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**NAVAL
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MONTEREY, CALIFORNIA

TECHNICAL REPORT

USER GUIDE

**Coupled Delft3D-Underwater Munition Scour Burial
(UnMUSB) Model**

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March 2022

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14. ABSTRACT A coupled Delft3D and underwater munition scour burial (UnMUSB) model has been developed and evaluated to assess sea-floor environment as well as munitions' migration and burial. The UnMUSB is a user-friendly physical based software written in MATLAB to help remediation management of the underwater munitions. This user guide documents the progress made in development of UnMUSB with coupling to the well-established Delft3D for nearshore environment during the project MR19-C1-1073 contract period (2019-2022), and includes a guide for detail and usage of UnMUSB with connection to Delft3D. The usage of coupled Delft3D-UnMUSB was demonstrated using the TREX13 data collected in the northern coastal region of the Gulf of Mexico near Panama City during the SERDP project MR-2320.					
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Couple Delft3D-Underwater Munition Scour Burial (UnMUSB) Model Demonstration and User's Guide

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Acronyms

AWAC	Acoustic Wave and Current Profiler
DDB	Delft3D Dash Board
Delft3D	Degree of Freedom
ECMWF	European Centre for Medium-Range Weather Forecasts
JONSWAP	Joint North Sea Wave Project
MOR	Morphology
MR	Munition Responses
MWL	Mean Water Level
NOAA	National Oceanic and Atmospheric Administration
NGDC	National Geophysical Data Center
SERDP	Strategic Environmental Research & Development Program
TREX	Target and Reverberation Experiment
UnMES	Underwater Munition Expert System
UnMUSB	Underwater Munition Scour Burial

Coupled Delft3D-UnMUSB Model: Demonstration and User's Guide

Abstract

A coupled Delft3D and underwater munition scour burial (UnMUSB) model has been developed and evaluated to assess sea-floor environment as well as munition migration and burial. The UnMUSB is a user-friendly physical based software written in MATLAB to help remediation management of the underwater munitions. This report documents the progress made in development of UnMUSB with coupling to the well-established Delft3D for nearshore environment during the project MR19-C1-1073 contract period (2019-2022), and includes a guide for detail and usage of UnMUSB with connection to Delft3D. The usage of coupled Delft3D-UnMUSB was demonstrated using the TREX13 data collected in the northern coastal region of the Gulf of Mexico near Panama City during the SERDP project MR-2320.

Objectives: Our objective is to develop an effective coupled Delft3D and underwater munition scour burial (UnMUSB) for the Munition Responses (MR) community to assess the environment and to predict munition mobility and burial. Since munition burial and mobility vary under various environment characteristics such as sediment types, sea floor morphology, and hydrodynamic conditions, timely prediction of seafloor environment and munition mobility including its location and burial is crucial to assess characteristics of munitions underwater and their environment.

The UnMUSB model contains four components: (a) munition physical parameters such as diameter, length, mass, and rolling moment, (b) dynamics of rolling munition around its major axis to predict munition mobility, (c) parameterized sediment scour model for predict munition burial, and (d) seabed environmental parameters such as currents, waves (peak period, significant wave height), sediment density, and median sediment grain size.

Technical Approach: The framework of UnMUSB is based on the dynamics of rolling rigid body on sea floor represented by analytical special Riccardi equation, along with the parameterized scour burial model popularly used in the SERDP MR community. With munition physical parameters such as diameter, length, mass, and rolling moment, the seabed environmental parameters, and as well as initial burial depth, solving the special Riccardi equation leads to the prediction of munition mobility; and using the parameterized scour burial model leads to the prediction of munition burial. The seabed environmental parameters can be obtained from field experimental data and from numerical modeling data such as Delft3D. This leads to the stand-alone UnMUSB model (experimental data) and coupled Delft3D-UnMUSB model.

Results: The UnMUSB and coupled Delft3D-UnMUSB models, written in Matlab, were

evaluated using the data of seabed environment and munition mobility and burial collected from a field experiment (SERDP-MR-2320) This experiment, called TREX13, was conducted from 21 April to 23 May 2013 off the coast of Panama City, Florida to measure both objects mobility using sector scanning and pencil beam sonars and simultaneous environmental time series data of the boundary layer hydrodynamics and sediment transport conditions for driving mobility.

Comparison between modeled and observed data show the model capability. The results were published in IEEE Journal of Oceanic Engineering (Appendix A) and MDPI Fluids (Appendix B). Besides, these two papers are used as examples to explain the UnMUSB and coupled Delft3D-UnMUSB model running.

Benefits: The UnMUSB and coupled Delft3D-UnMUSB models can be used for quick assessment of the environment and timely prediction of munition location, burial, and mobility in variety of aquatic environments. The whole package can be the part of the Underwater Munition Expert System (UnMES) (MR-2645, MR19-1126, PIs: Drs. Sarah Rennie and Alan Brandt at the John Hopkins University/APL).

1. Introduction

Development of the coupled Delft3D and Underwater Munition Scour Burial (UnMUSB) model has been supported by the United States Department of Defense Strategic Environmental Research and Development Program (SERDP) MR-19-C1-1073 (2019-2022). Munitions on seabed are less movable than sea mine in water column. Under the sponsorship of SERDP, experimental studies (Calantoni et al., 2014; Friedrichs et al. 2016), and statistical modeling (Rennie, 2017) focus on the determination of the conditions that determine the onset of a specific and important motion, i.e., roll of munition around its main axis, both on a hard surface and on a sand bed in the presence of concurrent scour burial. After initial efforts on seeking effective seabed scour burial models, we found the parameterized model developed under the SERDP project MR-2224 (Friedrichs et al., 2016) superior than other models, and made it the scour burial module of the UnMUSB model. We also developed a mathematical model based on the special Riccardi equation as the object module for predicting munition mobility. A preliminary model evaluation was conducted in collaboration with Dr. Joseph Calantoni (PI: SERDP MR-2320) using the TREX13 data (Calantoni et al., 2014).

This coupled Delft3D-UnMUSB can be used to assess the environment and to predict munition mobility. Since munition burial and mobility vary under various environment characteristics such as sediment types, sea floor morphology, and hydrodynamic conditions, timely prediction of seafloor environment and munition mobility including its location and burial is crucial to assess characteristics of munitions underwater and their environment. Furthermore, information obtained on munitions locations may be used to assess the utility of various underwater sensor and survey approaches. Figure 1 shows the model structure. If the environmental parameters are not obtained from Delft3D, it becomes stand-alone UnMUSB model. Since the Delft3D is a well-established community model for nearshore/coastal processes, it is roughly described in this manual.

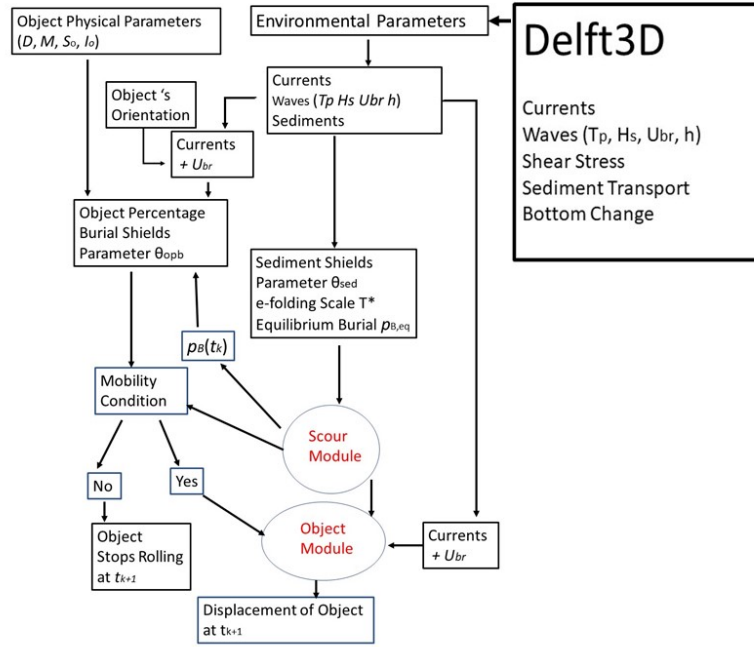


Figure 1. Flow chart of the coupled Delft3D-UnMUSB model to predict munition mobility and burial in seabed environment.

The nomenclature of the UnMUSB model is listed as follows:

(a_1, a_2, a_3)	Empirical Coefficients
B	Object Burial Depth (m)
b	Depth of Rotation Axis in Sediment (m)
(c_1, c_2)	Coefficients in e-Folding Time Scale for Scour Burial
(C_d, C_l)	(Drag, Lift) Coefficients
D	Diameter of Cylindrical Object (m)
d_{50}	Median of the Grain Diameter (m)
$(\mathbf{e}_h, \mathbf{e}_v)$	Horizontal and Vertical Unit Vectors
f	Wave Friction Factor
F_a	Added Mass (N)
F_d	Water Drag Force (N)
F_l	Water Lift Force (N)
F_w	Buoyancy Force (N)
h	Mean Water Level (m)
H_s	Significant Wave Height (m)
I_A	Rolling Moment of Object about b without Added Mass (kg m^2)
I_A^*	Rolling Moment of Object about b with Added Mass (kg m^2)
I_o	Rolling Moment of Object about its Symmetric Axis (kg m^2)
L	Length of Cylindrical Object (m)
l	Displacement of Object (m)
M	Mass of Object (kg)
$\mathbf{n}$	Unit Vector Normal to the Cylinder Surface

p_B	Object Percentage Burial
$p_{B,eq}$	Equilibrium Percentage Burial
p_b	Relative Depth of Rotation Axis in Sediment
S_o	Relative Object's Density Versus Water Density
S_{sed}	Relative Sediment's Density Versus Water Density
T^*	e-Folding Time Scale for Reaching Equilibrium Scour Burial (s)
T_B	Torque to Roll the Object Backward (N m)
T_F	Torque to Roll the Object Forward (N m)
T_P	Wave Peak Period (s)
$\mathbf{T}_S$	Sediment Torque on Object (N m)
U_c	Bottom Current Speed (m s^{-1})
U_{br}	Bottom Wave Orbital Velocity (m s^{-1})
u_o	Object's Rolling Horizontal Velocity (m s^{-1})
$\hat{u}_o$	Relative Object's Rolling Horizontal Velocity
U	Component of $\mathbf{V}_w$ Perpendicular to the Object's Main Axis (m s^{-1})
V	Components of $\mathbf{V}_w$ Parallel to the Object's Main Axis (m s^{-1})
$\mathbf{V}_w$	Bottom Water Velocity (Current and Wave) Vector (m s^{-1})
(v_e, v_n)	Bottom Current Velocity Vector (m s^{-1})
w	Weight Coefficient for Unburial Parameterization
(α, β)	Coefficients in the Object Mobility Model
γ	Ratio of Lift versus Drag Coefficient
θ_{opb}	Object Burial Percentage Shields Parameter
θ_{sed}	Sediment Shields Parameter
λ	Ratio between b and B
Π	Volume of Object (m^3)
ρ_o	Density of Object (kg m^{-3})
ρ_s	Density of Sediment (kg m^{-3})
ρ_w	Density of Sea Water (kg m^{-3})
ψ	Radial Angle Around the Center of Cylinder from the Vertical Axis
ω	Angular Velocity of Rolling Object (s^{-1})

2. UnMUSB Model

2.1. Object Module

The object module was developed on the base of the moment of momentum equation for the roll around munition main axis on seafloor (Figure 2)

$$\begin{cases} I_A \frac{d\omega}{dt} = T_F - T_B & \text{if } p_B < 0.5, \theta_{opb} > 1 \\ \omega = 0, & \text{otherwise} \end{cases} \quad (1)$$

where I_A is object's rolling moment without added mass; T_F is the torque to roll the object forward; T_B is torque to roll the object backward; p_B is the object percentage burial; and θ_{opb} is the object burial percentage Shields parameter.

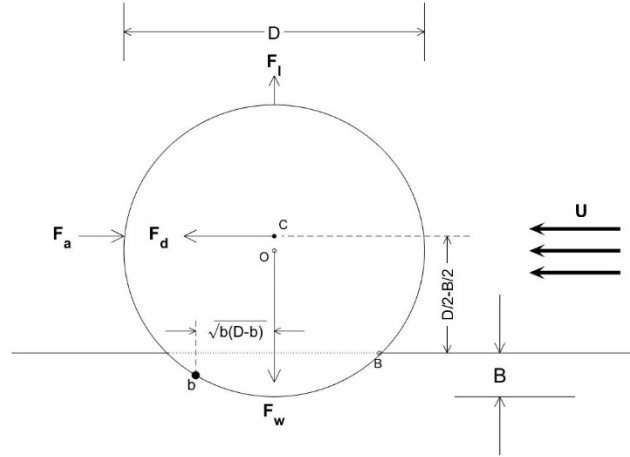


Figure 2. Forces and torques due to drag, lift, buoyancy, and added mass on a partially buried cylinder by combination of ocean currents and bottom wave orbital velocity (U) perpendicular to the major axis of the cylinder (see Figure 9 in Appendix A).

After mathematical manipulation, the basic equation (1) becomes a special Riccati equation [see Equation (20) in Appendix A]

$$\begin{cases} \frac{d(1-\hat{u}_o)}{dt} + \alpha(1-\hat{u}_o)^2 = \beta, & \text{if } p_B < 0.5, \theta_{opb}(t) > 1 \\ \hat{u}_o = 0, & \text{otherwise} \end{cases} \quad (2)$$

where $\hat{u}_o = u_o / U$, U is the current speed; α and β are model coefficients [see Equation (21) in Appendix A].

2.2. Scour Burial Module

Existing studies on scour burial were all concentrated on the motionless objects. The ratio between the fluid force (bottom shear stress) and the weight of the sediment particles, i.e., the sediment Shields parameter (θ_{sed}),

$$\theta_{sed} = \frac{F_w U_{br}^2}{g(S_{sed} - 1)d_{50}}, \quad S_{sed} = \frac{\rho_s}{\rho_w}, \quad F_w = \exp \left[5.5 \left(\frac{6U_{br}T_p}{\pi d_{50}} \right)^{-0.2} - 6.3 \right], \quad (3)$$

is crucial for scour burial of motionless object and in turn for prediction of the percentage burial parameter $p_B(t) = B/D$ (Rennie, 2017; Rennie et al., 2017; Friedrichs et al., 2016). Here, F_w is the wave friction factor (Nielsen, 1992), ρ_s the sediment grain density, and d_{50} the medium sand grain size. The bottom wave orbital velocity U_{br} represents interactions between surface waves and the seabed.

The scour burial module provides with two choices for the bottom wave orbital velocity (U_{br}): (a) a MATLAB function created by Wiberg and Sherwood (2008) using the parametric spectrum with the water depth (h), significant wave height (H_s), and peak period (T_p) as input, and (b) output from the Delft3D model.

As pointed in Rennie et al. (2017), the equilibrium percentage burial $p_{B,eq}$ for motionless cylinders induced by scour tends to increase as θ_{sed} increases. An empirical formula has been established,

$$p_{B,eq} = a_1 \theta_{sed}^{a_2} - a_3 \quad (4)$$

with different choices of the coefficients (a_1 , a_2 , a_3) determined experimentally for cylinders subject to steady currents. For motionless cylinders before scour burial reaches an equilibrium the percentage burial follows an exponential relationship (Whitehouse, 1998),

$$p_B(t_k) = p_{B,eq} \left[1 - \exp\left(-\frac{t_k}{T^*}\right) \right], \quad (5)$$

where T^* is the e-folding time scale.

3. Delft3D Model

The Delft3D version 4.04.01 with three major modules is used: Flow (tides and circulations), Wave (wave propagations, Morphology (sediment transport and morphological evolution). This area represents a typically lower energetic environment than other field sites on the east and west coast of the U.S. The Delft Dash Board (DDB) was used to create the grids and the tide forcing boundary conditions for the coupled wave and 2D flow model, including sediment transport and bottom change. Four nested grids compose the wave model, while two nested grids constitute the flow model. Wave boundary conditions were set up including Wavewatch III output and NOAA Buoy 42039 measurements. Moreover, wind data and bathymetric data from ECMWF and NOAA/NGDC, respectively, were incorporated as input data to the coupled model. The parameter settings for the flow, wave, and morphology modules are also provided. Detailed information on the Delft3D can be found from the website: <https://oss.deltares.nl/web/delft3d>.

4. Matlab Codes for the UnMUSB Model

4.1 General Description

UnMUSB.m is the main program to predict munition mobility and burial using the seabed environmental data. In the example depicted in Appendix A, the data files from the AWAC at the shallow quadpod are in netCDF format: [shallow.awac.stats.nc](#) for the wave parameters such as significant wave height (H_s), wave peak period (T_p) and mean water level, [shallow.awac.vel.nc](#) for current velocity. The environmental parameter data are taken as the input for the main program (UnMUSB.m). The model out put is Scourdata.nc.

4.2. Main Code UnMUSB.m

```
% Appendix A as example for running the UnMUSB model
% Equations mentioned here are referred to Appendix A
```

```
clear all; %close all;
```

```
gamma=0.2;
```

```

epps=0.0165;

Aff=0.9; % eq(29) w=0.90

Labels={'A2','A5','B5','C2','C4','C6','D3','D6'};
munidir=[20,15,15,-70,10,15,-60,15];
Rdir=(munidir-90)*pi/180;
muninm={'20mm','20mm','25mm','81mm','81mm','81mm','155mm','155mm'};

datest='20-Apr-2013 16:22:00';
datend='7-May-2013 11:30:00';

day0=datetime(datest);
daynd=datetime(datend)-day0;

temp=5;
nu= Nu(temp,1);

stc=sqrt(1.001);
rm=1025; % kg/m^3
g=9.806; % m/s^2

d50=0.23e-3; % m

a1=1.6; a2=0.85; a3=0; % eq(25)
c1=1.5; c2=0.11; % eq(27)
Ssed=2.7/1.025; % eq(24)

% load AWAC wave data
ncid=netcdf.open('shallow.awac.stats.nc','nowrite');
vid=netcdf.inqVarID(ncid,'date');
date_w=netcdf.getVar(ncid,vid);
vid=netcdf.inqVarID(ncid,'Hs');
hs=netcdf.getVar(ncid,vid);
vid=netcdf.inqVarID(ncid,'Tp');
tp=netcdf.getVar(ncid,vid);
vid=netcdf.inqVarID(ncid,'mwl');
h=netcdf.getVar(ncid,vid);

netcdf.close(ncid);

tp(tp<0)=NaN;

dayw=datetime(date_w)-day0;
ii=find(dayw>=0 & dayw<=daynd);
dayw=dayw(ii); date_w=date_w(ii,:); hs=hs(ii); tp=tp(ii); h=h(ii);
timew=dayw*24*3600;

```

```

[ubr,Tbr] = ubspecpar(hs,tp,h);

ubr=real(ubr); Tbr(Tbr<=0)=NaN;

% from SERDP MR-2319 Report

fw=exp(5.5*(6*ubr.*tp/(pi*d50)).^-0.2-6.3); % eq.(24)

stsed=fw.*ubr.^2/(g*(Ssed-1)*d50); % eq.(24)

pbeq=a1*stsed.^a2-a3; % eq.(25)

% load AWAC velocity data

ncid=netcdf.open('shallow.awac.vel.nc','nowrite');
vid=netcdf.inqVarID(ncid,'date');
date_v=netcdf.getVar(ncid,vid);
vid=netcdf.inqVarID(ncid,'vn');
vn=netcdf.getVar(ncid,vid);
vid=netcdf.inqVarID(ncid,'ve');
ve=netcdf.getVar(ncid,vid);

netcdf.close(ncid);
vn=vn(:,1); ve=ve(:,1);

dayv=datetime(date_v)-day0;

ii=find(dayv>=dayw(1) & dayv<=dayw(end));

dayv=dayv(ii); date_v=date_v(ii,:); vn=vn(ii); ve=ve(ii);

dt=diff(dayv)*24*3600;

daym=(dayv(1:end-1)+dayv(2:end))/2;
vnm=(vn(1:end-1)+vn(2:end))/2; vem=(ve(1:end-1)+ve(2:end))/2;

UM=sqrt(vnm.^2+vem.^2);
arfM=atan2(vnm,vem);

NM=length(daym);

Nwave=length(ubr); Nvel=length(ve);

ncid=netcdf.create('Scourdata.nc','clobber');

```

```

dimw=netcdf.defDim(ncid,'dim_wave',Nwave);
dimv=netcdf.defDim(ncid,'dim_vel',Nvel);
dim20=netcdf.defDim(ncid,'strin20',20);

datewid=netcdf.defVar(ncid,'date_w','char',[dimw,dim20]);
daywid=netcdf.defVar(ncid,'day_w','float',dimw);
datevid=netcdf.defVar(ncid,'date_v','char',[dimv,dim20]);
dayvid=netcdf.defVar(ncid,'day_v','float',dimv);

hid=netcdf.defVar(ncid,'h','float',dimw);
netcdf.putAtt(ncid,hid,'units','m');
Hsid=netcdf.defVar(ncid,'Hs','float',dimw);
netcdf.putAtt(ncid,Hsid,'units','m');
tpid=netcdf.defVar(ncid,'tp','float',dimw);
netcdf.putAtt(ncid,tpid,'units','s');
ubrid=netcdf.defVar(ncid,'ubr','float',dimw);
netcdf.putAtt(ncid,ubrid,'units','m/s');
stsedid=netcdf.defVar(ncid,'thetased','float',dimw);

veid=netcdf.defVar(ncid,'ve','float',dimv);
netcdf.putAtt(ncid,veid,'units','m/s');
vnid=netcdf.defVar(ncid,'vn','float',dimv);
netcdf.putAtt(ncid,vnid,'units','m/s');

netcdf.endDef(ncid);

netcdf.putVar(ncid,datewid,date_w);
netcdf.putVar(ncid,daywid,dayw);
netcdf.putVar(ncid,datevid,date_v);
netcdf.putVar(ncid,dayvid,dayv);

netcdf.putVar(ncid,hid,h);
netcdf.putVar(ncid,Hsid,hs);
netcdf.putVar(ncid,tpid,tp);
netcdf.putVar(ncid,ubrid,ubr);
netcdf.putVar(ncid,stsedid,stsed);

netcdf.putVar(ncid,veid,ve);
netcdf.putVar(ncid,vnid,vn);

netcdf.close(ncid);

for n=1:length(Labels)
[L,D,m,ro,Io]=getmunipara(Labels{n});

R=D/2;
IA=Io+m*R^2; VV=m/ro;

```

```

So=ro/rm;

TT=c1*stsed.^c2*D^2./sqrt((g*(Ssed-1)*d50^3)); % eq.(27)

pBB=pbeq.*(1-exp(-timew./TT)); % eq.(26)

pB=pBB;
for i=2:length(pB)

    if isnan(pB(i))
        pB(i)=pB(i-1);
    elseif pB(i)<pB(i-1)
        pB(i)=pB(i)*(1-Aff)+pB(i-1)*Aff; % eq.(29)

    end

    if pB(i)>=0.5
        pB(i:end)=0.5; break;
    end
end

lambda=getlambda(pB); % eq(28)

pb=lambda.*pB;

% -----

pB_m=interp1(dayw,pB,daym);
pb_m=interp1(dayw,pb,daym);
ubrm=interp1(dayw,ubr,daym);

pBall_m=pB_m;

i5=find(pB_m<0.5);
Ni5=length(i5);

Um=UM(i5)+ubrm(i5); arfm=arfM(i5); dtm=dt(i5);
pB_m=pB_m(i5); pb_m=pb_m(i5);

MVdir=ones(Ni5,1)*[cos(Rdir(n)),sin(Rdir(n))];
Umdir=[cos(arfm),sin(arfm)];
Smdir=sum(Umdir.*MVdir,2);
UmR=Um.*Smdir;

```

```

sig=sign(UmR);
AUmR=abs(UmR);

Re=max(0.001,AUmR*D/nu);
Re(Re<1)=1;
Cf=CdR28(AUmR,D,nu);
Cl=gamma*Cf;

theta0=Cf.*AUmR.^2/(g*D*(So-1));
theta=theta0/pi.*((1-pB_m).*(1+pB_m-2*pb_m))./sqrt(pb_m.*(1-pb_m))+2*gamma);

it1=find(theta>1);

Nit1=length(it1);

% only for theta>1
UmR=UmR(it1); AUmR=AUmR(it1); pB_m=pB_m(it1); pb_m=pb_m(it1); dtm=dtm(it1);
Cf=Cf(it1); Cl=Cl(it1); %theta=theta(it1); theta2=sqrt(theta);

Ia=rm/4*VV*D^2*(1-2*pb_m).*(1+pB_m-2*pb_m);

% calculate non-dimensional parameters
KI=rm*Cf*L.*AUmR*D^3.*dtm./(IA+Ia)/8;
Kw=g*dtm*rm*L*D^4./(IA+Ia)./AUmR/8;

alfa=KI.*(1-2*pb_m).*((1-pB_m).*(1+pB_m-2*pb_m)+2*gamma*sqrt(pb_m.*(1-pb_m)));
beta=Kw.*(So-1)*pi.*(1-2*pb_m).*sqrt(pb_m.*(1-pb_m));

ab2=sqrt(alfa.*beta);

ttmax=atanh(((1-epps)*ab2-(1-epps)*beta)./((epps+(1-epps)*sqrt(beta./alfa)).*alfa-beta))./ab2;
ttmax(ttmax>1)=1;

tanhabt=tanh(ab2.*ttmax);

uu=1-(ab2+beta.*tanhabt)./(ab2+alfa.*tanhabt);

u=uu.*UmR;
omega=u./(D*(0.5-pb_m));
t=ttmax.*dtm;

ds=u.*t/2;

cds=cumsum(ds);

DS=sum(ds);

```

```

disp([Labels{n},': ',num2str(DS)]);

ncid=netcdf.create(['Munition_',Labels{n},'.nc'],'clobber');

dimNM1=netcdf.defDim(ncid,'Ngrid',NM+1);
dimNM=netcdf.defDim(ncid,'Midgrid',NM);
dim20=netcdf.defDim(ncid,'string20',20);
dimNi5=netcdf.defDim(ncid,'Ni5',Ni5);
if DS~=0
    dimNit1=netcdf.defDim(ncid,'Ntheta>1',Nit1);
end

dateid=netcdf.defVar(ncid,'Date','char',[dimNM1,dim20]);
dayid=netcdf.defVar(ncid,'days','float',dimNM1);
umid=netcdf.defVar(ncid,'Um','float',dimNM);
netcdf.putAtt(ncid,umid,'units','m/s');
vmid=netcdf.defVar(ncid,'Vm','float',dimNM);
netcdf.putAtt(ncid,vmid,'units','m/s');

pBid=netcdf.defVar(ncid,'pB','float',dimNM);

I5id=netcdf.defVar(ncid,'I5','long',dimNi5);
netcdf.putAtt(ncid,I5id,'Description','Index for pB<0.5');
theta0id=netcdf.defVar(ncid,'Theta0','float',dimNi5);
thetaid=netcdf.defVar(ncid,'Theta','float',dimNi5);

if DS~=0
    It1id=netcdf.defVar(ncid,'It1','long',dimNit1);
    netcdf.putAtt(ncid,It1id,'Description','Theta>1 index in I5 index');
    dsid=netcdf.defVar(ncid,'ds','float',dimNit1);
    netcdf.putAtt(ncid,dsid,'units','m');
    omegaid=netcdf.defVar(ncid,'omega','float',dimNit1);
end

netcdf.endDef(ncid);

netcdf.putVar(ncid,dateid,date_v);
netcdf.putVar(ncid,dayid,dayv);
netcdf.putVar(ncid,umid,vem);
netcdf.putVar(ncid,vmid,vnm);
netcdf.putVar(ncid,pBid,pBall_m);
netcdf.putVar(ncid,I5id,i5);
netcdf.putVar(ncid,theta0id,theta0);
netcdf.putVar(ncid,thetaid,theta);
if DS~=0
    netcdf.putVar(ncid,It1id,it1);
    netcdf.putVar(ncid,dsid,cds);

```

```

        netcdf.putVar(ncid,omegaid,omega);
    end

    netcdf.close(ncid);

end

```

4.3. Matlab Function for Munition Parameters

```
function [L,D,m,ro,Io]=getmunipara(Labels)
```

% function [L,D,m,ro,Io]=getmunipara(Labels) is used for getting munition parameters
 % The munition parameters should be **user input**. The following parameters are obtained from
 % Table 1, Figures 1, 3, and 5 in the SERDP 2320 Interim Report and used for predicting
 % munition mobility and burial in the TREX13 area (see Appendix A).

```
switch Labels
```

```
case {'D5','D6'}
```

```

    L=584.2/1000;           % m
    D=152.2/1000;          % m
    m=75.3*0.4536/L;       % (mass of munition)/L, kg/m
    ro=0.161*0.4636/(0.0254^3); % density of munition, kg/m^3
    Io=316.3/L*0.4636*0.0254^2; % (rolling moment about the main axis)/L, kg*m

```

```
case {'D3','D4'}
```

```

    L=584.2/1000;           % m
    D=152.2/1000;          % m
    m=46.1*0.4536/L;       % (mass of munition)/L, kg/m
    ro=0.098*0.4636/(0.0254^3); % density of munition, kg/m^3
    Io=171.4/L*0.4636*0.0254^2; % (rolling moment about the main axis)/L, kg*m

```

```
case {'C3','C4'}
```

```

    L=524.6/1000;           % m
    D=80.1/1000;            % m
    m=8.3*0.4536/L;        % (mass of munition)/L, kg/m
    ro=0.112*0.4636/(0.0254^3); % density of munition, kg/m^3
    Io=8.471/L*0.4636*0.0254^2; % (rolling moment about the main axis)/L, kg*m

```

```
case {'C5','C6'}
```

```

    L=524.6/1000;           % m
    D=80.1/1000;            % m
    m=19.2*0.4536/L;       % (mass of munition)/L, kg/m
    ro=0.260*0.4636/(0.0254^3); % density of munition, kg/m^3
    Io=17.3/L*0.4636*0.0254^2; % (rolling moment about the main axis)/L, kg*m

```

```
case {'C1','C2'}
```

```

    L=524.6/1000;           % m

```

D=80.1/1000;	% m
m=3.2*0.4536/L;	% (mass of munition)/L, kg/m
ro=0.045*0.4636/(0.0254^3);	% density of munition, kg/m^3
Io=2.857/L*0.4636*0.0254^2;	% (rolling moment about the main axis)/L, kg*m
case {'B7','B8'}	
L=218.4/1000;	% m
D=37.8/1000;	% m
m=2.92*0.4536/L;	% (mass of munition)/L, kg/m
ro=0.284*0.4636/(0.0254^3);	% density of munition, kg/m^3
Io=0.680/L*0.4636*0.0254^2;	% (rolling moment about the main axis)/L, kg*m
case {'B5','B6'}	
L=218.4/1000;	% m
D=37.8/1000;	% m
m=0.86*0.4536/L;	% (mass of munition)/L, kg/m
ro=0.085*0.4636/(0.0254^3);	% density of munition, kg/m^3
Io=0.158/L*0.4636*0.0254^2;	% (rolling moment about the main axis)/L, kg*m
case {'B3','B4'}	
L=218.4/1000;	% m
D=37.8/1000;	% m
m=0.97*0.4536/L;	% (mass of munition)/L, kg/m
ro=0.098*0.4636/(0.0254^3);	% density of munition, kg/m^3
Io=0.235/L*0.4636*0.0254^2;	% (rolling moment about the main axis)/L, kg*m
case {'B1','B2'}	
L=218.4/1000;	% m
D=37.8/1000;	% m
m=0.52*0.4536/L;	% (mass of munition)/L, kg/m
ro=0.051*0.4636/(0.0254^3);	% density of munition, kg/m^3
Io=0.112/L*0.4636*0.0254^2;	% (rolling moment about the main axis)/L, kg*m
case {'A7','A8'}	
L=168.1/1000;	% m
D=29.4/1000;	% m
m=1.40*0.4536/L;	% (mass of munition)/L, kg/m
ro=0.284*0.4636/str(0.0254^3);	% density of munition, kg/m^3
Io=0.182/L*0.4636*0.0254^2;	% (rolling moment about the main axis)/L, kg*m
case {'A5','A6'}	
L=168.1/1000;	% m
D=29.4/1000;	% m
m=0.45*0.4536/L;	% (mass of munition)/L, kg/m
ro=0.096*0.4636/(0.0254^3);	% density of munition, kg/m^3
Io=0.046/L*0.4636*0.0254^2;	% (rolling moment about the main axis)/L, kg*m
case {'A3','A4'}	

```

L=168.1/1000; % m
D=29.4/1000; % m
m=0.44*0.4536/L; % (mass of munition)/L, kg/m
ro=0.098*0.4636/(0.0254^3); % density of munition, kg/m^3
Io=0.063/L*0.4636*0.0254^2; % (rolling moment about the main axis)/L, kg*m

case {'A1','A2'}
L=168.1/1000; % m
D=29.4/1000; % m
m=0.25*0.4536/L; % (mass of munition)/L, kg/m
ro=0.051*0.4636/(0.0254^3); % density of munition, kg/m^3
Io=0.033/L*0.4636*0.0254^2; % (rolling moment about the main axis)/L, kg*m

end

```

4.4. Matlab Function for Calculating Drag Coefficient

```

function [cd2,Re] = CdR28(v,d,nu)

% function cd2 = CdR28(v,d,nu);
Re=max(0.001,abs(v)*d/nu); %AR=L/d;
% disp(['Re= ',num2str(Re),' AR= ',num2str(AR)]);
cd2=zeros(size(v));

ii=find(Re>=350000);
if ~isempty(ii)
    cd2(ii)=1./(641550 ./ Re(ii) + 1.5);
end
ii=find(Re>=150000 & Re<350000);
if ~isempty(ii)
    cd2(ii) = 1.875 - .0000045 * Re(ii);
end
ii=find(Re>= 12000 & Re<150000);
if ~isempty(ii)
    cd2(ii) = 1.2;
end
ii=find(Re>= 2000 & Re<12000);
if ~isempty(ii)
    cd2(ii) = .84 + .00003 * Re(ii);
end
ii=find(Re >= 180 & Re<2000);
if ~isempty(ii)
    cd2(ii) = .86 + 89 / Re(ii);
end
ii=find(Re >= 12 & Re<180);
if ~isempty(ii)
    cd2(ii) = 1.26 + 16 / Re(ii);
end

```

```

ii=find(Re<12);
if ~isempty(ii)
    cd2(ii) = 1.8 + 8 / Re(ii);
end

```

4.5. Matlab Function for Determining λ

```

function lambda=getlambda(pB)
% function lambda=getlambda(pB)
lambda=0.4445+0.0286*pB+0.0165*pB.^2;

```

4.6. Matlab Function for Kinematic Viscosity

```

function nu = Nu(t, sea_water)
% function nu = Nu(t, sea_water);
%
% kinematic viscosity
% t: temperature(c) <-99 - Air
% sea_water: 0 - water, 1 - sea_water, -1 - air
%

if(~exist('t','var')); t=16; end

% Kinematic viscosity of air (m^2/s)
if(t<-90), nu=.0000146; return; end
if(exist('sea_water','var') && sea_water<0), nu=.0000146; return; end

% Corresponding kinetic viscosity of water

if(t<=10), nu=10^(-5.7471-0.0136*t); else
nu=10^(-5.77592-0.010718*t); end

if(exist('sea_water','var') && sea_water)
    nu=nu*1.0585*(1+t^2/27550);
end

```

4.7. Matlab Function for Calculating kh (Wavenumber × Depth) (from Wiberg and Sherwood 2008)

```
function kh = qkhfs( w, h )
% This Matlab function was obtained from Appendix E in Wiberg and Sherwood
% (2008)
% QKHFS—Quick iterative calculation of kh in dispersion relationship
% kh = qkhfs( w, h )
%
% Input:
% w= Angular wave frequency = 2*pi/T where T = wave period [1/s]
% h= Water depth [m]
% Returns:
% kh= wavenumber * depth [ ]
%
% Either w or h can be a vector, but not both.
% Hard-wired for MKS units.
% Orbital velocities from kh are accurate to 3e-12!
%
% RLSoulsby (2006)“Simplified calculation of wave orbital velocities”
% HR Wallingford Report TR 155, February 2006, Eqns. 12a–14
% csherwood@usgs.gov
% Sept 10, 2006
g = 9.81;
x = w.^2*h./g;
y = sqrt(x) .* (x<1)+x.* (x >= 1);
%this appalling bit of code is about 25% faster than a for loop
t = tanh( y );
y = y-( (y.*t -x)./(t+y.*(1-t.^2)));
t = tanh( y );
y = y-( (y.*t -x)./(t+y.*(1-t.^2)));
t = tanh( y );
y = y-( (y.*t -x)./(t+y.*(1-t.^2)));
kh = y;
return;
```

4.8. Matlab Function for Calculating Bottom Orbital Velocity Using a Parametric Spectrum (from Wiberg and Sherwood 2008)

```
function [ubr,Tbr,iter] = ubspecpar(hs,tp,h,specform)
% This Matlab function was obtained from Appendix D in Wiberg and Sherwood
% (2008)
% UBSPECPAR—Calculate ubr and Tbr from hs and tp using parametric
% spectrum
% [ubr,Tbr,iter] = ubspecform(hs,tp,h,specform)
%
% Input:
```

```

% hs—Significant wave height (m)
% tp—Peak period (s)
% h—Water depth (m)
% specform—spectral formulation to use
% specform = 'D' for Donelan spectrum (default)
% specform = 'J' for JONSWAP spectrum
% Returns:
% ubr = representative bottom orbital velocity (m/s)
% Tbr = representative bottom wave period (s)
% iter = number of iterations if Donelan spectrum is chosen
% If iter45 the calculation did not converge
% iter = 0 for the JONSWAP spectrum
%
% Patricia Wiberg, UVa
% Last modified 9 Mar 2007
if (~exist('specform','var'))
specform = 'D';
end
g = 9.81; dffp = 0.01;
ffp = 0.2:dffp:5; %nb = length(ffp);
nt = length(tp);
iter = zeros(nt,1);
ubr = zeros(nt,1); Tbr = ubr; %Tbz = Tbr;
for i = 1:nt
    if hs(i)>0
        m0 = hs(i).^2./16;
        fp = 1./tp(i);
        f = ffp.*fp; df = dffp.*fp;
        if length(h)==nt
            kh = qkhfs(2*pi*f,h(i));           %(see Appendix E)
        else
            kh = qkhfs(2*pi*f,h);             %(see Appendix E)
        end
        if specform=='D'
            xi = 1; tol = 1e-3; m0sfD = 0.;
            while abs((m0-m0sfD)/m0 > tol)
                fpbar = (m0*fp^4/(g^2*6.635e-6))^(1/.7);
                gam = 1.7;
                if fpbar >= 0.05
                    % alpha = 0.0165.*fpbar.^0.55;
                    if fpbar>0.159, gam = 6.5+2.606*log(fpbar); end
                    sig = 0.08+0.0013.*(fpbar).^-3;
                else
                    % alpha = 0.0165.*0.05.^0.55;
                    sig = 0.08+0.0013.*(0.05).^-3;
                end
            end
        end
    end
end

```

```

eterm = -((ffp-1).^2)./(2.*sig.^2);
ee = exp(eterm); t2 = gam.^ee;
t1 = -(ffp).^4;
sffpn = (ffp.^xi./ffp.^5).*exp(t1).*t2; %dimensionless spectrum
XD = 1./sum(sffpn.*dffp); %chi
sf = m0.*XD*fp.^4.*1./(f.^4*fp).*exp(t1).*t2; %Eq. 20
m0sfD = sum(sf.*df);
iter(i) = iter(i)+1;
if iter(i)>5, break; end
end
% m1sfD = sum(f.*sf.*df);
% m2sfD = sum(f.^2.*sf.*df);
% fmD = m1sfD./m0sfD; TmD = 1./fmD;
% fzD = sqrt(m2sfD./m0sfD); TzD = 1./fzD;
su = (2*pi.*f./sinh(kh)).^2.*sf; %Eq. 5
ubrD = sqrt(2.*sum(su.*df)); %Eq. 6
frD = sum(su.*f.*df)./sum(su.*df); %Eq. 9
% fzD = sqrt(sum(f.^2.*su.*df)./sum(su.*df));
TbrD = 1./frD; %TbzD = 1./fzD;
ubr(i) = ubrD; Tbr(i) = TbrD;
end
if specform=='J'
gam = 3.3; xi = 0; %XJ = 3.283; %The value of gam can be changed
jn = ffp41;
sig = 0.07*ones(size(ffp)); sig(jn) = 0.09;
eterm = -((ffp-1).^2)./(2.*sig.^2);
ee = exp(eterm); t2 = gam.^ee;
t1 = -1.25.*(ffp).^4;
sffpn = (ffp.^xi./ffp.^5).*exp(t1).*t2; %dimensionless spectrum
XJ = 1./sum(sffpn.*dffp); %chi
sf = m0.*fp.^4.*XJ./f.^5.*exp(t1).*t2; %Eq. 20
% m0sfJ = sum(sf.*df);
% m1sfJ = sum(f.*sf.*df);
% m2sfJ = sum(f.^2.*sf.*df);
% fmJ = m1sfJ./m0sfJ; TmJ = 1./fmJ;
% fzJ = sqrt(m2sfJ./m0sfJ); TzJ = 1./fzJ;
su = (2*pi.*f./sinh(kh)).^2.*sf; %Eq. 5
ubrJ = sqrt(2.*sum(su.*df)); %Eq. 6
frJ = sum(f.*su.*df)./sum(su.*df); %Eq. 9
% fzJ = sqrt(sum(f.^2.*su.*df)./sum(su.*df));
TbrJ = 1./frJ; %TbzJ = 1./fzJ;
ubr(i) = ubrJ; Tbr(i) = TbrJ;
end
end
end

```

5. Matlab Codes for Coupled Delft3D-UnMUSB Model

5.1. General Description

Delft3D-UnMUSB.m is the main Matlab program to predict munition mobility and burial with the UnMUSB model using the Delft3D output in the format '**_Delft3D.mat' as environmental parameter data: hs_Delft3D.mat for the significant wave height (HS), tp_Delft3D.mat for the peak period (TP), Ubr_Delft3D.mat for the bottom orbit velocity (Ubr), v_Delft3D.mat for the water velocity across the munition's main axis, and wl_Delft3D.mat for the water level. The Matlab functions CDR28.m, getlamda.m, getmunipara.m, and nu.m are the same as depicted in Subsections 4.3-4.7. The model output is in the netCDF format: Delft3D-UnMUSB.nc.

5.2. Main Code Delft3D-UnMUSB.m

% Coupled Delft3D-UnMUSB Model to Predict Munition Mobility and Burial on Sandy Seafloor

clear; %close all;

% r_delt=0; % drag force on the center
gamma=0.2; % without left force

epps=0.005;
% Rough=0.6; % mm
Rough=0.003; % mm

Aff=0.8;
%
% pBrough=Rough/1000/D; % mm / 1000 to m

Labels={'A2','A5','B5','C2','C4','C6','D3','D6'};
% xy0=[-1,-2.6; 1.1,-1.4; 1.6,-2.3; -2.8,-3.8; 0.3,-1.3; 0.3,-2.9; 1.4,-2.3; 0.2,-2.2];
% cols={'b','r','k','b','r','k','r','b'};
% munidir=[5,55,10,-55,10,15,-60,15];
% munidir=[5,55,50,-20,10,15,-60,15];
% munidir=[20,65,60,10,50,65,75,60]-40;
munidir=[20,15,15,-70,10,15,-60,15];
Rdir=(munidir-90)*pi/180;
muninm={'20mm','20mm','25mm','81mm','81mm','81mm','155mm','155mm'};
% enddays=['24-Apr-2013 12:00:00';
% '01-May-2013 12:00:00';
% '05-May-2013 12:00:00';
% '24-Apr-2013 12:00:00';
% '05-May-2013 12:00:00';

```

%      '05-May-2013 12:00:00';
%      '05-May-2013 12:00:00';
%      '05-May-2013 12:00:00'];

datest='20-Apr-2013 16:22:00';
% datend='23-May-2013 11:30:00';
datend='7-May-2013 11:30:00';
% datest='20-Apr-2013 16:30:00';
% datend='24-Apr-2013 00:00:00';

day0=datenum(datest);
daynd=datenum(datend)-day0;

reg='shallow'; % or 'deep';

temp=5;
nu= Nu(temp,1);

stc=sqrt(1.001);
% stc=sqrt(1.0006);
% stc=1;
rm=1025 % kg/m^3
g=9.806; % m/s^2

d50=0.23e-3; % m

a2=1.6; b2=0.85; % eq.(9) in Catano-Lopera et al [28]

N=1.5; M=0.11; % eq.(11 N-c1 & M-c2) Meyer-Peter Mueller
% Demir and Garcia (11)
Ssed=2.7/1.025; % eq. (8,11)

%% Loading Delft3D output (.mat file)
% ubr
load Ubr_Delft3D.mat;
date_w1=data.Time; %grabbing just time
date_w=datestr(date_w1,'dd-mmm-yyyy HH:MM:SS');
ubr=data.Val; %ubr
% Period
load tp_Delft3D.mat;
tp=data.Val; %grabbing just period

% Height
load hs_Delft3D.mat;

```

```

Hs=data.Val; %grabbing just height

%% Loading Model Results
load vl_Delft3D.mat;
h=data.Val;
h=h+7.6104;
% h=h+4.6889;

dayw=datetime(date_w)-day0;
ii=find(dayw>=0 & dayw<=daynd);
dayw=dayw(ii); date_w=date_w(ii,:);
timew=dayw*24*3600;
ubr=ubr(ii); tp=tp(ii); h=h(ii); Hs=Hs(ii);

ubr(ubr<1e-4)=1e-4;

% from paper MR-2319 report

fw=exp(5.5*(12*ubr.*tp/(2*pi*d50)).^-0.2-6.3); % eq (8) in Appendix B

stsed=fw.*ubr.^2/(g*(Ssed-1)*d50); % eq(8) in Appendix B

pbeq=a2*stsed.^b2; % eq(9) in Appendix B

%% loading Delft3D model output
data=load('v_Delft3D.mat');
date_vv1=data.data.Time;
date_v=datestr(date_vv1,'dd-mmm-yyyy HH:MM:SS');
vn=data.data.YComp;
ve=data.data.XComp;

vn=vn(:,end); ve=ve(:,end);

dayv=datetime(date_v)-day0;

ii=find(dayv>=dayw(1) & dayv<=dayw(end));

dayv=dayv(ii); date_v=date_v(ii,:); vn=vn(ii); ve=ve(ii);

dt=diff(dayv)*24*3600;

U=sqrt(ve.^2+vn.^2);

daym=(dayv(1:end-1)+dayv(2:end))/2;
vnm=(vn(1:end-1)+vn(2:end))/2; vem=(ve(1:end-1)+ve(2:end))/2;

```

```

UM=sqrt(vnm.^2+vem.^2);
arfM=atan2(vnm,vem);

ubrm=(ubr(1:end-1)+ubr(2:end))/2;

UM=UM+ubrm;

NM=length(daym);

ncid=netcdf.create('Delft3D-UnMUSB.nc','clobber');
dim1=netcdf.defDim(ncid,'Timedim',length(dayw));
dim20=netcdf.defDim(ncid,'dateleth',20);

dateid=netcdf.defVar(ncid,'date','char',[dim1,dim20]);
dayid=netcdf.defVar(ncid,'days','float',dim1);

hid=netcdf.defVar(ncid,'h','float',dim1);
netcdf.putAtt(ncid,hid,'units','m');
Hsid=netcdf.defVar(ncid,'Hs','float',dim1);
netcdf.putAtt(ncid,Hsid,'units','m');
tpid=netcdf.defVar(ncid,'tp','float',dim1);
netcdf.putAtt(ncid,tpid,'units','s');
Ubrid=netcdf.defVar(ncid,'Ubr','float',dim1);
netcdf.putAtt(ncid,Ubrid,'units','m/s');
stsedid=netcdf.defVar(ncid,'thetased','float',dim1);
veid=netcdf.defVar(ncid,'ve','float',dim1);
netcdf.putAtt(ncid,veid,'units','m/s');
vnid=netcdf.defVar(ncid,'vn','float',dim1);
netcdf.putAtt(ncid,vnid,'units','m/s');
Uid=netcdf.defVar(ncid,'U','float',dim1);
netcdf.putAtt(ncid,Uid,'units','m/s');

netcdf.endDef(ncid);

netcdf.putVar(ncid,dateid,date_w);
netcdf.putVar(ncid,dayid,dayw);
netcdf.putVar(ncid,hid,h);
netcdf.putVar(ncid,Hsid,Hs);
netcdf.putVar(ncid,tpid,tp);
netcdf.putVar(ncid,Ubrid,ubr);
netcdf.putVar(ncid,stsedid,stsed);
netcdf.putVar(ncid,veid,ve);
netcdf.putVar(ncid,vnid,vn);
netcdf.putVar(ncid,Uid,U);

```

```

netcdf.close(ncid);

for n=1:length(Labels)
    [L,D,m,ro,Io]=getmunipara(Labels{n});

    R=D/2;
    IA=Io+m*R^2; VV=m/ro;

    pBrough=Rough/1000/D;          % convert 'mm' to 'm'

    So=ro/rm;

    TT=M*stsed.^N*D^2./sqrt((g*(Ssed-1)*d50^3));          % eq(11) in Appendix B

    pBB=pbeq.*(1-exp(-timew./TT));          % eq(10) in Appendix B

    pB=pBB+pBrough;

    for i=2:length(pB)

        if isnan(pB(i))
            pB(i)=pB(i-1);
        elseif pB(i)<pB(i-1)
            pB(i)=pB(i)*(1-Aff)+pB(i-1)*Aff;          % eq(13) in Appendix B

        end

        if pB(i)>=0.5
            pB(i:end)=0.5; break;
        end
    end

    lambda=getlambda(pB);          % eq (12) in Appendix B

    pb=lambda.*pB;

    % -----

    pB_m=interp1(dayw,pB,daym);
    pb_m=interp1(dayw,pb,daym);
    % ubrm=interp1(dayw,ubr,daym);

    pBall_m=pB_m;

```

```

i5=find(pB_m<0.5);
Ni5=length(i5);

Um=UM(i5); arfm=arfM(i5); dtm=dt(i5);
pB_m=pB_m(i5); pb_m=pb_m(i5);

MVdir=ones(Ni5,1)*[cos(Rdir(n)),sin(Rdir(n))]; % munition rolling direction
Umdir=[cos(arfm),sin(arfm)]; % current velocity direction
Smdir=sum(Umdir.*MVdir,2);
UmR=Um.*Smdir; % current velocity component in munition roll direction

sig=sign(UmR);
AUmR=abs(UmR);

% calculation of the drag coefficient
Re=max(0.001,AUmR*D/nu);
Re(Re<1)=1;
Cf=CdR28(AUmR,D,nu);
Cl=gamma*Cf;

theta0=Cf.*AUmR.^2/(g*D*(So-1));
theta=theta0/pi.*((1-pB_m).*(1+pB_m-2*pb_m)/sqrt(pb_m.*(1-pb_m))+2*gamma);

it1=find(theta>1);

Nit1=length(it1);

% For theta>1
UmR=UmR(it1); AUmR=AUmR(it1); pB_m=pB_m(it1); pb_m=pb_m(it1); dtm=dtm(it1);
Cf=Cf(it1); Cl=Cl(it1); %theta=theta(it1); theta2=sqrt(theta);

Ia=rm/4*VV*D^2*(1-2*pb_m).*(1+pB_m-2*pb_m);

% calculate non-dimensional parameters
KI=rm*Cf*L.*AUmR*D^3.*dtm./(IA+Ia)/8;
Kw=g*dtm*rm*L*D^4./(IA+Ia)./AUmR/8;

alfa=KI.*(1-2*pb_m).*((1-pB_m).*(1+pB_m-2*pb_m)+2*gamma*sqrt(pb_m.*(1-pb_m)));
% eq(5) in Appendix B
beta=Kw.*(So-1)*pi.*(1-2*pb_m).*sqrt(pb_m.*(1-pb_m)); % eq(5) in Appendix B

ab2=sqrt(alfa.*beta);

ttmax=atanh(((1-epps)*ab2-(1-epps)*beta)./((epps+(1-epps)*sqrt(beta./alfa)).*alfa-beta))./ab2;
ttmax(ttmax>1)=1; % non-dimensional time

```

```

tanhabt=tanh(ab2.*ttmax);

uu=1-(ab2+beta.*tanhabt)./(ab2+alfa.*tanhabt);    % non-dimensional velocity

u=uu.*UmR;                                     % convert to dimensional velocity
omega=u./(D*(0.5-pb_m));                       % angle velocity
t=ttmax.*dtm;                                  % covert to dimensional time

ds=u.*t/2;                                     % displacement of munition at each step eq(7) in Appendix B

dpsi=omega.*t/2;
psi=cumsum(dpsi);

cds=cumsum(ds);

DS=sum(ds);                                     % total displacement

disp([Labels{n},': ',num2str(DS)]);

ncid=netcdf.create(['Munition_',Labels{n},'.nc'],'clobber');
netcdf.putAtt(ncid,-1,'description',['gamma=',num2str(gamma),' roughness=',...
    num2str(Rough),'(mm) eps=',num2str(epps)]);

dimNM1=netcdf.defDim(ncid,'Ngrid',NM+1);
dimNM=netcdf.defDim(ncid,'Midgrid',NM);
dim20=netcdf.defDim(ncid,'string20',20);
dimNi5=netcdf.defDim(ncid,'Ni5',Ni5);
if DS~=0
    dimNit1=netcdf.defDim(ncid,'Ntheta>1',Nit1);
end

dateid=netcdf.defVar(ncid,'Date','char',[dimNM1,dim20]);
dayid=netcdf.defVar(ncid,'days','float',dimNM1);
umid=netcdf.defVar(ncid,'Um','float',dimNM);
netcdf.putAtt(ncid,umid,'units','m/s');
vmid=netcdf.defVar(ncid,'Vm','float',dimNM);
netcdf.putAtt(ncid,vmid,'units','m/s');

pbid=netcdf.defVar(ncid,'pB','float',dimNM);

I5id=netcdf.defVar(ncid,'I5','long',dimNi5);
netcdf.putAtt(ncid,I5id,'Description','Index for pB<0.5');
theta0id=netcdf.defVar(ncid,'Theta0','float',dimNi5);
thetaid=netcdf.defVar(ncid,'Theta','float',dimNi5);

```

```

if DS~=0
    It1id=netcdf.defVar(ncid,'It1','long',dimNit1);
    netcdf.putAtt(ncid,It1id,'Description','Theta>1 index in I5 index');
    dsid=netcdf.defVar(ncid,'ds','float',dimNit1);
    netcdf.putAtt(ncid,dsid,'units','m');
    psiid=netcdf.defVar(ncid,'psi','float',dimNit1);
end

netcdf.endDef(ncid);

netcdf.putVar(ncid,dateid,date_v);
netcdf.putVar(ncid,dayid,dayv);
netcdf.putVar(ncid,umid,vem);
netcdf.putVar(ncid,vmid,vnm);
netcdf.putVar(ncid,pBid,pBall_m);
netcdf.putVar(ncid,I5id,i5);
netcdf.putVar(ncid,theta0id,theta0);
netcdf.putVar(ncid,thetaid,theta);
if DS~=0
    netcdf.putVar(ncid,It1id,it1);
    netcdf.putVar(ncid,dsid,cds);
    netcdf.putVar(ncid,psiid,psi);
end

netcdf.close(ncid);

end

```

6. Conclusions

The coupled Delft3D-UnMUSB model (written in Matlab) is used for predicting underwater munition mobility and burial in sandy bed, on the base of the results from SERDP sponsored projects such as MR-2224, MR-2227, MR-2319, and MR-2320, and ONR sponsored mine burial project. Roll of the munition is the major dynamics of this model with a new concept of its rolling center in the sediment. The munition displacement caused by rolling satisfies the Riccati equation with analytical solution. Along with the dynamical model, the parameterized scour model with re-exposure parameterization is used as part of the prediction system.

Data collected at the shallow quadpod during TREX13 (21 April to 23 May 2013) off the coast of Panama City, Florida from the SERDP project MR-2320 were used as model verification. The environmental data such as bottom currents, water depth (h), peak period (T_p), and significant wave height (H_s) are used to verify the Delft3D model. The munition positions tracked by the sector scanning sonar images and the maintenance divers are used to verify the munition mobility and burial model.

Although the coupled Delft3D-UnMUSB model has capability to predicts munition mobility, the

model has its own weakness such as it only considers cylindrical munition as well as the roll of the munition around its major axis. The UnMUSB model ignores pitch and yaw. Besides, the seabed is assumed flat. It is necessary to extend the UnMUSB modeling to more realistic seabed environment, munition shapes and more complete motion for operational use. Besides, it is noted that the observed data are quite crude and not sufficient to accurately verify the model prediction on an object's mobility and burial. Obviously both work on the model and more detailed data is required to validate UnMUSB.

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Appendix A. Journal Article (IEEE-JOE) for the UnMUSB Model

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Prediction of Mobility and Burial of Objects on Sandy Seafloor

Peter C. Chu , Chenwu Fan, Joseph Calantoni, *Member, IEEE*, and Alex Sheremet

Abstract—A numerical model was developed to predict mobility and burial of seafloor cylindrical objects. The model contains four components: 1) object's physical parameters such as diameter, length, mass, and rolling moment; 2) dynamics of rolling cylinder around its major axis; 3) empirical sediment scour model; and 4) seabed environmental characteristics such as currents, waves (peak period, significant wave height), sediment density, and medium sediment grain size. Under the sponsorship of the Department of Defense Strategic Environmental Research and Development Program, a field experiment was conducted from April 21 to May 23, 2013 off the coast of Panama City, FL, USA to measure both objects' mobility using sector scanning and pencil beam sonars and simultaneous environmental time-series data of the boundary layer hydrodynamics and sediment transport conditions for driving mobility. Comparison between modeled and observed data shows the model capability. Future work needs to consider more realistic object shapes and motions such as pitch and yaw, and wavy seabed.

Index Terms—Equilibrium burial percentage, munitions mobility and burial, object mobility model, object shields parameter, sediment shields parameter, sediment supporting point.

NOMENCLATURE

(a_1, a_2, a_3)	Empirical coefficients (25).
B	Object burial depth (m) [(8), (11), (12), (14), (15a), (17), (28)].
b	Depth of rotation axis in sediment (m) [(2), (8), (11), (14), (15a), (17)].
(c_1, c_2)	Coefficients in e-folding time scale for scour burial [see (27)].
(C_d, C_l)	(Drag, Lift) Coefficients [(12), (13)].

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D	Diameter of cylindrical object (m) [(2), (7), (8), (12), (14), (15a), (17), (18), (21), (27), (28)].
d_{50}	Medium of the grain diameter (m) [(1), (24), (27)].
$(\mathbf{e}_h, \mathbf{e}_v)$	Horizontal and vertical unit vectors [(4), (5)].
f	Wave friction factor [see (24)].
F_a	Added mass (N) [(12), (14)].
F_d	Water drag force (N) [(12), (14)].
F_l	Water lift force (N) [(12), (14)].
F_w	Buoyancy force (N) [(12), (14)].
h	Mean water level (m).
H_S	Significant wave height (m).
I_A	Rolling moment of object about b without added mass (kg m^2) [(16), (17), (18)].
I_A^*	Rolling moment of object about b with added mass (kg m^2) [see (17)].
I_o	Rolling moment of object about its symmetric axis (kg m^2) [see (18)].
L	Length of cylindrical object (m) [(12), (21)].
l	Displacement of object (m) [see (23)].
M	Mass of object (kg).
$\mathbf{n}$	Unit vector normal to the cylinder surface [(3), (4)].
p_B	Object percentage burial [(15a), (16), (17), (20), (21), (22), (26), (28)].
$p_{B,eq}$	Equilibrium percentage burial [(25), (26)].
p_b	Relative depth of rotation axis in sediment [(15a), (21), (28)].
S_o	Relative object's density versus water density [(15a), (21)].
S_{sed}	Relative sediment's density versus water density [see (24)].
T_*	e-Folding time scale for reaching equilibrium scour burial (s) [(26), (27)].
T_B	Torque to roll the object backward (N m) [see (16)].
T_F	Torque to roll the object forward (N m) [(16), (17)].
T_P	Wave peak period (s) [see (24)].
$\mathbf{T}_S$	Sediment torque on object (N m) [(6), (9)].
U_c	Bottom current speed (m s^{-1}).
U_{br}	Bottom wave orbital velocity (m s^{-1}) [see (24)].
u_o	Object's rolling horizontal velocity (m s^{-1}) [(2), (3), (5), (19), (23)].
$\hat{u}_o$	Relative object's rolling horizontal velocity [(19), (20), (22)].
U	Component of $\mathbf{V}_w$ perpendicular to the object's main axis (m s^{-1}) [(12), (15a), (21)].

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V	Components of $\mathbf{V}_w$ parallel to the object's main axis ($\text{m}\cdot\text{s}^{-1}$).
$\mathbf{V}_w$	Bottom water velocity (current and wave) vector ($\text{m}\cdot\text{s}^{-1}$).
(v_e, v_n)	Bottom current velocity vector ($\text{m}\cdot\text{s}^{-1}$).
w	Weight coefficient for unburial parameterization [see (29)].
(α, β)	Coefficients in the object mobility model [(20), (21), (22)].
γ	Ratio of lift versus drag coefficient [(13), (15a)].
θ_{opb}	Object burial percentage Shields parameter [(15a), (15b), (16), (17), (20), (22)].
θ_{sed}	Sediment Shields parameter [(24), (25), (27)].
λ	Ratio between b and B [(11), (28)].
Π	Volume of object (m^3) [(17), (18), (21)].
ρ_o	Density of object ($\text{kg}\cdot\text{m}^{-3}$) [(15a), (17), (18)].
ρ_s	Density of sediment ($\text{kg}\cdot\text{m}^{-3}$) [see (24)].
ρ_w	Density of seawater ($\text{kg}\cdot\text{m}^{-3}$) [(12), (17), (21), (24)].
ψ	Radial angle around the center of cylinder from the vertical axis.
ω	Angular velocity of rolling object (s^{-1}) [(2), (16), (17)].

I. INTRODUCTION

PREDICTION of the mobility and burial of a rigid object on the seafloor is confounded by the influence of environmental characteristics such as waves, currents, sediment dynamics, and seafloor morphology. The phenomenology of munitions, mines, and other objects in shallow underwater environments has broad military and civilian applications. For example, formerly used Defense sites have vast underwater environments that are contaminated with munitions and other unexploded ordnance that need to be managed and remediated [1].

We expect similarities exist between the prediction of burial and mobility for both underwater munitions and sea mines. Observations show that mine burial is sensitive to the sediment type, hydrodynamic forcing, and the size and shape of the mine [2]. A physical model to predict mobility and burial of an object on the seafloor has six degrees of freedom (DoF), and is usually called a 6-DoF model [3]. Three degrees of freedom refer to the position of center of mass of the object, and the other three degrees of freedom represent the orientation of the object (i.e., roll, yaw, and pitch). The 6-DoF model is based on the approach previously developed to predict sea mine impact burial, named IMPACT35, which predicts the trajectory of a mine through air and water, in addition to the amount of burial that occurs upon impact with the seafloor [4]–[6]. The IMPACT35 model contains full physics including nonlinear dynamics, fluid-structure interaction, instability theory [7], and bearing factor method to calculate the sediment force and torque with cavities for mine movements in sediment [8]. We have modified the existing 6-DoF model (IMPACT35) to predict the burial and mobility of idealized munitions (i.e., cylindrical objects) on the seafloor. Previous experimental observations [9]–[11] and statistical modeling [12] focus on the determination of the conditions that initiate motion, specifically, the roll of a

munition around its principle axis, both on a hard surface and on a sand bed in the presence of concurrent scour burial. Mobility of an object on the seafloor is characterized by the balance between the destabilizing hydrodynamics and the stabilizing weight of the object [13], [14]. The Shields parameter for object percentage burial θ_{opb} is estimated using the size and density of the object as well as the local velocity perpendicular to its main axis, U . The critical value for the mobility threshold of cylindrical objects was found to occur for, $\theta_{\text{opb}} > 1$.

In noncohesive sandy sediments, objects on the seafloor are subject to scour, which is a common burial mechanism that is controlled by the fluid velocity relative to other environmental characteristics of the sediments, regional pressure gradients, tidal forcing, and the orbital velocity of waves. The nondimensional sediment Shields parameter θ_{sed} is important for the scour burial. The existing experimental observations and statistical modeling [9]–[14] provide a useful framework for the development of a model for munitions.

A model to predict objects' mobility and burial on sandy seafloor has been developed in this study based on the following four assumptions:

- 1) cylindrical munition with a large aspect ratio (length L much larger than diameter D , $L \gg D$);
- 2) roll dominating the motion of the long cylinder ($L \gg D$) with no yaw and pitch;
- 3) burial depth (B) smaller than half of the diameter ($B < D/2$) [10];
- 4) flat seabed.

Fig. 1 shows the flowchart of this prediction system. It consists of four parts:

- 1) cylindrical object model with the burial percentage Shields parameter (θ_{opb});
- 2) sediment scour model with sediment Shields parameter (θ_{sed});
- 3) input data such as object's physical parameters (D , S_o , M , I_o), and environmental variables such as near seabed ocean currents, bottom wave orbital velocity (U_{br}) water depth (h), wave peak period (T_P), significant wave height (H_S), sediment characteristics;
- 4) model output such as the burial percentage p_B , and the object's displacement.

The rest of the article is outlined as follows. Section II describes the experimental design and methods. Section III presents the results of sedimentology and hydrodynamic observations from the field experiment. Section IV presents the sediment supporting point for estimating rolling motion. Section V defines the drag, lift, buoyancy forces, and added mass. Section VI presents the object mobility prediction model. Section VII presents the scour burial model. Section VIII shows the comparison between the field observations and model prediction of objects' mobility and burial. Finally, Section IX concludes this article.

II. FIELD EXPERIMENT

A field experiment was conducted from April 21, 2013 to May 23, 2013 to monitor the waves and currents as well as to simultaneously track the location of surrogate munitions on the seafloor [9]. The combined observations of munitions mobility

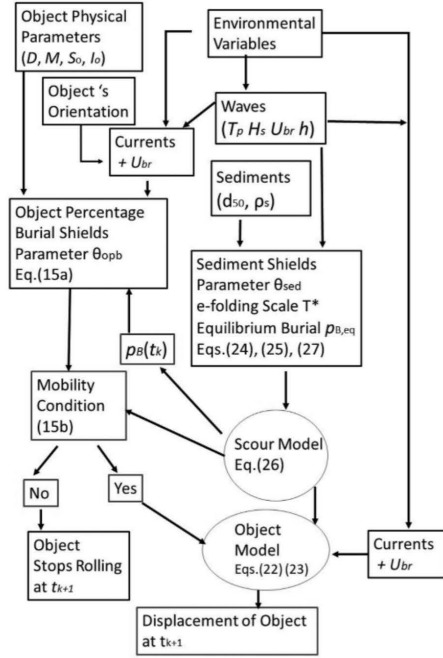


Fig. 1. Flowchart of the prediction system for objects' mobility and burial. See the nomenclature list and text for definitions and derivations of parameters.

and the driving hydrodynamic conditions provide data for model development to predict seafloor munitions' mobility and burial. During the field experiment, a range of surrogate munitions were deployed that include variations in caliber, bulk density, shape, and rolling moment.

A. Surrogate Munitions

Four types of surrogate and replica munitions that roughly represented the 155-mm HE M107, 81-mm mortar, 25-mm cartridge, and 20-mm cartridge were designed and fabricated using crude drawings and specifications provided by existing Army Technical Manuals (e.g., TM 43-0001-27 and TM 43-0001-28). Additional replica munitions for each of the four types listed above were purchased commercially. The purchased replicas were used to provide overall dimensions and shape details for the four types of munitions. These purchased replicas were constructed from solid urethane casts for the 155- and 81-mm calibers and solid aluminum for the 25- and 20-mm calibers (note the purchased 155-mm caliber was positively buoyant and not deployed).

A total of four surrogate munitions and nine replicas were deployed at each of two water depths adjacent to the quadpod instrument frames (described later). Replica munitions have the general dimensions and shape of their real counterparts;

however, replica munitions are constructed from a single solid material to cover a range of munitions' bulk density. Surrogate munitions are similar in size and shape to replicas except they were fabricated to have bulk densities and rolling moments that more closely match their real counterparts. The complete list of deployed and recovered objects along with brief descriptions and their material properties is given in Table I. The mass for each object was determined post fabrication before deployment. The density listed for the replicas fabricated from a single solid material is the known material density. The volume and rolling moments for all objects were estimated using CAD software. For the surrogates and replicas that are composites of more than one material the bulk density was estimated by combining the measured mass and the volume estimate from the CAD software. Photographs for the four types of surrogate munitions are shown in Fig. 2. The first type was designed to represent the projectile, 155 mm, HE, M107, typically fired from 155-mm howitzers [Fig. 2(a)]. The second type was designed to represent the 81-mm mortar [Fig. 2(b)]. The third and fourth types were designed to represent the cartridges, 25 and 20 mm [Fig. 2(c)]. Each of the photographs contains the fabricated surrogate, the purchased replica, and the replica fabricated from a single solid piece of aluminum [for 155 mm; see Fig. 2(a) bottom], with a solid stainless-steel body and aluminum tail section [for 81 mm; see Fig. 2(b) bottom], with a steel projectile fabricated with nearly identical density and dimensions to those found in the Army Technical Manual [for 25- and 20-mm cartridges; see Fig. 2(c) bottom].

Rolling moment for each surrogate or replica is also measured since it is an important parameter for the mobility. The difference in rolling moment calculated for the 81-mm surrogate with and without fins is only 4.4%. While the fins seemingly add little to the total rolling moment, their interaction with the seafloor and bottom currents should not be underestimated. For 25- and 20-mm cartridges, a solid piece of Delrin plastic cut in the shape of the shell casing very closely matches the weight of the remaining portion of the munitions. Note that while a steel projectile is very common for the 20-mm type, we realize that the projectiles for the 25-mm type may be much more sophisticated and varied. Here, we assumed the simplest solid steel projectile for the 25-mm type.

B. Instrumentation and Deployment

Instruments to observe the local hydrodynamics and resulting munitions' mobility were mounted on a pair of large rugged frames (herein referred to as "quadpods") that were deployed at two different water depths (herein referred to as "deep" and "shallow") during the Target and Reverberation Experiment (TRES13). The quadpods were deployed in the northern Gulf of Mexico offshore Panama City Beach, FL, USA [see Fig. 3(a)]. The deep quadpod was deployed at 30° 03.02330 N, 85° 41.33630 W, in about 20-m water depth, whereas the shallow quadpod was deployed at 30° 04.80994 N, 85° 40.41064 W, in about 7.5-m water depth. Fig. 3(b) shows the deployment of the quadpod from a research ship.

TABLE I
LIST OF SURROGATE AND REPLICA MUNITIONS USED DURING TREX13

Type with Diameter	Labels	Materials Type	Recovered	Rolling Moment (10^{-4} kg m ²)	Volume (10^{-5} m ³)	Mass (kg)	Density (kg m ⁻³)
155 mm, HE, M107	D5, D6	Delrin, 304 Stainless Surrogate	D5, D6	923.59	768.38	34.15	4,444
	D3, D4	Aluminum Replica	D3, D4	500.48	768.38	20.91	2,721
81 mm mortar	C3, C4	Delrin, 316 Stainless, Aluminum tail fins Surrogate	C3, C4	24.73	120.93	3.76	3,109
	C5, C6	304 Stainless, Aluminum tail fins Replica	C5, C6	50.51	120.93	8.70	7,194
	C1, C2	Urethane Replica		8.34	120.93	1.45	1,199
25 mm cartridge	B5, B6	Delrin, 316 Stainless Surrogate	B5, B6	0.46	16.55	0.39	2,356
	B7, B8	304 Stainless Replica	B7, B8	1.98	16.55	1.32	7,975
	B3, B4	Aluminum Replica	B3, B4	0.68	16.55	0.43	2,598
	B1, B2	Delrin Replica		0.35	16.55	0.23	1,390
20 mm cartridge	A5, A6	Delrin, 316 Stainless Surrogate	A6	0.13	7.70	0.20	2,597
	A7, A8	304 Stainless Replica	A7	0.53	7.70	0.63	8,181
	A3, A4	Aluminum Replica	A3, A4	0.18	7.70	0.19	2,468
	A1, A2	Delrin Replica		0.09	7.70	0.11	1,429

A total of 26 objects were deployed and 18 objects were recovered (from [9]). Type surrogate munitions were fabricated to have rolling moments within 10% of the estimated rolling moment of the real counterpart.

Waves and currents were obtained using both an acoustical surface tracking (Nortek AWAC) and pressure time series. A sector scanning sonar was mounted on one of the legs of each of the quadpods, scanning a 110° swath every 12 min. The surrogate and replica munitions (described above) were deployed within the view field of the sector scanning sonar. Divers laid the surrogate and replica munitions on the seafloor around each quadpod according to a predetermined schematic. Only objects laid by divers under the shallow quadpod were photographed [see Fig. 4(a)]. The light blue arc roughly denotes the field of view of the sector scanning sonar. The dark blue circle in the upper left denotes the location of the surrogates. The other replicas were grouped according to relative bulk density. In this case, the red boxes denote the objects that were not recovered from the shallow quadpod site. A maintenance dive was performed on the morning of May 8, 2013 immediately after the storm event and found a single clue that the surrogates and replicas may

have been buried in place as opposed to being transported away by the waves and currents [see Fig. 4(b)]. The 155-mm replica fabricated from solid aluminum was partially buried in the crest of a sand ripple (see Fig. 5). The sharp crest of the ripple (or bedform) is visible in the foreground of the image, indicated by the black arrow. The replica shown here was the only object not completely buried during the storm event at the shallow quadpod location (see Fig. 5). Excavating by hand, divers were able to recover a total of eight munitions buried just below the surface very near the known initial locations at the shallow quadpod during the maintenance dive performed on May 8, 2013.

III. OBSERVATIONAL DATA

A. Sedimentology

Sediment samples were collected at both shallow and deep quadpod locations during the deployment and retrieval. Two

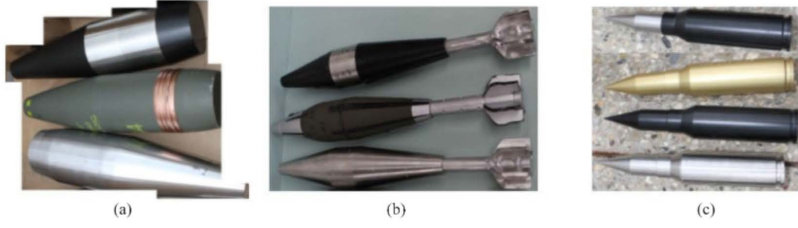


Fig. 2. Fabricated surrogate, purchased replica, and fabricated replica of (a) 155-mm HE M107; (b) 81-mm mortar; and (c) 25- and 20-mm cartridges (from [9]).

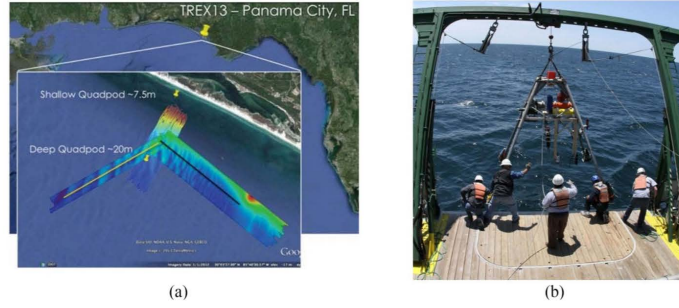


Fig. 3. (a) Deployment of the deep quadpod with near 3.3-m height into around 20-m depth through the A-frame of *R/V Smith* around 1240 local time on April 20, 2013; and (b) locations of deep and shallow quadpods (from [9]).

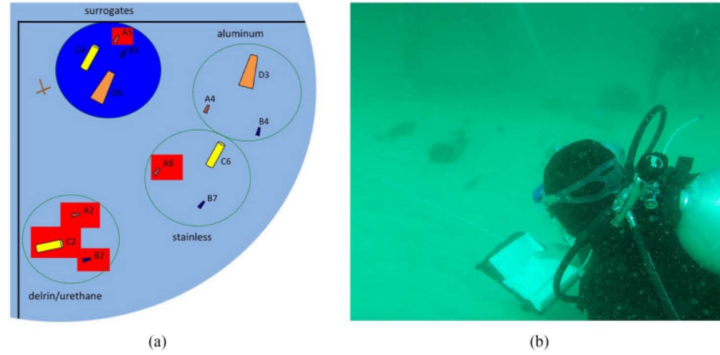


Fig. 4. (a) Layout of objects laid by divers under the shallow quadpod; and (b) the photograph of divers laying the object field during the shallow quadpod deployment (from [9]).

cores collected at shallow quadpod location have been found to contain nearly 100% sand. Grain size distributions were obtained with standard sieve techniques and results for porosity, bulk density, and void ratio were obtained by measuring the weight loss or water weight. We estimated the median grain diameter (d_{50}) and sediment density (ρ_s) in the upper 6 cm of the bed to be

$$d_{50} = 0.23 \times 10^{-3} \text{ m}, \quad \rho_s = 2.69 \times 10^3 \text{ kg} \cdot \text{m}^{-3}. \quad (1)$$

B. Waves and Currents

Since no similar initial objects' locations were given for the deep quadpod, we restrict our wave and current data depiction on the shallow quadpod. The waves were observed locally using the acoustical surface tracking of the Nortek AWAC with a frequency of 1 MHz installed at top of shallow and deep quadpods (one for each) from April 21, 2013 to May 23, 2013. The AWAC was placed as 2.3-m height above the bed on the shallow quadpod. Within every 30 min, the AWAC measures

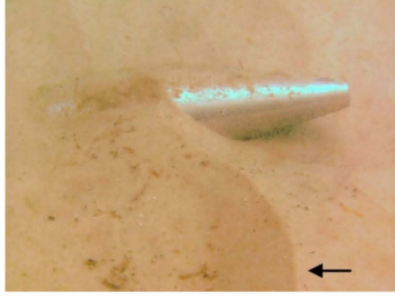


Fig. 5. Object field of the shallow quadpod taken on the morning of May 8, 2013 (from [9]).

high frequency (2 Hz) horizontal water velocity for 20 min. For 20 min starting at the beginning and midpoint of every hour, we obtain the wave statistical analysis to get peak period (T_p), significant wave height (H_S), and mean water level (h). The bottom wave orbital velocity (U_{br}) is the result of interactions between surface waves and the seafloor. A well-established linear wave model with MATLAB function [15] is used to calculate the bottom orbital velocity U_{br} with the water depth (h), significant wave height (H_S), and peak period (T_p) as input and the bottom orbital velocity (U_{br}) as output (see [15, App. D]).

The AWAC was installed at the shallow quadpod with the vertical distance of 0.5 m for two consecutive bins to obtain the current profile vector $\mathbf{U}_c = i\mathbf{v}_e + j\mathbf{v}_n$, with the temporal resolution of 1 min. Here, (i, j) are unit vectors in longitudinal and latitudinal directions. Let $U_c = \sqrt{v_e^2 + v_n^2}$ be the current speed. The currents at the lowest bin (2.3 m above the seabed) were directly taken for forcing surrogates or replicas with diameters around 0.15 m without using the log profile because 2.3 m is far above the seabed log layer. The bottom water velocity vector of combined current and waves is represented by $\mathbf{V}_w$ with $|\mathbf{V}_w| = U_c + U_{br}$ and the orientation $\psi = \tan^{-1}(v_n/v_e)$. Fig. 6 shows the time series of the environmental parameters [v_e , v_n , U_c , h , T_p , H_S , U_{br}]. Two consecutive atmospheric cold fronts passed by the area between May 5 and 7, 2013. The current speed U_c fluctuates from 0 to 0.5 m/s [see Fig. 6(c)] with no evident strengthening during the storm event. The bottom wave orbital velocity (U_{br}) was strengthened drastically from 0.2 m/s before the storm to 0.8 m/s on May 5, 2013 during the storm, and reduced to less than 0.2 m/s after the storm event on May 7, 2013 [see Fig. 6(g)].

IV. SEDIMENT SUPPORTING POINT

Roll of an object on the sandy floor needs a supporting point in sediment. Let the water velocity (consisting of current and waves) near the seabed ($\mathbf{V}_w$) be in the direction toward the cylinder with an angle ϕ perpendicular to the main axis of the cylinder, and be decomposed into $\mathbf{V}_w = (U, V)$ with U the

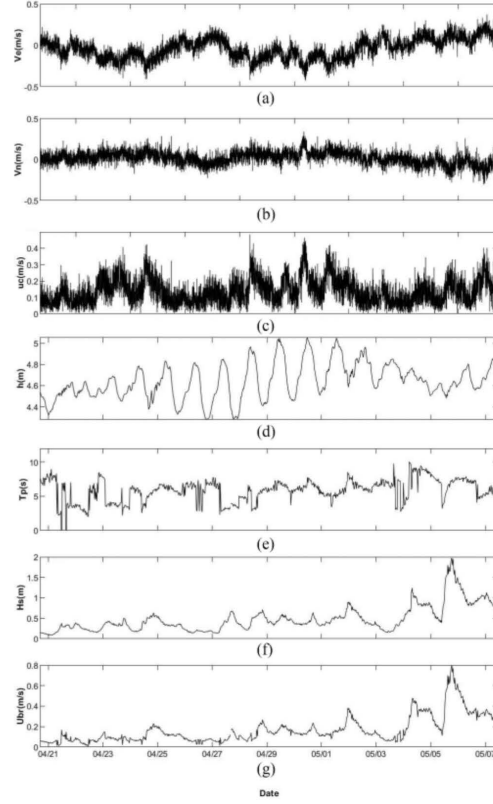


Fig. 6. Time series with 30-min increment of (a) near bed (~ 0.15 m) longitudinal current v_e (m/s); (b) near bed (~ 0.15 m) latitudinal current v_n (m/s); (c) near bed (~ 0.15 m) current speed U_c ; (d) water depth h (m) (mean water level plus quadpod's height, 2.3 m); (e) peak period T_p (s), (f) significant wave height H_S (m); and (g) computed bottom wave orbital velocity U_{br} (m/s), at the shallow quadpod from April 21 to May 23, 2013.

perpendicular component, and V the parallel component (see Fig. 7) to the main axis of the cylinder.

As the object rolls with angular velocity ω , let the axis of rotation inside the sediment be at depth b ($b < B$) and let ψ be the radial angle of the cylinder from the vertical axis with ψ_B at the top of sediment, and ψ_b at the depth b (see Fig. 8). The horizontal velocity of the object rolling around the point b is given by

$$u_o = \omega \left(\frac{D}{2} - b \right) \quad (2)$$

with the radial component of $u_o \sin \psi$ at any depth inside the sediment. Thus, the compressive normal stress of sediment on the object is represented by

$$\mathbf{F}_S = -n\kappa u_o \sin \psi \quad (3)$$

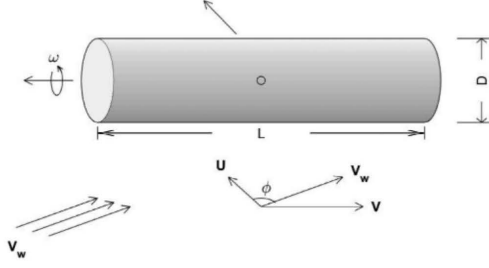


Fig. 7. Roll of a cylindrical object on the seafloor with a large aspect ratio forced by the combination of ocean currents and bottom wave orbital velocity. Here, $(\pi/2 - \phi)$ is the angle between $\mathbf{V}_w$ and the main axis of the cylinder.

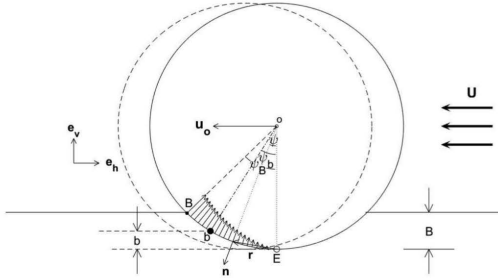


Fig. 8. Location of the axis of rotation of the cylinder in the sediment b is determined by the assumption of zero-sum torque to the roll.

where $\mathbf{n}$ is the unit vector normal to the cylinder surface, and κ is the compressive coefficient. Let $\mathbf{n}$ be decomposed into

$$\mathbf{n} = -\mathbf{e}_h \sin \psi - \mathbf{e}_v \cos \psi \quad (4)$$

where $(\mathbf{e}_h, \mathbf{e}_v)$ are horizontal and vertical unit vectors (see Fig. 2). The sediment compressive normal stress $\mathbf{F}_S$ is decomposed as

$$\mathbf{F}_S = \mathbf{e}_h \kappa u_o \sin^2 \psi + \mathbf{e}_v \kappa u_o \sin \psi \cos \psi. \quad (5)$$

With b as the axis of rotation, the sediment above (below) the depth b generates torque to resist (enhance) the rolling with the total torque from the sediment

$$\begin{aligned} \mathbf{T}_S &= \int_0^{\psi_B} [(\mathbf{r} - \mathbf{r}_b) \times \mathbf{F}_S] d\psi \\ &= \int_0^{\psi_B} [\mathbf{r} \times \mathbf{F}_S] d\psi - \mathbf{r}_b \times \int_0^{\psi_B} \mathbf{F}_S d\psi \end{aligned} \quad (6)$$

where $\mathbf{r}$ is the position vector at any point on the circle and $\mathbf{r}_b$ is the position vector at point b with point E as the origin

$$\begin{aligned} \mathbf{r} &= \frac{D}{2} (-\mathbf{e}_h \sin \psi + \mathbf{e}_v (1 - \cos \psi)) \\ \mathbf{r}_b &= \frac{D}{2} [-\mathbf{e}_h \sin \psi_b + \mathbf{e}_v (1 - \cos \psi_b)] \end{aligned} \quad (7)$$

The depths B and b are represented by

$$B = \frac{D}{2} (1 - \cos \psi_B), \quad b = \frac{D}{2} (1 - \cos \psi_b). \quad (8)$$

If we assume that at the depth b the total torque from the sediment is zero (i.e., zero-sum sediment torque for rolling), then we have

$$\mathbf{T}_S = 0. \quad (9)$$

Substitution of (5)–(7) into (9) gives

$$\psi_b = \tan^{-1} \left(\frac{\psi_B - \sin \psi_B \cos \psi_B}{\sin^2 \psi_B} \right). \quad (10)$$

The ratio $\lambda = b/B$ can be obtained from (8) and (10)

$$\lambda \equiv b/B = \frac{1 - \cos \left[\tan^{-1} \left(\frac{\psi_B - \sin \psi_B \cos \psi_B}{\sin^2 \psi_B} \right) \right]}{1 - \cos \psi_B}. \quad (11)$$

The ratio λ varies with the burial percentage $p_B = B/D$ mildly from near 0.4445 for $p_B = 0$ and 0.4630. Here, we take $\lambda = 0.453$ in this study.

V. DRAG, LIFT, BUOYANCY FORCES, AND ADDED MASS

The drag force (F_d), lift force (F_l), buoyancy force (F_w), and added mass (F_a) exerted on the object for rolling by the perpendicular component U are given by vv

$$\begin{aligned} F_d &= \frac{\rho_w C_d U^2 L (D - B)}{2}, \quad F_l = \frac{\rho_w C_l U^2 L D}{2} \\ F_w &= g(\rho_o - \rho_w) \Pi, \quad F_a = -\rho_w \Pi \frac{du_o}{dt} \end{aligned} \quad (12)$$

where $g = 9.81 \text{ m/s}^2$ is the gravitational acceleration; $\rho_w = 1025 \text{ kg/m}^3$ is the density of seawater; ρ_o is the density of the cylindrical object; (C_d, C_l) are the drag and lift coefficients across cylinder's main axis with vortex shedding caused by the oscillating flow (U) due to waves. If time averaged U within a certain time period being used, the mean coefficients for drag and lift (C_d, C_l), depending solely on the Reynolds number and aspect ratio (see Appendix A), can be used. Since the wave component, i.e., the bottom wave orbital velocity in $\mathbf{V}_w$ for the object model is computed from a linear wave model with the temporal resolution of 30 min, the mean coefficients for drag and lift are used. The vortex shedding from objects is neglected. Besides, the lift coefficient is less certain, we assume

$$C_l = \gamma C_d \quad (13)$$

where γ is the ratio of lift coefficient versus drag coefficient with γ being taken as value of 0.2.

The drag force rolls the cylinder forward around the axis of rotation b with the torque per unit length of $F_d(D/2 + B/2 - b)$. Here, $B/2$ is the distance between the volume center of the cylinder above the sediment (i.e., point C in Fig. 9) and the volume center of the whole cylinder (i.e., point O in Fig. 9). Note that point C is the center of the resultant drag force on the cylinder. The buoyancy force minus lift force resists the cylinder rolling forward caused by the drag with the torque per unit length of $F_w \sqrt{b(D-b)}$ (buoyancy force), $F_l \sqrt{b(D-b)}$

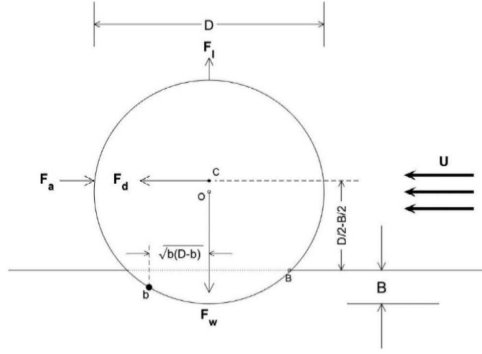


Fig. 9. Forces and torques due to drag, lift, buoyancy, and added mass on a partially buried cylinder by the combination of ocean currents and bottom wave orbital velocity (U) perpendicular to the major axis of the cylinder.

(lift), and the added mass $F_a(D/2 + B/2 - b)$. The threshold for the mobility of the object is represented by

$$F_d(D/2 + B/2 - b) + F_l\sqrt{b(D-b)} > F_w\sqrt{b(D-b)} + F_a(D/2 + B/2 - b) \quad (14)$$

which reduces to the mobility condition proposed in [10] if $F_l = 0$, $F_a = 0$, and the point C coincides with the point O.

VI. OBJECT MOBILITY MODEL

A. Core Dynamics

The cylindrical object with a large aspect ratio ($L \gg D$) is in motion when the object burial percentage Shields parameter θ_{opb} [13], [14]

$$\begin{aligned} \theta_{\text{opb}} &= \frac{\theta_0}{\pi} \left[\frac{1-p_b}{\sqrt{p_b(1-p_b)}} (1+p_B-2p_b) + 2\gamma \right] \\ \theta_0 &\equiv \left[\frac{C_d U^2}{gD(S_o-1)} \right], \quad p_B = \frac{B}{D} \\ p_b &= \frac{b}{D}, \quad S_o = \frac{\rho_o}{\rho_w} > 1 \end{aligned} \quad (15a)$$

satisfies the condition [(B6) in Appendix B]

$$\theta_{\text{opb}} > 1. \quad (15b)$$

Here, θ_0 is the object's Shields parameter; p_B is the percentage burial; p_b is the relative depth of the rotation axis in sediment; and S_o is the relative object's density versus water density. The corresponding moment of momentum equation including added mass is given by

$$\begin{cases} I_A \frac{d\omega}{dt} = T_F - T_B, & \text{if } p_B < 0.5, \theta_{\text{opb}} > 1 \\ \omega = 0, & \text{otherwise.} \end{cases} \quad (16)$$

Substitution of T_B in (B2) into (16) leads to

$$\begin{cases} I_A^* \frac{d\omega}{dt} = T_F - \Pi(\rho_o - \rho_w) \sqrt{b(D-b)}, & \text{if } p_B < 0.5, \theta_{\text{opb}} > 1 \\ \omega = 0, & \text{otherwise} \\ I_A^* = I_A + \left(\frac{D}{2} - b \right) \rho_w \Pi(D/2 + B/2 - b) \end{cases} \quad (17)$$

with

$$I_A = I_o + \rho_o \Pi D^2 / 4 \quad (18)$$

where I_o is the rolling moment about the symmetric axis of the munition; I_A is the rolling moment of munition about the point b (see Fig. 9) using the parallel axis theorem; and Π is the volume of the munition.

Let the relative horizontal velocity of the rolling object be defined by

$$\hat{u}_o = \frac{u_o}{U} \quad (19)$$

which makes $\hat{u}_o$ follow the direction of U . Substitution of (3), (B1), and (B2) into (17) and use of (19) lead to a special Riccati equation

$$\begin{cases} \frac{d(1-\hat{u}_o)}{dt} + \alpha(1-\hat{u}_o)^2 = \beta, & \text{if } p_B < 0.5, \theta_{\text{opb}}(t) > 1 \\ \hat{u}_o = 0, & \text{otherwise} \end{cases} \quad (20)$$

where

$$\begin{aligned} \alpha &= \frac{(1-2p_b)}{8I_A^*} \rho_w C_d |U| D^3 L \\ &\quad \times \left[(1-p_B)(1+p_B-2p_b) + 2\gamma\sqrt{p_b(1-p_b)} \right] > 0 \\ \beta &= \frac{(1-2p_b)}{2I_A^* |U|} g \rho_w \Pi (S_o - 1) D^2 \sqrt{p_b(1-p_b)} > 0. \end{aligned} \quad (21)$$

The special Riccati equation (20) integrated from t_k to t_k+1 ($k = 0, 1, 2, \dots, K-1$) has an analytical solution [16] with α_k and β_k as known constants during the integration. Substitution of (22), shown at the bottom of the next page, into (19) leads to the dimensional horizontal velocity of the rolling object, $u_o(t) = U \hat{u}_o(t)$, which should be used for each time interval Δt . The solution (22) depends on (α_k, β_k) , which involves the following three types of parameters:

- 1) time-independent physical parameters of the object for S_o , Π , L , and D ;
- 2) time-dependent water velocity $U(t_k)$ from observational data or numerical modeling;
- 3) time-dependent relative depth of sediment rolling axis $[p_b(t_k)]$, and burial percentage $[p_B(t_k)]$ determined using a scour burial model.

Let l be the displacement of object

$$dl/dt = u_o. \quad (23)$$

Integration of l with respect to time t leads to the munition's displacement.

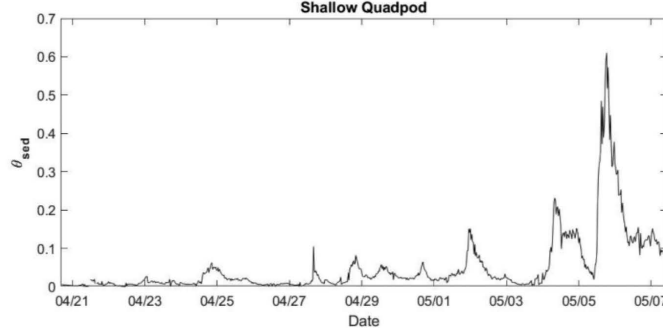


Fig. 10. Time series of sediment Shields parameter at the shallow quadpod.

VII. SCOUR BURIAL MODEL

Existing studies on scour burial were all concentrated on motionless objects. The ratio between the fluid force (bottom shear stress) and the weight of the sediment particles, i.e., the sediment Shields parameter (θ_{sed})

$$\theta_{sed} = \frac{fU_{br}^2}{g(S_{sed} - 1)d_{50}}, \quad S_{sed} = \frac{\rho_s}{\rho_w} \quad (24)$$

$$f = \exp \left[5.5 \left(\frac{6U_{br}T_P}{\pi d_{50}} \right)^{-0.2} - 6.3 \right]$$

is crucial for scour burial of motionless object and in turn for prediction of the percentage burial parameter $p_B(t) = B/D$ [12]–[14]. Here, f is the wave friction factor [17], ρ_s is the sediment grain density, and d_{50} is the median sand grain size. Using of the wave data (T_P , U_{br}) from Fig. 6(e) and (g), and sediment parameters (ρ_s , d_{50}) from (1), the sediment Shields parameter (θ_{sed}) are calculated from April 21 to May 23, 2013. It is less than 0.2 all the time except two atmospheric cold fronts passing by between May 4 and 6, 2013. The maximum value of θ_{sed} reached 0.61 (see Fig. 10).

As pointed in [13], the equilibrium percentage burial $p_{B,eq}$ for motionless cylinders induced by scour tends to increase as θ_{sed} increases. An empirical formula has been established

$$p_{B,eq} = a_1 \theta_{sed}^{a_2} - a_3 \quad (25)$$

with different choices of the coefficients (a_1 , a_2 , a_3) determined experimentally for cylinders subject to steady currents: $a_1 = 11$, $a_2 = 0.5$, $a_3 = 1.73$ [18], $a_1 = 0.7$, $a_2 = a_3 = 0$ [19], $a_1 = 2$, $a_2 = 0.8$, $a_3 = 0$ [20], and for cylinders under waves (depending on wave period): $a_1 = 1.6$, $a_2 = 0.85$, $a_3 = 0$ for T_p longer than 4 s [21]. For motionless cylinders before scour

burial reaching equilibrium, the percentage burial follows an exponential relationship [18]

$$p_B(t_k) = p_{B,eq} \left[1 - \exp \left(-\frac{t_k}{T^*} \right) \right] \quad (26)$$

where the e-folding time scale T^* is given by

$$T^* = \frac{c_1 \theta_{sed}^{c_2} D^2}{[g(S_{sed} - 1)d_{50}^3]^{0.5}}, \quad c_1 = 1.5, c_2 = 0.11. \quad (27)$$

With the bottom wave orbital velocity (U_{br}), sediment density (ρ_s), medium grain size (d_{50}) [see (1)], and in turn the sediment Shields parameters θ_{sed} , the equilibrium object percentage burial $p_{B,eq}$ are calculated using (25) with coefficients $a_1 = 1.6$, $a_2 = 0.85$, and $a_3 = 0$. The sediment supporting depth b (or p_b) is calculated from burial depth B (or p_B) using (11), i.e.,

$$b(t_k) = \lambda B(t_k), \quad p_b(t_k) = b(t_k)/D = \lambda p_B(t_k) \quad (28)$$

$$\lambda = 0.453.$$

It is noted that the predicted burial percentage (p_B) computed from (25) represents the depth that an object on the surface would bury to at that moment. But an object deployed at the beginning of the time sequence would always remain buried at the deepest burial it has reached so far. The burial depth of the base of the object below the ambient seabed is equivalent to the greatest depth that the scour pit has reached up to that point in time [22]. In other words, scour only acts to bury an object deeper. It can never unbury as the time series suggested by (26). Similar to [22], we propose a simple parameterization to represent the unburial process starting from k ($= 1, 2, \dots$): 1) doing nothing if $p_B(t_{k+1}) \geq p_B(t_k)$; and 2) computing the weighted average

$$\tilde{p}_B(t_{k+1}) = w p_B(t_k) + (1 - w) p_B(t_{k+1}) \quad (29)$$

$$\hat{u}_o(t) = \begin{cases} 1 - \frac{[1 - \hat{u}_o(t_k)] \sqrt{\alpha_k \beta_k} + \beta_k \tanh \left[\frac{(\sqrt{\alpha_k \beta_k}) (t - t_k)}{1} \right]}{\sqrt{\alpha_k \beta_k} + \alpha_k [1 - \hat{u}_o(t_k)] \tanh \left[\frac{(\sqrt{\alpha_k \beta_k}) (t - t_k)}{1} \right]}, & \text{for } p_B(t_k) < 0.5, \theta_{opb}(t_k) > 1 \\ 0, & \text{for } p_B(t_k) \geq 0.5, \theta_{opb}(t_k) \leq 1, \\ & t_k < t \leq t_{k+1}, \alpha_k = \alpha(t_k), \beta_k = \beta(t_k) \end{cases} \quad (22)$$

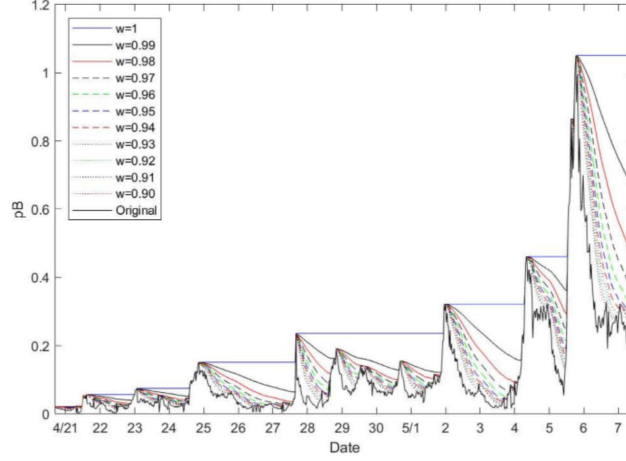


Fig. 11. Parameterization of the object's unburial with the weight coefficient varying from 0.90 to 1.00.

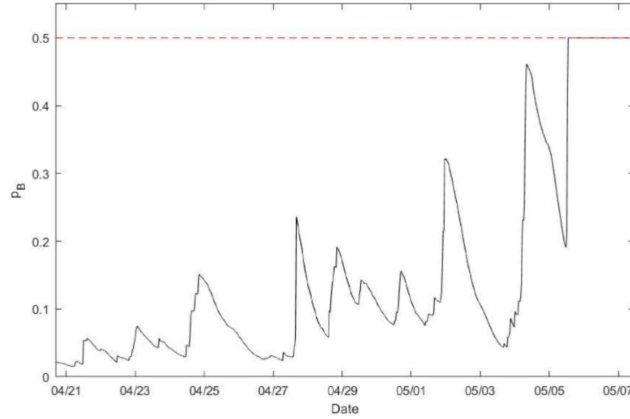


Fig. 12. Model predicted burial percentage $p_B(t)$ with unbury parameterization for each object at the shallow quadpod from April 20 to May 7, 2013. The predicted burial percentage $p_B(t)$ is less than 0.5 for all the munitions during the whole time period except during the storm event from 12:00 May 5 to 00:00 May 6, 2013.

if $p_B(t_{k+1}) < p_B(t_k)$ with w the weight coefficient. Fig. 11 shows the sensitivity of the parameterization on the weight coefficient. The stairway-type curve ($w = 1$) indicates the maximum percentage burial. For $w = 0.90$, the parameterized value $p_B(t_{k+1})$ is quite close to the original value (i.e., unparameterized). In this study, we take $w = 0.95$. The object mobility and burial models consist of (22), (23), and (25)–(29).

VIII. PREDICTION OF OBJECT'S MOBILITY AND BURIAL

Munition's mobility and burial were predicted using the object model consisting of (22), (25)–(29) with the environmental data

(see Fig. 6) used as model input. The model was integrated for each surrogate (or replica) from 12:40 (14:00) local time on April 20, 2013 at shallow quadpod with its orientation shown in Fig. 4(a). For each surrogate or replica deployed in the shallow quadpod, the angle between $\mathbf{V}_w$ [data represented in Fig. 6(a), (b), and (g)] and the direction perpendicular to cylinder's main axis is determined. The velocity vector $\mathbf{V}_w$ is then transformed into $\mathbf{V}_w = (U, V)$ with U the perpendicular component, and V the paralleling component. The component U is used in the model. The object's physical parameters such as the diameter (D), volume, mass (M), and density (ρ_o) are obtained from Table I.

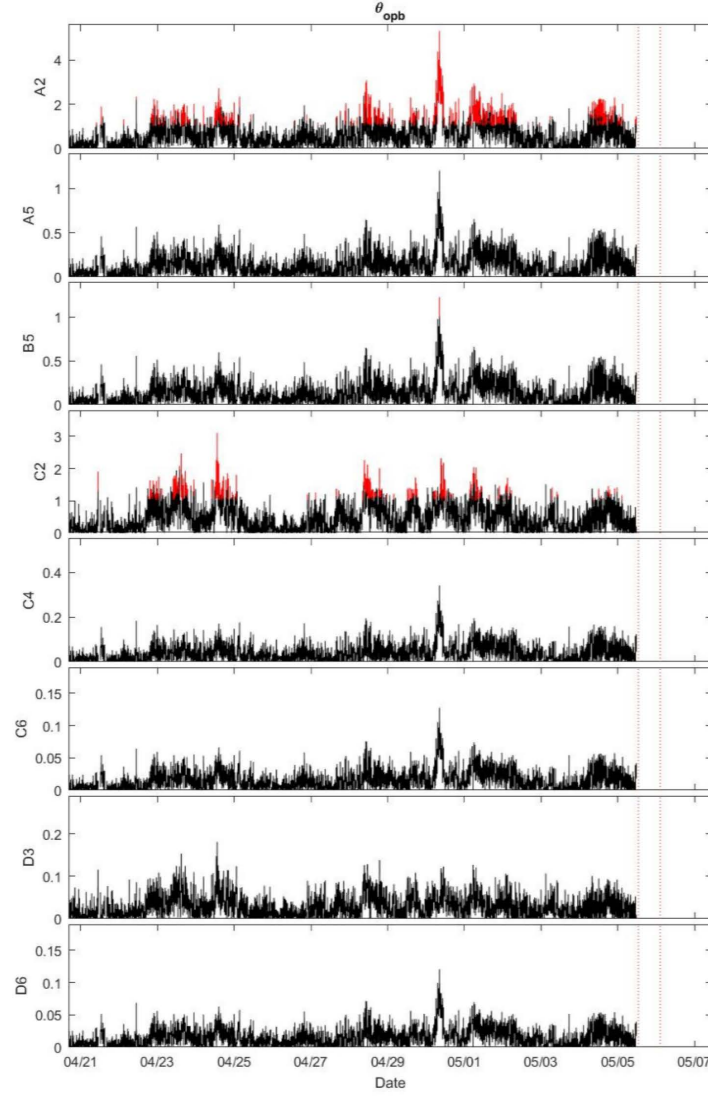


Fig. 13. Model predicted object percentage burial Shields parameters θ_{opb} at the shallow quadpod from April 20 to May 7, 2013. The red color shows that the condition for rolling the object [see (19)] is satisfied. The parameter θ_{opb} is not computed between 12:00 May 5 to 00:00 May 6, 2013 since the predicted burial percentage $p_B(t)$ is larger than 0.5. Among the eight objects, only A2 and C2 have evident time periods that the condition for rolling the object [see (19)] is satisfied.

The environmental data such as water depth (h), wave peak period (T_P), significant wave height (H_S), bottom wave orbital velocity (U_{br}) (shown in Fig. 6), and sediment characteristics [see (24)] are used for the scour model, i.e., (25)–(27) and (29), to get the burial percentage $p_B(t_k)$, and in turn the relative

rolling center depth $p_b(t_k)$ using (28). With the object's physical parameters (D , ρ_o , M), calculated $p_b(t_k)$, and observed (or modeled) bottom current velocity component perpendicular to the cylinder's main axis $U(t_k)$, the coefficients $[\alpha(t_k), \beta(t_k)]$ for the object model [i.e., (22) and (23)] are calculated using (20).

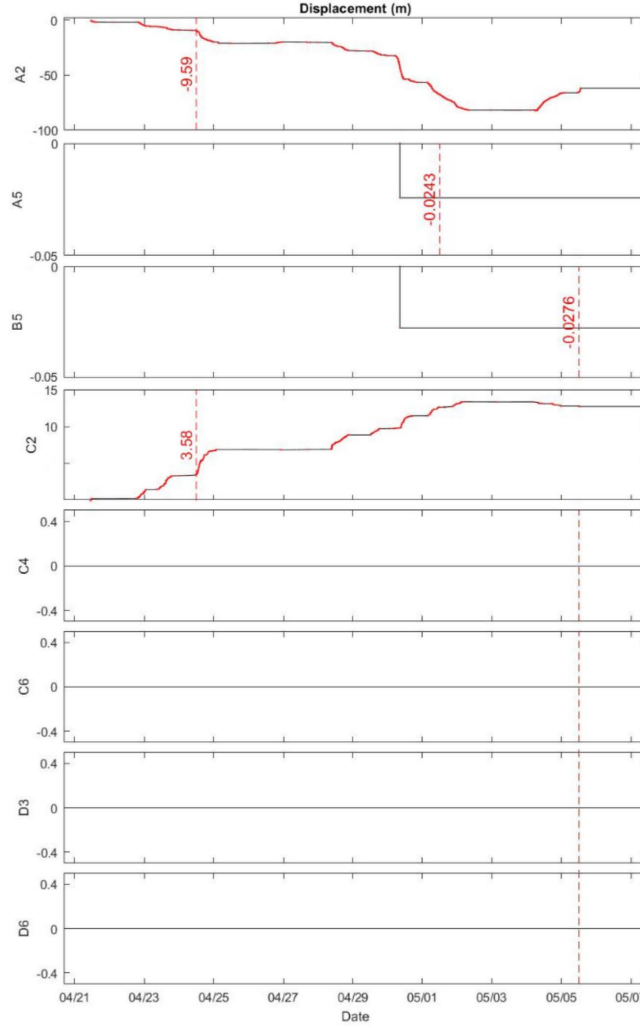


Fig. 14. Model predicted displacement $l(t)$ for each object at the shallow quadpod from April 20 to May 7, 2013. Among the eight objects, only A2 and C2 were immediately mobile and displaced 17.0 m (A2) and 6.38 m (C2) on 12:00 April 24, 2013 (dashed line); A5 and B5 were slightly mobile and displaced around 0.08 m after 12:00 April 30, 2013; C4, C6, D3, and D6 were completely motionless.

Based on the known initial locations of the objects at the shallow quadpod [see Fig. 4(a)], the model predicts the object's burial percentage $[p_B(t_k)]$ with the unburial parameterization (29) shown in Fig. 12, the object percentage burial Shields parameter $[\theta_{\text{opb}}(t_k)]$ shown in Fig. 13, and the object's displacement $[l(t_k)]$ shown in Fig. 14. The burial percentages p_B for all the objects were less than 0.5 except during the storm event on 12:00 May 5 to 00:00 May 6, 2013 local time (see Fig. 12). The red color in Fig. 13 shows that the object's rolling condition

$[\theta_{\text{opb}}(t) > 1]$ is satisfied. Here, the two dotted lines represent the storm event.

The surrogate and replica munitions' mobility and burial were observed by divers and sector scanning sonar images during the field experiment depicted in Section II and in [9]. A total of eight munitions in place at the shallow quadpod location were recovered by divers during the maintenance dive performed on May 8, 2013 [see Fig. 4(b)]. Note that the munitions excavated by the divers at the shallow quadpod location on May 8, 2013

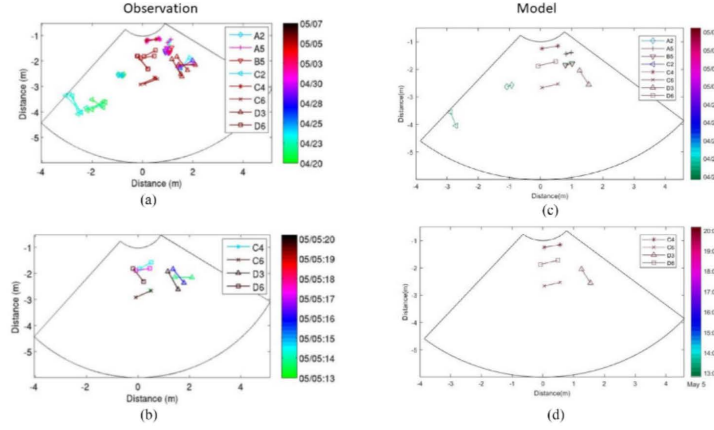


Fig. 15. Positions for all visible objects at the shallow quadpod location up to the maintenance diver performed on May 8: (a) observation for April 20–May 7, 2013; (b) observation for 13:00–20:00 on May 5, 2013; (c) model prediction for April 20–May 7, 2013; and (d) model prediction for 13:00–20:00 on May 5, 2013. Note that Fig. 15(a) and (b) was copied from [9]. The color bars denote the last time when each object was visible with dates for (a) and (c) and hour on May 5 for (b) and (d).

were immediately redeployed for the duration of the experiment. An overview of the observed munitions' mobility throughout the whole TREX13 experiment (April 20–May 7, 2013) is shown in Fig. 15(a), and during the storm event on 13:00–20:00 May 5, 2013 in Fig. 15(b). Objects A2 and C2 were immediately mobile and transported out of the field of view because they were last seen on April 23, 2013. However, the other objects were almost not mobile [see Fig. 15(a)]. The predicted objects' displacements (see Fig. 14) are consistent with the observations with large values of 9.59 m for A2 and 3.58 m for C2 on 12:00 April 24, 2012, and with near zero for the rest of the objects (see Fig. 14). Furthermore, overview of the modeled objects' mobility throughout the whole TREX13 experiment (April 20–May 7, 2013) is shown in Fig. 15(c), and during the storm event on 13:00–20:00 May 5, 2013 in Fig. 15(d). The similarity between the observation [Fig. 15(a) and (b)] and the model prediction [Fig. 15(c) and (d)] shows model capability.

However, the discrepancy between the two shows the model limitation. For example, yaw of munitions D3 and D6 was observed [Fig. 15(a) and (b)], but not predicted [Fig. 15(c) and (d)]. The munition D3 [rightmost triangles in Fig. 15(a) and (b)] moved, and then moved back; so, perhaps a net displacement of 0 m is correct, however the model predicted a value of $l = 0$ m by predicting that D3 never moved [see Figs. 14 and 15(c) and (d)]. The model limitation is due to the following four assumptions:

- 1) cylinder with a large aspect ratio ($L \gg D$);
- 2) no yaw and pitch;
- 3) percentage burial depth less than 0.5;
- 4) flat seabed.

Even if the bottom profile is flat when the object is deployed, the sand tends to accumulate in front of the object and to be eroded on the opposite side thus creating a wavy bed that affects

the dynamics of the object. The model will lose capability in the real world if the shape of munition is evidently different from the cylinder with $L \gg D$ and the effect of wavy seabed is large on the dynamics of the object.

IX. CONCLUSION

The following points summarize this article.

- 1) A numerical model was recently developed to predict underwater cylindrical objects' mobility and burial in sandy bed, on the basis of the results from SERDP sponsored projects such as MR-2224, MR-2227, MR-2319, and MR-2320, and ONR sponsored mine burial project. The roll of the object is the major dynamics of this model with a new concept of its rolling center in the sediment. The object's displacement caused by rolling satisfies the Riccati equation with analytical solution. Along with the dynamical model, the empirical scour model [13] is used as part of the prediction system.
- 2) Data collected during TREX13 off the coast of Panama City, FL, USA at shallow and deep quadpods from April 21 to May 23, 2013 from the SERDP project MR-2320 were used as model input or model verification depending on the data type. The environmental data such as bottom currents, water depth (h), peak period (T_p), significant wave height (H_s), sediment density (ρ_s), and median grain size (d_{50}) are taken as input to calculate the bottom wave orbital velocity U_{br} and sediment Shields parameter θ_{sed} , and in turn the burial percentage p_B and the sediment depth percentage (p_b). The object's physical characteristics data such as diameter (D), relative density (S_o), mass (M), and rolling moment (I_o) are taken as input to the object model. The objects' positions tracked by the sector scanning sonar

images are used for model evaluation. The predicted objects' positions agree qualitatively well with the observed surrogates (or replicas) data.

- 3) The model developed in this study is for cylindrical object only. Also, it only considers the roll of the cylinder around its major axis. The model ignores pitch and yaw. Besides, the seabed is assumed flat. It is necessary to extend the modeling effort to more realistic seabed environment, object shapes, and more complete motion for operational use.

APPENDIX A DRAG AND LIFT COEFFICIENTS

For cylindrical objects, the drag force is decomposed into along and cross-axis components. The drag coefficient across cylinder's main axis C_d depends on the Reynolds number

$$\text{Re} = \frac{UD}{\nu} \quad (\text{A1})$$

where $\nu = 0.8 \times 10^{-6} \text{ m}^2/\text{s}$ is the seawater kinematic viscosity; U is the horizontal water velocity perpendicular to the cylinder's main axis; and D is the cylinder's diameter (see Fig. 1). An empirical formula is used to calculate C_d [23]

$$C_d = \begin{cases} 1.9276 + 8/\text{Re}, & \text{if } \text{Re} \leq 12 \\ 1.261 + 16/\text{Re}, & \text{if } 12 < \text{Re} \leq 180 \\ 0.855 + 89/\text{Re}, & \text{if } 180 < \text{Re} \leq 2000 \\ 0.84 + 0.00003\text{Re}, & \text{if } 2000 < \text{Re} \leq 12000 \\ 1.2 - 4/\eta, & \text{if } 12000 < \text{Re} \leq 150000, \eta \geq 10 \\ 0.835 - 0.35/\eta, & \text{if } 12000 < \text{Re} \leq 150000, 2 \leq \eta < 10 \\ 0.7 - 0.08/\eta, & \text{if } 12000 < \text{Re} \leq 150000, \eta < 2 \\ 1.875 - 0.0000045\text{Re}, & \text{if } 150000 < \text{Re} \leq 350000 \\ 1/(641550/\text{Re} + 1.5), & \text{if } \text{Re} > 350000 \end{cases} \quad (\text{A2})$$

where $\eta = L/D$ is the cylinder's aspect ratio.

APPENDIX B THRESHOLD FOR INITIATION OF OBJECT MIGRATION

The drag and lift forces roll the object forward with the total torque (see Fig. 9)

$$\begin{aligned} T_F &= F_d (D/2 + B/2 - b) + F_l \sqrt{b(D-b)} \\ &= \frac{1}{2} C_d \rho_w U^2 L [(D-B)(D/2 + B/2 - b) \\ &\quad + \frac{C_l}{C_d} D \sqrt{b(D-b)}] \left(1 - \frac{u_o}{U}\right)^2. \end{aligned} \quad (\text{B1})$$

The buoyancy force and added mass roll the object backward with the torque $T_B = T_w + T_a$

$$\begin{aligned} T_B &= F_w \sqrt{b(D-b)} + F_a (D/2 + B/2 - b) \\ &= \frac{\pi}{4} g L D^2 (\rho_o - \rho_w) \sqrt{b(D-b)} \\ &\quad + \frac{du_o}{dt} \rho_w \Pi (D/2 + B/2 - b). \end{aligned} \quad (\text{B2})$$

When $T_F > T_B$, the object accelerates if it is in motion or starts to move if it is at rest ($u_o = 0$, $du_o/dt = 0$). When $T_F < T_B$, the object decelerates if it is in motion or keeps motionless if it is at rest. When $T_F = T_B$, the object keeps velocity constant if it is in motion or keeps motionless if it is at rest. Thus, the threshold for the munition's mobility becomes

$$T_F > T_B. \quad (\text{B3})$$

The acceleration-deceleration ratio is defined by

$$\frac{T_F}{T_B} = \theta_{\text{opb}} \left(1 - \frac{u_o}{U}\right)^2 \quad (\text{B4})$$

where

$$\begin{aligned} \theta_{\text{opb}} &= \frac{\theta_0}{\pi} \left[\frac{1-p_b}{\sqrt{p_b(1-p_b)}} (1+p_B-2p_b) + 2\gamma \right] \\ \theta_0 &\equiv \left[\frac{C_d U^2}{g D (S_o - 1)} \right], \quad p_b = \frac{b}{D}, \quad S_o = \frac{\rho_o}{\rho_w} > 1. \end{aligned} \quad (\text{B5})$$

Here, θ_{opb} is the object percentage burial Shields parameter based on p_b [13]; and S_o is the relative density of the object. For motionless munition ($u_o = 0$), the condition for the object to move is obtained through substituting (B4) into (B3) [10]

$$\theta_{\text{opb}} > 1. \quad (\text{B6})$$

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Prof. Sheremet was a corecipient of the 2015 Strategic Environmental Research and Development Program Project-of-the-Year Award for Munitions Response, together with Dr. Calantoni.

Appendix B. Journal Article (MDPI-Fluids) for the Coupled Delft3D-UnMUSB Model



Article

Coupled Delft3D-Object Model to Predict Mobility of Munition on Sandy Seafloor

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Abstract: The coupled Delft3D-object model has been developed to predict the mobility and burial of objects on sandy seafloors. The Delft3D model is used to predict seabed environmental factors such as currents, waves (peak wave period, significant wave height, wave direction), water level, sediment transport, and seabed change, which are taken as the forcing term to the object model consisting of three components: (a) physical parameters such as diameter, length, mass, and rolling moment; (b) dynamics of the rolling cylinder around its major axis; (c) an empirical sediment scour model with re-exposure parameterization. The model is compared with the observational data collected from a field experiment from 21 April to 13 May 2013 off the coast of Panama City, Florida. The experimental data contain both object mobility using sector scanning sonars and maintenance divers as well as simultaneous environmental time series data of the boundary layer hydrodynamics and sediment transport conditions. Comparison between modeled and observed data clearly shows the model's capabilities and limitations.



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Keywords: Delft3D; object mobility model; munitions mobility and burial; object mobility parameter for percentage burial; sediment Shields parameter; equilibrium burial percentage; sediment supporting point

1. Introduction

The U.S. Army Corps of Engineers and the Navy have identified more than 400 underwater sites potentially contaminated with munitions [1]. Thus, an efficient model to forecast mobility and burial of munitions on the seabed can improve risk assessment and reduce costs related to management and remediation actions. During the ONR accelerated research initiative (ARI) 2001–2005 “Mine Burial Prediction” [2], a physical model, called IMPACT35, was developed to predict the trajectory of a mine through air, water, and sediment to forecast the amount of burial that occurs upon impact with the seafloor [3–8]. IMPACT35 has six degrees of freedom (DoF). Three degrees of freedom refer to the position of the center of mass of the object, and the other three degrees of freedom represent the orientation of the object (i.e., roll, yaw, and pitch).

Munitions on the seabed are less movable than a sea mine in the water column. Therefore, the existing 6-DoF model (e.g., IMPACT35) for sea mine burial prediction needs modification. Moreover, the object model requires localized environmental parameters such as waves, currents, and sediment transport in order to accurately predict the location, mobility, and burial of underwater munitions. When wind transmits momentum to the water surface, it may form waves that produce near-seabed orbital motion responsible for stirred-up sediment and increased sediment transport. In contrast, wave orbital motion in the company of currents intensifies the bed shear stress and decreases the intensity of the current. Furthermore, the dissipation of wave energy in the surf zone induces currents along and across the shore. All the littoral flows carry a significant quantity of sediments.

Recently, an object model was developed to predict a munition's mobility and burial on sandy seafloor using observational environmental data (currents, waves, sediment, morphology) as the forcing term [9].

An open-source software, Delft3D, has been developed to predict currents, waves, sediment transport, and morphology in estuarine, fluvial, and littoral environments [10–12]. Delft3D output provides the environmental parameters around a munition, which are required by the 6-DoF model for predicting the munition's burial and mobility. Under the sponsorship of SERDP, experimental [13,14] and analytical [15,16] studies focus on the determination of the conditions that determine the onset of a specific and important motion, i.e., the roll of a munition around its main axis, both on a hard surface and on a sand bed in the presence of concurrent scour burial. In this study, a coupled Delft3D-object model has been developed to predict hydrodynamic and morphological processes, as well as munitions' burial and mobility on sandy seafloor. The Delft3D model output was taken as the forcing term for the object model (Figure 1).

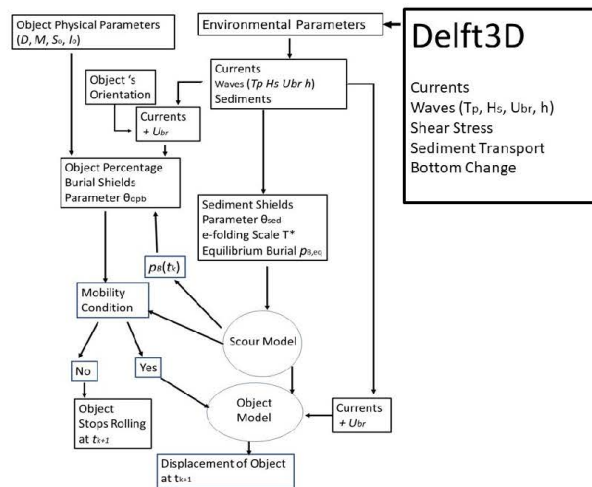


Figure 1. Flow chart of the coupled Delft3D-object model to predict an objects' mobility and burial.

The coupled system consists of two major components: Delft3D and object model. The object model has five parts: (a) a cylindrical object model with the burial percentage Shields parameter (θ_{obj}); (b) a sediment scour model with the sediment Shields parameter (θ_{sed}); (c) an object's physical parameters such as diameter (D), relative density versus water density (S_o), mass (M), and rolling moment about its symmetric axis (I_o); (d) environmental variables such as near seabed ocean currents, bottom wave orbital velocity (U_{br}), water depth (h), wave peak period (T_p), significant wave height (H_s), and sediment characteristics; (e) the model output such as the burial percentage p_B , and the object's displacement.

The Target Reverberation Experiment 2013 (TREX13) in Panama City, Florida from 21 April to 13 May 2013 produced a unique data set containing environmental measurements such as waves, currents, and sediment samples, as well as mobility and burial of munitions [13]. The TREX13 data were used to verify the coupled model. The remainder of the paper is outlined as follows. Section 2 depicts the study area. Section 3 describes the observational data from TREX13. Sections 4–6 present the Delft3D, the object mobility model, and the object scour model. Section 7 presents the prediction of an object's mobility and burial by the coupled Delft3D-object model. Section 8 presents the conclusions. Detailed object modeling information is included in Appendix A, Appendix B, Appendix C, Appendix D.

2. Study Area

The study area is off the coast of Panama City near the San Andrew Bay and is indicated by the region enclosed by the red lines in Figure 2. The tides are diurnal with an amplitude of highest astronomical tide (HAT) of 8.914 m and a maximum tidal range of 0.4 m [17]. The wind has strong seasonal variation: primarily from the north in winter and fall, and mostly from the south in summer and spring. The hurricane season in the Gulf of Mexico is typically from June to November with the peak occurring in August and September. During the off-hurricane season, the surface winds were not strong during 20 April–13 May 2013, with the east-west component (Figure 3a) and the north-south component (Figure 3b) from the NOAA buoy PACF1 (nearest to the study area) [18] and the ERA5 reanalysis data with 0.25° resolution from the European Centre for Medium-Range Weather Forecasts (ECMWF) [19]. It is noted that on 5 May 2013, a cold front was over northern Texas and passed over Panama City between 5–6 May 2013, causing a storm event and stronger waves. However, in general, the study area during the off-hurricane season represented a low-energy regime [13].

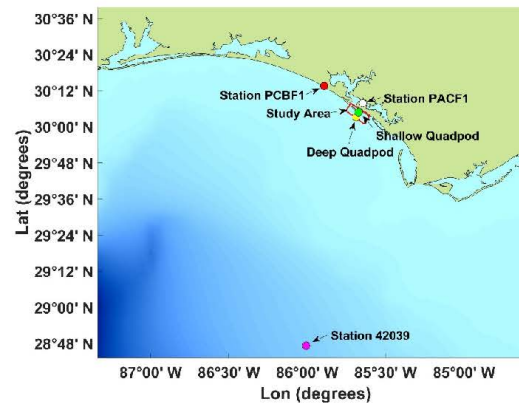


Figure 2. Northern Gulf of Mexico of the coast of Panama City with locations of the shallow quadpod (30°04.81' N, 85°40.41' W), deep quadpod (30°03.02' N, 85°41.34' W), and the NOAA buoy stations PACF1, PCBF1, and 42039. The study area is enclosed by the red lines. The NOAA buoy station PACF1 is nearest to the study area.

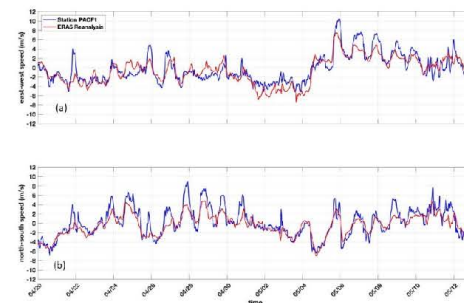


Figure 3. Surface (a) east-west wind component and (b) north-south wind component for the study area from the NOAA PACF1 buoy (blue curves) [18] and the ERA5 reanalysis data with 0.25° resolution from the European Centre for Medium-Range Weather Forecasts (ECMWF) (red curves) [19].

3. TREX13

3.1. Surrogate Munitions

During TREX13, four types of surrogate and replica munitions were used to roughly represent the 155 mm HE M107, 81 mm mortar, 25 mm cartridge, and 20 mm cartridge and were designed and fabricated using crude drawings and specifications provided by existing Army Technical Manuals (e.g., TM 43-0001-27 and TM 43-0001-28) [13] (Figure 4). Table 1 shows the complete list of deployed and recovered munitions along with brief descriptions and their physical properties, such as bulk density and rolling moment, which are closely matched to their real counterparts.



Figure 4. Fabricated surrogates, purchased replicas, and fabricated replicas of (left) 155 mm HE M107, (middle) 81 mm mortars, and (right) 25 mm and 20 mm cartridges (from [13]).

Table 1. List of surrogate and replica munitions used during TREX13. A total of 26 objects were deployed and 18 objects were recovered (from [13]). The surrogate munitions were fabricated to have rolling moments within 10% of the estimated rolling moment of the real counterpart.

Type with Diameter	Labels	Materials Type	Recovered	Rolling Moment (10^{-4} kg m ²)	Volume (10^{-5} m ³)	Mass (kg)	Density (kg m ⁻³)
155 mm, HE, M107	D5, D6	Delrin, 304 Stainless Surrogate	D5, D6	923.59	768.38	34.15	4444
	D3, D4	Aluminum Replica	D3, D4	500.48	768.38	20.91	2721
81 mm mortar	C3, C4	Delrin, 316 Stainless, Aluminum tail fins Surrogate	C3, C4	24.73	120.93	3.76	3109
	C5, C6	304 Stainless, Aluminum tail fins Replica	C5, C6	50.51	120.93	8.70	7194
	C1, C2	Urethane Replica		8.34	120.93	1.45	1199
25 mm cartridge	B5, B6	Delrin, 316 Stainless Surrogate	B5, B6	0.46	16.55	0.39	2356
	B7, B8	304 Stainless Replica	B7, B8	1.98	16.55	1.32	7975
	B3, B4	Aluminum Replica	B3, B4	0.68	16.55	0.43	2598
	B1, B2	Delrin Replica		0.35	16.55	0.23	1390

Table 1. Cont.

Type with Diameter	Labels	Materials Type	Recovered	Rolling Moment (10^{-4} kg m ²)	Volume (10^{-5} m ³)	Mass (kg)	Density (kg m ⁻³)
20 mm cartridge	A5, A6	Delrin, 316 Stainless Surrogate	A6	0.13	7.70	0.20	2597
	A7, A8	304 Stainless Replica	A7	0.53	7.70	0.63	8181
	A3, A4	Aluminum Replica	A3, A4	0.18	7.70	0.19	2468
	A1, A2	Delrin Replica		0.09	7.70	0.11	1429

3.2. Field Experiment

A field experiment was conducted to simultaneously collect both environmental (currents, waves, and sediment samples) data and the locations of surrogate/replica munitions on the seafloor from, 21 April 2013 to 13 May 2013, at two sites [13]. Instruments were mounted on a pair of large, rugged frames (herein referred to as “quadpods”) that were deployed at two different water depths (herein referred to as “deep” and “shallow”). The quadpods were deployed in the northern Gulf of Mexico offshore of Panama City Beach, FL, USA (Figure 5a). The deep quadpod was deployed at 30°03.02330 N, 85°41.33630 W, in about 20 m water depth, while the shallow quadpod was deployed at 30° 04.80994 N, 85° 40.41064 W, in about 7.5 m water depth. A sector scanning sonar was mounted on one of the legs of each of the quadpods, scanning a 110° swath every 12 min.

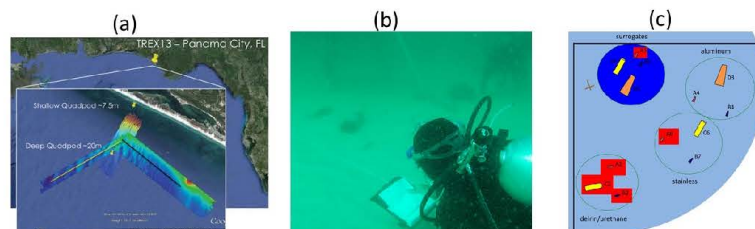


Figure 5. (a) Locations of deep and shallow quadpods; (b) photo of divers laying the object field during the shallow quadpod deployment; (c) layout of objects laid by divers under the shallow quadpod (from [13]).

The data captured by TREX13 were during the time when hurricanes are “least” expected to occur. Therefore, the environmental conditions during hurricanes were ignored. This implies the hypothesis that the munitions’ mobility is not influenced by hurricane-generated waves and currents. This is because in shallow-water depths of 10–20 m, extreme significant wave heights resulting from hurricanes cause large near-bed-orbital velocities, leading to rapid scouring and burial and in turn stopping the munitions’ mobility.

The divers’ performance confirmed this hypothesis. Divers laid four surrogate munitions and nine replica munitions on the seafloor near each of the shallow and deep quadpods, within the view field of the sector scanning sonar, on 21 April 2013 (Figure 5b). The location and orientation of surrogate and replica munitions were detected by the sector scanning sonar and a maintenance diver with a video camera. Only objects laid by divers under the shallow quadpod were photographed (Figure 5c). The field of view of the sector scanning sonar is roughly represented by the light blue. The locations of the surrogates are denoted by a dark blue circle in the upper left. The other replicas were grouped ac-

ording to relative bulk density. In this case, the red boxes denote the objects that were not recovered from the shallow quadpod site. Thus, the initial surrogate munitions' location and orientation provided from TREX13 are only for the shallow quadpod. Immediately after the storm event on 5–6 May 2013, a maintenance dive was performed in the morning of 8 May 2013 and found that the surrogates and replicas may have been buried in place as opposed to being transported away by the waves and currents. Excavating by hand, divers were able to recover a total of eight munitions buried just below the surface, very near to the known initial locations at the shallow quadpod. The observational period for the munitions' location and burial was 21 April–7 May 2013.

3.3. Data

As described in [13], the combined observations of munitions mobility and the driving environmental conditions were observed. Waves and currents were obtained using both acoustic surface tracking (Nortek AWAC) and a pressure time series. Two sediment cores were collected at the shallow quadpod location during deployment (core # D1) and retrieval (core # R1). It was found that both cores contained nearly 100% sand (Table 2). Therefore, the Shields parameter could be used for identifying the mobility of sediments. Grain size distributions were obtained with standard sieve techniques and results for porosity, bulk density, and void ratio were obtained by measuring the weight loss or water weight. The median grain diameter (d_{50}) is around 0.23 mm, and the sediment density (ρ_s) is about $2.69 \times 10^3 \text{ kg m}^{-3}$. These two parameters are most significant to influencing sediment mobility and are needed for the object scour burial model (see Section 6).

Table 2. Sediment properties from diver push cores taken during the deployment (D1) and the retrieval (R1) of the instrumentation at the shallow quadpod location (from [13]).

Depth Range (cm)	% Gravel		% Sand		Mean Phi-Value		Standard Deviation Phi-Value		% Porosity		Bulk Density (g/cc)		Void Ratio (e)	
Core #	D1	R1	D1	R1	D1	R1	D1	R1	D1	R1	D1	R1	D1	R1
0–2	0.00	0.04	100.00	99.96	2.14	2.06	0.39	0.40	38.35	39.55	2.04	2.02	0.62	0.65
2–4	0.00	0.00	100.00	100.00	2.12	2.04	0.40	0.40	39.28	40.14	2.03	2.02	0.65	0.67
4–6	0.00	0.02	100.00	99.98	2.13	2.08	0.42	0.46	39.13	38.96	2.03	2.03	0.64	0.64
6–8	0.02	0.01	99.98	99.99	2.23	2.21	0.43	0.44	38.84	39.46	2.04	2.03	0.63	0.65
8–10	0.13	0.01	99.87	99.99	1.94	2.24	0.62	0.40	37.62	39.26	2.06	2.03	0.60	0.65

4. Delft3D

4.1. Model Description

The open-source Delft3D version 4.04.01 was implemented in the TREX13 area to predict currents and waves. Under the wind and tidal forcing, the flow module [10] predicts the water level, currents, feeds the current data into the wave and morphology modules as input, computes the sediment transport, and updates the bathymetry. The wave module [11] is used to predict the wave generation, propagation, dissipation, and non-linear wave–wave interactions in the nearshore environment with inputs such as water level, bathymetry, wind, and currents from the flow module. The wave module uses Simulating Waves Nearshore (SWAN), which is a third-generation model derived from the Eulerian wave action balance equation [12]. Since we are only interested in the wave parameters such as the peak period (T_p), significant wave height (H_s), and bottom wave orbital velocity (U_{br}), with a temporal resolution of 1 h for the object model, the coupling time between the flow and wave modules was also set to 1 h. The morphology module works in an integrated way with the wave and flow modules in a cycle. This system is a process-based model that considers the impact of waves, currents, and sediment transport on morphological changes. In this study, the sediment module was not activated.

4.2. Model Grids and Time Steps

Two grids with different grid cell sizes were nested (Figure 6) to create a region with finer resolution. These rectangular grids compose the flow domain. The flow outer grid (coarser resolution) is composed of 137×75 grid points with spacing of 50 m in both longshore and cross-shore directions. The flow inner grid (finer resolution) has 20 m resolution and was divided into 139×124 grid points, equally spaced. The sediment transport and morphological evolution were computed only in the flow inner grid. The wave domain (Figure 6) was defined in order to avoid the boundary effect and allow the use of deep quadpod data to set up the wave boundary conditions. The wave grid is composed of 273×111 grid points with 50 m resolution. The bathymetric data (Figure 6) were from the Northern Gulf Coast Digital Elevation Model from the National Oceanic and Atmospheric Administration/National Geophysical Data Center (NOAA/NGDC) [20]. The resolution of this data set varies between 1/3 arc-second and 1 arc-second (around 10 and 30 m). The time step is set as 0.2 s for the coarse domain (grid size 50 m) and 0.1 s for the finer domain (grid size 20 m). The small time-steps (0.2 s, 0.1 s) are needed in order to satisfy the Courant–Friedrichs–Lewy (CFL) condition of computational stability for the Delft3D flow module.

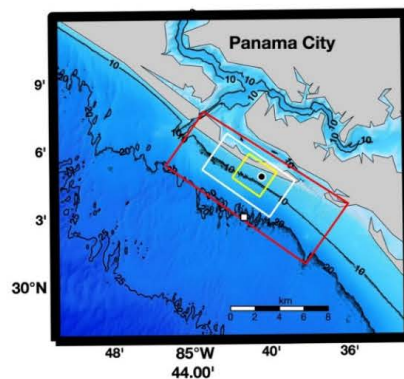


Figure 6. Study area with bathymetry, depth contours (10 m, 20 m, and 25 m), and computational grids for wave module (red), flow module with coarse resolution (white), and flow module with fine resolution (yellow). The black dot represents the shallow quadpod location, and the white square denotes the deep quadpod location (from [17]).

4.3. Wind and Tidal Forcing

The wind input files were set up using ERA5 reanalysis data from the European Centre for Medium-Range Weather Forecasts (ECMWF), with 0.25° resolution [19] for the flow and wave modules. The Global Inverse Tide Model TPXO 8.0 with 1/45-degree resolution was used to create the boundary conditions for the flow module. For the alongshore boundary, the water level with astronomic forcing was imposed. The water level gradient (a so-called Neumann boundary condition) was chosen with a constant zero water level slope in the longshore direction for both across-shore open boundaries. It allows for flow to leave and enter the lateral boundaries with no spurious circulation.

4.4. Initial and Boundary Conditions

As an initial condition, the water level and current velocity were set to zero. Additionally, the sediment transport boundary conditions were set by specifying the inflow concentration as zero kg/m^3 . The initial condition for the sand sediment was set as a uniform zero concentration, and the initial bed of sediment was set to 5 m. Wave boundary conditions were set based on the measurements from the deep quadpod location using the

significant wave height, wave period, wave directions, and directional spreading. These parameters were applied uniformly on the three open boundaries. A spin-up interval of 720 min was established to prevent any influence from a possible initial hydrodynamic instability on the bottom change calculation, which starts only after the spin-up interval. The sediment type was set as sand with a sediment-specific density of 2650 kg/m^3 .

The calibration was conducted to adjust the parameters to the best agreement between the modeled and observed water level, waves, and currents. For water level, calibration was done through minimizing the difference in amplitude and phase between predicted and measured tides. For waves, the calibration was to determine the optimal JONSWAP bottom friction coefficient and wave height to water depth ratio. For currents, the calibration was to identify the best Chézy bottom roughness as well as horizontal eddy viscosity and diffusivity. The calibration period was set up as 21–27 April 2013, which corresponds to 27% of the entire period of observations (21 April–13 May). During this process, parameters were adjusted separately. While one was fine-tuned, the others remained constant. The calibrated JONSWAP bottom friction coefficient was $0.067 \text{ m}^2/\text{s}^3$ and the wave height to water depth ratio was 0.7, for the wave module. The Chézy bottom roughness was $65 \text{ m}^{1/2}/\text{s}$, horizontal eddy viscosity was $0.5 \text{ m}^2/\text{s}$, and horizontal diffusivity was $10 \text{ m}^2/\text{s}$, for the flow module.

4.5. Model Output

The Delft3D output data with 1-h resolution are used as input for the object model. The output from the flow module includes the water depth (h) and the current velocity, $\mathbf{U}_c = i v_e + j v_n$, with (i, j) the unit vectors in longitudinal and latitudinal directions, and $U_c = (v_e^2 + v_n^2)^{1/2}$ as the current speed. The output from the wave module includes the wave peak-period (T_p), significant wave height (H_s), wave direction, and bottom wave orbital velocity (U_{br}). The bottom water velocity vector of combined current and waves is represented by $\mathbf{V}_w$ with $|\mathbf{V}_w| = U_c + U_{br}$ and the orientation $\psi = \tan^{-1}(v_n/v_e)$. Figure 7 shows the time series of the environmental parameters [v_e , v_n , h , T_p , H_s , U_{br}] predicted by the Delft3D (red curve) and observed by the AWAC (black curve). The AWAC only provides the observed data for [v_e , v_n , h , T_p , H_s], but not the bottom orbital velocity U_{br} , which was calculated using a well-established linear wave model with a Matlab function [21] using the observed water depth (h), significant wave height (H_s), and peak period (T_p) (see Appendix D in [21]).

Since the munitions were found totally buried, without mobility, on the morning of 8 May 2013 by the divers in TREX13, and TREX13 provides the munitions' mobility information from 21 April to 7 May 2013, the integration period for the coupled Delft3D-object model was set as 21 April–7 May 2013. The root-mean-square error (RMSE) between the Delft3D output and the TREX13 observations is 0.105 m for the water level, 0.111 m/s for the east-west current speed, 0.0641 m/s for the north-south current speed, 0.0946 m for the significant wave height, and 0.0928 m/s for the bottom wave orbital velocity. The bias between the Delft3D output and the TREX13 observations is -0.0244 m for the water level, -0.0367 m/s for the east-west current speed, 0.0055 m/s for the north-south current speed, 0.0429 m for the significant wave height, and -0.0786 m/s for the bottom wave orbital velocity. The correlation coefficient between the Delft3D output and the TREX13 observations is as high as 0.966 for the significant wave height, 0.941 for the bottom wave orbital velocity, reasonably high as 0.796 for the water depth, 0.571 for the north-south current speed, 0.551 for the east-west current speed, and the lowest at 0.373 for the peak wave period. The Delft3D modeling performed reasonably well according to the criteria presented in [22].

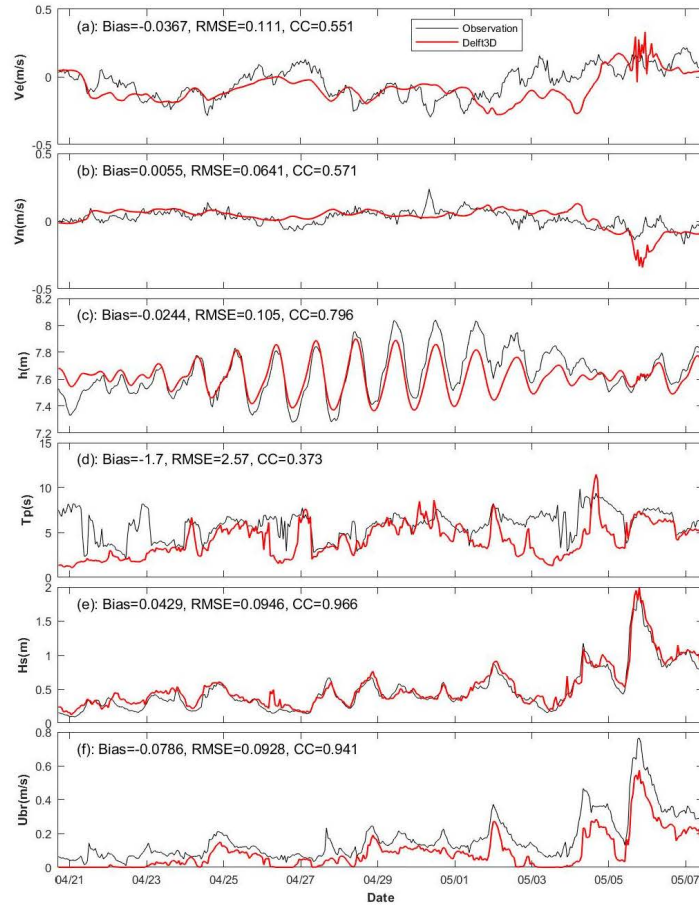


Figure 7. Comparison of Delft3D predictions (red) and observations during TREX13 (black) at the shallow quadpod from 21 April to 7 May 2013: (a) near bed (~ 0.15 m) longitudinal current v_e (m/s); (b) near bed (~ 0.15 m) latitudinal current v_n (m/s); (c) water depth h (m); (d) peak period T_p (s); (e) significant wave height H_s (m); (f) computed bottom wave orbital velocity U_{br} (m/s).

5. Object Mobility Model

Consider a cylindrical object with length L and diameter D buried in the seabed with the burial depth B ($B < D/2$). Let the water velocity (consisting of current and waves) near the seabed ($\mathbf{V}_w$) be in the direction towards the cylinder with an angle, ϕ , perpendicular to the main axis of the cylinder and be decomposed into $\mathbf{V}_w = (U, V)$, with U being the perpendicular component and V the parallel component (Figure 8) to the main axis of the cylinder. As the object rolls with angular velocity ω on the seabed with the object's burial, let the axis of rotation inside the sediment be at depth b ($b < B$) (see Appendix A).

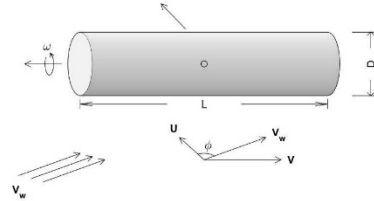


Figure 8. Roll of a cylindrical object on the seafloor with large aspect ratio forced by the combination of ocean currents and bottom wave orbital velocity. Here, $(\pi/2 - \phi)$ is the angle between $\mathbf{V}_w$ and the main axis of the cylinder.

As an object rolls around point b (see Figure A1 in Appendix A) with an angular velocity ω , the translation velocity of the object is given by:

$$u_o = \omega \left(\frac{D}{2} - b \right) \quad (1)$$

The corresponding moment of momentum equation of the rolling object is given by [see Equation (A17) in Appendix B]:

$$\begin{cases} I_A^* \frac{d\omega}{dt} = T_F - \Pi(\rho_o - \rho_w) \sqrt{b(D-b)} & \text{if } p_B < 0.5, \theta_{opb} > 1 \\ \omega = 0, & \text{otherwise} \end{cases} \quad (2)$$

where $I_A^* = I_o + \rho_o \Pi D^2/4 + (D/2 - b)\rho_w \Pi(D/2 + B/2 - b)$; I_o is the rolling moment of the munition about the symmetric axis of the munition (see Figure A2); T_F is the forward torque caused by the drag force (F_d) and lift force (F_l) (see Appendix C); $p_B = B/D$, is the percentage burial, and θ_{opb} is the object mobility parameter for percentage burial (see Appendix B); (ρ_o, ρ_w) are the densities of the object and water; Π is the volume of the munition.

Let the relative horizontal velocity of the rolling object be defined by:

$$\hat{u}_o = \frac{u_o}{U} \quad (3)$$

Substitution of (1) into (2) and use of (3) leads to a special Riccati equation,

$$\begin{cases} \frac{d(1 - \hat{u}_o)}{dt} + \alpha(1 - \hat{u}_o)^2 = \beta, & \text{if } p_B < 0.5, \theta_{opb}(t) > 1 \\ \hat{u}_o = 0, & \text{otherwise} \end{cases} \quad (4)$$

where

$$\begin{aligned} \alpha &= \frac{(1-2p_B)}{8I_A^*} \rho_w C_d |U| D^3 L \left[(1-p_B)(1+p_B-2p_B) + 2\gamma \sqrt{p_B(1-p_B)} \right] > 0 \\ \beta &= \frac{(1-2p_B)}{2I_A^* |U|} g \rho_w \Pi (S_o - 1) D^2 \sqrt{p_B(1-p_B)} > 0 \end{aligned} \quad (5)$$

The special Riccati Equation (4) has an analytical solution from integration from t_k to $t_k + 1$ ($k = 0, 1, 2, \dots, K-1$) [23]

$$\hat{u}_o(t) = \begin{cases} 1 - \frac{[1 - \hat{u}_o(t_k)] \sqrt{\alpha_k \beta_k} + \beta_k \tanh\left[\left(\frac{\sqrt{\alpha_k \beta_k}}{2}\right)(t - t_k)\right]}{\sqrt{\alpha_k \beta_k} + \alpha_k [1 - \hat{u}_o(t_k)] \tanh\left[\left(\frac{\sqrt{\alpha_k \beta_k}}{2}\right)(t - t_k)\right]}, & \text{for } p_B(t_k) < 0.5, \theta_{opb}(t_k) > 1 \\ 0, & \text{for } p_B(t_k) \geq 0.5, \theta_{opb}(t_k) \leq 1, \quad t_k < t \leq t_{k+1}, \quad \alpha_k = \alpha(t_k), \quad \beta_k = \beta(t_k) \end{cases} \quad (6)$$

with α_k and β_k as known constants during the integration. Substitution of (6) into (3) leads to the dimensional horizontal velocity of the rolling object, $u_o(t) = U \hat{u}_o(t)$, which should be used for each time interval Δt . The solution (6) depends on (α_k, β_k) which involve three types of parameters: (a) time-independent physical parameters of the object for S_o, Π, L , and D ; (b) time-dependent water velocity, $U(t_k)$, from the Delft3D model output;

(c) time-dependent relative depth of sediment rolling axis $[p_b(t_k)]$ and burial percentage $[p_B(t_k)]$ determined using a scour burial model. Let l be the displacement of the object,

$$dl/dt = u_o \quad (7)$$

Integration of l with respect to time t leads to the munition's displacement.

6. Object Scour Model

Existing studies on scour burial were all concentrated on motionless objects. The ratio between the fluid force (bottom shear stress) and the weight of the sediment particles, i.e., the sediment Shields parameter (θ_{sed}),

$$\theta_{sed} = \frac{fU_{br}^2}{g(S_{sed} - 1)d_{50}}, \quad S_{sed} = \frac{\rho_s}{\rho_w}, \quad f = \exp \left[5.5 \left(\frac{6U_{br}T_p}{\pi d_{50}} \right)^{-0.2} - 6.3 \right] \quad (8)$$

is crucial for scour burial of motionless objects and in turn for prediction of the percentage burial parameter $p_B(t) = B/D$ [15,16]. Here, f is the wave friction factor [24], ρ_s the sediment grain density, and d_{50} the medium sand grain size. Using the wave data (T_p , U_{br}) from Figure 6 e.g., and sediment parameters ($\rho_s = 2.69 \times 10^3 \text{ kg/m}^3$, $d_{50} = 0.23 \times 10^{-3} \text{ m}$) from TREX13 [9,13], the sediment Shields parameters (θ_{sed}) were calculated from 21 April to 7 May 2013. The results were less than 0.1 every time, except when atmospheric cold fronts passed by on 5–6 May 2013. The maximum value of θ_{sed} reached 0.33 (Figure 9).

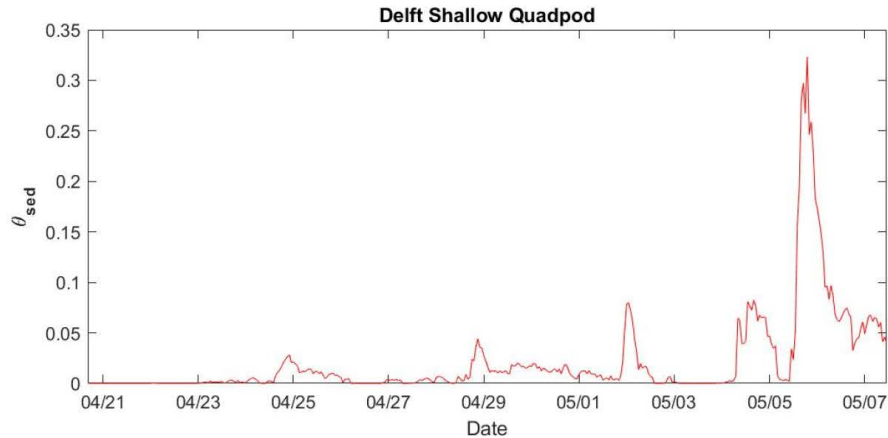


Figure 9. Time series of sediment Shields parameter (θ_{sed}) at the shallow quadpod computed from the Delft3D model output.

As pointed out in [24], the equilibrium percentage burial $p_{B,eq}$ for motionless cylinders induced by scouring tends to increase as θ_{sed} increases. An empirical formula has been established,

$$p_{B,eq} = a_1 \theta_{sed}^{a_2} - a_3 \quad (9)$$

with different choices of the coefficients (a_1 , a_2 , a_3) determined experimentally for cylinders subject to steady currents: $a_1 = 11$, $a_2 = 0.5$, $a_3 = 1.73$ [25], $a_1 = 0.7$, $a_2 = a_3 = 0$ [26], $a_1 = 2$, $a_2 = 0.8$, and $a_3 = 0$ [27]. For cylinders under waves (depending on wave period): $a_1 = 1.6$, $a_2 = 0.85$, and $a_3 = 0$ for T_p longer than 4 s [28]. For motionless cylinders, before scour burial reaches equilibrium, the percentage burial follows an exponential relationship [25],

$$p_B(t_k) = p_{B,eq} \left[1 - \exp \left(-\frac{t_k}{T^*} \right) \right] \quad (10)$$

where the e-folding time scale T^* is given by

$$T^* = \frac{c_1 \theta_{sed}^{c_2} D^2}{[g(S_{sed} - 1)d_{50}^3]^{0.5}}, \quad c_1 = 1.5, \quad c_2 = 0.11 \quad (11)$$

With the bottom wave orbital velocity (U_{br}), sediment density (ρ_s), medium grain size (d_{50}), and in turn the sediment Shields parameter (θ_{sed}), the equilibrium object percentage burial ($p_{B,eq}$) is calculated using (9) with coefficients $a_1 = 1.6$, $a_2 = 0.85$, and $a_3 = 0$. The sediment supporting depth b (or p_b) is calculated from burial depth B (or p_B) using (10), i.e.,

$$b(t_k) = \lambda B(t_k), \quad p_b(t_k) = b(t_k)/D = \lambda p_B(t_k), \quad \lambda = 0.453 \quad (12)$$

It is noted that the predicted burial percentage (p_B) computed from (10) represents the depth that an object on the surface would bury to at that moment, but an object deployed at the beginning of the time sequence would always remain buried at the deepest burial it has reached so far. The burial depth of the base of the object below the ambient seabed is equivalent to the greatest depth that the scour pit has reached up to that point in time [29]. In other words, scouring only acts to bury an object deeper. It can never be unburied (re-exposure), as the time series is suggested by (10). Similar to [29], a simple parameterization was proposed [9] to represent the re-exposure process starting from $k (= 1, 2, \dots)$: (a) doing nothing if $p_B(t_{k+1}) \geq p_B(t_k)$, (b) computing the weighted average

$$\tilde{p}_B(t_{k+1}) = wp_B(t_k) + (1-w)p_B(t_{k+1}) \text{ if } p_B(t_{k+1}) < p_B(t_k) \quad (13)$$

with w being the weight coefficient. In this study, we take $w = 0.80$. The object mobility and burial model consists of Equations (6)–(13).

7. Prediction of Object's Mobility and Burial

The munitions' mobility and burial were predicted using object mobility and burial models, with the environmental variables predicted by the Delft3D (Figure 1) as the forcing term. The model was integrated for each surrogate (or replica) munition deployed in the shallow quadpod with its initial location and orientation (Figure 5c) at 12:40 (14:00) local time on 20 April 2013. The angle between $\mathbf{V}_w$ (data represented by the red curves in panels a, b in Figure 7) and the direction perpendicular to cylinder's main axis is determined. The velocity vector of combined current and waves ($\mathbf{V}_w$) is then transformed into $\mathbf{V}_w = (U, V)$ with U being the perpendicular component, and V the paralleling component. The component U is used in the model. The object's physical parameters, such as the diameter (D), volume, mass (M), and density (ρ_o), are obtained from Table 1.

The environmental data, such as the water depth (h), wave peak period (T_p), significant wave height (H_s), bottom wave orbital velocity (U_{br}) (represented by the red curves in Figure 7), and sediment data (100% sand, $d_{50} = 0.23$ mm, $\rho_s = 2.69 \times 10^3$ kg m⁻³) are used by the sediment scour model (i.e., Equations (8)–(13)) to get the burial percentage $p_B(t_k)$, and in turn the relative rolling center depth $p_b(t_k)$. With the object's physical parameters (D , ρ_o , M), the calculated $p_b(t_k)$, model-predicted bottom current velocity component perpendicular to the object's main axis $U(t_k)$, and the coefficients [$\alpha(t_k)$, $\beta(t_k)$] for the object mobility model (i.e., Equation (5)), the object's displacement at the next time step $l(t_{k+1})$ can be predicted using (7).

Based on the known initial locations of the objects at the shallow quadpod (Figure 5c), the model predicts the objects' burial percentage ($p_B(t_k)$) shown in Figure 10, the objects' mobility parameter for percentage burial ($\theta_{opb}(t_k)$) shown in Figure 11, and the objects' displacement ($l(t_k)$) shown in Figure 12. The burial percentages p_B for all the objects were less than 0.5, except during the storm event at 12:00 on 5 May to 00:00 on 6 May 2013 local time (Figure 10). The red color in Figure 11 shows that the objects' rolling condition [$\theta_{opb} > 1$] is satisfied.

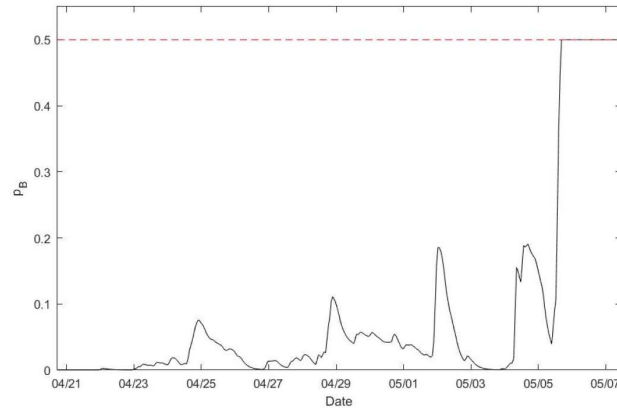


Figure 10. Model predicted burial percentage $p_B(t)$ with re-exposure parameterization (13) for each object at the shallow quadpod from 20 April to 7 May 2013. The predicted burial percentage $p_B(t)$ is less than 0.5 for all the munitions during the whole time period, except during the storm event from 12:00 5 May to 00:00 6 May 2013. The burial percentage $p_B(t)$ is the same for each object, since it only depends on the sediment characteristics [see Equations (9)–(11)].

The surrogate and replica munitions' mobility and burial were observed by divers and sector scanning sonar images during the field experiment depicted in Section 2 and in [13]. A total of eight munitions at the shallow quadpod location were recovered by divers during the maintenance dive performed on 8 May 2013 (Figure 5b). Note that the munitions excavated by the divers at the shallow quadpod location on 8 May 2013 were immediately redeployed for the duration of the experiment. An overview of the observed munitions' mobility throughout the whole TREX13 experiment (20 April to 7 May 2013) is shown in Figure 13a and during the storm event from 13:00–20:00 on 5 May 2013 in Figure 13b. The objects A2 and C2 were immediately mobile and transported out of the field of view; they were last seen on 23 April 2013 (very crude observational information). However, the other objects were nearly immobile (Figure 13a). The predicted large displacements of 20.7 m for A2 from 12:00 21 April to 12:00 24 April 2013 and 6.52 m for C2 from 12:00 21 April to 00:00 23 April 2013 (Figure 12) qualitatively agree with the crude observational information about A2 and C2.

Furthermore, overview of the modeled objects' mobility throughout the whole TREX13 experiment (20 April to 7 May 2013) is shown in Figure 13c and during the storm event from 13:00–20:00 on 5 May 2013 in Figure 13d. Similarity between observation (Figure 13a,b) and the model prediction (Figure 13c,d) shows model capability. However, model–data discrepancy exists. For example, the yaw of munitions D3 and D6 was observed (Figure 13a,b), but not predicted (Figure 13c,d). The munition D3 (rightmost triangles in Figure 13a,b) moved, but the model predicted a value of $l = 0$ m by predicting that D3 would never move (Figures 10 and 13c,d). The model limitation is due to these four assumptions: (a) cylinder with a large aspect ratio ($L \gg D$); (b) no yaw and pitch; (c) a percentage burial depth of less than 0.5; (d) a flat seabed. Even if the bottom profile is flat when the object is deployed, the sand tends to accumulate in front of the object and erodes on the opposite side, thus creating a wavy bed that affects the dynamics of the object. The model will lose capability in the real world if the shape of munitions is evidently different from the cylinder with $L \gg D$ and the effect of a wavy seabed is large on the dynamics of the object.

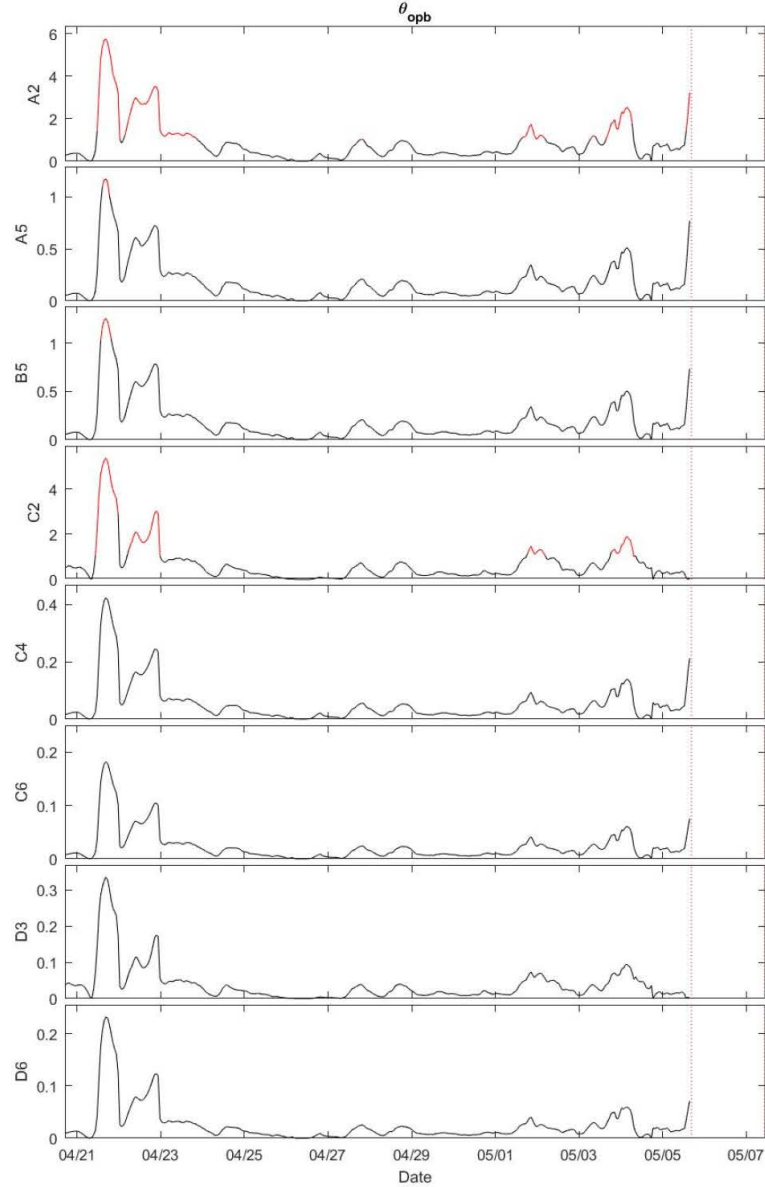


Figure 11. Model predicted objects' mobility parameters for percentage burial (θ_{opb}) at the shallow quadpod from 20 April to 7 May 2013. The red color shows that the condition for rolling an object [$\theta_{opb} > 1$] is satisfied. The parameters θ_{opb} is not computed between 12:00 5 May to 00:00 6 May 2013, since the predicted burial percentage $p_B(t)$ is larger than 0.5. Among the eight objects, only A2 and C2 have evident time periods when the condition for rolling an object [$\theta_{opb} > 1$] is satisfied.

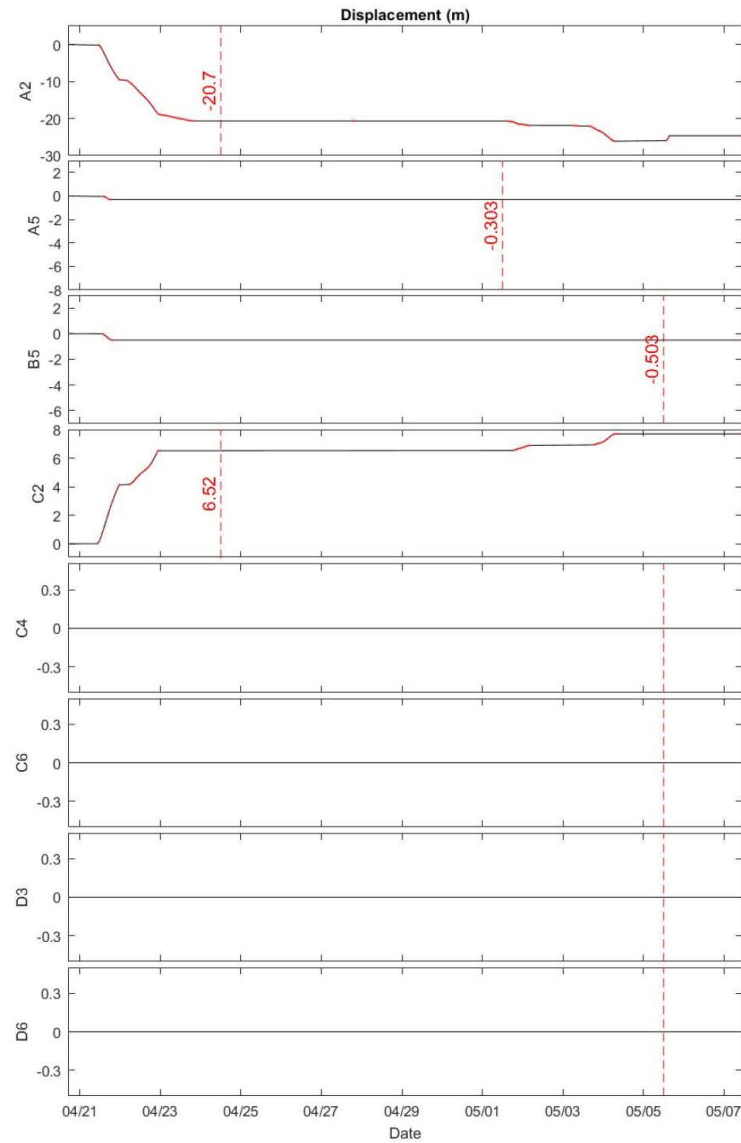


Figure 12. Model predicted displacement $l(t)$ for each object at the shallow quadpod from 20 April to 7 May 2013. Among the eight objects, only A2 and C2 were immediately mobile and displaced 20.7 m (A2) and 6.52 m (C2) on 12:00 24 April 2013 (dashed line). Other munitions A5, B5, C4, C6, D3, D6 were completely motionless. According to the TREX13 report [13], the objects A2 and C2 were immediately mobile and transported out of the field of view because they were last seen on 23 April 2013. After 23 April 2013, their locations have never been observed.

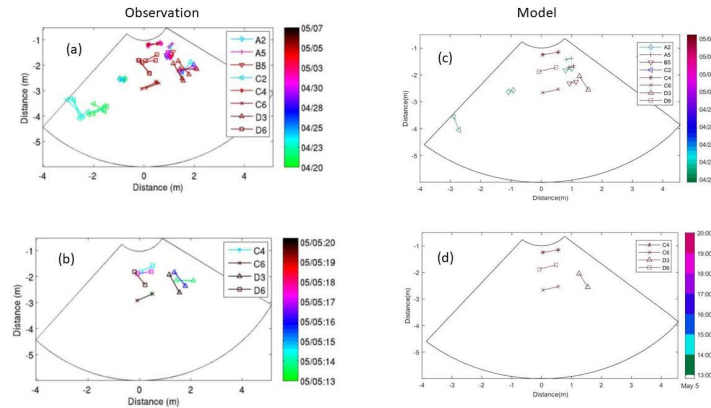


Figure 13. Positions for all visible objects at the shallow quadpod location up to the maintenance dive performed on 8 May: (a) observation for 20 April–7 May 2013; (b) observation for 13:00–20:00 on 5 May 2013; (c) model prediction for 20 April–7 May 2013; (d) model prediction for 13:00–20:00 on 5 May 2013. Note that Figure 13a,b were copied from [13]. The color bars denote the last time when each object was visible with dates for (a,c) and hour on 5 May for (b,d).

8. Conclusions

- (1) A coupled Delft3D-object model was recently developed to predict underwater cylindrical objects' mobility and burial in a sandy bed. The roll of the object is the major dynamic of this model, with a new concept of its rolling center in the sediment. The object's displacement caused by rolling satisfies the Riccati equation with an analytical solution. Along with the dynamical model, the empirical scour model with re-exposure parameterization is used as part of the prediction system.
- (2) Data collected at the shallow quadpod during TREX13 (21 April to 23 May 2013) off the coast of Panama City, Florida were used as model verification. The environmental data, such as bottom currents, water depth (h), peak period (T_p), and significant wave height (H_s), are used to verify the Delft3D model. The objects' positions tracked by sector scanning sonar images and maintenance divers are used to verify the object's mobility and burial model.
- (3) The model predicted object positions agree qualitatively well with the observed surrogates (or replicas) data. Observation shows that objects A2 and C2 were immediately mobile and transported out of the field of view, because they were last seen on 23 April 2013. The other objects were nearly immobile. The objects with large mobility are A2 (displaced 20.7 m from 12:00 21 April to 12:00 24 April 2013) and C2 (displaced 6.52 m for C2 from 12:00 21 April to 00:00 23 April 2013). A2 is a 20 mm cartridge with a mass of 0.11 kg and a density of 1429 kg m^{-3} . C2 is an 81 mm mortar with a mass of 1.45 kg and a density of 1.199 kg m^{-3} (see Table 1). The other objects, with almost no mobility, are A5 (density of 2597 kg m^{-3}), B5 (density of 2356 kg m^{-3}), C4 (density of 3109 kg m^{-3}), C6 (density of 7194 kg m^{-3}), D3 (density of 2721 kg m^{-3}), and D6 (density of 4444 kg m^{-3}). The larger the object's density, the smaller the object's mobility parameter for percentage burial (see Equation (A14)). However, it is noted that the observational object data are quite crude and not sufficient to accurately verify the model prediction on an object's mobility and burial.
- (4) Although the coupled Delft3D-object model has the capability to predict an object's mobility, the model has its own weakness specifically regarding cylindrical objects. It only considers the roll of the cylinder around its major axis. The object model ignores pitch and yaw. Besides, the seabed is assumed to be flat. It is necessary to extend

the object modeling to more realistic seabed environments, object shapes, and more complete motions for operational use.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Location of Object's Rotation Axis in Sediment.

Roll of an object on the sandy floor needs a supporting point in sediment. The compressive normal stress of sediment on the object is represented by

$$F_S = -n\kappa u_o \sin \psi \quad (A1)$$

where $\mathbf{n}$ is the unit vector normal to the cylinder surface, and κ is the compressive coefficient. Let $\mathbf{n}$ be decomposed into

$$\mathbf{n} = -\mathbf{e}_h \sin \psi - \mathbf{e}_v \cos \psi \quad (A2)$$

where $(\mathbf{e}_h, \mathbf{e}_v)$ are horizontal and vertical unit vectors (see Figure A1). The sediment compressive normal stress F_S is decomposed as,

$$F_S = \mathbf{e}_h \kappa u_o \sin^2 \psi + \mathbf{e}_v \kappa u_o \sin \psi \cos \psi \quad (A3)$$

With b as the axis of rotation, the sediment above (below) the depth b generates torque to resist (enhance) the rolling with the total torque from the sediment,

$$T_S = \int_0^{\psi_B} [(\mathbf{r} - \mathbf{r}_b) \times F_S] d\psi = \int_0^{\psi_B} [\mathbf{r} \times F_S] d\psi - \mathbf{r}_b \times \int_0^{\psi_B} F_S d\psi \quad (A4)$$

where $\mathbf{r}$ is the position vector at any point on the circle and $\mathbf{r}_b$ is the position vector at point b with point E as the origin,

$$\mathbf{r} = \frac{D}{2} (-\mathbf{e}_h \sin \psi + \mathbf{e}_v (1 - \cos \psi)), \quad \mathbf{r}_b = \frac{D}{2} [-\mathbf{e}_h \sin \psi_b + \mathbf{e}_v (1 - \cos \psi_b)] \quad (A5)$$

The depths B and b are represented by

$$B = \frac{D}{2} (1 - \cos \psi_B), \quad b = \frac{D}{2} (1 - \cos \psi_b) \quad (A6)$$

If we assume that at the depth b the total torque from the sediment is zero (i.e., zero-sum sediment torque for rolling),

$$T_S = 0 \quad (A7)$$

Substitution of (A3)–(A5) into (A7) gives

$$\psi_b = \tan^{-1} \left(\frac{\psi_B - \sin \psi_B \cos \psi_B}{\sin^2 \psi_B} \right) \quad (\text{A8})$$

The ratio $\lambda = b/B$ can be obtained from (A6) and (A8),

$$\lambda \equiv b/B = \frac{1 - \cos \left[\tan^{-1} \left(\frac{\psi_B - \sin \psi_B \cos \psi_B}{\sin^2 \psi_B} \right) \right]}{1 - \cos \psi_B} \quad (\text{A9})$$

The ratio, λ , varies with the burial percentage $p_B = B/D$ mildly from near 0.4445 for $p_B = 0$ and 0.4630. Here, we take $\lambda = 0.453$ in this study.

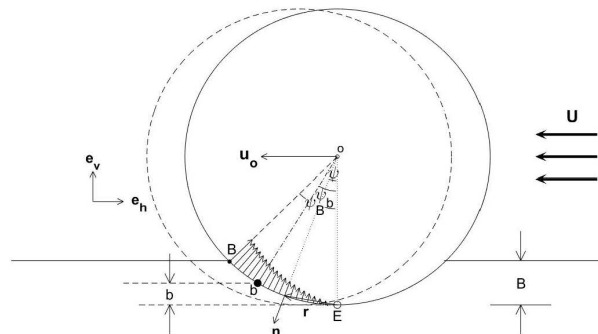


Figure A1. The location of the axis of rotation of the cylinder in the sediment, b , is determined by the assumption of zero-sum torque to the roll.

Appendix B

Dynamics of Rolling Object.

The drag force (F_d) and lift force (F_l) (see Appendix C) roll the object forward with the torque T_F (Figure A2),

$$\begin{aligned} T_F &= F_d(D/2 + B/2 - b) + F_l \sqrt{b(D-b)} \\ &= \frac{1}{2} C_d \rho_w U^2 L \left[(D-B)(D/2 + B/2 - b) + \frac{C_l}{C_d} D \sqrt{b(D-b)} \right] \left(1 - \frac{u_o}{U} \right)^2 \end{aligned} \quad (\text{A10})$$

The buoyancy force and added mass roll the object backward with the torque, $T_B = T_w + T_a$,

$$\begin{aligned} T_B &= F_w \sqrt{b(D-b)} + F_a(D/2 + B/2 - b) \\ &= \frac{\pi}{4} g L D^2 (\rho_o - \rho_w) \sqrt{b(D-b)} + \frac{du_o}{dt} \rho_w \Pi (D/2 + B/2 - b) \end{aligned} \quad (\text{A11})$$

When $T_F > T_B$ the object accelerates if it is in motion or starts to move if it is at rest ($u_o = 0$, $du_o/dt = 0$). When $T_F < T_B$ the object decelerates if it is in motion or keeps motionless if it is at rest. When $T_F = T_B$ the object keeps velocity constant if it is in motion or keeps motionless if it is at rest. Thus, the threshold for the munition's mobility becomes

$$T_F > T_B \quad (\text{A12})$$

The acceleration-deceleration ratio is defined by

$$\frac{T_F}{T_B} = \theta_{opb} \left(1 - \frac{u_o}{U} \right)^2 \quad (\text{A13})$$

4.2. Model Grids and Time Steps

Two grids with different grid cell sizes were nested (Figure 6) to create a region with finer resolution. These rectangular grids compose the flow domain. The flow outer grid (coarser resolution) is composed of 137×75 grid points with spacing of 50 m in both longshore and cross-shore directions. The flow inner grid (finer resolution) has 20 m resolution and was divided into 139×124 grid points, equally spaced. The sediment transport and morphological evolution were computed only in the flow inner grid. The wave domain (Figure 6) was defined in order to avoid the boundary effect and allow the use of deep quadpod data to set up the wave boundary conditions. The wave grid is composed of 273×111 grid points with 50 m resolution. The bathymetric data (Figure 6) were from the Northern Gulf Coast Digital Elevation Model from the National Oceanic and Atmospheric Administration/National Geophysical Data Center (NOAA/NGDC) [20]. The resolution of this data set varies between 1/3 arc-second and 1 arc-second (around 10 and 30 m). The time step is set as 0.2 s for the coarse domain (grid size 50 m) and 0.1 s for the finer domain (grid size 20 m). The small time-steps (0.2 s, 0.1 s) are needed in order to satisfy the Courant–Friedrichs–Lewy (CFL) condition of computational stability for the Delft3D flow module.

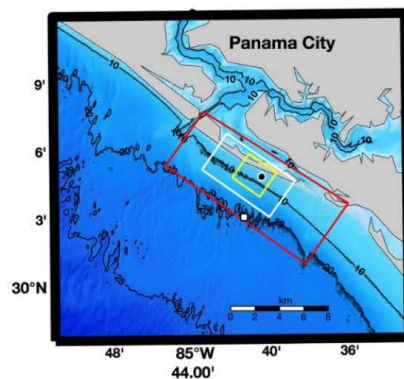


Figure 6. Study area with bathymetry, depth contours (10 m, 20 m, and 25 m), and computational grids for wave module (red), flow module with coarse resolution (white), and flow module with fine resolution (yellow). The black dot represents the shallow quadpod location, and the white square denotes the deep quadpod location (from [17]).

4.3. Wind and Tidal Forcing

The wind input files were set up using ERA5 reanalysis data from the European Centre for Medium-Range Weather Forecasts (ECMWF), with 0.25° resolution [19] for the flow and wave modules. The Global Inverse Tide Model TPXO 8.0 with 1/45-degree resolution was used to create the boundary conditions for the flow module. For the alongshore boundary, the water level with astronomic forcing was imposed. The water level gradient (a so-called Neumann boundary condition) was chosen with a constant zero water level slope in the longshore direction for both across-shore open boundaries. It allows for flow to leave and enter the lateral boundaries with no spurious circulation.

4.4. Initial and Boundary Conditions

As an initial condition, the water level and current velocity were set to zero. Additionally, the sediment transport boundary conditions were set by specifying the inflow concentration as zero kg/m^3 . The initial condition for the sand sediment was set as a uniform zero concentration, and the initial bed of sediment was set to 5 m. Wave boundary conditions were set based on the measurements from the deep quadpod location using the

where

$$\begin{aligned}\theta_{opb} &= \frac{\theta_0}{\pi} \left[\frac{1-p_b}{\sqrt{p_b(1-p_b)}} (1+p_b-2p_b) + 2\gamma \right], \\ \theta_0 &\equiv \left[\frac{C_d U^2}{g D (S_o - 1)} \right], \quad p_b = \frac{b}{D}, \quad S_o = \frac{\rho_o}{\rho_w} > 1\end{aligned}\quad (\text{A14})$$

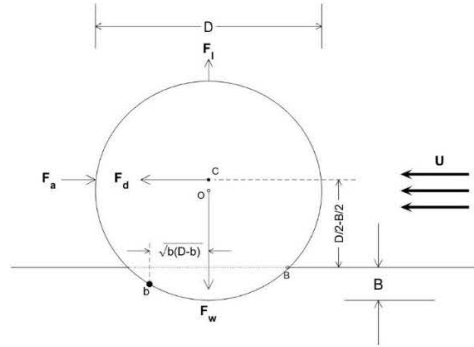


Figure A2. Forces and torques due to drag, lift, buoyancy, and added mass on a partially buried cylinder by combination of ocean currents and bottom wave orbital velocity (U) perpendicular to the major axis of the cylinder.

Here, θ_{opb} is the object's mobility parameter for percentage burial p_b [14] and S_o is the relative density of the object. For motionless munition ($u_o = 0$), the condition for the object to move is obtained through substituting (A13) into (A12)

$$\theta_{opb} > 1 \quad (\text{A15})$$

The corresponding moment of momentum equation of the rolling object is given by

$$\begin{cases} I_A \frac{d\omega}{dt} = T_F - T_B & \text{if } p_b < 0.5, \theta_{opb} > 1 \\ \omega = 0, & \text{otherwise} \end{cases} \quad (\text{A16})$$

Substitution of (T_F , T_B) in (A10) (A11) into (16) leads to

$$\begin{cases} I_A^* \frac{d\omega}{dt} = T_F - \Pi(\rho_o - \rho_w) \sqrt{b(D-b)} & \text{if } p_b < 0.5, \theta_{opb} > 1 \\ \omega = 0, & \text{otherwise} \\ I_A^* = I_A + \left(\frac{D}{2} - b \right) \rho_w \Pi (D/2 + B/2 - b) \end{cases} \quad (\text{A17})$$

with

$$I_A = I_o + \rho_o \Pi D^2 / 4, \quad (\text{A18})$$

where I_o is the rolling moment about the symmetric axis of the munition; I_A is the rolling moment of munition about the point b (see Figure A2) using the parallel axis theorem; Π is the volume of the munition.

Appendix C

Drag, Lift, Buoyancy Forces, and Added Mass.

The drag force (F_d), lift force (F_l), buoyancy force (F_w), and added mass (F_a) exerted on the object for rolling by the perpendicular component, U , are given by

$$\begin{aligned}F_d &= \frac{\rho_w C_d U^2 L (D-b)}{2}, & F_l &= \frac{\rho_w C_l U^2 L D}{2}, \\ F_w &= g(\rho_o - \rho_w) \Pi, & F_a &= -\rho_w \Pi \frac{du_o}{dt}\end{aligned}\quad (\text{A19})$$

where $g = 9.81 \text{ m/s}^2$ is the gravitational acceleration; $\rho_w = 1025 \text{ kg/m}^3$ is the density of seawater; ρ_o is the density of the cylindrical object; (C_d, C_l) are the drag and lift coefficients across-cylinder's main axis, with vortex shedding caused by the oscillating flow (U) due to waves. If time averaged U within a certain time period being used, the mean coefficients for drag and lift (C_d, C_l), depending solely on the Reynolds number and aspect ratio (see Appendix D), can be used. Since the wave component (i.e., the bottom wave orbital velocity) in $\mathbf{V}_w$ for the object model is computed from a linear wave model with the temporal resolution of 30 min, the mean coefficients for drag and lift are used. The vortex shedding from objects is neglected. Besides, when the lift coefficient is less certain, we assume

$$C_l = \gamma C_d \quad (\text{A20})$$

where γ is the ratio of lift coefficient versus drag coefficient with γ being taken as value of 0.2.

Appendix D

Drag Coefficient.

For cylindrical objects, the drag force is decomposed into along and cross axis components. The drag coefficient across-cylinder's main axis C_d depends on the Reynolds number,

$$\text{Re} = \frac{UD}{\nu} \quad (\text{A21})$$

where $\nu = 0.8 \times 10^{-6} \text{ m}^2/\text{s}$, is the sea water kinematic viscosity; U is the horizontal water velocity perpendicular to the cylinder's main axis; and D is the cylinder's diameter (see Figure 6). For an empirical formula used to calculate C_d [30].

$$C_d = \begin{cases} 1.9276 + 8/\text{Re}, & \text{if } \text{Re} \leq 12 \\ 1.261 + 16/\text{Re}, & \text{if } 12 < \text{Re} \leq 180 \\ 0.855 + 89/\text{Re}, & \text{if } 180 < \text{Re} \leq 2000 \\ 0.84 + 0.00003\text{Re}, & \text{if } 2000 < \text{Re} \leq 12000 \\ 1.2 - 4/\eta & \text{if } 12000 < \text{Re} \leq 150000, \eta \geq 10 \\ 0.835 - 0.35/\eta, & \text{if } 12000 < \text{Re} \leq 150000, 2 \leq \eta < 10 \\ 0.7 - 0.08/\eta, & \text{if } 12000 < \text{Re} \leq 150000, \eta < 2 \\ 1.875 - 0.0000045\text{Re}, & \text{if } 150000 < \text{Re} \leq 350000 \\ 1/(641550/\text{Re} + 1.5), & \text{if } \text{Re} > 350000. \end{cases} \quad (\text{A22})$$

where $\eta = L/D$, is the cylinder's aspect ratio.

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