded. - Fired B scoots, but R holds its position. - R's surveillance assets do not detect enemy.	- If R's position is detected by B's surveillance assets, its position is kept track by surveillance assets until it decides to scoot. - Non-scooting B can shoot inaccurate fire even if fired B scoots because both B share the surveillance assets and radar, and R holds its position. - If B's location is not detected by R's surveillance assets, R cannot shoot.	$(1 - \beta_1)b(1 - \gamma)(1 - \varepsilon_1)$
	F_1, I, F_1	- No kill is recorded. - Fired B scoots, but R holds its position. - R's surveillance assets detect enemy.	- If R's position is detected by B's surveillance assets, its position is kept track by surveillance assets until it decides to scoot. - Non-scooting B can shoot inaccurate fire even if Fired B scoots because both B share the surveillance assets and radar, and R holds its position. - R can shoot at non-scooting B because R holds its position. - If non-scooting B is detected by R's surveillance assets, R can shoot inaccurate fire.	$(1 - \beta_1)b(1 - \gamma)\varepsilon_1$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	F_1, F_1, I	- No kill is recorded. - Neither fired B nor R scoots. - R's surveillance assets and radar do not detect enemy.	- If R's position is detected by B's surveillance assets, its position is kept track by surveillance assets until it decides to scoot. Therefore, B can shoot inaccurate fire repeatedly. - Both B shoot the same type of fire simultaneously because they share the surveillance assets and radar. - If B's location is not detected by R's surveillance assets and radar, R cannot shoot.	$\begin{aligned} &(1 - \beta_1) \\ &(1 - b)(1 - \gamma) \\ &(1 - \varepsilon_1)(1 - \varepsilon_2) \end{aligned}$
	F_1, F_1, F_1	- No kill is recorded. - Neither fired B nor R scoots. - R's surveillance assets detect enemy. - R's radar does not detect enemy.	- If R's position is detected by B's surveillance assets, its position is kept track by surveillance assets until it decides to scoot. Therefore, B can shoot inaccurate fire repeatedly. - Both B shoot the same type of fire simultaneously because they share the surveillance assets and radar. - If B and R do not scoot and R detects enemy using surveillance assets, R can shoot inaccurate fire. However, R cannot shoot accurate fire because R's radar fails to detect enemy.	$\begin{aligned} &(1 - \beta_1) \\ &(1 - b)(1 - \gamma) \\ &\varepsilon_1(1 - \varepsilon_2) \end{aligned}$
	F_1, F_1, F_2	- No kill is recorded. - Neither fired B nor R scoots. - R's radar detects enemy.	- Once R's position is detected by B's surveillance assets, its position is kept track by surveillance assets until it decides to scoot. Therefore, B can shoot inaccurate fire repeatedly. - Both B shoot the same type of Fire simultaneously because they share the surveillance assets and radar. - If B and R do not scoot and R detects enemy using radar, R can shoot accurate fire.	$\begin{aligned} &(1 - \beta_1) \\ &(1 - b)(1 - \gamma)\varepsilon_2 \end{aligned}$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	$*, *, K$	- B kills R	-	β_1
	* No transitions: $I, I, F_1 / F_2, I, I / F_2, I, F_1 / F_2, I, F_2 / F_2, I / F_2, F_2, F_1 / F_2, F_2, F_2 / K, I, I / K, I, F_1 / K, F_1, I / K, F_1, F_1 / K, F_1, F_2 / K, F_2, I / K, F_2, F_1 / K, F_2, F_2 / K, K, * / K, K, K$			
F_1, I, F_1	I, I, I	- No kill is recorded. - R scoots.	- If at least one of the two sides decides to scoot, information from surveillance assets is lost.	$(1 - \beta_1)(1 - \rho_1)\gamma$
	F_1, I, I	- No kill is recorded. - Attacked B scoots, but R holds its position. - B's radar does not detect enemy. - R's surveillance assets do not detect enemy.	- If R's position is detected by B's surveillance assets, its position is kept track by surveillance assets until it decides to scoot. - Non-scooting B can shoot inaccurate fire even if attacked B scoots because both B share the surveillance assets and radar, and R holds its position. However, B cannot shoot accurate fire because B's radar fails to detect enemy. - R can shoot at non-scooting B because R holds its position. However, R cannot fire because it does not detect non-scooting B.	$(1 - \beta_1)(1 - \rho_1)$ $c(1 - \gamma)$ $(1 - \delta_2)(1 - \varepsilon_1)$
	F_1, I, F_1	- No kill is recorded. - Attacked B scoots, but R holds its position. - B's radar does not detect enemy. - R's surveillance assets detect enemy.	- If R's position is detected by B's surveillance assets, its position is kept track by surveillance assets until it decides to scoot. - Non-scooting B can shoot inaccurate fire even if attacked B scoots because both B share the surveillance assets and radar, and R holds its position. However, B cannot shoot accurate fire because B's radar fails to detect enemy. - R can shoot at non-scooting B because R holds its position. - If non-scooting B is detected by R's surveillance assets, R can shoot inaccurate fire.	$(1 - \beta_1)(1 - \rho_1)$ $c(1 - \gamma)$ $(1 - \delta_2)\varepsilon_1$
	F_1, F_1, F_1	- No kill is recorded. - Neither fired B nor R scoots. - B's radar does not detect enemy.	- If B and R's position is detected by surveillance assets, their position is kept track by surveillance assets until at least one of them decides to scoot.	$(1 - \beta_1)(1 - \rho_1)$ $(1 - c)(1 - \gamma)$ $(1 - \delta_2)(1 - \varepsilon_2)$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
		- R's radar does not detect enemy.	Therefore, they can shoot inaccurate fire repeatedly. - Both B shoot the same type of fire simultaneously because they share the surveillance assets and radar. - They cannot shoot accurate fire because their radars fail to detect enemy.	
	F_1, F_1, F_2	- No kill is recorded. - Neither fired B nor R scoots. - B's radar does not detect enemy. - R's radar detects enemy.	- If B and R's position is detected by surveillance assets, their position is kept track by surveillance assets until at least one of them decides to scoot. Therefore, they can shoot inaccurate fire repeatedly. - Both B shoot the same type of fire simultaneously because they share the surveillance assets and radar. - If B and R do not scoot and R detects enemy using radar, R can shoot accurate fire. However, B cannot shoot accurate fire because its radar fails to detect enemy.	$(1 - \beta_1)(1 - \rho_1)$ $(1 - c)(1 - \gamma)$ $(1 - \delta_2)\varepsilon_2$
	F_2, I, I	- No kill is recorded. - Attacked B scoots, but R holds its position. - B's radar detects enemy. - R's surveillance assets do not detect enemy.	- Even if attacked B scoots, non-scooting B can shoot accurate fire if B's radar detects enemy because R holds its position. - R can shoot at non-scooting B because R holds its position. However, R cannot fire because it does not detect non-scooting B.	$(1 - \beta_1)(1 - \rho_1)$ $c(1 - \gamma)$ $\delta_2(1 - \varepsilon_1)$
	F_2, I, F_1	- No kill is recorded. - Attacked B scoots, but R holds its position. - B's radar detects enemy. - R's surveillance assets detect enemy.	- Even if attacked B scoots, non-scooting B can shoot accurate fire if B's radar detects enemy because R holds its position. - R can shoot at non-scooting B because R holds its position. - If non-scooting B is detected by R's surveillance assets, R can shoot inaccurate fire.	$(1 - \beta_1)(1 - \rho_1)$ $c(1 - \gamma)\delta_2\varepsilon_1$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	F_2, F_2, F_1	- No kill is recorded. - Neither fired B nor R scoots. - B's radar detects enemy. - R's radar does not detect enemy.	- If B and R do not scoot and B detects enemy using radar, B can shoot accurate fire. - Both B shoot the same type of fire simultaneously because they share the surveillance assets and radar. - If B's location is not detected by R's radar while keeping track by R's surveillance assets, R cannot shoot accurate fire. Therefore, R can shoot only inaccurate fire.	$(1 - \beta_1)(1 - \rho_1)$ $(1 - c)(1 - \gamma)$ $\delta_2(1 - \varepsilon_2)$
	F_2, F_2, F_2	- No kill is recorded. - Neither fired B nor R scoots. - B's radar detects Enemy. - R's radar detects enemy.	- If B and R do not scoot and B detects enemy using radar, B can shoot accurate fire. - Both B shoot the same type of fire simultaneously because they share the surveillance assets and radar. - If B and R do not scoot and R detect enemy using radar, R can shoot accurate fire.	$(1 - \beta_1)(1 - \rho_1)$ $(1 - c)(1 - \gamma)\delta_2\varepsilon_2$
	K, I, I	- B does not kill R. - R kills one of B. - R scoots.	- If at least one of the two sides decides to scoot, information from surveillance assets is lost.	$(1 - \beta_1)\rho_1\gamma$
	K, F_1, I	- B does not kill R. - R kills one of B. - R does not scoot. - B's radar does not detect enemy. - R's surveillance assets do not detect enemy.	- R can shoot at alive B because R holds its position. However, R cannot fire because it does not detect alive B. - Alive B can shoot inaccurate fire if B's surveillance assets detect enemy because R holds its position. However, B cannot shoot accurate fire because B's radar fails to detect enemy.	$(1 - \beta_1)\rho_1(1 - \gamma)$ $(1 - \delta_2)(1 - \varepsilon_1)$
	K, F_1, F_1	- B does not kill R. - R kills one of B. - R does not scoot. - B's radar does not detect enemy. - R's surveillance assets detect enemy.	- R can shoot at alive B because R holds its position. - If alive B is detected by R's surveillance assets, R can shoot inaccurate fire. - Alive B can shoot inaccurate fire if B's surveillance assets detect enemy because R holds its position. However, B cannot shoot accurate fire because B's radar fails to detect enemy.	$(1 - \beta_1)\rho_1$ $(1 - \gamma)(1 - \delta_2)\varepsilon_1$
	K, F_2, I	- B does not kill R.	- R can shoot at alive B because	$(1 - \beta_1)\rho_1$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
		- R kills one of B. - R does not scoot. - B's radar detects enemy. - R's surveillance assets do not detect enemy.	- R holds its position. However, R cannot fire because it does not detect alive B. - Alive B can shoot accurate fire if B's radar detects enemy because R holds its position.	$(1 - \gamma) \delta_2 (1 - \varepsilon_1)$
	K, F_2, F_1	- B does not kill R. - R kills one of B. - R does not scoot. - B's radar detects Enemy. - R's surveillance assets detect enemy.	- R can shoot at alive B because R holds its position. - If alive B is detected by R's surveillance assets, R can shoot inaccurate fire. - Alive B can shoot accurate fire if B's radar detects enemy because R holds its position.	$(1 - \beta_1) \rho_1$ $(1 - \gamma) \delta_2 \varepsilon_1$
	$*, *, K$	- B kills R.	-	β_1
	* No transitions: $I, I, F_1 / F_1, I, F_2 / F_1, F_1, I / F_2, I, F_2 / F_2, F_2, I / K, I, F_1 / K, F_1, F_2 / K, F_2, F_2 / K, K, * / K, K, K$			
F_1, I, F_2	I, I, I	- No kill is recorded. - R scoots.	- If at least one of the two sides decides to scoot, information from surveillance assets and radar is lost.	$(1 - \beta_1)(1 - \rho_2)\gamma$
	F_1, I, I	- No kill is recorded. - Attacked B scoots, but R holds its position. - B's radar does not detect enemy. - R's surveillance assets do not detect enemy.	- If R's position is detected by B's surveillance assets, its position is kept track by surveillance assets until it decides to scoot. - Non-scooting B can shoot inaccurate fire even if attacked B scoots because both B share the surveillance assets and radar, and R holds its position. However, B cannot shoot accurate fire because B's radar fails to detect enemy. - R can shoot at non-scooting B because R holds its position. However, R cannot fire because it does not detect non-scooting B.	$(1 - \beta_1)(1 - \rho_2)$ $c(1 - \gamma)$ $(1 - \delta_2)(1 - \varepsilon_1)$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	F_1, I, F_1	- No kill is recorded. - Attacked B scoots, but R holds its position. - B's radar does not detect enemy. - R's surveillance assets detect enemy.	- If R's position is detected by B's surveillance assets, its position is kept track by surveillance assets until it decides to scoot. - Non-scooting B can shoot inaccurate fire even if attacked B scoots because both B share the surveillance assets and radar, and R holds its position. However, B cannot shoot accurate fire because B's radar fails to detect enemy. - R can shoot at non-scooting B because R holds its position. - If non-scooting B is detected by R's surveillance assets, R can shoot inaccurate fire.	$\frac{(1 - \beta_1)(1 - \rho_2)}{c(1 - \gamma)(1 - \delta_2)\varepsilon_1}$
	F_1, F_1, F_2	- No kill is recorded. - Neither B nor R scoots. - B's radar does not detect enemy.	- If B and R's position is detected by surveillance assets and radar, they can shoot until one of them decides to scoot. Therefore, they can shoot at each other repeatedly. - Both B shoot the same type of fire simultaneously because they share the surveillance assets and radar. - B cannot shoot accurate fire because B's radar fails to detect enemy.	$\frac{(1 - \beta_1)(1 - \rho_2)}{(1 - c)(1 - \gamma)(1 - \delta_2)}$
	F_2, I, I	- No kill is recorded. - Attacked B scoots, but R holds its position. - B's radar detects Enemy. - R's surveillance assets do not detect enemy.	- Even if attacked B scoots, non-scooting B can shoot accurate fire if B's radar detects enemy because R holds its position. - R can shoot at non-scooting B because R holds its position. However, R cannot fire because it does not detect non-scooting B.	$\frac{(1 - \beta_1)(1 - \rho_2)}{c(1 - \gamma)\delta_2(1 - \varepsilon_1)}$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	F_2, I, F_1	- No kill is recorded. - Attacked B scoots, but R holds its position. - B's radar detects Enemy. - R's surveillance assets detect enemy.	- Even if attacked B scoots, non-scooting B can shoot accurate fire if B's radar detects enemy because R holds its position. - R can shoot at non-scooting B because R holds its position. - If non-scooting B is detected by R's surveillance assets, R can shoot inaccurate fire.	$(1 - \beta_1)(1 - \rho_2)$ $c(1 - \gamma)\delta_2\varepsilon_1$
	F_2, F_2, F_2	- No kill is recorded. - Neither B nor R scoots. - B's radar detects enemy.	- If B and R do not scoot and B detects enemy using radar, B can shoot accurate fire. - Both B shoot the same type of fire simultaneously because they share the surveillance assets and radar. - If B and R do not scoot and R was shooting accurate fire, R can shoot accurate fire repeatedly.	$(1 - \beta_1)(1 - \rho_2)$ $(1 - c)(1 - \gamma)\delta_2$
	K, I, I	- B does not kill R. - R kills one of B. - R scoots.	- If at least one of the two sides decides to scoot, information from surveillance assets and radar is lost.	$(1 - \beta_1)\rho_2(\gamma + (1 - \gamma)(1 - \delta_1))$ $(1 - \delta_2)(1 - \varepsilon_1)$
		- B does not kill R. - R kills one of B. - R does not scoot. - B's surveillance assets and B's radar do not detect enemy. - R's surveillance assets do not detect enemy.	- R can shoot at alive B because R holds its position. However, R cannot fire because it does not detect alive B. - If R's location is not detected by B's surveillance assets and radar, B cannot shoot.	
	K, I, F_1	- B does not kill R. - R kills one of B. - R does not scoot. - B's surveillance assets and B's radar do not detect enemy. - R's surveillance assets detect enemy.	- R can shoot at alive B because R holds its position. - If alive B is detected by R's surveillance assets, R can shoot inaccurate fire. - If R's location is not detected by B's surveillance assets and radar, B cannot shoot.	$(1 - \beta_1)\rho_2(1 - \gamma)$ $(1 - \delta_1)(1 - \delta_2)\varepsilon_1$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	K, F_1, I	- B does not kill R. - R kills one of B. - R does not scoot. - B's surveillance assets detect enemy. - B's radar does not detect enemy. - R's surveillance assets do not detect enemy.	- R can shoot at alive B because R holds its position. However, R cannot fire because it does not detect alive B. - Alive B can shoot inaccurate fire if B's surveillance assets detect enemy because R holds its position. However, B cannot shoot accurate fire because B's radar fails to detect enemy.	$(1 - \beta_1)\rho_2(1 - \gamma)\delta_1(1 - \delta_2)(1 - \varepsilon_1)$
	K, F_1, F_1	- B does not kill R. - R kills one of B. - R does not scoot. - B's surveillance assets detect enemy. - B's radar does not detect enemy. - R's surveillance assets detect enemy.	- R can shoot at alive B because R holds its position. - If alive B is detected by R's surveillance assets, R can shoot inaccurate fire. - Alive B can shoot inaccurate fire if B's surveillance assets detect enemy because R holds its position. However, B cannot shoot accurate fire because B's radar fails to detect enemy.	$(1 - \beta_1)\rho_2(1 - \gamma)\delta_1(1 - \delta_2)\varepsilon_1$
	K, F_2, I	- B does not kill R. - R kills one of B. - R does not scoot. - B's radar detects enemy. - R's surveillance assets do not detect enemy.	- R can shoot at alive B because R holds its position. However, R cannot fire because it does not detect alive B. - Alive B can shoot accurate fire if B's radar detects enemy because R holds its position.	$(1 - \beta_1)\rho_2(1 - \gamma)\delta_2(1 - \varepsilon_1)$
	K, F_2, F_1	- B does not kill R. - R kills one of B. - R does not scoot. - B's radar detects enemy. - R's surveillance assets detect enemy.	- R can shoot at alive B because R holds its position. - If alive B is detected by R's surveillance assets, R can shoot inaccurate fire. - Alive B can shoot accurate fire if B's radar detects enemy because R holds its position.	$(1 - \beta_1)\rho_2(1 - \gamma)\delta_2\varepsilon_1$
	$*, *, K$	- B kills R.	-	β_1
	* No transitions: $I, I, F_1 / F_1, I, F_2 / F_1, F_1, I / F_1, F_1, F_1 / F_2, I, F_2 / F_2, F_2, I / F_2, F_2, F_1 / K, F_1, F_2 / K, F_2, F_2 / K, K, * / K, K, K$			

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
F_1, F_1, I	I, I, I	- No kill is recorded. - Two B scoot, but R holds its position.	- If at least one of the two sides decides to scoot, information from surveillance assets is lost.	$(1 - \beta_1)^2$ $(b^2(1 - \gamma) + \gamma)$
		- No kill is recorded. - R scoots.		
	F_1, I, I	- No kill is recorded. - One of fired B scoots, but R holds its position. - R's surveillance assets and R's radar do not detect enemy.	- If R's position is detected by B's surveillance assets, its position is kept track by surveillance assets until it decides to scoot. - Non-scooting B can shoot inaccurate fire again because R holds its position. - R can shoot at non-scooting B because R holds its position. However, R cannot fire because it does not detect B.	$(1 - \beta_1)^2$ $2b(1 - b)(1 - \gamma)$ $(1 - \varepsilon_1)(1 - \varepsilon_2)$
	F_1, I, F_1	- No kill is recorded. - One of fired B scoots, but R holds its position. - R's surveillance assets detect enemy. - R's radar does not detect enemy.	- If R's position is detected by B's surveillance assets, its position is kept track by surveillance assets until it decides to scoot. - Non-scooting B can shoot inaccurate fire because R holds its position. - R can shoot at non-scooting B because R holds its position. - If non-scooting B is detected by R's surveillance assets, R can shoot inaccurate fire. However, R cannot shoot accurate fire because R's radar fails to detect enemy.	$(1 - \beta_1)^2$ $2b(1 - b)(1 - \gamma)$ $\varepsilon_1(1 - \varepsilon_2)$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	F_1, I, F_2	- No kill is recorded. - One of fired B scoots, but R holds its position. - R's radar detects enemy.	- If R's position is detected by B's surveillance assets, its position is kept track by surveillance assets until it decides to scoot. - Non-scooting B can shoot inaccurate fire because R holds its position. - R can shoot at non-scooting B because R holds its position. - If non-scooting B is detected by R's radar, R can shoot accurate fire.	$(1 - \beta_1)^2$ $2b(1 - b)$ $(1 - \gamma)\varepsilon_2$
	F_1, F_1, I	- No kill is recorded. - R and two B do not Scoot. - R's surveillance assets and R's radar do not detect enemy.	- If R's position is detected by B's surveillance assets, its position is kept track by surveillance assets until it decides to scoot. - If B's location is not detected by R's surveillance assets and radar, R cannot shoot.	$(1 - \beta_1)^2$ $(1 - b)^2(1 - \gamma)$ $(1 - \varepsilon_1)(1 - \varepsilon_2)$
	F_1, F_1, F_1	- No kill is recorded. - R and two B do not Scoot. - R's surveillance assets detect enemy. - R's radar does not detect enemy.	- If R's position is detected by B's surveillance assets, its position is kept track by surveillance assets until it decides to scoot. - If B and R do not scoot and R detects enemy using surveillance assets, R can shoot inaccurate fire. However, R cannot shoot accurate fire because R's radar fails to detect enemy.	$(1 - \beta_1)^2$ $(1 - b)^2(1 - \gamma)$ $\varepsilon_1(1 - \varepsilon_2)$
	F_1, F_1, F_2	- No kill is recorded. - R and two B do not scoot. - R's radar detects enemy.	- If R's position is detected by B's surveillance assets, its position is kept track by surveillance assets until it decides to scoot. - If B and R do not scoot and R detects enemy using radar, R can shoot accurate fire.	$(1 - \beta_1)^2$ $(1 - b)^2(1 - \gamma)\varepsilon_2$
	$*, *, K$	- B kills R.	-	$1 - (1 - \beta_1)^2$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	* No transitions: $I, I, F_1 / F_2, I, I / F_2, I, F_1 / F_2, I, F_2 / F_2, F_2, I / F_2, F_2, F_1 / F_2, F_2, F_2 / K, I, I / K, I, F_1 / K, F_1, F_1 / K, F_1, F_2 / K, F_2, I / K, F_2, F_1 / K, F_2, F_2 / K, K, * / K, K, K$			
F_1, F_1, F_1	I, I, I	- No kill is recorded. - Two B scoots, but R holds its position.	- If at least one of the two sides decides to scoot, information from surveillance assets is lost.	$(1 - \beta_1)^2(1 - \rho_1)$ $(bc(1 - \gamma) + \gamma)$
		- No kill is recorded. - R scoots.		
	F_1, I, I	- No kill is recorded. - Attacked B scoots, but R and non-attacked B hold their positions. - R's surveillance assets and R's radar do not detect enemy.	- If R's position is detected by B's surveillance assets, its position is kept track by surveillance assets until it decides to scoot. - Non-scooting B can shoot inaccurate fire because R holds its position. However, B cannot shoot accurate fire because B's radar fails to detect enemy. - R can shoot at non-scooting B because R holds its position. However, R cannot fire because it does not detect B.	$(1 - \beta_1)^2(1 - \rho_1)$ $(1 - b)c(1 - \gamma)$ $(1 - \delta_2)$ $(1 - \varepsilon_1)(1 - \varepsilon_2)$
	F_1, I, F_1	- No kill is recorded. - Attacked B scoots, but R and non-attacked B hold their positions. - B's radar does not detect enemy. - R's surveillance assets detect enemy. - R's radar does not detect enemy.	- If R's position is detected by B's surveillance assets, its position is kept track by surveillance assets until it decides to scoot. - R can shoot at non-scooting B because R holds its position. - If non-scooting B is detected by R's surveillance assets, R can shoot inaccurate fire.	$(1 - \beta_1)^2(1 - \rho_1)$ $(1 - b)c(1 - \gamma)$ $(1 - \delta_2)\varepsilon_1(1 - \varepsilon_2)$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	F_1, I, F_2	- No kill is recorded. - Attacked B scoots, but R and non-attacked B hold their positions. - B's radar does not detect enemy. - R's radar detects Enemy.	- If B and R's position is detected by surveillance assets, their position is kept track by surveillance assets until at least one of them decides to scoot. - R can shoot at non-scooting B because R holds its position. - If B and R do not scoot and R detects enemy using radar, R can shoot accurate fire. However, B cannot shoot accurate fire because its radar fails to detect enemy.	$(1 - \beta_1)^2(1 - \rho_1)$ $(1 - b)c(1 - \gamma)$ $(1 - \delta_2)\varepsilon_2$
	F_1, F_1, F_1	- No kill is recorded. - Neither B nor R scoots. - B's radar does not detect enemy. - R's radar does not detect enemy.	- If B and R's position is detected by surveillance assets, their position is kept track by surveillance assets until at least one of them decides to scoot. - They cannot shoot accurate fire because their radars fail to detect enemy.	$(1 - \beta_1)^2(1 - \rho_1)$ $(1 - c)(1 - \gamma)$ $(1 - \delta_2)(1 - \varepsilon_2)$
	F_1, F_1, F_2	- No kill is recorded. - Neither B nor R scoots. - B's radar does not detect enemy. - R's radar detects enemy.	- If B and R's position is detected by surveillance assets, their position is kept track by surveillance assets until at least one of them decides to scoot. - If B and R do not scoot and R detects enemy using radar, R can shoot accurate fire. However, B cannot shoot accurate fire because its radar fails to detect enemy.	$(1 - \beta_1)^2(1 - \rho_1)$ $(1 - c)(1 - \gamma)$ $(1 - \delta_2)\varepsilon_2$
	F_2, I, I	- No kill is recorded. - Attacked B scoots, but R and non-attacked B hold their positions. - B's radar detects enemy. - R's surveillance assets and R's radar do not detect enemy.	- Even if attacked B scoots, non-scooting B can shoot accurate fire if B's radar detects enemy because R holds its position. - R can shoot at non-scooting B because R holds its position. However, R cannot fire because it does not detect another B.	$(1 - \beta_1)^2(1 - \rho_1)$ $(1 - b)c(1 - \gamma)$ $\delta_2(1 - \varepsilon_1)(1 - \varepsilon_2)$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	F_2, I, F_1	- No kill is recorded. - Attacked B scoots, but R and non-attacked B hold their positions. - B's radar detects enemy. - R's surveillance assets detect enemy. - R's radar does not detect enemy.	- Even if attacked B scoots, non-scooting B can shoot accurate fire if B's radar detects enemy because R holds its position. - R can shoot at non-scooting B because R holds its position. - If non-scooting B is detected by R's surveillance assets, R can shoot inaccurate fire.	$\frac{(1 - \beta_1)^2(1 - \rho_1)(1 - b)c(1 - \gamma)}{\delta_2 \varepsilon_1(1 - \varepsilon_2)}$
	F_2, I, F_2	- No kill is recorded. - Attacked B scoots, but R and non-attacked B hold their positions. - B's radar detects enemy. - R's radar detects enemy.	- Even if attacked B scoots, non-scooting B can shoot accurate fire if B's radar detects enemy because R holds its position. - R can shoot at non-scooting B because R holds its position even if attacked B scoots. - If non-scooting B is detected by R's radar, R can shoot accurate fire.	$\frac{(1 - \beta_1)^2(1 - \rho_1)(1 - b)c(1 - \gamma)}{\delta_2 \varepsilon_2}$
	F_2, F_2, F_1	- No kill is recorded. - Neither B nor R scoots. - B's radar does not detect enemy. - B's radar detects enemy. - R's radar does not detect enemy.	- If B and R do not scoot and B detects enemy using radar, B can shoot accurate fire. - Both B shoot the same type of fire simultaneously because they share the surveillance assets and radar. - If B's location is not detected by R's radar while keeping track by R's surveillance assets, R cannot shoot accurate fire. Therefore, R can shoot only inaccurate fire.	$\frac{(1 - \beta_1)^2(1 - \rho_1)(1 - c)(1 - \gamma)}{\delta_2(1 - \varepsilon_2)}$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	F_2, F_2, F_2	- No kill is recorded. - Neither B nor R Scoots. - B's radar detects enemy. - R's radar detects enemy.	- If B and R do not scoot and B detects enemy using radar, B can shoot accurate fire. - Both B shoot the same type of fire simultaneously because they share the surveillance assets and radar. - If B and R do not scoot and R detect enemy using radar, R can shoot accurate fire.	$(1 - \beta_1)^2(1 - \rho_1)$ $(1 - c)(1 - \gamma)\delta_2\varepsilon_2$
	K, I, I	- B does not kill R. - R kills one of B. - R and fired B scoot.	- If at least one of the two sides decides to scoot, information from surveillance assets is lost.	$(1 - \beta_1)^2\rho_1$ $(b\gamma + b(1 - \gamma) + (1 - b)\gamma)$
		- B does not kill R. - R kills one of B. - Fired B scoots, but R holds its position.		
		- B does not kill R. - R kills one of B. - R scoots, but fired B holds its position.		
	K, F_1, I	- B does not kill R. - R kills one of B. - Neither B nor R Scoots. - B's radar does not detect enemy. - R's surveillance assets and R's radar do not detect enemy.	- R can shoot at alive B because R holds its position. However, R cannot fire because it does not detect alive B. - Alive B can shoot inaccurate fire if B's surveillance assets detect enemy because R holds its position. However, B cannot shoot accurate fire because B's radar fails to detect enemy.	$(1 - \beta_1)^2\rho_1$ $(1 - b)(1 - \gamma)$ $(1 - \delta_2)$ $(1 - \varepsilon_1)(1 - \varepsilon_2)$
	K, F_1, F_1	- B does not kill R. - R kills one of B. - Neither B nor R scoots. - B's radar does not detect enemy. - R's surveillance assets detect enemy. - R's radar does not detect enemy.	- R can shoot at alive B because R holds its position. - If alive B is detected by R's surveillance assets, R can shoot inaccurate fire. - Alive B can shoot inaccurate fire if B's surveillance assets detect enemy because R holds its position. However, B cannot shoot accurate fire because B's radar fails to detect enemy.	$(1 - \beta_1)^2\rho_1$ $(1 - b)(1 - \gamma)$ $(1 - \delta_2)\varepsilon_1(1 - \varepsilon_2)$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	K, F_1, F_2	- B does not kill R. - R kills one of B. - Neither B nor R scoots. - B's radar does not detect enemy. - R's radar detects enemy.	- R can shoot at alive B because R holds its position. - If alive B is detected by R's radar, R can shoot accurate fire. - Alive B can shoot inaccurate fire if B's surveillance assets detect enemy because R holds its position. However, B cannot shoot accurate fire because B's radar fails to detect enemy.	$\frac{(1 - \beta_1)^2 \rho_1}{(1 - b)(1 - \gamma)(1 - \delta_2)\epsilon_2}$
	K, F_2, I	- B does not kill R. - R kills one of B. - Neither B nor R scoots. - B's radar detects enemy. - R's surveillance assets and R's radar do not detect enemy.	- R can shoot at alive B because R holds its position. However, R cannot fire because it does not detect alive B. - Alive B can shoot accurate fire if B's radar detects enemy because R holds its position.	$\frac{(1 - \beta_1)^2 \rho_1}{(1 - b)(1 - \gamma)\delta_2(1 - \epsilon_1)(1 - \epsilon_2)}$
	K, F_2, F_1	- B does not kill R. - R kills one of B. - Neither B nor R scoots. - B's radar detects enemy. - R's surveillance assets detect enemy. - R's radar does not detect enemy.	- R can shoot at alive B because R holds its position. - If alive B is detected by R's surveillance assets, R can shoot inaccurate fire. - Alive B can shoot accurate fire if B's radar detects enemy because R holds its position.	$\frac{(1 - \beta_1)^2 \rho_1}{(1 - b)(1 - \gamma)\delta_2\epsilon_1(1 - \epsilon_2)}$
	K, F_2, F_2	- B does not kill R. - R kills one of B. - Neither B nor R scoots. - B's radar detects enemy. - R's radar detects enemy.	- R can shoot at alive B because R holds its position. - If alive B is detected by R's radar, R can shoot accurate fire. - Alive B can shoot accurate fire if B's radar detects enemy because R holds its position.	$\frac{(1 - \beta_1)^2 \rho_1}{(1 - b)(1 - \gamma)\delta_2\epsilon_2}$
	$*, *, K$	- B kills R.	-	$1 - (1 - \beta_1)^2$
	* No transitions: $I, I, F_1 / F_1, F_1, I / F_2, F_2, I / K, I, F_1 / K, K, * / K, K, K$			
F_1, F_1, F_2	I, I, I	- No kill is recorded. - Two B scoot, but R holds its position.	- If at least one of the two sides decides to scoot, information from surveillance assets and radar is lost.	$(1 - \beta_1)^2(1 - \rho_2)(bc(1 - \gamma) + \gamma)$
		- No kill is recorded. - R scoots.		

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	F_1, I, I	- No kill is recorded. - Attacked B scoots, but R and non-attacked B hold their positions. - B's radar does not detect enemy. - R's surveillance assets and R's radar do not detect enemy.	- If R's position is detected by B's surveillance assets, its position is kept track by surveillance assets until it decides to scoot. - Non-scooting B can shoot inaccurate fire again because R holds its position. - R can shoot at non-scooting B because R holds its position. However, R cannot fire because it does not detect B.	$(1 - \beta_1)^2(1 - \rho_2)$ $(1 - b)c(1 - \gamma)$ $(1 - \delta_2)$ $(1 - \varepsilon_1)(1 - \varepsilon_2)$
	F_1, I, F_1	- No kill is recorded. - Attacked B scoots, but R and non-attacked B hold their positions. - B's radar does not detect enemy. - R's surveillance assets detect enemy. - R's radar does not detect enemy.	- If R's position is detected by B's surveillance assets, its position is kept track by surveillance assets until it decides to scoot. - Non-scooting B can shoot inaccurate fire because R holds its position. - R can shoot at non-scooting B because R holds its position. - If non-scooting B is detected by R's surveillance assets, R can shoot inaccurate fire. However, R cannot shoot accurate fire because R's radar fails to detect enemy.	$(1 - \beta_1)^2(1 - \rho_2)$ $(1 - b)c(1 - \gamma)$ $(1 - \delta_2)\varepsilon_1(1 - \varepsilon_2)$
	F_1, I, F_2	- No kill is recorded. - Attacked B scoots, but R and non-attacked B hold their positions. - B's radar does not detect enemy. - R's radar detects enemy.	- If R's position is detected by B's surveillance assets, its position is kept track by surveillance assets until it decides to scoot. - Non-scooting B can shoot inaccurate fire because R holds its position. - R can shoot at non-scooting B because R holds its position. - If non-scooting B is detected by R's radar, R can shoot accurate fire.	$(1 - \beta_1)^2(1 - \rho_2)$ $(1 - b)c(1 - \gamma)$ $(1 - \delta_2)\varepsilon_2$
	F_1, F_1, F_2	- No kill is recorded. - Neither B nor R scoots. - B's radar does not detect enemy.	- If R's position is detected by B's surveillance assets, its position is kept track by surveillance assets until it decides to scoot. - If B and R do not scoot and R detects enemy using radar, R can shoot accurate fire.	$(1 - \beta_1)^2(1 - \rho_2)$ $(1 - c)(1 - \gamma)$ $(1 - \delta_2)$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	F_2, I, I	- No kill is recorded. - Attacked B scoots, but R and non-attacked B hold their positions. - B's radar detects enemy. - R's surveillance assets and R's radar do not detect enemy.	- Even if attacked B scoots, non-scooting B can shoot accurate fire if B's radar detects enemy because R holds its position. - R can shoot at non-scooting B because R holds its position. However, R cannot fire because it does not detect non-scooting B.	$\begin{aligned} &(1 - \beta_1)^2(1 - \rho_2) \\ &(1 - b)c(1 - \gamma) \\ &\delta_2(1 - \varepsilon_1)(1 - \varepsilon_2) \end{aligned}$
	F_2, I, F_1	- No kill is recorded. - Attacked B scoots, but R and non-attacked B hold their positions. - B's radar detects enemy. - R's surveillance assets detect enemy. - R's radar does not detect enemy.	- Even if attacked B scoots, non-scooting B can shoot accurate fire if B's radar detects enemy because R holds its position. - R can shoot at non-scooting B because R holds its position. - If non-scooting B is detected by R's surveillance assets, R can shoot inaccurate fire.	$\begin{aligned} &(1 - \beta_1)^2(1 - \rho_2) \\ &(1 - b)c(1 - \gamma) \\ &\delta_2\varepsilon_1(1 - \varepsilon_2) \end{aligned}$
	F_2, I, F_2	- No kill is recorded. - Attacked B scoots, but R and non-attacked B hold their positions. - B's radar detects enemy. - R's radar detects enemy.	- Even if attacked B scoots, non-scooting B can shoot accurate fire if B's radar detects enemy because R holds its position. - R can shoot at non-scooting B because R holds its position even if attacked B scoots. - If non-scooting B is detected by R's radar, R can shoot accurate fire.	$\begin{aligned} &(1 - \beta_1)^2(1 - \rho_2) \\ &(1 - b)c(1 - \gamma) \\ &\delta_2\varepsilon_2 \end{aligned}$
	F_2, F_2, F_2	- No kill is recorded. - Neither B nor R scoots. - B's radar detects enemy.	- If B and R do not scoot and B detects enemy using radar, B can shoot accurate fire. - Both B shoot the same type of fire simultaneously because they share the surveillance assets and radar. - If B and R do not scoot and R detect enemy using radar, R can shoot accurate fire.	$\begin{aligned} &(1 - \beta_1)^2(1 - \rho_2) \\ &(1 - c)(1 - \gamma)\delta_2 \end{aligned}$
	K, I, I	- B does not kill R. - R kills one of B. - R and non-attacked B scoot.	- If at least one of the two sides decides to scoot, information from surveillance assets and radar is lost.	$\begin{aligned} &(1 - \beta_1)^2\rho_2 \\ &(b\gamma + b(1 - \gamma) \\ &+ (1 - b)\gamma) \end{aligned}$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
		- B does not kill R. - R kills one of B. - non-attacked B scoots, but R holds its position.		
		- B does not kill R. - R kills one of B. - R scoots, but non-attacked B holds its position.		
	K, F_1, I	- B does not kill R. - R kills one of B. - Neither B nor R scoots. - B's radar does not detect enemy. - R's surveillance assets and R's radar do not detect enemy.	- R can shoot at alive B because R holds its position. However, R cannot fire because it does not detect alive B. - Alive B can shoot inaccurate fire if B's surveillance assets detect enemy because R holds its position. However, B cannot shoot accurate fire because B's radar fails to detect enemy.	$(1 - \beta_1)^2 \rho_2$ $(1 - b)(1 - \gamma)$ $(1 - \delta_2)$ $(1 - \varepsilon_1)(1 - \varepsilon_2)$
	K, F_1, F_1	- B does not kill R. - R kills one of B. - Neither B nor R scoots. - B's radar does not detect enemy. - R's surveillance assets detect enemy. - R's radar does not detect enemy.	- R can shoot at alive B because R holds its position. - If alive B is detected by R's surveillance assets, R can shoot inaccurate fire. - Alive B can shoot inaccurate fire if B's surveillance assets detect enemy because R holds its position. However, B cannot shoot accurate fire because B's radar fails to detect enemy.	$(1 - \beta_1)^2 \rho_2$ $(1 - b)(1 - \gamma)$ $(1 - \delta_2)\varepsilon_1(1 - \varepsilon_2)$
	K, F_1, F_2	- B does not kill R. - R kills one of B. - Neither B nor R scoots. - B's radar does not detect enemy. - R's radar detects enemy.	- R can shoot at alive B because R holds its position. - If alive B is detected by R's radar, R can shoot accurate fire. - Alive B can shoot inaccurate fire if B's surveillance assets detect enemy because R holds its position. However, B cannot shoot accurate fire because B's radar fails to detect enemy.	$(1 - \beta_1)^2 \rho_2$ $(1 - b)(1 - \gamma)$ $(1 - \delta_2)\varepsilon_2$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	K, F_2, I	- B does not kill R. - R kills one of B. - Neither B nor R scoots. - B's radar detects enemy. - R's surveillance assets and R's radar do not detect enemy.	- R can shoot at alive B because R holds its position. However, R cannot fire because it does not detect alive B. - Alive B can shoot accurate fire if B's radar detects enemy because R holds its position.	$(1 - \beta_1)^2 \rho_2$ $(1 - b)(1 - \gamma)$ $\delta_2(1 - \varepsilon_1)(1 - \varepsilon_2)$
	K, F_2, F_1	- B does not kill R. - R kills one of B. - Neither B nor R scoots. - B's radar detects enemy. - R's surveillance assets detect enemy. - R's radar does not detect enemy.	- R can shoot at alive B because R holds its position. - If alive B is detected by R's surveillance assets, R can shoot inaccurate fire. - Alive B can shoot accurate fire if B's radar detects enemy because R holds its position.	$(1 - \beta_1)^2 \rho_2$ $(1 - b)(1 - \gamma)$ $\delta_2 \varepsilon_1(1 - \varepsilon_2)$
	K, F_2, F_2	- B does not kill R. - R kills one of B. - Neither B nor R scoots. - B's radar detects enemy. - R's radar detects enemy.	- R can shoot at alive B because R holds its position. - If alive B is detected by R's radar, R can shoot accurate fire. - Alive B can shoot accurate fire if B's radar detects enemy because R holds its position.	$(1 - \beta_1)^2 \rho_2$ $(1 - b)(1 - \gamma)$ $\delta_2 \varepsilon_2$
	$*, *, K$	- B kills R.	-	$1 - (1 - \beta_1)^2$
	* No transitions: $I, I, F_1 / F_1, F_1, I / F_1, F_1, F_1 / F_2, F_2, I / F_2, F_2, F_1 / K, I, F_1 / K, K, * / K, K, K$			
F_2, I, I	I, I, I	- No kill is recorded. - R and fired B scoot.	- If at least one of the two sides decides to scoot, information from radar is lost.	$(1 - \beta_2)$ $(b\gamma + (1 - b)\gamma)$
		- No kill is recorded. - R scoots, but fired B holds its position.		
	F_2, I, I	- No kill is recorded. - Fired B scoots, but R holds its position. - R's surveillance assets do not detect enemy.	- If R's position is detected by B's radar, information is valid until it decides to scoot. - If B's location is not detected by R's surveillance assets, R cannot shoot.	$(1 - \beta_2)$ $b(1 - \gamma)(1 - \varepsilon_1)$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	F_2, I, F_1	- No kill is recorded. - Fired B scoots, but R holds its position. - R's surveillance assets detect enemy.	- If R's position is detected by B's radar, information is valid until it decides to scoot. - Non-scooting B can shoot accurate fire because both B share the surveillance assets and radar, and R holds its position. - R can shoot at non-scooting B because R holds its position. - If non-scooting B is detected by R's surveillance assets, R can shoot inaccurate fire.	$\frac{(1 - \beta_2)}{b(1 - \gamma)\epsilon_1}$
	F_2, F_2, I	- No kill is recorded. - Neither B nor R scoots. - R's surveillance assets and R's radar do not detect enemy.	- If R's position is detected by B's radar, information is valid until it decides to scoot. - Both B shoot the same type of fire simultaneously because they share the surveillance assets and radar. - If B's location is not detected by R's surveillance assets and radar, R cannot shoot.	$\frac{(1 - \beta_2)}{(1 - b)(1 - \gamma)(1 - \epsilon_1)(1 - \epsilon_2)}$
	F_2, F_2, F_1	- No kill is recorded. - Neither B nor R scoots. - R's surveillance assets detect enemy. - R's radar does not detect enemy.	- If R's position is detected by B's radar, information is valid until it decides to scoot. - Both B shoot the same type of fire simultaneously because they share the surveillance assets and radar. - If B and R do not scoot and R detects enemy using surveillance assets, R can shoot inaccurate fire. However, R cannot shoot accurate fire because R's radar fails to detect enemy.	$\frac{(1 - \beta_2)}{(1 - b)(1 - \gamma)\epsilon_1(1 - \epsilon_2)}$
	F_2, F_2, F_2	- No kill is recorded. - Neither B nor R scoots. - R's radar detects enemy.	- If R's position is detected by B's radar, information is valid until it decides to scoot. - Both B shoot the same type of Fire simultaneously because they share the surveillance assets and radar. - If B and R do not scoot and R detects enemy using radar, R can shoot accurate fire.	$\frac{(1 - \beta_2)}{(1 - b)(1 - \gamma)\epsilon_2}$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	$*, *, K$	- B kills R.	-	β_2
	* No transitions: $I, I, F_1 / F_1, I, I / F_1, I, F_1 / F_1, I, F_2 / F_1, F_1, I / F_1, F_1, F_1 / F_1, F_1, F_2 / F_2, I, F_2 / K, I, I / K, I, F_1 / K, F_1, I / K, F_1, F_1 / K, F_1, F_2 / K, F_2, I / K, F_2, F_1 / K, K, * / K, K, K$			
F_2, I, F_1	I, I, I	- No kill is recorded. - R and attacked B scoot. - No kill is recorded. - R scoots, but attacked B holds its position.	- If at least one of the two sides decides to scoot, information from surveillance assets and radar is lost.	$(1 - \beta_2)(1 - \rho_1)$ $(c\gamma + (1 - c)\gamma)$
	F_2, I, I	- No kill is recorded. - Attacked B scoots, but R holds its position. - R's surveillance assets do not detect enemy.	- If R's position is detected by B's radar, information is valid until it decides to scoot. - Non-scooting B can shoot accurate fire because R holds its position. - R can shoot at non-scooting B because R holds its position. However, R cannot fire because it does not detect another B.	$(1 - \beta_2)(1 - \rho_1)$ $c(1 - \gamma)(1 - \varepsilon_1)$
	F_2, I, F_1	- No kill is recorded. - Attacked B scoots, but R holds its position. - R's surveillance assets detect enemy.	- If R's position is detected by B's radar, information is valid until it decides to scoot. - Non-scooting B can shoot accurate fire because R holds its position. - R can shoot at non-scooting B because R holds its position even if attacked B scoots. - If non-scooting B is detected by R's surveillance assets, R can shoot inaccurate fire.	$(1 - \beta_2)(1 - \rho_1)$ $c(1 - \gamma)\varepsilon_1$
	F_2, F_2, F_1	- No kill is recorded. - Neither B nor R scoots. - R's radar does not detect enemy.	- If R's position is detected by B's radar, information is valid until it decides to scoot. - Both B shoot the same type of fire simultaneously because they share the surveillance assets and radar. - R cannot shoot accurate fire because R's radar fails to detect enemy.	$(1 - \beta_2)(1 - \rho_1)$ $(1 - c)(1 - \gamma)$ $(1 - \varepsilon_2)$
	F_2, F_2, F_2	- No kill is recorded. - Neither B nor R scoots. - R's radar detects Enemy.	- If R's position is detected by B's radar, information is valid until it decides to scoot. - Both B shoot the same type of fire simultaneously because they share the surveillance	$(1 - \beta_2)(1 - \rho_1)$ $(1 - c)(1 - \gamma)\varepsilon_2$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
			assets and radar. - If B and R do not scoot and R detect enemy using radar, R can shoot accurate fire.	
	K, I, I	- R kills B. - B does not kill R. - R scoots.	- If at least one of the two sides decides to scoot, information from surveillance assets and radar is lost.	$(1 - \beta_2)\rho_1\gamma$
	K, F_2, I	- R kills B. - B does not kill R. - R does not scoot. - R's surveillance assets do not detect enemy.	- R can shoot at alive B because R holds its position. However, R cannot fire because it does not detect alive B. - Alive B can shoot accurate fire if B's radar detects enemy because R holds its position.	$(1 - \beta_2)\rho_1$ $(1 - \gamma)(1 - \varepsilon_1)$
	K, F_2, F_1	- R kills B. - B does not kill R. - R does not scoot. - R's surveillance assets detect enemy.	- R can shoot at alive B because R holds its position. - If alive B is detected by R's surveillance assets, R can shoot inaccurate fire. - Alive B can shoot accurate fire if B's radar detects enemy because R holds its position.	$(1 - \beta_2)\rho_1$ $(1 - \gamma)\varepsilon_1$
	$*, *, K$	- B kills R.	-	β_2
	* No transitions: $I, I, F_1 / F_1, I, I / F_1, I, F_1 / F_1, I, F_2 / F_1, F_1, I / F_1, F_1, F_1 / F_1, F_1, F_2 / F_2, I, F_2 / F_2, F_2, I / K, I, F_1 / K, F_1, I / K, F_1, F_1 / K, F_1, F_2 / K, F_2, F_2 / K, K, * / K, K, K$			
F_2, I, F_2	I, I, I	- No kill is recorded. - R and attacked B scoot.	- If at least one of the two sides decides to scoot, information from surveillance assets and radar is lost.	$(1 - \beta_2)(1 - \rho_2)$ $(c\gamma + (1 - c)\gamma)$
		- No kill is recorded. - R scoots, but attacked B holds its position.		

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	F_2, I, I	- No kill is recorded. - Attacked B scoots, but R holds its position. - R's surveillance assets do not detect enemy.	- Even if attacked B scoots, non-scooting B can shoot accurate fire if B's radar detects enemy because R holds its position. - R can shoot at non-scooting B because R holds its position even if attacked B scoots. However, R cannot fire because it does not detect another B.	$(1 - \beta_2)(1 - \rho_2)$ $c(1 - \gamma)(1 - \varepsilon_1)$
	F_2, I, F_1	- No kill is recorded. - Attacked B scoots, but R holds its position. - R's surveillance assets detect enemy.	- Even if attacked B scoots, non-scooting B can shoot accurate fire if B's radar detects enemy because R holds its position. - R can shoot at another B because R holds its position even if attacked B scoots. - If non-scooting B is detected by R's surveillance assets, R can shoot inaccurate fire.	$(1 - \beta_2)(1 - \rho_2)$ $c(1 - \gamma)\varepsilon_1$
	F_2, F_2, F_2	- No kill is recorded. - Neither B nor R scoots.	- If B and R do not scoot and B detects enemy using radar, B can shoot accurate fire. - Both B shoot the same type of fire simultaneously because they share the surveillance assets and radar. - If B and R do not scoot and R was shooting accurate fire, R can shoot accurate fire repeatedly.	$(1 - \beta_2)(1 - \rho_2)$ $(1 - c)(1 - \gamma)$
	K, I, I	- R kills B. - B does not kill R. - R scoots.	- If at least one of the two sides decides to scoot, information from surveillance assets and radar is lost.	$(1 - \beta_2)\rho_2\gamma$
	K, F_2, I	- R kills B. - B does not kill R. - R does not scoot. - R's surveillance assets do not detect enemy.	- R can shoot at alive B because R holds its position. However, R cannot fire because it does not detect alive B. - Alive B can shoot accurate fire if B's radar detects enemy because R holds its position.	$(1 - \beta_2)\rho_2$ $(1 - \gamma)(1 - \varepsilon_1)$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	K, F_2, F_1	- R kills B. - B does not kill R. - R does not scoot. - R's surveillance assets detect enemy.	- R can shoot at alive B because R holds its position. - If alive B is detected by R's surveillance assets, R can shoot inaccurate fire. - Alive B can shoot accurate fire if B's radar detects enemy because R holds its position.	$(1 - \beta_2)\rho_2$ $(1 - \gamma)\varepsilon_1$
	$*, *, K$	- B kills R.	-	β_2
	* No transitions: $I, I, F_1 / F_1, I, I / F_1, I, F_1 / F_1, I, F_2 / F_1, F_1, I / F_1, F_1, F_1 / F_1, F_1, F_2 / F_2, I, F_2 / F_2, F_2, I / F_2, F_2, F_1 / K, I, F_1 / K, F_1, I / K, F_1, F_1 / K, F_1, F_2 / K, F_2, F_2 / K, K, * / K, K, K$			
F_2, F_2, I	I, I, I	- No kill is recorded. - Two B scoot, but R holds its position.	- If at least one of the two sides decides to scoot, information from radar is lost.	$(1 - \beta_2)^2$ $(b^2(1 - \gamma) + \gamma)$
		- No kill is recorded. - R scoots.		
	F_2, I, I	- No kill is recorded. - One of fired B scoots, but R holds its position. - R's surveillance assets and R's radar do not detect enemy.	- If R's position is detected by B's radar, information is valid until it decides to scoot. - If B's location is not detected by R's surveillance assets and radar, R cannot shoot.	$(1 - \beta_2)^2$ $2b(1 - b)(1 - \gamma)$ $(1 - \varepsilon_1)(1 - \varepsilon_2)$
	F_2, I, F_1	- No kill is recorded. - One of fired B scoots, but R holds its position. - R's surveillance assets detect enemy. - R's radar does not detect enemy.	- If R's position is detected by B's radar, information is valid until it decides to scoot. - Non-scooting B can shoot accurate fire because R holds its position. - R can shoot at non-scooting B because R holds its position. - If non-scooting B is detected by R's surveillance assets, R can shoot inaccurate fire.	$(1 - \beta_2)^2$ $2b(1 - b)(1 - \gamma)$ $\varepsilon_1(1 - \varepsilon_2)$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	F_2, I, F_2	- No kill is recorded. - One of fired B scoots, but R holds its position. - R's radar detects enemy.	- If R's position is detected by B's radar, information is valid until it decides to scoot. - Non-scooting B can shoot accurate fire because R holds its position. - R can shoot at non-scooting B because R holds its position. - If non-scooting B is detected by R's radar, R can shoot accurate fire.	$\frac{(1 - \beta_2)^2}{2b(1 - b)(1 - \gamma)\epsilon_2}$
	F_2, F_2, I	- No kill is recorded. - Neither B nor R scoots. - R's surveillance assets and R's radar do not detect enemy.	- If R's position is detected by B's radar, information is valid until it decides to scoot. - If B's location is not detected by R's surveillance assets and radar, R cannot shoot.	$\frac{(1 - \beta_2)^2}{(1 - b)^2(1 - \gamma)(1 - \epsilon_1)(1 - \epsilon_2)}$
	F_2, F_2, F_1	- No kill is recorded. - Neither B nor R scoots. - R's surveillance assets detect enemy. - R's radar does not detect enemy.	- If R's position is detected by B's radar, information is valid until it decides to scoot. - If B and R do not scoot and R detects enemy using surveillance assets, R can shoot inaccurate fire. However, R cannot shoot accurate fire because R's radar fails to detect enemy.	$\frac{(1 - \beta_2)^2}{(1 - b)^2(1 - \gamma)\epsilon_1(1 - \epsilon_2)}$
	F_2, F_2, F_2	- No kill is recorded. - Neither B nor R scoots. - R's radar detects enemy.	- If R's position is detected by B's radar, information is valid until it decides to scoot. - If B and R do not scoot and R detects enemy using radar, R can shoot accurate fire.	$\frac{(1 - \beta_2)^2}{(1 - b)^2(1 - \gamma)\epsilon_2}$
	$*, *, K$	- B kills R.	-	$1 - (1 - \beta_2)^2$
	* No transitions: $I, I, F_1 / F_1, I, I / F_1, I, F_1 / F_1, I, F_2 / F_1, F_1, I / F_1, F_1, F_1 / F_1, F_1, F_2 /$ $K, I, I / K, I, F_1 / K, F_1, I / K, F_1, F_1 / K, F_1, F_2 / K, F_2, I / K, F_2, F_1 /$ $K, F_2, F_2 / K, K, * / K, K, K$			
F_2, F_2, F_1	I, I, I	- No kill is recorded. - Two B scoot, but R holds its position.	- If at least one of the two sides decides to scoot, information from surveillance assets and radar is lost.	$\frac{(1 - \beta_2)^2(1 - \rho_1)}{(bc(1 - \gamma) + \gamma)}$
		- No kill is recorded. - R scoots.		

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	F_2, I, I	- No kill is recorded. - Attacked B scoots, but R and non-attacked B hold their positions. - R's surveillance assets and R's radar do not detect enemy.	- Even if attacked B scoots, non-scooting B can shoot accurate fire because R holds its position. - R can shoot at non-scooting B because R holds its position. However, R cannot fire because it does not detect another B.	$(1 - \beta_2)^2(1 - \rho_1)$ $(1 - b)c(1 - \gamma)$ $(1 - \varepsilon_1)(1 - \varepsilon_2)$
	F_2, I, F_1	- No kill is recorded. - Attacked B scoots, but R and non-attacked B hold their positions. - R's surveillance assets detect enemy. - R's radar does not detect enemy.	- Even if attacked B scoots, non-scooting B can shoot accurate fire because R holds its position. - R can shoot at non-scooting B because R holds its position. - If non-scooting B is detected by R's surveillance assets, R can shoot inaccurate fire.	$(1 - \beta_2)^2(1 - \rho_1)$ $(1 - b)c(1 - \gamma)\varepsilon_1(1 - \varepsilon_2)$
	F_2, I, F_2	- No kill is recorded. - Attacked B scoots, but R and non-attacked B hold their positions. - R's radar detects enemy.	- Even if attacked B scoots, non-scooting B can shoot accurate fire because R holds its position. - R can shoot at non-scooting B because R holds its position. - If non-scooting B is detected by R's radar, R can shoot accurate fire.	$(1 - \beta_2)^2(1 - \rho_1)$ $(1 - b)c(1 - \gamma)\varepsilon_2$
	F_2, F_2, F_1	- No kill is recorded. - Neither B nor R scoots. - R's radar does not detect enemy.	- If B and R do not scoot and B detects enemy using radar, B can shoot accurate fire. - If B's location is not detected by R's radar while keeping track by R's surveillance assets, R cannot shoot accurate fire. Therefore, R can shoot only inaccurate fire.	$(1 - \beta_2)^2(1 - \rho_1)$ $(1 - c)(1 - \gamma)(1 - \varepsilon_2)$
	F_2, F_2, F_2	- No kill is recorded. - Neither B nor R scoots. - R's radar detects enemy.	- If B and R do not scoot and B detects enemy using radar, B can shoot accurate fire. - If B and R do not scoot and R detect enemy using radar, R can shoot accurate fire.	$(1 - \beta_2)^2(1 - \rho_1)$ $(1 - c)(1 - \gamma)\varepsilon_2$
	K, I, I	- R kills B. - B does not kill R.	- If at least one of the two sides decides to scoot, information	$(1 - \beta_2)^2\rho_1$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
		- R and non-attacked B scoot. - R kills B. - B does not kill R. - Non-attacked B scoots, but R holds its position. - R kills B. - B does not kill R. - R scoots, but non-attacked B holds its position.	from surveillance assets and radar is lost.	$(b\gamma + b(1 - \gamma) + (1 - b)\gamma)$
	K, F_2, I	- R kills B. - B does not kill R. - Neither B nor R scoots. - R's surveillance assets and R's radar do not detect enemy.	- R can shoot at alive B because R holds its position. However, R cannot fire because it does not detect alive B. - Alive B can shoot accurate fire if B's radar detects enemy because R holds its position.	$(1 - \beta_2)^2 \rho_1 (1 - b)(1 - \gamma) (1 - \varepsilon_1)(1 - \varepsilon_2)$
	K, F_2, F_1	- R kills B. - B does not kill R. - Neither B nor R scoots. - R's surveillance assets detect enemy. - R's radar does not detect enemy.	- R can shoot at alive B because R holds its position. - If alive B is detected by R's surveillance assets, R can shoot inaccurate fire. - Alive B can shoot accurate fire if B's radar detects enemy because R holds its position.	$(1 - \beta_2)^2 \rho_1 (1 - b)(1 - \gamma) \varepsilon_1(1 - \varepsilon_2)$
	K, F_2, F_2	- R kills B. - B does not kill R. - Neither B nor R scoots. - R's radar detects enemy.	- R can shoot at alive B because R holds its position. - If alive B is detected by R's radar, R can shoot accurate fire. - Alive B can shoot accurate fire if B's radar detects enemy because R holds its position.	$(1 - \beta_2)^2 \rho_1 (1 - b)(1 - \gamma) \varepsilon_2$
	$*, *, K$	- B kills R.	-	$1 - (1 - \beta_2)^2$
	* No transitions: $I, I, F_1 / F_1, I, I / F_1, I, F_1 / F_1, I, F_2 / F_1, F_1, I / F_1, F_1, F_1 / F_1, F_1, F_2 / F_2, F_2, I / K, I, F_1 / K, F_1, I / K, F_1, F_1 / K, F_1, F_2 / K, K, * / K, K, K$			
F_2, F_2, F_2	I, I, I	- No kill is recorded. - Two B scoot, but R holds its position. - No kill is recorded. - R scoots.	- If at least one of the two sides decides to scoot, information from surveillance assets and radar is lost.	$(1 - \beta_2)^2 (1 - \rho_2) (bc(1 - \gamma) + \gamma)$
	F_2, I, I	- No kill is recorded. - Attacked B scoots, but R and non-attacked B hold their positions. - R's surveillance	- Even if attacked B scoots, non-scooting B can shoot accurate fire because R holds its position. - R can shoot at non-scooting B because R holds its position.	$(1 - \beta_2)^2 (1 - \rho_2) (1 - b)c(1 - \gamma) (1 - \varepsilon_1)(1 - \varepsilon_2)$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
		assets and R's radar do not detect enemy.	However, R cannot fire because it does not detect another B.	
	F_2, I, F_1	- No kill is recorded. - Attacked B scoots, but R and non-attacked B hold their positions. - R's surveillance assets detect enemy. - R's radar does not detect enemy.	- Even if attacked B scoots, non-scooting B can shoot accurate fire. - R can shoot at non-scooting B because R holds its position even if attacked B scoots. - If non-scooting B is detected by R's surveillance assets, R can shoot inaccurate fire.	$(1 - \beta_2)^2(1 - \rho_2)$ $(1 - b)c(1 - \gamma)$ $\varepsilon_1(1 - \varepsilon_2)$
	F_2, I, F_2	- No kill is recorded. - Attacked B scoots, but R and non-attacked B hold their positions. - R's radar detects Enemy.	- Even if attacked B scoots, non-scooting B can shoot accurate fire because R holds its position. - R can shoot at non-scooting B because R holds its position. - If non-scooting B is detected by R's radar, R can shoot accurate fire.	$(1 - \beta_2)^2(1 - \rho_2)$ $(1 - b)c(1 - \gamma)\varepsilon_2$
	F_2, F_2, F_2	- No kill is recorded. - Neither B nor R scoots.	- If B and R do not scoot and B detects enemy using radar, B can shoot accurate fire. - If B and R do not scoot and R detect enemy using radar, R can shoot accurate fire.	$(1 - \beta_2)^2(1 - \rho_2)$ $(1 - c)(1 - \gamma)$
	K, I, I	- R kills B. - B does not kill R. - R and non-attacked B scoot.	- If at least one of the two sides decides to scoot, information from surveillance assets and radar is lost.	$(1 - \beta_2)^2\rho_2$ $(b\gamma + b(1 - \gamma) + (1 - b)\gamma)$
		- R kills B. - B does not kill R. - Non-attacked B scoots, but R holds its position.		
		- R kills B. - B does not kill R. - R scoots, but non-attacked B holds its position.		
	K, F_2, I	- R kills B. - B does not kill R. - Neither B nor R scoots. - R's surveillance assets and R's radar do not detect enemy.	- R can shoot at alive B because R holds its position. However, R cannot fire because it does not detect alive B. - Alive B can shoot accurate fire if B's radar detects enemy because R holds its position.	$(1 - \beta_2)^2\rho_2$ $(1 - b)(1 - \gamma)$ $(1 - \varepsilon_1)(1 - \varepsilon_2)$
	K, F_2, F_1	- R kills B. - B does not kill R.	- R can shoot at alive B because R holds its position.	$(1 - \beta_2)^2\rho_2$ $(1 - b)(1 - \gamma)$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
		- Neither B nor R scoots. - R's surveillance assets detect enemy. - R's radar does not detect enemy.	- If alive B is detected by R's surveillance assets, R can shoot inaccurate fire. - Alive B can shoot accurate fire if B's radar detects enemy because R holds its position.	$\varepsilon_1(1 - \varepsilon_2)$
	K, F_2, F_2	- R kills B. - B does not kill R. - Neither B nor R scoots. - R's radar detects enemy.	- R can shoot at alive B because R holds its position. - If alive B is detected by R's radar, R can shoot accurate fire. - Alive B can shoot accurate fire if B's radar detects enemy because R holds its position.	$(1 - \beta_2)^2 \rho_2$ $(1 - b)(1 - \gamma)\varepsilon_2$
	$*, *, K$	- B kills R.	-	$1 - (1 - \beta_2)^2$
	* No transitions: $I, I, F_1 / F_1, I, I / F_1, I, F_1 / F_1, I, F_2 / F_1, F_1, I / F_1, F_1, F_1 / F_1, F_1, F_2 / F_2, I, I / F_2, F_2, F_1 / K, I, F_1 / K, F_1, I / K, F_1, F_1 / K, F_1, F_2 / K, K, * / K, K, K$			
K, I, I	K, I, I	- Both B's surveillance assets and R's surveillance assets do not detect enemy.	- If enemy's location is not detected by surveillance assets, they cannot shoot.	$(1 - \delta_1)(1 - \varepsilon_1)$
	K, I, F_1	- B's surveillance assets do not detect enemy. - R's surveillance assets detect enemy.	- If B's location is detected by R's surveillance assets, R can shoot inaccurate fire.	$(1 - \delta_1)\varepsilon_1$
	K, F_1, I	- B's surveillance assets detect enemy. - R's surveillance assets do not detect enemy.	- If R's location is detected by B's surveillance assets, B can shoot inaccurate fire.	$\delta_1(1 - \varepsilon_1)$
	K, F_1, F_1	- Both B's surveillance assets and R's surveillance assets detect enemy.	- If B and R detect the enemy by their surveillance assets, they can shoot inaccurate fire.	$\delta_1\varepsilon_1$
	* No transitions: $I, I, I / I, I, F_1 / F_1, I, I / F_1, I, F_1 / F_1, I, F_2 / F_1, F_1, I / F_1, F_1, F_1 / F_1, F_1, F_2 / F_2, I, I / F_2, I, F_1 / F_2, I, F_2 / F_2, F_2, I / F_2, F_2, F_1 / F_2, F_2, F_2 / K, F_1, F_2 / K, F_2, I / K, F_2, F_1 / K, F_2, F_2 / K, K, * / *, *, K / K, K, K$			
K, I, F_1	K, I, I	- No kill is recorded. - At least one of the two sides scoots.	- If at least one of the two sides decides to scoot, information from surveillance assets is lost.	$(1 - \rho_1)$ $(\alpha c + (1 - \alpha)c$ $+ \alpha(1 - c))$
	K, I, F_1	- No kill is recorded. - Neither B nor R scoots. - B's surveillance assets and radar do not detect enemy.	- If R's location is not detected by B's surveillance assets and radar, B cannot shoot.	$(1 - \rho_1)$ $(1 - \alpha)(1 - c)$ $(1 - \delta_1)(1 - \delta_2)$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	K, F_1, F_1	- No kill is recorded. - Neither B nor R scoots. - B's surveillance assets detect enemy. - B's radar do not detect enemy.	- If B and R do not scoot and B detect enemy using surveillance assets, B can shoot inaccurate fire. However, B cannot shoot accurate fire because B's radar fails to detect enemy.	$(1 - \rho_1)$ $(1 - \alpha)(1 - c)$ $\delta_1(1 - \delta_2)$
	K, F_2, F_1	- No kill is recorded. - Neither B nor R scoots. - B's radar detect enemy.	- If B and R do not scoot and B detect enemy using radar, B can shoot accurate fire.	$(1 - \rho_1)$ $(1 - \alpha)(1 - c)\delta_2$
	$K, K, *$	- R kills B.	-	ρ_1
	* No transitions: $I, I, I / I, I, F_1 / F_1, I, I / F_1, I, F_1 / F_1, I, F_2 / F_1, F_1, I / F_1, F_1, F_1 / F_1, F_1, F_2 / F_2, I, I / F_2, I, F_1 / F_2, I, F_2 / F_2, I, F_2 / F_2, I / F_2, F_2, F_1 / F_2, F_2, F_2 / K, F_1, I / K, F_1, F_2 / K, F_2, I / K, F_2, F_2 / *, *, K / K, K, K$			
K, F_1, I	K, I, I	- No kill is recorded. - At least one of the two sides scoots.	- If at least one of the two sides decides to scoot, information from surveillance assets is lost.	$(1 - \beta_1)$ $(b\gamma + (1 - b)\gamma + b(1 - \gamma))$
	K, F_1, I	- No kill is recorded. - Neither B nor R scoots. - R's surveillance assets and R's radar do not detect enemy.	- If B's location is not detected by R's surveillance assets and radar, R cannot shoot.	$(1 - \beta_1)$ $(1 - b)(1 - \gamma)$ $(1 - \varepsilon_1)(1 - \varepsilon_2)$
	K, F_1, F_1	- No kill is recorded. - Neither B nor R scoots. - R's surveillance assets detect enemy. - R's radar do not detect enemy.	- If B and R do not scoot and R detect enemy using surveillance assets, R can shoot inaccurate fire. However, R cannot shoot accurate fire because R's radar fails to detect enemy.	$(1 - \beta_1)$ $(1 - b)(1 - \gamma)$ $\varepsilon_1(1 - \varepsilon_2)$
	K, F_1, F_2	- No kill is recorded. - Neither B nor R scoots. - R's radar detect enemy.	- If B and R do not scoot and R detect enemy using radar, R can shoot accurate fire.	$(1 - \beta_1)$ $(1 - b)(1 - \gamma)\varepsilon_2$
	$*, *, K$	- B kills R.	-	β_1
	* No transitions: $I, I, I / I, I, F_1 / F_1, I, I / F_1, I, F_1 / F_1, I, F_2 / F_1, F_1, I / F_1, F_1, F_1 / F_1, F_1, F_2 / F_2, I, I / F_2, I, F_1 / F_2, I, F_2 / F_2, F_2, I / F_2, F_2, F_1 / F_2, F_2, F_2 / K, I, F_1 / K, F_2, I / K, F_2, F_1 / K, F_2, F_2 / K, K, * / K, K, K$			

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
K, F_1, F_1	K, I, I	- No kill is recorded. - At least one of the two sides scoots.	- If at least one of the two sides decides to scoot, information from surveillance assets is lost.	$(1 - \beta_1)(1 - \rho_1)$ $(c\gamma + (1 - c)\gamma + c(1 - \gamma))$
	K, F_1, F_1	- No kill is recorded. - Neither B nor R scoots. - Neither B's radar nor R's Radar detects enemy.	- If B and R's position is detected by their surveillance assets, their position is kept track by surveillance assets until at least one of them decides to scoot. Therefore, they can shoot inaccurate fire repeatedly. - B and R cannot shoot accurate fire because they fail to detect enemy's location by their radar.	$(1 - \beta_1)(1 - \rho_1)$ $(1 - c)(1 - \gamma)$ $(1 - \delta_2)(1 - \varepsilon_2)$
	K, F_1, F_2	- No kill is recorded. - Neither B nor R scoots. - B's radar does not detect enemy. - R's radar detects enemy.	- If B and R's position is detected by their surveillance assets, their position is kept track by surveillance assets until one of them decides to scoot. - R can shoot accurate fire because its radar detects enemy. - If B's location is detected by R's radar while keeping track by R's surveillance assets, R shoot accurate fire, not inaccurate fire.	$(1 - \beta_1)(1 - \rho_1)$ $(1 - c)(1 - \gamma)$ $(1 - \delta_2)\varepsilon_2$
	K, F_2, F_1	- No kill is recorded. - Neither B nor R scoots. - B's radar detects enemy. - R's radar does not detect enemy.	- If B and R's position is detected by their surveillance assets, their position is kept track by surveillance assets until one of them decides to scoot. - B can shoot accurate fire because its radar detects enemy. - If R's location is detected by B's radar while keeping track by B's surveillance assets, B shoot accurate fire, not inaccurate fire.	$(1 - \beta_1)(1 - \rho_1)$ $(1 - c)(1 - \gamma)$ $\delta_2(1 - \varepsilon_2)$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	K, F_2, F_2	- No kill is recorded. - Neither B nor R scoots. - B's radar detects enemy. - R's radar detects enemy.	- If B and R's position is detected by their surveillance assets, their position is kept track by surveillance assets until one of them decides to scoot. - If enemy's location is detected by radar while keeping track by surveillance assets, they shoot accurate fire, not inaccurate fire.	$(1 - \beta_1)(1 - \rho_1)$ $(1 - c)(1 - \gamma)$ $\delta_2 \varepsilon_2$
	$K, K, *$	- R kills B. - B does not kill R.	-	$(1 - \beta_1)\rho_1$
	$*, *, K$	- B kills R. - R does not kill B.	-	$\beta_1(1 - \rho_1)$
	K, K, K	- Simultaneous kill of both sides.	-	$\beta_1\rho_1$
	* No transitions: $I, I, I / I, I, F_1 / F_1, I, I / F_1, I, F_1 / F_1, I, F_2 / F_1, F_1, I / F_1, F_1, F_1 / F_1, F_1, F_2 / F_2, I, I / F_2, I, F_1 / F_2, I, F_2 / F_2, F_2, I / F_2, F_2, F_1 / F_2, F_2, F_2 / K, I, F_1 / K, F_1, I / K, F_2, I$			
K, F_1, F_2	K, I, I	- No kill is recorded. - At least one of the two sides scoots.	- If at least one of the two sides decides to scoot, information from surveillance assets and radar is lost.	$(1 - \beta_1)(1 - \rho_2)$ $(c\gamma + (1 - c)\gamma + c(1 - \gamma))$
	K, F_1, F_2	- No kill is recorded. - Neither B nor R scoots. - B's radar does not detect enemy.	- If B and R's position is detected by their surveillance assets or radar, their position is kept track until at least one of them decides to scoot. Therefore, they can shoot repeatedly. - If R's location is not detected by B's radar while keeping track by B's surveillance assets, B cannot shoot accurate fire. Therefore, B can shoot only inaccurate fire.	$(1 - \beta_1)(1 - \rho_2)$ $(1 - c)(1 - \gamma)$ $(1 - \delta_2)$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	K, F_2, F_2	- No kill is recorded. - Neither B nor R scoots. - B's radar detects enemy.	- If B and R's position is detected by their surveillance assets or radar, their position is kept track until at least one of them decides to scoot. Therefore, they can shoot repeatedly. - If R's location is detected by B's radar while keeping track by B's surveillance assets, B shoot accurate fire, not inaccurate fire.	$(1 - \beta_1)(1 - \rho_2)$ $(1 - c)(1 - \gamma)\delta_2$
	$K, K, *$	- R kills B. - B does not kill R.	-	$(1 - \beta_1)\rho_2$
	$*, *, K$	- B kills R. - R does not kill B.	-	$\beta_1(1 - \rho_2)$
	K, K, K	- Simultaneous kill of both sides.	-	$\beta_1\rho_2$
	* No transitions: $I, I, I / I, I, F_1 / F_1, I, I / F_1, I, F_1 / F_1, I, F_2 / F_1, F_1, I / F_1, F_1, F_1 / F_1, F_1, F_2 / F_2, I, I / F_2, I, F_1 / F_2, I, F_2 / F_2, I, F_2 / F_2, F_1 / F_2, F_2, F_1 / F_2, F_2, F_2 / K, I, F_1 / K, F_1, I / K, F_1, F_1 / K, F_2, I / K, F_2, F_1$			
K, F_2, I	K, I, I	- No kill is recorded. - At least one of the two sides scoots.	- If at least one of the two sides decides to scoot, information from radar is lost.	$(1 - \beta_2)$ $(b\gamma + (1 - b)\gamma + b(1 - \gamma))$
	K, F_2, I	- No kill is recorded. - Neither B nor R scoots. - R's surveillance assets and R's radar do not detect enemy.	- If B's location is not detected by R's surveillance assets and radar, R cannot shoot.	$(1 - \beta_2)$ $(1 - b)(1 - \gamma)$ $(1 - \varepsilon_1)(1 - \varepsilon_2)$
	K, F_2, F_1	- No kill is recorded. - Neither B nor R scoots. - R's surveillance assets detect enemy. - R's radar does not detect enemy.	- If B and R do not scoot and R detect enemy using surveillance assets, R can shoot inaccurate fire. However, R cannot shoot accurate fire because R's radar fails to detect enemy.	$(1 - \beta_2)$ $(1 - b)(1 - \gamma)$ $\varepsilon_1(1 - \varepsilon_2)$
	K, F_2, F_2	- No kill is recorded. - Neither B nor R scoots. - R's radar detects enemy.	- If B and R do not scoot and R detect enemy using radar, R can shoot accurate fire.	$(1 - \beta_2)$ $(1 - b)(1 - \gamma)\varepsilon_2$
	$*, *, K$	- B kills R.	-	β_2
	* No transitions: $I, I, I / I, I, F_1 / F_1, I, I / F_1, I, F_1 / F_1, I, F_2 / F_1, F_1, I / F_1, F_1, F_1 / F_1, F_1, F_2 / F_2, I, I / F_2, I, F_1 / F_2, I, F_2 / F_2, F_2, I / F_2, F_2, F_1 / F_2, F_2, F_2 / K, I, F_1 / K, F_1, I / K, F_1, F_1 / K, F_1, F_2 / K, K, * / K, K, K$			

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
K, F_2, F_1	K, I, I	- No kill is recorded. - At least one of the two sides scoots.	- If at least one of the two sides decides to scoot, information from surveillance assets and radar is lost.	$(1 - \beta_2)(1 - \rho_1)$ $(c\gamma + (1 - c)\gamma + c(1 - \gamma))$
	K, F_2, F_1	- No kill is recorded. - Neither B nor R scoots. - R's radar does not detect enemy.	- If B and R's position is detected by surveillance assets or radar, they can shoot until at least one of them decides to scoot. Therefore, they can shoot at each other repeatedly. - If B's location is not detected by R's radar while keeping track by R's surveillance assets, R cannot shoot accurate fire. Therefore, R can shoot only inaccurate fire.	$(1 - \beta_2)(1 - \rho_1)$ $(1 - c)(1 - \gamma)$ $(1 - \varepsilon_2)$
	K, F_2, F_2	- No kill is recorded. - Neither B nor R scoots. - R's radar detects enemy.	- If B and R's position is detected by surveillance assets or radar, they can shoot until at least one of them decides to scoot. Therefore, they can shoot at each other repeatedly. - If B's location is detected by R's radar while keeping track by R's surveillance assets, R shoot accurate fire, not inaccurate fire.	$(1 - \beta_2)(1 - \rho_1)$ $(1 - c)(1 - \gamma)\varepsilon_2$
	$K, K, *$	- R kills B. - B does not kill R.	-	$(1 - \beta_2)\rho_1$
	$*, *, K$	- B kills R. - R does not kill B.	-	$\beta_2(1 - \rho_1)$
	K, K, K	- Simultaneous kill of both sides.	-	$\beta_2\rho_1$
	* No transitions: $I, I, I / I, I, F_1 / F_1, I, I / F_1, I, F_1 / F_1, I, F_2 / F_1, F_1, I / F_1, F_1, F_1 / F_1, F_1, F_2 / F_2, I, I / F_2, I, F_1 / F_2, I, F_2 / F_2, F_2, I / F_2, F_2, F_1 / F_2, F_2, F_2 / K, I, F_1 / K, F_1, I / K, F_1, F_1 / K, F_1, F_2 / K, F_2, I$			
K, F_2, F_2	K, I, I	- No kill is recorded. - At least one of the two sides scoots.	- If at least one of the two sides decides to scoot, information from radar is lost.	$(1 - \beta_2)(1 - \rho_2)$ $(c\gamma + (1 - c)\gamma + c(1 - \gamma))$
	K, F_2, F_2	- No kill is recorded. - Neither B nor R scoots.	- If B and R's position is detected by their surveillance assets or radar, their position is kept track until at least one of them decides to scoot. Therefore, they can shoot repeatedly.	$(1 - \beta_2)(1 - \rho_2)$ $(1 - c)(1 - \gamma)$
	$K, K, *$	- R kills B. - B does not kill R.	-	$(1 - \beta_2)\rho_2$

Present State (B, B, R)	Future State (B, B, R)	Situations	Assumptions	Transition Probability
	$*, *, K$	- B kills R. - R does not kill B.	-	$\beta_2(1 - \rho_2)$
	K, K, K	- Simultaneous kill of both sides.	-	$\beta_2\rho_2$
	* No transitions: $I, I, I / I, I, F_1 / F_1, I, I / F_1, I, F_1 / F_1, I, F_2 / F_1, F_1, I / F_1, F_1, F_1 / F_1, F_1, F_2 / F_2, I, I / F_2, I, F_1 / F_2, I, F_2 / F_2, F_2, I / F_2, F_2, F_1 / F_2, F_2, F_2 / K, I, F_1 / K, F_1, I / K, F_1, F_1 / K, F_1, F_2 / K, F_2, I / K, F_2, F_1$			

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