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THE REFLECTION OF A SOUND PULSE NORMALLY INCIDENT IN A LIQUID MEDIUM UPON A PLANE SOLID PLATE

NAVAL RESEARCH LABORATORY

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Report On

The Reflection of a Sound Pulse Normally Incident in a Liquid Medium Upon a Plane Solid Plate

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TABLE OF CONTENTS

PLATES

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ABSTRACT

The reflection of plane wave sound pulses normally incident in a liquid medium upon a plane solid non-dissipative plate backed by a medium of negligible elasticity is theoretically analyzed. A general solution of the problem is obtained by resolving the incident pulse into its steady state components and recombining them after reflection for the reflected pulse. Certain special cases, such as the rectangular pulse and the damped sine pulse, are given special consideration.

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I. INTRODUCTION

1. In this report the effect of reflection by a plane solid plate on the characteristics of a plane wave sound pulse is theoretically investigated. The discussion is restricted to a normally incident pulse and to a non-dissipative plate backed by a medium of negligible elasticity. It is believed that the analytic solution of the problem presented in this report possesses some advantages over that of Muscat* in respect to physical interpretability. In any case, this solution warrants presentation as a preliminary to the discussion of the specific pulses considered herein.

2. No assumptions are made concerning the behavior of purely transient pressure waves as such. A rectangular pulse is resolved into a Fourier integral of continuous waves, the widely used reflection laws for continuous waves are applied, and the reflected spectrum of continuous waves is recomposed into the reflected pulse. By an examination of the details of the method and of the results, a generalization applicable to any pulse shape is deduced.

3. In the following discussion there is postulated a medium of liquid to the left of $x = 0$, bounded by a solid plate with plane parallel sides at $x = 0$ and $x = L$ respectively. The medium which extends to the right of $x = L$ is assumed to have a characteristic acoustic impedance which is negligible compared with that of the solid plate. This set-up is illustrated in Plate 1, Fig. 1. There is also illustrated a rectangular pressure pulse travelling through the liquid towards the discontinuities in medium at $x = 0$ and $x = L$. The reflected pulse for this type of incident wave is first obtained.

II. REFLECTION COEFFICIENT FOR CONTINUOUS WAVES

4. If the plate is considered to be a smooth dissipationless transmission line of length L terminated at the right by zero impedance, the input impedance, at the left, is

$$z' = i z_1 \tan nL/C_1 \qquad (1)$$

in which z_1 $(= \rho_1 c_1)$ is the characteristic impedance of the solid medium, n is 2π times frequency, and C_1 is the plane wave velocity in the solid medium.

5. Next, if the liquid is considered to be a smooth transmission line of characteristic impedance z_0 $(= \rho_0 c_0)$ terminated

*Morris Muscat "The Reflection of Plane Wave Pulses from Plane Parallel Plates," J.A.P. 9(275), April 1938.

[redacted]

at the right by an impedance z', the pressure reflection coef-
ficient γ is given by

$$\gamma = \frac{z' - z_0}{z' + z_0} = -\frac{1 - i\,K \tan nL/C_1}{1 + i\,K \tan nL/C_1} \qquad (2)$$

in which

$$K = z_1/z_0 \qquad (3)$$

γ has a magnitude of unity indicating total reflection of a continuous wave. It may be rewritten

$$\gamma = e^{i\phi} \qquad (4)$$

in which

$$\phi = -\pi - 2 \tan^{-1} (K \tan nL/C_1) \qquad (5)$$

ϕ is the phase shift at reflection.

III. RECTANGULAR PULSE

6. Consider a rectangular pulse $f(t)$ defined as follows at $x = 0$:

$$f(t) = F_{-\epsilon}^{+\epsilon} \qquad (6)$$

This means that $f(t)$ is the constant F during the time interval between $t = -\epsilon$ and $t = +\epsilon$, and that $f(t) = 0$ for all t outside this interval. Under this symbolism, a retardation of the pulse by a time α would be indicated by

$$f(t-\alpha) = F_{\alpha-\epsilon}^{\alpha+\epsilon}$$

meaning that $f(t-\alpha)$ is the constant F during the time interval between $t = \alpha-\epsilon$ and $t = \alpha+\epsilon$ and that $f(t-\alpha) = 0$ for all t outside this interval. This pulse is plotted in Plate I, Fig. 2(a). Applying the Fourier Integral theorem to equation (6)

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} g(n) \exp(int)\, dn \qquad (7)$$

and

$$g(n) = \int_{-\infty}^{+\infty} f(t) \exp(-int)\, dt = \frac{2F \sin(n\epsilon)}{n}. \qquad (8)$$

[redacted]

Substituting (8) in (7)

$$f(t) = \frac{F}{\pi}\int_{-\infty}^{+\infty} \frac{\sin(n\varepsilon)}{n} \quad \exp\,(int)\; dn \tag{9}$$

$$= \frac{2F}{\pi}\int_{0}^{+\infty} \frac{\sin(n\varepsilon)}{n} \quad \cos\,(nt)\; dn$$

The quantity $\frac{2F}{\pi} \cdot \frac{\sin(n\varepsilon)}{n}$, plotted in Plate 1, Fig. 2(b), shows the dependence of the amplitude on frequency in the spectrum of the pulse.

7. If each frequency present in the integrand of f(t) has the phase shift ϕ of equation (5) introduced into it on reflection, the function f(t) is transformed to R(t), the reflected wave. Thus

$$R(t) = -\frac{2F}{\pi}\int_{0}^{\infty} \frac{\sin(n\varepsilon)}{n} \quad \cos\left[nt - 2\tan^{-1}\,(K\tan nL/C_1\right] dn \tag{10}$$

8. With some trigonometric manipulation (10) may be expressed as

$$R(t) = \frac{F}{\pi}\int_{0}^{\infty}\left[\frac{\sin n\,(t-\varepsilon)}{n} - \frac{\sin n\,(t+\varepsilon)}{n}\right] \frac{1-K^2\tan^2(nL/C_1)\;dn}{1+K^2\tan^2(nL/C_1)}$$

$$-\frac{F}{\pi}\int_{0}^{\infty}\left[\frac{\cos n\,(t+\varepsilon)}{n} - \frac{\cos n\,(t-\varepsilon)}{n}\right] \frac{2\,K\tan\,(nL/C_1)\;dn}{1+K^2\tan^2(nL/C_1)}. \tag{10(a)}$$

After expanding the factors

$$\frac{1-K^2\tan^2(nL/C_1)}{1+K^2\tan^2(nL/C_1)}$$

and

$$\frac{2\,K\tan\,(nL/C_1)}{1+K^2\tan^2\,(nL/C_1)}$$

in Fourier series, the integration may be carried out. If the first of these factors has the form

$$-a_0 - a_1\cos nL/C_1 - a_2\cos 2\,nL/C_1 - \;.....,$$

the coefficients of the second factor are simply related to the a's, and the whole solution is expressible in terms of the a's.

[redacted]

$$R(t) = a_o \, \Gamma_{-\varepsilon}^{+\varepsilon} + \sum_{m=0}^{m=\infty} a_m \, \Gamma_{\frac{2mL}{C_1}-\varepsilon}^{\frac{2mL}{C_1}+\varepsilon} \qquad (11)$$

where

$$a_o = \frac{K-1}{K+1}\,, \text{ and } a_m = \frac{-4K}{K^2-1}\left(\frac{K-1}{K+1}\right)^m, \; m \neq 0 \qquad (12)$$

9. The reflections of rectangular pulses of various durations relative to the transit time through the reflecting plate and back are shown in Plates 2 and 3. The transit time is $\frac{2L}{C_1}$, the duration of the pulse is 2ε. In Plate 2, Fig. 2, for example, the transit time is twice the duration of the pulse and the various sections of R(t) are distinct. In Figures 4 through 7, the transit time is so short that the sections overlap. Plate 2 applies to a reflecting plate of characteristic impedance equal to 20 times the characteristic impedance of the liquid (i.e., K = 20). This is approximately representative of sea water in contact with a steel reflecting plate. For this case, since C_1 is of the order of 5×10^5 cm/sec, the plate thickness for a 100 microsecond pulse must be about 50 cm. in Figure 2, and about 0.8 cm. in Figure 6. This latter thickness is of the order of magnitude of the hull thickness of a submarine.

10. Figure 7 on Plate 2 shows a case where the energy density in the reflected wave greatly exceeds that in the incident wave during a short interval. (Of course the total incident energy equals the total reflected energy). This phenomenon is readily explainable. If the pulse is sufficiently long compared to the transit time through the plate, a condition approximating the steady state is reached in which the reflected wave is inverted but has no change in amplitude. In Fig. 7 this condition is approached as t approaches ε. In this state there are contained in the reflected wave a number of sections, all but one of which are inverted. The single uninverted pulse, which we have called section 1, subtracts from the others to produce less pressure amplitude than there would be without it. Therefore, if the incident pulse is suddenly terminated, the single uninverted section is simultaneously terminated and the reflected wave as a whole undergoes a sharp increase in amplitude. This effect is greatly reduced with incident pulses tailing off less steeply.

11. Plate 3 might well apply to sea water in contact with a plate of some plastic material such that K = 5. In a plastic the velocity of sound, C_1 is less than in steel, and the thickness of the plastic material for a particular figure of Plate 3 would therefore be less than the thickness of steel for the corresponding figure of Plate 2.

[redacted]

12. Referring to formula (11), it is seen that the incident wave undergoes partial reflection upon its arrival at $x = 0$. This portion of the reflected wave is represented by the first term of the right side of equation (11), namely

$a_0 F_{-\varepsilon}^{+\varepsilon}$ where $a_0 = \frac{K - 1}{K + 1}$ is just the reflection coefficient which would exist if the solid medium extended from $x = 0$ to $x = \infty$. Thus the effect of the interface at $x = L$ is not felt by this part of the reflected wave. The whole wave undergoes this partial reflection without change of phase for $K > 1$.

13. The transmitted portion of the wave has a pressure amplitude $F\sqrt{K(1 - \gamma^2)}$. At reflection at $X = L$, the sign is changed, and upon return to $X = 0$), a fraction

$$\sqrt{\frac{1 - \gamma^2}{K}} \quad \text{of this wave, or}$$

$$\sqrt{\frac{1 - \gamma^2}{K}} \cdot (-F) \sqrt{(1 - \gamma^2)K} = - F(1 - \gamma^2)$$

is transmitted through the interface and adds a second section to the reflected wave in the liquid medium. This is the first term of the summation, namely

$$a_1 F_{\frac{2L}{c_1} - \varepsilon}^{\frac{2L}{c_1} + \varepsilon}$$

where the time lag introduced by the transit time of the round trip through the plate and back is now indicated. Continuing with successive reflections within the plate and successive transmissions through the left face into the liquid medium, the various sections of $R(t)$ can be identified, term by term.

IV. PULSE OF ANY SHAPE

14. It has been shown that the rectangular pulse is divided into sections at reflection, each section being represented by one of the terms of (11), and that each section is a rectangular pulse. Since every section has the same shape as the incident pulse except for change in amplitude (with change of sign), it follows that the ratio of the amplitudes of the different Fourier components is the same for every section, and that the time lag introduced in each Fourier component of any one section is the same. This is equivalent to stating that the Fourier components of each section travel together at the same velocity, undergoing the same time lag and the same phase shift at reflection at any interface. Since the components of a section behave in this manner, the sections of a

[redacted]

pulse of any shape (not necessarily rectangular) must have the same shape as that of the incident pulse, and formula (11) without limiting f(t) to the form $F^{+\epsilon}_{-\epsilon}$ must still apply.

The generalized form is

$$R(t) = a_0 f(t) + \sum_{m=1}^{m=\infty} a_m f(t - 2mL/C_1)$$

where the a's are defined by (12) and the function $f(t - 2mL/C_1)$ is delayed in time by $2mL/C_1$ behind f(t).

15. What really has been shown is that the whole pulse obeys the same reflection and transmission laws as a continuous wave, except that each interface acts like an interface between media of infinite extent. To have assumed this at the start would have been less satisfying and would not have eliminated the work of computation of reflection coefficients for the various sections of the reflected pulse.

V. CAUSE OF DISTORTION AT REFLECTION

16. Equation (2) shows, by the fact that its numerator and denominator have the same magnitude, that there is no change in amplitude of the components of different frequencies at reflection. This is so because it has been assumed that there is no dissipation in the plate, and because no energy can be fed through the boundary at $X = L$ into zero impedance. Equation (2) shows also that a phase shift is introduced into these components, which is different for components of different angular velocities, n. One must therefore conclude that any distortion of the pulse is caused by phase shifts of the components, a point which warrants further discussion.

17. An insight into the production of distortion of the pulse by phase shifts of its components may be gained by the following considerations:-

a. The constant phase shift of $-\pi$ for all the components (see Equation 5) produces an inversion of the pulse, such as takes place in the limiting case of zero thickness of the reflecting plate. This statement may be verified by substituting $\cos(nt - \pi)$ for $\cos nt$ in equation (9) and observing that, since $\cos(nt - \pi)$ equals $-\cos nt$, the substitution merely introduces a change in sign.

b. Any linear variation of phase with frequency simply displaces the pulse in time without change in shape, since such a variation of phase transforms $\cos nt$ to $\cos(nt - n\theta)$ or $\cos n(t - \theta)$. The transformed expression is identical* at time $t = t_1 + \theta$ with the untransformed expression at a different time t ≠ t, for every n.

c. Since the last term of Equation 5 is not linear in n, change in shape of the reflected pulse is to be expected, and generally distortion other than inversion is great if the deviation of this term from linearity is great.

VI. DAMPED SINE PULSE

18. A pulse which may occur in practice is the damped sine pulse. Plates 4 and 5 apply to this case.

19. In Plate 4, Fig. 1 shows plots of damped sine waves, all of the form $e^{-\alpha nt}$ sin nt, for different values of α, the damping coefficient per <u>radian</u>. Figure 2 of Plate 4 shows in solid lines the amplitude distribution with frequency in the incident wave for different values of α. This distribution remains unchanged in the reflected wave.

20. The dotted curves of Fig. 2 of Plate 4 show the phase lags introduced into the different component frequencies at reflection for several special cases as indicated. These phase lags are the ϕ's obtained from Equation 5 with sign reversed. In view of the dependence of distortion on phase shift, as discussed in paragraph 17, it should be clear that any deviation of the dotted curve of phase lag versus frequency from a straight line through zero frequency and zero phase lag represents distortion in shape. (This distortion amounts to inversion of the wave for a straight line through zero frequency and π phase lag). Of the cases covered by these curves, more distortion would be expected for the case where $2L/C_1 = \pi/8n$ than for the case where $2L/C_1 = \pi/32n$ because the dotted curve for the former case deviates more from a straight line within the frequency range which contains the components of significant amplitude.

21. The observed vibration of a magnetostriction projector of stacked laminated rings when excited by a sharp pulse of current approximates the case where $\alpha = .1$, α being the damping coefficient per radian. If this pulse is reflected from a submarine for which $2L/C_1 = \pi/8n$ its form is distorted as illustrated in Plate 5. This distortion has the appearance of phase modulation. Also, the amplitude of the reflected wave builds up at first instead of decaying from the start like the incident wave.

VII. CONCLUSIONS

22. Under the given conditions, the Fourier components of a pulse are reflected with change of phase but without change in amplitude. There may be a considerable change in the shape of a

[redacted]

pulse at reflection with no loss of energy content.

23. The mathematical expression for the reflected wave is a series, in which each term may be interpreted as corresponding to a "section" of the whole reflected pulse. Successive sections have greater and greater time lags behind the incident pulse, and each section is a pulse of the same shape as the incident pulse except for inversion. This generalization follows from the treatment of the rectangular pulse. The simplicity of the treatment of the rectangular pulse justifies the method although the rectangular pulse is not physically realizable.

24. The successive sections of a reflected pulse may be distinct from each other, in which case a single incident pulse is reflected as a series of pulses, the first of which duplicates the incident pulse in shape, and all others of which have the same shape as the incident pulse but are inverted. The amplitudes of the successive sections after the first decrease progressively in amplitude. If successive sections overlap, a single incident pulse is reflected as a single pulse of modified shape.

25. The behavior of the individual sections of the reflected pulse contrasts with that of the whole reflected pulse in that, although the phase changes in the sections lead only to retardation in time and to inversion, the phase changes in the whole pulse lead to distortion in shape.

26. A phenomenon which may occur with an abruptly terminated incident pulse is the production, at the instant of its termination, of a peak pressure amplitude in the reflected pulse which substantially exceeds the maximum pressure amplitude in the incident pulse.

27. In an echo of the type now utilized in service echo-ranging equipment, the pulse is an undamped sinusoid of relatively long duration. Since the band spread of the spectrum of such a signal or pulse is very small, it appears unlikely that the distortion considered in this report would be observable.

28. The sinusoidal pulse with considerable damping, upon reflection from a submarine, should exhibit distortion and this might be detectable. Construction of a receiver to detect such distortion is at least conceivable.

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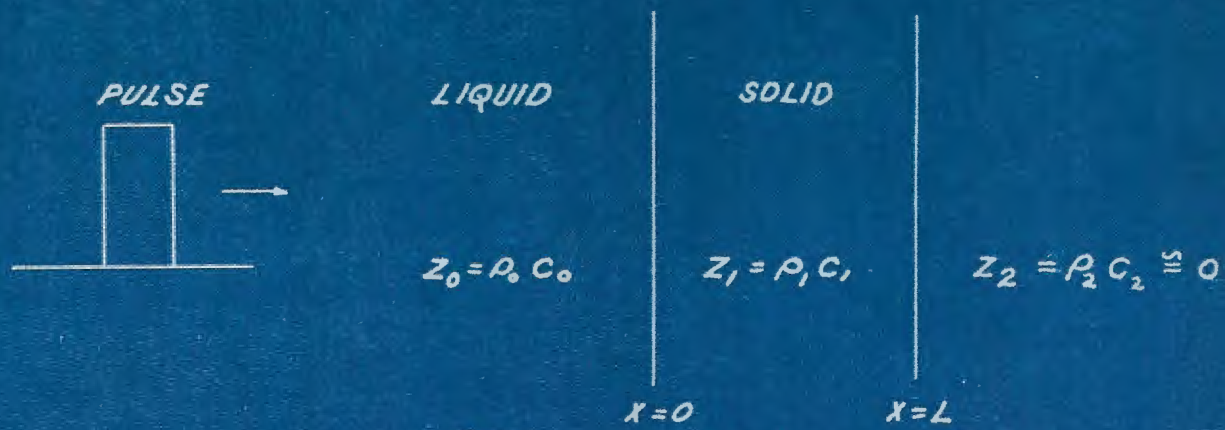


FIG. 1

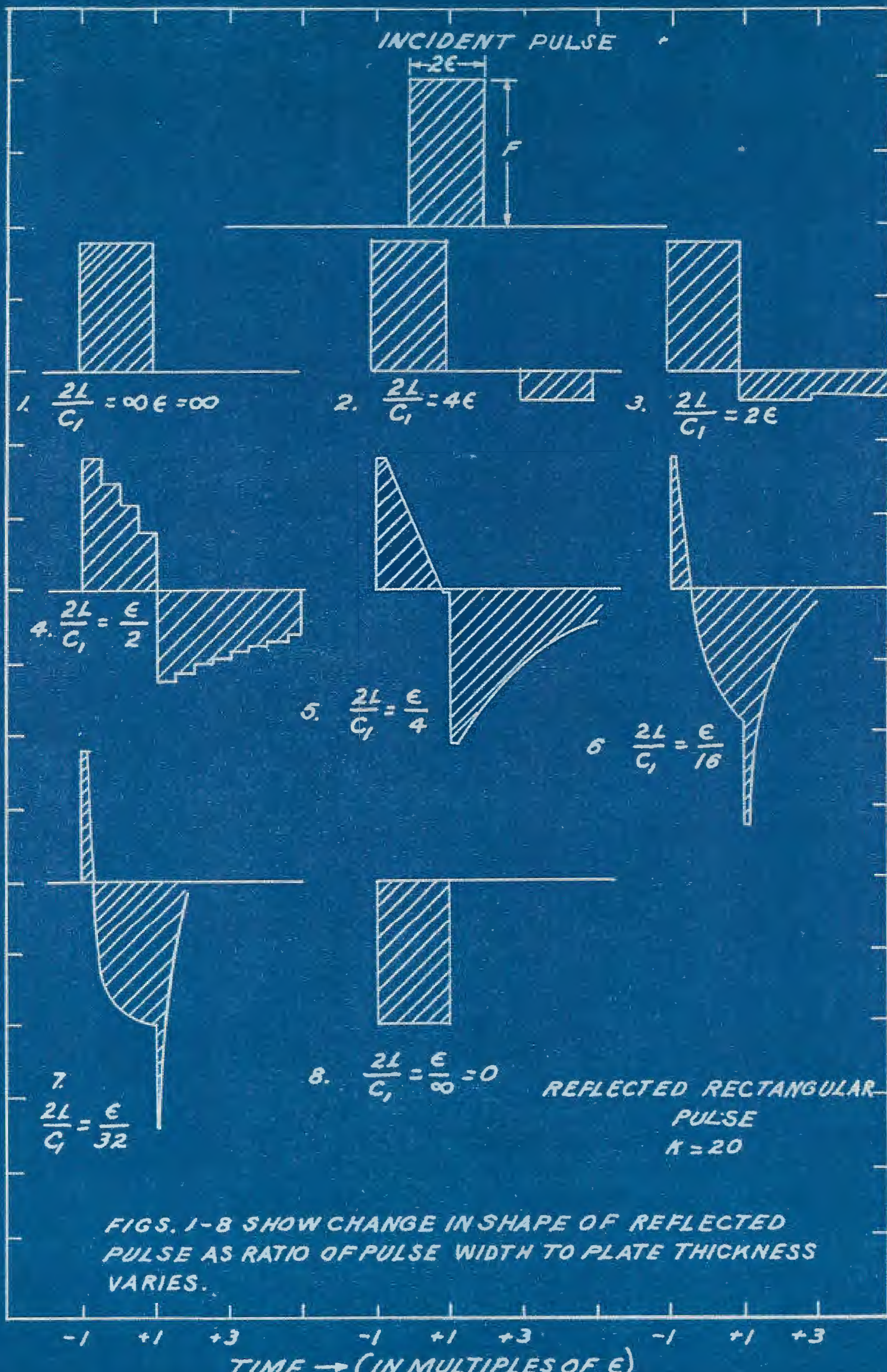
SYSTEM UNDER DISCUSSION

2(a)
INCIDENT PULSE

2(b)
FREQUENCY SPECTRUM
OF INCIDENT PULSE

FIG. 2

PLATE I



FIGS. 1-8 SHOW CHANGE IN SHAPE OF REFLECTED PULSE AS RATIO OF PULSE WIDTH TO PLATE THICKNESS VARIES.

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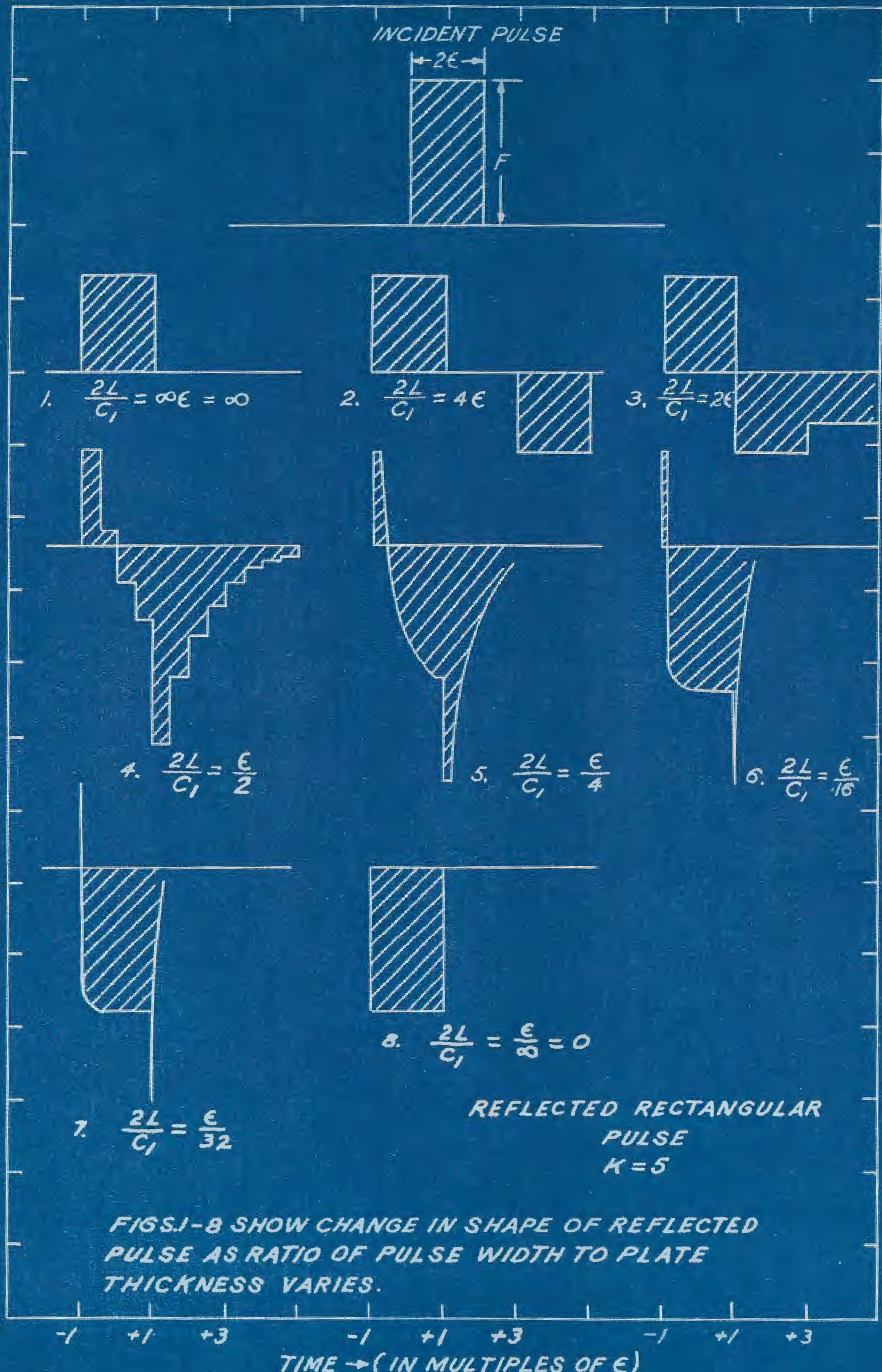
PLATE 2



FIGS.1-8 SHOW CHANGE IN SHAPE OF REFLECTED PULSE AS RATIO OF PULSE WIDTH TO PLATE THICKNESS VARIES.

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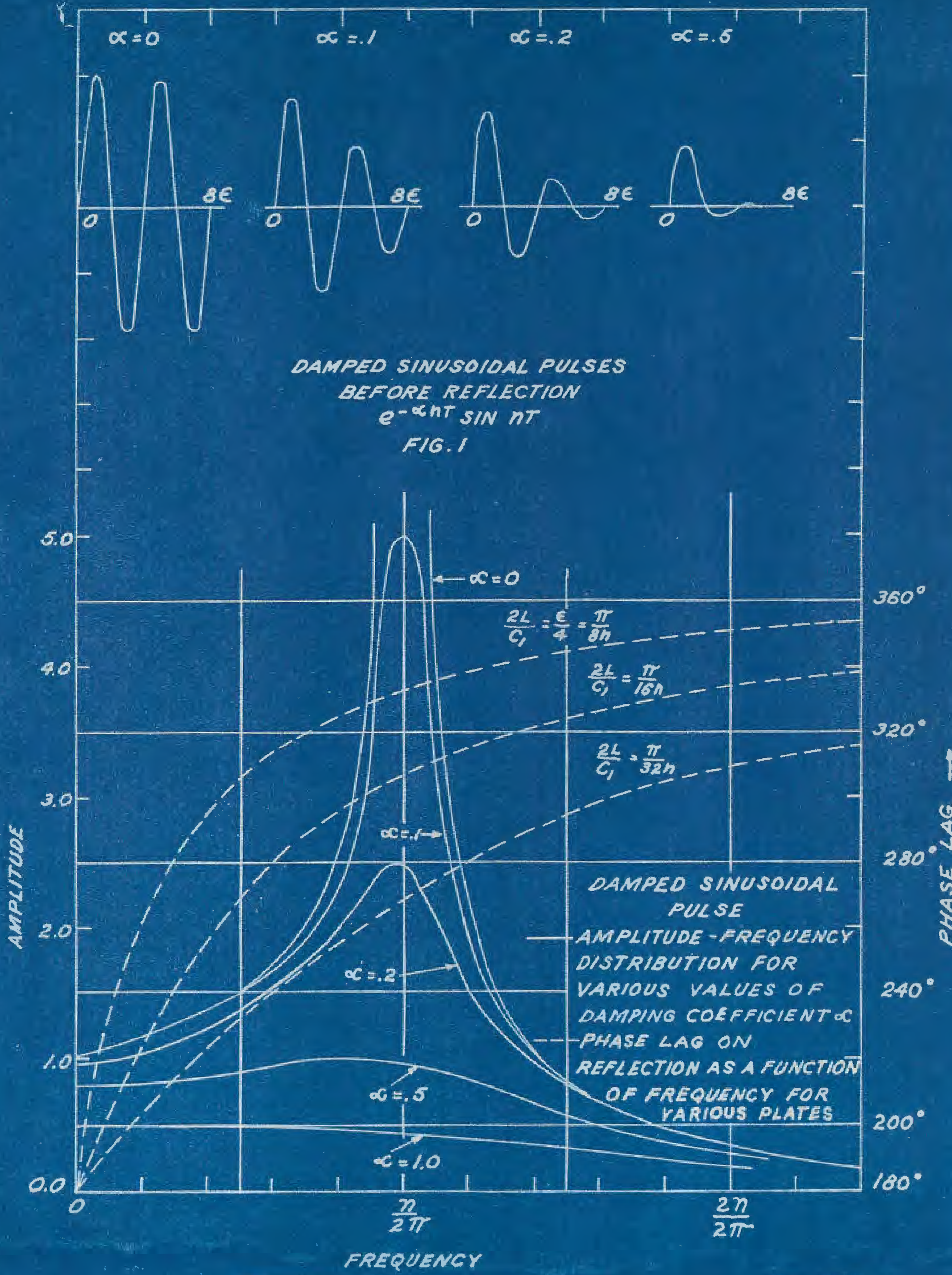
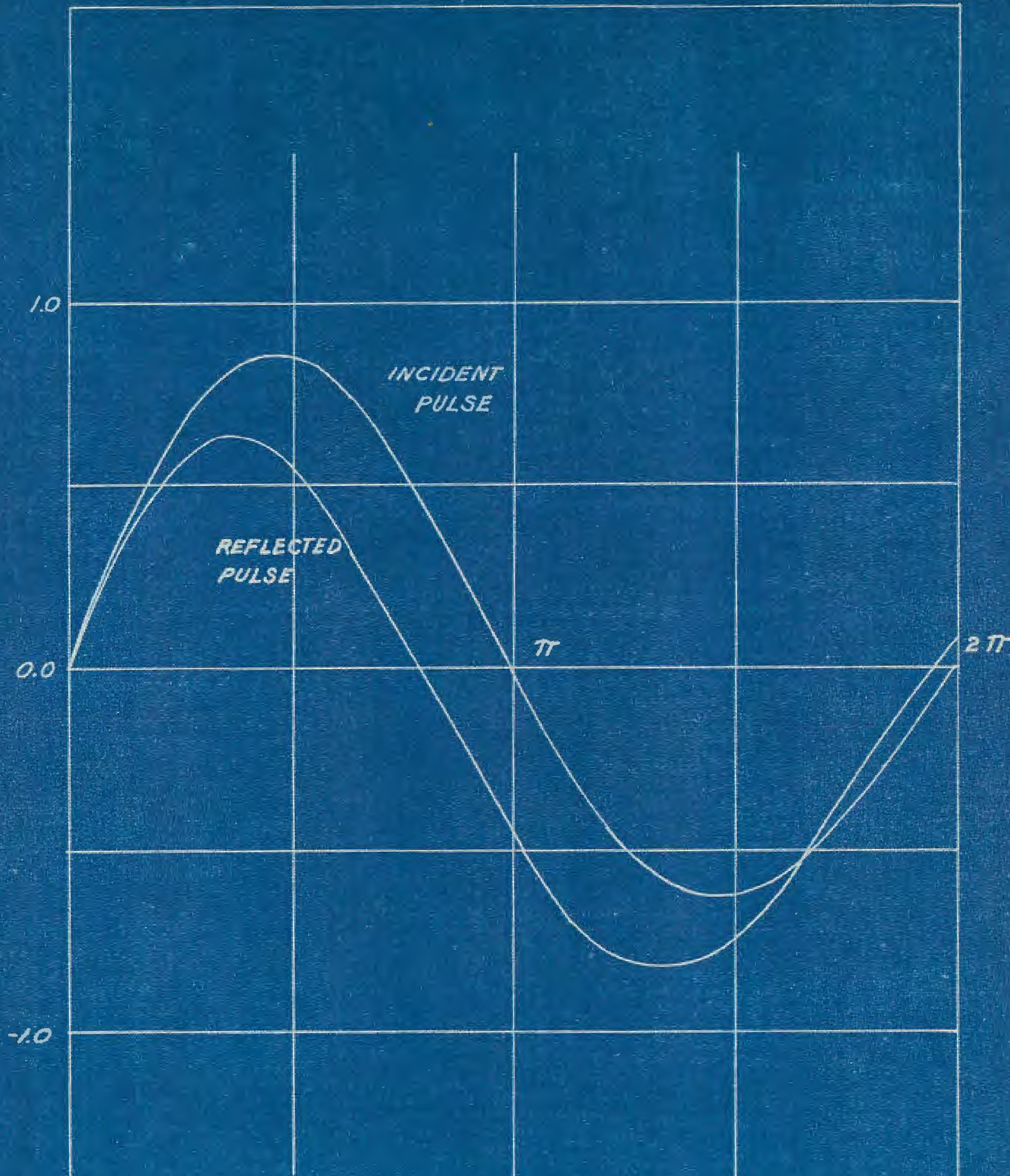
PLATE 3



α = 0
α = .1
α = .2
α = .5
8ε
0
DAMPED SINUSOIDAL PULSES
BEFORE REFLECTION
$e^{-\alpha n T} \sin nT$
FIG. 1
5.0
4.0
3.0
2.0
1.0
0.0
AMPLITUDE
360°
320°
280°
240°
200°
180°
PHASE LAG →
← α = 0
α = .1 →
α = .2
α = .5
α = 1.0
$\frac{2L}{C_1} = \frac{\epsilon}{4} = \frac{\pi}{8n}$
$\frac{2L}{C_1} = \frac{\pi}{16n}$
$\frac{2L}{C_1} = \frac{\pi}{32n}$
DAMPED SINUSOIDAL PULSE
AMPLITUDE - FREQUENCY DISTRIBUTION FOR VARIOUS VALUES OF DAMPING COEFFICIENT α
PHASE LAG ON REFLECTION AS A FUNCTION OF FREQUENCY FOR VARIOUS PLATES
0
$\frac{n}{2\pi}$
$\frac{2n}{2\pi}$
FREQUENCY

PLATE 4



INCIDENT AND REFLECTED
DAMPED SINUSOIDAL PULSE
FOR CASE $\begin{cases} K = 20 \\ \alpha = .1 \end{cases}$ $\frac{2L}{C_1} = \frac{\epsilon}{3} = \frac{\pi}{6n}$

PLATE 5