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Toward Modeling of Coulomb Explosions

by Stephan Bilyk and Michael Grinfeld

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14. ABSTRACT

Simple continuum models of Coulomb explosions are suggested. The purpose of this effort is twofold. First, we would like to combine the classical continuum models of explosions with Maxwell (or electrostatic) theories of electromagnetism of continuum physics and mechanics. The second goal is to provide models, permitting analytical studies, that can be used for validation and verification purposes of the corresponding numerical codes.

15. SUBJECT TERMS

Coulomb explosions; electromagnetism; laser radiation; exact solutions; validation and verification; Electromagnetic Spectrum Sciences; Energy Sciences; Photonics, Electronics, and Quantum Sciences; Sciences of Extreme Materials; Terminal Effects

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1. Introduction

First efforts in continuum-based studies of Coulomb-forces-driven destabilization of liquid substances were pioneered by Rayleigh (1882), whose studies found multiple applications in different industries including US Army science. Later on, Rayleigh's efforts were significantly expanded by Wigner (1934) and his followers (see, for instance, Foldy [1978] and references therein) to cover crystalline substances.

More recently, the problem of Coulomb-forces-driven destabilization and explosions found multiple application in conjunction with the development of laser-based technologies (see, for instance, Bulgakova et al. [2005] and references therein.) The phenomenon of our particular interest is the considerable growth of explosion intensity due to the electrostatic forces accompanying penetration of solid projectiles through metals. Such evidence has been noticed in the publications of Marakhtanov and Marakhtanov (2002).

In this report we begin with a simple model of a radial Coulomb explosion of an electrically charged spherical gaseous layer. This model permits quite elementary technical analysis. It is particularly important for understanding the mostly robust features of Coulomb explosions. Also, it provides a relevant exact solution for validation and verification purposes when dealing with more sophisticated models.

2. Model of Coulomb Explosions

Normally, a substance consists of positively and negatively charged particles. There are different forces acting between particles. Some appear due to the Coulomb electric interaction. In our model, we distinguish between two sorts of particles, positively and negatively charged. Particles carrying similar charges repel each other, while particles of opposite charges attract each other.

Particles of the same charge, when placed in a closed, electrically neutral vessel, accumulate in vicinity of the vessel's walls. In open vessels, they move away from each other, often going to infinity. Typically, electrostatic (Coulomb) forces are much greater than other forces, acting between particles. Therefore, the integrity of a crystalline substance can be maintained if the net charge of the substance is close or equal to zero. Therefore, the external field of all the electric charges vanish.

When an electrically neutral substance is exposed to external electromagnetic field $\boldsymbol{E}_{out}$, the electric charges inside the substance get redistributed. The result is that there appears an effective electromagnetic field $\boldsymbol{E}_{in}$ caused by the charges of the body. The two fields act in concert. Each of them exposed electric forces acting on electrically charged particles inside and outside the body under study.

Under the action of strong electromagnetic pulses, electrons can be pushed out of the initially neutral body. This usually happens with free or weakly bounded electrons. Thus, the body in question becomes positively charged. The charge balance becomes essentially violated, so the generated electrostatic field prevails over other forces maintaining the integrity of the body. Sometimes the body under electromagnetic radiation gets destroyed. The destruction may cause explosions, coined Coulomb explosions.

The Coulomb explosions in nature are multifaceted phenomena, involving many peculiarities that depend on the specific features of the substance under study. As always, before building the relevant synthetic picture useful for engineering applications, it makes sense to begin with analytical studies emphasizing only a few of the many relevant phenomena. The same is true regarding the development of numerical codes.

In the following, we suggest, probably, the simplest model of Coulomb explosion that allows avoidance of any sophisticated methods of theoretical and mathematical physics. We believe, though, that this model is rather instructive in various respects.

3. Radially Symmetric Model of a Bubble Coulomb Explosion

Consider a radially symmetric bubble that originally was electrically neutral and had initial radius R° (Fig. 1).

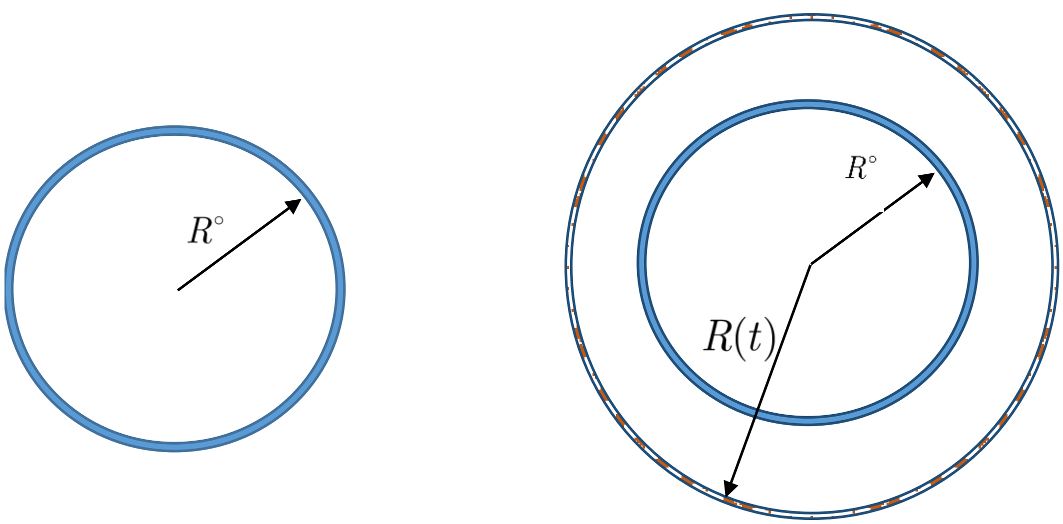

Fig. 1 Radial explosion of a charged bubble

Let us postulate the following model of this phenomenon. The total energy of the charged dust consists of two main ingredients: the electrostatic energy and the kinetic energy.

We assume that the fixed total charge Q is evenly distributed over the spherical surface S_t with the variable radius $R(t)$. Also, let the fixed total mass M of the layer be evenly distributed over the same spherical surface. Then the surface charge density $\zeta = \zeta(t)$ is equal to

$$\zeta(t) = \frac{Q}{4\pi R^2} \tag{1}$$

whereas the actual mass density $\rho = \rho(t)$ is equal to

$$\rho(t) = \frac{M}{4\pi R^2} \tag{2}$$

Then the radial velocity of the material particles is equal to $R_t(t) \equiv \partial R(t) / \partial t$, and the total kinetic energy $E_{kin}(t)$ is given by the relationship

$$E_{kin}(t) = \frac{M}{2} R_t^2 \tag{3}$$

The total electrostatic energy E_{elec} in the purely radial motion can be taken in the following form:

$$E_{elec}(t) = \frac{Q^2}{2R(t)} \tag{4}$$

Thus, the total energy of the charged bubble in purely radial motion reads

$$E_{tot} = \frac{Q^2}{2R} + \frac{M}{2} R_t^2 \tag{5}$$

Imagine that under the action of electromagnetic pulse the bubble's skin loses its negative charges and the bubble appears to be positively charged. Then the positive particles repel each other and effectively try to make the bubble's radius bigger. The mechanical integrity of the bubble is supported by the internal connecting links. As a result, the connecting links get stretched and stressed. As long as the net charge is sufficiently small, the induced deformations and stresses remain small and reversible, and they can be analyzed by means of models based on elasticity theory. But when the charge exceeds certain critical values, the internal links get broken and the bubble's links are broken. It seems reasonable to then use the model of electrically charged dust, which ignores completely those internal links.

When using the dust model, the total energy E_{tot} remains fixed and the radius of the bubble should satisfy the following ordinary differential equation

$$R_t^2 = \frac{2E_{tot}}{M} - \frac{Q^2}{M}\frac{1}{R} \tag{6}$$

Equation 6 can be rewritten in the following form:

$$\frac{dR}{dt} = \sqrt{\alpha - \gamma R^{-1}} \tag{7}$$

where the parameters α and β are defined as

$$\alpha \equiv \frac{2E_{tot}}{M},\ \gamma \equiv \frac{Q^2}{M} \tag{8}$$

We assume that the initial velocity at $t = 0$ of the bubble's particles vanishes. Then the total energy, which remains constant, is given by the relationship

$$E_{tot} = \frac{Q^2}{2R^\circ} \tag{9}$$

We can now rewrite Eq. 6 in the following form:

$$R_t^2 = \frac{Q^2}{M}\frac{R - R^\circ}{RR^\circ} \tag{10}$$

We can now rewrite Eq. 10 as follows:

$$\frac{dY}{dt} = \sqrt{\frac{Y - Y^\circ}{Y}} \tag{11}$$

where we introduce parameters

$$Y(t) = R(t)\sqrt{\frac{MR^\circ}{Q^2}},\ Y^\circ = R^\circ\sqrt{\frac{MR^\circ}{Q^2}} \tag{12}$$

We proceed as

$$\sqrt{\frac{Y}{Y-Y^{\circ}}}\frac{dY}{dt}=1 \tag{13}$$

We can rewrite Eq. 13 as

$$\sqrt{\frac{y}{y-1}}\frac{dy}{d\tau}=1,\, y(0)=1 \tag{14}$$

where

$$y=\frac{Y}{Y^{\circ}},\quad \tau=\frac{t}{Y^{\circ}} \tag{15}$$

To explore the solution of Eq. 14 in the vicinity of the initial moment of explosion, let us rewrite it as

$$\frac{dy}{d\tau}=\sqrt{\frac{y-1}{y}},\, y(0)=1 \tag{16}$$

Equation 16 obviously implies

$$\frac{dy(0)}{d\tau}=0 \tag{17}$$

Differentiating the former of Eq. 17, we get

$$\frac{d^2y}{d\tau^2}=\frac{1}{2y^2} \tag{18}$$

as implied by the following chain:

$$\frac{d^2y}{d\tau^2}=\frac{dy}{dt}\frac{d}{dy}\sqrt{\frac{y-1}{y}}=\sqrt{\frac{y-1}{y}}\frac{d}{dy}\sqrt{\frac{y-1}{y}}=\frac{y-1}{y}\frac{d}{dy}\ln\sqrt{\frac{y-1}{y}}=$$
$$\frac{y-1}{2y}\frac{d}{dy}\ \ln(y-1)-\ln y\ =\frac{y-1}{2y}\left(\frac{1}{y-1}-\frac{1}{y}\right)=\frac{1}{2y^2}$$

Combining Eqs. 16 and 18, we get

$$\frac{d^2y(0)}{d\tau^2}=\frac{1}{2} \tag{19}$$

Using Eqs. 16, 17, and 19, we can find the for the Taylor series approximation of $y(\tau)$ in small vicinity of $\tau = 0$:

$$y(\tau) \approx 1 + \frac{1}{4}\tau^2 + ... \tag{20}$$

The exact implicit solutions of Eq. 16 reads

$$\sqrt{y-1}\sqrt{y} + \log(\sqrt{y-1} + \sqrt{y}) = \tau \tag{21}$$

or

$$\sqrt{Y - Y^\circ}\sqrt{Y} + Y^\circ \log \frac{\sqrt{Y - Y^\circ} + \sqrt{Y}}{\sqrt{Y^\circ}} = t \tag{22}$$

The numerical solution of Eq. 21 is shown in Fig. 2.

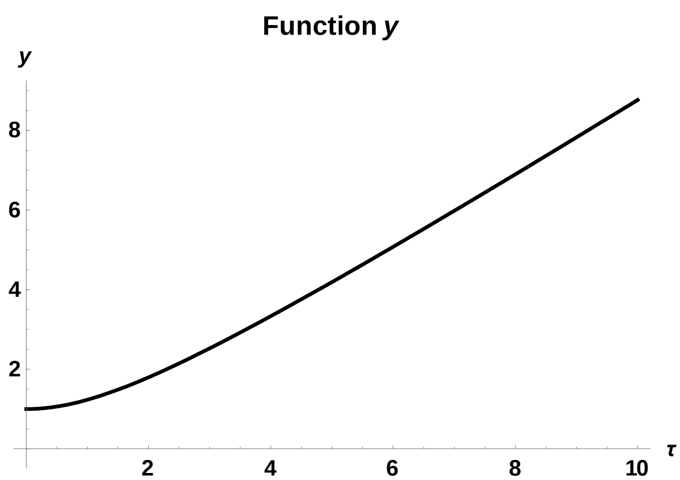

Fig. 2 Graph of solution $\tau(y)$ of Eq. 21

We see that for large τ, the exact solution approaches a straight line with the tangent 1, or

$$R(t) \approx \frac{Q^2}{MR^{\circ 2}} t \tag{23}$$

4. Conclusion

We suggested a continuum model of Coulomb explosions. The model permits a rather simple analytical solution. We recommend its use for validation and verification purposes. The model shows that right after the explosion, the radius grows as a quadratic parabola (Eq. 20). At the late stage, the radius grows linearly (Eq. 23). The solution for all $t > 0$ can be determined exactly in the nonexplicit form (Eq. 21 or equivalently Eq. 22).

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