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Report on some Considerations concerning Vibration Machines  
and vibration measurements

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ABSTRACT

Three specific factors concerning vibration machines and vibration measurements have been considered because of their immediate interest at this time. These factors are:

- (a) Effects of a flexibly mounted load on the vibration characteristics of a reaction type vibration machine.
- (b) Effects of a flexibly mounted load on the bearing forces of a direct drive vibration machine.
- (c) Significance of the commonly made vibration measurements of acceleration and displacement amplitudes.

It is concluded (a) The vibration characteristics of a reaction type machine cannot generally be predicted. (b) The maximum bearing pressure of a direct drive machine is determined principally by resonant amplitudes of heavy flexibly mounted apparatus. (c) RMS measurements of vibration amplitude should be in terms of displacement amplitude rather than acceleration amplitude unless wave analyzers are used.

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## INTRODUCTION

### (A) Authorization and References

1. This work was authorized by reference (a).

Ref. (a) Bureau of Ships Project Order No. 1433/42  
of 24 September 1941.

Ref. (b) Mechanical Vibration, Den Hartog, 2nd edition,  
McGraw-Hill, 1940.

Ref. (c) Tentative Shock & Vibration Specifications  
for Shipborne Naval Radar Equipment, Bureau  
of Ships, RE9284A, 7 October 1943.

### (B) Statement of Problem

2. The purpose of this report may be divided into three parts:

(a) Analysis of vibration characteristics of a reaction type vibration machine under load conditions. A reaction type machine is driven by one or more attached rotating unbalanced weights.

(b) Analysis of possible bearing pressures for the direct drive type vibration machine. The direct drive machine is forced to vibrate at some given amplitude by mechanical linkages between the table top and the source of driving power, which is attached to a heavy base.

(c) Discussion of the significance of commonly made vibration measurements of acceleration and displacement amplitudes.

## GENERAL DISCUSSION

3. The construction, installation, and probable upkeep difficulties encountered in direct drive machines appear to be enormously greater than those for reaction type machines designed for the same load capacity. No attempt will be made to weigh the various factors involved to determine which is the more suitable machine for any given situation. The three points referred to in paragraph 2 will be followed quite closely. They are taken as the subjects for consideration as they appear to be of prime importance at this time.

### (A) Loaded Reaction Type Vibration Machine

4. The problem of flexibly mounted apparatus being tested on a reaction type vibration machine may be represented in part

by Figure 1. The table mass is represented by  $M$  which is flexibly supported by a spring of constant  $K$ . The apparatus mass and the spring constant of its coupling to the table are represented by  $m$  and  $k$  respectively. The system is assumed constrained to motions along the mounting axes. Viscous damping is represented by the dashpot  $D$ . An exciting force

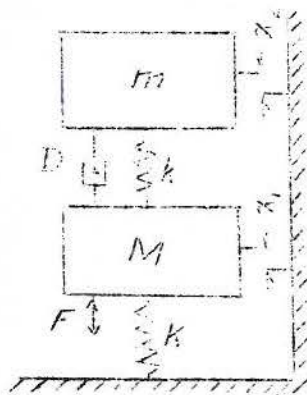


Figure 1. Schematic model of reaction type machine with damped flexibly mounted load.

$$F = K'F_0\omega^2\sin\omega t$$

(1)

is applied to  $M$ . This force is taken proportional to  $\omega^2$  as the force exerted by an unbalanced rotating mass varies in this manner. Thus actual working conditions are assumed for the applied force. The constant  $K'$  is included mainly for dimensional reasons.  $K'$  has the dimensions of time squared. If  $F_0$  is defined as the magnitude of the force vector when  $\omega$  is equal to unity, then  $K'$  can be taken as unity. The displacements of  $m$  and  $M$  from their equilibrium positions are given by  $x_2$  and  $x_1$  respectively. The viscous damping is represented by the constant  $c$ .

5. The equations of motion of this system are given by equations (2 and (3 below.

$$M\ddot{x}_1 + Kx_1 + k(x_1 - x_2) = c(\dot{x}_1 - \dot{x}_2) = K'F_0\omega^2\sin\omega t \quad (2)$$

$$m\ddot{x}_2 + k(x_2 - x_1) + c(\dot{x}_2 - \dot{x}_1) = 0 \quad (3)$$

These are very nearly the same as those given by Den Hartog, reference (b), on page 115. Nomenclature similar to that used in this reference has therefore been used.

6. A solution of the above equations, transient terms neglected, is a sinusoidal function of frequency represented by  $\omega$ . In place of being concerned with the instantaneous values of displacements only the magnitude of the displacement vectors will be considered. These will be represented by  $X_1$  and  $X_2$  corresponding to  $x_1$  and  $x_2$ . A dimensionless form of the solution of equations (2 and (3 is

$$\frac{X_1}{K'F_0/M} = \left[ \frac{\frac{\omega^4}{\omega_1^4} \left[ \left(1 - \frac{\omega^2}{\omega_2^2}\right)^2 + \frac{4\omega^2 c^2}{\omega_2^2 c_c^2} \right]}{\left[ \left(1 - \frac{\omega^2}{\omega_1^2}\right) \left(1 - \frac{\omega^2}{\omega_2^2}\right) - \mu \frac{\omega^2}{\omega_1^2} \right]^2 + \frac{4\omega^2 c^2}{\omega_2^2 c_c^2} \left[1 - \frac{\omega^2}{\omega_1^2} - \mu \frac{\omega^2}{\omega_1^2}\right]^2} \right]^{\frac{1}{2}} \quad (4)$$

$$\frac{X_2}{K'F_0/M} = \left[ \frac{\frac{\omega^4}{\omega_1^4} \left[ 1 + \frac{4\omega^2 c^2}{\omega_2^2 c_c^2} \right]}{\left[ \left(1 - \frac{\omega^2}{\omega_1^2}\right) \left(1 - \frac{\omega^2}{\omega_2^2}\right) - \mu \frac{\omega^2}{\omega_1^2} \right]^2 + \frac{4\omega^2 c^2}{\omega_2^2 c_c^2} \left[1 - \frac{\omega^2}{\omega_1^2} - \mu \frac{\omega^2}{\omega_1^2}\right]^2} \right]^{\frac{1}{2}} \quad (5)$$

Where

$$x_1 = X_1 \sin(\omega t + \alpha)$$

$$x_2 = X_2 \sin(\omega t + \beta)$$

$$\omega = 2\pi f \text{ and } f = \text{applied force frequency}$$

$$\omega_1 = \sqrt{\frac{K}{M}}$$

$$\omega_2 = \sqrt{\frac{k}{m}}$$

$$\mu = \frac{m}{M}$$

$$c_c = \frac{2k}{\omega_2} = \text{critical damping constant}$$

7. The ratios of the amplitudes are given rather simply as:

$$\frac{X_2}{X_1} = \left[ \frac{1 + \frac{4\omega^2 c^2}{\omega_2^2 c_c^2}}{\left(1 - \frac{\omega^2}{\omega_2^2}\right)^2 + \frac{4\omega^2 c^2}{\omega_2^2 c_c^2}} \right]^{\frac{1}{2}} \quad (6)$$

This equation is identical to that given in Den Hartog, reference (b), page 37, for the transmissibility of a flexible mounting with a single degree of freedom. It is interesting to note that the ratio of the amplitudes of vibration of  $m$  and  $M$  is entirely independent of the magnitude of vibration of  $M$  and the spring constant  $K$ .

8. An important point in this respect is that this ratio reaches a maximum value at a frequency that is independent of  $M$  and  $K$ . Thus the resonant frequency is the same regardless of the size of  $M$ , and for no damping it occurs at  $\omega = \omega_2$ . This may be considered the resonant point of  $m$  on  $M$ , but this cannot be considered a resonant point for the  $m - M$  system. An examination of equation (4 and (5 shows that there are several values of  $\omega$  at which the denominators will have minimum values, or zero values if no damping is assumed. At these points the values of  $X_1$  and  $X_2$  will become large. These points, however, are a function of both  $M$  and  $m$  and therefore have no general significance to shipboard mounted equipment. It should be remembered that the resonant value referred to in the tentative shock and vibration specifications RE9284A, reference (c), can only mean the value,  $\omega = k/m$ , where the ratio of  $X_2/X_1$  is a maximum value. This resonant value is independent of the testing machine.

9. Reaction type machines are generally designed so that the resonant frequency of the unloaded machine (that of  $M$  on  $K$ ) is as low as practical. Equations (4 and (5 will be considerably simplified if it is assumed that  $\omega$  approaches zero. In this case these equations reduce to

$$\frac{X_1}{K'F_0/M} = \left[ \frac{\left(1 - \frac{\omega^2}{\omega_2^2}\right)^2 + \frac{4\omega^2 c^2}{\omega_2^2 c_c^2}}{\left(\frac{\omega^2}{\omega_2^2} - 1 - \mu\right)^2 + \frac{4\omega^2 c^2}{\omega_2^2 c_c^2} (1 + \mu)^2} \right]^{\frac{1}{2}} \quad (7)$$

$$\frac{X_2}{K'F_0/M} = \left[ \frac{1 + \frac{4\omega^2 c^2}{\omega_2^2 c_c^2}}{\left(\frac{\omega^2}{\omega_2^2} - 1 - \mu\right)^2 + \frac{4\omega^2 c^2}{\omega_2^2 c_c^2} (1 + \mu)^2} \right]^{\frac{1}{2}} \quad (8)$$

10. Equations (6 and (7 are plotted for various values of  $\mu$  and  $c_c$  on Plates 1 through 4. These curves show the following features:

- (a) For no load, i.e.,  $\mu = 0$ , the left hand side of equation (7), which is proportional to the table amplitude, is of unit value independent of frequency. In what follows the left hand part of equation (7) shall be referred to as the table amplitude factor.
- (b) For a loaded table the table amplitude factor is  $1/(1 + \mu)$  part of a unit value at very low frequencies. As the frequency increases the table amplitude factor goes through a minimum near  $\omega = \omega_2$  after which it rises to a maximum near  $\omega = 1 + \mu$ . As the frequency increases still further the amplitude factor asymptotically approaches unity.
- (c) The ratio between the apparatus and table amplitude increases from unity, at the very low frequencies, to a maximum near  $\omega = \omega_2$  after which it decreases asymptotically to zero. The ratio passes through unit value at  $\omega/\omega_2 = \sqrt{2}$ . If the spring constant  $K$ , had not been assumed of zero value there would have been another system resonance at a low value of  $\omega/\omega_2$ .

(B) Bearing Load for Direct Drive Type Vibration Machine

11. A direct drive vibration machine of vibration mass  $M$  driven sinusoidally through an amplitude  $X$  is shown in Figure 2. An apparatus of mass  $m$ , is flexibly attached to  $M$  by a spring of constant  $k$ . The system is constrained to move along the axis of the spring.

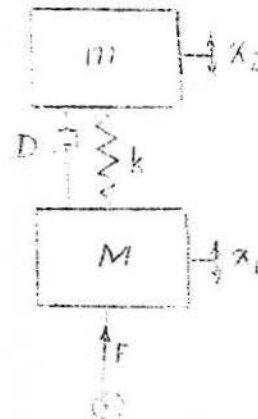


Figure 2. Schematic model of a direct drive machine with damped flexibly mounted load.

12. The equation of motion of  $m$  and  $M$  are given by

$$m\ddot{x}_2 + k(x_2 - x_1) + c(\dot{x}_2 - \dot{x}_1) = 0 \quad (9)$$

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$$M\ddot{x}_1 + k(x_1 - x_2) + c(\dot{x}_1 - \dot{x}_2) = F(\omega) \sin(\omega t + \beta) \quad (10)$$

where  $F(\omega)$ , or simply  $F$ , is the force exerted by the crank on the vibrating mass  $M$ . It is assumed that this force is sinusoidal and directed along the axis of the spring. Effects of gravity are neglected, but should be considered in final calculations.

13. As the motion of  $M$  is completely determined by the crank, the values of  $x_1$  can be assigned, so

$$x_1 = X_1 \sin \omega t \quad (11)$$

Equation (9) can now be solved for  $x_2$ , this solution is:

$$x_2 = X_1 D \sin(\omega t + \alpha) \quad (12)$$

where

$$D = \left[ \frac{1 + \frac{4c^2\omega^2}{c_c^2\omega_r^2}}{\left(1 - \frac{\omega^2}{\omega_r^2}\right)^2 + \frac{4c^2\omega^2}{c_c^2\omega_r^2}} \right]^{\frac{1}{2}}$$

and

$$\alpha = \tan^{-1} \left[ \frac{\frac{-2c\omega^3}{c_c\omega_r^3}}{1 - \frac{\omega^2}{\omega_r^2} + \frac{4c^2\omega^2}{c_c^2\omega_r^2}} \right]$$

14. With the values of  $x_1$  and  $x_2$  completely determined by means of equations (9) and (11) these values can be substituted in the left hand part of equation (10) which is then identically equal to the right hand part of equation (10). The solution of  $F$  so obtained and put in dimensionless form is:

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$$\frac{\mu F}{kX_1} = \left[ \left( \mu - \frac{\omega^2}{\omega_r^2} - \mu D \cos \alpha + 2\mu \frac{c\omega D}{c_c \omega_r} \sin \alpha \right)^2 + \left( 2\mu \frac{c\omega}{c_c \omega_r} - \mu D \sin \alpha - 2\mu \frac{c\omega}{c_c \omega_r} D \cos \alpha \right)^2 \right]^{\frac{1}{2}}$$

where

$$\mu = \frac{m}{M}$$

$$\omega_r^2 = k/m = \omega_2^2$$

$$c_c = \frac{2k}{\omega_r}$$

F = force on bearing

X<sub>1</sub> = displacement amplitude of vibration table.

15. The ratio of the amplitudes X<sub>2</sub> and X<sub>1</sub> is the same as given by equation (6) if  $\omega_r$  is substituted for  $\omega_2$ .

16. Curves showing values proportional to the bearing force F, as a function of  $\omega/\omega_r$ , are plotted on Plates 5 and 6. Plate 5 illustrates these values for  $c/c_c = 0.05$  and  $\mu = 1$ . The ratio of the amplitudes X<sub>2</sub> and X<sub>1</sub> is also plotted on this plate. Bearing forces for the unloaded vibration machine are shown to the same scale. This curve is proportional to the square of the frequency.

17. A study of the equations and curves allows the following generalizations:

- (a) A maximum of bearing force, for a flexibly loaded direct drive vibration machine, occurs near the resonant value,  $\omega_r = \sqrt{k/m}$ , when the damping is reasonably small and when the mass of the load is of the same order of magnitude as that of the vibrating table.
- (b) When the damping is small and the load is large (approximately that of the table) the maximum bearing force, neglecting gravity forces, can be taken to be

$$\begin{aligned} F &= m \omega_r^2 X_2 \\ &= m \omega_r^2 R X_1 \end{aligned}$$

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For many shear type rubber mounts  $R$  can be taken as about 20, hence if the load,  $m$ , were equal to the table mass the bearing force at resonance would be 20 times that of the unloaded table at the same frequency. At this resonant point the phase difference between the motion of  $m$  and  $M$  is  $90^\circ$ . The contribution of  $M$  to the bearing force may be vectorially added to that of  $m$  without much difficulty.

- (c) The bearing force decreases toward zero as the frequency is increased above resonance, until a frequency ratio near  $\sqrt{1 + \mu}$  is reached. This is the resonant frequency of the free  $m - M$  system, and is a point of infinite amplitude for the undamped case of the reaction type machine. After the bearing force has reached this minimum its value increases with frequency finally approaching the force curve of the unloaded machine. It is interesting to compare the reaction type machine displacement curve with the bearing pressure curve of the direct drive machine. The bearing pressure is a maximum where the reaction type vibration displacement is a minimum, and the bearing pressure is a minimum where the reaction type vibration displacement is a maximum.

### (C) Significance of Commonly Made Vibration Measurements

18. It is conventional to plot vibration amplitudes, or ratios of vibration amplitudes, as a function of the frequency of the driving force. This practice often involves the tacit assumption that both the driving force and the vibration are simple harmonic. There is a natural tendency to make the same assumption in connection with graphs or discussions of acceleration as a function of frequency.

19. In the preceding discussions of both reaction and direct drive vibration machines, it was explicitly assumed that the driving force was simple harmonic. It followed that the vibrations maintained, and also the reaction forces, were of the same form. Amplitudes of displacement, acceleration and force were computed on this basis. In actual practice, appearance of harmonics of the driving force frequency or fundamental frequency is to be expected. The effect of such high frequency components upon acceleration amplitudes is at once made apparent by observing that a displacement containing a fundamental and 5 percent of the 9th harmonic is represented as

$$x = x_0 (\sin \omega t + 0.05 \sin 9 \omega t)$$

The acceleration is then

$$\ddot{x} = -\omega^2 x_0 (\sin \omega t + 4.05 \sin 9 \omega t)$$

which has a 9th harmonic 4 times as large as the fundamental. Thus the effect of the  $n$ th harmonic upon the acceleration amplitude is  $n^2$  times as great as its effect on the displacement amplitude.

20. Plate 7 illustrates acceleration curves and displacement curves obtained at one point on a vibration machine. This was a small direct drive type with poor wave form. However, the wave forms of several reaction type machines were found to be of a similar nature. The measurements were made with a G.R. Type 761-A vibration meter. The displacement curves are reasonably sinusoidal, but the acceleration curves exhibit a high frequency "hash" superimposed on the fundamental, and in some cases even masking it. This hash may be associated with the noise of the machine. In general it is nonperiodic, so that standing patterns can not be obtained on the oscillograph viewing screen. The curves shown are all single sweep patterns. It is to be expected, of course, that some of the high frequency components may be periodic. Displacements caused by this hash are small compared to the fundamental sinusoidal displacements.

21. Plate 8 by curves a and a' illustrates the acceleration wave form on either side of a flexible mounting. The mount evidently filters out nearly all of the high frequency components. The ratio of the root-mean-square values of accelerations on either side of the mount was 0.93. Corresponding displacement wave forms are shown in curves b , b' . The ratio of RMS displacements was 1.80.

22. Plates 9 and 10 present more detailed information on the motion of the table of a typical vibration machine without load. These also afford a comparison of two methods of measurement. The more elementary method consists of direct observation of the double amplitude of displacement by means of a traveling microscope. Comparative measurements were made with a G.R. 761-A vibration meter. This of course involved use of a piezoelectric pickup responsive to acceleration followed by amplification and two-fold integration to yield an output reading proportional to the RMS displacement. Prior calibration had established a correction factor of 1.2 for output readings of acceleration, velocity, and displacement by this particular meter. Plate 10 shows the ratio of displacement amplitude measurements by the two methods, plotted as a function of driving frequency. These ratios approach the value of 1.2 at higher frequencies, in accord with the normal correction factor for this G.R. meter. At lower frequencies the response of the G. R. meter falls off, so the ratio curves rise in this region. It is to be noted that when the meter control was set for frequency response down to 2 cps, the displacement ratio curves were horizontal to below 10 cps. The displacement ratios at the minimum amplitude of 0.011 inch were consistently low. This was caused by a readjustment of the balance of the G.R. meter between this amplitude and

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the higher amplitude. However, vibration displacements of more than a few thousandths of an inch appear to be sufficiently sinusoidal for agreement between amplitudes observed directly with the microscope and the RMS amplitudes values derived by the meter in response to acceleration. Use of a wave analyzer appears unnecessary in such a comparison of displacement amplitude measurements. This generalization should be checked to determine its validity for any given machine.

23. In Plate 9 acceleration ratios are plotted as a function of driving frequency. The numerator of the ratio was calculated from traveling microscope measurements of displacement amplitude and observations of driving frequency, assuming sinusoidal wave form. The denominator of the ratio was supplied by direct readings of the G.R. 761-A vibration meter. At large amplitudes and high frequencies, the acceleration amplitude ratios approach 1.2, which is the normal correction factor for the G.R. meter used. At low frequencies, and especially at small amplitudes, the ratios become much less than 1.2. This lack of agreement can be interpreted in various ways, depending on the point of view. The curves of Plates 7 and 8 indicate reasonably sinusoidal displacements, but in contrast there are also shown acceleration traces in which sinusoidal variation at the driving frequency may be completely masked by high frequency harmonics and hash. Therefore the assumption of sinusoidal wave form used in computing acceleration amplitudes from microscopic measurements of displacement amplitudes did not apply. The trend of the curves of Plate 9 may then be taken as evidence that the motion of the table deviated from simple harmonic, more so at low than high frequencies, and much more so at low amplitudes than high ones.

24. Another interpretation of more general interest is equally valid. The microscopic measurements of displacement amplitudes at driving frequency  $\omega_0$ , with assumption of sinusoidal motion, are a satisfactory basis for calculation of the component of acceleration amplitude at frequency  $\omega_0$ . The displacement measurements, however, afford no information about component accelerations caused by harmonics of  $\omega_0$ , or by hash. The wide frequency response of the G. R. meter causes its acceleration reading at driving frequency  $\omega_0$  to include not only the fundamental component at  $\omega_0$  but also the higher ones. Suppression of the vibration meter response to frequencies above  $\omega_0$ , or use of a wave analyzer, would yield an RMS acceleration amplitude value more truly comparable with that computed from the direct displacement measurements.

25. Yet another inference from the data of Plates 7, 8, and 9 is that graphs of acceleration amplitude as a function of driving frequency  $\omega_0$  are open to misinterpretation. If the acceleration graphed is calculated from displacement amplitude measurements and assumed sinusoidal motion, it represents only the  $\omega_0$  component of acceleration, which may be only a small

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fraction of the actual total resultant acceleration at that driving frequency. If, on the other hand, the graph is based on readings from a vibration meter with a wide frequency response, there may be some question of whether these readings should be shown as a function of driving frequency  $\omega_0$ .

### SUMMARY

26. Three specific factors concerning vibration machines and vibration measurements have been considered because of their immediate interest at this time. These three factors are:

- (a) Effects of a flexibly mounted load on the vibration characteristics of a reaction type vibration machine.
- (b) Effects of a flexibly mounted load on the bearing forces of a direct drive type vibration machine.
- (c) Significance of commonly made vibration measurements of acceleration and displacement amplitudes.

### CONCLUSIONS

27. It is concluded that vibration characteristics of a reaction type machine cannot generally be predicted when the machine is loaded with heavy flexibly mounted masses. The theory of dynamic vibration absorbers shows that the vibration of the machine is nearly eliminated at resonant points of the load on the machine. With a mounted mass of several degree of freedom there will result several machine frequencies of small vibrations. There will also be resonant values of the mass-table system where the amplitude of the machine and of the mounted system may become very large.

28. For the direct drive machine the amplitude of vibration must remain at some given value regardless of load conditions. A flexibly mounted load will react, at certain frequencies, so that the bearing forces are many times those which would result if the load were rigidly attached to the vibration machine. When the damping of a heavy flexibly mounted load is small the maximum heavy force is nearly equal to  $m \omega_r^2 R X_1 + mg$  (See paragraph 17 (b)).

29. Commonly made measurements of vibration amplitudes are usually r-m-s values of acceleration or displacement amplitudes. For small displacement amplitudes and low fundamental frequencies, high frequency harmonics cause r-m-s values of acceleration amplitudes to have little meaning unless a wave analyzer is employed. The high frequency vibrations are generally of such

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small amplitudes that the displacement wave form of most vibration machines is essentially sinusoidal and the r-m-s displacement amplitude is sufficiently accurate without the use of a wave analyzer. The validity of this statement should be, of course, be determined for any given machine.

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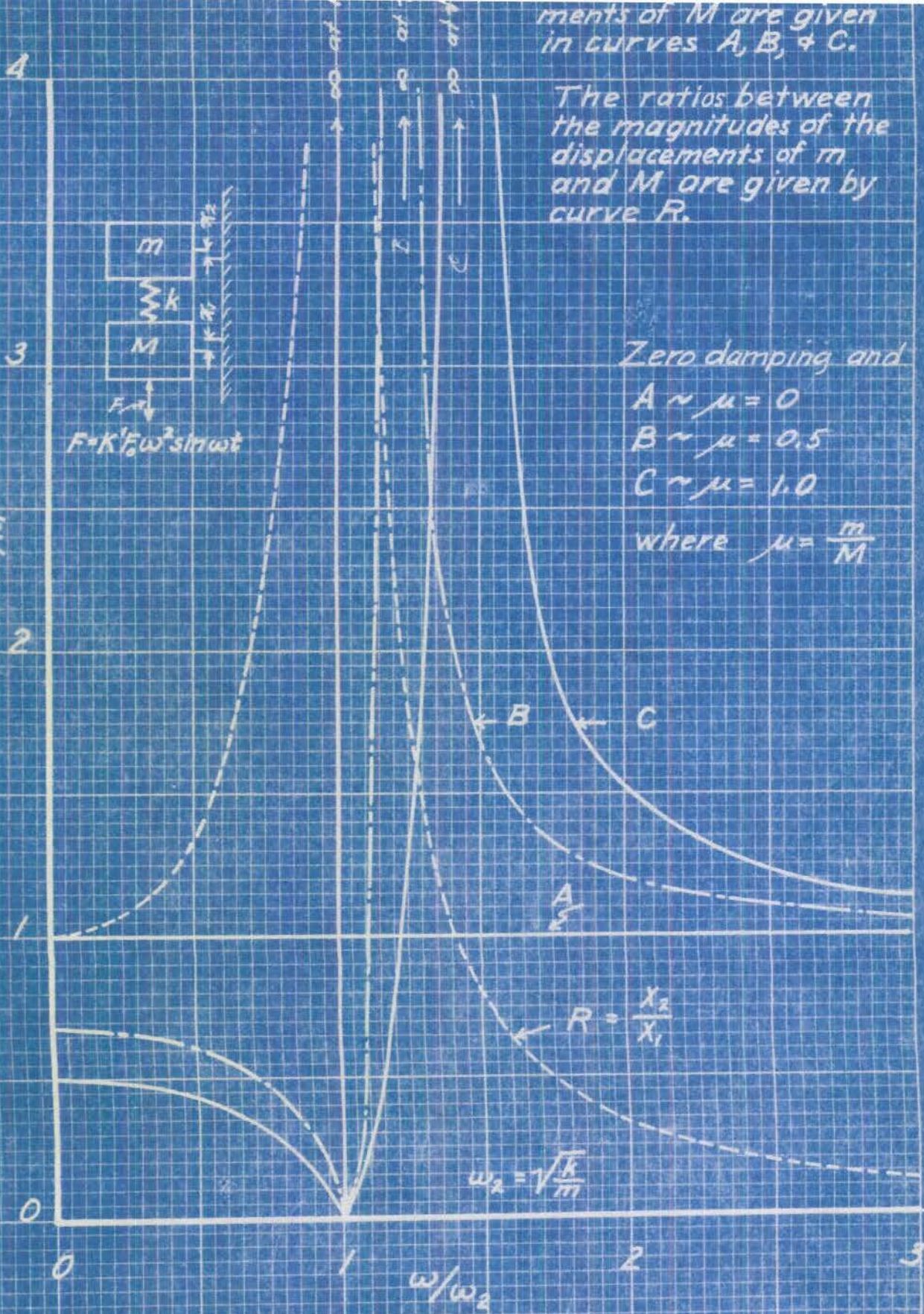
ments of  $M$  are given in curves A, B, & C.

The ratios between the magnitudes of the displacements of  $m$  and  $M$  are given by curve R.

Zero damping and  
 $A \sim \mu = 0$   
 $B \sim \mu = 0.5$   
 $C \sim \mu = 1.0$   
 where  $\mu = \frac{m}{M}$

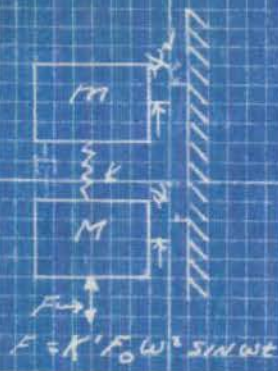


$\frac{x_1}{K' F_0 / M}$  or  $R$



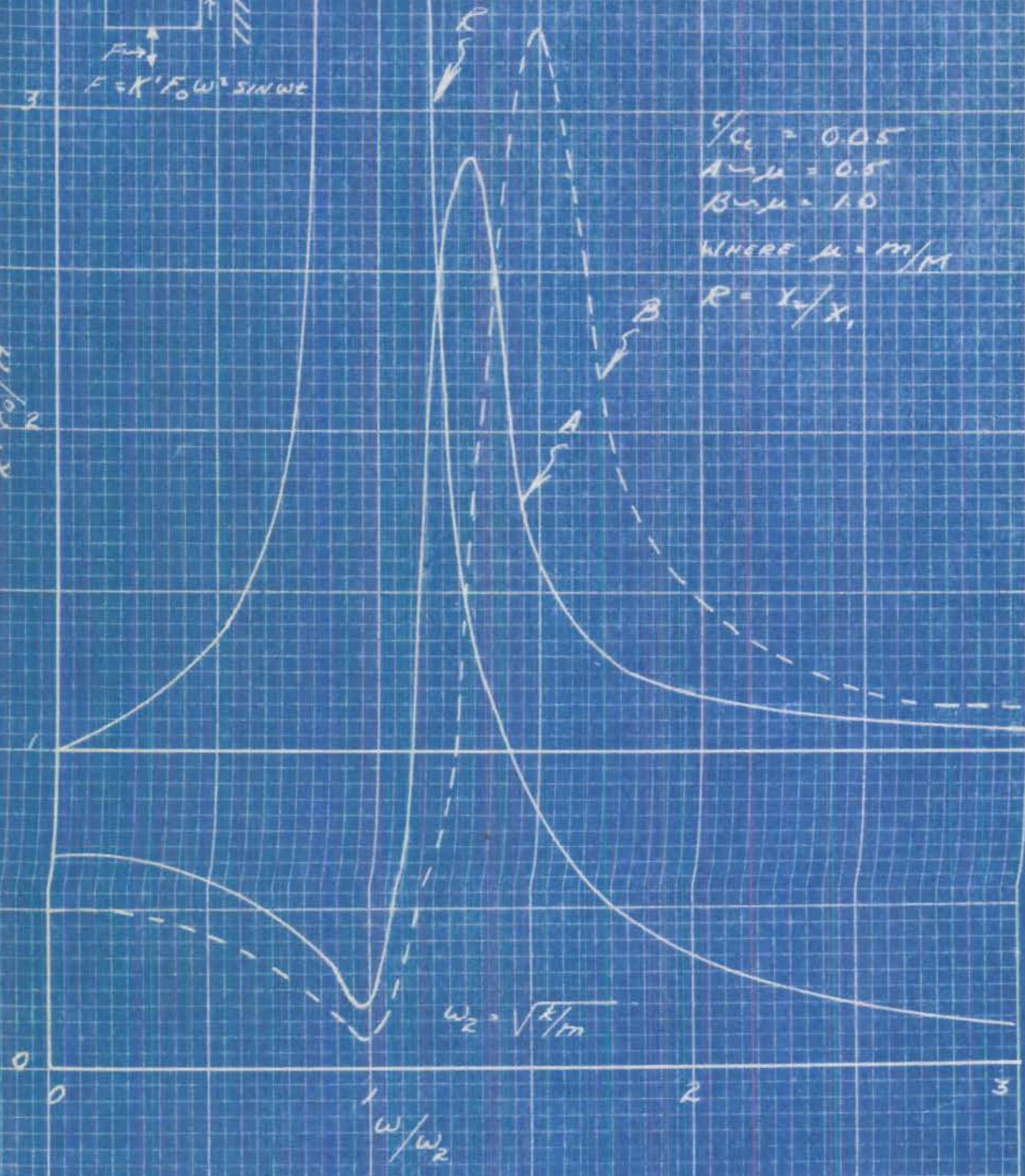
VALUES OF  $M$  ARE GIVEN IN CURVES A, B.

THE RATIOS BETWEEN THE MAGNITUDES OF THE DISPLACEMENTS OF  $m$  AND  $M$  ARE GIVEN BY CURVE R.



$\frac{x_1}{K' F_0 / M} \text{ OR } R$

$\zeta/c_c = 0.05$   
 $A \sim \mu = 0.5$   
 $B \sim \mu = 1.0$   
 WHERE  $\mu = m/M$   
 $R = x_2/x_1$



AMPLITUDES OF DISPLACEMENTS OF  $M$  ARE GIVEN IN CURVES A, B.

THE RATIOS BETWEEN THE MAGNITUDES OF THE DISPLACEMENTS OF  $m$  AND  $M$  ARE GIVEN BY CURVE R.



$$F = K' F_0 \omega^2 \sin pt$$

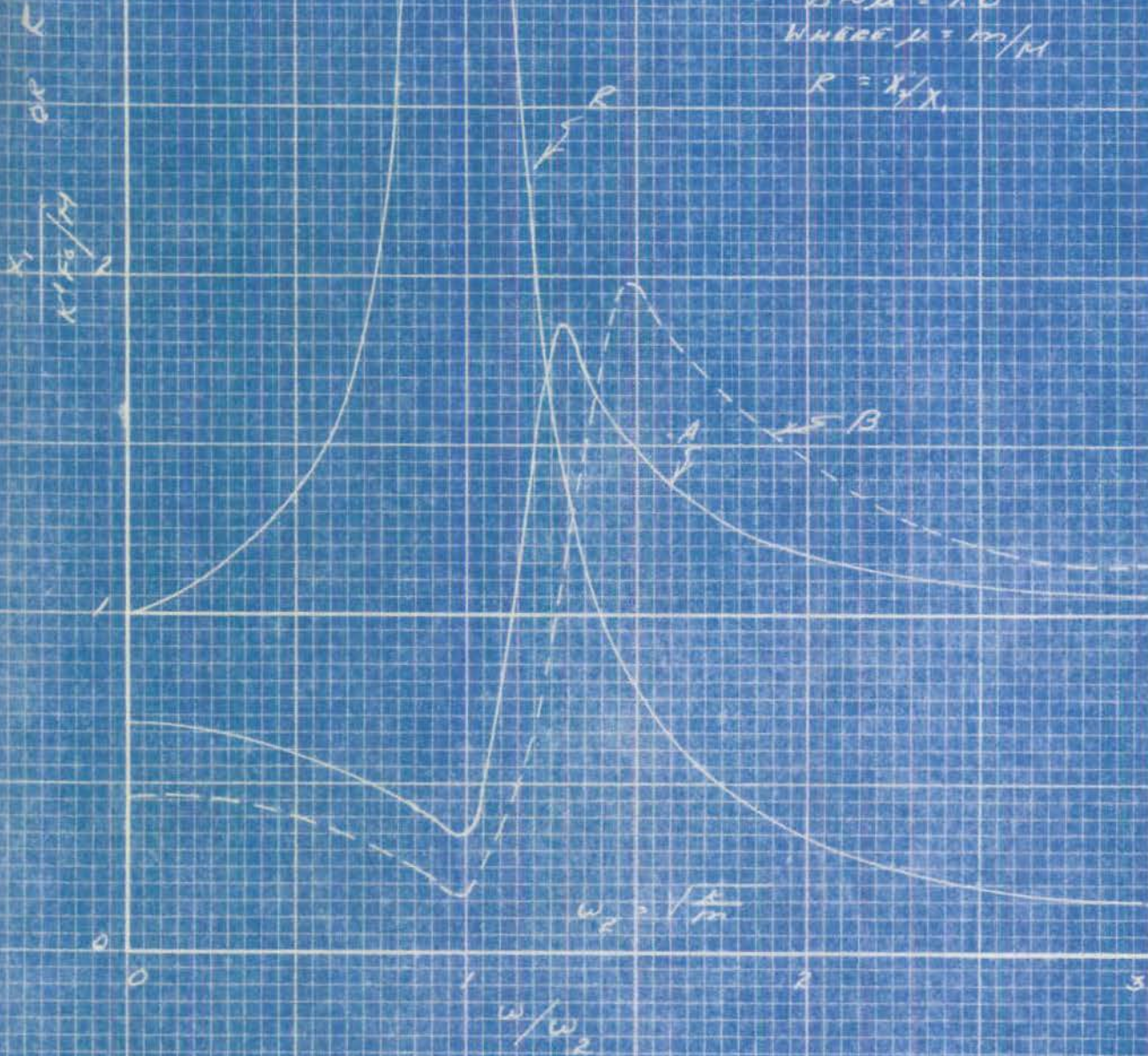
$$c/L_0 = 0.1$$

$$A \sim \mu = 0.5$$

$$B \sim \mu = 1.0$$

WHERE  $\mu = m/M$

$$R = x_1/x_2$$



AMPLITUDES OF DISPLACEMENTS OF  $m$  ARE GIVEN IN CURVES A, B

THE RATIOS BETWEEN THE MAGNITUDES OF THE DISPLACEMENTS OF  $m$  AND  $M$  ARE GIVEN BY CURVE R.



$$F = K' F_0 \sin pt$$

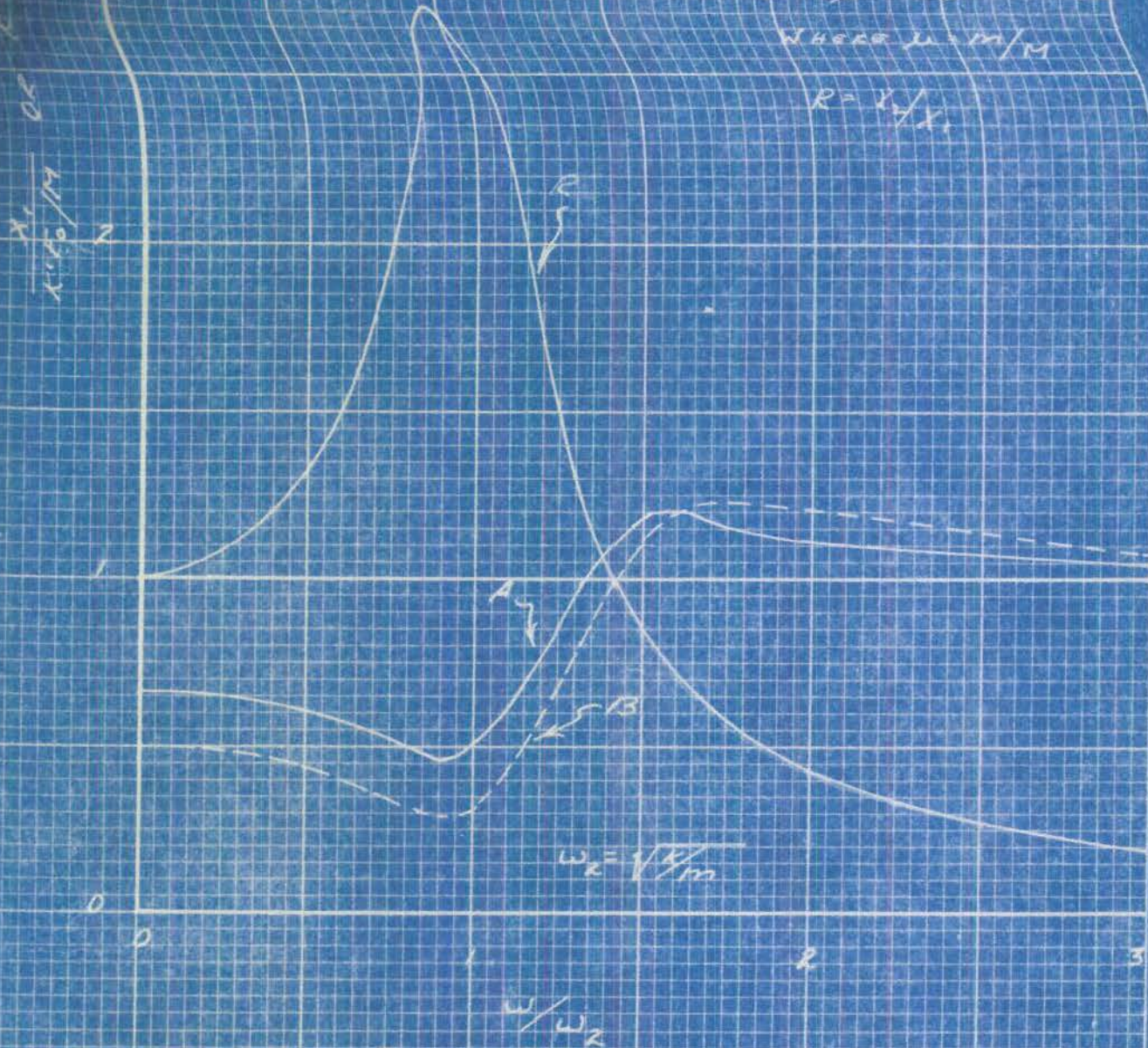
$$c/c_c = 0.2$$

$$A \mu = 0.5$$

$$B \mu = 1.0$$

WHERE  $\mu = m/M$

$$R = x_1/x_2$$



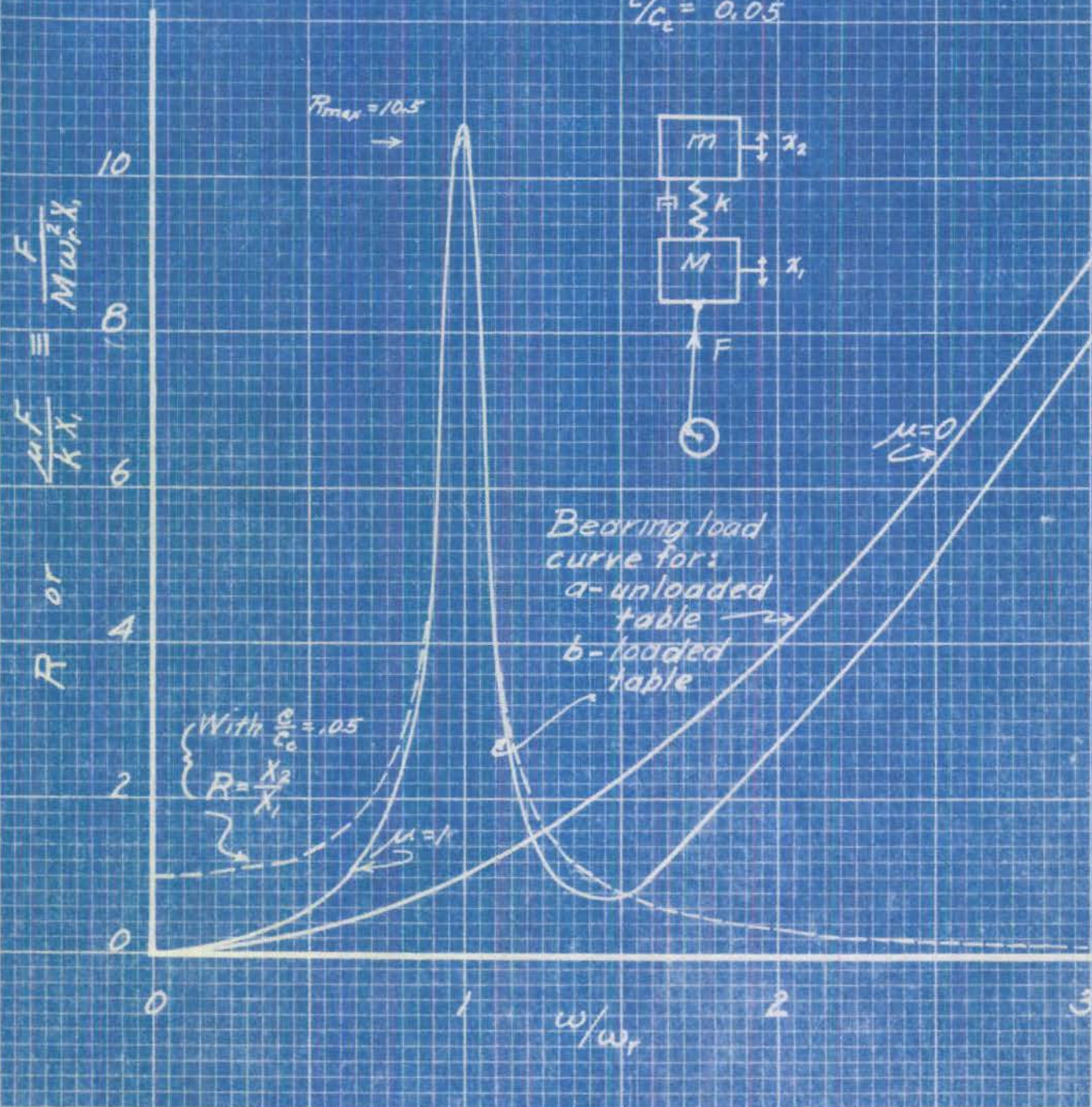
# Bearing Load for a Direct Drive Vibration Machine

$F = \text{Bearing load}$

$$\mu = \frac{m}{M} = 1$$

$$\omega_r = \sqrt{\frac{k}{m}}$$

$$c/c_c = 0.05$$

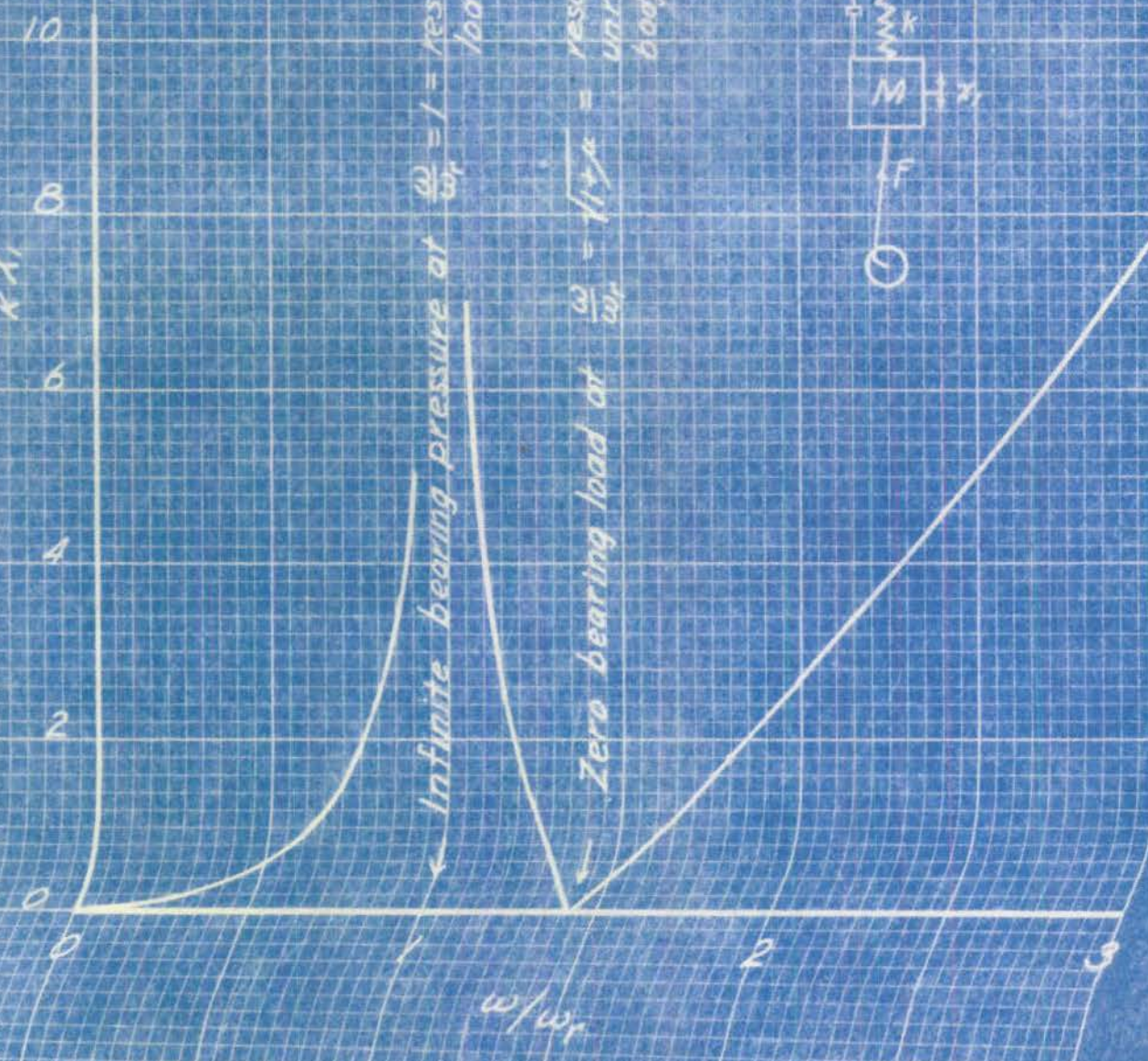


# Bearing Loads for a Direct Drive Vibration Machine

$F$  = bearing load  
Zero damping

$$\mu = \frac{m}{M} = 1$$

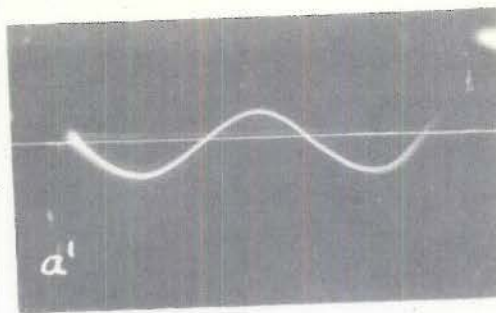
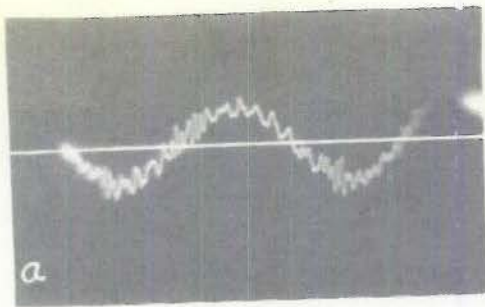
$$\omega_r = \sqrt{\frac{k}{m}}$$



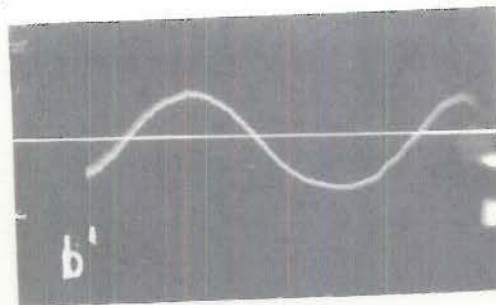
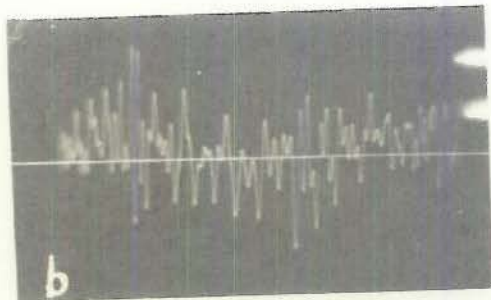
← Infinite bearing pressure at  $\frac{\omega}{\omega_r} = 1$  = resonant point of load on machine

← Zero bearing load at  $\frac{\omega}{\omega_r} = 1.5$  = resonant point of unrestrained two-body system

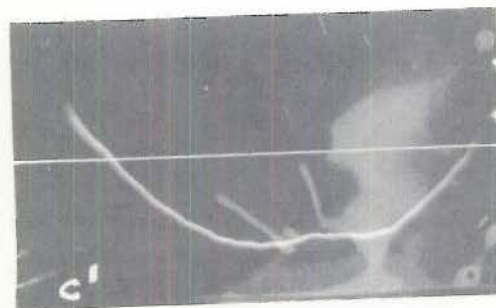
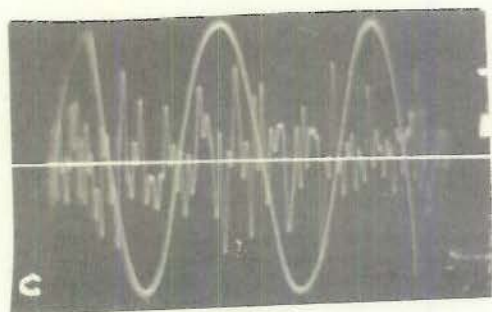




Double displacement amplitude is  
0.18 inches. Freq. 30 c.p.s.

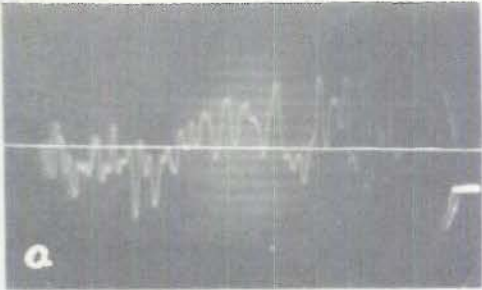


Double displacement amplitude is  
0.011 inches. Freq. 30 c.p.s.

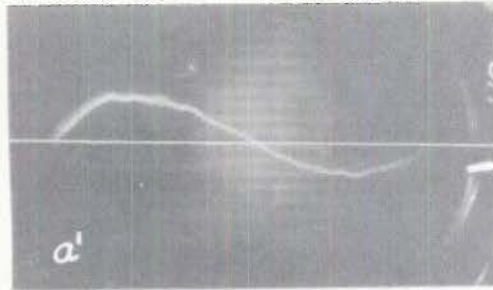


Double displacement amplitude is  
0.011 inches. Freq. 10 c.p.s.

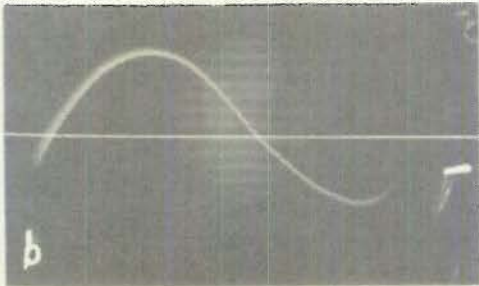
Acceleration curves, *a*, *b*, and *c*; and displacement curves, *a'*, *b'*, and *c'*, for a given point on a vibration machine. A 60 cycle timing trace is superimposed on *c*. The curves are single sweep oscillographic patterns.



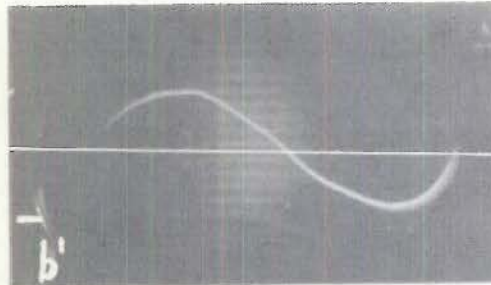
Acceleration curve of vibration at base of a flexibly mounted apparatus.



Acceleration curve on the flexibly mounted apparatus. Near resonant conditions existed.



Displacement curve on the flexibly mounted apparatus.



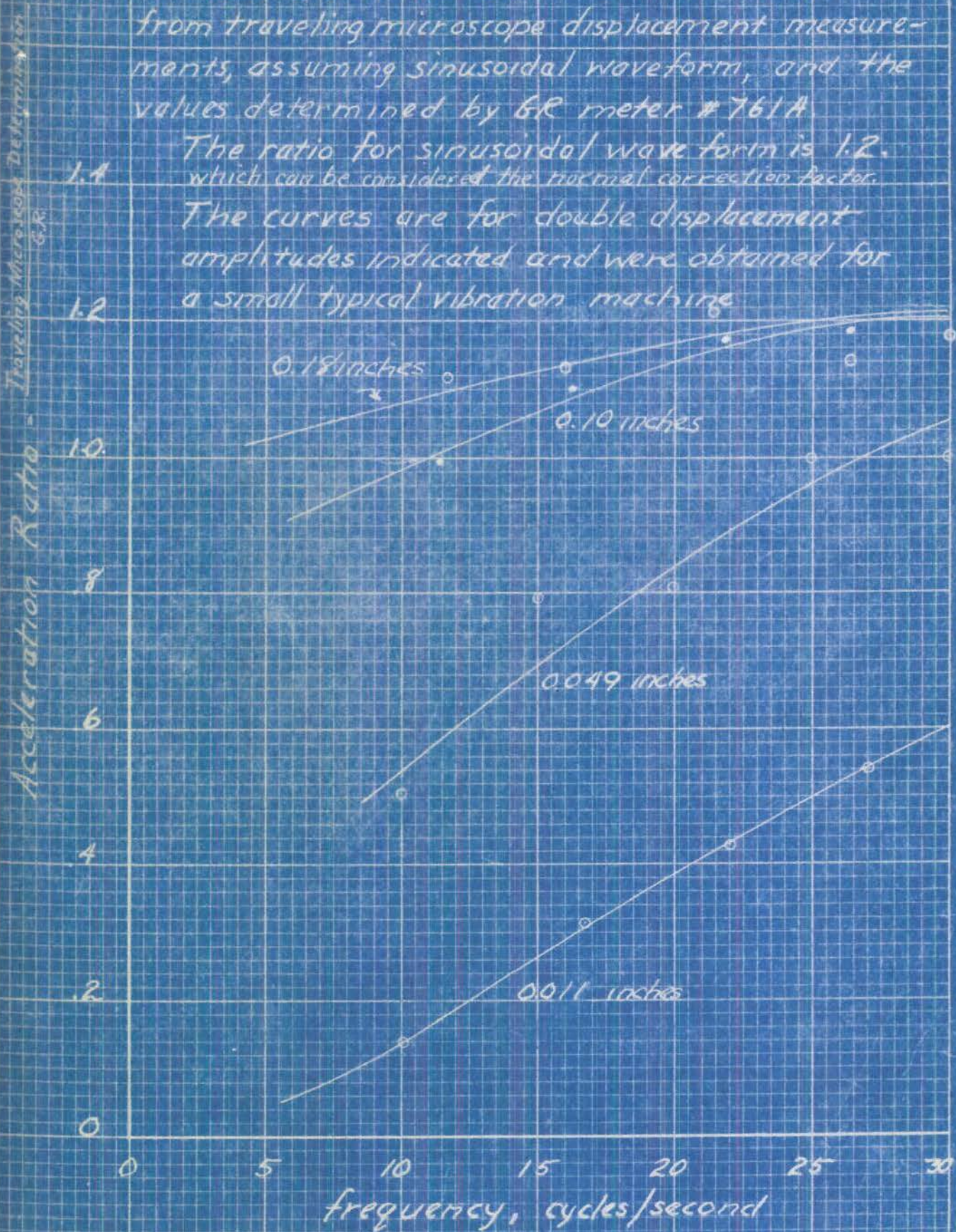
Displacement curve of vibration at the base of the flexibly mounted apparatus.

Acceleration and displacement curves on either side of a mounting of a flexibly mounted apparatus. The curves are single sweep oscillographic patterns. Frequency is 20 c.p.s. Displacement double amplitude of the base is about 0.025 inches and that of the apparatuses about 0.045 inches.

Acceleration Ratios of the values calculated from traveling microscope displacement measurements, assuming sinusoidal waveform, and the values determined by GR meter # 761A.

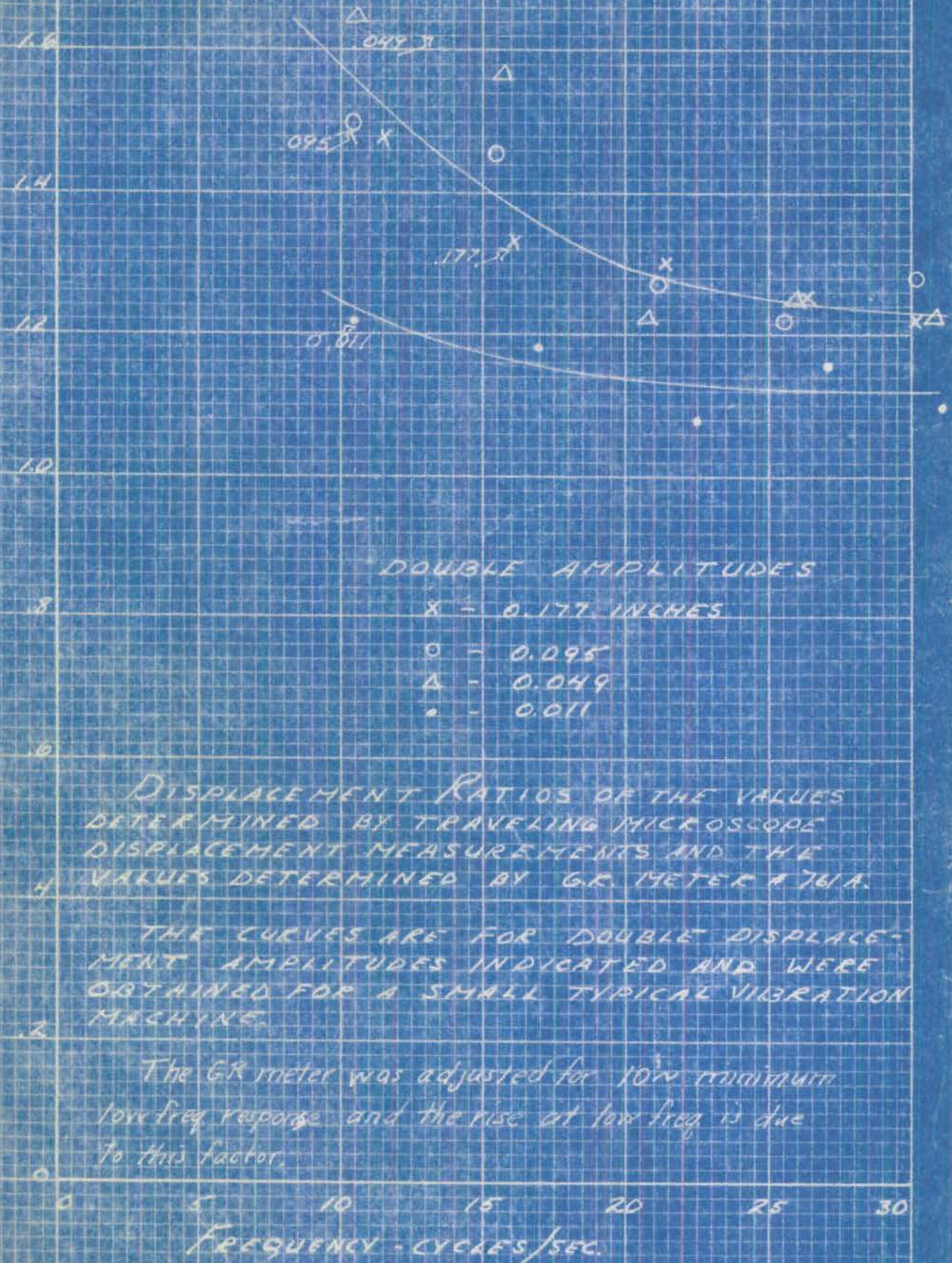
The ratio for sinusoidal waveform is 1.2, which can be considered the normal correction factor.

The curves are for double displacement amplitudes indicated and were obtained for a small typical vibration machine.



Traveling Microscope  
GR

DISPLACEMENT RATIO

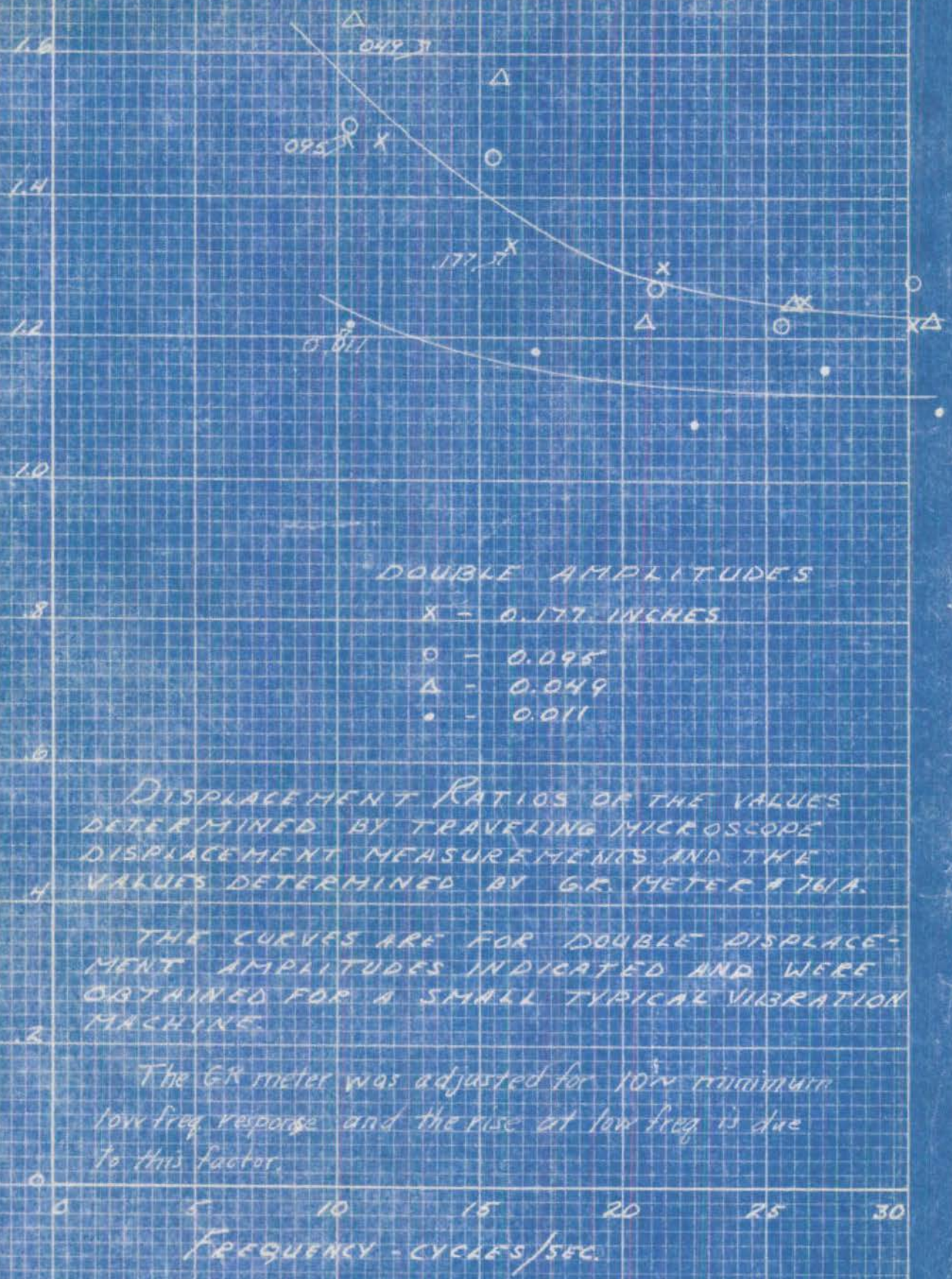


DISPLACEMENT RATIOS OF THE VALUES DETERMINED BY TRAVELING MICROSCOPE DISPLACEMENT MEASUREMENTS AND THE VALUES DETERMINED BY GR. METER #761A.

THE CURVES ARE FOR DOUBLE DISPLACEMENT AMPLITUDES INDICATED AND WERE OBTAINED FOR A SMALL TYPICAL VIBRATION MACHINE.

The GR meter was adjusted for 10<sup>10</sup> minimum low freq response and the rise at low freq is due to this factor.

Displacement Ratio =  $\frac{\text{Traveling Microscope}}{G.R. \text{ Meter}}$



DOUBLE AMPLITUDES  
X = 0.177 INCHES  
O = 0.095  
Δ = 0.049  
• = 0.011

DISPLACEMENT RATIOS OF THE VALUES DETERMINED BY TRAVELING MICROSCOPE DISPLACEMENT MEASUREMENTS AND THE VALUES DETERMINED BY G.R. METER #761A.

THE CURVES ARE FOR DOUBLE DISPLACEMENT AMPLITUDES INDICATED AND WERE OBTAINED FOR A SMALL TYPICAL VIBRATION MACHINE.

The G.R. meter was adjusted for 10<sup>1/2</sup> minimum low freq response and the rise at low freq is due to this factor.