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Antenna Duplexing

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I. INTRODUCTION.

The term "duplexing", as used in this report, refers to the use of a single antenna for transmission and reception on a radio system, the requisite switching operations being performed automatically by other than mechanical means. Advantages, such as saving of construction and installation time and cost, of single antenna operation with any radio system are obvious; where both transmitting and receiving antennas are directive arrays whose directions of effect must be jointly variable, it becomes almost a necessity. Ordinary communications radio systems may be able to utilize mechanical switches for obtaining single antenna operation. A radar system using one antenna, however, requires that the antenna be switched from transmitter to receiver in an extremely short time after the termination of a transmitted pulse, a time which, ideally, should be a small fraction of a micro-second. Mechanical switches cannot operate in this time, so that single antenna operation in the case of a radar system requires the use of electronic switches. Accordingly, since such single antenna operation is virtually essential in modern radar, work was undertaken at the Naval Research Laboratory on the development of duplexing systems for radar, employing electronic switching devices, under Bureau of Ships letter of authorization L1-2/NP14(10-19-FS) of October 21, 1938. This report will be concerned only with duplexing systems developed at this Laboratory for use on frequencies below those which require the use of cavity resonators, although some of the principles brought out will apply to systems for the higher frequencies.

II. THEORY.

The duplexing system must perform two functions: it must disconnect the receiver from the antenna during transmission to prevent over-loading of the receiver as well as voltage breakdown of the tube insulation structures in particular, and to prevent loss of transmitted power into the receiver, and it must remove the transmitter from the antenna during reception to prevent loss of received signal energy into the transmitter circuits. In the ideal, there would be no insertion loss in the network connecting a given unit to the antenna, and no leakage loss into the unit which is "disconnected". The equivalent circuit of the ideal system would, therefore, be simply a double pole, double throw switch connecting transmitter and receiver alternately to the antenna and completely disconnecting the other at the same time. Actual electronic switching systems, however, fall short of the ideal in that they introduce both types of loss referred to above. Duplexing systems developed by this Laboratory are switching devices employing transmission line networks and electronic devices, which have an equivalent circuit using mechanical switches shown in Figure 1. The various impedances that are present in addition to the necessary switches, exist inherently by the nature of the system, and give rise to the undesired losses.

The figure shows an unbalanced circuit, but the same principles will apply to a balanced one. During reception the mechanical

[redacted]

switch is thrown to terminals (1) and the antenna is coupled to the receiver across a mutual resistance R_m. At the same time a resistance R_d is placed in series with the branch line to the transmitter, reducing the amount of received signal power going into this branch. During transmission the switch is thrown to terminals (2). Note that the receiver is still coupled to the antenna system, but only through an isolating network. Across the low side of the transformer T_1 is connected a small impedance Z_t; the transformed value of this impedance, Z_h, will be high relative to the surge impedance of the antenna feed line across which it is connected, and the power absorbed from this line will be small. The voltage developed across Z_t will be $E/\sqrt{Z_h/Z_t}$. This voltage will be applied to the receiver input through a reactive voltage divider Z, which further reduces the voltage to a safe value for the receiver input. Also during transmission, resistance R_d is short circuited by the switch, and a second high impedance Z_h is placed across the antenna line by transformer T_2. The network associated with switch section I has been called the receiver protective system, that with switch section II the transmitter decoupler.

This switching system would be reduced to the ideal if the receiver were completely disconnected from the antenna line during transmission and the transmitter during reception, R_m were of infinite value, and switch section II threw zero load across the line during transmission. The actual arrangement cannot meet these conditions, and an improvement in the duplexing efficiency consists in approaching them more closely, i.e., in making R_m and R_d larger to reduce losses during reception, and in making the transformation ratio $n = \sqrt{Z_h/Z_t}$ and the voltage division in Z larger, to increase receiver protection and to reduce power absorption during transmission. At the frequencies at which these systems have been most used at this Laboratory, transmission line networks have been of convenient physical size to form the impedance elements, and have been used therefor since they show more nearly ideal characteristics of the various elements. The transforming property of quarter-wave lines has been used in place of the lumped transformers T, and a resonant circuit, either two wire or concentric, into which a spark gap or gaseous discharge tube is connected, has been used to supply the coupling and isolating resistances R_m and R_d during reception, and the elements Z_t and Z during transmission.

III. SIMPLE DUPLEXING CIRCUITS.

A duplexing action can be obtained by connecting the input circuit of the receiver to the antenna line with a line section which is electrically an odd number of quarter-wave lengths long (See Figure 2). A protective system only is shown - no transmitter decoupling network is connected. The receiver input tube will act in a similar capacity to the spark gaps or gaseous discharge tubes used in better systems, dropping to a lower impedance during transmission, and raising the input impedance to the connecting line, which in turn decreases the power absorbed by the receiver circuit, and drops the voltage applied to the receiver input below the value existing on the antenna line. The system is poor because it uses the very effect which a duplexing arrangement should prevent, viz., overloading of the receiver, to accomplish its

[redacted]

purpose, and because the first receiver tube will not drop in impedance to values as low as could be desired, and power absorption will be appreciable. Referring to the equivalent circuit, the transformer ratio will be low, R_t will be high, and Z will be absent altogether. For relatively low-power circuits, where the first receiver tube can handle the grid current required by this arrangement, the scheme is applicable and affords voltage protection for the receiver. Decoupling can be obtained, using no added external circuit, since the main transmission line will be mismatched at the transmitter coupling point while the transmitter is "cold" during reception, by picking a point (P) for connection of the line to the receiver, where the impedance looking toward the transmitter is high; but a system employing a decoupler as such allows connection of the duplexing equipment at any point in the main line, eliminates the necessity for making measurements to obtain the main line impedance distribution while the transmitter is non-oscillating, and does not change its effectiveness if this distribution is changed due to a change in coupling conditions at the transmitter. Another method which will take care of changes in the impedance distribution along the line is the use of a "trombone" for varying the line length between the transmitter and the connection point (P) to protective system. It must allow a variation in line length of $\pm \lambda/4$ from the length at its neutral position. Such a system is bulky and hard to keep free from resistive joints, especially where extreme powers are used. It also requires a more elaborate mechanism if it is to be tuned accurately and smoothly, and is more apt to require retuning if transmitter loading is changed than a decoupling system proper.

Another arrangement is connection of a spark gap or similar device across the receiver input line at a point an odd number of quarter-wave lengths from the junction to the antenna feed line. (Refer to Figure 3). Here again, the impedance placed across the termination of the quarter-wave section by the ionized gap during transmission will not be nearly so low as desirable - the transformation ratio will be low and R_t high - and the receiver is subjected to the full voltage across the spark gap - Z is absent. Furthermore, the initial voltage across the gap is only that of the main transmission line, and to keep the initial "spike" voltage reaching the receiver as low as possible, as well as for lowest fired impedance of the gap, the gap should be subjected to a multiplied value of the main line voltage. One way of accomplishing this is to use a line whose length is a half-wavelength multiple, and whose surge impedance is higher than that of the main line, for connecting the receiver to the antenna feeder, placing the gap at an odd-quarter wave point as before. The available multiplication in this method is rather limited, however.

IV. EFFICIENT DUPLEXING SYSTEMS.

A. General Circuit & Mode of Operation.

To overcome the faults of the primitive systems, the spark gap should be connected into an anti-resonant circuit, where the voltage step-up obtainable is used to accomplish the results noted in the previous paragraph. Such a circuit aids also by making the termination of the transforming connecting line during transmission a lower

[redacted]

impedance than that of the gap itself.

The anti-resonant circuit supplies the mutual resistance R_m when receiving and the resistance R_t and reactive network Z of the protective system when transmitting, and a second such circuit can be used for the decoupling system, which supplies the resistance R_d during reception, as well as R_t for that system during transmission. The block diagram of the complete duplexing system developed and most commonly employed at this Laboratory, showing both protective and decoupling arrangements, is shown in Figure 4. The tanks are shown as block units, since they may be of various styles. Figure 5 shows the use of specific tanks to be described.

During reception, when the spark gaps are not ionized, the circuits or tanks are tuned by means of the gap capacity plus any desired additional variable tuning capacity to the operating frequency. The impedances measured across any part of the anti-resonant circuits are then pure resistances; in particular, those across the gaps are the highest impedances in the circuits, that across the input terminals of the protective tank (when the receiver load is disconnected) is the mutual resistance R_m of the equivalent circuit, and that across the input terminals of the transmitter decoupler tank has a value of the same order as R_m (around 1000 ohms if z_o for the transmission line is around 50 ohms, giving a theoretical loss of 0.2 db in each tank). The protective tank is then a coupling network, and if the receiver connection tap on the tank is placed symmetrically with the input tap, is a one-to-one transformer with low loss, and the circuit through to the receiver is correctly matched. The input resistance of the decoupling tank is transformed by its connecting line, shunting a very low resistance across the main line at point P. Consequently a high value of resistance, R_d of the equivalent circuit, is seen looking into the branch going to the transmitter from point Q, a quarter-wave length away, and little received signal energy will be lost into this branch.

During transmission the spark gaps ionize, detuning the tanks. Impedances measured across the input terminals of these tanks are then made up of small resistances in series with small inductances. The lengths of the connecting lines ℓ_1 and ℓ_2 are adjusted to be anti-resonant with these impedances and form the transformers T_1 and T_2, throwing high resistances R_h across the line. The decoupling circuit is simply a light load, absorbing a little transmitted energy, and performing no useful function. The protective system performs its nominal purpose: transformer T_1 reduces the main line voltage to a low value across the input to the protective tank, and this tank, now detuned, is the reactive network Z, through which this lower voltage is further reduced before it is applied to the receiver.

A commonly employed early form of protective system using an anti-resonant tank was a two-wire affair, as shown in Figure 6. This circuit represents a large step in the right direction. Here the gap is excited by a higher voltage than that on the antenna line, and the termination of the quarter-wave section when transmitting is much lower

[redacted]

than in the previous systems. In addition, the receiver coupling point on the anti-resonant circuit is decoupled from the input coupling point when the gap is ionized; this means that the receiver will exert less loading effect on the connecting line (this point will be discussed more in detail later) and also that there is now a voltage drop between connecting line termination and receiver connection point - the voltage divider Z of the equivalent circuit is now present. An open wire system, however, is much less desirable than a concentric, due to the existence of greater losses and consequently lower anti-resonant impedances in the tanks, which results in a poorer effectiveness as will be brought out presently (See pp 7, 10). We will concern ourselves henceforth with concentric systems, for this reason, and also because most of the radio systems with which the duplexing equipment has been used at this Laboratory employ concentric transmission line systems, and it is undesirable to introduce the necessity for balanced-to-unbalanced coupling networks. The principles brought out will apply as well to two-wire systems as to concentric, however.

B. Necessary Conditions for the System to be Effective.

The general requirements for an improvement in the effectiveness of a duplexing system have been pointed out earlier in terms of the equivalent circuit. These requirements must now be made specific as regards the actual system to be used - the one shown in Figure 4.

The following terminology will be adopted:

Duplexing tank - the anti-resonant circuit containing the spark gap or tube. This means either the protective tank or the decoupler tank.

Transformer - the nominal quarter wave (or odd multiple thereof) connecting section between antenna line and tank.

z_o - surge impedance of the antenna line, transformer, and receiver input line.

Z_o - surge impedance of the line of which the tank is constructed.

X_c - reactance required to tune the tank to resonance when gap is not fired. This is in parallel with the gap and includes the capacitive reactance of the gap itself.

$Z_T = R_T + jX_T$ - maximum impedance across the tank itself (between the two conductors) when the gap is not fired. At anti-resonance it is a pure resistance.

$Z_g = R_g + jX_g$ - impedance across gap when the latter is not fired. At anti-resonance, it is a pure resistance.

[redacted]

$Z_m = R_m + jX_m$ - impedance between input tap point of tank and ground, gap not fired. It has a value $R_m + j0$ at the resonant frequency.

R_{g_f} - parallel resistive component of impedance of the gap when fired.

$Z_t = R_t + jX_t$ - impedance between input tap point and ground, of the tank, gap fired.

$Z_h = R_h + jX_h$ - input impedance to the transformer, with gap fired. It has the value $R_h + j0$ at the fundamental operating frequency.

$n = \sqrt{\frac{|Z_h|}{|Z_t|}}$ - voltage transformation ratio of the transformer, gap fired.

The necessary conditions for a good duplexing system may now be summarized in more exact working terms as follows, assuming R_m established several times the magnitude of z_o so that the insertion loss on receiving is low:

(1) as great a ratio R_m/R_t as possible.

(2) as great a voltage drop in the tank itself, between input and output coupling points, as possible during transmission.

(3) high impedance Z_g and hence high voltage step-up to the gap before firing.

(4) wide frequency pass band.

The second and third requirements need not be elaborated on; the fourth is necessary when the duplexing equipment is to be used with a pulse transmission system, in order that the pulses may be passed with low distortion. Various types of tanks will vary in their fulfillment of these conditions. The first requirement amounts to saying that the transformer should be terminated in as low an impedance Z_t as can be obtained during transmission, keeping R_m fixed for reception, but requires further discussion. Consider the system during transmission - See Figure 7. We wish the transformed impedance Z_h to be high and a pure resistance. The attenuation of the line itself may be neglected.

$$Z_h = z_o \frac{(R_t + jX_t) + jZ_o \tan \theta}{z_o + j(R_t + jX_t) \tan \theta}$$

$$= z_o \frac{R_t + j(X_t + z_o \tan \theta)}{z_o - X_t \tan \theta + jR_t \tan \theta}$$

If we adjust the length ℓ, so that $z_o \cot \theta = X_t$, that is, make the capacitive reactance looking back into the line from the receiving terminals equal to the inductive reactance of the termination, Z_h reduces to

[redacted]

$$Z_h = \frac{X_t^2 + z_o^2}{R_t} - jX_t = \frac{1 + z_o^2/X_t^2}{R_t/X_t^2} - jX_t$$

It is necessary, therefore, that both X_t and R_t be as small as possible, and that the ratio X_t/R_t be as large as possible. In practical tanks, having reasonable values of Q, placement of the input tap at points which give usual values of R_m will make X_t fairly small relative to Z_o^2, for all the types of tanks to be described, and selection primarily of a type which gives the smallest R_t will produce the greatest value of Z_h, since R_t then will have the greater effect on the value of Z_h. It is the ratio R_m/R_t which must be considered, and not R_t alone, for the tap on the tank must be placed high enough to raise R_m to a value giving requisite low loss in the tank itself during reception. Different styles of duplexing tanks will, for the same R_m, show different values of R_t when the gap fires. The effect of the receiver load connected to the tank must be considered here - any shunt resistance connected across the input tap is a loading resistance on the transformer which effectively increases the impedance of the quarter-wave line termination and decreases the transformation ratio, and is not a shunt on a low impedance termination of a true quarter-wave transformer. Consider the admittance

$$Y_t = \frac{1}{Z_t} = G_t - jB_t$$

Now $Z_t = R_t + jx_t = \dfrac{G_t}{G_t^2 + B_t^2} + j\,\dfrac{B_t}{G_t^2 + B_t^2}$. (See Fig. 8).

If G is smaller than B, as it will be found to be here, then if connecting the receiver load increases G the series resistance R_t is made larger. Here again, various types of tanks will be variously affected by the receiver load. Some will have much better isolation between input and output tap points under transmitting conditions, and the value of R_t will be less adversely affected; in one style of tank it will actually be improved by the receiver connection.

R_h must be high not only to make for good protection of the receiver as well as small loss of transmitter power, but also to keep the power dissipated in the spark gap at a reasonably low value. Since this device is the predominant dissipative element in the tank, it absorbs nearly all of the power drawn from the main line. The life of the gap is an inverse function of the power it dissipates; a gaseous discharge tube is particularly limited in its dissipation capabilities.

C. Concentric Duplexing Tanks.

Three types of concentric tanks have been commonly employed in duplexing systems developed by this Laboratory. They are shown, together with their transforming lines and receiver connections, in Figure 9. The first will be referred to as a "quarter-wave" tank, the second as a "half-wave" with shunt gap and the third as a "half-wave" with series gap, although the actual lengths of the tanks are foreshortened from these designated ones. The properties of these tanks will now be discussed individually and it will be shown how they compare

[redacted]

in meeting the necessary conditions for an effective system. They are most easily analyzed for the receiving condition case in which the spark gap is not ionized, by considering the tanks as made up of short-circuited sections of concentric line, of characteristic impedance Z_o. The circuits can then be designed and calculated as simple anti-resonant circuits, for the relations for resistance and inductance of the line sections are well known. For the case in which the spark gap is ionized, the tanks are detuned, and must be handled by point-to-point transmission line calculations. General relations for various quantities such as input impedance, etc., pertaining to a given tank, in terms of the variables such as R_g, X_c, position of the connecting taps, etc., are too complex for interpretation, and each individual case must be calculated.

1. Quarter-Wave Tank.

See Figure 10. Let ℓ be the inner conductor length (considering spark gap to be at the extreme end).

$\beta = 2\pi/\lambda$ is the phase shift constant.

$\beta\ell =$ electrical length of the tank (in radians).

$Z_o =$ surge impedance of line of which tank is constructed.

$r_o =$ static resistance per unit length of tank - assuming uniform current distribution.

$X_c =$ reactance of tuning and spark gap capacitance.

The inductive reactance of the tank itself as seen from the condenser terminals is $Z_o \tan \beta\ell$ ohms, and since the resistance is negligible, relatively, the condition for anti-resonance is

$$X_c = Z_o \tan \beta\ell$$

For designing, C, Z_o and the frequency will be known, and ℓ can be calculated. This is made the distance from bottom of tank to the actual spark gap position. For exact work, the inductive reactance of the spark gap electrode above the actual gap (the reactance of the short section of tank at the top) must be subtracted from the gap capacitive reactance to give the effective capacitive reactance; the same must be done in the case of the tuning condenser leads, if one is used.

The impedance across the gap at anti-resonance, which is equal to the tank impedance, is

$Z_g = Z_T = X_c^2/R$ where R is the total effective resistance in the circuit. That part of R due to the line sections composing the tank, which is the only part that can be calculated, is, for all but a negligibly capacitively loaded tank[1]

$$R = \frac{r_o\,(2\beta\ell + \sin 2\beta\ell)}{4\beta \cos^2 \beta\ell}$$

[1]See list of references.

[redacted]

The impedance at the connection tap, point P, is

$$R_m = \frac{2\beta Z_o^2 \sin^2 \theta}{r_o \tan^{-1}\left(\frac{X_c}{Z_o}\right)}$$

if $\theta \angle\angle \beta\ell$, where $\theta = \beta x$ is the electrical distance of the tap above the shorting disc. This expression may be derived by considering the two impedances seen looking away from each side of the tap point, which are in parallel. The derivation will not be given here.

The frequency band width of a duplexing system employing this tank, as well as that of one using the other styles of tanks considered, is discussed in another part of this paper.

An improvement in the effectiveness of the circuit results when the inductive part of Z_t and the reactance of the lead into the tank are tuned out by means of a condenser, as shown in Figure 11, leaving point P removed from ground by a small pure resistance. The length of line from P to Q can then be a full quarter-wave length, and the shunting effect of the receiver load is beneficial.

2. Half-Wave Tank with Shunt Gap.

See Figure 12. Let ℓ be one-half the total length of the inner conductor. $\beta\ell$ is then one-half the electrical length. Other symbols are as defined for the quarter-wave tank. Consider the tank as made up of two short-circuited sections of line of characteristic impedance Z_o in parallel, across which the reactance X_c of the spark gap and tuning condenser is connected. The inductive reactance of each short circuited section will be $Z_o \tan \beta\ell$, that of the two in parallel will be half of this, and for anti-resonance

$$X_c = \frac{Z_o \tan \beta\ell}{2}$$

from which the tank may be designed. For exact work, again, the inductances of spark gap and condenser leads must be taken into account.

The impedance across the gap is given by

$Z_g = Z_T = 2X_c^2/R$, where R is the effective resistance of one half of the circuit, that part of which that is due to the line sections of the tank being given by the same expression as in the case of the quarter-wave tank.

The tap point impedance at anti-resonance is

$$R_m = \beta \frac{Z_o^2 \sin^2 \theta}{r_o \tan^{-1} \frac{2X_c}{Z_o}}$$

[redacted]

if $\theta \ll \beta l$, where $\theta = \beta x$ is the electrical distance of the tap above the end plate. If the tank is to work between the same two values of impedance, θ will be the same on each end of the tank.

3. Half-Wave Tank with Series Gap.

See Figure 13. Let l be one-half the distance between inside edges of the end plates. βl is then the corresponding electrical length. Other symbols are as defined previously. Consider the tank as a series circuit of two short-circuited sections of line and the capacitive reactance X_c of spark gap and tuning condenser. Each of the shorted sections has a reactance of $X_L = Z_0 \tan \beta l$ ohms, looking away from the condenser terminals, and, for anti-resonance

$$X_c = 2Z_0 \tan \beta l$$

The impedance across the spark gap will be

$$Z_g = X_c^2/2R$$

where R is the effective resistance of one-half of the tank, given by the previous expression.

The impedance across the tank itself will be

$$Z_T = X_c^2/8R$$

or one-fourth Z_g. This may be obtained by considering the tank, as before, to be made up of two inductive sections, for each one of which the reactance is one-half X_c, and each of which has resistance R. Hence

$$Z_T = \frac{X_L^2}{2R} = \frac{(X_c/2)^2}{2R} = \frac{X_c^2}{8R}$$

is the impedance across one of these sections at anti-resonance.

The impedance between the connecting tap point, P, and ground, is

$$R_m = \beta \frac{Z_0^2 \sin^2 \theta}{r_0 \tan^{-1} \frac{X_c}{2Z_0}}$$

if $\theta \ll \beta l$, where $\theta = \beta x$ is the electrical distance of the tap above the end plate.

The effectiveness of this tank as a protector will depend upon its length, among other factors - it should be neither too short (too heavily capacitively loaded) nor too long (too lightly loaded), but preferably about a quarter-wave length long at the operating frequency. This can be seen by considering the voltage distribution along the tank with the gap ionized. See Figure 14. E_t is the impressed voltage between the input tap and ground. E_r is the resultant voltage applied to the receiver cable. The voltage will fall off in either direction from the

[redacted]

input tap along sections of a sine curve, that toward the receiver tap undergoing a sudden drop which is not drawn (with some shift in phase if X_c is of the same order as R_{gf}) as it passes the spark gap. Now if the tank is shorter than the optimum length the voltage will not have dropped to as low a value at the receiver connection, and if the tank is longer than optimum, the voltage will first rise as we proceed toward the output end, then fall, and again will not have as low a value at the receiver tap as with correct length.

The development of this tank grew out of an investigation of the effect of the electrode lengths of gaseous discharge tubes when such tubes are used in a quarter-wave tank. These lengths are an appreciable fraction of the length of the tank at the higher frequencies, and the position of the gap is not actually at the end of the quarter-wave affair. The question arose as to the effect of moving the gap position along the inner conductor (keeping the system anti-resonant by varying overall length) on the impedance Z_g. An analysis showed that this impedance would be a maximum when the gap were in the lengthwise center of the assembly. Note that the impedance concerned is that across the gap, not that across the tank, which is a maximum with the gap placed at the extreme end of a quarter-wave tank. (We are here considering X_c to be held constant).

D. Comparative Performance of Duplexing Systems Employing Concentric Tanks.

Some quantitative comparisons between the three styles of tanks will now be made.

1. Consider each of the three tanks to be designed for the same frequency. Each will be constructed of line for which $Z_0 = 100\ \Omega$, and each will be tuned by a reactance $X_c = 300\ \Omega$. Then theoretically the following relations will hold true for receiving conditions:

	$\beta\ell$	Relative R	Relative Z_T	Relative Z_g
$\lambda/4$ Tank	$1.25^{(r)}$	0.27	1.9	1.9
$\lambda/2$ Shunt Gap Tank	1.4	1	1	1
$\lambda/2$ Series Gap Tank	0.98	0.08	0.78	3.1

2. Consider, at the same frequency, the three tanks to have the same value of $\beta\ell$, say $1.25^{(r)}$, that all three have $Z_0 = 100\ \Omega$, and that the $\lambda/4$ tank is tuned by $X_c = 300\ \Omega$. Each of the other two will require a different tuning capacity. Relative quantities for receiving will be

	X_c	Relative R	Z_T	Z_g
$\lambda/4$ Tank	300 Ω	1	2	2
$\lambda/2$ Shunt Gap Tank	150	1	1	1
$\lambda/2$ Series Gap Tank	600	1	1	4

3. The data in this paragraph are for three tanks built for operation at 515 mc., each tuned by a gaseous discharge tube having a reactance of 258 Ω at this frequency. Each tank was constructed using 2-7/16" i.d. tubing as the outer conductor, 3/8" o.d. tubing as the inner conductor, making Z_o nominally 108 Ω. The tanks were silver plated. The input tap on each was adjusted so that R_m was the same in all three cases, approximately 1000 Ω.

A. For receiving conditions, calculations give:

	Z_T	Z_g
$\lambda/4$ Tank	670,000 Ω	670,000 Ω
$\lambda/2$ Shunt Gap Tank	370,000	370,000
$\lambda/2$ Series Gap Tank	260,000	1,040,000

Experimentally determined values of these quantities are not available.

B. If we assume that the discharge tubes, when fired, drop to a value $R_{gf} = 100\,\Omega$, the data for transmitting conditions are (transformer length adjusted to the proper value in each case):

1. Without receiver connected:

	Z_t	R_h	Voltage Ratio in Transformer $\sqrt{\frac{R_h}{\lvert Z_t \rvert}}$
$\lambda/4$ Tank	1.5 + j 11.5 Ω	1750 Ω	12.3
$\lambda/2$ Shunt Gap Tank	3.1 + j 17.5	920	7.2
$\lambda/2$ Series Gap Tank	0.8 + j 15	3410	15.1

2. With receiver connected (50 load):

	Z_t	R_h	Voltage Ratio $\sqrt{\frac{R_h}{\lvert Z_t \rvert}}$
$\lambda/4$ Tank	3.8 + j 10.3 Ω	690 Ω	7.9
$\lambda/2$ Shunt Gap Tank	2.9 + j 17.6	970	7.4
$\lambda/2$ Series Gap Tank	0.88 + j 15	3110	14.4

[redacted]

Note that while it is primarily the value of R_t which determines R_h and the equivalent transformer ratio, it is $|Z_t| = \sqrt{R_t^2 + X_t^2}$ which determines the voltage developed across the tank input tap, for Z_t is the complete impedance between this point and ground.

For use as a protective device, the $\lambda/4$ tank is not as efficient as either of the $\lambda/2$ affairs. The voltage reduction by the <u>transformer</u> for the $\lambda/4$ style will be slightly better than that in the case of the $\lambda/2$ shunt gap tank, but this is overshadowed by the existence of a large additional voltage drop in the $\lambda/2$ tank itself, which has no counterpart in the $\lambda/4$ style.

A choice between the half-wave styles, if one of them is to be used, will be dictated to some extent by considerations of the radio system with which the duplexing equipment is to be used. If the transmitter power is relatively low, the half-wave series gap style would probably be desirable since it will produce highest impedance across the spark gap and thus cause breakdown of the gap at transmitter powers which would not operate the half-wave shunt gap style. For higher powers, as regards protection, the wave shape of the voltage pulse reaching the receiver as a result of the transmitted pulse should be considered. It will be made up of an initial "spike" of voltage existing during the time taken to ionize the spark gap, and a "plateau" during the remainder (and most) of the pulse after the gap ionizes, which is lower in amplitude because the sustaining voltage across the gap is lower than the firing voltage. With either half-wave tank, the spike voltage reaching the receiver will remain constant regardless of the transmitter power, but it will be higher with the shunt gap than with the series gap tank, for, because of the greater impedance across the gap (greater step-up ratio) in the latter case, breakdown voltage across the gap (which is the same in either case) will be reached at a lower value of voltage in the wave-front of the transmitter pulse than it will in the case of the shunt gap tank. The plateau voltage, on the other hand, will be more or less constant with transmitter power when a shunt gap tank is used, since the gap when ionized is approximately a constant voltage device; it will rise as transmitter power is increased when a series gap tank is used. With values of transmitter power in use at present, measurements have shown that both tanks of the half-wave style limit the receiver voltage during the plateau to a safe value of a few volts, although this voltage is higher at the higher powers with the series gap than with the shunt gap tank. However, for very high powers, the shunt gap design might be preferable. Present knowledge of the relative danger to the receiver due to high spike voltage and to high plateau voltage is limited, so that no definite statement can be made as to the desirability of one half-wave tank rather than the other.

Power absorbed from the main transmission line will be less in the case of the $\lambda/2$ affairs, particularly the series gap style, than in the case of the $\lambda/4$ tank.

For decoupling, the $\lambda/4$ style is very good, for in this application no loading circuit such as the receiver input is added and the transformer will show quite high input impedance when the gap

[redacted]

fires. The $\lambda/2$ series gap style is better yet. Figure 5 illustrates the employment of a $\lambda/2$ shunt gap protective tank and a $\lambda/4$ decoupling tank in a complete duplexing system.

E. Frequency Selectivity of the Duplexing Network.

The duplexing network, if it is to be employed in a pulse transmission system, must have sufficiently low frequency selectivity during reception to preserve the pulse shape. We may arbitrarily define the band-width of the duplexing system as that frequency band about the fundamental operating frequency within which the receiver is supplied one-half or more of the maximum possible power which would be delivered to it in the absence of a duplexing system with its attendant insertion loss. The band-width of the duplexing tank itself, as an anti-resonant circuit, will determine the band-width of the system, but the latter will be much greater than the band-width of the tank alone, because, taking the quarter-wave tank as an example, the tank is (in the ideal) connected at the tap point merely as a shunt across the receiver load, as shown in Figure 16, and the impedance of this shunt at resonance is very high relative to the receiver input impedance. The effect of inductances of the leads into the tank will modify the actual circuit slightly, as shown later. The actual band-width will not be computed here, but enough data will be given to show the truth of the statement just made.

The antenna and transmission line system are equivalent to a generator of generated voltage E and internal impedance z_o ohms. The receiver input impedance will be equal to z_o. Let Z_m be the value of tank tap point impedance, a function of frequency. Then the voltage appearing at the receiver input will be

$$V_R = E \frac{\dfrac{z_o Z_m}{z_o + Z_m}}{z_o + \dfrac{z_o Z_m}{z_o + Z_m}} = E \frac{Z_m}{z_o + 2Z_m}$$

and the power delivered to the receiver is

$$P_R = \frac{|V_R|^2}{z_o} = \frac{E^2}{z_o} \cdot \left| \frac{Z_m}{z_o + 2Z_m} \right|^2$$

With $Z_m = \infty$, the maximum power that could be delivered to the receiver would be $P_{R_o} = \frac{E^2}{4z_o}$

Hence the power loss due to the shunting effect of the tuned tank is given by

$$\frac{P_{R_o}}{P_R} = \frac{|z_o + 2Z_m|^2}{4|Z_m|^2}$$

If the tap point impedance for the resonant condition of the tank has been adjusted to be, say, $20.z_o$ (1000 Ω for a 50 Ω system),

[redacted]

then $\frac{P_{R_0}}{P_R}$ at resonance will be 1.0506 and the db loss will be 0.214.

When the frequency has changed from the resonant value far enough to make the magnitude of the tap impedance 70.7% of its peak value, its resistive and reactive components each equal to 10 z_0, which condition specifies the frequency selectivity of the tank itself according to the usual convention, the value of $\frac{P_{R_0}}{P_R}$ has increased only to 1.0513 and the db loss to 0.217.

In the appendix is derived an expression for the frequency selectivity of the duplexing tank itself, which is the same for each of the three types of tanks discussed, being $\frac{\Delta f}{f} = \frac{r_0}{\beta Z_0}$

where r_0 = static resistance per unit length of tank
Z_0 = surge impedance of line used for tank
$\beta = 2\pi/\lambda$

In this expression, Δf is defined as the difference between the two frequencies at which the tank impedance is down to 70.7% of its resonant value.

As an example, take the 515 mc. tanks discussed as an illustration above. These were built with 2-7/16" i.d. silver plated brass shells and 3/8" silver plated inner conductors. Z_0 is then 108 Ω, and r_0 at the frequency specified is 21.05×10^{-4} ohms/cm. This gives for the band-width of such a tank itself

$$\Delta f = \frac{r_0 f}{\beta Z_0} = 93.1 \text{ kc.}$$

In using the tank in a duplexing system as described such that the tap point impedance at resonance is 1000 Ω, the insertion loss at each edge of the 93 kc. band in the ideal will be merely 0.217 db as noted above, so that the effective band-width of the system is much greater than 93 kc.

The overall selectivity of duplexing systems employing different styles of resonant tanks will differ somewhat due to the different values of series reactance through the tank itself when it is detuned (which comes into the picture of the half-wave tanks in addition to the shunting effect) but it will still be much more favorable than the selectivity of the tank taken by itself. Also, the decoupling system selectivity tends to sharpen the overall value for the duplexing system, but the latter will be sufficiently wide in spite of this factor also. The tanks may then be designed on the basis of maximum impedance, and band-width will be found to be adequate.

F. Design of Duplexing Tank.

Having picked a certain style of tank for use in a duplexing system, it should be made as effective as possible. It should have low losses, so that, other factors being the same, it will have a high anti-resonant impedance, enabling the tap to be placed low for a given value of R_m. Also, consistent with maintenance of sufficient impedance step-up to the gap, the tank should be as heavily capacity loaded as possible, for it has been shown (see individual discussion of the three styles of tank) that, keeping the connection tap a fixed electrical distance above ground, R_m will be inversely proportional to the electrical length of the tank, and hence, for heavier loading capacity, the tap can again be placed lower to give the same R_m. For a lower tap, the tap-point impedance will drop to a lower value when the gap ionizes.

1. For a given value of loading capacity to be used, the tank should be designed for a certain Z_0 if it is to show greatest anti-resonant impedance. This is true because of the form of the expression for the resistive component of the input impedance of a short-circuited section of line, which resistance determines the dynamic impedance for a given reactance. This resistive component is given by[1]

$$R = \frac{r_0}{4\beta} \; \frac{(2\beta\ell + \sin 2\beta\ell)}{\cos^2 \beta\ell}$$

The trigonometric factor is an increasing function of ℓ. Now,

$$Z_0 = k_1 \log\left(\frac{b}{a}\right)$$

$$r_0 = k_2 \sqrt{f}\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{k_2 \sqrt{f}}{b}\left(\frac{b}{a} + 1\right)$$

$$\beta\ell = \tan^{-1} \frac{X_c}{Z_0}$$

where b = inner diameter of outer conductor, a = outer diameter of inner conductor, f = frequency. With X_c, b, and the frequency fixed, a change in Z_0 will produce changes of opposite directions in the magnitudes of r_0 and $\beta\ell$ (and hence in the magnitude of $\frac{2\beta\ell + \sin 2\beta\ell}{\cos^2 \beta\ell}$) and there will exist a value of Z_0 giving a minimum for the product of r_0 and the trigonometric factor. The curve of Figure 21 shows the surge impedance to be used for a concentric tank for various values of loading capacity. This curve has been obtained by minimizing R in terms of Z_0. (The final solution of the conditional equation must be obtained graphically). Individual curves of R vs. Z_0 have fairly broad peaks at minima, so it is necessary to adhere to a value of Z_0 only within about 5 percent of the optimum for a given X_c.

2. With Z_0 kept constant, the outer conductor of the tank should be of the largest permissible diameter to obtain the greatest anti-resonant impedance. This may be seen from the equation just given for r_0. This parameter is inversely proportional to b, if b/a is constant. Reference is also made to the article by F.E. Terman, "Resonant Lines in Radio Circuits".[2]

[1] List of numbered references at end.

[2] List of references at end.

[redacted]

3. The tank should be made of the best obtainable radio-frequency conducting material. Brass, silver-plated on the conducting surfaces, has been much used.

4. In the case of the half-wave tank with series gap, the length of the tank has an optimum value, as has been pointed out. This should be in the neighborhood of a quarter-wave length at the operating frequency. The length will be determined by the choice of X_c and Z_o.

A concentric duplexing tank of any style may be designed according to the following steps:

1. Choose the tuning capacity, including that of the spark gap, if appreciable (as it is in the case of gaseous discharge tubes).

2. Calculate the tank length from $\tan \beta \ell = X/Z_o$. If X_c is the tuning capacity, then for the quarter-wave tank $X = X_c$; for the half-wave shunt gap, $X = 2X_c$; for the half-wave series gap, $X = X_c/2$. Having found $\beta\ell$, $\ell = \frac{\beta\ell}{1.57} \cdot \frac{\lambda}{4}$ inches.

3. Determine Z_o for optimum impedance from the curve of Fig. 21.

4. Make the outer shell diameter as large as permissible - compromise between greater tank impedance and a saving of material and space. Determine the inner conductor diameter from Z_o and the diameter of the outer shell.

5. Set the required resistance looking into the tap points for connecting lines according to the allowable total losses in the duplexing system during reception (the loss in protective tank plus that into the transmitter branch) - again this is a compromise between low losses in receiving, and sufficient protection plus sufficiently low transmitted power loss and low power dissipation in the spark gaps, especially if they are gaseous discharge tubes. Determine the physical position of the connecting tap (see notes following).

6. The critical dimensions of the tank are now determined. It remains to complete the mechanical design details, such as those of the tuning condenser, connecting attachments, etc.

The tanks have been customarily constructed of brass tubing, the outer shells varying in diameter from 2" to 4". Inner conductors have been brass tubing with inserts where necessary, or brass rod, and the end plates have been machined disks, held in place by machine screws through the shell. For external connection to the input line, split fittings plugging into the inner conductor of the line and sleeves for the outer conductor, sometimes split and having a locking screw, or more elaborate and positive clamps made up as parts of gas barrier couplings, have been attached to the tanks. It is important that this connection be positive - of very low loss - if best efficiency is to be obtained. Connections for the receiver line have, as a rule, been

[redacted]

commercial transmission line cable connectors. For production of the units in large quantity, methods of construction and assembly involving less machine work would be desirable.

It has been found most convenient to use variable spacing condensers as tuning elements. In the cases of the quarter-wave and half-wave shunt gap circuits, the condenser plates are circular disks, one fixed in position on the inner conductor, the other mounted on a threaded shaft which extends through a tapped collar on the outer shell. A locking nut or similar means such as a split collar with a clamping screw, is provided for fixing the capacity. The split collar is preferable. For the half-wave series gap affair, one plate again has been fixed (or semi-fixed using a collar with set screw) to one section of the inner conductor, and the other has been arranged to slide concentrically along the opposite inner conductor section by means of a screw adjustment controlled at the end of the tank, on the outside, by a knob. This arrangement can be used only when the spark gap is as small in diameter as the tank inner conductor, to allow the condenser plate to move over it. It has been used at this Laboratory, actually, only with an open style of gap, using tungsten points sparking in air. Where a glass enclosed gaseous discharge gap has been used in this style of tank, which happens to have been at rather high frequencies where additional loading capacity was undesirable, the tank has been arranged to have no tuning condenser, but an adjustable length, controlled also by a screw attachment at one or both ends.

When using this series gap style of tank, it is very undesirable to have any lumped capacity between the ends of the inner conductors and the shell, i.e. shunt capacity across the tank, for a mode of resonance is then possible in which each half of the tank is anti-resonant at the frequency of operation, and the two are <u>in phase</u>, the series condenser acting as a coupling instead of a tuning capacity (See Figure 15). This mode has been observed in addition to the desired one in experimental tanks. If it exists, the gap will have very little exciting voltage, and even if it does fire, the circuits will be detuned only slightly, allowing no protection for the receiver. Hence, it is recommended that this type of circuit be tuned only by a series capacitance. It would, of course, be quite all right if the undesired mode of resonance existed within the tuning range of the tank, provided the operator always picked the correct mode; the danger is that in practice the two will be indistinguishable when lining up the equipment on the basis of strongest received signal.

It is difficult to calculate the actual position of the connection taps for a required value of R_m. The inner conductor of the tank has discontinuities, and the stub coming out for the connector introduces a discontinuity in the internal field. A better procedure is to place the tap at what seems a likely point, then measure the input impedance with the tank in tune and with no output connection. If, as in the neighborhood of 200 mc., the tank is around 20 to 25 inches long in the case of the half-wave tanks, or around 10 inches in the case of the quarter-wave tanks, a tap about 3/4 inch from the end plate gives

R_m a value on the order of 1000 ohms, and as the tank becomes shorter going to the higher frequencies, the actual distance of the tap from the plate will not vary a great deal for the same impedance because the impedance of the tank is falling off. Some experience will enable the taps to be placed on the first trial at points which give measured impedances sufficiently close to those desired. The connecting stub into the tank has appreciable inductance, and if its length is more than about 10% of a quarter-wave length, a more accurate measurement of the tap-point impedance is obtained by connecting a line section to the tank, tuning it by means of a movable short-circuit to an electrical length of 3/4 wave length, then, leaving the system set, measuring the impedance across the line at a point 1/4 wave length toward the tank from the plunger, which point will be 1/2 wavelength electrically from the tank tap point. See Figure 17. This method introduces a small error since part of the connecting line (the part inside the tank) will be of different surge impedance from the remainder, but the error will be less than that incurred in measuring the impedance at the stub coming out from the tank, without the use of the 1/2 wave line. A measurement of the insertion loss of the tank when connected as in practice to its intended load will also yield the desired impedance.

The position of the tap on the tank is of importance, of course, only insofar as it is one of the things which contribute to the values of the insertion losses of the duplexing system. Measurements of the latter are of fundamental importance, and should be made on any newly constructed system that is quite different in frequency range and physical dimensions from any built before. Measurements should be made both under receiving conditions, with the tanks tuned to resonance, and under transmitting conditions, with the spark tubes or gaps shunted by resistances approximately equal to their fired resistances (short-circuited in the case of air gaps), the signal path being in each case the one taken in actual operation, and with the system in each case working into its intended load value. Provided these losses are satisfactorily low, the necessity for measuring the impedances of the tanks at the taps is obviated, of course, but a preliminary knowledge of these impedances is necessary if the system is to be designed systematically as outlined above, to give specified insertion losses.

The effect of the inductance of the lead into the tank will increase the losses during reception, as can conveniently be brought out by a vector diagram. The tank will automatically be tuned off resonance when it is adjusted to give maximum received signal, in order to produce unity power factor conditions for the system as a whole. Taking a quarter-wave tank as illustration, the equivalent circuit is as shown in Figure 18. Capacitance C will be of such magnitude as to bring E and I_1 in phase. R_m will not change substantially from its antiresonant value. X_L will be of the same order of magnitude as the receiver input resistance, R_L. Taking the two equal for purposes of illustration, the vector diagram can be drawn by starting with E_m, and adjusting magnitudes and angles of the arbitrary vectors to produce the required unity power factor. Figure 19 shows the diagram for this condition.

[redacted]

Then $E_m = \sqrt{2}\,E_1$, since $I_2R_L = I_2X_L$, and the power lost in the tank is $\frac{(\sqrt{2}E_1)^2}{R_m} = \frac{2E_1^2}{R_m}$, whereas if there were no lead inductance the power lost would be $\frac{E_1^2}{R_m}$. Hence, the effect of a lead reactance equal to the receiver input impedance is to double the loss in the protector tank. This fact should be borne in mind when choosing the value of R_m.

G. Connecting Line System.

The connecting line lengths for a duplexing system are rather critical and those connecting the tanks to the main line should be determined experimentally. They can be made of variable length and adjusted in actual operation for best signal, or in some cases the lines can be preset for operation over a limited frequency range as follows:

1. Replace or shunt the spark gap by a resistance approximately equal to R_g. Actually the use of a "short-circuit" of metal rod has given satisfactory results at the lower frequencies, i.e., 200 mc and below. If the gap consists of open air spark electrodes, simply screw these into the short circuit position.

2. Connect to the tank input a line section equipped with a movable plunger short-circuit, and having air dielectric only. The tank should have two small holes drilled on opposite sides through the outer shell at about the level of the connecting tap points. Insert through one a loop coupled to an oscillator operating on the system frequency, and through the other a loop coupled to a pick-up device. (Sometimes better results are had by coupling the pickup through a hole in the connecting line itself, close to the tank). The planes of the loops should be parallel to the length of the tank. Adjust the plunger on the line section for resonance of the system as indicated on the pick-up device. The distance from plunger to the effective ground point inside the tank is then a multiple of 1/2 wave-length - a single 1/2 wave length for the closest tuning point to the tank, and if we move in from the plunger toward the tank a true 1/4 wave-length (since the line is of low loss and contains only air dielectric) we are 1/4 wave-length away from the ground point, or effective short-circuit produced by the tank on the resonant line section. This quarter-wave point, or high impedance point, can be located in reference to a point such as the nearest outside edge of the tank, for example, and the connecting line should be constructed of such length that the quarter-wave point just determined falls on the nearer edge of the main transmission line inner conductor.

3. If the tank having a line fitted is to be a decoupler, it should now tune with its spark gap re-inserted and with a short-circuit placed at the high impedance point on the connecting line. This tuning can be checked as above by the signal source and pickup. The idea is that the connecting line, adjusted for transmitting conditions as specified, will be shorter than a true quarter-wave length, so that

under receiving conditions its input will be reactive if the decoupler tank itself is tuned to anti-resonance, and therefore the tank must be capable of being tuned far enough off anti-resonance to bring the system as a whole into tune. (This detuning, of course, will lower the effectiveness of the decoupling system).

4. The center section of the line system, between center lines of connection of the protective and decoupler tank lines, should be made a full quarter-wave long if it contains only air dielectric, as it preferably should, and requires no experimentation.

The line system can be permanently assembled as an integral unit, composed of the main line section and the two tank connecting-lines and then is ready for insertion at any point in the antenna line of the equipment with which it is to operate.

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APPENDIX

I. RADIO FREQUENCY RESISTANCE OF THE TANKS.

It is not often desired to calculate the effective resistance of a duplexing tank; the only part of this resistance that can be conveniently calculated is that due to the short-circuited line sections themselves, as has been noted in the main body of this paper. However, for purposes of specifying the depth of silver plating on the systems, it is desirable to know the depth of penetration of the r.f. currents on the conductor surfaces, and for this reason a brief discussion of the skin effect is here given, which is carried through to yield the static resistance per unit length of the transmission line sections composing the tanks.

In a conductor carrying alternating current, the current density at a distance x below the surface is

$$i_x = i_o e^{-x\sqrt{\frac{\pi f \mu}{\rho}}}$$

where i_o = current density at surface

f = a.c. frequency

μ = permeability of conducting material

ρ = resistivity of conducting material

e = base of system of natural logarithms.

The distance into the conductor at which the current density has decreased to a value $\frac{1}{e} \cdot i_o$ is usually called the "depth of penetration", d, and is given by

$$x = d = \sqrt{\frac{\rho}{\pi f \mu}}$$

The concept of depth of penetration, while not literally correct, of course, is useful since the a.c. resistence of a conductor is equal to the d.c. resistance of a surface layer having a depth equal to d, the depth of penetration.

For a radio frequency conductor, now, the actual static resistance (assuming skin effect to be the only cause of deviation from the d.c. value of resistance) will be:

$$R = \rho \frac{\ell}{A}$$ where ρ = conductor resistivity

ℓ = current path length

A = cross sectional area of surface layer having a depth d.

[redacted]

If the conductor is a straight wire or rod (See Figure 20) having a circular cross section of radius r, the actual area of the surface layer concerned is $\pi r^2 - \pi(r - d)^2 = \pi(2r - d)d$, but if $r >> d$, a sufficiently close expression is $A = 2\pi rd$.

The static resistance per unit length for such a conductor is then

$$r_0 = \rho \cdot \frac{1}{2\pi r} \cdot \sqrt{\frac{\pi f \mu}{\rho}} = \frac{1}{r} \cdot \sqrt{\frac{\mu \rho f}{4\pi}}$$

Silver has, at 20° C, the constants $\mu = 4\pi \times 10^{-7}$ henry/meter and $\rho = 1.63 \times 10^{-8}$ ohm meter, and for a linear round silver conductor of radius r the resistance per unit length is

$$r_0 = \frac{40.4 \sqrt{f}}{r} \times 10^{-9} \text{ ohms/inch.}$$

For other non-magnetic materials, the resistance will be proportional to the square root of the resistivity. The following table gives data on the high frequency resistance of linear round conductors of various metals. The data are for a temperature of 20° C, except in the case of chromium where the temperature is 0° C.

Material	Resistivity	High Frequency Resistance Per Unit Length
Aluminum	2.83×10^{-8} ohm meter	$53.3 \frac{\sqrt{f}}{r} \times 10^{-9}$ ohms/inch
Brass	7 approximately	84 approximately
Cadmium	7.6	87.4
Chromium	2.6	51.1
Copper	1.724	41.6
Gold	2.44	49.5
Rhodium	5.11	71.6
Zinc	5.8	76.4

It is obvious that, to take full advantage of the decreased losses obtainable by plating a radio frequency conductor with material of low resistivity, the depth of plating must be such that almost all of the current flows in the plated layer. This condition is met by making the thickness of plating about three times the computed depth of penetration, for if we compute the depth at which the current density is down to 10% of i_0, we find

$$x_{10\%} = \sqrt{\frac{\rho}{\pi f \mu}} \cdot \log_{10} e = 2.3026\ d.$$

[redacted]

As an example, for silver at 200 mc.,

$d = 0.179$ mil.

$x_{10\%} = 0.412$ mil.

and silver plating to a depth of 0.5 mil is adequate even though the current distribution is not quite the same in the plated conductor and in a solid silver conductor. (One mil is 0.001 inch). In the case of concentric tank duplexers for use at ultra-high frequencies, the assumptions made above in the derivation of the expression for r_o are justified, and this expression may be used to compute the resistance of the line sections composing the tanks. If the conducting surfaces of the tanks are plated with a material of low resistivity, use of r_o for the plating material will yield the approximate resistance of the plated conductors, provided the plating is sufficiently thick as specified. The approximation will, of course, be optimistic.

The components of resistance due to the inner and the outer conductors must be added, giving for example for conductors silver plated to a sufficient depth as specified above

$$r_o = 40.4\sqrt{f}\left(\frac{1}{a} + \frac{1}{b}\right) \times 10^{-9} = \frac{40.4}{b}\sqrt{f}\left(1 + \frac{b}{a}\right) \times 10^{-9} \text{ ohms/inch}$$

where a = outer radius of inner conductor in inches.

b = inner radius of outer conductor in inches.

f = frequency in cycles/second.

II. SHARPNESS OF RESONANCE OF DUPLEXING TANKS.

The expression for the sharpness of resonance of a resonant tank composed of a short-circuited line section tuned by a lumped capacitance,

$$\frac{\Delta f}{f} = \frac{r_o}{\beta Z_o}$$

is derived here. The case is that of a parallel resonant circuit, with inductive reactance supplied by the transmission line.

In the case of the ordinary parallel resonant circuit using lumped parameters and having a reasonable value of Q, the parallel impedance in the neighborhood of the anti-resonant frequency may be expressed sufficiently exactly as

$$Z_p = \frac{X_L X_c}{R_s + j(X_L - X_c)} = R_p + j X_p = \frac{R_s X_L X_c}{R_s^2 + (X_L - X_c)^2} - j\frac{X_L X_c (X_L - X_c)}{R_s^2 + (X_L - X_c)^2}$$

where R_s, X_L, X_c, are the parameter values measured around the L-C loop.

Since R_s and $X_L X_c = L/C$ are fixed, when on either side of resonance the reactance $(X_L - X_c)$ has risen to a value equal to R_s, the resistive component R_p of parallel impedance has dropped to a value

[redacted]

$R_p = \frac{X_L\, X_c}{2\, R_s} = X_p$, one half its value at resonance, the magnitude of Z_p is 70.7% of its resonant value, and with a constant current flowing through the circuit, the power dissipated therein is one-half its peak (resonance) value. The two frequencies on either side of the resonant frequency at which the foregoing condition holds, are customarily taken to define the limits of the frequency "pass band" of the circuit.

The same definition will be used for the pass band of a capacity-tuned tank. The same expression as in the lumped parameter case holds for the parallel impedance Z_p, and

$$R_p = \frac{R_s\, X_L\, X_c}{R_s^{\,2} + (X_L - X_c)^2} \; .$$

All of the circuit resistance R_s will be assumed to be in the shorted line section. Here, however, R_s is not constant with frequency, nor is the product $X_L\, X_c$ since the two reactances vary according to different laws, so that the numerator of the expression for R_p is not fixed, but increases with frequency. However, the effect of the variation of R_s in the denominator is to compensate for this, so that the approximation may be made that R_p has been reduced to one-half its peak value when we have gone off the anti-resonant frequency far enough to increase $X_L - X_c$ from zero (at resonance) to a value equal to R_s. Under this condition R_p will be equal to X_p, and the magnitude of Z_p will be 70.7% of maximum. The frequencies at which this is true will define the limits of the pass band under the assumptions made. Hence the necessary condition defining these limits is

$$\Delta X = R_s$$

where $R_s = \frac{r_o}{4\beta}\left(\frac{2\beta\ell + \sin 2\beta\ell}{\cos^2 \beta\ell}\right)$

is the input resistance of the short-circuited line section. ΔX is the change in reactance measured <u>around</u> the L-C loop, from the value $X = 0$ at resonance.

The inductive reactance supplied by the line section is

$$X_L = Z_o \tan\beta\ell = Z_o \tan \frac{2\pi f\ell}{c}$$

where c is the phase velocity of the wave in the tank. Then

$$\frac{\partial X_L}{\partial f} = \frac{1}{f} \;\; \frac{Z_o\, \beta\ell}{\cos^2\beta\ell}$$

Multiplying numerator and denominator by $\tan \beta\ell$, this is

$$\frac{\partial X_L}{\partial f} = \frac{X_L \;\; 2\beta\ell}{f \sin 2\beta\ell}$$

[redacted]

The change in X_L produced by a small change of frequency $\frac{\Delta f}{2}$ will be approximately

$$\Delta X_L = X_L \frac{2\,\beta l}{\sin\,2\,\beta l} \cdot \frac{\Delta f}{2f}$$

The capacitive reactance used to tune the tank is

$$X_c = \frac{1}{2\,\pi\, f\, C}$$

$$\frac{\partial X_c}{\partial f} = \frac{-1}{f}\, X_c$$

and, approximately, for a small change of frequency $\frac{\Delta f}{2}$

$$\Delta X_c = -X_c \cdot \frac{\Delta f}{2f}$$

Now if $\frac{\Delta f}{2}$ is the change in frequency from the parallel resonant frequency of the line and condenser combination to the 70% impedance frequency

$$\Delta X = \Delta X_L - \Delta X_c = X_L \frac{2\,\beta l}{\sin\,2\beta l}\frac{\Delta f}{2f} + X_c \cdot \frac{\Delta f}{2f} = \frac{r_o}{4\beta}\left(\frac{2\,\beta l + \sin\,2\,\beta l}{\cos^2\,\beta l}\right)$$

In the vicinity of resonance X_L and X_c may be taken to be equal, so that

$$X_L\left(\frac{2\,\beta l}{\sin\,2\,\beta l} + 1\right)\frac{\Delta f}{2f} = \frac{r_o}{4\,\beta}\left(\frac{2\,\beta l + \sin\,2\,\beta l}{\cos^2\,\beta l}\right)$$

$$\frac{\Delta f}{f} = \frac{r_o}{2\,\beta X_L} \cdot \frac{\sin\,2\,\beta l}{\cos^2\,\beta l} = \frac{r_o}{\beta\, X_L}\,\tan\beta l = \frac{r_o}{\beta\, Z_o}$$

Δf is here defined as the difference in the 70% impedance frequencies on each side of resonance, i.e., as the total bandwidth. The selectivity is not a function of the amount of capacity loading. It depends at a given frequency only on the radii of the tank conductors, will have a maximum value (most narrow pass band) when $b/a = 3.6$, $Z_o = 76.5\,\Omega$ where b is the inner radius of the tank outer shell, and a is the outer radius of the inner conductor, and with Z_o fixed, the bandwidth will decrease with increase of b.

As applied to the concentric duplexing tanks discussed in the body of this paper, the derivation must take into account the different expressions for the net inductance of the tanks. These expressions are, for the quarter-wave tank, $X_L = Z_o \tan \beta l$, for the half-wave shunt gap, $X_L = \frac{Z_o \tan\,\beta l}{2}$, and for the half-wave series gap, $X_L = 2Z_o \tan \beta l$, l being defined under the discussions of these tanks. The frequency discrimination in all three cases turns out to be the same, $\frac{r_o}{\beta\, Z_o}$,

[redacted]

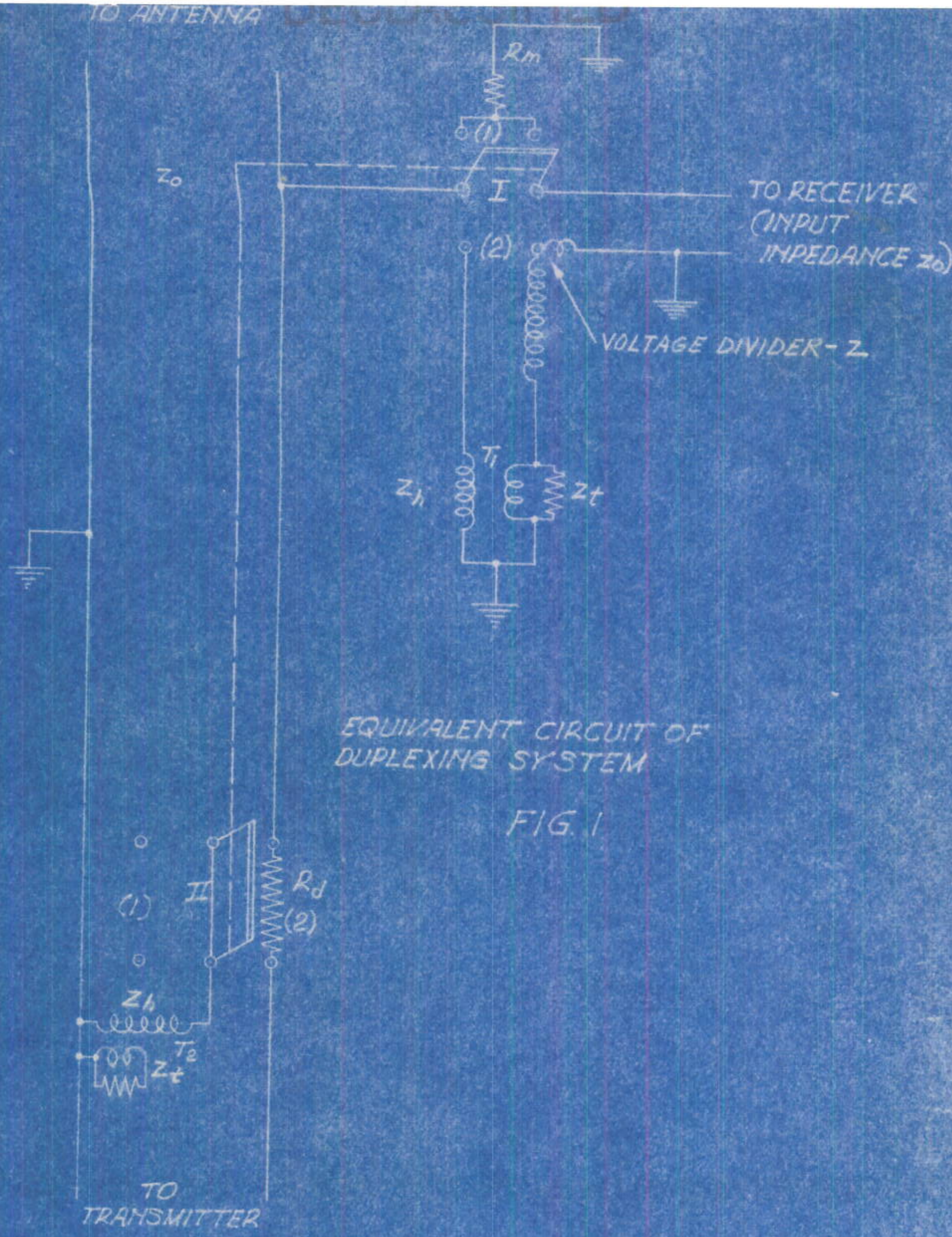
because the relative resistances also differ. Hence all three tanks, if formed of line sections of the same radial dimensions, will have the same frequency discrimination, and this will not depend on their electrical lengths.

List of numbered references:

1. L.S. Nergaard, "Survey of U.H.F. Measurements", R.C.A. Review, Oct. 1938. Reprint appears in "Radio at Ultra-High-Frequencies", R.C.A.I. Tech. Press.

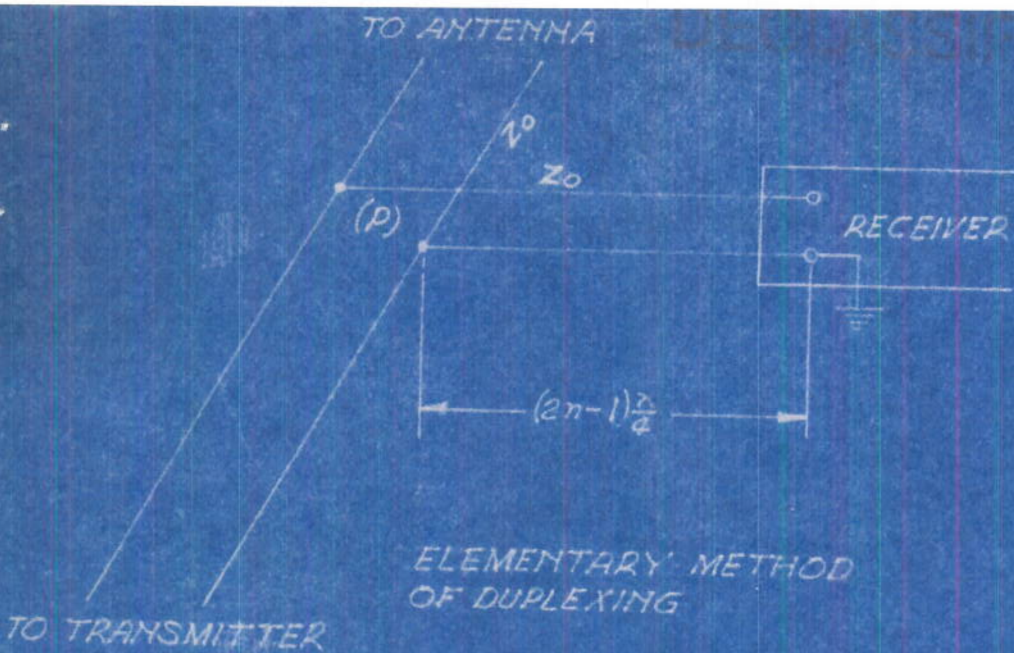
2. F.E. Terman, "Resonant Lines in Radio Circuits," Electrical Engineering, July 1934, p. 1046.

[redacted]



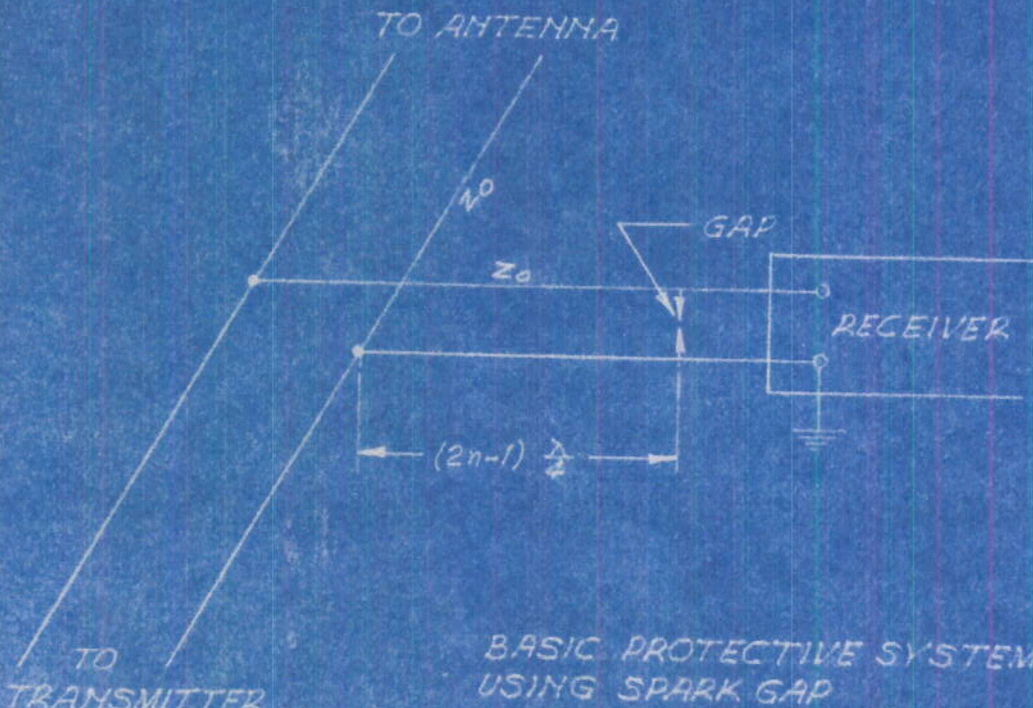
EQUIVALENT CIRCUIT OF DUPLEXING SYSTEM

FIG. 1



ELEMENTARY METHOD
OF DUPLEXING

FIG. 2



BASIC PROTECTIVE SYSTEM
USING SPARK GAP

FIG. 3

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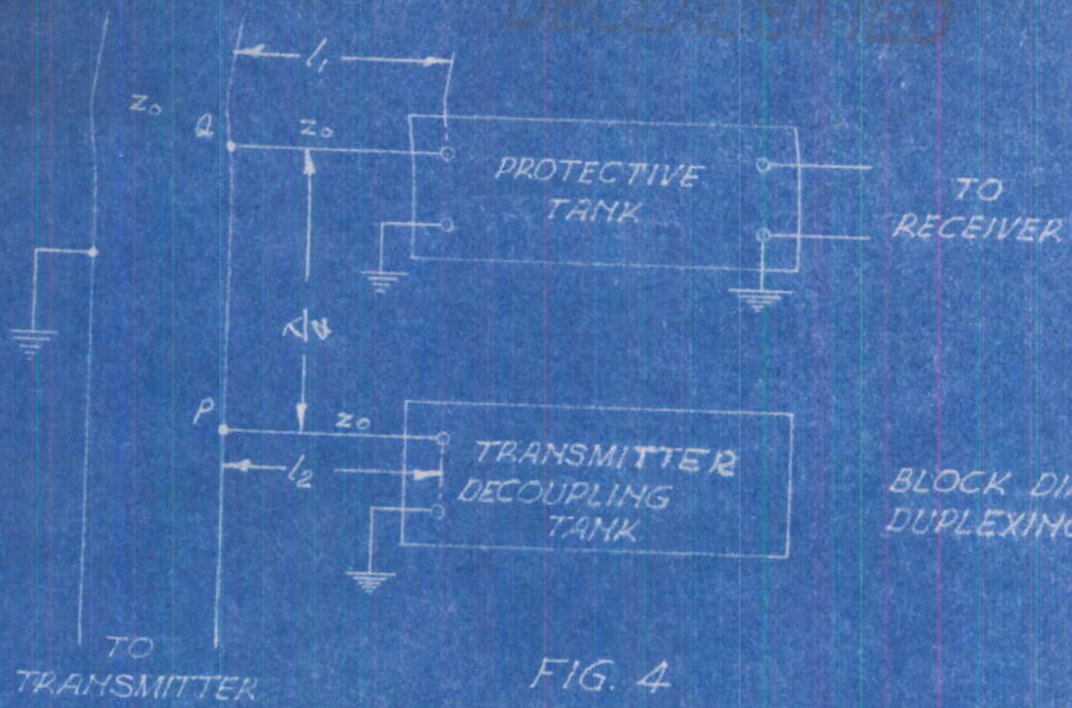
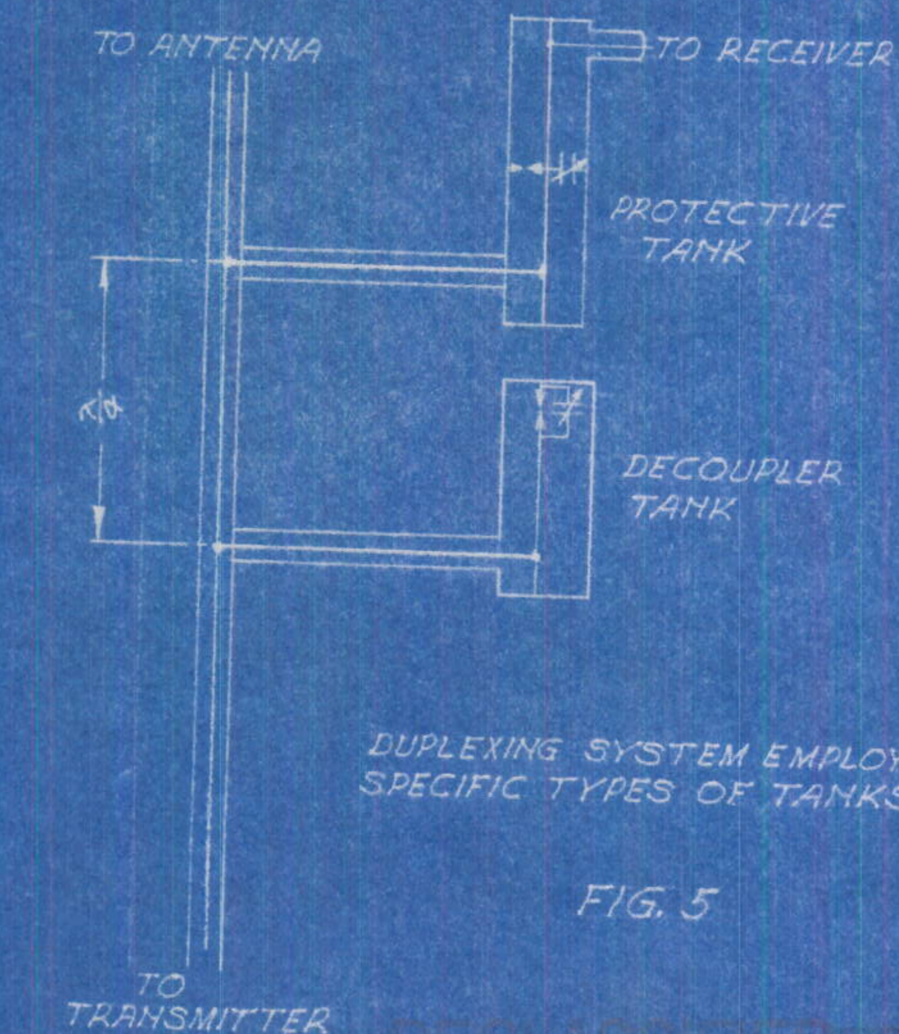


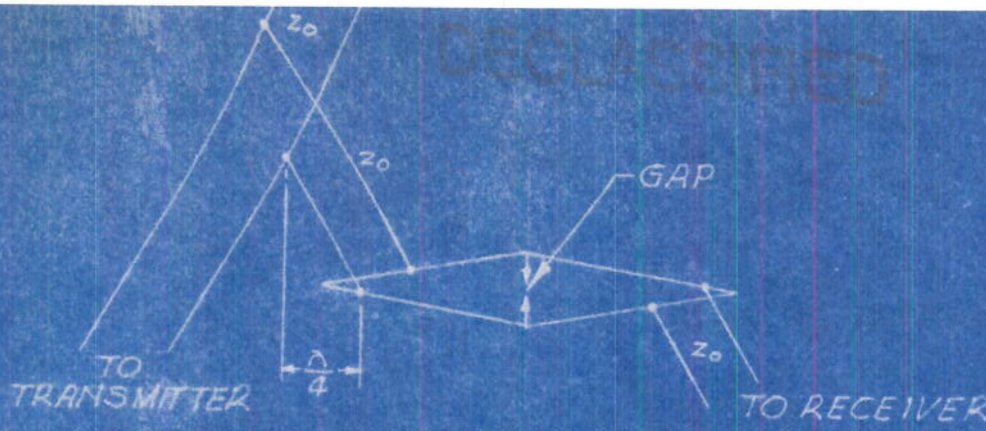
BLOCK DIAGRAM OF DUPLEXING SYSTEM

FIG. 4



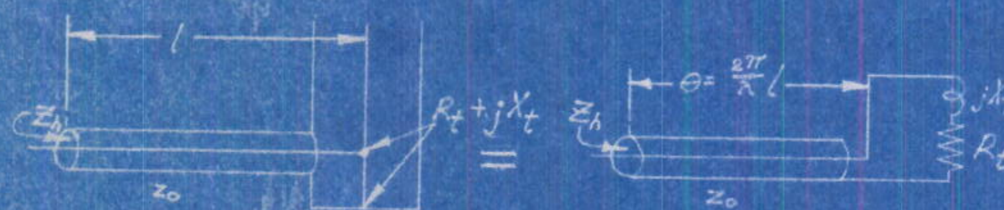
DUPLEXING SYSTEM EMPLOYING SPECIFIC TYPES OF TANKS

FIG. 5



PROTECTIVE SYSTEM EMPLOYING
TWO-WIRE ANTI-RESONANT CURENT

FIG. 6



TRANSFORMING INPUT LINE
TO DUPLEXING TANK

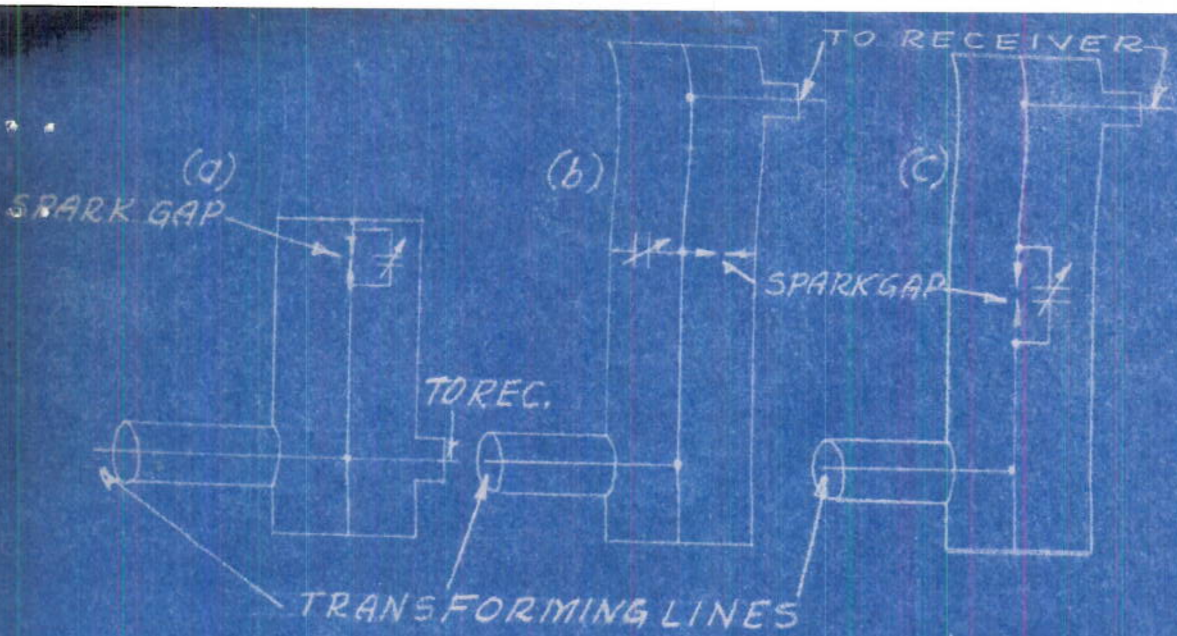
FIG. 7

TANK TAP POINT IMPEDANCE,
SPARK GAP IONIZED.

FIG. 8

[redacted]

PLATE 4

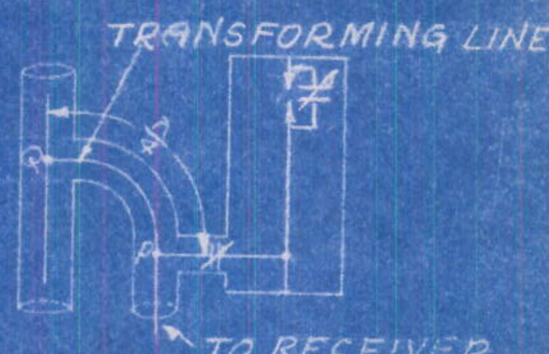


CONCENTRIC DUPLEXING TANKS
& TRANSFORMING LINES

FIG. 9

QUARTER WAVE TANK

FIG. 10



IMPROVED CIRCUIT FOR
QUARTER WAVE TANK

FIG. 11

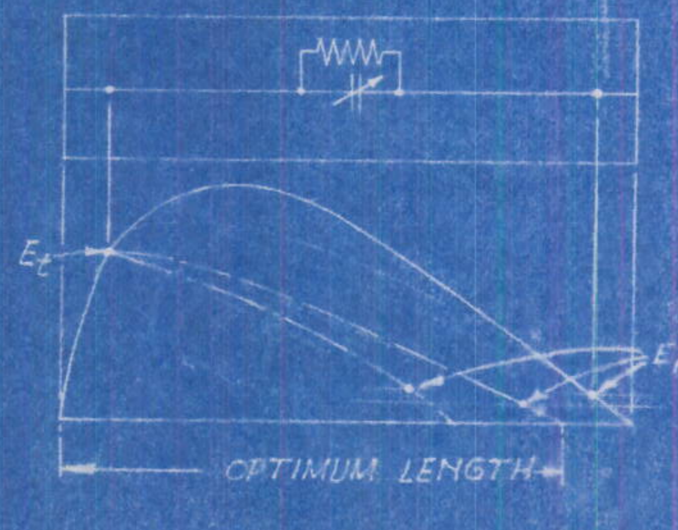
PLATE 5

HALF-WAVE SHUNT GAP TANK

FIG. 12

HALF-WAVE SERIES GAP TANK

FIG. 13

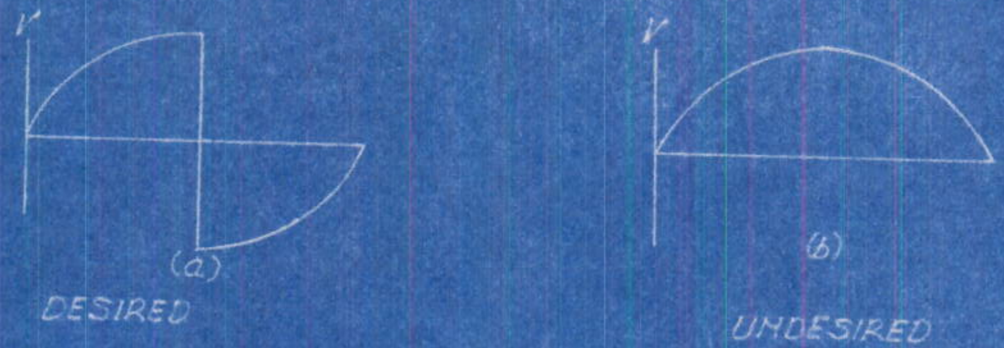


VOLTAGE DISTRIBUTION IN SERIES
GAP TANK DURING TRANSMISSION

FIG. 14

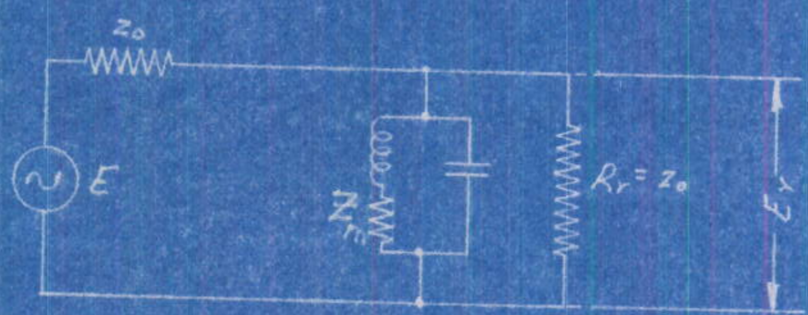
[redacted]

PLATE 6



MODES OF RESONANCE
IN SERIES GAP TANK

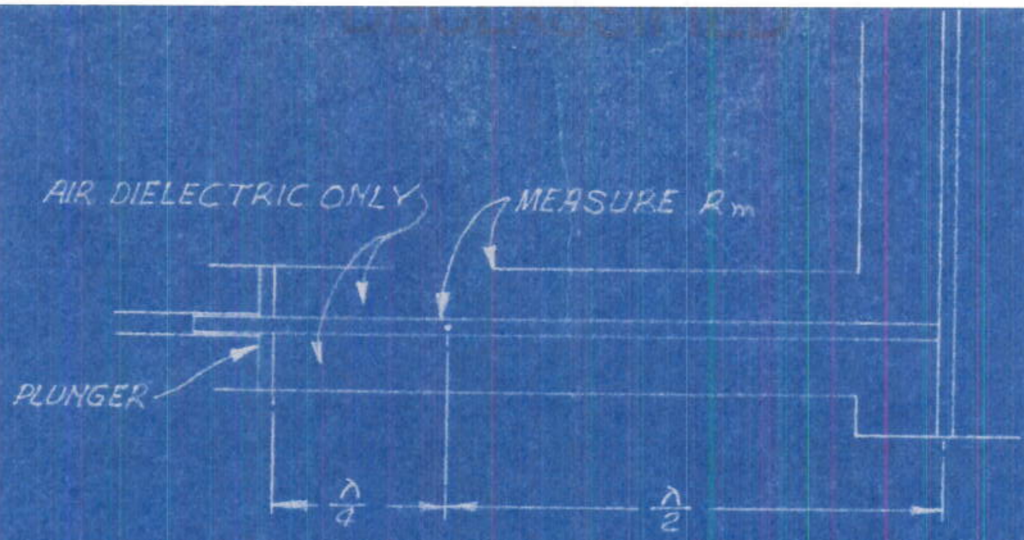
FIG. 15



SHUNTING EFFECT OF DUPLEXING
SYSTEM ON RECEIVER INPUT

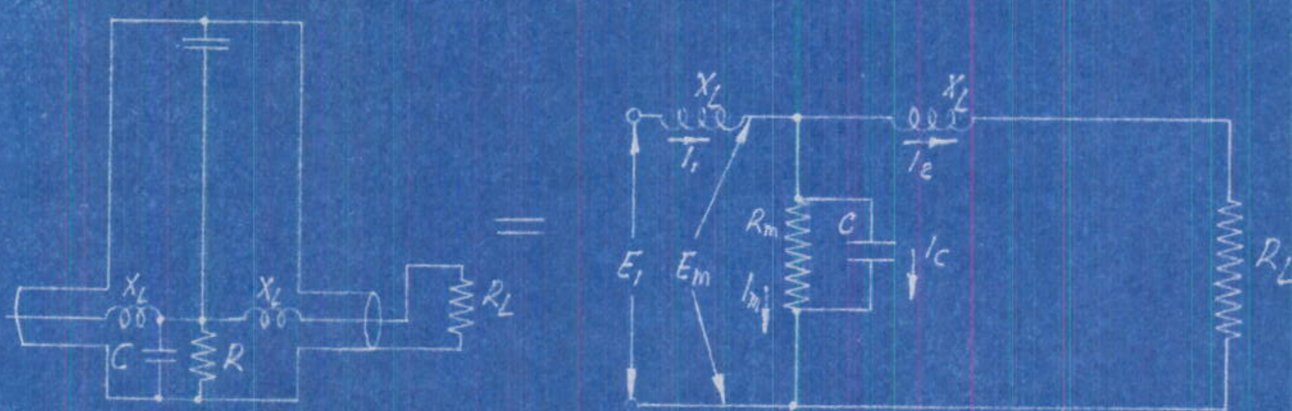
FIG. 16

 [redacted]



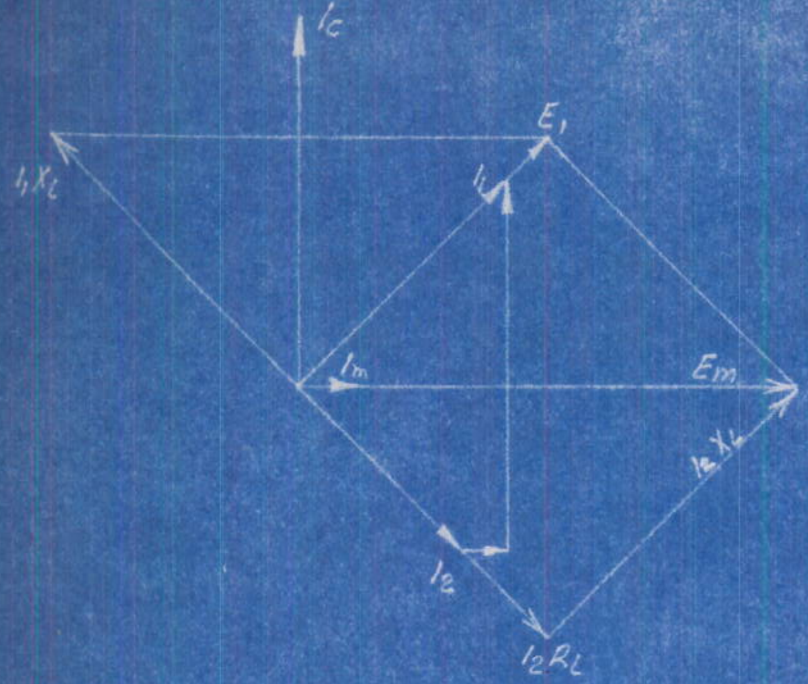
MEASUREMENT OF TAP-POINT
IMPEDANCE

FIG. 17



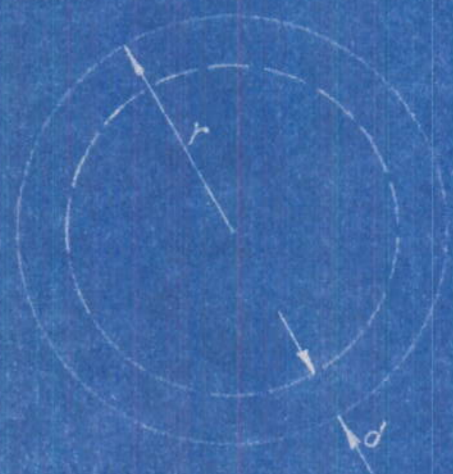
LEAD INDUCTANCE IN
DUPLEXING TANK

FIG. 18



VECTOR DIAGRAM FOR RECEIVING CONDITIONS IN DUPLEXING TANK, SHOWING EFFECT OF LEAD INDUCTANCE

FIG. 19



CROSS SECTION
OF A.C. CONDUCTOR

FIG. 20

[redacted]

PLATE 9

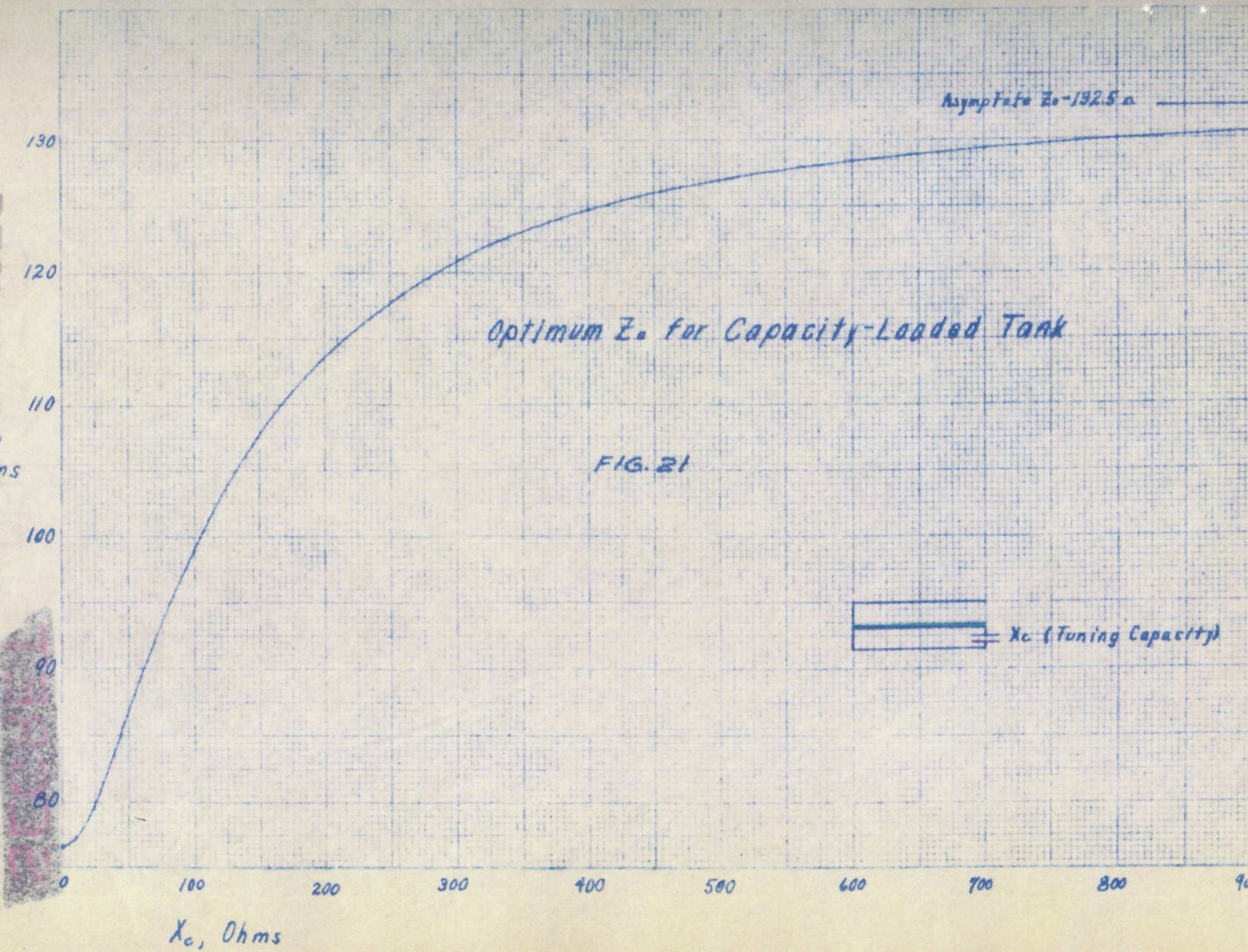


FIG. 21

PLATE 10