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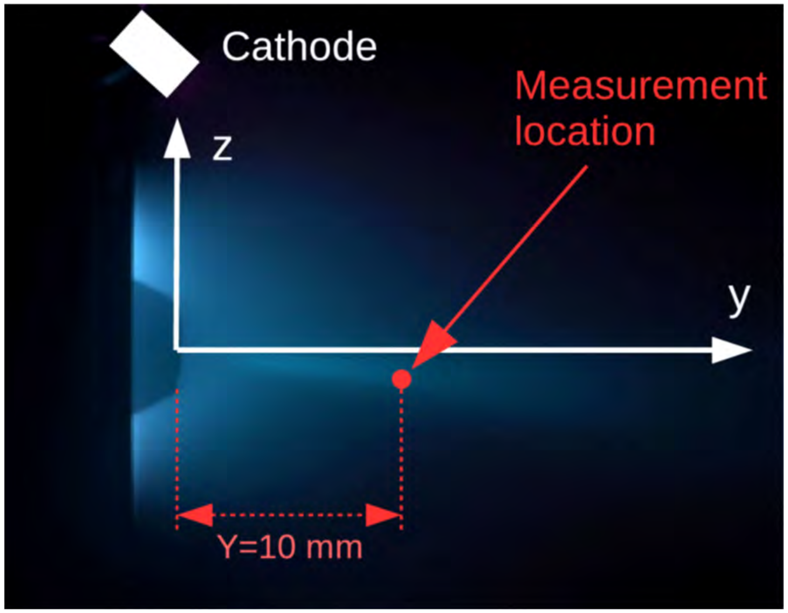
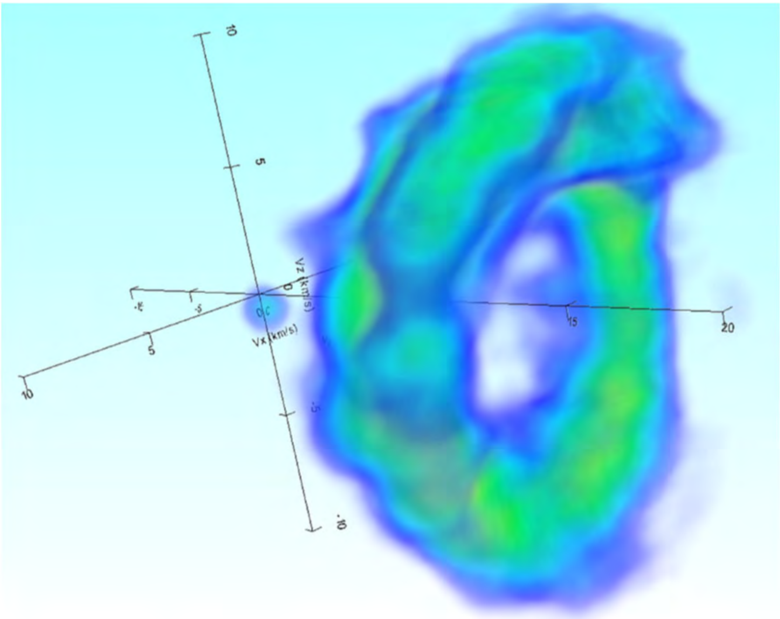
# Adaptive and Compressed Kinetic Simulations

Robert Martin[1], Will Taitano[2] , Alex Alekseenko[3]
In-space Propulsion Branch (AFRL/RQRS)[1], Exquadrum Inc.[2], AFRL SFFP Program (CSUN)[3]
Rocket Propulsion Division
Aerospace Systems Directorate

# Plasma Thruster Flow Highly Non-Equilibrium Velocity Distributions



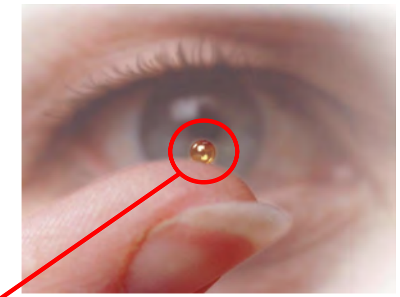



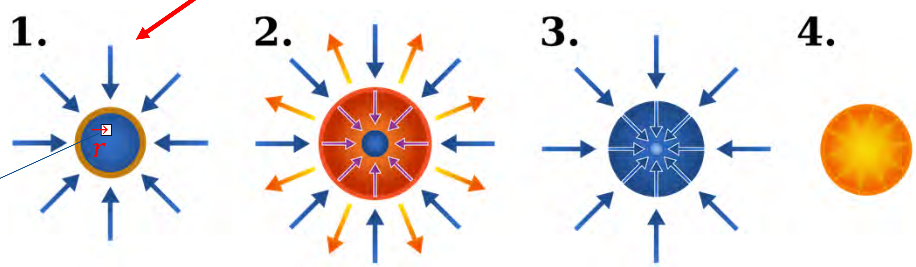
Elias, Jarrige, Cucchetti, Cannat, Packan, Rev. Scientific Instruments 88, 093511 (2017)

Solution Requires Boltzmann Equation: $\partial_t f_\alpha + \nabla_x \cdot (\vec{v} f_\alpha) + \nabla_v (\vec{a} f_\alpha) = \left.\frac{df}{dt}\right|_{Coll.}$

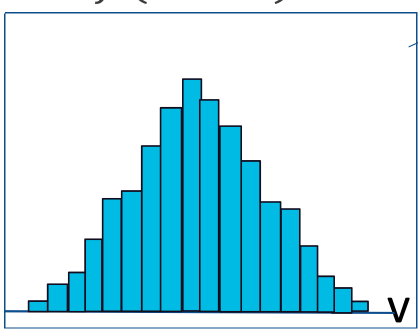
**Computationally Intractable** with Naïve Discretization: 6D VDF (3D×3V)

# LANL Kinetic Modeling: Vlasov-Fokker-Planck and relevance to ICF






$f(\vec{r}, \vec{v}, t)$

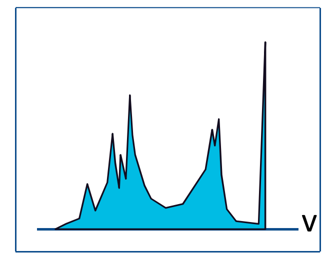



$$\partial_t f_\alpha + \nabla_x \cdot (\vec{v} f_\alpha) + \nabla_v \cdot (\vec{a} f_\alpha) = \nabla_v \cdot \left[ \overline{\overline{D}}_\beta \cdot \nabla_v f_\alpha - \vec{A}_\beta f_\alpha \right]$$

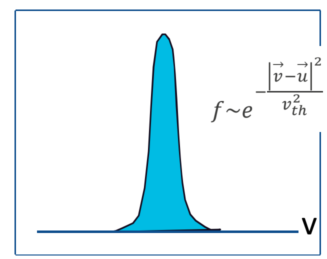
$$= \frac{q}{m} \left[ \vec{E} + \vec{v} \times \vec{B} \right]$$

( Fokker-Planck: $\left.\frac{df}{dt}\right|_{Coll.}$ )

$f(\vec{r}, \vec{v}, t_0)$




$f(\vec{r}, \vec{v}, t_0 + \tau_{col})$



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# Adaptive Kinetic Simulation

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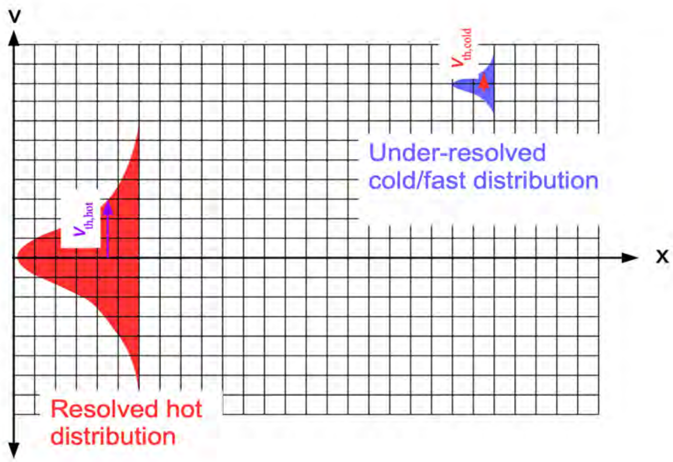
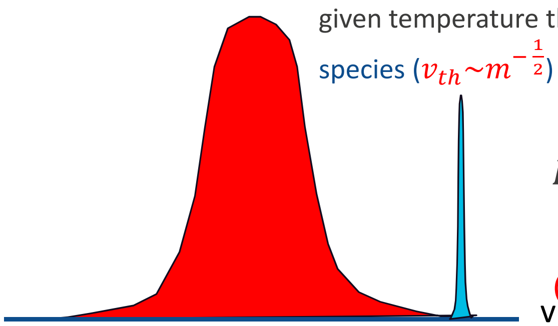
# Curse of Dimensionality: Kinetic Description impossible in High-D

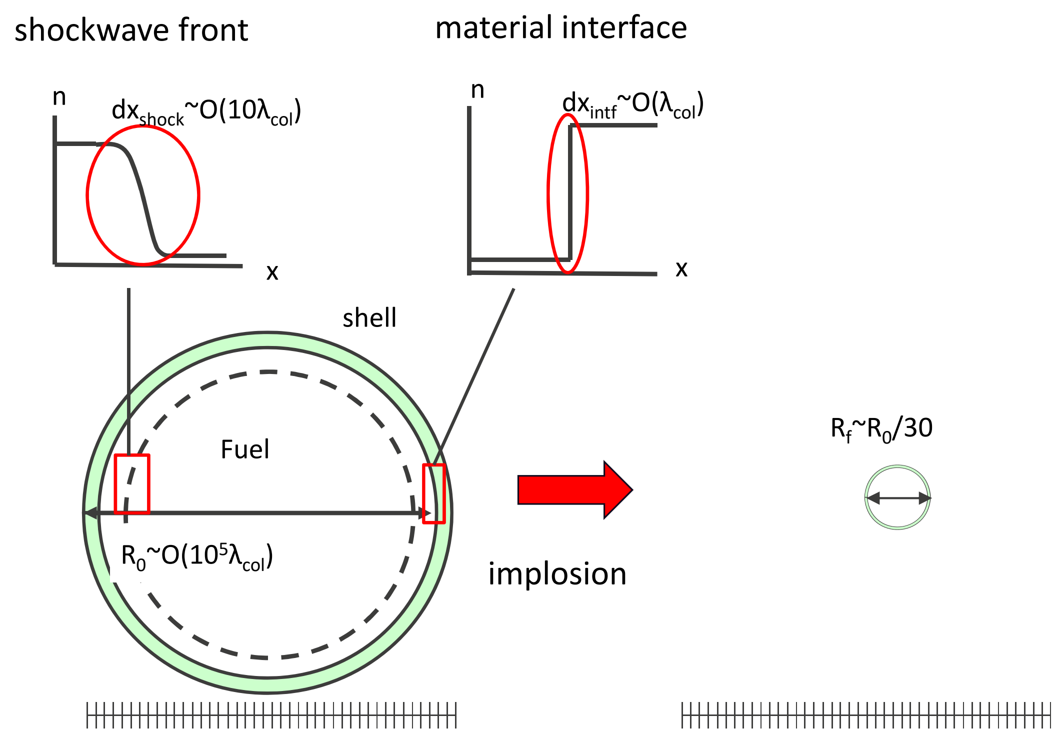



Heavy species have a narrower distribution for a given temperature than light species ($v_{th} \sim m^{-\frac{1}{2}}$)

$$R_v = \frac{v_{th,max}}{v_{th,min}} \sim 4000$$

$$(L_v N_v R_v)^3 \approx 10^{16}$$



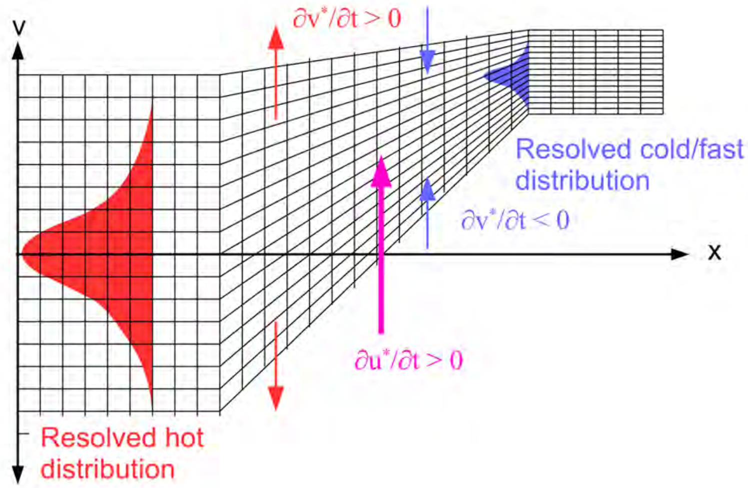

$$N_x^3 \approx 10^{15}$$

# Moving phase-space grid strategy to address challenge

Velocity space transformation: $\vec{v} = v^* \vec{\tilde{v}} + \vec{u}^*$

$v^* \sim v_{th}$ (variance of the distribution)

$\vec{u}^* \sim \vec{u}$ (the mean of the distribution)




Taitano *et al.*, CPC **258**, 107547 (2021)

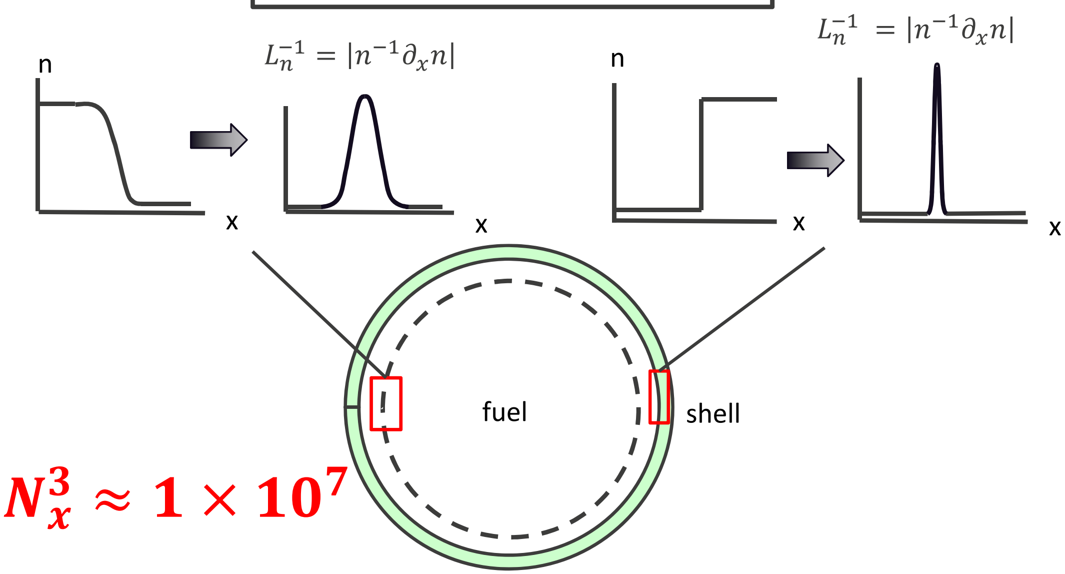
$$(L_v N_v)^3 \approx 1 \times 10^5$$

We solve for x-grid position based on error equidistribution principle with a nonlinear optimization step to stabilize grid evolution:

$$\partial_t x^* = \tau_g^{-1} \partial_\xi \left[ L_n^{-1} \partial_\xi x^* \right]$$

$\min \mathbb{F}(x^*) \to x$ where $\mathbb{F}$ is a cost function which leads to a corrected grid, x, which leads to an optimal balance between grid quality and grid evolution

Taitano *et al.*, CPC **263**, 107861




$$N_x^3 \approx 1 \times 10^7$$

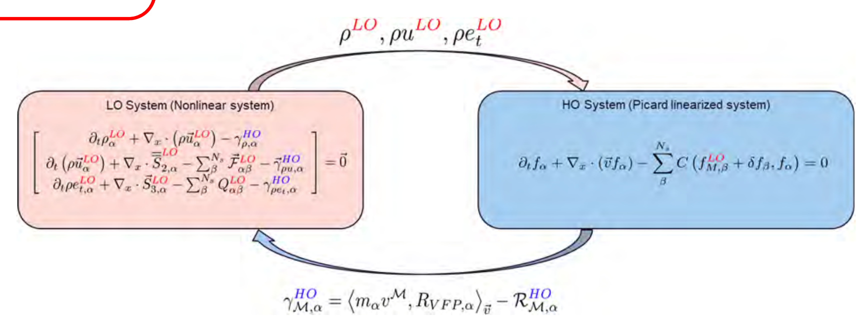
# Transform VFP Eq. to Curvilinear Coordinate + Inertial Terms

$$\partial_t f_\alpha + \nabla_x \cdot (\vec{v} f_\alpha) + \nabla_v \cdot (\vec{a} f_\alpha) = \sum_\beta^{N_s} \nabla_v \cdot \left[\vec{J}_{\alpha\beta}\right]$$

$$\partial_t (\mathbb{J} f_\alpha) + \nabla_\xi \cdot \left[\mathbb{J}\left(\vec{v}^\xi - \dot{\vec{r}}^\xi\right) f_\alpha\right] + \nabla_{\tilde{v}} \cdot \left[\mathbb{J}\left(\vec{\tilde{a}} - \vec{\tilde{a}}^\dagger\right) f_\alpha\right] = \sum_\beta^{N_s} \nabla_{\tilde{v}} \cdot \left[\mathbb{J}\vec{\tilde{J}}_{\alpha\beta}\right]$$

inertial terms/Christoffel symbols

+ Efficient HOLO Implicit Nonlinear Convergence Accelerator

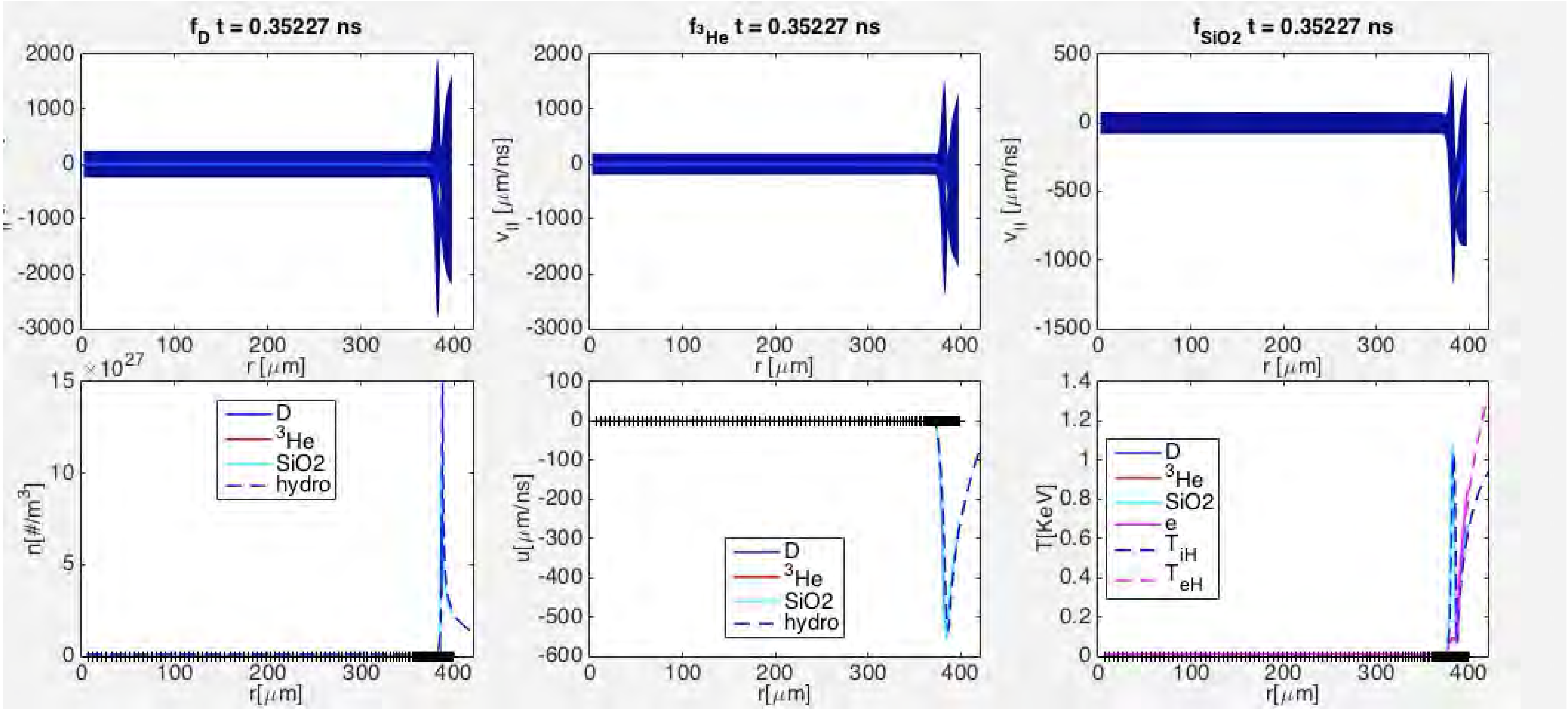



- $\mathbb{J}$: Jacobian of transformation
- $\dot{\vec{r}}^\xi$: contravariant component of configuration grid velocity
- $\vec{v}^\xi$: contravariant component of particle velocity
- $\vec{\tilde{a}}$: Lorentz force in the new velocity coordinate system
- $\vec{\tilde{a}}^+$: Fictitious acceleration terms due to expansion/contraction of velocity grid

Required for Stepping Over Stiff HEDP Collisional Terms

$$\frac{\tau_{plasma}}{\tau_{collision}} \approx O(10^5)$$

# Algorithms Deployed into LANL Production Code, iFP



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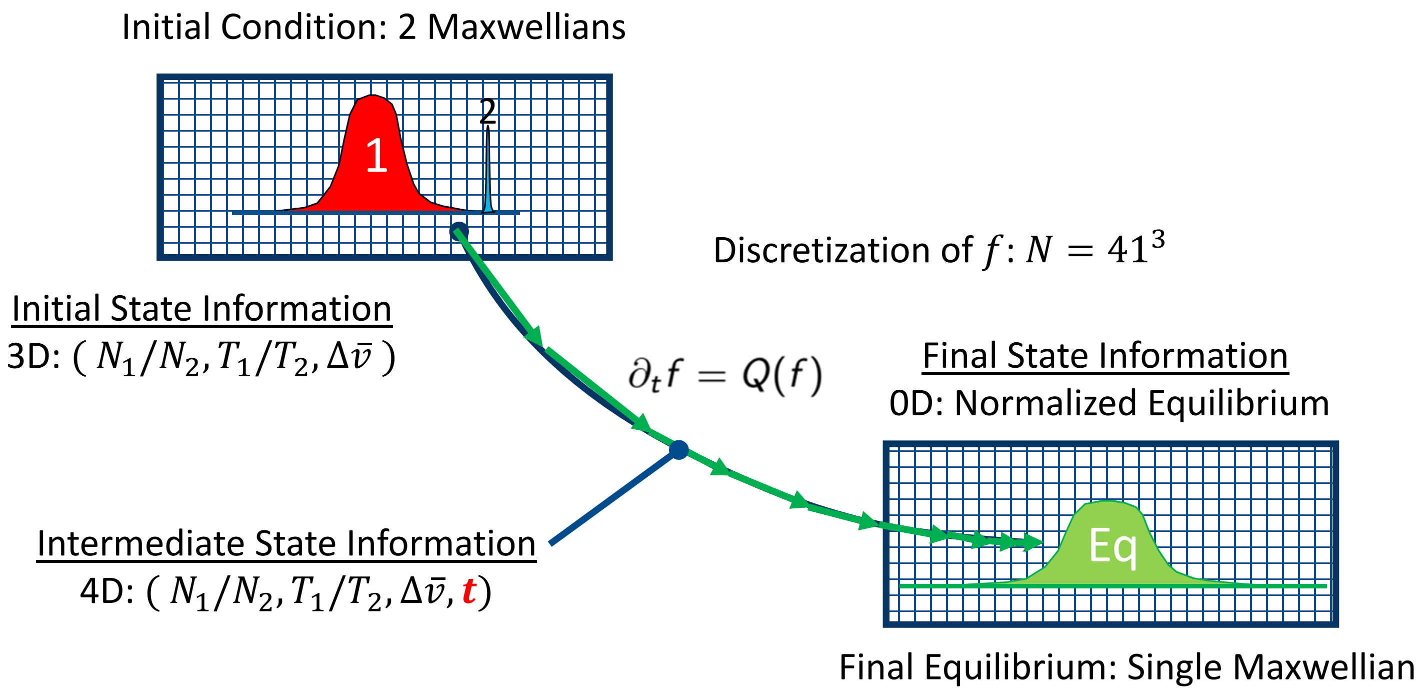
# Compressing Homogenous Relaxation

THE AIR FORCE RESEARCH LABORATORY

# Complexity Reduced in Restricted Problem Classes

**Example:** Homogenous Relaxation from 2 Maxwellians



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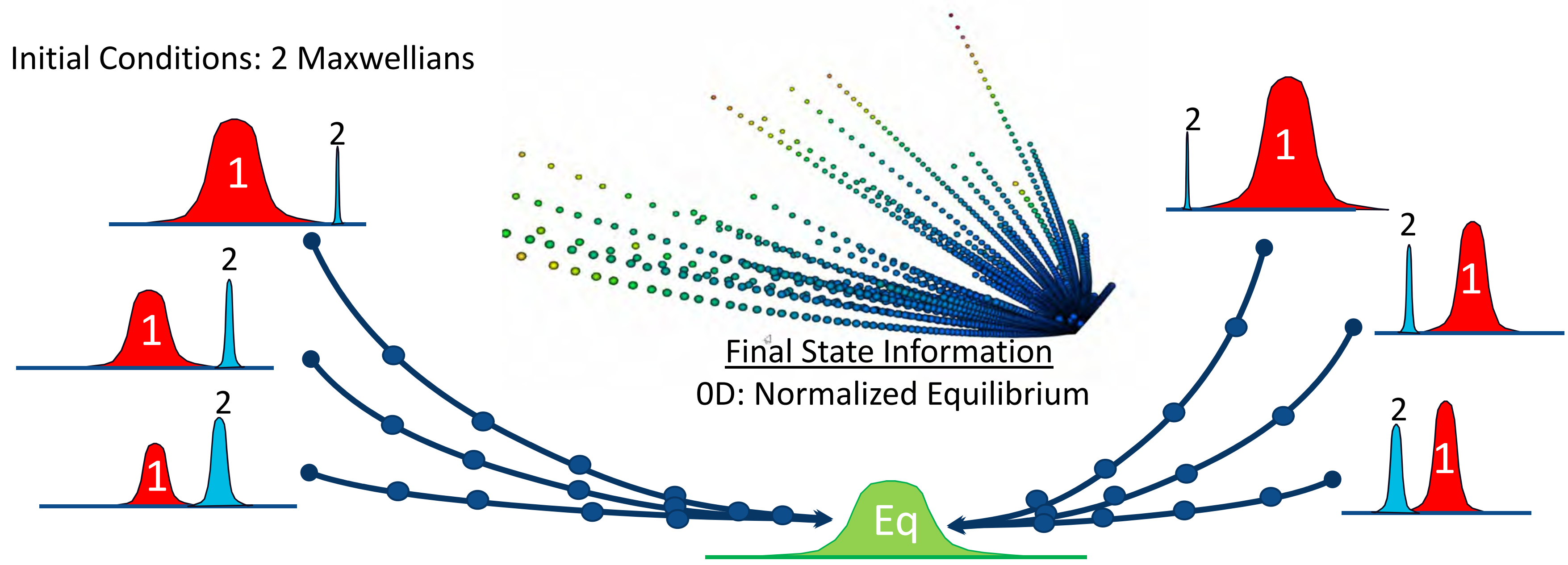
# Complexity Reduced in Restricted Problem Classes

**Example:** Homogenous Relaxation from 2 Maxwellians



# Accelerating the Boltzmann Collision Integral using ROMs and Artificial Neural Networks

Collaboration: Dr. A. Alekseenko, T. Nguyen, CSUN, Dr. A. Wood, AFIT

Boltzmann collision integral (BCI) describes effects of molecular interactions. Use of BCI can improve simulation of plasma, high speed high altitude flows and slow flows. However, it is prohibitively expensive computationally.

$$\partial_t f = Q(f)$$

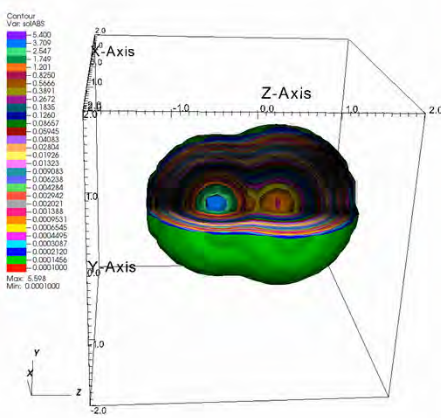
$$Q(f,f) = \int_{\mathbb{R}^3} \int_0^{2\pi} \int_0^{b_*} (f' f_1' - f f_1) |g| \, b \, db \, d\varepsilon \, dv_1$$

Use of ROMs and ANN can improve computation time for BCI:

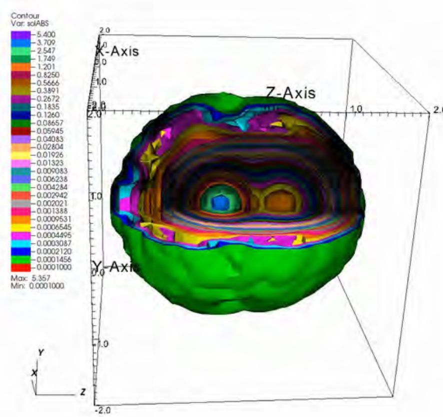
| Approach | Cost | CPU Time | Speed-up | % |
|---|---|---|---|---|
| Nodal-DG | O($M^6$) | 151 sec | | |
| ROM | O($K^3$) | 0.35 sec | 431 | 0.2% |
| CNN | O(N) | 0.18 sec | 838 | 0.1% |

CPU time to evaluate collision operator. Acceleration is relative to O($n^6$) nodal-DG discretization on $41^3$ velocity mesh (151 sec).

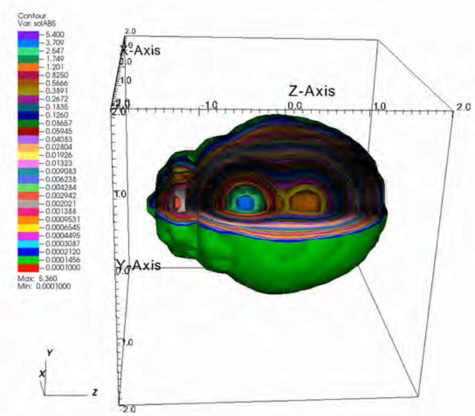
Spatially Homogeneous Relaxation of two Homogeneous Gauss Densities




Nodal-DG Velocity Discretization




ZCMP ROM




Convolutional NN

**Benefits:**

- Fast

**Concerns:**

- Is model accurate when data that is not familiar to the learned model?
- Are solutions asymptotically stable in time?

# ROM Off-Line Training Stage: Low Rank Class of Solutions

$$D_{ij} \approx \sum_{i=1}^{K} \sigma_i u_i v_i^T$$

A good SVD truncation is achieved at K = 40

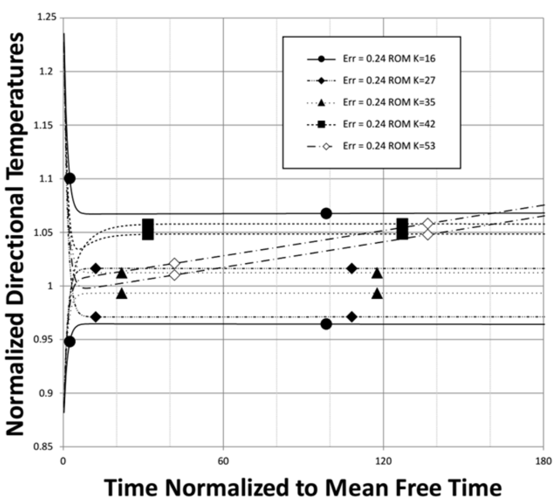
$$\partial_t f = Q(f)$$

**Last Year:** ROM is the Galerkin discretization of the BCI using first $K$ right singular vectors $v_i$ .

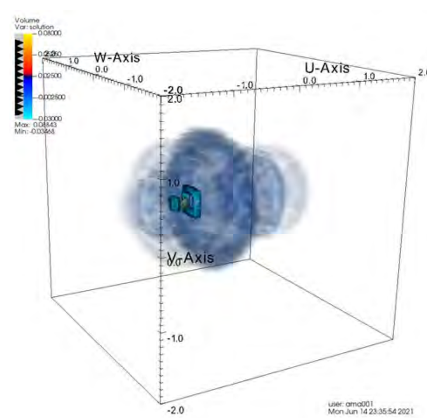
$$H^T = \sum_{i=1}^{K} e_i v_i^T, \quad e_i \text{ is basis in } \mathbb{R}^K$$

ROM variable: $y=H^Tf$

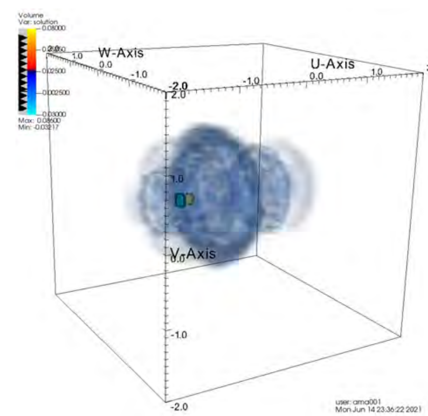
ROM: $$\partial_t(y)_k = \partial_t(Hf)_k = \sum_{k'=1}^{K}\sum_{k''=1}^{K} y_{k'} y_{k''} \hat{A}_{k',k'',k}$$



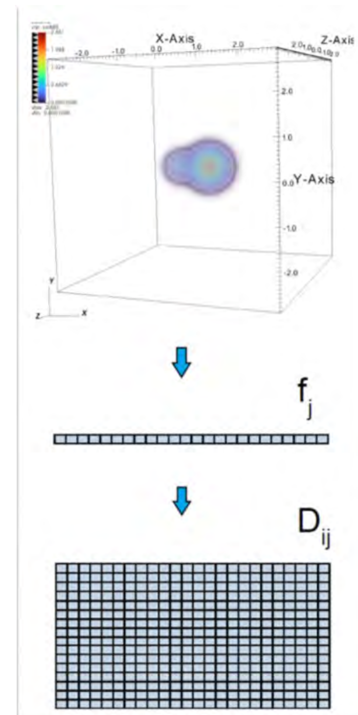
SVD ROM Non-physical moments; growing instabilities



Error in the steady state



ROM residual



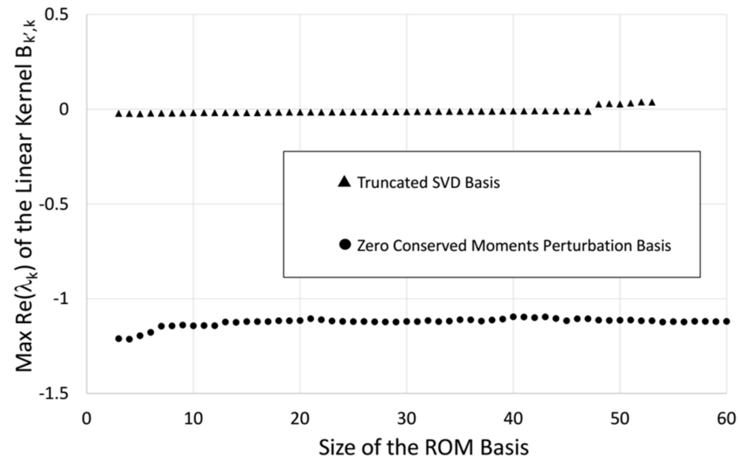
Data points are time saves of solutions

# Stabilizing and Correcting the SVD Reduced Order Model

ROM can be re-formulate to evolve deviation from the steady state, $\varepsilon_k = y_k - w_k$, where $w_k$ is the ROM projection of the steady state:

$$\partial_t \epsilon_k = \sum_{k'=1}^{K} B_{k',k}\epsilon_{k'} + \sum_{k',k''=1}^{K} \epsilon_{k'}\epsilon_{k''}\hat{A}_{k',k'',k}$$

$$B_{k',k} = 2\sum_{k''=1}^{K} \hat{A}_{k',k'',k} w_{k''}$$




Largest real parts of eigenvalues of $B_{k',k''}$

**Enforcing Stability:** Zero Conserved Moments Perturbation Basis :

- Include steady state as the first ROM basis vector, so $w_k$=(1,0,..,0)
- Orthogonalize
- Perturb vectors with k>1 so that their conserved moments are zero.
- Orthogonalize

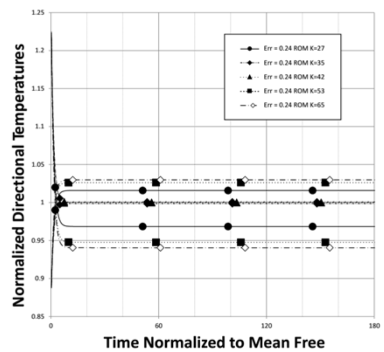
**Damping ROM Residual:**

- Define

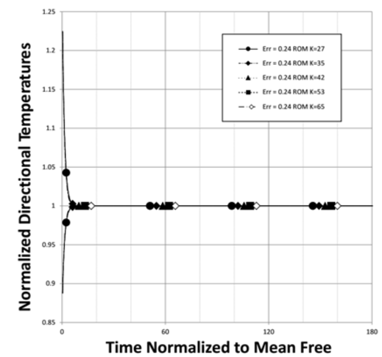
$$\epsilon^{\perp} = (I - HH^T)f$$

- Evolve

$$\partial\epsilon^{\perp} = -\nu\epsilon^{\perp}$$




No residual damping




Residual damping

# Acceleration Using Convolutional Neural Networks

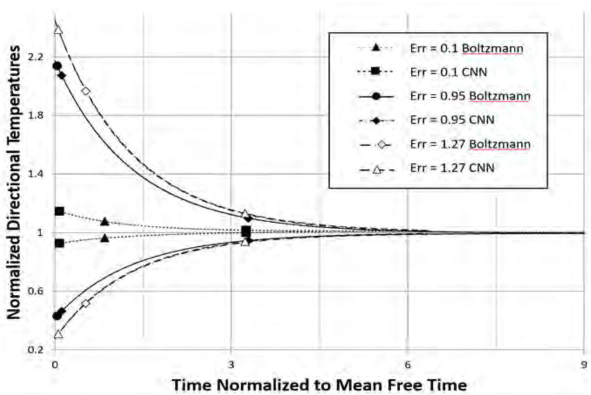
Results: Tom Nguyen, CSUN

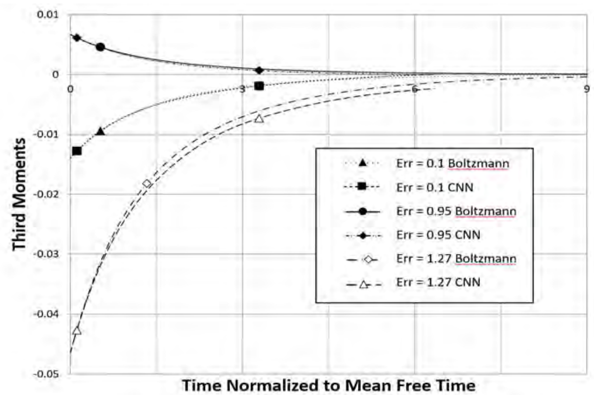
Convolutional Neural Network (CNN) was trained to predict values of the collision operator for the class of solutions discussed above. The CNN predictions were used to approximate collision operator in Euler time stepping scheme to solve

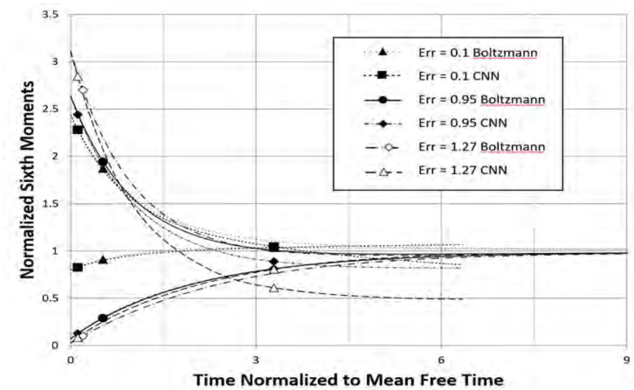
$$\partial_t f = Q_{CNN}(f)$$

Structure of the CNN:

- Input: discrete f at $41^3$ points
- 1st hidden layer: 4 filters; 5x5x5 kernel
- 2nd hidden layer: 8 filters; 3x3x3 kernel
- 3rd hidden layer: max pooling; 2x2x2 kernel
- 4th hidden layer: 16 filters; 3x3x3 kernel
- 5th hidden layer: max pooling; 2x2x2 kernel
- 6th hidden layer: 32 filters; 3x3x3 kernel
- Output: fully connected, discrete collision $Q_{CNN}$ on $41^3$ points
- parametric leaky ReLU, MAE loss, adamax optimizer.

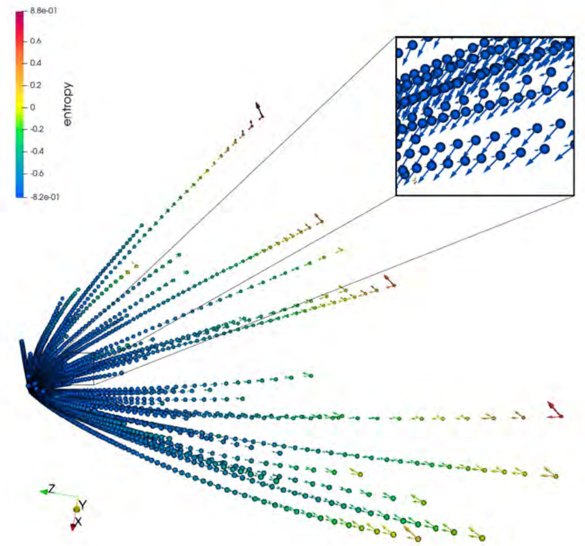


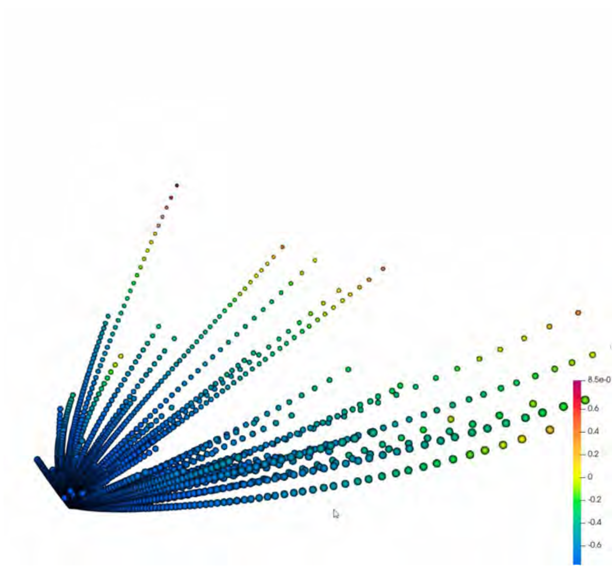

Relaxation of moments in solutions obtained using CNN approximation of collision operator

# Exploring Flow Structure of Low Dimensional of Solutions




**Observations**

- Class of solutions depends on three parameters and time
- Trajectories look very simple in SVD basis
- Kinetic Entropy $H(f) = \int_{\mathbb{R}^3} f(\vec{v}) \ln f(\vec{v})\, dv$ decreases monotonically in time and could be a candidate for the potential function.



Trajectories of solutions in the basis of first three singular vectors of $D_{ij}$ .

**Future Work**

- Find NN approximation to s.p.d. matrix $\Theta(f)$ such that
  $$\langle \partial_t f(t, \vec{v}), \phi(v) \rangle = -\langle \nabla H, \Theta(f)\phi(\vec{v}) \rangle$$
- Recover trajectories by integrating along $\Theta(f)\nabla H$
- Alternatively, formulate a minimization problem using appropriate action, see e.g. Erbar 2016

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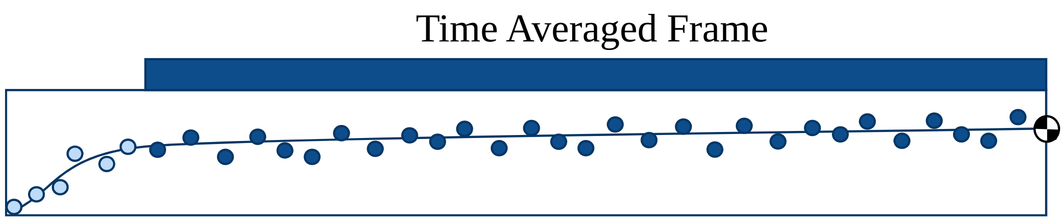
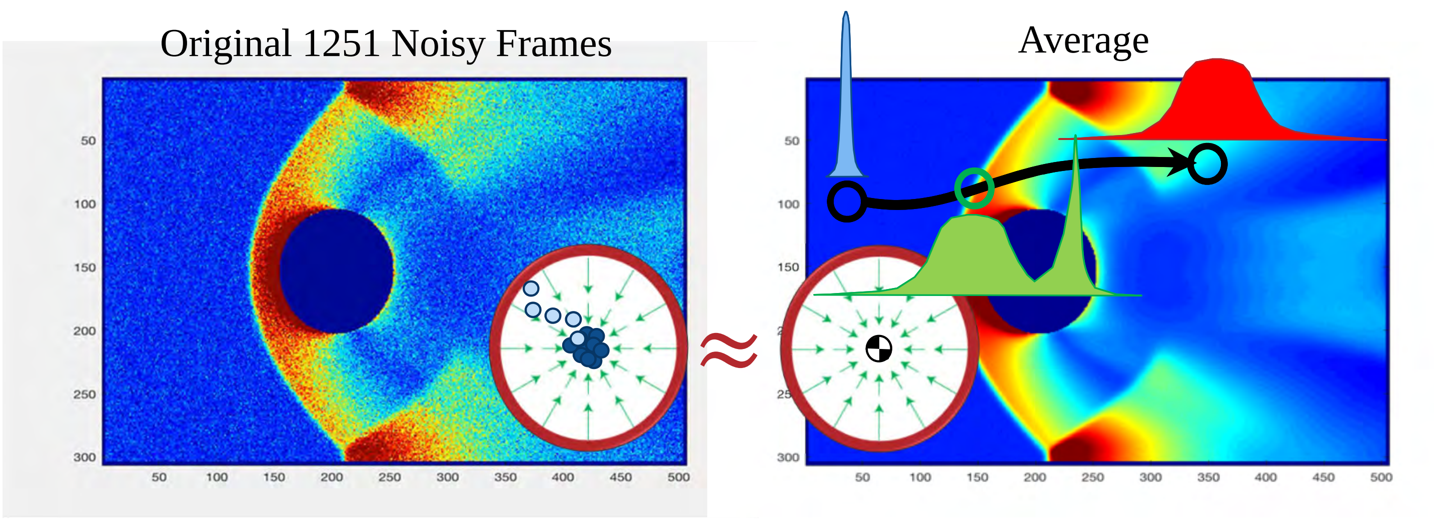
# Spatiotemporal Compression of Kinetic Solutions

# Noise in Particle Simulations Mitigated via Temporal Average



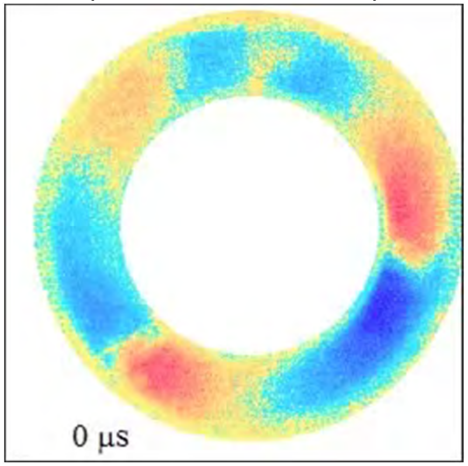




Steady State Flow with only Local Non-Equilibrium

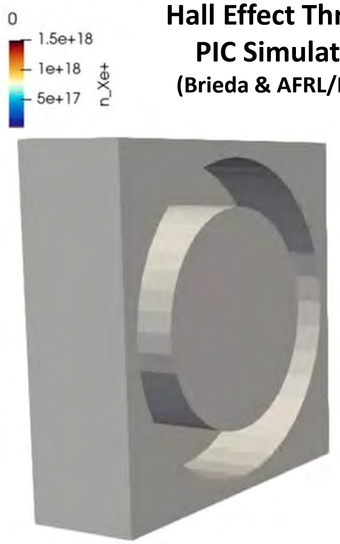
# Insufficient Dissipation → Emergent Coherent Structures

**Mode Locking: Energy → Large Scales**

**Hall Effect Thruster**
**Highspeed Video**
**(McDonald, PhD Dis., UMich)**




**Hall Effect Thruster**
**PIC Simulation**
**(Brieda & AFRL/RQRS)**

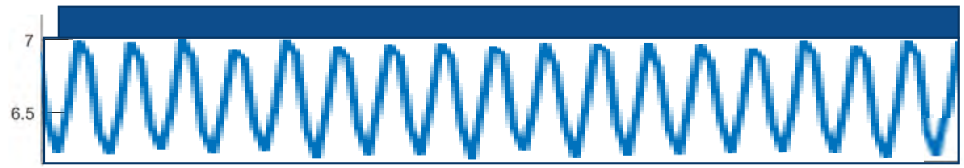



**Ansatz: After Initial Transient,**
**Kinetic 'Observables' Nonlinear Function of Relatively Low-D State**
**(i.e. Dissipation Enables System to Forget Initial State & Collapse to Low-D State)**

# **Toy Example:** Stable Flow Perturbed by Low Frequency Sine Wave

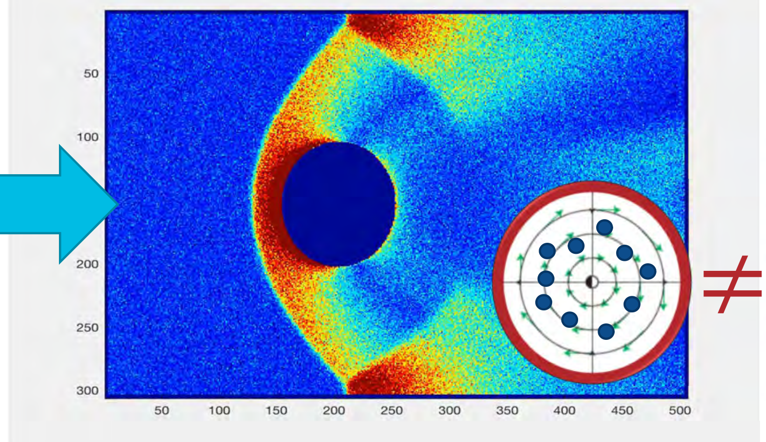
## Why Should We Care?

Time Averaged Frame



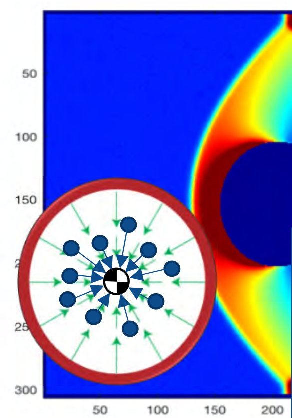

**Dynamics**
Emergent
Or
Driven

Original 1251 Noisy Frames

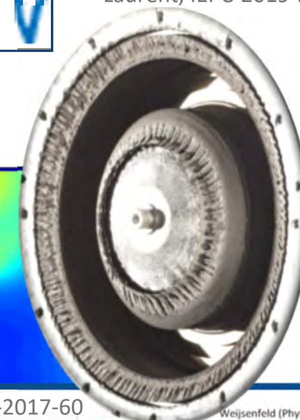



$\neq$

Average




Laurent, IEPC-2019-274



$$Y(\bar{v}) \neq \overline{Y(v)}!$$

Yim, IEPC-2017-60

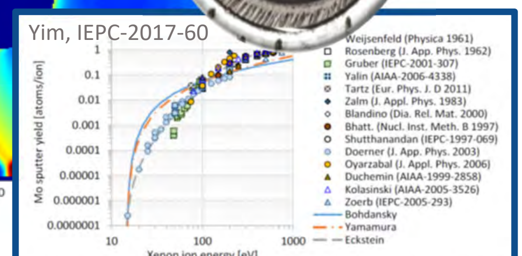
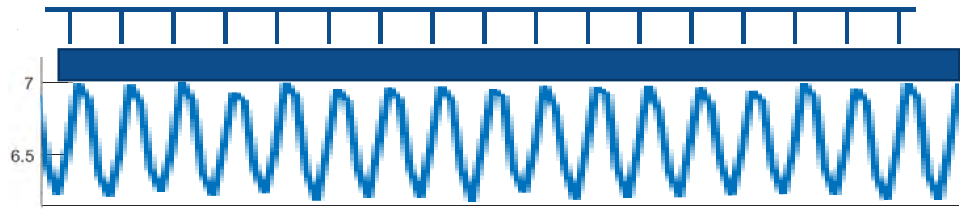



**Figure 6: Molybdenum sputter yields from low energy xenon ions at normal incidence and compared to different sputter yield formulas**

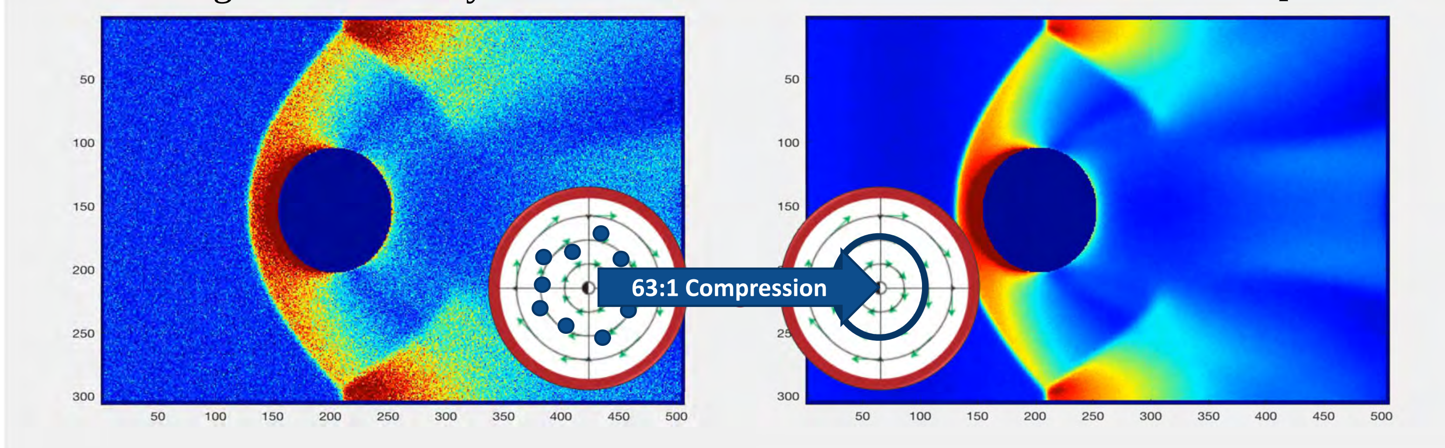
# **Toy Example:** Stable Flow Perturbed by Low Frequency Sine Wave

Boxcar Averaged Frames




Original 1251 Noisy Frames

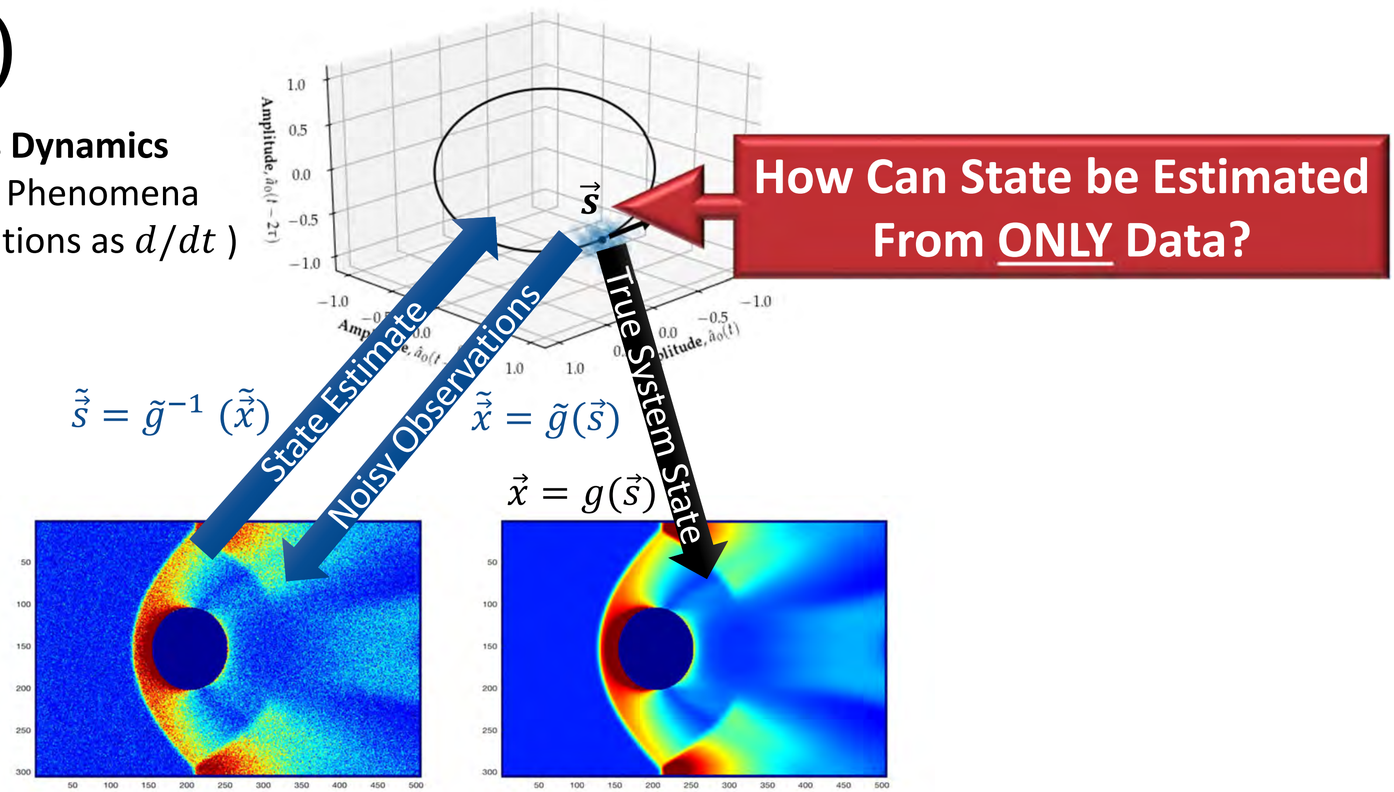
De-noised 20 Frame Loop



# Dynamical State Based Data Compressor

$$\dot{\vec{s}} = f(\vec{s})$$

**Note: Assumes Autonomous Dynamics**
No Absolute Time in Physical Phenomena
(i.e. Time Only Enters Equations as $d/dt$ )

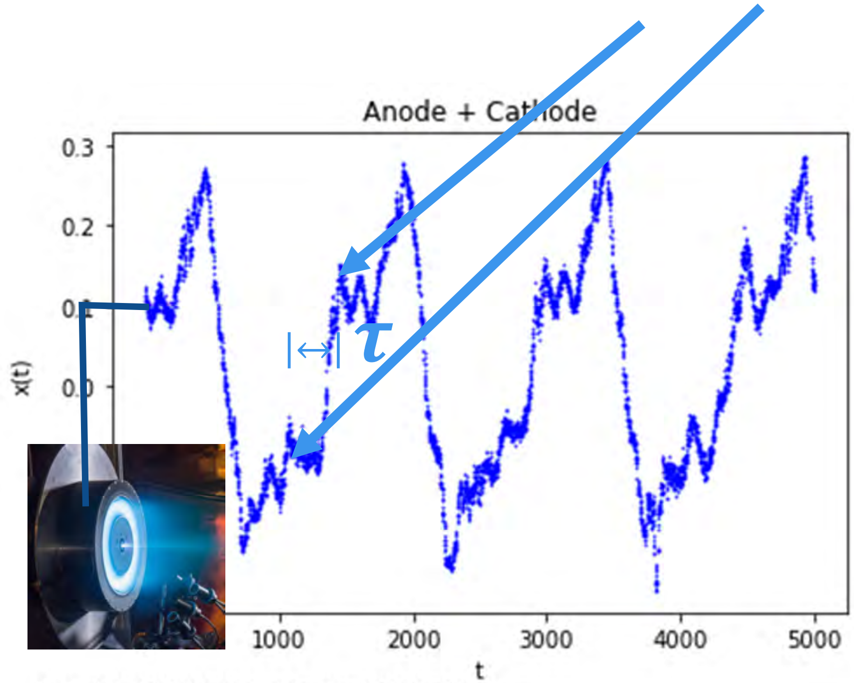


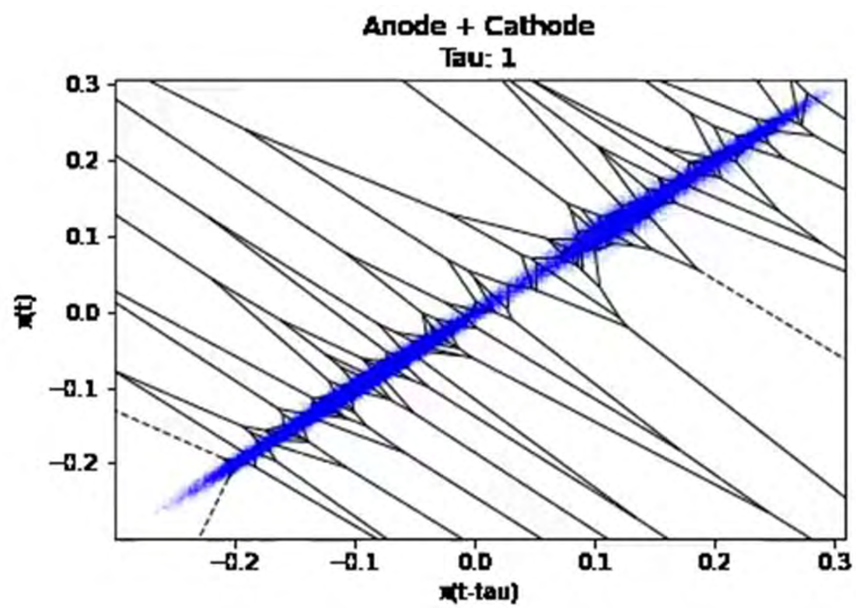
# Evidence of Low-D State from Experimental Data

## **Time-Delay Embedding to Access State**

Given Generic Observable Time Series, $x(t)$

Build Delay Vectors: $\vec{x}(t) = [x(t), x(t-\tau)]$



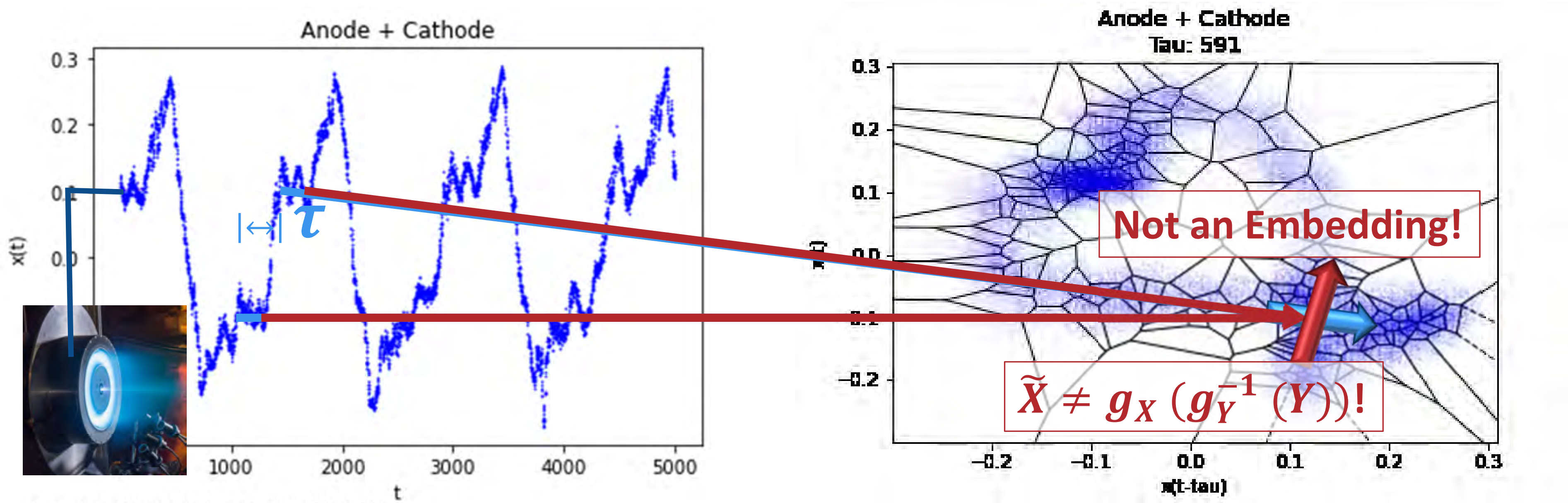
# Evidence of Low-D State from Experimental Data

## Time-Delay Embedding to Access State

Given Generic Observable Time Series, $x(t)$

Build Delay Vectors: $\vec{x}(t) = [x(t), x(t-\tau)]$

$$\dot{\vec{s}} \neq f(\vec{s})!$$



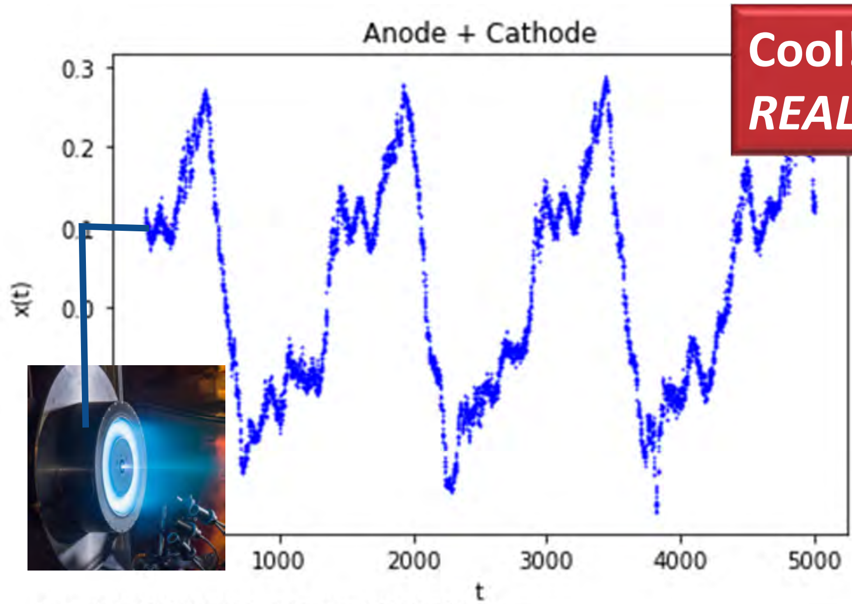
# Evidence of Low-D State from Experimental Data

## **Time-Delay Embedding to Access State**

Given Generic Observable Time Series, $x(t)$

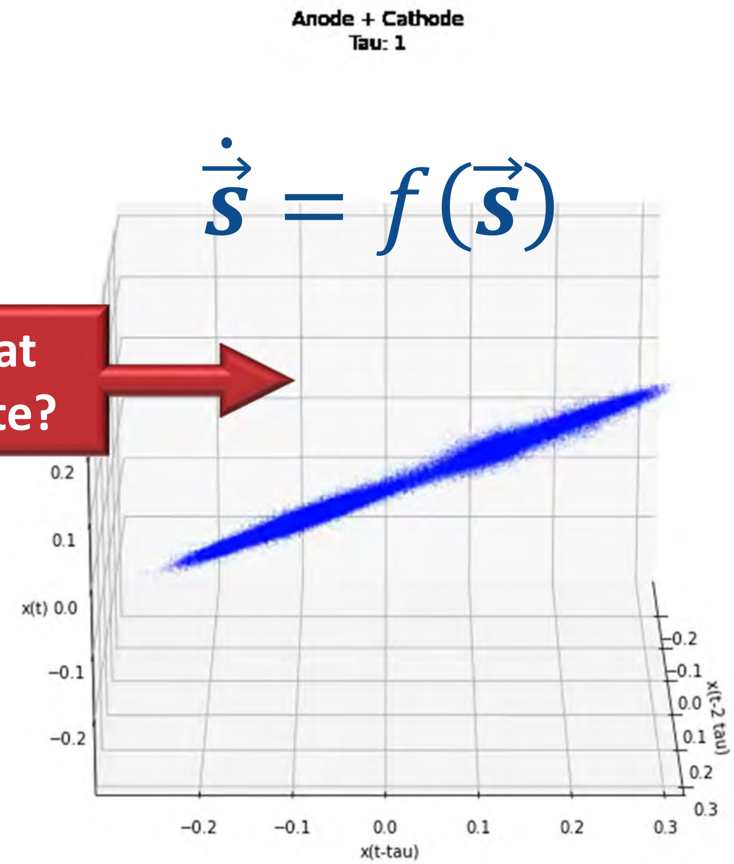
Build Delay Vectors: $\vec{x}(t) = [x(t), x(t-\tau), \boldsymbol{x(t-2\tau)}]$

Takens Theorem: $(2d_A + 1)$ Needed for Embedding




**Cool! But is that *REALLY* the state?**

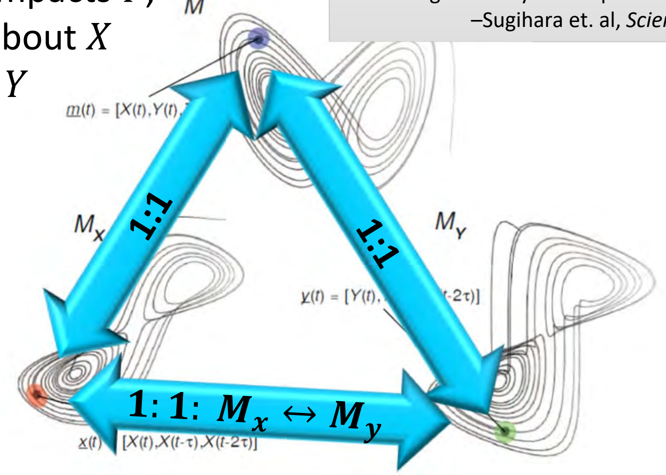
$$\dot{\vec{s}} = f(\vec{s})$$



# Convergent Cross Mapping (CCM) for Detecting Causality

If $X$ causally impacts $Y$, information about $X$ is encoded in $Y$

Detecting Causality in Complex Ecosystems
–Sugihara et. al, *Science*, 2012




CCM Constructs a Convergent NONLINEAR Map:

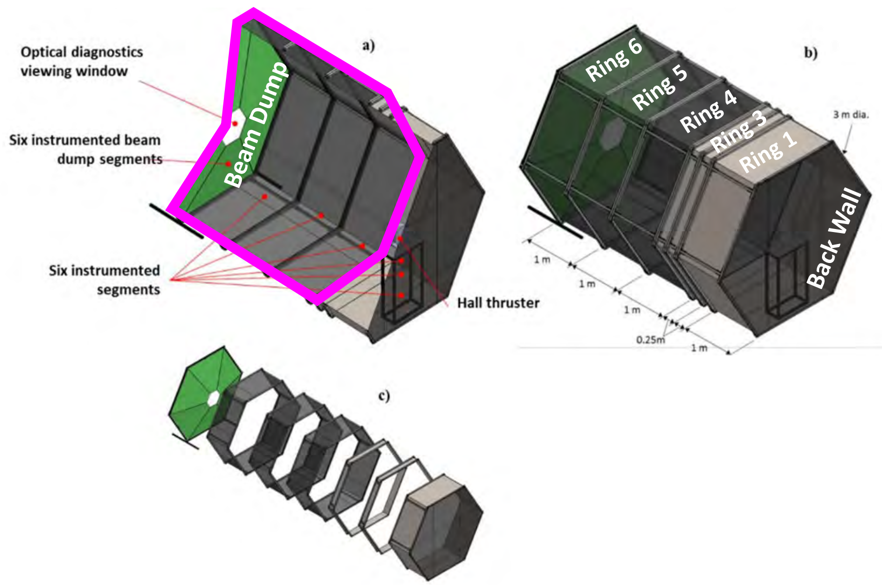
$$\tilde{X} = g_X(g_Y^{-1}(Y))$$

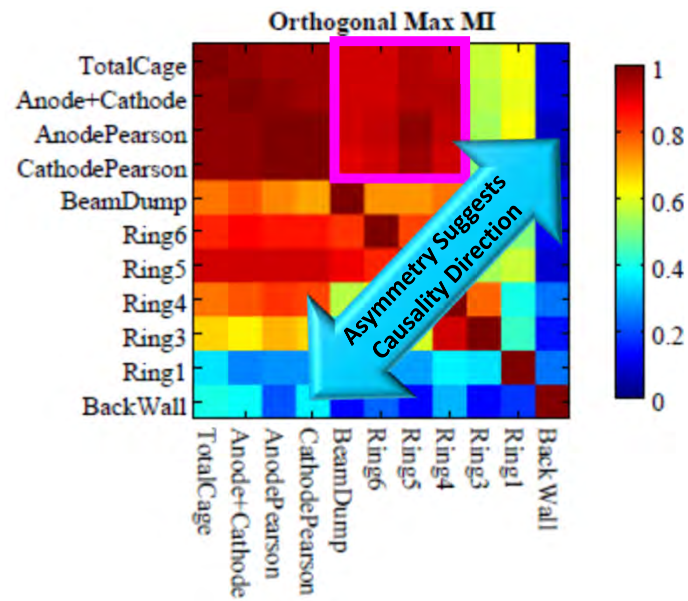
Without Knowledge of $\vec{s}$ or $f(\vec{s})$!

**Causation IMPLIES Existence of Nonlinear Map**

If: $\tilde{X} \rightarrow X$ as $t \rightarrow \infty$, $X$ causally impacts $Y$

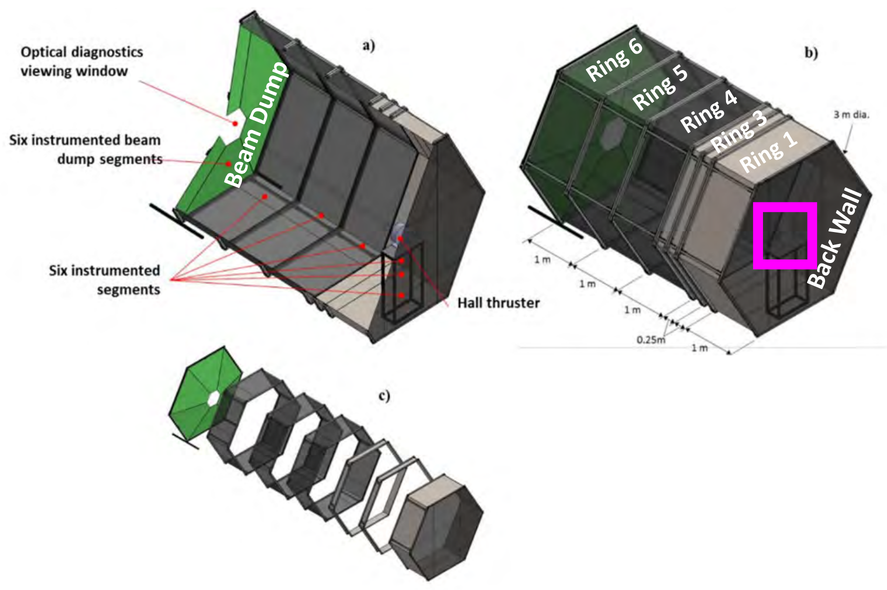
# **Causal Relationships:** Discovered in EP-TEMPEST Data

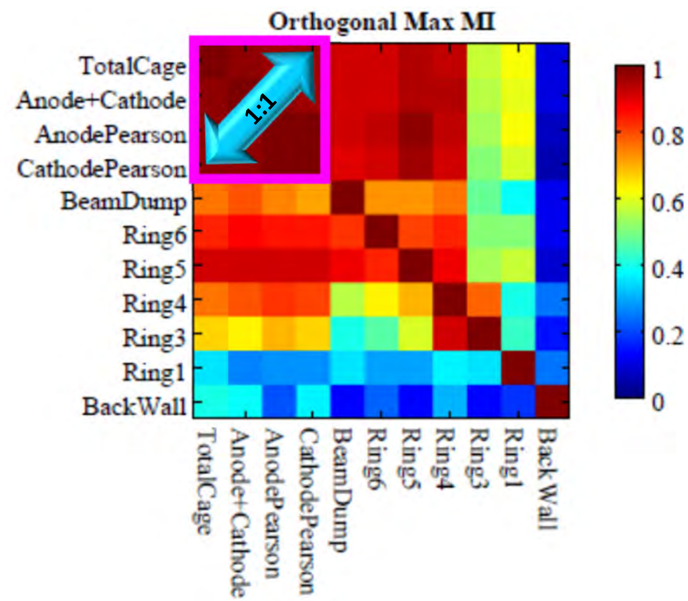



Majority of 5D Thruster Signal Energy Encoded in Walls

# **Causal Relationships:** Discovered in EP-TEMPEST Data




Bidirectional 1:1 Maps Imply
Thruster Observables Governed by <u>same</u> 5D State.

# Dynamical State Based Data Compressor

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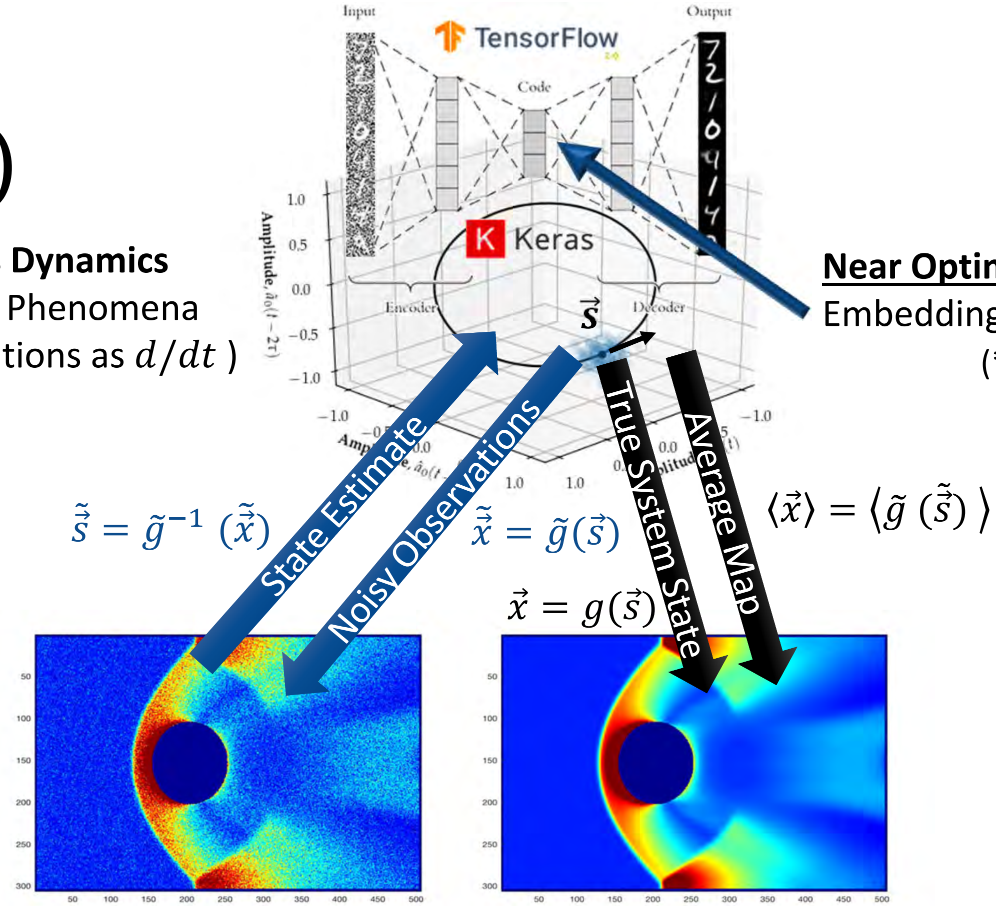
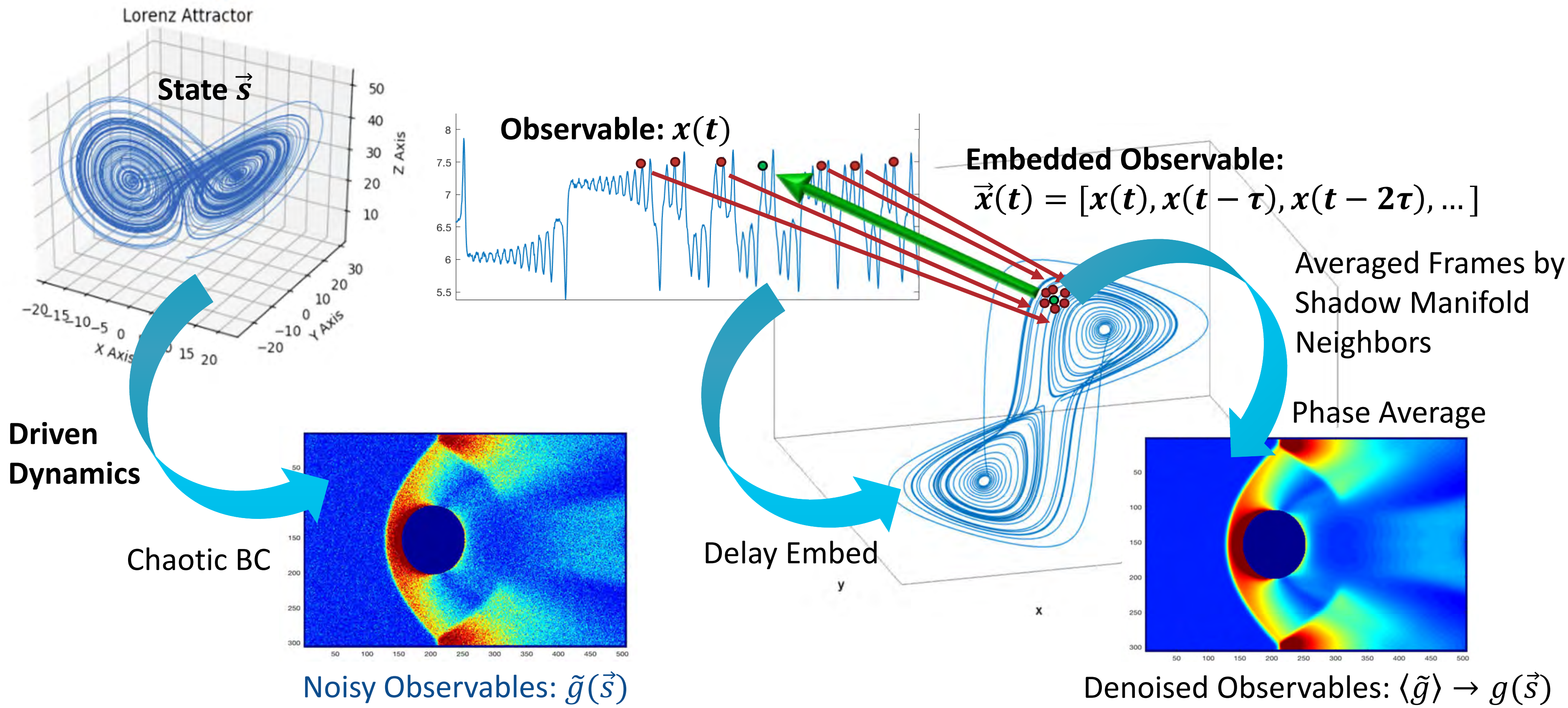
**Note: Assumes Autonomous Dynamics**
No Absolute Time in Physical Phenomena
(i.e. Time Only Enters Equations as $d/dt$ )

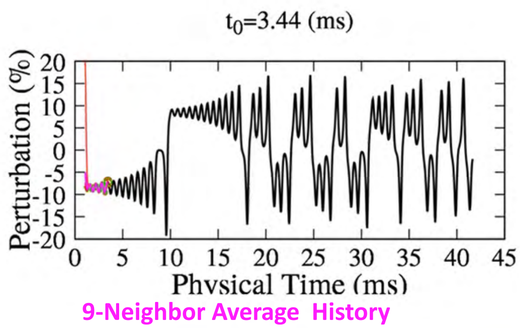


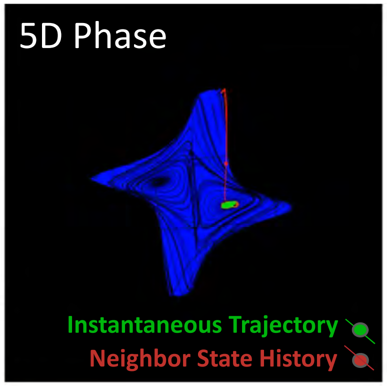

**Near Optimal* State Compression**
Embedding is Constant → 2x State Dim
(*By Construction… No Training!)

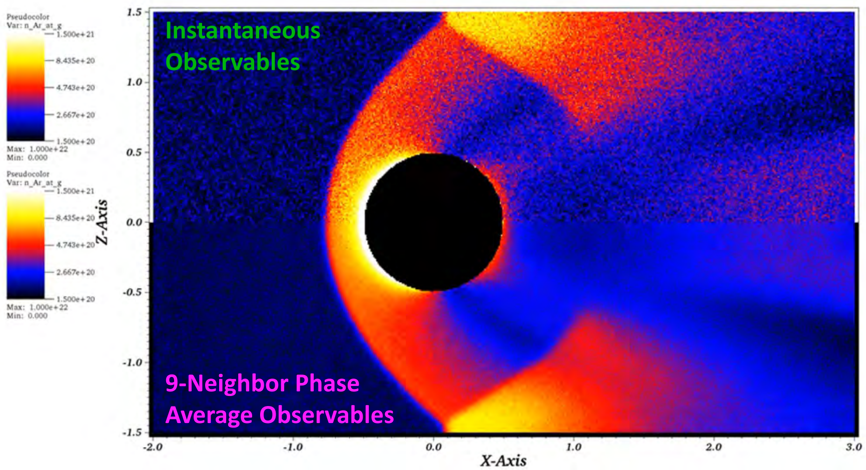
# Extending Phase Average to Multi-Dimensional State



# Extending Phase Average to Multi-Dimensional State

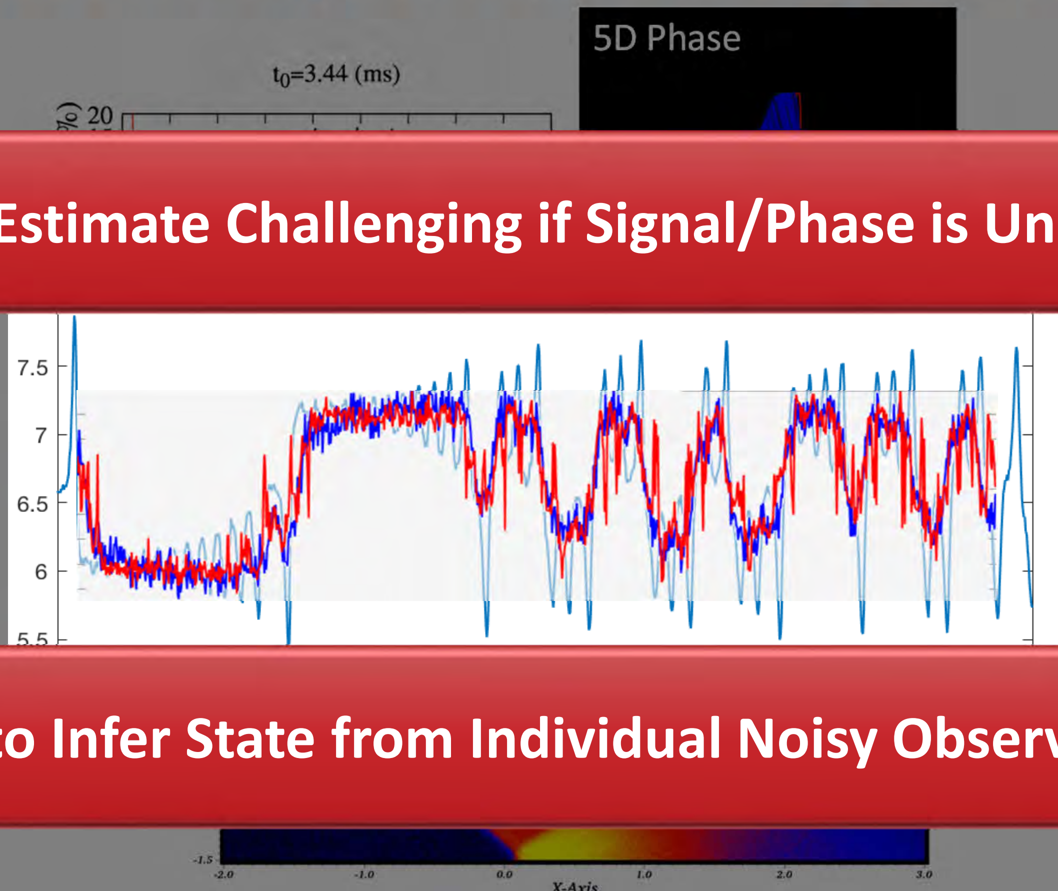


# Extending Phase Average to Multi-Dimensional State

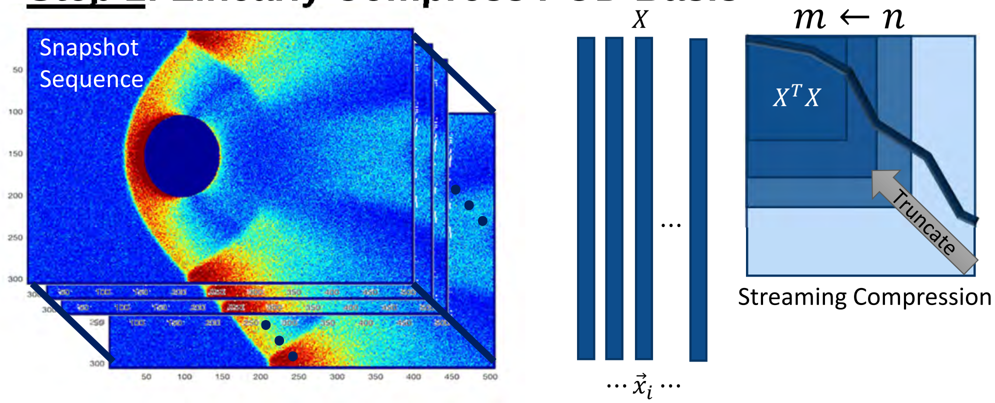
**State Estimate Challenging if Signal/Phase is Unknown**



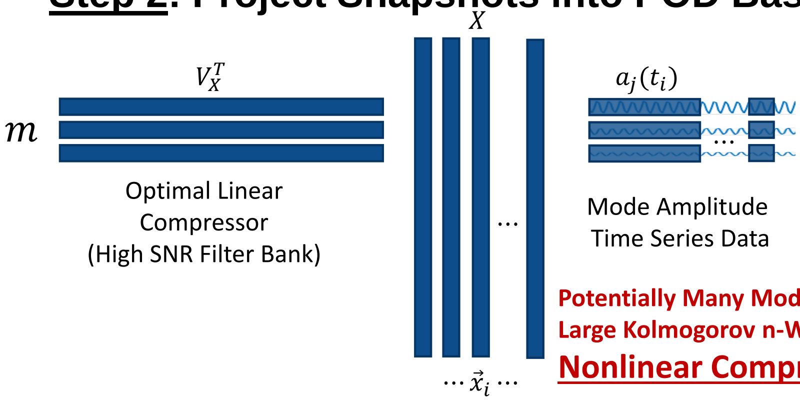

**Hard to Infer State from Individual Noisy Observations**

# Dynamics Based Nonlinear Compressor

## Step 1: Linearly Compress POD Basis



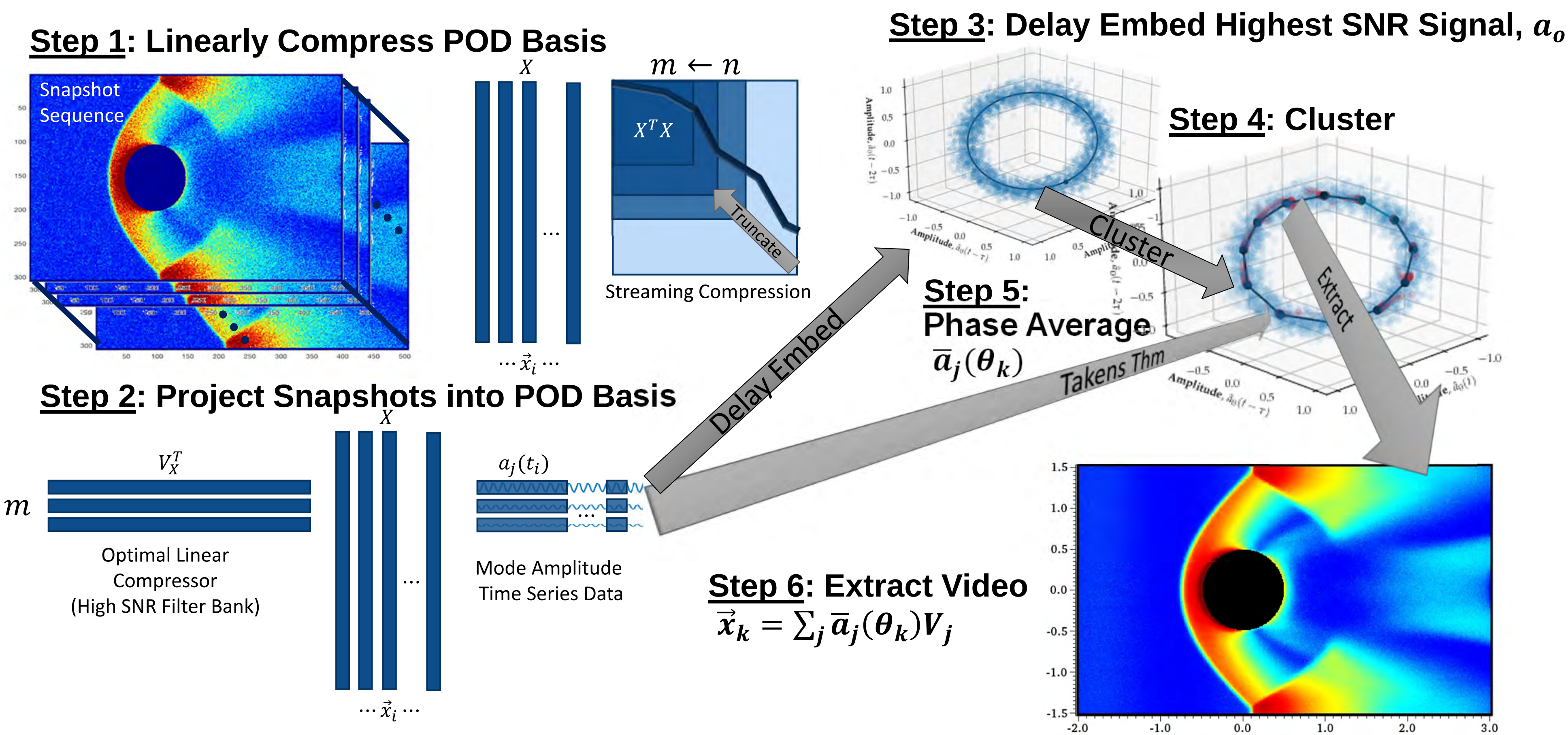

## Step 2: Project Snapshots into POD Basis



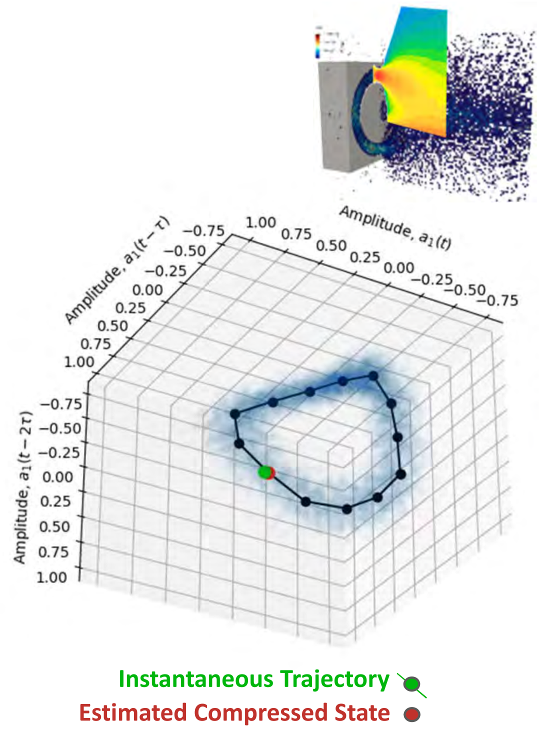

**Potentially Many Modes for**
**Large Kolmogorov n-Width Problems**

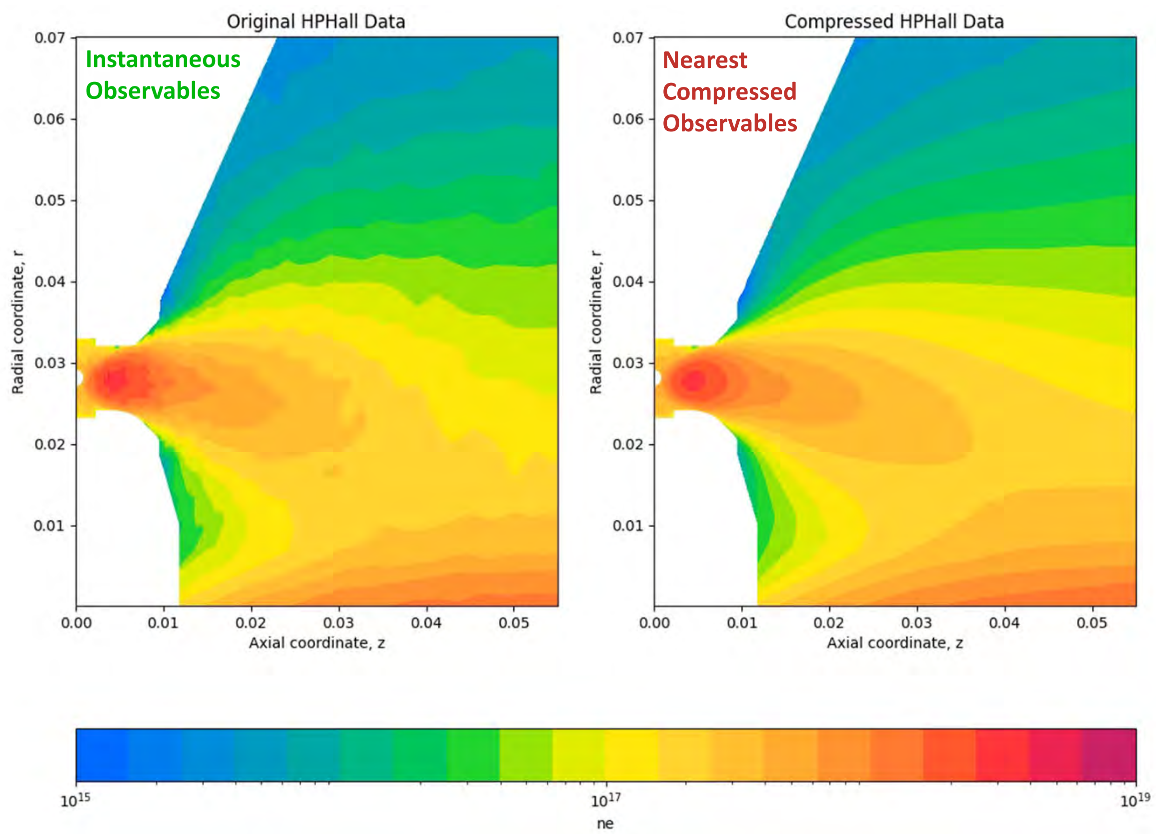
**Nonlinear Compressor Required**

# Dynamics Based Nonlinear Compressor



# Synchronizing Compressed Model with Emergent Dynamics

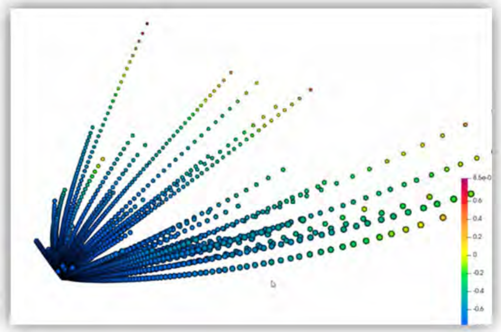


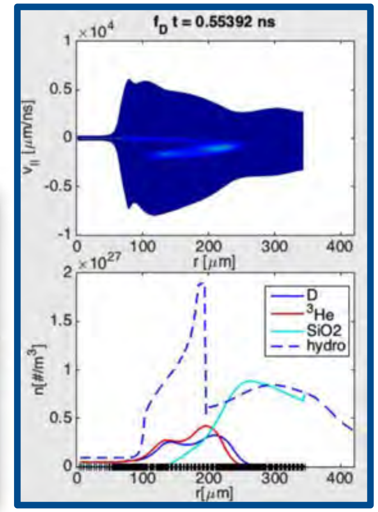



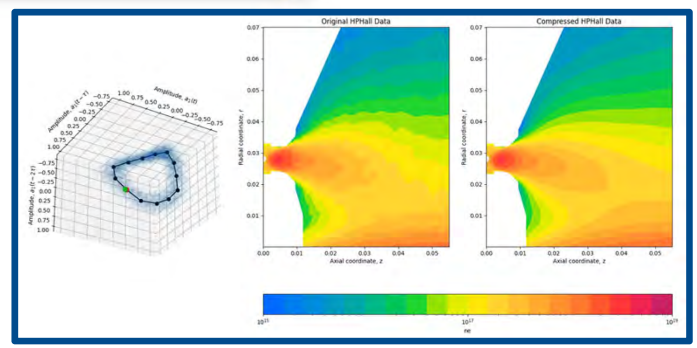
# Conclusion

# Summary

- Adaptive Kinetic Simulations
  - R-adaptive (moving) phase-space grid
  - Fully nonlinear implicit time integration via HOLO nonlinear accelerator
  - $10^{12}$ savings enables practical ICF implosion simulation to aid experimental design
- Accelerating Homogenous Relaxation
  - NN and ROMs can approximate Boltzmann collision operator.
  - More work to be done to improve accuracy, develop error indicators, enforce smoothness.
- Spatiotemporal Compression
  - Insufficient Dissipation → Emergent Dynamics
  - Evidence for Low-D Thruster State
  - Low-D Enables Nonlinear State Compression

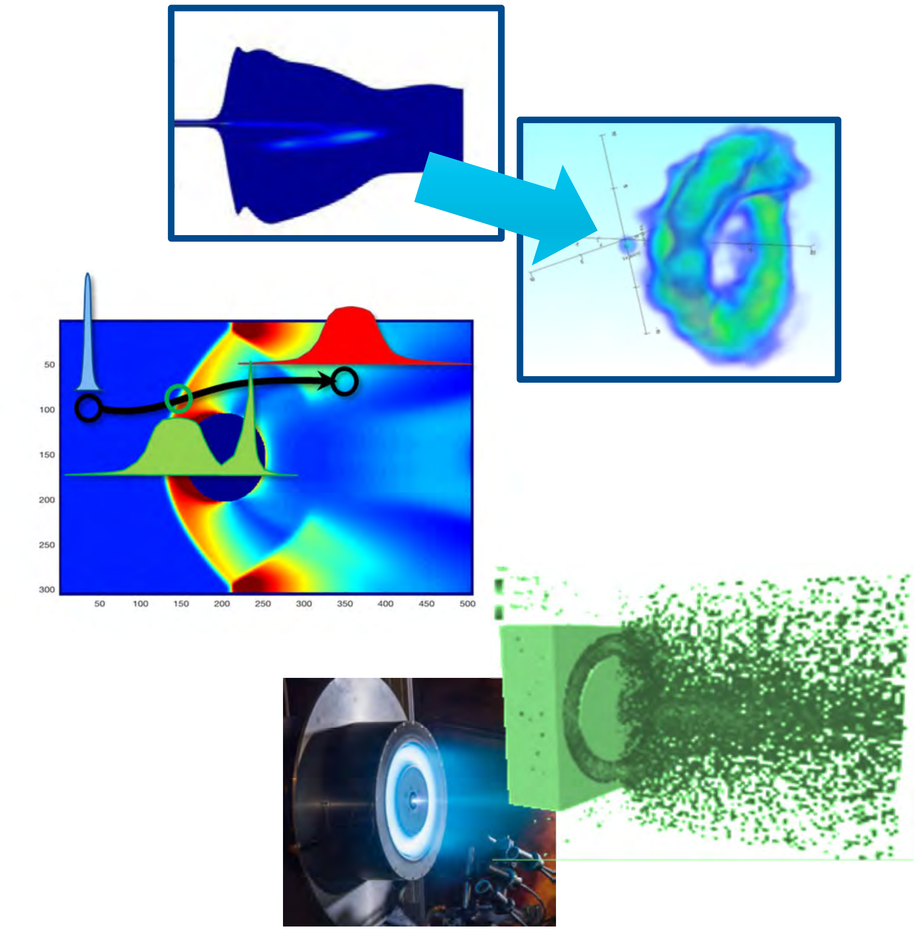




# Next Steps

- Adaptive Kinetic Simulations
  - Extend capability to 3D3V
  - Further address Curse of Dimensionality via low rank tensor approximation of VFP operator (Future Collaboration w/ J. Qiu – U. Delaware)
  - Translate from ICF to Thruster Conditions
- Accelerated Boltzmann-ROM Relaxation
  - Test NN and ROM approaches spatially dependent flows in 1D and 2D.
  - Determine if NN and ROM methods are efficient enough to allow modeling non-equilibrium hypersonic flight.
- Spatiotemporal Compression
  - Synchronize Experiment & Model via Compressed State → Dynamical Digital Twin
  - Integrate and Test Predictions of Quantities Sensitive to Dynamic Kinetic Distributions



# Acknowledgements


**Thank You Collaborators:**

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AFRL Dynamical Analysis Workgroup, AFRL/IPAM RIPS Teams

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AFRL

# Questions?

THE AIR FORCE RESEARCH LABORATORY

AFRL

# Backup Slides

# Abstract

This presentation overviews recent progress and future directions at the Air Force Research Laboratory's In-Space Propulsion Branch in developing accelerated, compressed models of rarefied fluid and plasma flows. The talk first describes recent work by Taitano while working at Los Alamos National Laboratory for development of adaptive moving mesh discretization of the Vlasov-Fokker-Planck Equation to be adapted to thruster applications. The talk then describes progress in accelerating solution of the Boltzmann collisional integral through the construction of stabilized Galerkin reduced order models (ROMs) and Neural Network approximations of the collision operator. Finally, streaming nonlinear state-based spatiotemporal variance reduction and the further extension of these concepts towards the construction of transient data-driven compression of kinetic flow information extracted from noisy particle simulations is described.

# The equation is high-dimensional, integro-differential and heavily constrained (conservation and positivity)

$$\partial_t f_\alpha + \vec{v} \cdot \nabla_x f_\alpha + \vec{a}_\alpha \cdot \nabla_v f_\alpha = \underbrace{\nabla_v \cdot \left[ \overline{\overline{D}}_\beta \cdot \nabla_v f_\alpha - \frac{m_\alpha}{m_\beta} \vec{A}_\beta f_\alpha \right]}_{C_{\alpha\beta}}$$

$$\overline{\overline{D}}_\beta = \nabla_v \nabla_v G_\beta \qquad \vec{A}_\beta = \nabla_v H_\beta$$

$$\nabla_v^2 G_\beta = H_\beta \qquad \nabla_v^2 H_\beta = -8\pi f_\beta$$

integral nature

$$\left\langle \begin{bmatrix} 1 \\ \vec{v} \\ v^2 \end{bmatrix}, C_{\alpha\beta} + C_{\beta\alpha} \right\rangle_v = \vec{0} \qquad f_\alpha \geq 0 \;\forall\; \{\vec{r}, \vec{v}, t\}$$

constraints

+ Maxwell's equations…

High dimensionality (3D+3V), exceedingly multiscale

# Nonlinearly implicit solvers can stably and accurately step over stiff time-scales

$$\Delta \mathbf{t}_{exp} \propto \tau_{col,pusher} \Delta \mathbf{v}^2 \sim 10^{-12}$$ **[ns] (but dynamically irrelevant)**

$$\tau_{dynamical} \sim 10^{-3}$$ **[ns]**

$$t_{sim} = 1 \sim 10$$ **[ns]**

$N_t >= 10^{14}$-$10^{15}$ with explicit time integration

$N_t \sim 10^3$-$10^4$ with fully implicit solvers

# we exploit the relationship between kinetic and moment equations to develop an efficient nonlinear convergence accelerator

Kinetic system (3D3V)

Moment system (3D)

$$\mathbb{B}_\alpha = \partial_t f_\alpha + \vec{v}\cdot\nabla_x f_\alpha - \sum_{\beta}^{N_s} C\left(f_\beta, f_\alpha\right) = 0$$

consistent representation

$$\langle m, \mathbb{B}_\alpha\rangle_{\vec{v}} = \partial_t \rho_\alpha + \nabla_x \cdot \left(\rho \vec{u}_\alpha\right) = 0$$

$$\langle m\vec{v}, \mathbb{B}_\alpha\rangle_{\vec{v}} = \partial_t \rho \vec{u}_\alpha + \nabla_x \cdot \left(\overline{\overline{S}}_{2,\alpha}\right) - \sum_{\beta}^{N_s} F_{\alpha\beta} = 0$$

$$\left\langle m\frac{v^2}{2}, \mathbb{B}\right\rangle_{\vec{v}} = \partial_t \rho U_\alpha + \nabla_x \cdot \left(\vec{S}_{3,\alpha}\right) - \sum_{\beta}^{N_s} W_{\alpha\beta} = 0$$

$$\vdots$$

$$\langle m v^l, \mathbb{B}_\alpha\rangle_{\vec{v}} = \partial_t \mathcal{M}_\alpha^l + \nabla_x \cdot \vec{S}_l - \sum_{\beta}^{N_s} \mathbb{W}_{l,\alpha\beta} = 0$$

closure

consistent physics

for collisionally dominated system, stiff physics are picked up by first three moments (Navier Stokes)

# Iterate HO and LO system to convergence

$$\rho^{LO}, \rho u^{LO}, \rho e_t^{LO}$$

**LO System (Nonlinear system)**

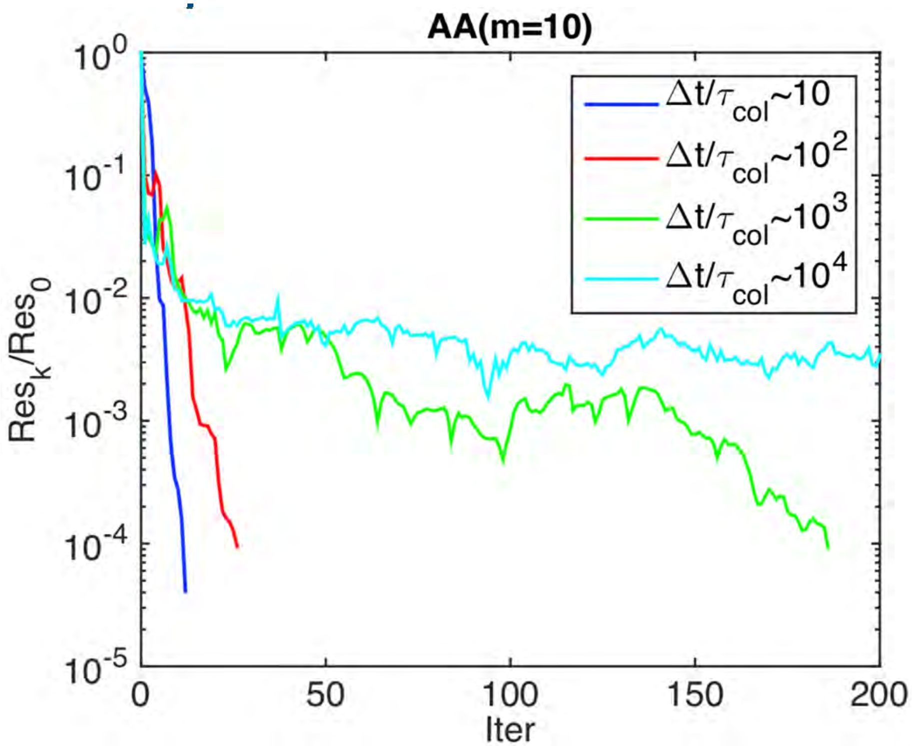
$$\begin{bmatrix} \partial_t \rho_\alpha^{LO} + \nabla_x\cdot\left(\rho\vec{u}_\alpha^{LO}\right) - \gamma_{\rho,\alpha}^{HO} \\ \partial_t\left(\rho\vec{u}_\alpha^{LO}\right) + \nabla_x\cdot \overline{\overline{S}}_{2,\alpha}^{LO} - \sum_\beta^{N_s}\vec{\mathcal{F}}_{\alpha\beta}^{LO} - \vec{\gamma}_{\rho u,\alpha}^{HO} \\ \partial_t\rho e_{t,\alpha}^{LO} + \nabla_x\cdot\vec{S}_{3,\alpha}^{LO} - \sum_\beta^{N_s} Q_{\alpha\beta}^{LO} - \gamma_{\rho e_t,\alpha}^{HO}\end{bmatrix} = \vec{0}$$

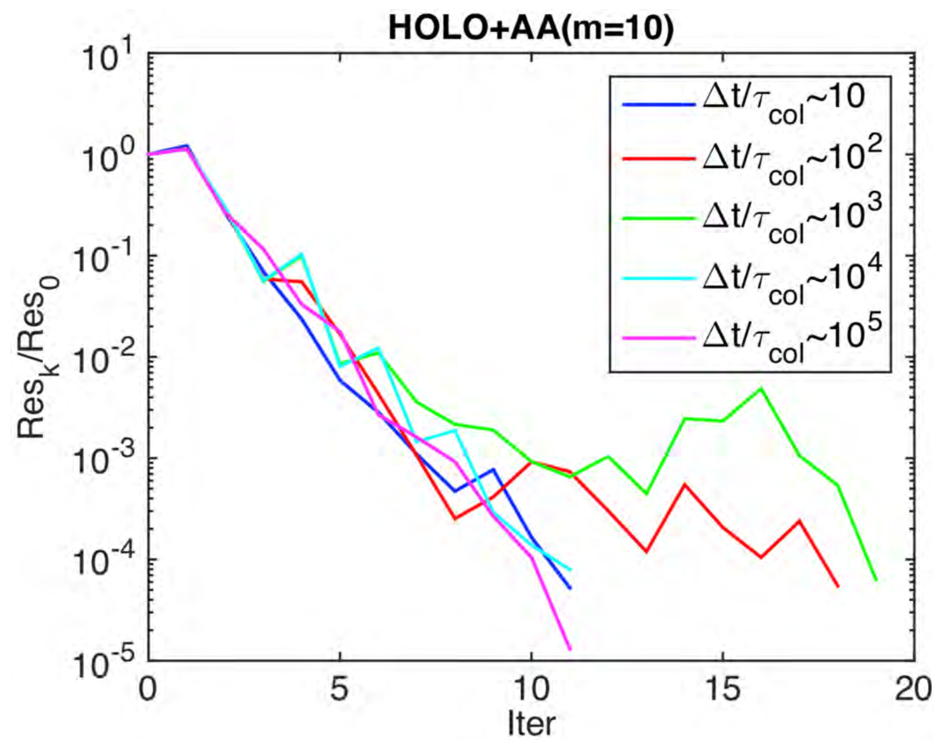
**HO System (Picard linearized system)**

$$\partial_t f_\alpha + \nabla_x\cdot(\vec{v} f_\alpha) - \sum_\beta^{N_s} C\left(f_{M,\beta}^{LO}+\delta f_\beta, f_\alpha\right) = 0$$

$$\gamma_{\mathcal{M},\alpha}^{HO} = \left\langle m_\alpha v^{\mathcal{M}}, R_{VFP,\alpha}\right\rangle_{\vec{v}} - \mathcal{R}_{\mathcal{M},\alpha}^{HO}$$

# HOLO truly enables integrated problems by being able to efficiently step over dynamically irrelevant stiff collision time



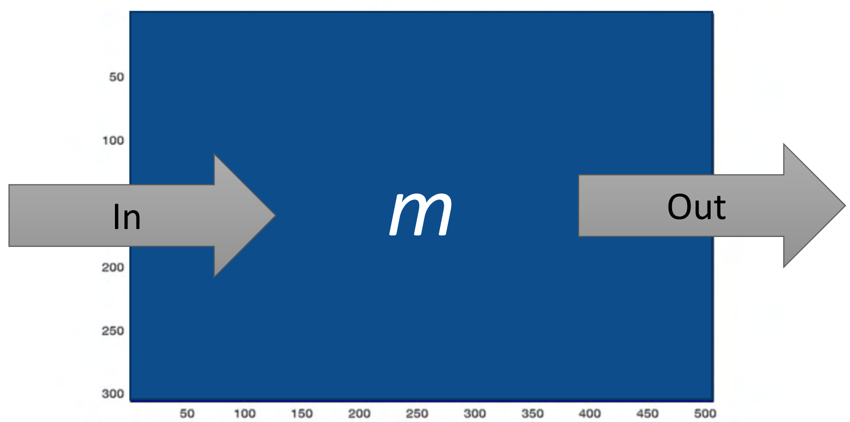

AA (m=10): Anderson acceleration with m=10 nonlinear histories on a Quasi-Newton (sparse Jacobian representation) fixed point iteration scheme

# The combined algorithm provides us with over $10^{12}$ reduction in computational complexity

$$\frac{N_{v,static}N_{x,static}}{N_{v,adapt}N_{x,adapt}} = \left( \underbrace{\sqrt{\frac{v_{th,max}}{v_{th,min}}}}_{\sim 300} \times \underbrace{\sqrt{\frac{m_{SiO2}}{m_D}}}_{\sim 3} \right)^2 \times \underbrace{\frac{\Delta x_{max}}{\Delta x_{min}}}_{\sim 100} \sim 10^7$$

$$\frac{\langle \Delta t \rangle_{HOLO}}{100 \times \Delta t_{exp}} \sim 10^5,$$

# How Much Information is in a State?




***Definition:*** *The state of a dynamic system is the smallest set of variables (called state variables) such that knowledge of these variables at $t = t_0$ together with input for $t \geq t_0$, completely determine the behavior of the system for any time $t \geq t_0$.*

-Modern Control Engineering (Ogata)

**Model Degrees of Freedom**
**≠**
**Dimension of State!**

Approx Models

**Nature:** $O(10^{23})$-p ticles x 6 (3xPostion,3xVelocity)?

**DSMC:** $O(10^5)$-pa xPostion,3xVelocity)?

**Fluid:** $O(10^4)$-cells x s,3xMomentum,Energy)?

⋮

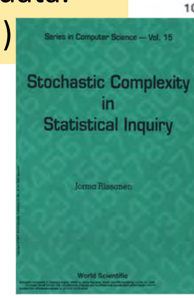
**Lump Mass (0D):** 1 (Total Mass in Control Volume)

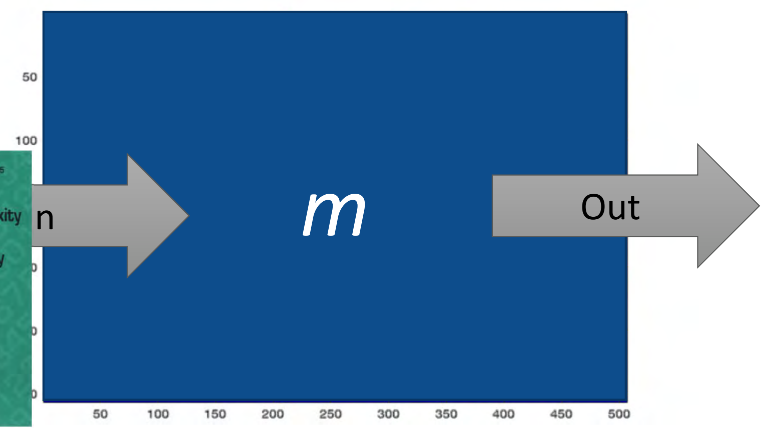
# How Much Information is in a State?

***Stochastic Complexity:*** (Minimum Data Description Size):

1. The code length for the noninformative 'noise' that remains when the regular features captured by the optimal model are taken away
2. The code length for the optimal model as seen through the data.

-Stochastic Complexity in Statistical Inquiry (Rissanen)

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**$\neq$**
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⋮

**Lump Mass (0D):** (Total Mass in Control Volume)

# Finding "All" the System Information

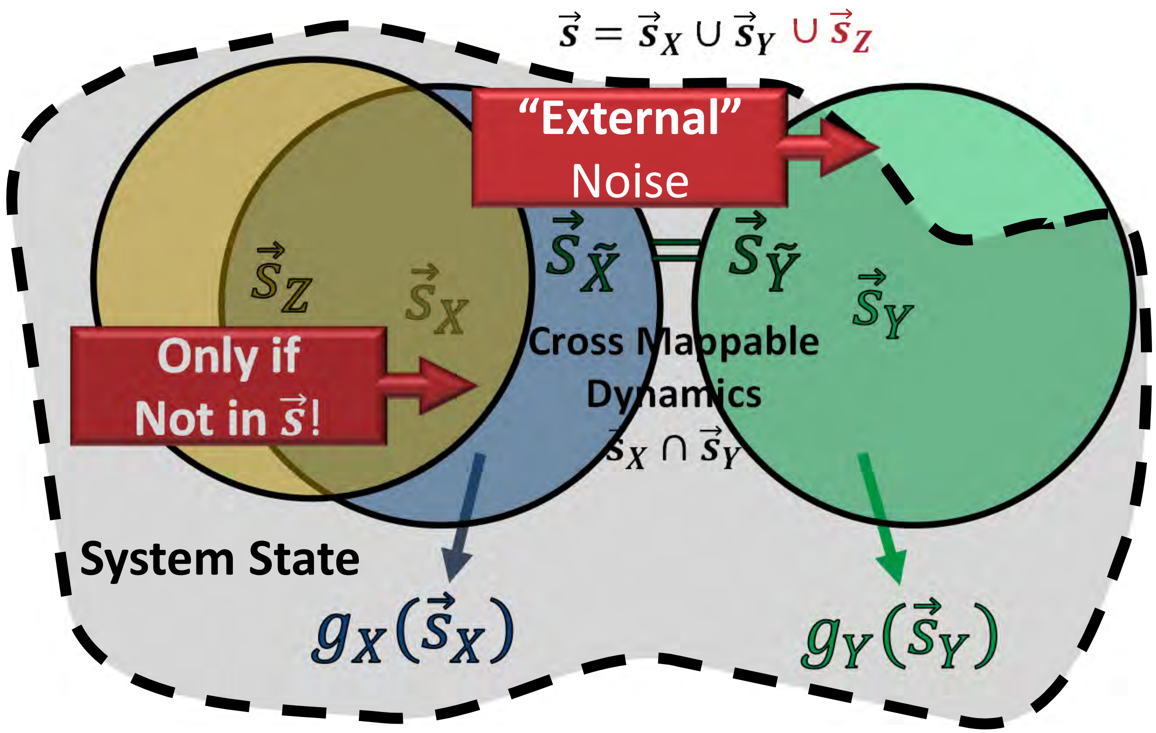
**Why Correlations below 100%?**

$$\tilde{X} = g_X\left(g_Y^{-1}(Y)\right)$$

**What does this mean?**

$$SNR = \frac{\|\vec{s}_{\tilde{X}}\|}{\|\vec{s}_X\|}\ ?$$

**Dynamical System:** $\dot{\vec{s}} = f(\vec{s})$



# From Trajectories back to Probability Flows

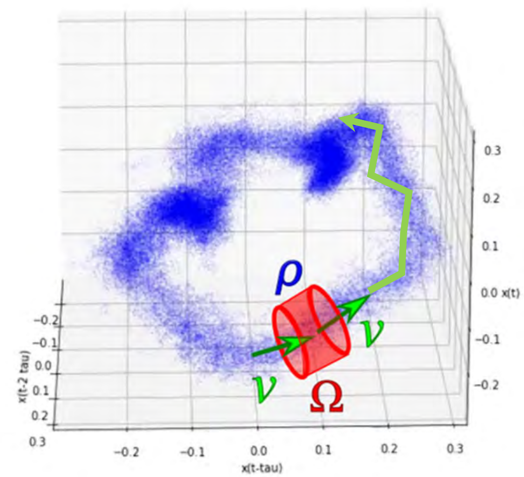
Embedded Flow ≈ State Space Flow

Flow Stays on Attractor: $\phi: M \to M$

Trajectories Entering a Volume Leave
- No Sources or Sinks for State-Space
- Control Volumes in Shadow State-Space

Lagrangian ODE → Eulerian PDE… **Why?**

**Eularian Model Linear in $\boldsymbol{\rho}$: $\boldsymbol{\rho}^{k+1} = \mathbf{A}\boldsymbol{\rho}^{k}$**
- $\mathbf{A}$ Discretized 0th-Order Finite Volume
- Upwind Flux: $\mathbf{A}$ Stochastic Transition Matrix
- Unique Steady State $\rho^{Eq}$: $(\mathbf{A} - \mathbf{I})\boldsymbol{\rho}^{Eq} = \mathbf{0}$ (i.e. 0-Eigenmode from Frobenius-Perron)




**Shadow Trajectory:** $\dot{\vec{s}} = f(\vec{s})$

Lagrangian→Eularian

**Continuity Eqn:** ~~$\frac{\partial \rho}{\partial t}$~~ $+ \nabla \cdot (\rho\vec{v})$ **= 0**

Plus Noise: $+\epsilon$

**Fokker-Planck:** ~~$\frac{\partial \rho}{\partial t}$~~ $+ \nabla \cdot (\rho\vec{v}) + \nabla \cdot (\bar{\bar{D}}\nabla\rho)$ **= 0**