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NAVY DEPARTMENT

Report on

A PRESSURE-ACTUATED DEVICE FOR ARMING MINES

NAVAL RESEARCH LABORATORY
ANACOSTIA STATION
WASHINGTON, D. C.

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TABLE OF CONTENTS

ABSTRACT	
PERSONNEL	
I. INTRODUCTION	1
II. PRESSURE EFFECTS	1
III. PRINCIPLE OF THE ARMING DEVICE	1
IV. COMPENSATION FOR DEPTH	2
V. COMPENSATION FOR WAVE AND TIDAL EFFECTS	3
VI. PROBLEM OF SWEEPING	4
APPENDIX A. ORIGIN OF THE PRESSURE EFFECT	5
APPENDIX B. EFFECT OF WAVES AND TIDES	8
APPENDIX C. THEORY OF THE PRESSURE ARMING DEVICE	9
1. General Considerations	9
2. The Mechanical System	9
3. The Equivalent Electrical Circuit	10
4. Units Used in the Calculations	11
5. Calculation of Circuit Parameters	11
6. The Approximate Circuit	18
7. Steady State Solution	18
8. Step Function Solution	19
9. External Pressure Variation Required to Produce Contact	20
10. Indeterminate Elements	21
BIBLIOGRAPHY	24

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TABLES

- | | | |
|-----|---|----|
| I. | Steady State Response of the Pressure Arming Device | 22 |
| II. | Response of the Pressure Arming Device to a Step Function | 23 |

PLATES

1. Pressure Arming Device Assembled
2. Details of Upper Chamber
3. Pressure Arming Device Unassembled
4. Source-Sink Distribution about a Ship
Schematic Diagram of Disarming Circuit
5. Schematic Diagram of Pressure Arming Device
Schematic Diagram of Equivalent Mechanical Arrangement
6. Analogous Electrical Circuit - Mobility Basis
Direct Analogous Electrical Circuit
Approximate Analogous Electrical Circuit
7. Equivalent Circuit for Compressibility of a Gas
Section for Calculating Viscous Flow through Pipe
Circuit for Calculating Steady State Response
8. Response to a Unit Step Function
9. Pressure Variations with Depth
10. Pressure Required for Arming vs. Depth
11. Sensitivity vs. Depth
12. Schematic Sectional View
- 12A. Sketch of Arming Mechanism

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ABSTRACT

A mine-arming device, operated by the hydrostatic pressure effects produced beneath a moving vessel, is described. The problem of compensating the device for depth changes and for hydrostatic pressure changes by waves and tides is discussed, and it is stated that sweeping for mines armed with the subject device would appear to be very difficult. The appendices present a review of the mathematical analyses of the effects produced by a ship, the effects of waves and tides, and the mechanical behavior of the device.

PERSONNEL

The device described and discussed in this report was designed at the Naval Research Laboratory by Mr. G. M. Cross, who also performed the major part of the development and experimental work on which the report is based. The preliminary description of the device and of its functions was written by Mr. William W. Stifler and the mathematical analyses were prepared by Dr. Horace M. Trent.

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I. INTRODUCTION

1. The subject of this report is a pressure-actuated device which arms a mine on the approach of a ship. The instrument described is simple and compact, and should lend itself to quantity production. It can be employed with any type of electrical firing circuit and is designed to operate in 40 to 120 feet of water. An average cargo ship, 200 feet in length, passing directly over this device at a speed of 6 knots produces sufficient pressure change to actuate it and thus to arm the mine.

II. PRESSURE EFFECTS

2. Considerable research has been done on the hydrostatic pressure variations produced on the ocean floor when a surface ship passes overhead. It has been established experimentally that there are regions of increased pressure on the ocean floor ahead and astern of a surface ship, and a decreased pressure region somewhere amidships. The maximum negative pressure variation from the mean is greater than either of the positive variations, and is therefore more susceptible of mine warfare applications.

III. PRINCIPLE OF THE ARMING DEVICE

3. This negative pressure variation has been considered as a means for actuating both mine-firing and mine-arming mechanisms. The function of the subject device, a mine-arming mechanism, is to turn on power supplies and to make the mine stand by to fire when given the signal by some other device, such as a sound or magnetic detector.

4. Plate 1 shows the arming device assembled; Plate 2 is a view of the inside of the head, Plate 3 shows the major parts disassembled, and Plate 12-A is a sketch of the mechanism within the head.

5. The principle of operation of the device is shown schematically in Plate 12, wherein numerals (1) and (2) represent two chambers separated by an intervening diaphragm (3) which responds to a differential of pressure between the two chambers to move a platinum wire contact (4) through cooperative action between the membrane stilus (5), a pivot bearing (6), and the lever arm (7). A hair spring (8) operates to exert a light pressure between members (5) and (7).

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6. A second electrode (9) is carried by arm (10), attached to insulating collar (11). This collar rotates about pivot (12) through pressure action on a Bourdon gauge (13), the interior of which communicates through tube (14) with an atmospheric pressure chamber in the mine itself.

7. When the mine is planted, sea water enters chamber (2) through screen (15), thereby compressing the air in a thin rubber sack (16), flexing diaphragm (3) upwards against supporting plate (17), and increasing the air gap between (9) and (4) to a maximum.

8. During the mine's descent to its operating depth, the excess pressure on the lower side of diaphragm (3) tends to disappear through passage of air from within sack (16) to chamber (1) through a slow leak (18) adjusted by screw (19). This passage of air through the slow leak continues after the mine has reached the ocean floor until a pressure equilibrium condition between the two chambers obtains.

9. Since the interior of the Bourdon gauge (13) remains at atmospheric pressure, the increased pressure within chambers (1) causes a counterclockwise rotation of arm (20) about pivot (21), and, through action of the coupling arms (23) and (22), a counterclockwise rotation of collar (11). This rotation moves contact (9) along a path shown by the dotted arc.

10. After equilibrium is established, and the position of contact (9) is fixed, any subsequent relatively sudden pressure drop on the ocean floor is transmitted to the air within the rubber sack. This pressure drop produces a downward deflection of diaphragm (3), which, in turn, produces a counterclockwise rotation of contact arm (4). If the pressure difference between chambers (1) and (2) is sufficiently great, contact (4) touches contact (9), closing the power supply switch which arms the mine. The contact leads (24) and (25) connect with a source of s.m.f. within the mine case.

IV. COMPENSATION FOR DEPTH

11. Since the effects of surface disturbances are attenuated with depth, the sensitivity of the mechanism must be made to increase with depth. This is accomplished by the action of the Bourdon gauge. As the pressure in chamber (1) increases, arm (10) is made to rotate in a counterclockwise direction as explained in paragraph 9. This rotation lessens

the static distance between contacts (9) and (4) and makes the device more sensitive to pressure variations.

12. The proper operation of the device depends primarily upon making contact (4) of a form that gives a relation between contact air gap and depth of mine substantially as shown by the curve of Plate 10, and setting air leak adjustment (19) to a proper value.

V. COMPENSATION FOR WAVE AND TIDAL EFFECTS

13. Natural pressure variations produced by wave and tidal effects are frequently as great as those produced by surface ships. To build a mine capable of differentiating between ships and natural phenomena under all possible conditions is very difficult. The subject device is, however, designed to filter out the tidal effects and to remain completely inoperative during stormy weather, as will be explained in the following paragraphs.

14. The difficulty introduced by tidal effects depends upon the maximum rate of change of depth during ebb tide. This difficulty is solved by the slow leak. The size of the leak is adjusted so that air flows from the head into the rubber sack rapidly enough so that a pressure difference great enough to actuate the mechanism is not established merely by tidal flow.

15. To conserve batteries during stormy weather, the device is made to operate the one-tube circuit shown in Plate 4, Fig. 2, in which K represents the contacts that are closed by the pressure sensitive device. The tube is of the cold cathode glow type which requires no filament current for its operation. At the instant K closes, the potential of the grid is raised to the value of the B supply and the tube becomes conducting.

16. Since the tube draws its plate current through the relay, the contacts are closed, supplying power to the firing mechanism of the combination mine. At the same time C is charged to near 135 volts. This condition continues to exist until the TDM opens the cathode circuit approximately 20 seconds later. The tube cannot be rendered conducting again until C has partially discharged through R_1 , the approximate time required for this delay being 45 seconds. If K closes before 45 seconds have elapsed, C is recharged without the tube's conducting again. In this event 45 seconds must elapse again before the auxiliary apparatus can be powered.

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17. Even in stormy weather, the swells have periods mainly between 15 and 20 seconds. It is easily perceived that, except for one 20 second period, the apparatus will be entirely disarmed during periods of high waves. It is estimated that this condition will exist between 15 and 20 percent of the time.

VI. PROBLEM OF SWEEPING

18. As has been stated, a relatively sudden drop in hydrostatic pressure, followed immediately by the disturbance which actuates the detonating mechanism, is required to explode a mine armed with this device. Therefore, to sweep such a mine, it becomes necessary to produce both a drop in hydrostatic pressure and the required acoustic or magnetic phenomenon at approximately the same time.

19. This can be done during a period when water conditions are becoming rough, as, for example, before a storm. Since the mine remains continuously unarmed after rough water conditions have once been established, it is necessary to make use of the initial 20 second period during which the mine is armed by the first large wave.

20. This can be accomplished if the disturbance required for actually firing the mine is produced continuously in the mined area during the approach of a storm. In the case of acoustically detonated mines, this may prove possible. It appears to be difficult, if not impossible, in the case of magnetically detonated mines.

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APPENDIX A.ORIGIN OF THE PRESSURE EFFECT

21. The action of the subject device is based upon the well established fact that there is a reduction of the pressure on the ocean floor when a ship passes directly overhead. It has been established experimentally that the effect increases as the square of the speed of the ship for ship speeds up to about 10 knots (MUR #324), while the variation with depth is a function of the length and shape of the ship and the depth of the water at that point.

22. The theory of the production of the pressure effect has been presented adequately both by the Naval Ordnance Laboratory and the British Admiralty (Bibliography 1 and 2). For the purposes of this report, a resume of the assumptions and findings reported will be sufficient.

23. The methods of analysis employed in the above references are essentially the same. By the use of Bernoulli's theorem, the pressure at any point in an incompressible fluid can be expressed in terms of the head of the fluid, the externally applied pressure, and the velocity of the fluid at the point. For the purposes of the present analysis, the first two parameters are assumed to remain constant. It should be emphasized that this assumption requires that the wave motion initiated by the passage of the ship shall be neglected. Since the size and form of this wave motion is a complicated function of the outline of the ship's hull and the speed at which it is traveling, simplicity of analysis requires that this assumption be made. Some difference between the theoretical predictions and the experimental findings is therefore to be expected.

24. It is assumed that the hydrodynamical action of a ship is equivalent to a source located just aft of the bow and an equal sink located just forward of the stern. In order to assure that the motion of the water at the bottom shall have no normal component, it is necessary to postulate an identical image source and sink distribution as far below the bottom as the original distribution is above. This assumption of a rigid bottom may be questionable with a soft muddy bottom, but is probably accurate enough at the low frequencies involved. The resulting configuration is illustrated in Plate 4, Fig 1. Some

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Improvement in the theory would be obtained if distributed sources and sinks were used instead of a point distribution, but the improvement in the accuracy so obtained would not be worth the increase in the complexity of the analysis.

25. In order that the equations describing the motion of the water shall be independent of time, coordinates are assigned which have their origin moving with the ship. It is further assumed the motion of the water is irrotational and that $\frac{1}{2}\rho v_x^2 \ll \rho v_x V$ where ρ is the density of water, v_x is the horizontal component of the water velocity, and V is the ship's velocity. Under these conditions, Bernoulli's equation can be reduced to

$$P = \text{Constant} + \rho v_x V \quad (1)$$

where P is the pressure at the given point. If the value of v_x can be calculated, the change in pressure can be deduced at any point.

26. The source strength, S , is assumed to be proportional to AV , where A is the maximum cross-section of the ship below the waterline. The velocity of the water due to a source S at a distance r is given by $v = S/2\pi r^2$. Using this expression for S , it is easy to show that for the configuration of Plate 4, Fig 1, the change in pressure at a point on the bottom at the center of the source-sink distribution is given by the expression

$$\Delta P_h = -\frac{2S\rho}{\pi} \frac{V}{(l^2 + h^2)^{3/2}} \propto -\frac{2\rho l AV^2}{\pi(l^2 + h^2)^{3/2}} \quad (2)$$

wherein the negative sign arises from the fact that v_x and V are in opposite directions.

27. This expression illustrates the dependence of the effect on the ship's speed and the depth of the water. Different writers introduce slightly different proportionality constants in the above expression. The actual value of the proportionality constant need not concern us here since we are primarily interested in the variation of the effect with depth. A useful way of showing this relation can be arrived at as follows. Let ΔP_0 represent the value of the pressure change for $h=0$, and ΔP_h the pressure change on the bottom where the depth of the water is h . From the geometry of Plate 4, Fig 1, the value of ΔP_0 is seen to be

$$\Delta P_0 = -\frac{2S\rho}{\pi} \frac{V}{l^2} \propto -\frac{2\rho l AV^2}{\pi l^2} \quad (3)$$

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The ratio of ΔP_h to ΔP_s becomes

$$\frac{\Delta P_h}{\Delta P_s} = \left[1 + \left(\frac{h}{r} \right)^2 \right]^{\frac{1}{2}} \quad (4)$$

The above relations are modified in practice by the wave motion set up by the ship, the turbulence about the hull, especially near the propeller, and the wake behind the ship.

28. Actual measurements of the pressure variations on the bottom caused by the passage of a ship show a small excess pressure under the bow, a strong pressure minimum under the center of the ship, a small excess pressure under the stern, followed by small oscillations caused by the wave train left by the ship. The pressure arming device is designed so that an electrical contact is made by the action of the large pressure reduction occurring amidships.

APPENDIX B.EFFECT OF WAVES AND TIDES

29. Unfortunately, both waves and tides can produce pressure fluctuations as great as those produced by the passage of a ship. Fluctuations caused by waves are particularly troublesome, for the periods of the large waves appearing during stormy weather are comparable in duration to the effect caused by the passage of a ship. Because the period of tidal fluctuation is extremely long, the apparatus is able to filter out the effect. On the other hand, it is not easy to separate the effects of waves and ships on the basis of their periodicities.

30. The theory of the pressure effects caused by waves has been treated by Naval Ordnance Laboratory (Bibliography 3). Briefly, the mathematical technique consists of setting up a velocity potential function for the wave motion, from which the particle velocity at any point is derived by forming the directional derivative of the potential function. The particle velocity so derived is substituted in Bernoulli's equation, thus yielding an expression for the pressure. This analysis yields the following formulas:

$$A = \frac{B}{c} \sinh kh,$$

$$c^2 = \frac{g}{k} \tanh kh,$$

$$T = \frac{2\pi}{kc} \quad (\text{when } h > \lambda), \text{ and}$$

$$P = \text{Constant} + \frac{\rho g A}{\cosh kh} \cos kx,$$

where

A = Amplitude of the wave motion,

c = Velocity of the waves,

λ = Wave length,

$k = \frac{2\pi}{\lambda}$,

h = Depth of water,

B = Constant,

T = Period of the wave motion,

g = Acceleration due to gravity,

P = pressure at the bottom, and

ρ = Density of the water

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APPENDIX C.

THEORY OF THE PRESSURE ARMING DEVICE

General Considerations

31. Both wave and tidal effects can be considered as harmonic variations of pressure. The former have periods ranging in general between 4 and 20 seconds and the latter have periods of about 12 hours and 25 minutes. The pressure fluctuations caused by the passage of a ship have much the appearance of harmonic variations, but the phenomenon is over before a steady state can be realized in the arming device. The reaction of the device to waves and tidal effects will be shown by calculating its steady state response to harmonic pressure fluctuations covering the significant frequency ranges and its transient nature by calculating its response to a step function.

The Mechanical System.

32. A schematic diagram of the pressure-sensitive device is shown in Plate 5, Fig 1. It consists of four Helmholtz resonators interconnected. As long as the pressure fluctuations are very small compared to the mean pressure, a Helmholtz resonator can be shown to be equivalent to a mass acted upon by a spring, a driving force, and a viscous force (Bibliography 4). The viscous force is made up of two components, a radiation resistance and a viscosity. Bibliography (4) gives for the former

$$R = \frac{\rho \omega^2}{2\pi c} S^2,$$

where

ρ = Density of the fluid,
 ω = $2\pi \times$ frequency,
 c = velocity of sound in the fluid,
 S = Cross-sectional area of the orifice, and
 R = Radiation resistance.

At the frequencies of importance for this device, this radiation component becomes entirely insignificant. The only viscous forces that need to be considered are those arising from the effect of the two gauges and the external leak.

33. In a Helmholtz resonator, the fluid enclosed within the chamber acts as a spring. The fluid will be compressed in general by a combination of isothermal and of adiabatic processes. This is dynamically equivalent to a spring acting in parallel with

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a combination consisting of a spring and a viscous conductivity in series. It will be shown later that the external leak is equivalent to a viscous conductance.

34. From the foregoing brief introduction, it is apparent that a mechanical assembly constructed of conventional springs, masses, and viscosities can be constructed which will have for its description the same integro-differential equations as the subject device. Such an arrangement is shown in Plate 5, Fig 4, in which conventional symbols are used (Bibliography 7). The symbols have the following meanings:

$F =$	external driving pressure,
$M_1 =$	effective mass of water in the lowest orifice,
$M_2 =$	effective mass of air in the opening between V_2 and V_3 ,
$M_3 =$	effective mass of the diaphragm,
$M_4 =$	effective mass of air in opening between V_4 and V_5 ,
$S_1 =$	elastic effect of water in V_1 ,
$S_2 \text{ \& } S'_2$	elastic effect of air in V_2 ,
$S_3 \text{ \& } S'_3$	elastic effect of air in V_3 ,
$S_4 =$	elastic effect of the diaphragm,
$S_5 \text{ \& } S'_5$	elastic effect of air in V_4 ,
$S_6 \text{ \& } S'_6$	elastic effect of air in V_5 ,
$G_1 =$	viscous effect of lower gauze,
$G_2 =$	thermal conductivity in walls surrounding V_2 ,
$G_3 =$	viscous effect of upper gauze,
$G_4 =$	viscous effect of the external leak,
$G_5 =$	thermal conductivity in walls surrounding V_3 ,
$G_6 =$	thermal conductivity in walls surrounding V_4 , and
$G_7 =$	thermal conductivity in walls surrounding V_5 .

The Equivalent Electrical Circuit

35. It is possible to establish an electrical circuit which will have the same integro-differential equations for its description as would be used in the description of a given mechanical arrangement. Firestone (Bibliography 5), and Miles (Bibliography 6), have pointed out that if force is made analogous to current and velocity analogous to voltage, the equivalent electrical array can be drawn by inspection. Such an

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equivalent electrical circuit is shown in Plate 6, Fig 5, in which the subscripts have the same meanings as in the equivalent mechanical structure. It must be remembered that in this analogy, masses become capacitances, and springs becomes inductances. The quantity of interest is the current through the parallel combination of C_3 and L_4 , for this will be equal to the net force acting on the diaphragm.

36. Gardener and Barnes (Bibliography 7) describe a simple graphical method of constructing a circuit which will have descriptive integro-differential equations of the same form as a given circuit, but with the roles of current and voltage interchanged. The circuit shown in Plate 6, Fig 6, was constructed in this manner from Fig 5. With the proper assignment of circuit parameters, it will have the same set of descriptive equations as the original mechanical device. In this circuit, voltage is analogous to pressure, current to velocity, capacity to elastance, inductance to mass, and resistance to viscosity. Since this so-called "Direct Analogy" is the analogy most widely known, all subsequent calculations will be based upon it. The important item to be calculated is the voltage across C_4 , since this is proportional to the diaphragm displacement.

Units Used in the Calculations.

37. In the calculations that follow, the unit of pressure will be taken as the pressure due to an inch head of water, and the unit of volume will be the cubic inch. The density of water then becomes approximately $1/385$, and the unit of mass 13.91 lbs. A change of pressure of 1 inch of water will be taken to be equivalent to 1 volt, and a negative change in volume of 1 cubic inch to be equivalent to one coulomb.

Calculation of Circuit Parameters

38. Constants of the Fluid Volumes.

The circuit contains four combinations of the type shown in Plate 7, Fig 8, which represents the elastic properties of the fluid and the thermal conductivity at the walls of the vessel. In order that this circuit may be a true representation of the action of the fluid, the following conditions must be satisfied:

- 1) when $R = 0$, the gas reacts isothermally,
 - 2) when $R = \infty$, the gas reacts adiabatically,
- and

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- 3) when R is finite, the rate of change of charge q arising from a unit step function of voltage must be the negative of the rate of change of volume v arising from a unit step function of pressure. The negative sign appears because dq is analogous to $-dv$.

Condition 1) gives $Pv = \text{Constant}$,

$$Pdv + vdP = 0,$$

$$-dv = \frac{v}{P} dP \approx \frac{v_0}{P_0} dP, \quad (1)$$

Therefore

$$C = \frac{v_0}{P_0}. \quad (2)$$

Condition 2) gives

$$Pv^\gamma = \text{Constant}$$

$$\gamma v^{\gamma-1} P dv + v^\gamma dP = 0$$

(3)

$$-dv = \frac{v}{\gamma P} dP \approx \frac{v_0}{\gamma P_0} dP$$

$$\frac{1}{C} + \frac{P_0}{v_0} = \frac{\gamma P_0}{v_0}$$

$$\frac{1}{C} = (\gamma - 1) \frac{P_0}{v_0}$$

$$C' = \frac{1}{\gamma - 1} \frac{v_0}{P_0}$$

(4)

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From condition 3) the value of R can be established if it is assumed the temperature of the gas follows Newton's law of cooling, which can be written

$$-\frac{d\theta}{dt} = K(\theta - \theta_0) \quad (5)$$

where θ_0 is the equilibrium temperature, θ is the temperature at any instant t, and K is a constant. For the gas,

$$-\frac{dv}{dt} = K \frac{\gamma-1}{\gamma} \frac{v_0}{P_0} e^{-Kt} \quad (6)$$

represents the response to a unit step function; for the electrical circuit the equivalent expression is

$$\frac{dq}{dt} = \frac{1}{R} \left(\frac{C}{C+C'} \right)^2 e^{-\frac{t}{RC+C'C}} \quad (7)$$

These two equations are consistent, provided

$$R = \frac{1}{K(C+C')} = \frac{\gamma-1}{\gamma K} \frac{P_0}{v_0} \quad (8)$$

Since the walls of the device are made of thick bronze, which is a good conductor of heat, it can be assumed that K is large. This makes R a small quantity, and at the frequencies of interest it can be assumed that time constants of all RC combinations are so small that they do not materially affect the final results. Such combinations will be neglected hereafter.

39. From the design dimensions of the device, C_3 and C_5 are very small compared to C_2 , C_4 , and C_6 . Since they are parallel elements, they can be neglected in an approximate solution.

40. The elastance of the volume of water is represented by C_1 . This parameter can be shown to be negligible by the following analysis. By definition,

$$\text{Compressibility, } \eta = -\frac{1}{v_0} \frac{dv}{dP}$$

Also, $c^2 = \frac{1}{\eta \rho}$ where c is the velocity of sound in sea water (57,800 inches/second).

$$-dv = \eta v_0 dP = \frac{v_0}{c^2 \rho} dP \quad (9)$$

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$$C_1 = \frac{V_0}{c^2 \rho} \quad (10)$$

This quantity is of the order of $\frac{50 \times 385}{(57600)^2}$. Since this capacity is a parallel element, it can be neglected at the frequencies of immediate interest.

41. Calculation of the Equivalent Inductances.

In three instances, the inductances are due to the motion of a fluid through an orifice. In Bibliography 4, it is shown that the values of these parameters can be calculated by use of the relation

$$L = \frac{\rho \left(l + \frac{16a}{3\pi} \right)}{\pi a^2}$$

where, L = inductance of the fluid in the orifice,
 l = length of the orifice,
 a = radius of the orifice, and
 ρ = density of the fluid.

$16a/3\pi$ is twice the familiar end correction for a tube. The dimensions of the lowest orifice are

$$a = \frac{3}{16} \text{ "}$$

$$l = \frac{5}{16} \text{ "}$$

The value of L_1 is given by

$$L_1 = \frac{\frac{5}{16} + \frac{16 \times 3}{\pi \times 16 \times 3}}{\pi \times 9 \times 585} \times 256 = .01465 .$$

42. The other two orifices have dimensions of the same order of magnitude, but since the motion in these cases involves air instead of water, their inductances will be smaller by a factor equal to the specific gravity of air. The specific gravity of air is .001293 at 10^0 C. Hence the values of L_2 and L_4 are about .001 that of L_1 . At the frequencies of interest they can be neglected.

43. The equivalent inductance of the diaphragm can be shown to be negligible by the following reasoning. According

to Crandall (Bibliography 8), the deflection, y , of a point located at distance r from the center of a circular plate of radius a , clamped at the edge is, if pressure is uniformly applied, closely approximated by

$$y = y_0 \left(1 - \frac{r^2}{a^2}\right)^2 \quad (11)$$

The change in volume from the mean position is given by the value of the definite integral

$$V = \int_0^a 2\pi y_0 \left(1 - \frac{r^2}{a^2}\right)^2 r dr = \frac{1}{3} \pi a^2 y_0 \quad (12)$$

The kinetic energy, T , of the plate, is determined by the definite integral

$$T = \frac{1}{2} \int_0^a 2\pi \rho' \dot{y}^2 r dr = \frac{1}{2} \left(\frac{1}{3} \pi a^2 \rho'\right) \dot{y}_0^2 \quad (13)$$

where ρ' is the density per unit area.

If the kinetic energy is expressed in terms of the rate of change of volume instead of the velocity of the center of the diaphragm, the expression becomes

$$T = \frac{1}{2} \left(\frac{9}{5} \frac{\rho'}{\pi a^2}\right) \dot{V}^2 \quad (14)$$

Therefore, the equivalent inductance of the diaphragm is given by

$$L_4 = \frac{9}{5} \frac{\rho'}{\pi a^2} = \frac{9}{5} \frac{M}{S} \quad (15)$$

where M = total mass of the diaphragm, and
 S = area of the diaphragm.

When the constants of the device are substituted in the relation (15), the value of L_4 is found to be .0000103, which is entirely negligible.

44. Calculation of the Resistances

The viscous effect of the lower screen can be found directly by putting a known head of water in the lowest chamber and

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measuring the rate of discharge of water into an infinite volume of water. This value is found to be about .18 cubic inches/second/unit pressure. The equivalent resistance is the reciprocal of this quantity, or 5.55 ohms.

45. In the case of the upper gauge, the viscous effect arises from the passage of air, which has a viscosity of only .01342 times that of water. The equivalent resistance will be reduced an equivalent amount below that of the lower gauge, or approximately .075 ohms. The time constant introduced by this quantity is negligible compared to the duration of the phenomenon being detected. Consequently, this resistance can be neglected.

46. The equivalent resistance of the leak will be calculated by the use of Fig 9 on Plate 7, in which it is assumed the kinetic reaction of the gas is negligible compared to the viscous forces. The viscous force is assumed in the conventional manner to be proportional to the velocity of the flow. The partial differential equation describing the motion of the gas in the tube is

$$PA - (P + \frac{\partial P}{\partial x} \Delta x)A - k'Au \Delta x = \rho A \Delta x \frac{\partial u}{\partial t}, \quad (16)$$

$$-\frac{\partial P}{\partial x} - k'u = \rho \frac{\partial u}{\partial t} = \rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x}. \quad (17)$$

In the above relations

- P = pressure at the point x,
- A = cross-section area of the tube,
- u = velocity of the gas, and
- k' = viscous force per unit velocity per unit length.

When a steady state has been reached, $\frac{\partial u}{\partial t} = 0$. In such a device, both u and $\frac{\partial u}{\partial x}$ are such small quantities that their product may be neglected in comparison with k'u. Equation (17) reduces to

$$\frac{\partial P}{\partial x} + k'u = 0. \quad (18)$$

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If equation (18) is multiplied by ρ , and ρu is set equal to m , where m is the mass of gas passing a given cross-section per second, it is found that

$$\rho \frac{\partial P}{\partial x} + k' \rho u = 0 = \rho \frac{\partial P}{\partial x} + k' m. \quad (19)$$

In such a device, the gas will expand isothermally so that

$$\rho = \frac{\rho_0}{P_0} P. \quad (20)$$

The equation of motion becomes, by substitution of (20) into (19),

$$\frac{1}{2} \frac{\rho_0}{P_0} \frac{d}{dx} (P^2) + k' m = 0. \quad (21)$$

If equation (21) is integrated with respect to x over the entire length of the tube (3), the following equation results

$$\frac{1}{2} \frac{\rho_0}{P_0} (P^2 - P_0^2) + k' m \ell = 0 \quad (22)$$

where P_0 = pressure at position $x = 0$, and
 P = pressure at position $x = \ell$.

Let $k' \ell = k$ and $P = P_0 + \Delta P$ where $\Delta P \ll P_0$.
 Equation (22) becomes, after the substitutions are made,

$$\frac{1}{2} \frac{\rho_0}{P_0} (P_0^2 + 2P_0 \Delta P + \overline{\Delta P}^2 - P_0^2) + k' m = 0. \quad (23)$$

$\overline{\Delta P}^2$ can be neglected in comparison with $2P_0 \Delta P$, and the equation (23) reduces to

$$\rho_0 \Delta P = -k m = -k \rho_0 \frac{\Delta V}{\Delta t}, \quad \text{or}$$

$$\Delta P = -k \frac{\Delta V}{\Delta t}. \quad (24)$$

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Equation (24) shows that the volume of air passing through the leak per second is independent of the mean pressure, a result that has been verified experimentally. The equivalent resistance is seen to be equal to k . Its value can be found by applying to the ends of the leak a difference of pressure equal to a 1 inch head of water and measuring the volume of gas issuing per second. The leak is adjusted to give a rate of flow of .00141 cu. in./sec/unit pressure. The value of the equivalent resistance, or k , is the reciprocal of the rate of flow, or 709.2.

The Approximate Circuit

47. From the above analysis, it is obvious that the neglecting of many of the parameters in the exact circuit equivalent introduces an error of only a few percent. If these parameters are neglected the circuit can be reduced to the fairly simple one shown in Plate 6, Fig 7. Actual numerical results will now be calculated for this arrangement. As stated before, the desired quantity is the ratio of the voltage across C_4 to the driving voltage, for this will show the fraction of the external pressure fluctuation that appears across the diaphragm. It should be pointed out that the values of the circuit parameters vary with temperature. This is particularly true for the capacities which represent the elastances of the air volumes, in which case the values are inversely proportional to the absolute temperature of the air. The calculations which follow will be based upon the assumption that the equilibrium temperature of the air is the same as it was when the mine was first planted.

Steady State Solution

48. As pointed out in Appendix B, the actions of waves and tides are equivalent to harmonic pressure fluctuations. A steady state solution is of value because it shows how the device will react to such stimulations. Such calculations can be based upon conventional complex ac vectors. The circuit of Plate 6, Fig 7, can be reduced to four impedance as shown in Plate 7, Fig 10. It is easily seen that the value of the ratio V_3/E is given by

$$\frac{V_3}{E} = \frac{Z_2 Z_3}{(Z_1 + Z_2)(Z_2 + Z_3 + Z_4) - Z_2^2} \quad (25)$$

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Values of this ratio for three different frequencies and five different depths are given in Table I. The phase differences between the voltages are also shown. This table shows clearly the difference between the reactions of the device to frequencies equivalent to wave effects and to those equivalent to tidal effects. In the former cases, between 1/3 and 1/5 of the external pressure fluctuation appears across the diaphragm, and the response is almost in phase with the external variation. In the latter case, the fraction is between 1/700 and 1/1200 and the response is almost 90° out of phase with the external variation.

Step Function Solution

49. A step function solution is of importance because it shows the magnitudes and time constants of the principal transient effects, and gives some information concerning the duration of the contact resulting from a sudden decrease in pressure. Such a solution can be obtained most effectively by the use of the Laplace transformation.

50. Referring again to Fig 7 of Plate 6, the integro-differential equations describing the circuit on a nodal basis are:

$$\left. \begin{aligned} G_1 V_1 + \Gamma \int V_1 dt - G_1 V_2 &= \Gamma \int E dt, \\ G_1 V_1 - G_1 V_2 - (C_2 + C_6) \frac{dV_2}{dt} + C_6 \frac{dV_3}{dt} &= 0, \text{ and } \\ C_6 \frac{dV_2}{dt} - (C_4 + C_6) \frac{dV_3}{dt} - G_4 V_3 &= 0, \end{aligned} \right\} (26)$$

where $\Gamma = \frac{1}{L}$,
 $G_1 = \frac{1}{R_1}$, and
 $G_4 = \frac{1}{R_4}$,

If the Laplace transformation is performed on equations (26), and V' represents the Laplace transformation of V , the transformed equations become

$$\left. \begin{aligned} (G_1 + \frac{\Gamma}{s}) V_1' - G_1 V_2' &= \frac{\Gamma E}{s^2}, \\ G_1 V_1' - [G_1 + (C_2 + C_6)s] V_2' + C_6 s V_3' &= 0, \\ C_6 s V_2' - [G_4 + (C_4 + C_6)s] V_3' &= 0. \end{aligned} \right\} (27)$$

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In equations (27), s is the complex parameter arising from a Laplace transformation of a derivative or an integral (Bibliography 7). Equations (27) are now solved simultaneously for $\frac{V_3'}{E}$ yielding the equation

(28)

$$\frac{V_3'}{E} = \frac{\Gamma G_1 C_0}{(G_2 s + \Gamma) [G_1 + (C_2 + C_0) s] [G_1 + (C_2 + C_0) s] - C_0^2 s^2 (G_2 s + \Gamma) - G_1^2 s [G_1 + (C_2 + C_0) s]}$$

In order to perform the inverse Laplace transformation on Equation (28), and obtain an expression for V_3 in terms of the circuit parameters and time, it is necessary to find the roots of the denominator. Since this expression is a cubic in s it is easier to determine the roots by the Graeffe Method after actual numerical values have been substituted for the circuit constants. This has been done for five different depths with the results shown in Table II.

51. A typical plot of the step function solution for an equilibrium pressure of 1200 inches of water is shown in Plate 8. The pressure across the diaphragm rises to a maximum in .56 seconds, after which it decays at a rate determined essentially by the first term of the equation for V_3 . A finite time is required to reach this maximum because of the inertia and viscosity of the water entering the device. The final decay is due to the external leak. It should be pointed out that the important coefficients appearing in the step function solution are almost equal to the coefficients for the steady state solution with the larger values of ω .

External Pressure Variation Required to Produce Contact.

52. Equation (4) of Appendix A gives an expression for the decrease of the pressure effect with depth. If a typical value of 90 feet be assigned for l , the effect will decrease with depth according to the upper curve of Plate 9. The calculations listed in Table I show that the sensitivity of the diaphragm decreases with depth according to the lower curve of Plate 9. If a constant pressure differential were required across the diaphragm to produce contact, the curves of Plate 9 show that the sensitivity of the device would decrease rapidly with the depth of the water. To compensate for this effect the initial spacing between the contacts is made to decrease with increasing depth according to the curve of Plate 10. From the curves of Plates 9 and 10 and equation (4) of Appendix A, it is possible to deduce the pressure change just under a ship

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that will be required to produce contact. Plate 11 shows such a curve for a ship whose value of l is 90 feet. This curve shows that the device is slightly over-compensated for this type of ship. If l is smaller, the compensation is more perfect.

Indeterminate Elements.

53. Sack.

The device includes a composition sack which separates the water from the air in the lowest chamber. The original mechanical design assumed this sack to be completely "floppy", i.e. it offers no appreciable resistance to the motion of the water. This same assumption has been made in the theoretical treatment. There is some question as to whether the sack folds up properly so that the above assumption is valid at all depths. If this assumption is not quite valid, the measured pressure differentials required to produce contact will be larger than the predicted values. Data to be collected soon are expected to shed some light on this question.

54. Thermal Effects

The theoretical treatment of the approximate equivalent circuit assumes that the air always expands isothermally. This amounts to assuming that the rate of heat conduction through the walls of the device is infinite. No evidence collected to date indicates that this assumption introduces any appreciable error. If later tests indicate that the effect must be included, experiments will have to be made to determine the rate of cooling of the air inside the device.

55. Friction Effects

The motion of the diaphragm is communicated to the contact through a multiplying device which obviously has some friction. This has been neglected in the theoretical treatment because tests have shown that the contacts open and close on the same pressure differential across the diaphragm to within .01 inch of water.

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TABLE I

Steady State Response of the Pressure Arming Devices.

ω = frequency of the pressure variations,
 P_0 = mean pressure in inches of water,
 R = ratio of differential pressures across
the diaphragm to the external pressure amplitude, and
 θ = phase difference between the pressure
differential across the diaphragm and
the external pressure.

P_0	ω	R	θ
865	1	.3375	-9.34°
865	1/3	.3401	1.95°
865	1/7000	.00148	89.90°
1050	1	.2991	-5.62
1050	1/3	.2995	3.50°
1050	1/7000	.001283	89.96°
1200	1	.2569	-3.67°
1200	1/3	.2725	4.35°
1200	1/7000	.001066	89.96°
1450	1	.2375	-1.72°
1450	1/3	.2368	5.27°
1450	1/7000	.0005568	89.96°
1850	1	.1968	-.57°
1850	1/3	.1957	6.76°
1850	1/7000	.0003873	90°

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TABLE II

Response of the Pressure Arming Device to a Step Function.

P_0 = Equilibrium pressure in inches of water.

V_3 = Pressure differential across the diaphragm.

E = External pressure variation.

Expression for

P_0	Expression for		
865	$.3450e^{-.0331t}$	$-.3499e^{-5.104t}$	$+.00479e^{-470.2t}$
1050	$.3026e^{-.03531t}$	$-.3089e^{-7.624t}$	$+.006369e^{-368.1t}$
1200	$.2754e^{-.03686t}$	$-.2831e^{-10.04t}$	$+.00773e^{-366.7t}$
1450	$.2390e^{-.03859t}$	$-.2493e^{-15.02t}$	$+.01035e^{-360.7t}$
1850	$.1976e^{-.04076t}$	$-.2150e^{-25.34t}$	$+.0154e^{-350.3t}$

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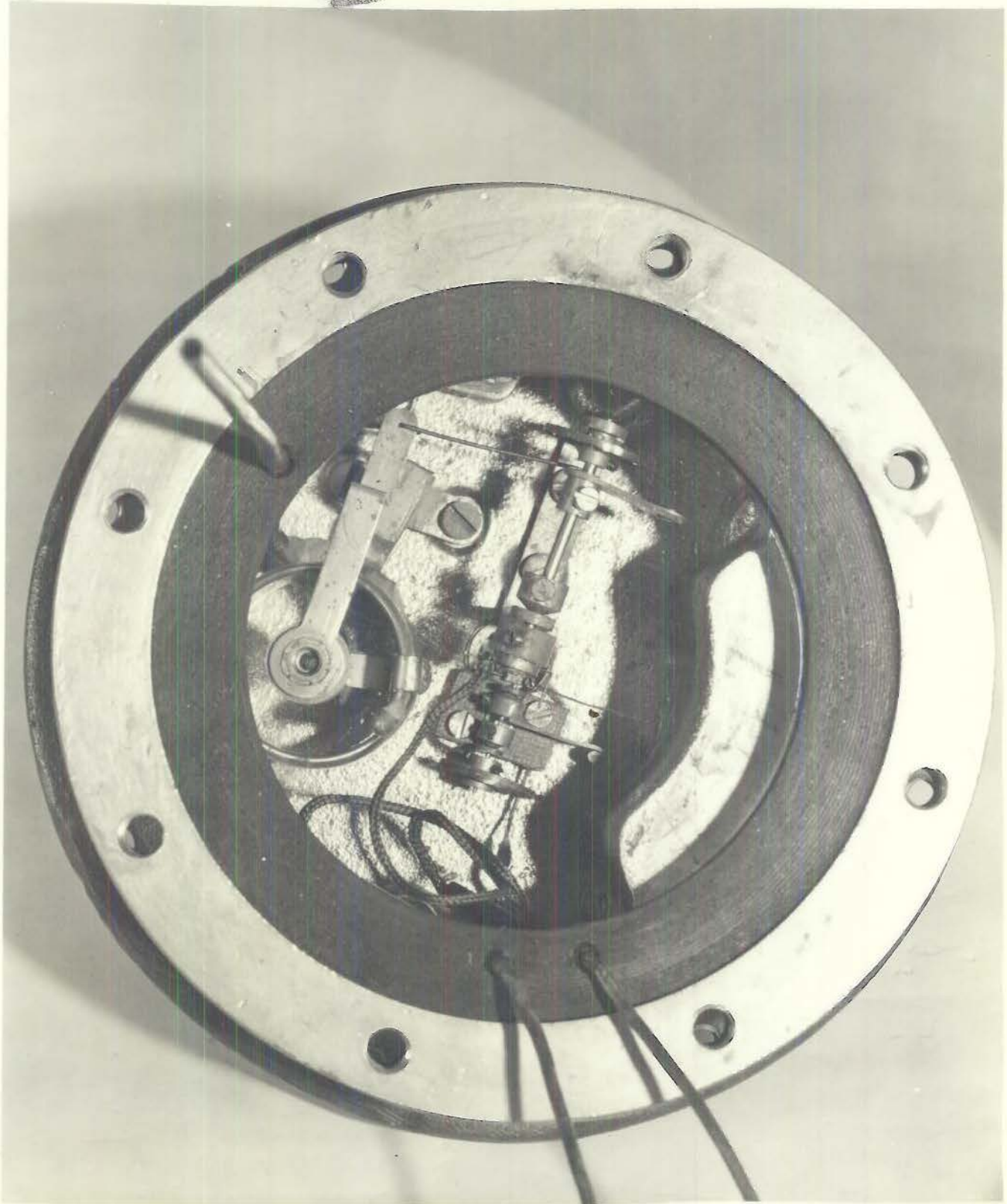
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PLATE I

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PLATE 2

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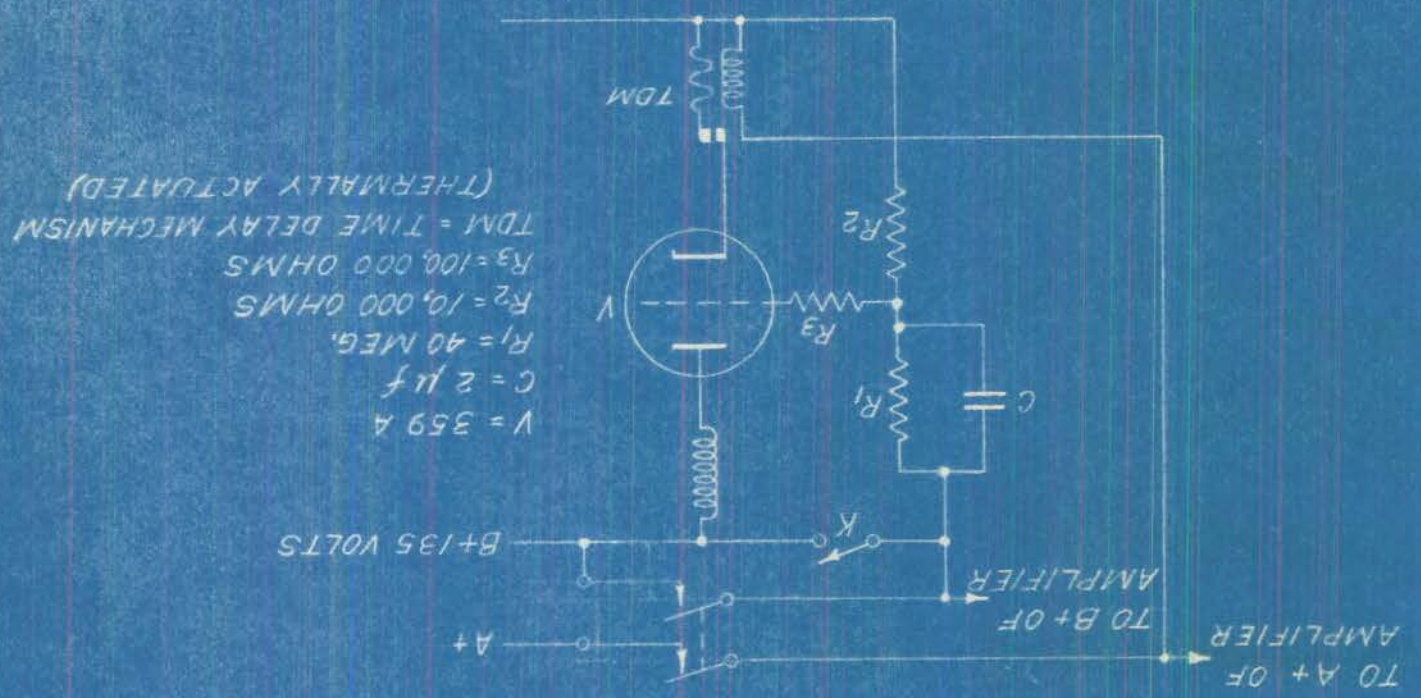


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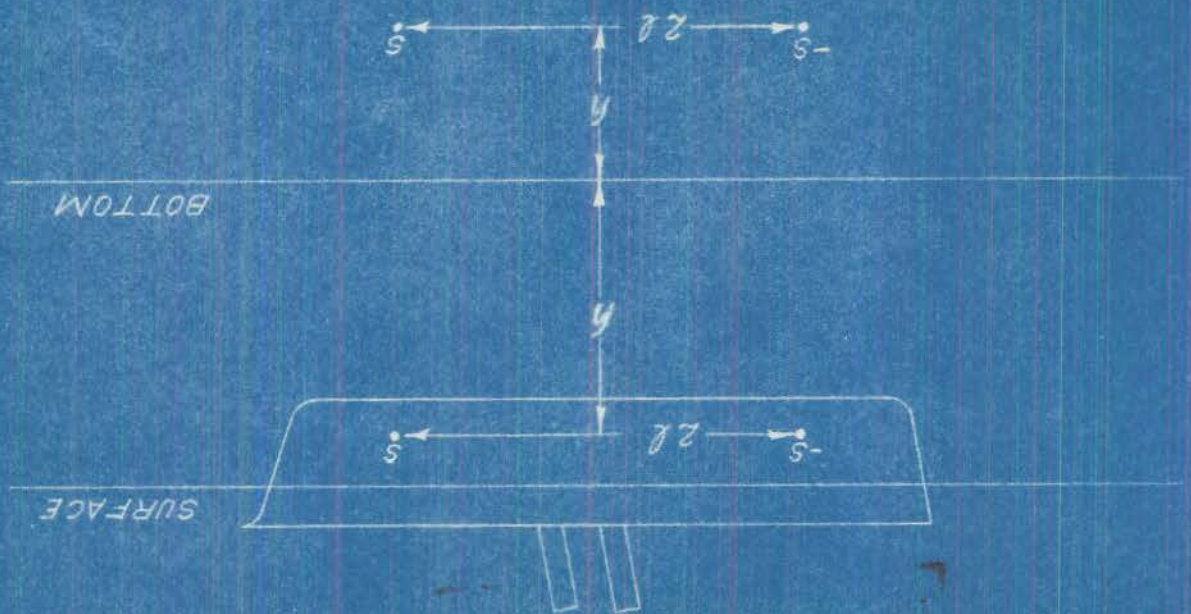
PLATE 3

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DISARMING CIRCUIT
FIG. 2

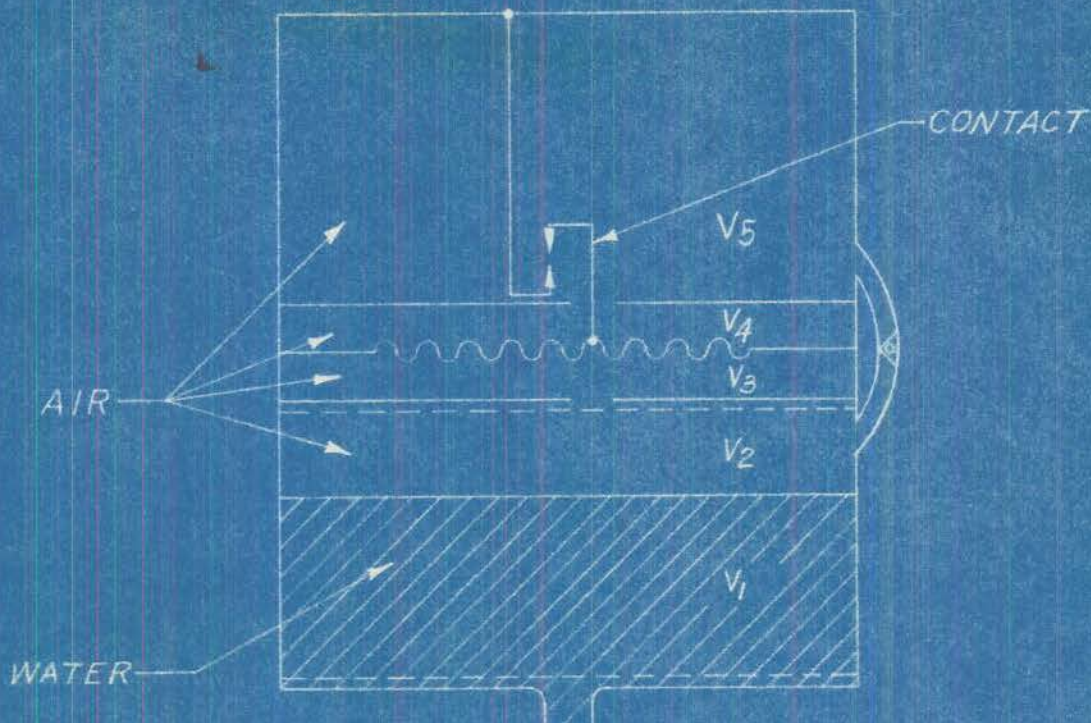


SOURCE-SINK DISTRIBUTION ABOUT A SHIP
FIG. 1

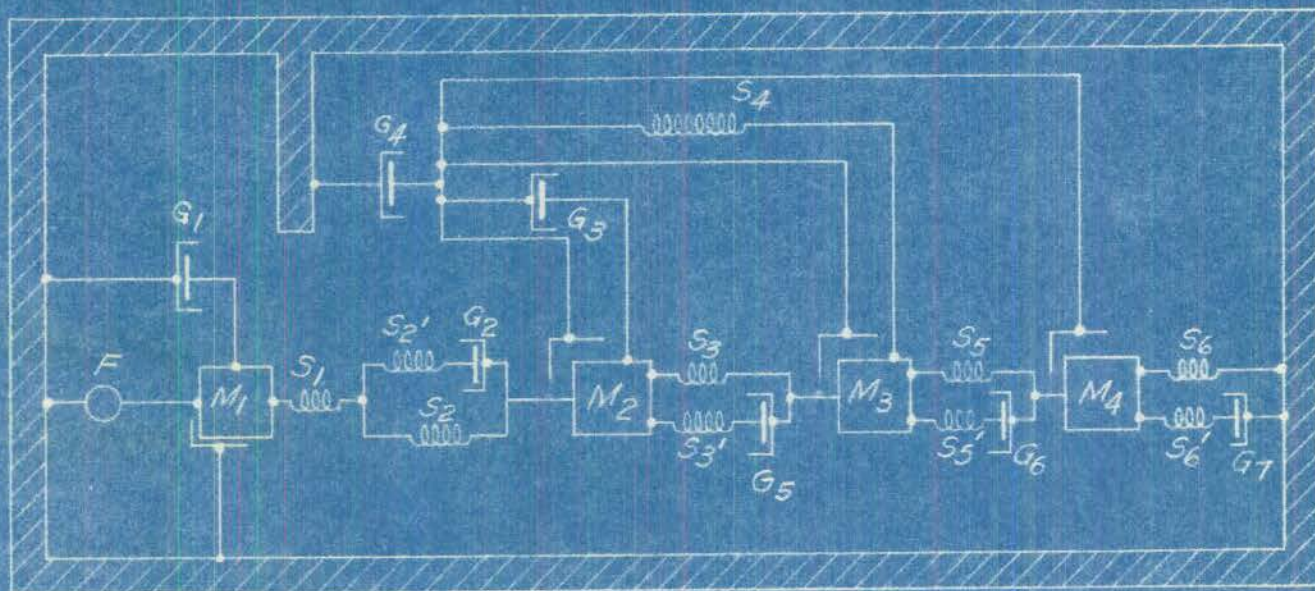


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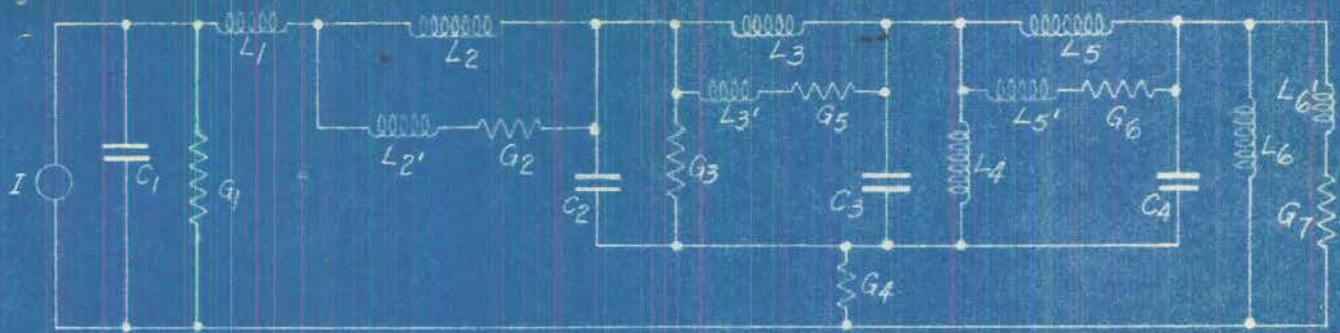
SCHEMATIC DIAGRAM OF PRESSURE ARMING DEVICE
FIG. 3



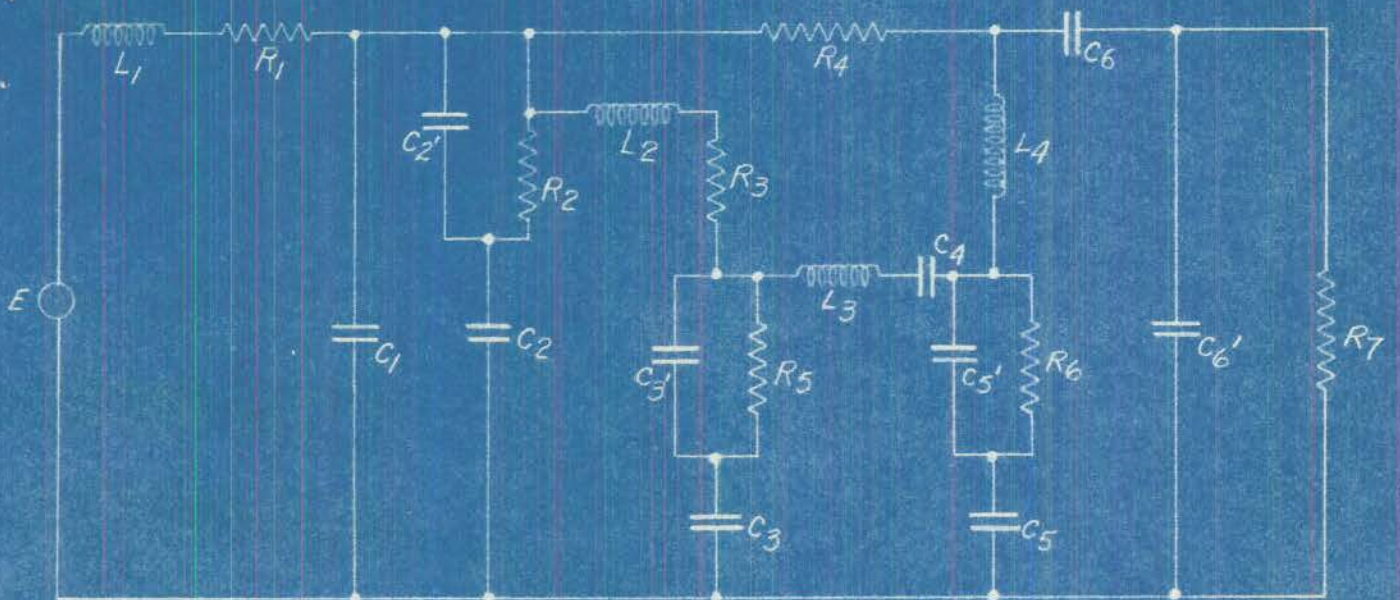
SCHEMATIC OF AN EQUIVALENT MECHANICAL ARRANGEMENT
FIG. 4

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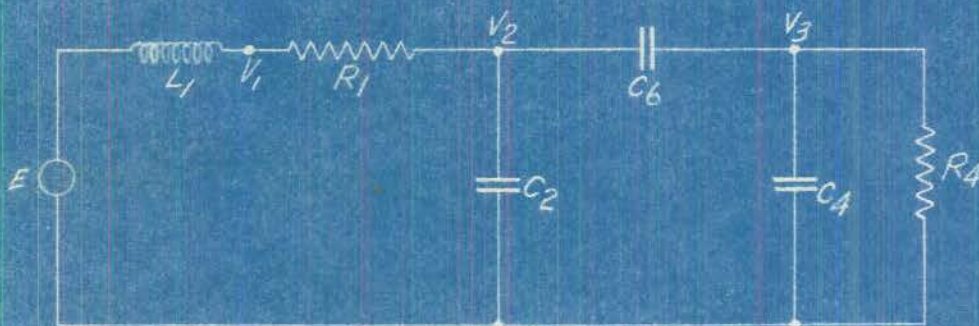
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ANALOGOUS ELECTRICAL CIRCUIT - MOBILITY BASIS
FIG. 5



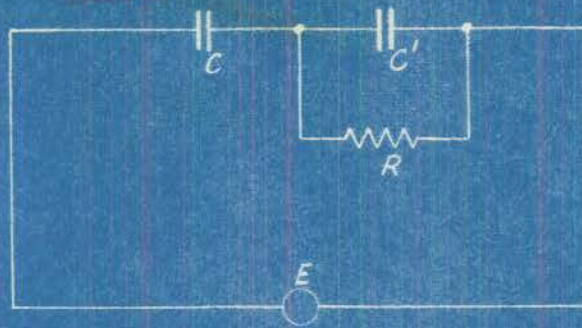
DIRECT ANALOGOUS ELECTRICAL CIRCUIT
FIG. 6



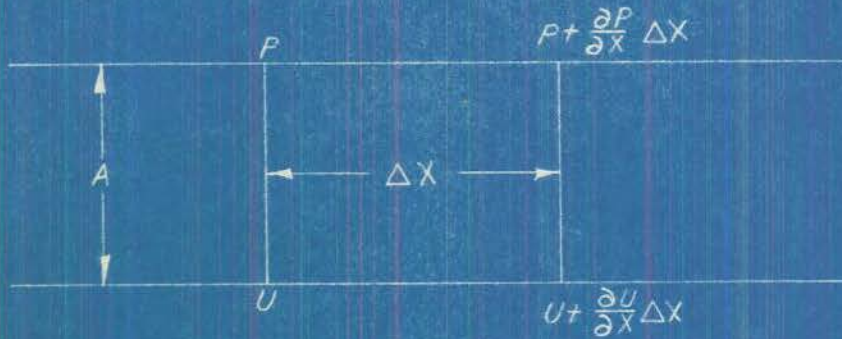
APPROXIMATE ANALOGOUS ELECTRICAL CIRCUIT
FIG. 7

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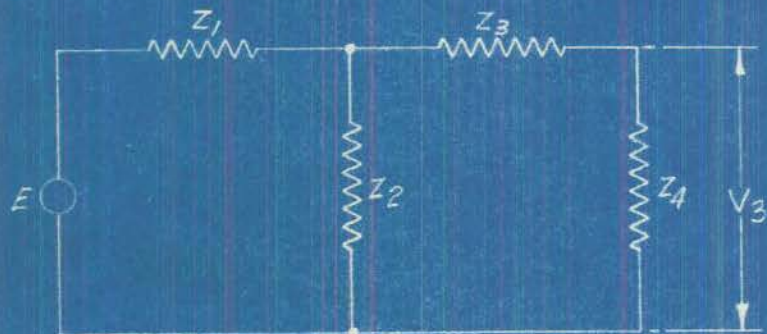
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EQUIVALENT CIRCUIT FOR THE COMPRESSIBILITY OF A GAS
FIG. 8



SECTION FOR CALCULATING THE VISCOUS FLOW THROUGH A PIPE
FIG. 9

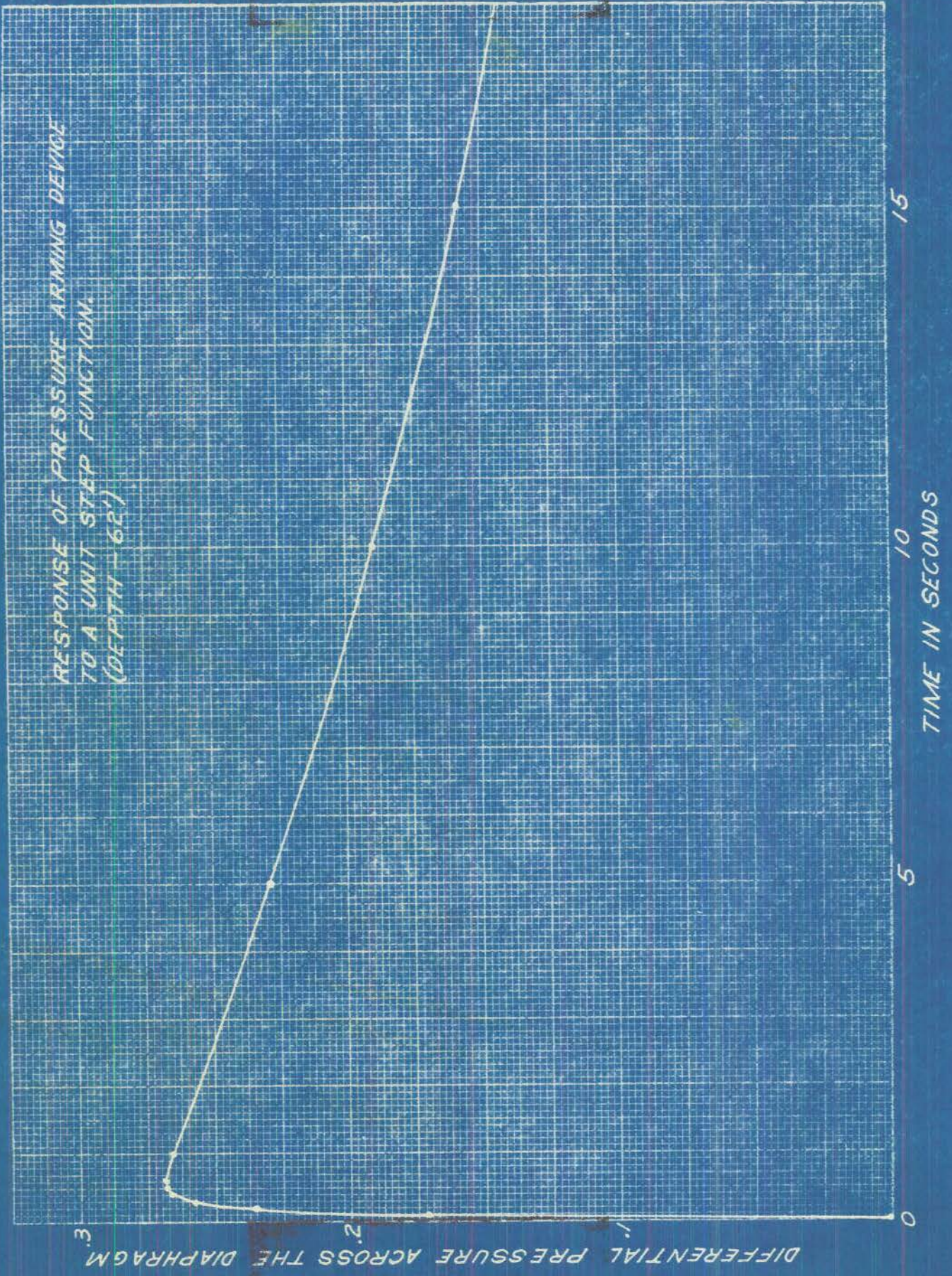


CIRCUIT FOR CALCULATION OF THE STEADY STATE RESPONSE
FIG. 10

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RESPONSE OF PRESSURE ARMING DEVICE
TO A UNIT STEP FUNCTION
(DEPTH - 62')



N. P. L. 31A

DIFFERENTIAL PRESSURE ACROSS THE DIAPHRAGM

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ΔP_0 - PRESSURE CHANGE UNDER THE SHIP
 ΔP_1 - PRESSURE CHANGE AT DEPTH h
 ΔP_2 - PRESSURE DIFF-FERENTIAL ACROSS THE
DIAPHRAGM

1.0

0.5

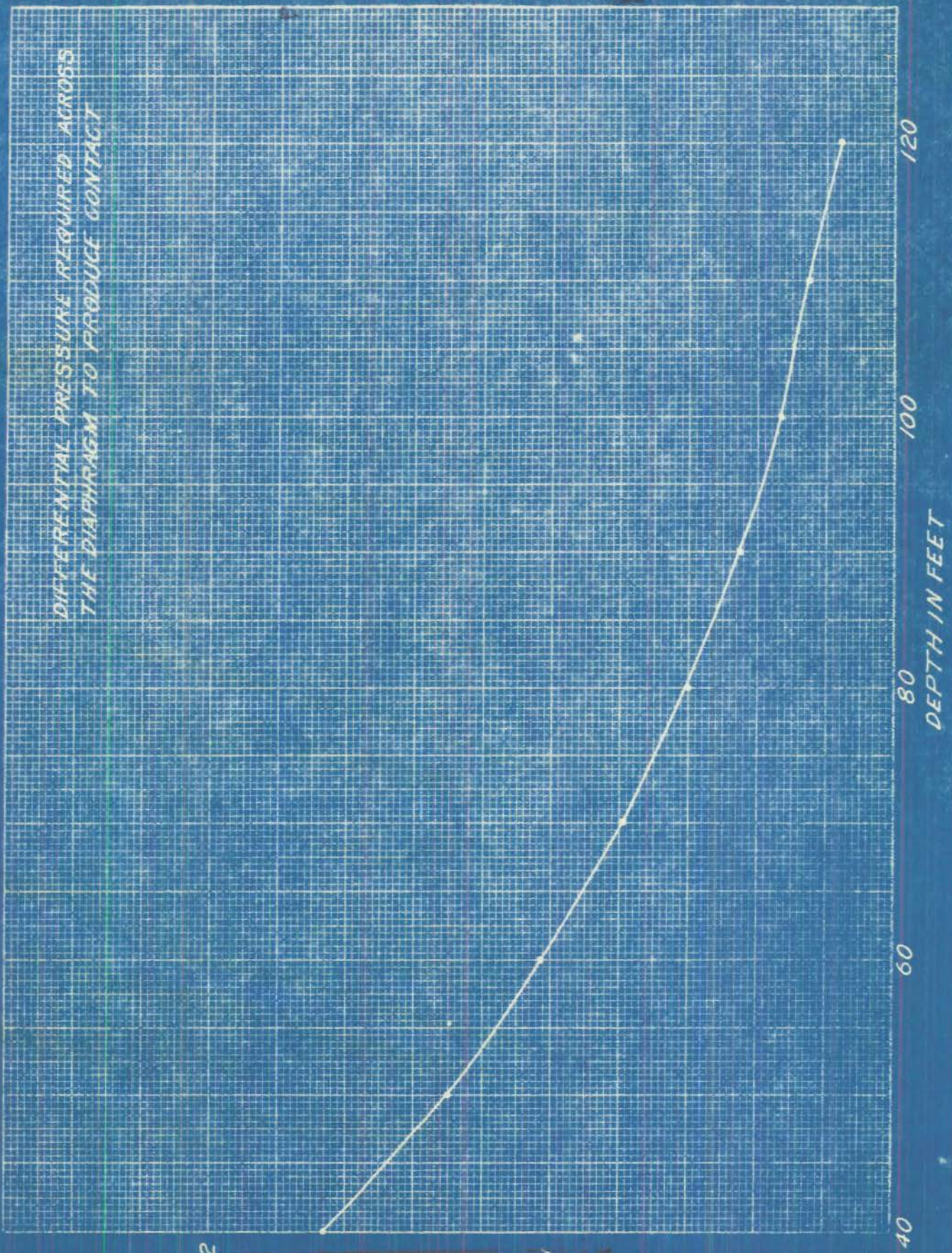
40 60 80 100 120
DEPTH IN FEET

$\frac{\Delta P_0}{\Delta P_1}$
 $\frac{\Delta P_1}{\Delta P_2}$

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DIFFERENTIAL PRESSURE REQUIRED ACROSS
THE DIAPHRAGM TO PRODUCE CONTACT



PRESSURE DIFFERENTIAL IN INCHES OF WATER

40

60

80

100

120

DEPTH IN FEET

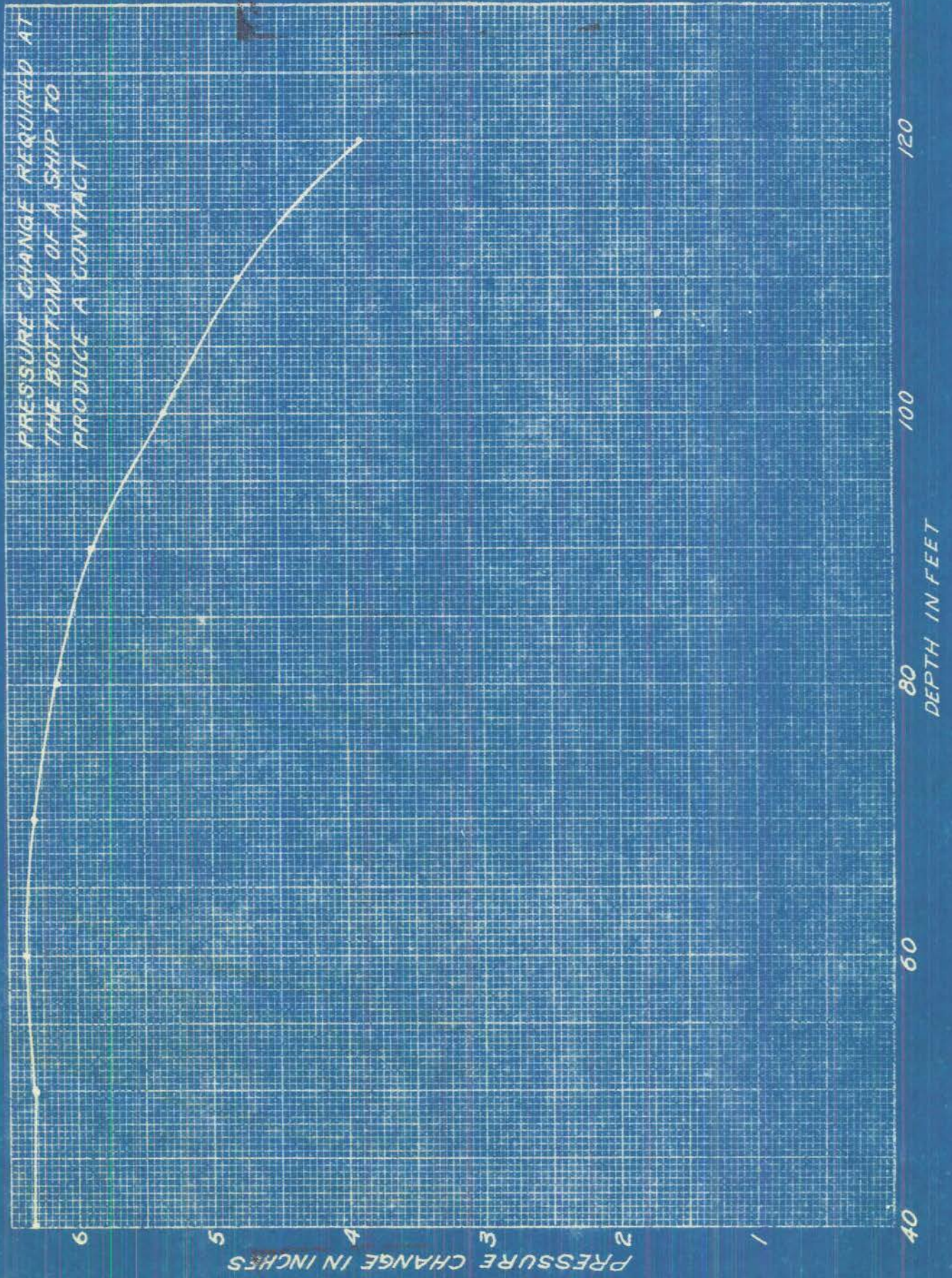
N. R. L. 34A

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PLATE 10

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PRESSURE CHANGE REQUIRED AT
THE BOTTOM OF A SHIP TO
PRODUCE A CONTACT

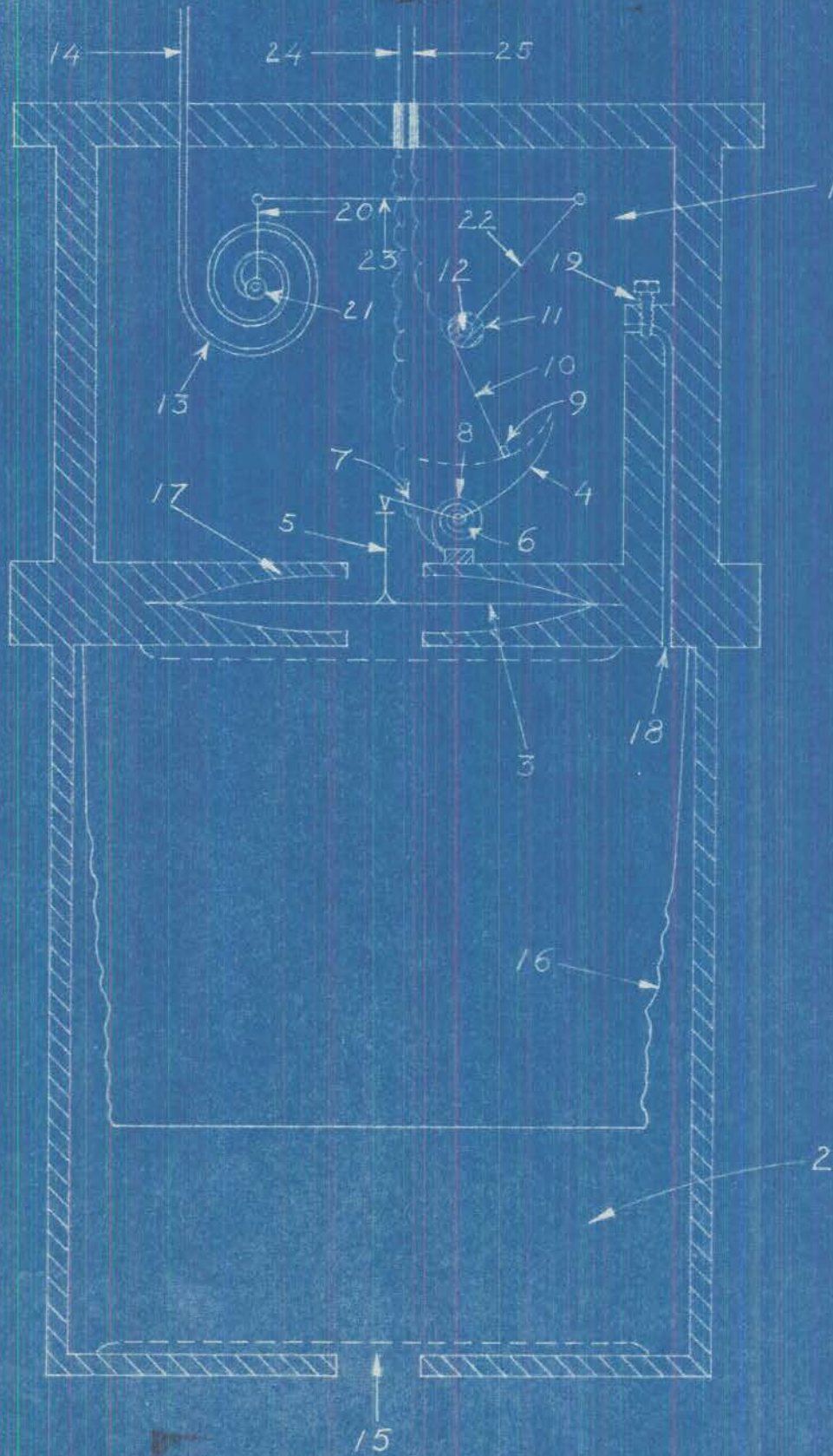


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PLATE II

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