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As a Function of Bottom Impedance

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NRL Report No. 8-2032

NAVY DEPARTMENT

Report On

UNDERWATER SOUND PRESSURES BENEATH A SHIP-MOUNTED SOURCE
AS A FUNCTION OF BOTTOM IMPEDANCE.

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NAVAL RESEARCH LABORATORY
ANACOSTIA STATION
WASHINGTON, D. C.

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ABSTRACT

The sound pressure distribution directly beneath a ship-mounted source has been investigated, and over 200 experimental records have been made, in summer and in winter, at various frequencies from 36 cps to 2000 cps, and at various locations in the Potomac River area. The technique and the analysis are of general applicability, and most of the findings are believed to be typical.

Real and imaginary components of the bottom impedance were determined by comparison of the experimental records with plotted curves, computed in accordance with a theoretical analysis. The results show that "free-boundary" reflection of sound pressure waves, with attenuation of about 6 db per reflection, is characteristic of most of the Potomac River bottom used in summer. Limited areas near the river mouth, where the bottom is hard sandy mud, gave records showing "rigid-boundary" reflection of sound pressure waves. In most locations the winter records showed much greater attenuation per reflection than the summer records, and the character of the reflection changed from "free-boundary" in summer, to "transitional" in winter.

The applications of these results to the design of acoustic mines and to the experimental study of ship noises or acoustic minesweeping devices are discussed.

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TABLE OF CONTENTS

	<u>PAGE</u>
ABSTRACT	
I.	INTRODUCTION 1
II.	EXPERIMENTAL PROCEDURE 1
III.	ANALYSIS OF RECORDS. 3
IV.	DISCUSSION OF RESULTS. 5
	A. Classification of Records 5
	B. Results of Analysis; Proposed Explanations 6
	C. Variation in Records with Experimental Conditions 10
	1. Variation of Source Depth. 10
	2. Variation of Angle of Incidence at Bottom. 10
	D. Comparison of Records from Two Types of Hydrophones 11
	E. Sound Pressure Distribution Below the River Bottom 11
V.	APPLICATIONS OF THE RESULTS 13
VI.	CONCLUSIONS. 15
	APPENDIX A. FREE FIELD ACOUSTIC IMPEDANCE THEORY. 17
	APPENDIX B. AVERAGE VERTICAL GRADIENT OF SOUND
	PRESSURE LEVEL 24
	APPENDIX C. NOTES ON NOL REPORT NO. 488 29
	BIBLIOGRAPHY OF REFERENCES 31

TABLES

- I. Low Frequency Standing Wave Data for the Potomac River Area.
- II. Low Frequency Bottom Impedance Data for the Potomac River Area.

PLATES

- 1 Sound Pressure Beneath a Ship-Mounted Source.
- 2,3,4 Standing Waves of Sound Pressure Beneath a Ship-Mounted Source.
- 5 Comparison of Sound Pressure and Pressure Gradient Beneath a Ship-Mounted Source.
- 6,7,8 Computed Curves for Determination of Absorption and Phase Constants.



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I. INTRODUCTION:

1. The distribution of sound pressures to be expected between a ship-mounted source and the sea bottom is an important aspect of the general problem of the propagation of sound in water. Increased understanding of the vertical distribution of sound pressure should be of value for studies of ship noise and should facilitate the design of acoustic mines and of the means for sweeping them.

2. An experimental investigation of the sound pressure distribution, and of the impedance of the bottom, directly beneath a ship-mounted source is described in this report. The technique and the analysis are of general applicability and most of the findings are believed to be typical, although the actual experimental results were all obtained in the Potomac River area. Data were obtained at frequencies as low as 36 cps and as high as 2000 cps, although most of the records were made at frequencies in the range 100 cps through 750 cps. Extension of the results described in this report is expected from additional experiments now in progress. Other aspects of the propagation of low frequency sound in water, illustrated by range run recordings, will be considered in a report now in preparation.

II. EXPERIMENTAL PROCEDURES:

3. The experimental procedure consisted of setting up an underwater sound field of high intensity, and recording the sound pressure level registered by a hydrophone as it was raised vertically from the river bottom to the surface. The sound source was mounted on the USS AQUAMARINE (PYc 7), a converted yacht. With the ship at anchor, and the sound source operating at each of several frequencies in the lower audio range, the acoustic field beneath the source was probed and recorded. This was done, in summer and in winter, at a variety of locations in the Potomac River area.

4. Many of the experiments employed as sound source the NRL Model X-3 magnetic type underwater loud-speaker. The design and construction of this speaker, as well as performance tests comprising acoustic field measurements and range run recordings, have been described in NRL Reports (See Bibliography(1)). The speaker produces pressure levels of 130-150 db (630-6300 dynes/sq.cm.) at a distance of 12 ft, in the frequency range 150-500 cps. Higher frequencies up to 2000 cps are obtainable at lower level. The speaker is driven by an audio oscillator and power amplifier.

5. Some of the experiments, particularly at the lowest frequencies, employed as sound source a mechanically driven underwater sound

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generator developed at NRL, and designated Model XUR-2. The acoustic output from this device, measured at 6 ft distance, attains levels of 163-168 db (20-50 kilodynes per sq. cm.) for driving frequencies in the range 70-110 cps. Fundamental and successive harmonics may be used to cover a frequency range of 70-400 cps. The device has been described briefly (Bibliog. (2)), and will be discussed in greater detail in another report.

6. Both the loud-speaker and the mechanical generator were in effect point sources of sound, since their dimensions were small in comparison with the wavelength. The output from both may be characterized as polyphonic, a word here used to designate a fundamental frequency accompanied by harmonics the intensity of which decreases with the order.

7. Most of the records were made with a pressure-actuated tourmaline hydrophone, designated NRL No.6. A few records were made with the NRL tourmaline "watch-case" hydrophone, and a few with a pressure-gradient or velocity hydrophone designated SC-10 (Bell Telephone Laboratories).

8. For each record the hydrophone was dropped to the bottom on a suspension line, and then raised at constant speed by means of the anchor winch of the AQUAMARINE. The electric output of the hydrophone was fed into an ERPI Sound Frequency Analyzer (RA 277 F) and Graphic Level Recorder (RA 246). Index marks were placed on the records at intervals corresponding to the emergence of marks at 10 ft intervals along the suspension line. When provided with suitable scales, the records are quantitative graphs of sound pressure level (in DB above 0.0002 dynes/sq.cm.) versus vertical distance (in ft), referred either to the water surface or to the bottom.

9. Some of the measurements were made with both source and hydrophone rigged over the side of the AQUAMARINE, about 3 ft apart in the horizontal plane. Others were made with the source mounted in the large sound well of the AQUAMARINE, and the hydrophone rigged over the side. In a few cases both hydrophone and source were rigged in the well. The greatest departure from vertical incidence was 4° to 7°, when the hydrophone and speaker were about 10 ft apart in the horizontal plane. Below the first 20 ft from the surface, the records were essentially the same for all methods of rigging. Various anomalies occur in the upper 20 ft, but these do not affect the conclusions of this report.

10. Measurements were made only in locations where the bottom was flat over a reasonable area beneath the source. The depth of the sound generator was held constant at 10 ft. The effective distance from the

source to the surface must have been somewhat less than this, owing to yielding of the ship's hull. The consistency of the records indicated that the source-depth relations required by the theoretical analysis were realized at frequencies between 100 and 400 cps. At higher frequencies a close check with theory was not to be expected, owing to complexities introduced by reflections from the hull of the AQUAMARINE.

III. ANALYSIS OF RECORDS:

11. Theoretical expressions, valid within certain limitations, may be derived for the vertical distribution of sound pressure in a homogeneous medium bounded by a free upper surface and a bottom with normal impedance Z . A treatment of this problem by the Harvard NDRC group (Bibliog. (3)), has been clarified and adapted to the conditions of these experiments. A summary of the derivations and limiting conditions will be found in Appendix A.

12. According to the theory, the vertical distribution of underwater sound pressure directly beneath a ship-mounted source should form a standing wave pattern. The character of the pattern is a function of the bottom impedance, which in general is complex. The real part determines the limits between which the pattern oscillates; the imaginary part determines the initial phase of the pattern, i.e. whether it shall have a node or a loop at the bottom. An open system, such as any extended body of water bounded by surface and bottom, is characterized by horizontal spreading of acoustic energy. The standing waves which may be set up in an open system differ from those in a closed system, such as a pipe with rigid walls, by showing a rise in average sound pressure as the source is approached. The actual pressure distribution may be analyzed into an expression involving a sinusoidal term, expressing the standing wave; and a hyperbolic cosine term, expressing the gradual rise toward the source.

13. The modified standing wave system described above is expressed mathematically by equation (12) of Appendix A. This equation gives relative values of rms sound pressure at various points below an underwater source emitting at wavelength (λ). Values were computed from equation (12), and reduced to the form of curves of pressure level (in DB referred to the sound pressure at the bottom) versus distance above the bottom in half wavelengths. Plates 6, 7, and 8 illustrate curves computed for values of three parameters. These are α_0 and β_0 , the real and imaginary parts, respectively, of the effective propagation constant; and the ratio of the wavelength to 2π times the vir-

tual height of the source above the bottom. The virtual height, in accordance with the theory, is taken as five/fourths of the water depth.

14. Analysis of the experimental records proceeded as follows. For each record the third parameter listed above ($\lambda/2\pi\alpha_0$) was calculated, and the family of computed curves corresponding to this value was picked out. That curve in the family which corresponded most closely with the experimental record was chosen, and α_0 and β_0 were read from it. Some interpolation was usually required. Once α_0 and β_0 were known, the bottom impedance was computed from the relation:

$$Z = R + jX = \rho c \tanh(\alpha_0 + j\beta_0)$$

where R and X are the resistive and reactive components of Z, ρc is the radiation resistance of water, α_0 is the absorption constant, and β_0 is the initial phase constant for the standing wave system. The hyperbolic tangent was evaluated by means of Kennelly's "Chart Atlas of Complex Hyperbolic and Circular Functions", or from Plates 1 and 2 at the end of Bibliog. (6).

15. It follows from the properties of the hyperbolic tangent that, for absolute values of β_0 between zero and 0.5, the sign of the bottom reactance will be the same as the sign of β_0 . The latter may be either negative or positive, negative values being taken to correspond to mass reactance, and positive values to stiffness reactance in the components of the bottom impedance. This is in accordance with Morse's convention (Bibliog. (6)). Since the reactive component of the impedance is identically equal to zero for $\beta_0 = 0$ or $\beta_0 = 0.5$, it is immaterial what sign is taken at these points. In this report the sign of β_0 is arbitrarily taken as negative for the value of 0.5. For all other values the sign of β_0 indicates mass or stiffness reactance in accordance with the convention given above.

16. Since the maxima (and also the minima) of the standing wave system are approximately a half wavelength apart, the phase constant β_0 is roughly equal to the distance of the first pressure minimum above the bottom, measured in half wavelengths. The condition for "soft" or "free-boundary" bottom reflections (sound pressure node at the bottom) is $\beta_0 \approx 0$, and for "hard" or "rigid-boundary" reflection (pressure antinode at the bottom) is $\beta_0 \approx 1/2$.

17. For acoustically "soft" bottoms, the bottom impedance is predominantly real and smaller than the radiation resistance of the water; for acoustically "hard" bottoms, the bottom impedance is predominantly real and larger than the radiation resistance of the water; and for all other acoustic behavior the bottom impedance is complex. The latter type of bottom reflection is labelled "transitional".

18. Some of the results are expressed in terms of the attenuation per bottom reflection. From the definition of the reflection coefficient for sound pressure the loss from absorption at a boundary may be written in the form:

$$\text{DB loss per bottom reflection} = 54.6 \alpha.$$

IV. DISCUSSION OF RESULTS:

A. Classification of Records.

19. Standing wave patterns of the expected type were obtained in seven locations in the Potomac River area, and at various frequencies from 38 cps to 2000 cps. The records are illustrated in Plates 1-5 inclusive. A summary of the data obtained from about 200 records is given in Table I. A more detailed analysis, with computed bottom impedances, is given in Table II. A study of the records showed that three types of acoustic behavior at the river bottom were represented on the various sets of records. The first group indicated acoustically "soft" or "free-boundary" bottom reflection, the second group indicated acoustically "hard" or "rigid-boundary" reflection, and the remaining group showed initial reflection phases which were a function of frequency. The latter indicated bottoms which became acoustically "harder" as frequency increased, with a reversal of initial phase shown by the records in the frequency range 300-500 cps.

20. "Free-boundary" reflection is illustrated by Plate 1, curves (a), (c), and (d), and by the uppermost records on Plates 2 and 3. "Rigid-boundary" bottom reflection is illustrated by Plate 1, (b), the middle record of Plate 2, the lower record of Plate 3, and the first and third records on Plate 4. Intermediate or "transitional" initial phase is illustrated by the middle record on Plate 3, and by the second record from the top of Plate 4. The records on Plate 5 illustrate "free-boundary" reflection at low frequency, and a change in initial phase as the frequency increases.

21. The three types of acoustic behavior at the river bottom are indicated in Table I, in the column labelled "Character of Bottom Reflection". Record sets (a), (d), (i), and (k) showed sound pressure nodes at the bottom for all frequencies below 1000 cps; record sets (g), (h), and (m) showed pressure anti-nodes at the bottom; and records sets (b), (c), (e), (f), and (j) showed pressure nodes at the bottom for some frequencies and anti-nodes for others, with intermediate initial phases.

B. Results of Analysis; Proposed Explanations.

22. In order to meet the conditions required by the theory the records chosen for detailed analysis were limited to those made at frequencies in the range 100-400 cps. The phase and absorption constant could be determined from most of the records with an accuracy of about 20%. In many cases much greater accuracy was obtained. Inaccurate location of the bottom, and the sinking of the hydrophone in the mud, accounted for anomalies on some of the earlier records. Records made under favorable conditions of wind and tide, or in locations where the bottom was definitely either "soft" or "hard", gave the best results. Values derived for components of bottom impedance were, of course, no more accurate than the phase and absorption constants from which they were computed. Although precision cannot be claimed for measurements of this character, the final results are close approximations, and give a clear picture of the physical phenomena.

23. The quantitative results obtained from the analysis of all the experimental records are listed in Table II. For each set of records the absorption constant α_0 , the phase constant β_0 , and the components of the bottom impedance are given in Table II for several frequencies. The real and imaginary components and the absolute magnitudes of the bottom impedance determinations are expressed in the form of ratios to the impedance of the water. The latter is equal to the radiation resistance, ρc .

24. As stated in Section III - Analysis of Records, values of $\beta_0 \approx 0$ indicate acoustically "soft" bottoms, values of $|\beta_0| \approx 1/2$ indicate acoustically "hard" bottoms, and intermediate value of β_0 indicate "transitional" conditions. Similarly the values of $Z/\rho c$ indicate whether the bottom is acoustically "hard" or "soft", depending upon whether the impedance of the bottom is greater or less than that of the water. The attenuation per reflection at the bottom, expressed in terms of the absorption constant, is $54.6\alpha_0$ in db.

25. Table II shows that absorption constants ranging between 0.05 and 0.30 were encountered on the experimental records. These limits correspond to attenuations of 3 db and 17 db per bottom reflection, and pressure reflection coefficients of 75% and 15%, respectively. Records were found corresponding to phase constants ranging from +0.4 to -0.5, although most of the values were negative. Positive values indicated exceptional bottom conditions. Bottom impedances ranging from $1/7$ to more than 3 times the water impedance were encountered, and in most locations both real and imaginary com-

ponents of the impedance increased with frequency. Reactive components of the bottom impedance ranged from +0.7 to -1.4 times the water impedance, depending upon the value of the phase constant. Although the bottom was non-reactive in many cases, the sign of the reactance most commonly encountered was negative (mass reactance) with typical values ranging between zero and $1/3$ the water impedance. At some frequencies, particularly at the river mouth where the bottom was hard sandy mud, the reactance of the bottom was positive (stiffness reactance). Stiffness reactance was also found on the winter records for the soft mud bottom locations, although in these cases the bottom became non-reactive and then mass-reactive as frequency increased.

26. In a previous section, the records were divided into groups representing three types of bottom reflection, "free-boundary", "rigid-boundary", and "transitional". These types may be characterized more clearly after a study of the data in Table II. "Free-boundary" reflection corresponds to values of phase constant between 0 and -0.2, and to values of bottom impedance less than $2/3$ the water impedance. "Rigid-boundary" reflection corresponds to values of phase constant in the neighborhood of $1/2$ (irrespective of sign), and to values of bottom impedance greater than the water impedance. "Transitional" reflection corresponds to intermediate values of phase constant, and to absolute magnitudes of bottom impedance nearly equal to the water impedance. Some "transitional" records indicated anomalous acoustic conditions; others showed an orderly sequence of phase changes as frequency increased.

27. Study of the results in Tables I and II shows that there was a pronounced difference between the acoustic behavior of the river bottom in summer and in winter. The records made in August at all the locations studied except the known hard area near the river mouth showed "free-boundary" reflection with the phase constant approximately zero and a pressure node of sound at the bottom for frequencies below 400 cps. In most cases the change of impedance with frequency was slight. The values of absorption constant for this type of bottom were low, ranging from 0.05 to 0.10; and the bottom impedance was real and equal to $1/7$ to $1/3$ the impedance of the water. The lowest absorption, and the most complete reflection at the bottom, was encountered at Glymont, where the attenuation per reflection amounted to less than 3 db per reflection. These records indicate that, in summer, "free-boundary" reflection with relatively low absorption (3-6 db per reflection) is the characteristic acoustic behavior of the Potomac River mud.

28. The records made in January and February, compared with those made in August, showed an upward trend in all cases in both the real and imaginary components of the bottom impedance, correlative with an increase in absorption constant and a shift in phase constant. At the locations where the bottom was soft mud, the character of the reflection changed from "free-boundary" in summer toward "transitional" in mid-winter. At locations where the bottom was hard sandy mud the character of the bottom reflection did not change, although the absorption constant increased. Typical winter records are shown in Plate 4, and superposed summer and winter records for the same location are shown in Plate 1, (c) and (d).

29. A plausible explanation of the observed low values of bottom impedance observed in many locations in summer may be found in the effect of organic activity in the mud of the bottom. The gas bubbles associated with chemical or biological activity may alter the acoustic properties of the upper layers of bottom mud in such a way as to cause the observed effects. The action of a complex of gas bubbles in the mud, and particularly adjacent to the mud-water interface, should be to lower the effective impedance of the bottom for sound pressure waves incident from above.

30. This explanation is supported by the observed seasonal changes of bottom impedance. Chemical and biological reaction rates at the bottom must be at a minimum in winter, with near freezing water temperatures; and at a maximum in summer, with water temperatures of 25°-30°C. The records demonstrate that the bottoms which were acoustically "soft" in summer, were much "harder" in winter. Between summer and winter, the absorption constant and the magnitude of the impedance increased, and the character of the bottom reflection changed in most cases from "free-boundary" to "transitional".

31. The explanation of seasonal effects in terms of organic activity is also supported by the observation on the Potomac River that large quantities of bubbles rise to the surface when the bottom is disturbed by such operations as weighing or dropping anchor. This is much more noticeable in summer than in winter.

32. A reasonable explanation of the "transitional" bottom impedances may be given in terms of the reflections from lower layers in a complex or stratified bottom. If sound waves are reflected at normal incidence at the boundary between two media, the phase constant must be either 0 or 1/2. Intermediate values of phase constant and reactive components of boundary impedance can be obtained only if



reflections from other boundaries contribute to the impedance at the boundary under consideration. It is shown in theoretical discussions (Bibliog. (4) and (7)) of the normal transmission of sound through three media arranged in layers, that the properties of the intermediate layer enter into the transmission equations in a term involving the ratio of the thickness of the layer to the wavelength. As a special case, when this thickness is equal to one half wavelength the character of the reflections (and the impedance) at the first boundary will be determined by the properties of the third medium, and is independent of the properties of the intermediate layer.

33. It is probable, therefore, that the river bottom in locations where "transitional" records were obtained is stratified, or arranged in layers with different acoustic properties. In some instances, notably at the River Bridge (B) in summer, and at Glymont in winter (See Plate 5), the absorption was relatively small and constant, whereas the phase constant increased systematically with frequency. In these locations the reflections from the underlying layers appear to have been regular in character.

34. In other instances, notably at the River Bridge (A) in winter, the absorption was high (16 db per reflection), and the records at certain frequencies were erratic and indistinct. The erratic character of these particular records cannot be attributed to experimental inadequacies, but may be explained in terms of reflections from uneven underlying layers. Clear records obtained at some frequencies indicate that the first boundary, the actual bottom, was smooth. If the boundaries of the lower layers were diffuse or uneven, the reflections from these layers at certain frequencies would have varied widely in intensity and phase over the small area directly below the source. Since the absorption at the water-mud boundary was high, a large fraction of the energy was transmitted into the mud, and the effective reflected sound field in the water was in large part determined by reflections from the lower mud layers. The resultant field was therefore an erratic function of position above the bottom, and anomalous records were obtained. It is to be expected that the above effect will occur only at certain frequencies and under conditions of high absorption at the water-mud boundary.

35. Correlation of the acoustic results with the known data from hydrographic charts and from sampling shows that most of the Potomac River bottom consists of soft mud, and that the reflections from this mud were of the "free-boundary" type in summer, and of the "transitional" type in winter. The mud is harder at the River Bridge and at Piney Point than in the upper river near Glymont. If seasonal variations in

organic content or activity of the bottom mud are responsible for the observed changes in acoustic impedance, it is to be expected that such effects will be larger in rivers and bays than along the sea-coast. The Potomac River area should be typical of the former type of location.

36. The hydrographic charts of the Potomac show a few spots marked "hard", particularly near Piney Point and St. George Island near the mouth of the river. Sampling shows well packed sandy mud in these locations, and correlation with acoustic results shows that the reflections were of the "rigid-boundary" type both in summer and in winter. There is no known rock bottom in the Potomac River area.

C. Variation in Records with Experimental Conditions:

(1) Variation of Source Depth.

37. Records were made at the Potomac River Bridge with the source at depths of 6 ft and 12 ft, corresponding to $1/4$ wavelength and $1/2$ wavelength, respectively, at 200 cps. The portions of the records for depths greater than 30 ft below the surface were identical in shape. The upper portions were different, as might be expected. The tests indicated that determinations of absorption constant and initial phase, from the lower portions of the records, should not be greatly in error if the source depth is not an exact quarter wavelength.

(2) Variation of Angle of Incidence at the Bottom.

38. In order to determine whether the initial phase of the standing wave pattern was dependent on the angle of incidence of the sound pressure wave, several records were made by raising and lowering the hydrophone from a small boat placed 130 ft distant horizontally from the ship-mounted source. The experiment was made at the Potomac River Bridge, where the water depth was 66 ft. In these tests the angle of incidence at the bottom directly below the small boat was about 70° . The records made under these conditions showed the same character of bottom reflection, "free-boundary" in this instance, as the records for normal incidence. For a given frequency, however, the pattern was simpler and contained fewer nodes and loops than the corresponding pattern for normal incidence. From the theory (Bibliog. (4)), it is to be expected that at considerable horizontal distance from the source the higher modes will be more rapidly attenuated than the lower ones. The result should be a simplification of the vertical sound pressure pattern as hori-

zontal distance increases. Additional experimental verification is in progress.

D. Comparison of Records from Two Types of Hydrophones.

39. In order to confirm the results obtained with the pressure hydrophone employed in most of this investigation, a few records were made using a pressure-gradient hydrophone (SC-10), a moving coil instrument built by the Bell Telephone Laboratories. The readings of this hydrophone, when oriented face up as in these tests, were proportional to the vertical component of the particle velocity in the sound field. Since this type of hydrophone is directional, care was taken to raise it vertically along a line through the sound source, and the records were discontinued when it reached a position 6 ft directly beneath the source. Since the SC-10 was found to be very sensitive to water currents, the records were made at slack tide, on an unusually calm day, February 22, 1943. Records under identical conditions, with the AQUAMARINE at anchor in 63 ft of water near Glymont, were made alternately with the pressure hydrophone and with the pressure-gradient hydrophone. The XUR sound generator was employed as source.

40. The records obtained with the two hydrophones for frequencies of 100 cps, 200 cps, and 300 cps, are reproduced in Plate 5. An additional sound pressure record for 400 cps is included. The absolute values recorded by the two hydrophones, when expressed as pressure level, differ by a few db, presumably because the field corrections which should be applied to a velocity hydrophone in a standing wave system are lacking. These corrections are not easily computed. Comparison of the records from the two types of hydrophones graphically demonstrates that in the standing wave system which exists beneath the ship-mounted sound source, the pressure and pressure-gradient are 90° out of space-phase with each other. Plate 5 also demonstrates that excellent records may be obtained by careful attention to detail in the adjustment of experimental conditions. Unfortunately this is not always possible, owing to the vagaries of wind and tide.

E. Sound Pressure Distribution Below the River Bottom.

41. A few experiments were made in which a hydrophone, the NRL tourmaline "watch-case", was pushed several feet into the bottom mud by means of a special pipe fitting. The pipe was then removed, leaving the hydrophone buried, but with a line attached to it. Records were then taken as the hydrophone was pulled up slowly through the mud at a constant rate by means of a chain fall. The results will be described briefly, although their interpretation may be open to question, pending confirmation by additional investigations with improved technique.

42. These records were made in February, 1943, in a location near Glymont, where the water depth was 20 ft. As in most of the upper Potomac River area, the bottom consisted of soft mud. The XUR-2, mounted on the AQUAMARINE, was employed as sound source. Additional records of this type will be made from time to time, in various locations, in order to determine the variations with season and character of bottom. It is to be expected that the decrease in sound pressure on entering the bottom mud will be much greater in summer than in winter.

43. The record for 100 cps, illustrated in Plate 5 (lower right), showed an increase in sound pressure of 14 db in the first foot below the bottom, and a decrease of the same amount in the succeeding foot. A similar cycle of pressure increase and decrease followed in the succeeding 6 ft. When the hydrophone was embedded to a depth of 9 ft in the mud, the sound pressure was still 6 db greater than that recorded at the water-mud boundary which constitutes the normal river bottom. The pressure maxima in the two media, water and mud, differed by about 15 db.

44. The records for 200 cps and for 300 cps showed similar but less well defined patterns of maxima and minima continuing into the mud as far as the hydrophone was pushed (about 8 ft). The distance between sound pressure maxima (or minima) in the mud just beneath the water-mud boundary was about 2 ft at 100 cps, 1 ft at 200 cps, and 8 inches at 300 cps. Assuming that these distances represent half wavelengths on a standing wave pattern in the mud, the velocity of sound in the medium would appear to be about $1/12$ the velocity of sound in water.

45. If this interpretation of the observations is correct, the existence of a standing wave pattern in the mud argues the presence of reflecting layers lying beneath the water-mud boundary. The scale of the observed pattern suggests that the velocity of sound in the mud is very low compared with that in water. The suggestion that the bottom consists of a mud layer of very low velocity, underlaid by harder layers, is consistent with the interpretations made previously from the recorded standing wave patterns in water.

43. The records also showed an anomaly, which will be mentioned although no adequate explanation is available at this time. In addition to the characteristics described above, the record for 200 cps showed 12 db sudden decrease, and the record for 300 cps 30 db sudden decrease, in sound pressure on entering the bottom. The record for 100 cps showed no such discontinuity. If relatively large pressure

discontinuities at the bottom actually occur at some frequencies and not at others, the phenomenon might have important implications for mine design. It is suggested that, if the mine head were embedded in mud or "silted-up", a pressure-actuated acoustic mine designed for 100 cps might retain its sensitivity more effectively than units designed for higher frequencies.

V. APPLICATIONS OF THE RESULTS:

47. The results described in the foregoing sections have important applications to acoustic mine design and to experimental studies of underwater sound at low frequencies.

48. It has been shown that the reflections from the bounding surfaces (top and bottom) give rise to standing wave patterns beneath a ship-mounted sound source. The sound pressure in the neighborhood of the bottom may represent a minimum, a maximum, or an intermediate position in the pattern, depending upon whether the bottom is acoustically "soft", "hard", or "transitional". It has been shown that these differences may be large. For example, typical records for "hard" and a "soft" bottom are shown superposed in Plate 1, (a) and (b). The average pressure level gradients shown by these records are much the same. The initial phases of the standing wave systems are such, however, that a pressure-actuated hydrophone placed on the bottom at Glymont would record 16 db lower sound level than the same unit placed on the bottom at Piney Point (A), assuming the same sound source in both cases. Although these records illustrate extreme cases, it is clear that the bottom impedance should be taken into account if advantageous placement is desired for pressure-actuated hydrophones, acoustic mines, and similar devices.

49. The differences in sound pressure level shown by the August records between the bottom and a position a quarter wavelength above it ranged between 6 db and 23 db for the "soft" bottom locations which were found to be typical of the Potomac River area. The increases in level obtained by moving the receiving unit from the bottom to a position a quarter wavelength above it were, on the average, 18 db at Glymont, 8 db at Potomac River Bridge (A), 10 db at Piney Point (C), and 14 db at Piney Point (B). At Piney Point (A), over the "hard" bottom, a reduction of 6 db occurred upon raising the hydrophone a quarter wavelength. The reduction was smaller than the increase obtained by raising the hydrophone above the "soft" bottoms.

50. It may be concluded from the measurements that for a sound source approximately a quarter wavelength below the surface, maximum

response will be obtained from a pressure-actuated receiving unit near the bottom, when the latter is placed on a "hard" bottom, or a quarter wavelength above a "soft" bottom. The differences in level to be expected from placement within this range will be most pronounced over soft mud bottoms, particularly in summer, and they may amount to 8-23 db.

51. The records obtained with the pressure-gradient hydrophone, illustrated in Plate 5, show that maximum response will be obtained from a velocity-actuated receiving unit near the bottom, when the latter is placed on a "soft" bottom, or a quarter wavelength above a "hard" bottom. This behavior is the reverse of that to be expected from a pressure-activated device. Advantage could therefore be taken of the phase relations near the bottom by employing velocity-actuated receiving mechanisms in the design of acoustic mines for use in areas where the bottom is known to be predominantly "soft". Such devices would be effectively more sensitive than sound pressure actuated units in the same position, and their effectiveness would be less critically dependent upon the position near the bottom.

52. Although the analysis presented in this report has been made for single frequencies, similar considerations should apply to the reaction of a resonant acoustic mine receiver to complex ship noise. The German acoustic mines, for example, are actuated by a frequency band only a few cycles wide. Some of the results are applicable even to a mine unit with a broad response, since the character of the bottom reflection, and the initial phase of the standing wave pattern, is in many cases independent of frequency. Sound pressure minima (antinodes) were found at the bottom at all frequencies studied, in most of the Potomac River locations in summer.

53. The observation that the average sound pressure level (standing wave pattern averaged out) decreases 2-3 db in each 10 ft of vertical distance beneath a ship-mounted source, and that this figure is relatively independent of the type of bottom, should be helpful in making rough estimates of the pressure levels to be expected below the underwater sound sources employed in acoustic mine-sweeping. The theory of the average pressure level gradient, and its possible application to a new method of estimating α_s , is discussed in Appendix B.

54. It was inferred from the records that standing wave effects may exist in the mud beneath the bottom, as well as in the water above it. These phenomena, as well as the observed discontinuities in sound pressure at the water-mud boundary, may have important implications for

the problems of "silting-up" of acoustic mine units.

55. In experimental studies of underwater sound propagation, it has long been understood that the interpretation of sound pressure measurements made near the surface of the water may be complicated by the cancellation which occurs in the neighborhood of a low impedance boundary. It has not been generally realized, however, that the bottom may also be a low impedance boundary, at which similar effects will be found. It is obvious from the results of this study that hydrophones for measuring underwater sound should be located a substantial distance (at least an eighth of the wavelength of the lowest frequency expected) away from both surface and bottom, if they are to measure the representative average values of sound pressure in the water. Special precautions may be required to accomplish this, and to prevent the hydrophone from sinking into soft mud. Misleading results, particularly at low frequencies, may be obtained from a hydrophone placed too close to a "soft" bottom. Some of the discrepancies between measurements made at different acoustic ranges may be attributed to differences in hydrophone placement.

56. It is probable that well defined standing wave effects are limited to frequencies below 2000 cps, since the patterns will tend to average out as the geometric irregularities and non-homogeneities of the media become comparable to the wavelengths.

57. The acoustic impedance data in this report, limited to the Potomac River below Glymont, should be extended by surveys of bottom impedance in other areas. The same methods could be employed to obtain information on acoustic conditions for analysis and correlation with hydrographic data for important navigable rivers, harbors, and coastal strips. Such surveys should be of value to scientific groups working on problems of acoustic mine design and acoustic minesweeping, and might also be of interest to laboratories concerned with oceanographic studies.

VI. CONCLUSIONS:

(a) It is concluded that the character of the bottom reflections, the attenuation per reflection, and the components of the bottom impedance, may be obtained from the analysis of experimental records of sound pressures in the standing wave patterns directly beneath a ship-mounted source.

(b) It is concluded from the analysis of more than 200 experimental records, made at various locations in the Potomac River area,

that "free-boundary" reflection of sound pressure waves, with attenuation of about 6 db per reflection, is characteristic of most of the Potomac River bottom mud, particularly in summer. This is believed to be typical of similar areas.

(c) The results of the survey indicate, for maximum sensitivity, mine units operated by audio frequency sound pressures should be placed with the receiving element directly on a hard bottom or a quarter wavelength above a soft bottom. Advantage can be taken of the phase relations near the bottom by employing velocity-actuated receiving mechanisms in acoustic mine units for use in areas where the bottom is known to be soft. The sound pressure variations near a soft bottom may amount to as much as 35 db at the water-mud boundary and 23 db in the first quarter wavelength above the bottom.

(d) In order to measure representative average values of sound pressures, measuring hydrophones for the study of underwater sounds should be located a substantial fraction of a wavelength away from the bounding surfaces (surface and bottom). For studies at low frequencies, especially over bottoms which are acoustically "soft", the hydrophone may require placement several feet above the bottom.

APPENDIX A.

FREE FIELD ACOUSTIC IMPEDANCE THEORY

58. The theory relating the pressure distribution about an underwater sound source to the acoustic impedance of the sea bottom has been treated. (Bibliog. (3) and (4)). It is the purpose of this appendix to restate this theory briefly, emphasizing the conditions and limitations of the derivations, and arranging the results for convenient comparison with the experimental records. Full credit is given to the Harvard NDRC group for the derivations in Appendix A. The extension of the theory to a special case of practical interest, included in Appendix B, was added during the course of the NRL investigation. In order to facilitate the reduction of results, curves were computed from the final formulas of Appendix A. These curves, unavailable in the literature, are reproduced in Plates 6, 7, and 8 of this report, in the belief that they may be useful in future experimental surveys of bottom impedance.

59. Outline of mathematical procedure. The sound pressure field from a point source radiating into an infinite medium is first written in spherical coordinates. This is transformed into the definite integral of an expression involving cylindrical coordinates and a variable of integration. Cylindrical coordinates about a vertical axis are employed in order to simplify the formulation of boundary conditions. Additional terms are added to the integrand to take account of the "net upward reflected waves" and "net downward reflected waves", which arise when boundaries (surface and bottom) are introduced into the medium. The effect of the boundaries enters into the integrand through certain functions, $F_1(u)$ and $F_2(u)$, which may be determined by applying appropriate boundary conditions. At the upper boundary, the surface, the sound pressure is equal to zero. At the lower boundary, the sea bottom, the pressure is expressed in terms of a normal acoustic impedance, Z .

60. The concept of an acoustic impedance for the sea bottom results from an extension of the impedance concept which is familiar from the standard theory for the reflection of plane waves at the boundary between two media, (Bibliog (6)). For plane waves at normal incidence the acoustic impedance is defined as the ratio between pressure and normal particle velocity at the boundary. When the waves are spherical and their spreading is cylindrical, as in the case under discussion, certain limitations and approximations may be introduced which permit the boundary condition at the bottom to be evaluated in terms of the plane wave impedance. The boundary impedance, Z , for plane waves at normal incidence on the boundary between two media, is:

$$Z = R + jX = \rho c \tanh \pi(\alpha_0 + j\beta_0) \quad (1)$$

(1)

where R and X are the resistive and reactive components of Z_0 , ρc is the radiation resistance of the first medium (1.43×10^5 for fresh water), α_0 is the absorption constant, and β_0 is the phase constant for plane waves at the boundary, $j = \sqrt{-1}$. Mass reactance is taken negative, in accordance with Morse's convention.

61. Reverting to the outline of mathematical procedure, the impedance condition at the lower boundary, expressed in terms of α_0 and β_0 , is put into the integrand of the equation for sound pressure. After making various simplifications and reductions in the integrand, the pressure expression is transformed again into spherical coordinates. The final equation for sound pressure distribution is then analyzed in some detail, for the special case when the source depth is approximately a quarter wavelength. Curves of sound pressure versus depth are computed and plotted (Plates 6, 7, and 8) for various assigned values of the parameters, and values of α_0 and β_0 are then determined by comparison of the computed curves with the experimental records.

62. Mathematical Derivations. Following the method of bibliography (a), consider a point source of sound, of strength A , at any point in an infinite homogeneous medium. The sound pressure, p , at a distance r from the source is:

$$p = A \frac{e^{jkr}}{kr} \quad (2)$$

where $k = 2\pi/\lambda$ and λ is the wavelength of the wave radiated in the medium. The above expression defines the direct spherical wave which arrives at the point under observation.

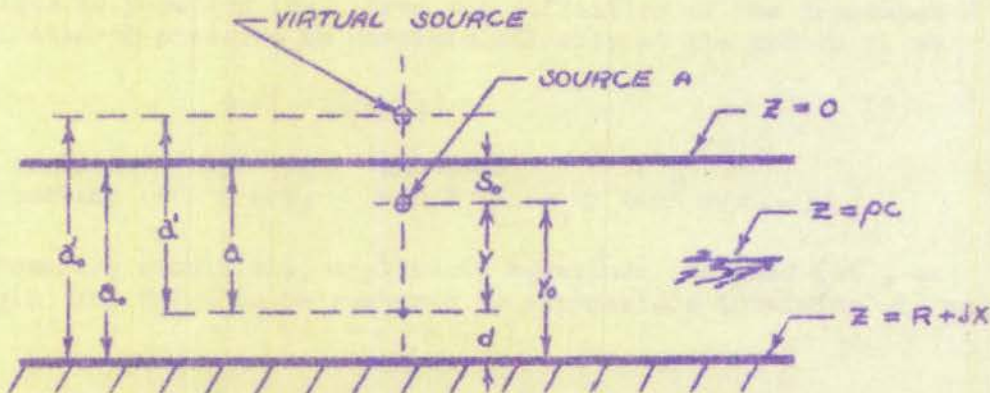


Figure 1.

If two plane parallel bounding surfaces, representing the water surface and the sea bottom, be introduced as in the accompanying figure, there may also be present an infinite number of reflected waves. The phase and amplitude of the reflections will be determined by the boundary impedances. In order to simplify the formulation of boundary conditions, it is desirable to express the direct spherical wave (Equation 2) in terms of circular cylindrical coordinates y and σ . This is done by making use of the following transformation, (Bibliography (8) p. 546):

$$P = f(y, \sigma) = A \int_0^{\infty} \frac{e^{-k\gamma\sqrt{u^2-1}}}{\sqrt{u^2-1}} J_0(k\sigma u) u du \quad [y > 0] \quad (3)$$

$$P = f(y, \sigma) = A \int_0^{\infty} \frac{e^{+k\gamma\sqrt{u^2-1}}}{\sqrt{u^2-1}} J_0(k\sigma u) u du \quad [y < 0]$$

63. The complete expression for pressure, including direct and reflected waves, is given by a similar integral:

$$P = \phi(y, \sigma) = \int_0^{\infty} [e^{-k\gamma\sqrt{u^2-1}} (1 + F_2(u)) + e^{+k\gamma\sqrt{u^2-1}} F_1(u)] J_0(k\sigma u) \frac{u du}{\sqrt{u^2-1}} \quad (4a) \quad [y > 0]$$

$$P = \phi(y, \sigma) = \int_0^{\infty} [e^{+k\gamma\sqrt{u^2-1}} (1 + F_1(u)) + e^{-k\gamma\sqrt{u^2-1}} F_2(u)] J_0(k\sigma u) \frac{u du}{\sqrt{u^2-1}} \quad (4b) \quad [y < 0]$$

where $F_2(u)$, as yet undefined, is a function which determines the character of the "net downward reflected waves", and $F_1(u)$ is a similar function for the "net upward reflected waves". These functions are to be determined by applying the appropriate boundary conditions to Equation (4). From the definition of the impedance Z as the ratio of pressure to particle velocity at the boundary, we have

$$p = (-jZ/k\rho c) \quad (5)$$

At the surface: $y = -S_0$ and $Z = 0$

At the bottom: $y = y_0$ and $Z = Z_0 = \rho c \tanh \pi(\alpha_0 + j\beta_0)$

These boundary conditions, applied to Equations (4a) and (4b), enable $F_2(u)$ and $F_1(u)$ to be replaced by expressions involving α_0 and β_0 .

64. If the solutions be restricted to points directly below the source, and if the source is taken to be more than $3/4 \lambda$ above the bottom, several simplifications and reductions may be made in equations (4a) and (4b). The final expression for sound pressure, transformed again into spherical coordinates, then becomes:

$$P = A \left\{ \left[\frac{e^{jkY}}{Ky} \quad \frac{e^{jk(Y+2S_0)}}{K(Y+2S_0)} \right] - e^{-2\pi(\alpha_0 + j\beta_0)} \left[\frac{e^{jk(2Y_0 - Y)}}{K(2Y_0 - Y)} - \frac{e^{jk(2S_0 + 2Y_0 - Y)}}{K(2S_0 + 2Y_0 - Y)} \right] \right. \\ \left. + e^{-2\pi(\alpha_0 + j\beta_0)} \left[\frac{e^{jk(2S_0 + 2Y_0 + Y)}}{K(2S_0 + 2Y_0 + Y)} - \frac{e^{jk(4S_0 + 2Y_0 + Y)}}{K(4S_0 + 2Y_0 + Y)} \right] - \dots \dots \dots \right\} \quad (6)$$

The first term in brackets represents the direct wave from the effective dipole formed by the source and its image in the surface, the second term represents the first bottom-reflected wave from the dipole, and the third term represents the wave from the dipole reflected once at the bottom and once at the top surface.

65. The above expression in its complete form does not easily lend itself to analysis. If the source depth, S_0 , be taken as one quarter wavelength, and the origin of coordinates be shifted by letting $y + \lambda/4 = a$ and $y_0 + \lambda/4 = a_0$, equation (6) may be simplified to:

$$P = \frac{-jA}{K} \left\{ e^{jka} \left[\frac{1}{a - \lambda/4} \quad \frac{1}{a + \lambda/4} \right] - e^{-2\pi(\alpha_0 + j\beta_0)} e^{jk(2a_0 - a)} \right. \\ \cdot \left[\frac{1}{(2a_0 - a) - \lambda/4} \quad \frac{1}{(2a_0 - a) + \lambda/4} \right] + e^{-2\pi(\alpha_0 + j\beta_0)} e^{jk(2a_0 + a)} \\ \cdot \left[\frac{1}{(2a_0 + a) - \lambda/4} \quad \frac{1}{(2a_0 + a) + \lambda/4} \right] - \dots \dots \dots \left. \right\} \quad (7)$$

For points considerably deeper than a quarter wavelength beneath the water surface, $a \gg \lambda/4$, the pressure equation further simplifies to:

$$P \approx \frac{-2jA}{K} \left\{ \left[\frac{e^{jka}}{a} \right] - e^{-2\pi(\alpha_0 + j\beta_0)} \left[\frac{e^{jk(2a_0 - a)}}{2a_0 - a} \right] \right. \\ \left. + e^{-2\pi(\alpha_0 + j\beta_0)} \left[\frac{e^{jk(2a_0 + a)}}{2a_0 + a} \right] - \dots \dots \dots \right\} \quad (8)$$

86. The above equations show that, when the source is located at a depth of a quarter wavelength, the pressure distribution is that which would be obtained from a series of virtual simple sources of decreasing strengths, located at the surface and above and below it in equal steps of twice the depth. It can be shown that this picture is still essentially correct when the source depth is intermediate between a small fraction of a wavelength and almost a half wavelength. It was found experimentally that the source depth was not very critical for the frequencies studied in this report.

87. It is shown (Bibliog. (3)), that for the values of absorption which actually occur, the effect of the multiple reflections beyond the second (terms beyond the second in equation 6) may be taken into account by considering the sound source to be at an effective height above the bottom equal to somewhat more than the water depth. The virtual height depends chiefly upon the absorption, and partly on the phase change at the bottom. For the values of absorption encountered in the Potomac River area it was found sufficiently accurate to assume an effective height (a_0') 25% greater than the water depth (a_0). This was done in making the computations described below.

88. To summarize the limitations and conditions under which the theoretical derivations are valid:-

- (a) The expressions for sound pressure apply only to points which are directly or nearly below the source, and in the lower half of the region enclosed by the bounding surfaces.
- (b) The source must be at least three-quarters of a wavelength above the bottom.
- (c) The source depth should be taken to be approximately a quarter wavelength below the surface.
- (d) The depth should be at least one wavelength, so that (b) and (c) may be maintained and also $a^2 \gg \lambda^2/16$.
- (e) The effective distance of the source from the bottom (a_0'), should be taken to be 10% to 25% greater than the water depth (a_0). Higher terms may be neglected if this is done.

Restrictions (a), (b), and (d) ensure that variations of bottom impedance with angle of incidence do not need to be considered. Restrictions (c) and (d) together permit the dipole to be reduced to an equivalent simple source.

69. Computation of Sound Pressure Curves. In order to determine α_0 and β_0 from the experimental records, curves of sound pressure distribution near the sea bottom were computed for assigned values of the parameters. The actual pressure field is to be probed by a hydrophone which measures rms values of pressure, and the pressure distribution of interest is concerned only with the relative values of this quantity in the neighborhood of the sea bottom. In order to reduce equation (8) to a form convenient for computation, it is factored and re-arranged; with a_0 replaced by a_0' :

$$p \approx -2jA e^{ik_0 a_0'} e^{-\pi(\alpha_0 + j\beta_0)} \left[\frac{e^{\pi\{\alpha_0 + j[\beta_0 - 2/\lambda(a_0' - a')]\}}}{2} - \frac{e^{-\pi\{\alpha_0 + j[\beta_0 - 2/\lambda(a_0' - a')]\}}}{2a_0' - a'} \right] \quad (9)$$

The rms value of pressure in equation (9) varies as follows:

$$|p| \sim \frac{e^{\tanh^{-1}(1 - a_0'/a_0')}}{a_0'(2 - a_0'/a_0')} \sqrt{\cosh 2\pi[\alpha_0 + \frac{1}{\pi} \tanh^{-1}(1 - a_0'/a_0')] - \cos 2\pi[\beta_0 - 2a_0'/\lambda(1 - a_0'/a_0')]} \quad (10)$$

The approximation $\tanh^{-1}(1 - \frac{a_0'}{a_0}) \approx 1 - \frac{a_0'}{a_0}$ is valid in this case, since the solutions are restricted to points where $(a_0' - a) \ll a_0'$. The expression for rms pressure becomes:

$$|p| \sim \sqrt{\cosh 2\pi[\alpha_0 + \frac{1}{\pi}(\frac{a_0' - a}{a_0})] - \cos 2\pi[\beta_0 - \frac{2a_0'}{\lambda}(\frac{a_0' - a}{a_0})]} \quad (11)$$

It can be shown that this formula is very nearly correct for values of (a') up to one half of the depth (a_0).

69. For computation it is convenient to let $(a_0' - a) = d$, the distance above the bottom, and to let the independent variable have the form $2d/\lambda$. Thus:

$$|p| \sim \sqrt{\cosh 2\pi[\alpha_0 + \frac{1}{\pi} \frac{2d}{\lambda}] - \cos 2\pi[\beta_0 - \frac{2d}{\lambda}]} \quad (12)$$

70. The above expression, reduced to the form of relative sound pressure in db, has been computed for the first half wavelength above the bottom, for various values of the parameters α_0 , β_0 and $\sqrt{2\pi}\alpha_0$. The computed curves, plotted in Plates 6, 7, and 8, were compared with the experimental records to yield the values of α_0 and β_0 which characterized the bottom over which the data were taken.

Note. Since the reflection coefficient for sound pressure at a bottom of absorption coefficient α_0 is

$$U = e^{-2\pi\alpha_0} \quad (13)$$

the loss from absorption at the boundary may be written in the form: DB loss per bottom reflection = $54.6 \alpha_0$. The character of the bottom reflection is determined by the phase constant β_0 . The phase constant is also a function of the velocity distribution in the second medium, which may be complicated owing to stratification of the bottom material. The experimental results and their probable explanations are discussed in the body of the report.

Note. The finding of records indicating both stiffness and mass reactance at the bottom differs from the conclusion of bibliography (3). It was stated there that "the bottom of the harbor has a mass reactance in every case". The pressure curve chosen to illustrate the Harvard - NDRC report, however, seems also to illustrate stiffness reactance.

APPENDIX B.

AVERAGE VERTICAL GRADIENT OF SOUND PRESSURE LEVEL

71. The average vertical gradient of sound pressure level beneath a ship-mounted source may be estimated from the theory outlined in Appendix A. This gradient is the slope of the pressure level curve which would be obtained if the sinusoidal components of the standing wave pattern were averaged out. Since this is also the sort of pressure distribution which should be produced by a multi-frequency source such as a hammerbox, its study may have applications to acoustic mine-sweeping

72. For large values of absorption coefficient, and for the source depth and distance conditions of Appendix A, the reflected wave from the bottom becomes negligible and the pressure distribution is that for a simple spherical wave. In this case only the first term of equation (8) need be considered:

$$p = \frac{(2A/k) e^{j\omega t}}{a}$$

from which $|p| \sim 1/a$ (14)

From this expression the ratio of pressure at two points on the axis at distances a_1 and a_2 ($a_2 > a_1$) is:

$$\frac{P_1}{P_2} = \frac{a_2}{a_1} = \frac{a_1 + \Delta a}{a_1} = 1 + \frac{\Delta a}{a_1} \quad (15)$$

73. For example, the gradient of sound pressure level, expressed in db, at a depth of 40 ft, when bottom reflections are absent, is $db_1 - db_2 = 20 \log_{10}(1 + \Delta a/a_1) = 0.20$ db per ft.

74. In general, however, the absorption at the bottom is not complete, and at least two terms of equation (8) must be taken into account. It will be shown by derivation from equation (9) that the average gradient of the sound pressure level is larger for finite values of bottom absorption than it is for complete absorption. This result is derived as follows:

Considering only absolute magnitudes, equation (9) gives

$$|P| \sim \left\{ \frac{e^{\pi[a_0 + j(a_0 - \frac{3}{2}[a'_0 - a'])]}}{a'} - \frac{e^{-\pi[a_0 + j(a_0 - \frac{3}{2}[a'_0 - a'])]}}{2a'_0 - a'} \right\} \quad (16)$$

This expression contains a slowly varying portion and a periodically varying portion. Average pressure amplitudes may be obtained by evaluation for points at which the periodic portion is zero. For such points

$$|P| \sim \left\{ \frac{e^{\pi\alpha_0}}{a'} - \frac{e^{-\pi\alpha_0}}{2a'_0 - a'} \right\} \sim \left\{ \frac{1}{a'} - \frac{U}{2a'_0 - a'} \right\} \quad (17)$$

The ratio of pressures at two such points, at a'_1 and a'_2 , is:

$$\frac{P_1}{P_2} = \left[\frac{a'_2}{a'_1} \right] \cdot \left[\frac{2a'_0 - a'_2}{2a'_0 - a'_1} \right] \cdot \left[\frac{2a'_0 - (1+U)a'_1}{2a'_0 - (1+U)a'_2} \right] \quad (18)$$

where U , the reflection coefficient, is defined as in equation (13) in terms of α_0 .

75. Let the points be taken fairly close together (depth equal to several wavelengths), and let the difference $a'_2 - a'_1$ be designated $\Delta a'$. The pressure ratio becomes:

$$\frac{P + \Delta P}{P} \approx \left[\frac{a'_1 + \Delta a'}{a'_1} \right] \cdot \left[\frac{2a'_0 - a'_1 - \Delta a'}{2a'_0 - a'_1} \right] \cdot \left[\frac{2a'_0 - (1+U)a'_1}{2a'_0 - (1+U)(a'_1 + \Delta a')} \right] \quad (19)$$

By applying successive valid approximations, the expression may be shown to closely approximate

$$\frac{P + \Delta P}{P} \approx \left\{ 1 + \frac{\Delta a'}{a'_1} \left[\frac{(U+1)a'_1 - 4a'_0 a'_1 + 4(a'_1)^2}{(U+1)(a'_1)^2 - 2(U+2)a'_0 a'_1 + 4(a'_1)^2} \right] \right\} \quad (20)$$

Further simplification may be obtained by letting $a'_1 = m a'_0$ where $0 < m < 1$

$$\frac{P + \Delta P}{P} \approx \left\{ 1 + \frac{\Delta a'}{m a'_0} \left[\frac{(U+1)m^2 - 4m + 4}{(U+1)m^2 - 2(U+2)m + 4} \right] \right\} \quad (21)$$

When converted to db, this equation expresses the average pressure level gradient for any point, below the source, at which the theory previously derived is valid. The above expression is independent of the phase constant, β_0 , and it will be shown below that the level gradient is relatively constant between the bottom and a position somewhat more than half way to the surface.

76. The quadratic form of the bracketted term in equation (21) suggests that values of U might be found which will give the same value of pressure level gradient for two arbitrarily chosen points in the field. Let the bottom and a point part way to the surface be chosen, for which $m = 1$ and $m = m_1$, respectively. A solution for U may be obtained by setting the value of equation (21) at the point ($m = m_1$) equal to its value at the bottom ($m = 1$).

$$\frac{(U+1)m_1^2 - 4m_1 + 4}{(U+1)m_1^3 - 2(U+2)m_1^2 + 4m_1} = \frac{1+U}{1-U} \quad (22)$$

Combining terms and simplifying, this reduces to:

$$(U+1)^2 m_1^2 - (U+1)^2 m_1^3 + 4(U+1)m_1^2 - 4(U+1) - 8m_1 + 8 = 0$$

77. This equation defines the relation which must exist between m and U , in order that the gradient of sound pressure level shall be the same at some specified point as it is at the bottom. For the special case of the point corresponding to $m_1 = \frac{1}{2}$, ($a' = \frac{1}{2} a_0'$), a position somewhat more than half way to the surface, the above expression reduces to

$$(U+1)^2 - 24(U+1) + 32 = 0, \text{ from which } U = 0.417 \text{ and}$$

$$\alpha_0 = \log_e [1/U] / 2\pi = .12$$

78. This value of α_0 , curiously enough, is representative of those obtained from experimental records in most locations. Returning to equation (21), and substituting in the above value of U and $m = \frac{1}{2}$, an expression is obtained for the pressure level gradient at the point corresponding to $a' = \frac{1}{2} a_0'$

$$\frac{P + \Delta P}{P} = 1 + 2.43 \frac{\Delta a'}{a'}$$

For a water depth of 50 ft ($a_0' = 63$ ft) this is equal to 1.0386 or, expressed in db: 0.33 db per ft.

79. To estimate how much the gradient varies at intermediate points between the chosen points, it may be evaluated for example at $a' = 3/4 a_0'$. At this point the pressure level gradient is 0.387 db per ft.

80. Evaluation of the average pressure level gradient for other intermediate points shows that it is substantially constant over the entire range. Furthermore, for other values of α_0 , in the neighborhood of 0.12, it may be shown that the gradient will be reasonably constant for all points lying between the bottom and somewhat more than half way to the surface. Over this range, pressure level curves (db vs distance) taken over bottoms for which α_0 is approximately 0.12, will rise almost linearly with distance above the bottom. This will be true, independently of the value of β_0 , and for a fairly wide range of values of α_0 . This behavior is well substantiated by the experimental records. See for example the upper records on Plate 3.

81. Since the average pressure level gradient is nearly independent of position for points in the vicinity of the bottom in many cases, the value at the bottom will often be representative of the value at some distance above it. The pressure level gradient at the bottom may be obtained by evaluating equation (21) for $m = 1$:

$$\frac{P + \Delta P}{P} \approx \left\{ 1 + \frac{\Delta a'}{a_0'} \left[\frac{1+U}{1-U} \right] \right\} \quad (23)$$

For the case of large absorption, $\alpha_0 \gg 0$, U approaches zero, and the above expression reduces to the pressure relation for the simple spherical field, equation (15).

82. Equations (21) and (23) show that the average vertical gradient of the pressure level in the complex acoustic field beneath a ship-mounted source is in general greater than that which would be found if the medium extended downward toward infinity (great water depth), or if the absorption at the bottom were complete.

83. Comparison with Experimental Results. It was shown by the experimental records that the average gradient of the sound pressure level beneath the ship-mounted source was independent of the initial phase constant (β_0), and that it varied with absorption constant in the manner indicated by equation (23). Quantita-

tive agreement was obtained between measured and computed values of the gradient. The agreement was satisfactory for those records which were made under ideal conditions, for example the series illustrated on Plate 5. For these records $\alpha_0 = 0.10$, $U = 0.543$, $a_0' = 79$ ft, and the computed gradient (equation (23)) was 3.6 db per 10 ft. The measured gradient was 3.3 db per 10 ft. In general the measured values of average gradient were in the range 2 db - 3.5 db per 10 ft, and the agreement with computation was within 0.8 db per 10 ft. The measured values were usually low. Some of the deviation between measured and computed values of gradient may be explained as due to the inadequacy of the simplifications which were introduced to make allowance for the influence of higher terms than the second in equation (8). Some of the deviation was experimental, and was due to departures from normal incidence. The best agreement should be expected from records made with hydrophones and source located accurately along a vertical line, as in the above example. From records made under suitable experimental conditions, it should be possible to estimate the absorption coefficient, α_0 , from the measured average vertical gradient of sound pressure level. Such a method could be employed to determine α_0 rapidly and with a minimum of analysis, at frequencies high enough (wavelength less than roughly one third the depth) to permit averaging of the standing wave pattern on the record.

APPENDIX C.NOTES ON NOL REPORT NO. 488.

83. NOL Report No. 488, entitled "Acoustic Pressure at Harbor Bottom of Finite Impedance", (Bibliography (5)), gives a theoretical solution for the sound pressure to be expected at a harbor bottom due to a source located below and near the surface. Since this is a special case of the problem considered in this report, a comparison of the NOL treatment with that of the Harvard NDRC group (Bibliog. (3)) is a relevant addition to the present discussion. It will be shown below that essentially the same results were obtained by entirely different methods.

84. The mathematical device employed in the NOL report is the expression of the original spherical wave in the form of an infinite number of elementary plane waves. This gives rise to a complicated integral which is evaluated by imposing essentially the same limitations as those required for the derivations in bibliography (3) and Appendix A. The limitations differ in one respect, however. The lowest frequency for which the final expression is valid, at any given depth, is specified differently in the two treatments. For example, at 40 ft depth, the low frequency limit is 300 cps for the NOL formula, and approximately 100 cps for the Harvard-NDRC formula. This suggests the possibility that the final integral of the NOL report may be evaluated for lower frequencies than those indicated.

85. In the NOL report, the final result gives the pressure on the bottom directly below the source, in terms of a series of simple source images each multiplied by the proper coefficient. Subject to the proper limitations, the exact expression derived is:

$$P(a_0) = C_1(1+K) [P_f(a_0) - KP_f(3a_0) + K^2P_f(5a_0) - \dots]$$

In this equation C_1 is a proportionality constant, including the source strength, etc. $K = -e^{-2\pi(\alpha_0 + j\beta_0)}$ is the complex reflection coefficient, given as negative to take care of the phase change at reflection. $P_f(a_0)$ is the "free field" sound pressure at distance (a_0) due to the source and its image by reflection at the surface, in the absence of any other boundaries. $P(a_0)$ is the sound pressure on the bottom at depth (a_0) .

86. By making use of the relations $a_0 = y_0 + s_0$, and

$$P_f(a_0) = \left[\frac{e^{jk(a_0 - s_0)}}{a_0 - s_0} - \frac{e^{jk(a_0 + s_0)}}{a_0 + s_0} \right]$$

and expanding the pressure expression in terms of the parameters, the complete expression for pressure may be obtained in the form:

$$P(a_0) = C_1 \left[\left[\frac{e^{jk y_0}}{y_0} - \frac{e^{jk(y_0 + 2s_0)}}{y_0 + 2s_0} \right] - e^{-2\pi(\alpha_0 + j\beta_0)} \left[\frac{e^{jk}}{y_0} - \frac{e^{jk(y_0 + 2s_0)}}{y_0 + 2s_0} \right] \right. \\ \left. + e^{-2\pi(\alpha_0 + j\beta_0)} \left[\frac{e^{jk(3y_0 + 2s_0)}}{3y_0 + 2s_0} - \frac{e^{jk(3y_0 + 4s_0)}}{3y_0 + 4s_0} \right] - \dots \right]$$

87. The comparison of this expression with equation (6) of Appendix A, evaluated at $y = y_0$, shows that the two formulations are identical in form. Since the two treatments involved the use of entirely different mathematical means, the agreement between the results is gratifying.

BIBLIOGRAPHY OF REFERENCES

1. NRL Reports No. S-1900 and S-1924.
 S-1900: "Underwater Loud-Speaker For Low Audio Frequencies, Model X-3." John M. Ide.
 S-1924: "Performance Tests of Underwater Loud-Speaker Model X-3." John M. Ide.
2. Memos for Director NRL, C-S81(476) dated 29 Oct 1942, "Preliminary Report on XUR-2 Equipment"; and C-S81(476) dated 4 December 1942, "Sound Output of XUR-2 Equipment at 6 ft Distance."
3. OSRD Report No. 598, "Underwater Impedance Measurement." R. L. Brown and J. R. Pellam, of Harvard University.
4. Report No. V by MIT Research Group DIC 5985 Appendix L, "Reflection and Absorption of Underwater Sound by the Sea Bottom", pp 301-325.
5. NOL Report No. 488, "Acoustic Pressure at Harbor Bottom of Finite Impedance."
6. "Vibration and Sound." P. M. Morse, (McGraw Hill, 1936).
7. "Acoustics." G. W. Stewart and R. B. Lindsay (Van Nostrand, 1930).
8. "Differentialgleichungen der Physik" - II, p. 546. Riemann-Webers.

TABLE I

LOW FREQUENCY STANDING WAVE DATA FOR THE POTOMAC RIVER AREA.

<u>Record Group</u>	<u>Location in River</u>	<u>Water Depth</u>	<u>Date</u>	<u>Type of Bottom</u>	<u>Character of Bottom Reflection</u>	<u>Loss per Bottom Reflection</u>
a	Glymont A	40'	Aug.	Very Soft Mud	FB	2.7 DB
b	Glymont A	40'	Jan.	Very Soft Mud	TR	6-9 DB
c	Glymont B	63'	Feb.	Very Soft Mud	FB to TR	6 DB
d	PR Bridge A	56'	Aug.	Soft Mud	FB	6-9 DB
e	PR Bridge A	56'	Jan.	Soft Mud	TR	16 DB
f	PR Bridge B	73'	Aug.	Mud	TR	3-6 DB
g	Piney Pt. A	48'	Aug.	Hard Sandy Mud	RB	6-8 DB
h	Piney Pt. A	46'	Jan.	Hard Sandy Mud	RB	11-22 DB
i	Piney Pt. B	58'	Aug.	Soft Mud	FB	6 DB
j	Piney Pt. B	46'	Jan.	Soft Mud	TR to RB	8-16 DB
k	Piney Pt. C	80'	Aug.	Soft Mud	FB	6 DB
m	River Mouth	55'	Jan.	Hard Sandy Mud	RB	8-14 DB

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TABLE I (Con'td.)

Note:

(1) In the above table, FB indicates "free-boundary" reflection at the bottom, RB indicates "rigid-boundary" reflection at the bottom, and TR indicates that the character of the reflection is intermediate between FB and RB, or that it changes with frequency.

(2) All locations were in the Potomac River area as follows:

Glymont (A) - in mid-channel 500 yards up river from the dock at Indian Head, Md.

Glymont (B) - 200 yards north of the flashing red light at Glymont, Md.

FR Bridge (A) - on the NRL range course 300 yards south of the Potomac River Bridge near Morgantown, Md.

FR Bridge (B) - 200 yards east of the range course and 300 yards south of the River Bridge.

Piney Pt (A) - 600 yards south of Piney Point light (Lat. $38^{\circ}07'50''$, Long. $76^{\circ}32'0''$) where the chart shows hard bottom.

Piney Pt (B) - in mid-channel east of Ragged Point (Lat. $38^{\circ}09'22''$, Long. $76^{\circ}33'59''$).

Piney Pt (C) - 3000 yards south of Piney Point light (Lat. $38^{\circ}06'40''$, Long. $76^{\circ}32'0''$).

River Mouth - 3000 yards south of St. George Island (Lat. $38^{\circ}04'16''$, Long. $76^{\circ}28'08''$), where the chart shows hard bottom, and where range runs were made in performance tests on the XUR-2 sound generator.

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TABLE II

LOW FREQUENCY BOTTOM IMPEDANCE DATA FOR THE POTOMAC RIVER AREA.

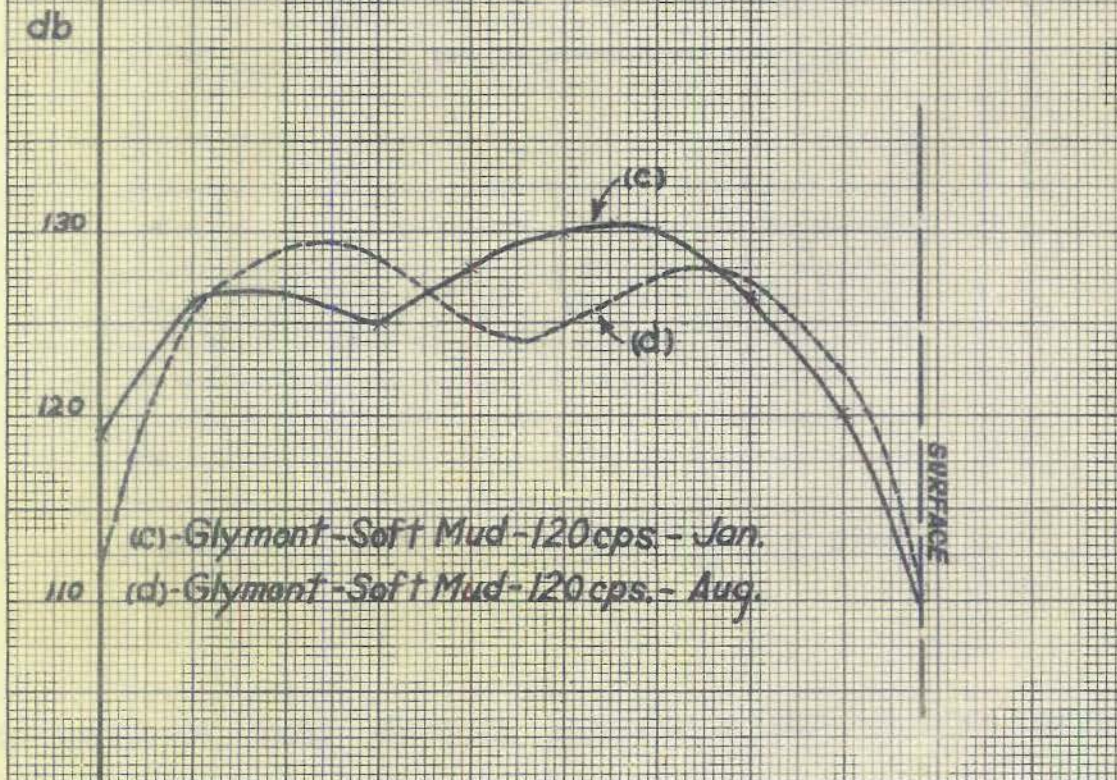
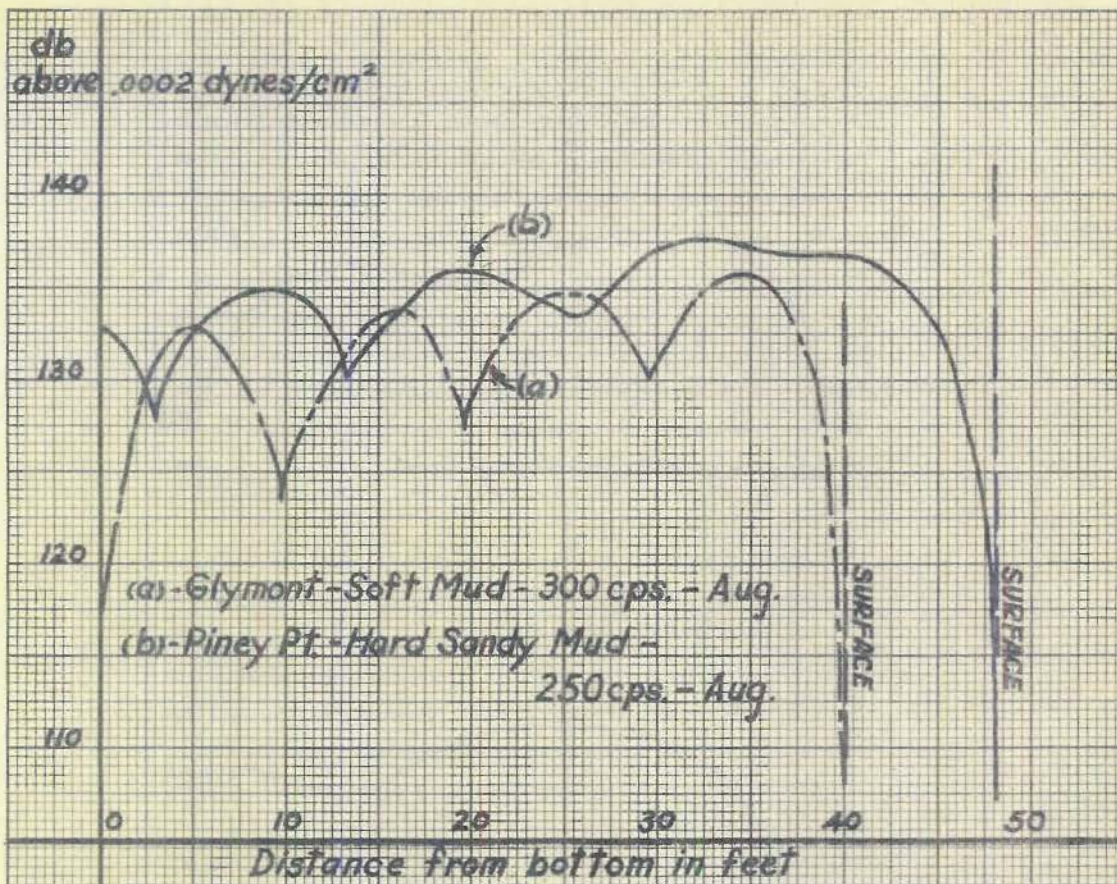
Record Group	Location and Date	Frequency cps	α_0	β_0	$R/\rho c$	$X/\rho c$	$IZ/\rho c$
a	Glymont A - Aug.	120	0.05	0.00	0.15	0.00	0.15
		200	0.05	0.00	0.15	0.00	0.15
		300	0.05	0.00	0.15	0.00	0.15
b	Glymont A - Jan.	100	0.16	0.10	0.50	0.25	0.56
		120	0.13	-0.15	0.42	-0.47	0.65
		240	0.10	-0.50	0.75	-1.00	1.25
c	Glymont B - Feb.	100	0.10	0.00	0.30	0.00	0.30
		200	0.10	-0.08	0.33	0.25	0.41
		300	0.10	-0.15	0.38	-0.45	0.59
		400	0.10	-0.20	0.45	-0.62	0.77
d	PR Bridge A - Aug.	100	0.12	-0.04	0.34	-0.10	0.35
		200	0.12	-0.04	0.34	-0.10	0.35
		300	0.14	-0.06	0.41	-0.14	0.43
		400	0.16	-0.10	0.50	-0.25	0.56
e	PR Bridge A - Jan.	100	0.30	0.10	0.77	0.14	0.78
		200	0.30	-0.20	0.88	-0.26	0.92
f	PR Bridge B - Aug.	100	0.08	-0.04	0.25	-0.10	0.27
		200	0.05	-0.10	0.18	-0.32	0.37
		250	0.05	-0.12	0.18	-0.37	0.41
		300	0.07	-0.25	0.40	-0.90	1.00
		400	0.08	-0.35	1.00	-1.40	1.70
		500	0.10	-0.50	3.20	0.00	3.20
g	Piney Pt. A - Aug.	100	0.15	-0.35	1.30	-0.80	1.50
		150	0.12	-0.50	2.70	0.00	2.70
		200	0.12	-0.50	2.70	0.00	2.70
		250	0.12	-0.50	2.70	0.00	2.70
		400	0.14	0.35	1.20	1.00	1.60
h	Piney Pt. A - Jan.	100	0.40	-0.46	1.20	-0.05	1.20
		200	0.25	-0.50	1.50	0.00	1.50
		300	0.20	0.45	1.70	0.30	1.70

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TABLE II (Con'td.)

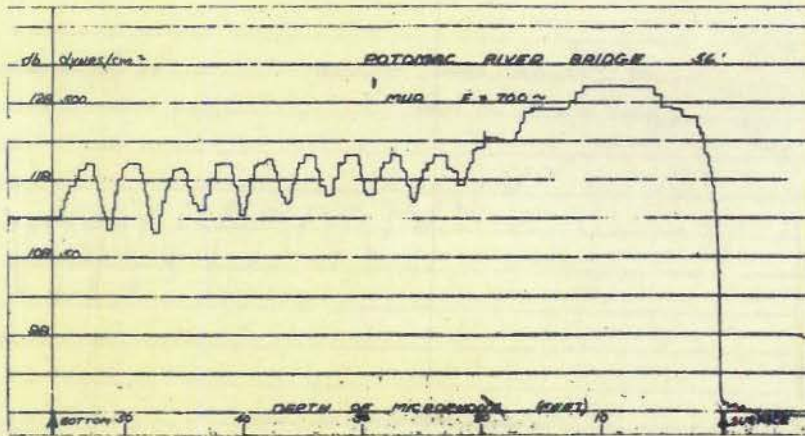
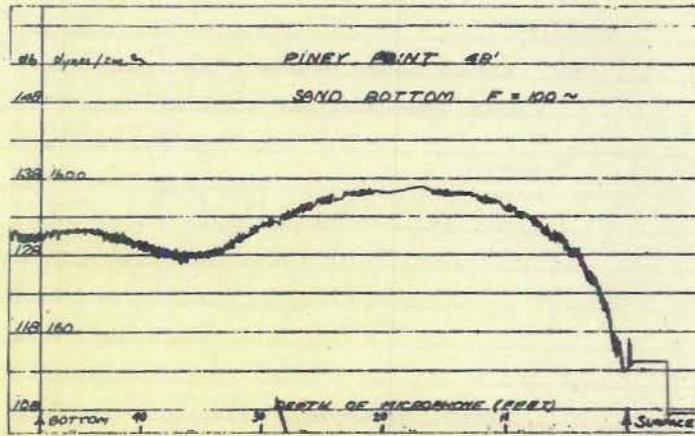
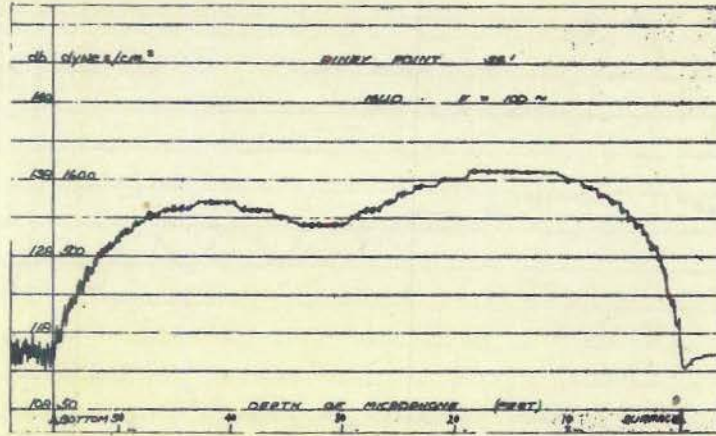
Record Group	Location and Date	Frequency cps	α_0	β_0	R_{pc}	X_{pc}	$ Z _{pc}$
i	Piney Pt. B - Aug.	100	0.10	-0.04	0.30	-0.10	0.32
j	Piney Pt. B - Jan.	100	0.30	0.42	1.30	0.20	1.30
		200	0.30	-0.30	1.10	-0.30	1.10
		300	0.25	-0.30	1.10	-0.45	1.20
		400	0.15	-0.20	0.60	-0.53	0.80
k	Piney Pt. C - Aug.	100	0.09	0.00	0.28	0.00	0.28
		150	0.10	0.00	0.30	0.00	0.30
		200	0.10	0.00	0.30	0.00	0.30
		300	0.10	0.00	0.30	0.00	0.30
m	River Mouth Jan.	100	0.25	-0.50	2.30	0.00	2.30
		200	0.17	-0.50	2.00	0.00	2.00
		300	0.18	0.30	1.00	0.70	1.20
		400	0.25	0.10	0.30	0.30	0.40
		500	0.17	0.40	1.60	0.70	1.80

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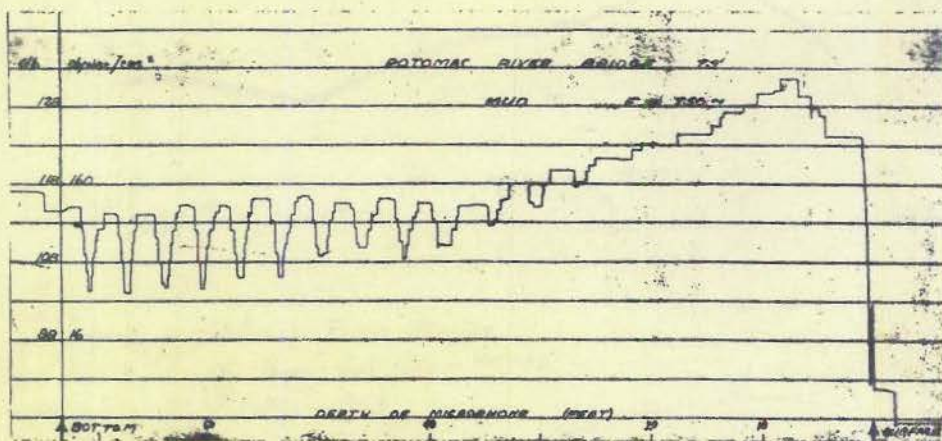
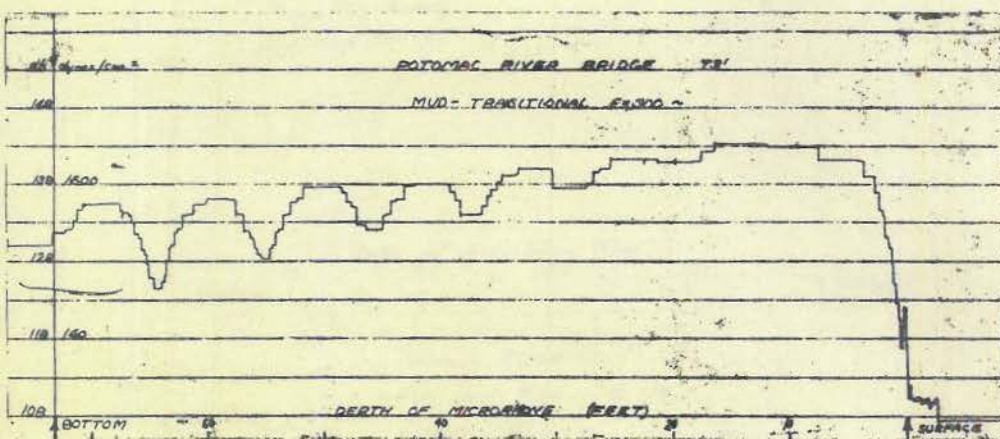
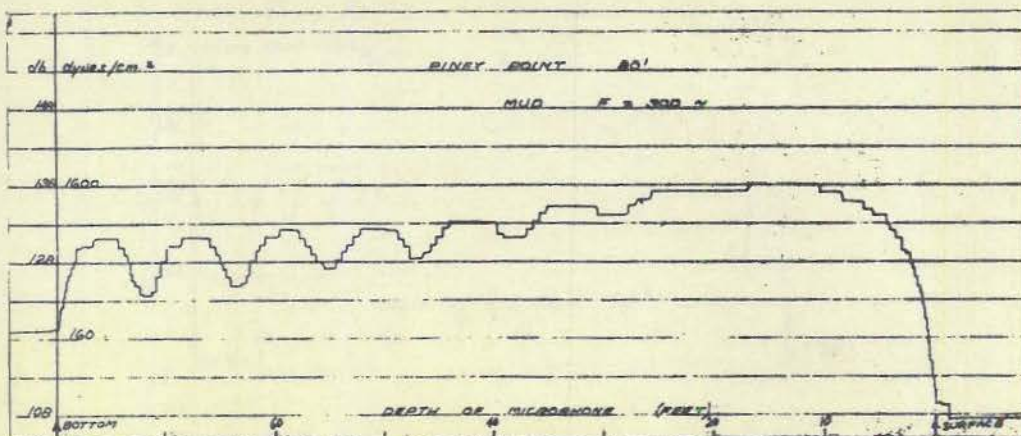


SOUND PRESSURE BENEATH A SHIP-MOUNTED SOURCE
PLATE I.

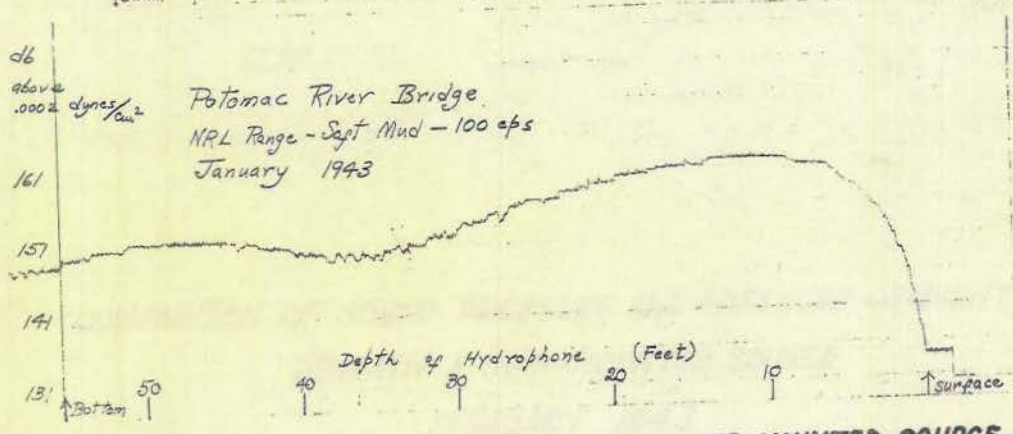
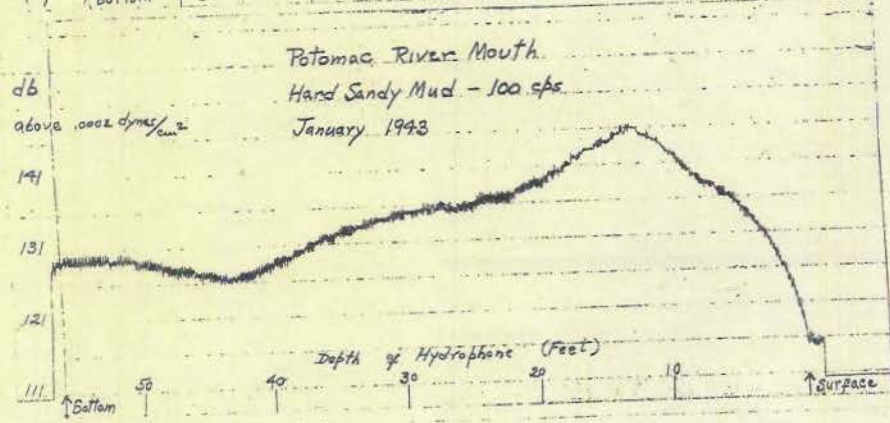
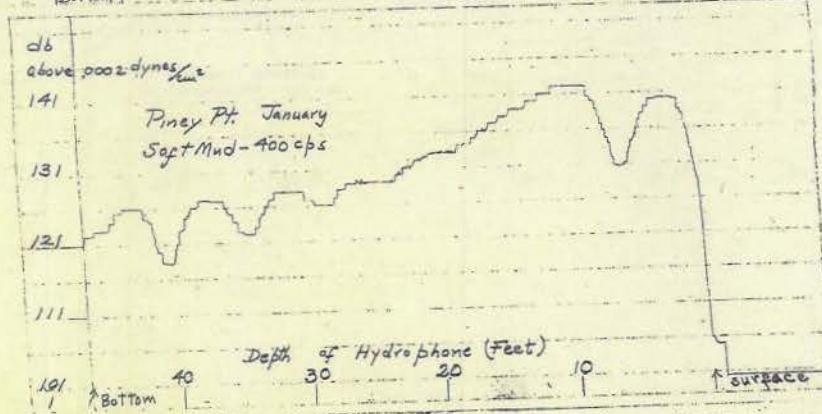
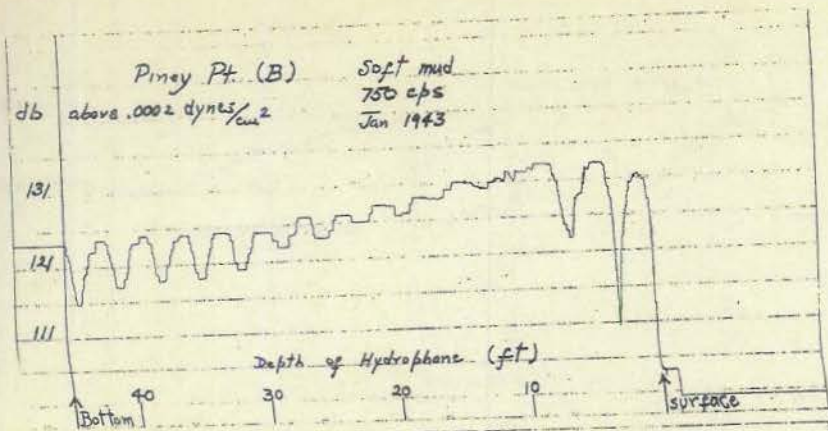
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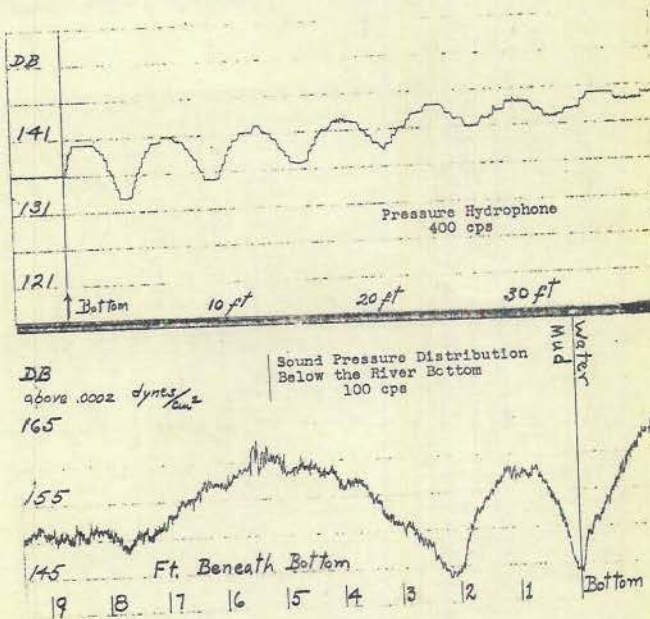
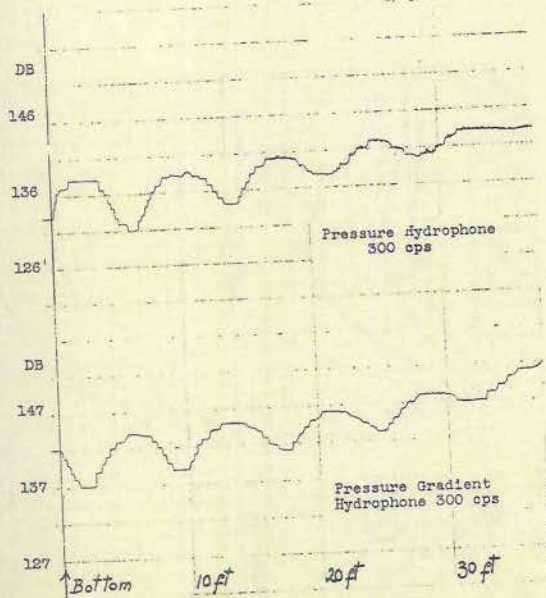
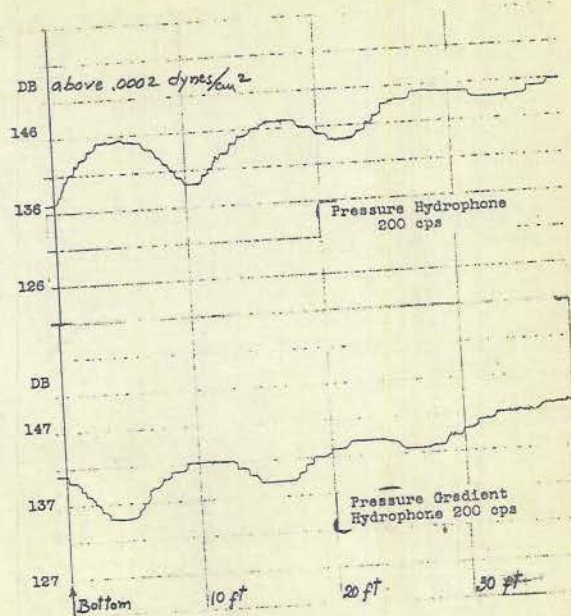
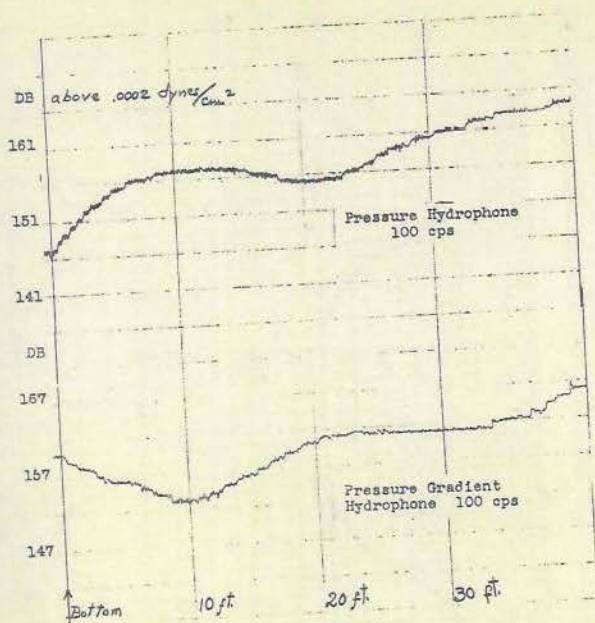
STANDING WAVES OF SOUND PRESSURE BENEATH A SHIP-MOUNTED SOURCE



STANDING WAVES OF SOUND PRESSURE BENEATH A SHIP-MOUNTED SOURCE



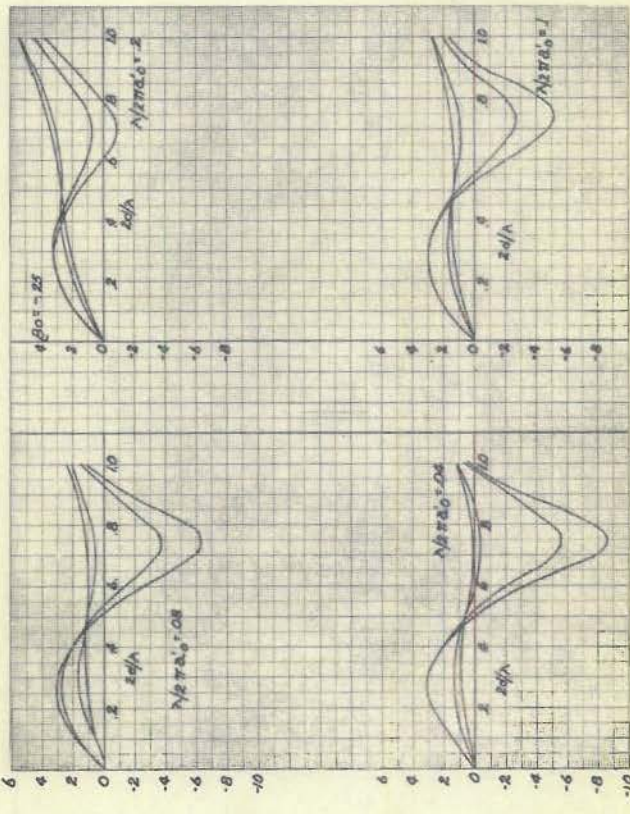
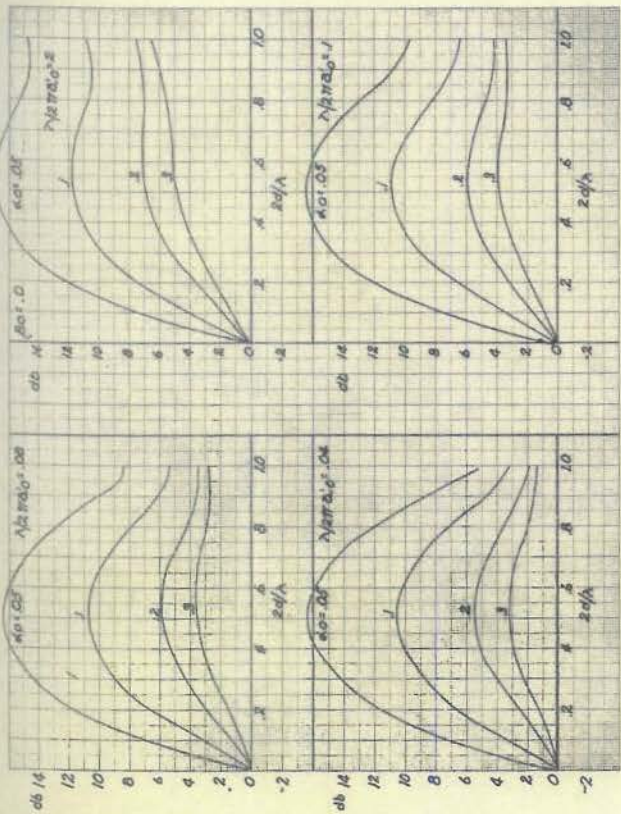
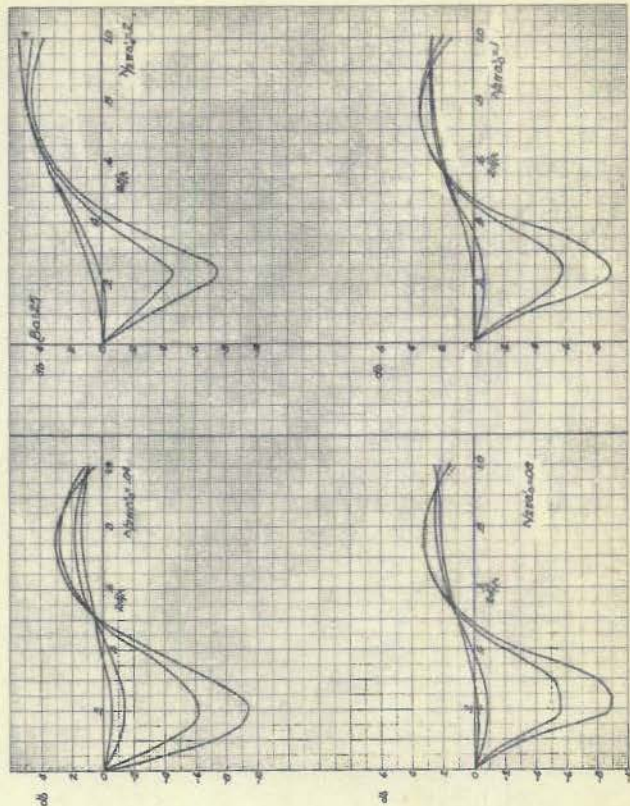
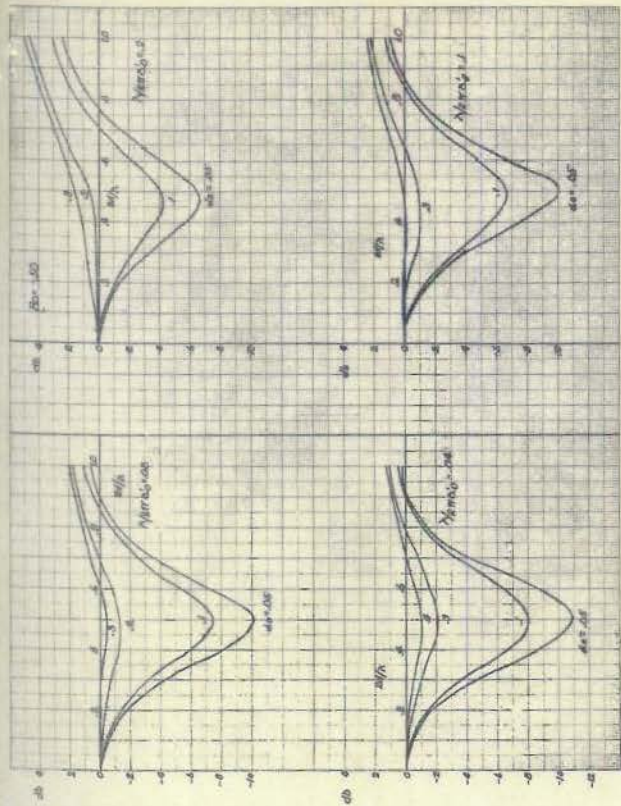
STANDING WAVES OF SOUND PRESSURE BENEATH A SHIP-MOUNTED SOURCE



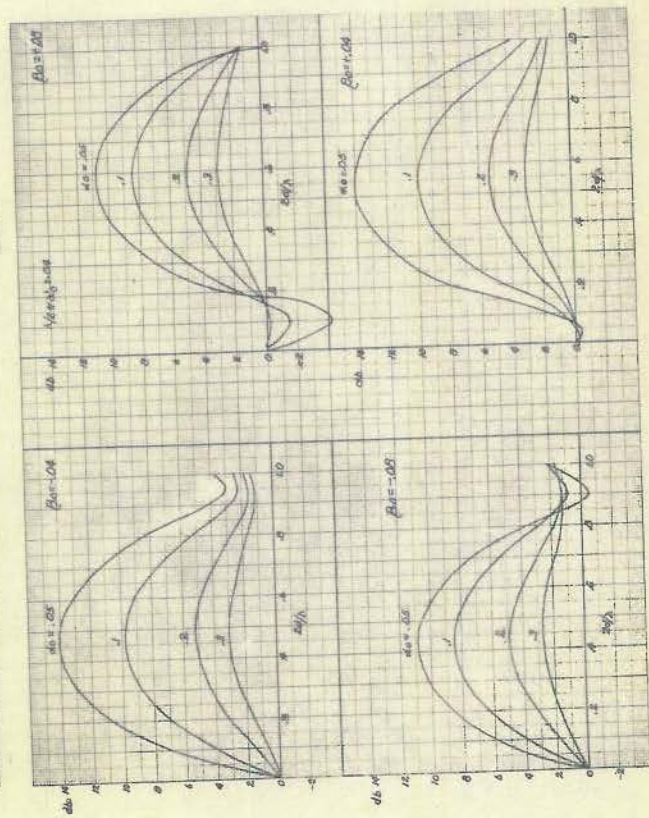
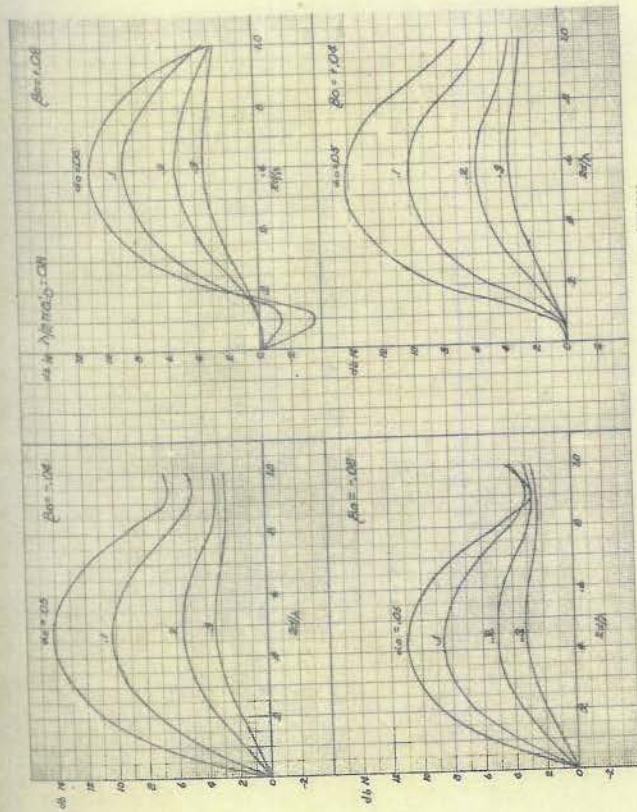
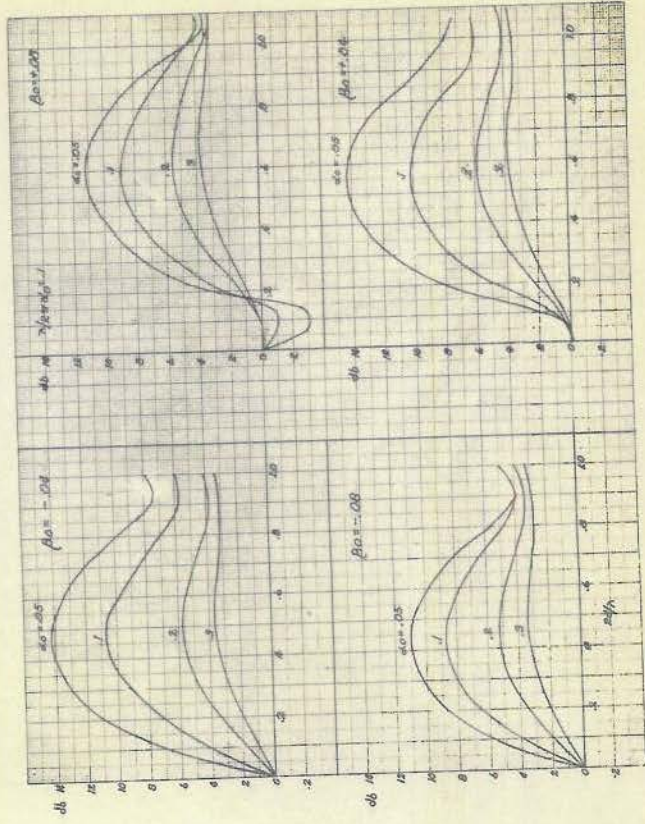
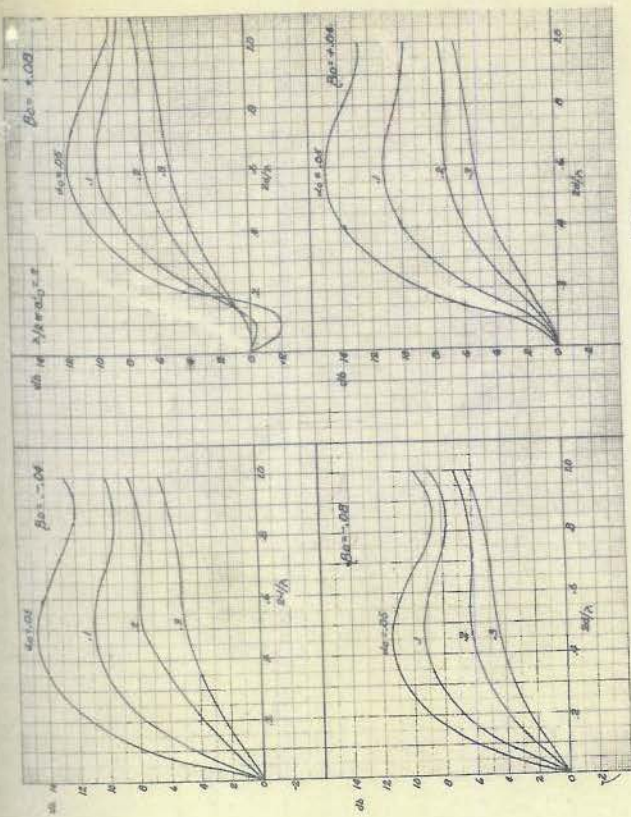
COMPARISON OF SOUND PRESSURE AND PRESSURE GRADIENT
BENEATH A SHIP-MOUNTED SOURCE
FEBRUARY 1943

PLATE 5

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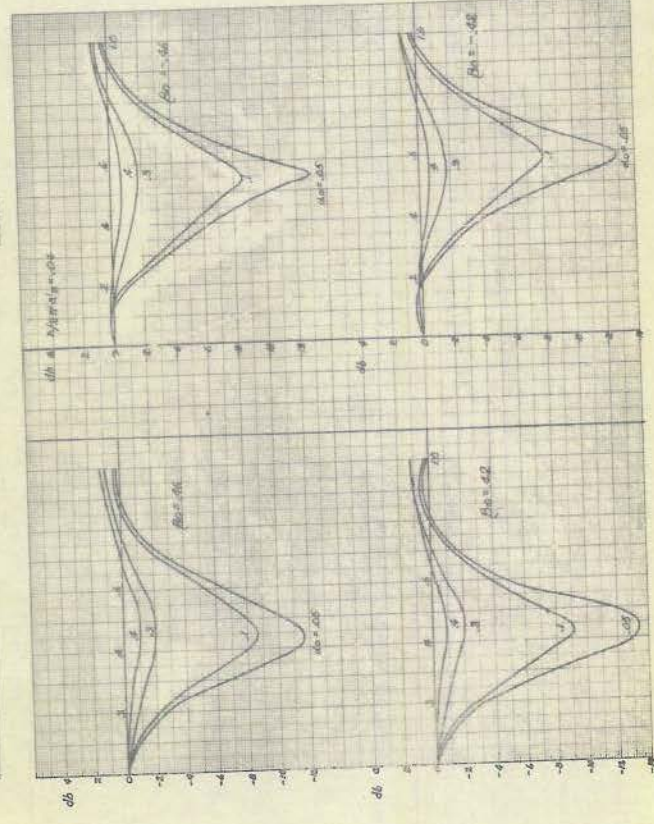
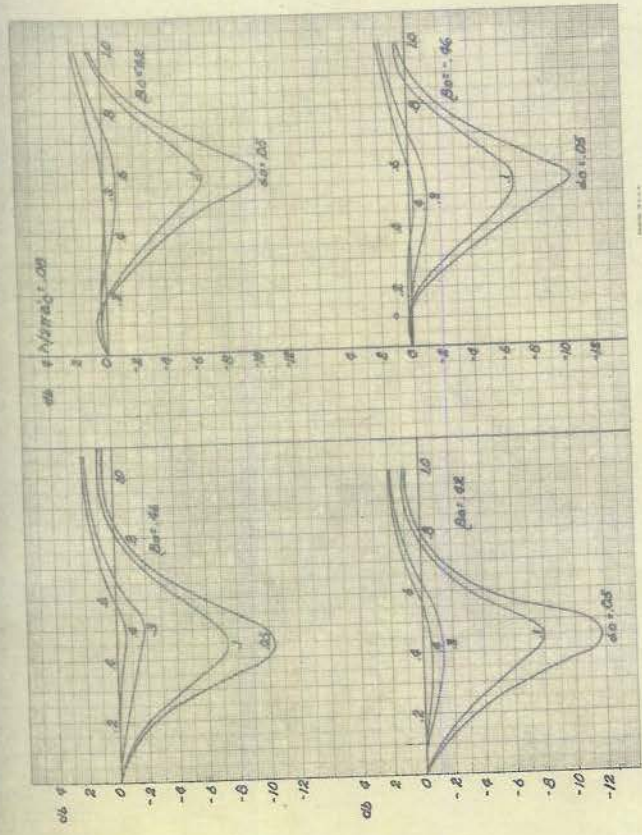
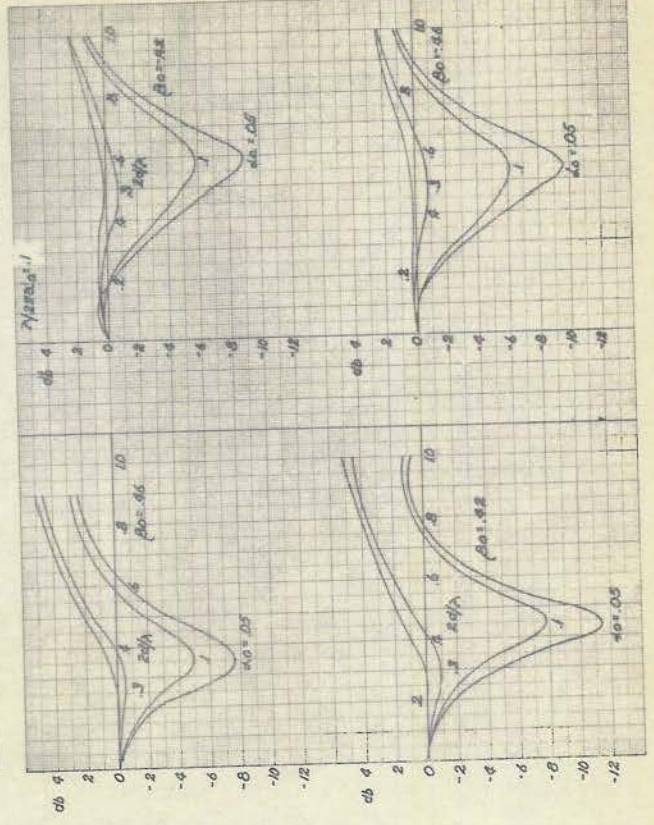
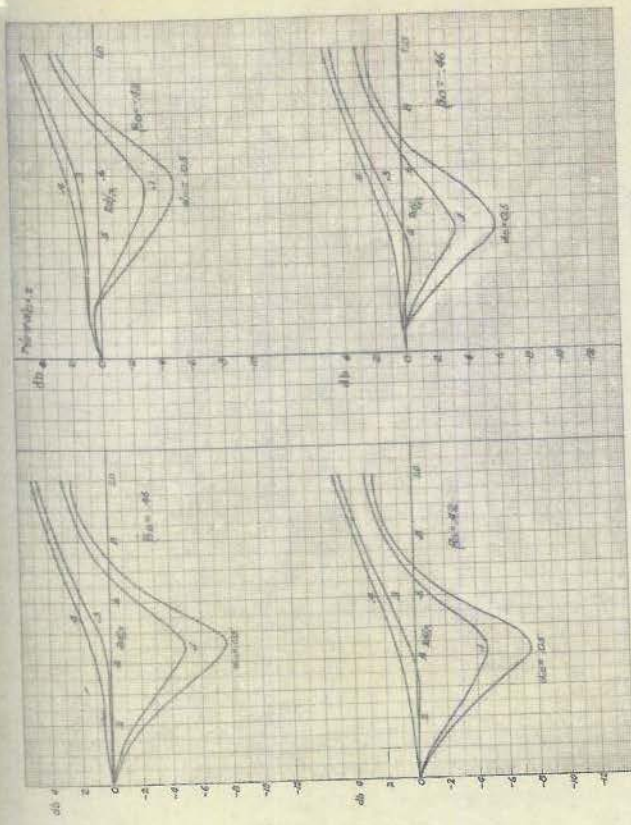


COMPUTED CURVES FOR DETERMINATION OF ABSORPTION AND PHASE CONSTANTS



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