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Silver Spring, Maryland

APL/JHU TG-20
January 14, 1947

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DEPARTMENT OF ENERGY DECLASSIFICATION REVIEW

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NUCLEAR-POWERED FLIGHT

By

An Informal Committee

of

THE APPLIED PHYSICS LABORATORY

of

THE JOHNS HOPKINS UNIVERSITY

A. E. Ruark, Chairman

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NUCLEAR POWERED FLIGHT

AS OF APRIL 1, 1947 THE CLASSIFICATION OF THIS DOCUMENT HAS BEEN CHANGED TO [redacted]

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~~Upgrade Authority: EMAIL - John McFadden (HQ) TG-NPP-2; DOE OC Oct 2010; TG-NAS-2; DOE OC; May 1996~~
~~Upgraded By: David C. Dallis (OSTI) Classification Analyst~~

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FIRST PROGRESS REPORT

[redacted]

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14 January 1947

TO: L. R. Hafstad

FROM: A. E. Ruark

SUBJECT: Transmittal of Progress Report entitled "Nuclear-Powered Flight", by an Informal Committee of the Applied Physics Laboratory of the Johns Hopkins University.

In accordance with your verbal instructions of about 9 June 1946, the Committee has considered the general problem of air vehicles driven by nuclear power. Three copies of the subject report are respectfully submitted herewith. A first draft was submitted October 25, 1946. Since that time many errors have been corrected and much new material has been added. The initial distribution is indicated in the report.

Your comments and those of other interested persons will be appreciated by the Committee. Review by suitable members of APL is hereby requested.

It is believed that any further work on this subject at APL should be carried on by a small staff with fresh instructions, and that the existing large committee should be discharged in the near future.

FOR THE COMMITTEE

Arthur E. Ruark, Chairman;
Technical Supervisor
for Research Laboratory.

AER:rh

Encl. 3 -- Copies 1, 2, and 3 of subject report.

[redacted]

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SECRET

CHAPTER II

PART A. CYLINDRICAL ENRICHED REACTORS

By Nicholas M. Smith, Jr.

We consider the slow-neutron reactor fueled with enriched fissionable material (U^{235} or Pu^{239}) mixed in a moderator. We dispense with the use of a tamper because the tamper adds to the total weight and size of the reactor. The assumed surface temperature of the reactor is 3000°K at full power. Important factors in the neutron design of a reactor for power production are

a. The critical dimensions,

b. The total mass of fissionable material,

c. Extra uranium for control, and the time of response to the controls,

d. The effect of poisoning by the products,

e. Depletion of uranium,

1. Solution of Differential Equations for a Cylindrical Reactor in the Steady State.

We follow elementary diffusion theory as in the computation of the critical size of a spherical reactor (see Appendix 1). For the steady state, we have the equation:

$$\triangle n + q^2 n = 0, \qquad (1)$$

where n is the number of neutrons per c. c.;

$$q^2 \equiv \frac{K - 1}{L^2};$$

K is the ratio of the creation and absorption rates; and

L is the diffusion length, discussed in detail further on,

In cylindrical coordinates eq. 1, becomes:

$$\frac{d^2n}{dr^2} + \frac{1}{r}\frac{dn}{dr} + \frac{d^2n}{dz^2} + q^2n = 0, \qquad (2)$$

Solving by separation of variables, let $n = R\,Z$,

where R and Z are, respectively, functions of r and z only. Then

$$r\frac{d^2R}{dr^2} + \frac{dR}{dr} + a^2rR = 0, \qquad (3a)$$

and

$$\frac{d^2Z}{dz^2} + b^2Z = 0 \qquad (3b)$$

where a and b must satisfy the relation

$$a^2 + b^2 = q^2 = \frac{K - 1}{L_d^2}, \qquad (3c)$$

It follows that

$$R = J_o(ar), \qquad (4a)$$

the Bessel function of zero order, and that

$$Z = A\cos bz + B\sin bz. \qquad (4b)$$

Putting the center of the reactor at $z = 0$, we see that the concentration must be an even function of z, so that the sine term drops out. For the steady state the neutron density drops off in all directions from the center. It is not true that the density is zero at the surface, since there is a finite flow of neutrons to the outside. However, in designing the reactor, it is convenient and satisfactory to use the boundary condition $n = 0$, because the neutron density is relatively low at the boundary when the mean free path is reasonably small compared to the smallest dimension of the reactor. This condition will be satisfied for the dimensions we shall consider. The theory should not be used for bodies whose smallest dimension is of the order of the mean free path.

Taking the zeros of the solution to determine the critical boundaries, we get, since $J_0(x) = 0$ for $x = 0.7655\,\pi$,

$$\left.\begin{aligned} ar_c &= 0.7655\,\pi, \\ bh_c &= \pi\,; \\ a^2 + b^2 &= q^2, \\ a &\geq 0, \\ b &\geq 0. \end{aligned}\right\} \qquad (5)$$

Here r_c is the critical radius and h_c the critical length.

$$K = V\,\frac{\text{Absorption in fissionable material}}{\text{Total absorption}} \qquad (6)$$

Here we mean by absorption (in U_{235}, for example) the quantity $N_U\ \sigma_{aU}$, where N_U is the number of atoms of U_{235} per cc., and σ_{aU} the absorption cross-section. V is the average number of neutrons emitted per fission. We shall assume that V lies between 2.0 and 2.4.

Putting $C = \dfrac{\text{Absorption in uranium 235}}{\text{Absorption in other materials}}$

K takes the form

$$K = V\,\frac{C}{1+C}. \qquad (6a)$$

2. The Reactor Containing Tubes.

Now if the reactor, instead of being solid, contains tubes for heating a gas stream, we may consider that we have a new reactor of reduced density of both U^{235} and moderator. Let Γ denote the ratio of the empty corss-section to the total cross-section in the reactor; so that $1 - \Gamma$ is the solid cross-section over the total cross-section. Suppose that the length of each tube is great compared with its diameter. Then, we may assume that all mean free paths increase by a factor $1/(1 - \Gamma)$, so that the dimensions of

a critical reactor increase by the same factor. Averaging over the whole volume of the tubes and the solid material, we see that the effective densities of uranium and moderator vary directly with $1 - \Gamma$. The concentration factor C which is defined as the ratio of absorption in U_{235} to the absorption in moderator will remain constant as long as the composition of the mixture is not altered. The reproduction factor K is independent of Γ.

However, the diffusion length varies inversely as the overall average density, being

$$L/1 - \Gamma. \qquad (7)$$

L is the diffusion length for a reactor devoid of tubes.

3. Outline of the Design.

Now we shall find the mass of uranium in the tube-filled reactor of critical size. Then we shall pick the "optimum" value of C, which permits the use of the smallest quantity of uranium. These steps do not complete the design. The critical reactor of optimum concentration has an arbitrary power output, depending on the initial neutron flux. This is clear from the linearity of the differential equation for the neutron concentration. To provide a finite power output, it is necessary to use more than the critical amount of uranium and to introduce control rods. Also, the effect of poisoning by fission products during operation, and the effect of temperature increase on neutron mean free paths (and therefore reactor size) must be considered. Allowance for the depletion of the uranium during the total time of operation must be made. All these factors result in increase in the quantity of uranium needed. They will not be considered in our initial discussion of the steady state.

4. Optimum Concentration.

For the tube-filled reactor the critical dimensions become

$$\left.\begin{aligned} ar_c &= 0.7655\pi \\ bh_c &= \pi \\ a^2+b^2 &= (1-\Gamma)^2\left(\frac{K-1}{L_{do}^2}\right) \\ &= \frac{(1-\Gamma)^2}{L_{do}^2}\left(\frac{VC}{1+C}-1\right) \end{aligned}\right\} \quad (8)$$

Now let

$$\frac{r_c}{h_c} = \mu \quad (9a)$$

so that

$$\mu = 0.7655\ b/a, \quad (9b)$$

and

$$q^2 = \frac{(1-\Gamma)^2}{L_{do}^2}\left(\frac{VC}{1+C}-1\right) = \left(1+\frac{0.586}{\mu^2}\right)b^2 \quad (9c)$$

$$= \left(1+\frac{\mu^2}{0.586}\right)a^2 \quad (9d)$$

From the definition of C, and for the case of a pure moderator, without poisoning material produced by fission,

$$C = \frac{N_U\ \sigma_{aU}}{N_M\ \sigma_{aM}} = \frac{(\rho_{U/235})N_U\ \sigma_{aU}}{(\rho_M/W_M)N_M\ \sigma_{aM}}, \quad (10)$$

The subscript M refers to the moderator material. The density of a constituent of the mixture expressed as mass per unit volume, and the molecular weight are indicated by ρ and W, while N_A is the Avogadro number. We abbreviate (10) in the form

$$\rho_U = \beta C \quad (10a)$$

where

$$\beta = \frac{\rho_{M235}}{W_M}\ \frac{\sigma_{aM}}{\sigma_{aU}} \quad (10b)$$

$= 4.72 \times 10^{-4}$ in cgs units for graphite

The total mass of fissionable material in the reactor is given by

$$m = \pi r_c^2 h_c \rho_U (1 - \Gamma)$$

$$= \pi \frac{0.586\,\pi^2}{a^2} \frac{\pi}{b} \rho_U (1 - \Gamma)$$

$$= \frac{0.586\,\pi^4 \left(1 + \frac{\mu^2}{0.586}\right)\left(1 + \frac{0.586}{\mu^2}\right)^{\frac{1}{2}} L^3}{(1-\Gamma)^2 \left[\frac{VC}{1+C} - 1\right]^{3/2}} \beta C. \qquad (11)$$

Equation (11) does not show the total dependence of the mass of fissionable material on C, for L, the diffusion length is a function of C. Only in special cases, where the contribution to L^2 made by the diffusion at thermal velocities can be neglected (as for the case of a water moderator treated by Christy) is L independent of C.

We set

$$(L^2)\ \text{total} = (L^2)\ \text{slowing down} + (L^2)\ \text{thermal} \qquad (12a)$$

and consider (L^2) thermal:

$$(L^2)\ \text{thermal} = \frac{\lambda_{th}\ \lambda_{th}}{3} \qquad (12b)$$

where λ_{th} = thermal mean free path for scattering,

and λ_{th} = thermal mean free path for absorption,

Now
$$\lambda_{th} = \frac{1}{N_M\ \sigma_{sM} + N_U\ \sigma_{sU}}$$

and
$$\lambda_{th} = \frac{1}{N_M\ \sigma_{aM} + N_U\ \sigma_{aU}} \qquad (13)$$

where the subscript s refers to scattering, and a to absorption. In the case of a graphite reactor we shall find on page 2.13 that $N_U\ \sigma_{aU}$ should be about 18 times $N_M\ \sigma_{aM}$. It follows that

$$N_M\ \sigma_{sM} \gg N_U\ \sigma_{sU}.$$

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and it is permissible to write

$$\lambda_{th} = \frac{1}{N_M \sigma_{sM}}$$

but $$\lambda_{th} = \frac{1}{1+C} \frac{1}{N_M \sigma_{aM}}$$

and we have

$$(L^2)_{th} = \frac{(L^2)_{\text{mod, thermal}}}{1+C} \qquad (14)$$

where the subscript "mod" refers to a uranium-free moderator. For the same reason that we neglect $N_U \sigma_{sU}$ in comparison with $N_M \sigma_{sM}$, we can neglect the entire contribution to (L^2) slowing made by the uranium cross-section;--in other words, because the σ's are of the same magnitude and $N_M >> N_U$. Whenever these conditions do not hold, we no longer have a slow-neutron reactor, but a slow-and-fast-neutron reactor. For the conditions of a slow-neutron reactor considered here, we can rewrite (12a) in the form

$$(L^2)_{\text{total}} = (L^2)_{\text{slowing down}} + \frac{(L^2)_{\text{mod. therm.}}}{1+C} \qquad (15a)$$

$$= D + \frac{E}{1+C} \qquad (15b)$$

where $D = (L^2)$ slowing down

and $E = (L^2)$ mod. therm.

Now, Equation (11) becomes

$$m = \frac{F\left(D + \frac{E}{1+C}\right)^{3/2} C}{\left(\frac{VC}{1+C} - 1\right)^{3/2}} \qquad (16)$$

where

$$F = \frac{0.586\,\pi^4}{(1-i)^2}\left(1 + \frac{\mu^2}{0.586}\right)\left(1 + \frac{0.586}{\mu^2}\right)^{\frac{1}{2}} \beta \qquad (16a)$$

This factor segregates all length ratios and other quantities which are not used in finding the optimum value of C. Equation (16) has a minimum as C is varied,

which depends on D, E, and V, and which gives an optimum concentration of fissionable material.

We compute $\partial M/\partial C$, let it equal zero and solve for $C_{optimum}$:

$$C_{opt} = \frac{(E+D)(V-1)+5D+\{[(E+D)(V+1)+5D]^2+16(E+D)D(V-1)\}^{\frac{1}{2}}}{4D(V-1)} \quad (17)$$

For the case in which L^2 is independent of C (Equation 11), $E = 0$, and we obtain

$$C_{opt} = \frac{V+4+(V^2+24V)^{\frac{1}{2}}}{4(V-1)} \quad (18)$$

Equation (18) yields the following values:

V:	2	2.1	2.2	2.3	2.4
C_{opt}:	3.30	3.07	2.88	2.72	2.58

These values would be useful for a reactor moderated with hydrogen or water, but for a graphite moderator (and presumably, one of beryllium oxide), eq. (17) must be employed, so we proceed to consider the values of D and E.

5. Computation of Diffusion Length in Pure Moderator.

For use in Equation 15b we desire to compute D and E separately, where now we desire these for the case of the pure moderator, having already made allowance for the concentration of uranium by introducing the factor $1 + C$. The calculations will be approximate. We restrict our attention to carbon and U_{235}. Table 1 gives absorption cross sections and total cross sections. In carbon of density 2.25 the mean path for a cross section of 1 barn is 8.85 cm, so mean paths are given by

$$\lambda(\text{cm}) = \frac{8.85}{\sigma(\text{barns})}.$$

We write λ_{sl} to indicate that the free path we need is that for the slowing-down process; no matter whether scattering be elastic or inelastic λ_{sl} is taken as $1/(N_M \sigma_t)$.

Table 1. Neutron Cross-Sections in Carbon and U_{235}

Element	Neutron Energy, Electron Volts	Cross-Sections in Units of 10^{-24} cm^2		Mean path, in cm.	
		σ_a	σ_t	λ	λ
Carbon	1/30	0.0045	4.84	1970	1.83
	1		4.8		1.84
	$50\ 10^3$		4.7		1.88
	$100\ 10^3$		4.55		1.94
	$200\ 10^3$		4.13		2.14
	$300\ 10^3$		3.7		2.39
	$500\ 10^3$		3.23		2.74
	$1\ 10^6$		2.41		3.67
	$2\ 10^6$		1.67		5.30
	$3\ 10^6$		Resonance		----
			(scattering)		
U_{235}	1/30	420	17		
	Fast	2.4	6		

References: The uranium cross-sections and the carbon absorption cross-sections are from "Nuclear Fission and Atomic Energy," University of Pennsylvania Staff; Tables 9.2 and 9.4. The total cross-sections for carbon were kindly supplied by D. H. Frisch of MIT.

We have $D = 1/3\ N\ \overline{\lambda^2}_{sl}$

where N is the average number of collisions required to reduce the energy of the neutron from 1.2 MEV to RMS thermal value for 3000°K, or 1/3 EV.

$\overline{\lambda^2}_{sl}$ should be the average of λ^2_{sl} over all collisions, weighting each collision equally. As an approximation we shall use

$$E_o\ (1 - \bar{f})^N = E_N \qquad (19)$$

as the relation defining N. Here E_o and E_N are the initial and final energies respectively, while $\bar{f}$ is the arithmetic average fractional loss of energy in a single collision. From the paper of Condon and Breit (Phys. Rev. 49, 229 (1936)), we have for carbon

$$\bar{f} = 0.142$$

We compute:

$$N = \frac{\log\left(\frac{1.2 \quad 10^6}{0.33}\right)}{\log\left(\frac{1}{0.858}\right)} = 97 \text{ collisions.} \qquad (20)$$

To get λ_{sl}^2 we have plotted λ_{sl} against log E and have picked off values at equal increments of log E, which means roughly, equal increments of the collision-number. The average of λ_{sl}^2 is computed from the values thus chosen. That is,

$$N\ \overline{\lambda_{sl}^2} = \sum \lambda_{sl}^2\ \Delta N. \qquad (21)$$

We carry this summation from an assumed average initial energy of 1.2 MEV down to the RMS energy corresponding to the reactor temperature. The results for D are given in Table 2. Inasmuch as the reactor must start at room temperature and must run at high temperature we consider the values of D and E under both conditions. D becomes larger by about 10 per cent when the temperature falls to 300°K. The reason is that more collisions are required to reach thermal energy values. We have

$$E = \lambda_{th}\ \ \lambda_{th}/3. \qquad (22)$$

and from page 2.9, we find that E = 1200 sq. cm. at room temperature. We assume that in the region considered, the cross-section for absorption in the moderator follows the 1/v law and is therefore proportional to $1/T^{\frac{1}{2}}$. Thus

$$E(T) = \left(\frac{T}{T_o}\right)^{\frac{1}{2}} \frac{\lambda_{T_o}\,\lambda_{T_o}}{3}. \qquad (23)$$

Finally, in evaluating $D + E/(1 + C_{opt})$, which is L^2 for the optimum critical reactor, we use several values of C_{opt}, corresponding to different values of V (see Equation 17), so that the uncertainties connected with our lack of knowledge of V can be appraised. The results for critical reactors with optimized uranium concentrations are given in Table 2.

We note here that in addition to an optimum concentration factor there exists an optimum radius/length ratio, independent of the value of C. In Equation (16a) we set $\partial F/\partial \mu = 0$, finding that the uranium mass will be a minimum when

$$\frac{\mu^2}{(.7655)^2} = \frac{1}{2}; \quad \mu = 0.542; \qquad (24)$$

so that the best diameter is 1.084 times the length. The shapes dealt with in Table 2 range from the compact optimum one to slender ones. (See also Part B, and Figure 2.3). The latter require more uranium, but they may be necessary in order to provide sufficient temperature rise of the gas in some missile reactors. The uranium masses given, $m(1 - \Gamma)^2$, are, of course, the masses of the solid reactors. To get the mass of a reactor with holes, we divide the figures in the table by $(1 - \Gamma)^2$.

From Equations (8) and (9) we obtain the values of the critical radius and length, namely,

$$r_c = 2.405\ (1 + \mu^2/0.586)^{\frac{1}{2}}\ L/(K - 1)^{\frac{1}{2}}, \qquad (24a)$$

$$h_c = \pi\ (1 + 0.586/\mu^2)^{\frac{1}{2}}\ L/(K - 1)^{\frac{1}{2}}. \qquad (24b)$$

These dimensions are also presented in Table 2. To obtain the dimensions of a reactor with holes, divide the given values by $1 - \Gamma$.

The results in Table 2 for the 3000° temperature are dependent on the quite-plausible assumption about the temperature variation of the thermal-neutron diffusion length in the moderator expressed by Equation (23). The presence of a carbon resonance near the room-temperature thermal region (1/30 volt) could bring it about that concentrations considerably higher than those in Table 2 would be necessary to minimize the uranium mass for the hot reactor. An experiment on neutron absorption in graphite, performed by Bernstein (Reference 5), indicates that resonance in graphite is a minor effect, if it exists at all. When the uranium needed to make the hot reactor operate is provided, then the body will be overcritical when cold, on the basis of any reasonable assumption about the velocity-dependence of the absorption. Increase of temperature helps to stabilize the reactor and makes the control problem easier.

Table 2. Critical Design Values for Cylindrical Carbon Reactors
(Dimensions in cm, masses in grams)

Temperature			300°K			3000°K		
μ	$F(1-\Gamma)^2$		V = 2	2.2	2.4	V = 2	2.2	2.4
0.05	0.414	m_c(gr)	23110	17040	13090	53550	40170	31540
		r_c(cm)	46	42	40	46	42	39
		h_c(cm)	916	846	789	915	840	782
0.0667	0.313	"	17470	12880	9900	40490	30370	23840
			46	43	40	46	42	39
			689	637	594	688	632	588
0.1	0.212	"	11840	8730	6700	27420	20570	16150
			46	43	40	46	42	39
			462	427	398	461	424	394
0.2	0.114	"	6360	4690	3600	14750	11060	8680
			47	44	41	47	43	40
			237	219	204	236	217	202
0.542	0.070	"	3910	2880	2210	9050	6790	5330
			56	52	48	56	52	48
			103	96	89	103	95	88
	D			151			133	
	E			1200			3800	
	C_{opt}		8.09	7.55	7.16	18.8	18.2	17.7
	L^2		283	292	298	325	331	336
	K		1.78	1.94	2.11	1.90	2.09	2.27

6. Total Uranium Needed for Fuel.

Rocket.

From page 4.17 the total energy needed to propel the rocket is given as

$$E_g = 3.93 \times 10^{11} \text{ ergs/gram of fuel.}$$

Now, one electron-volt is 1.60×10^{-12} ergs and one fission produces 170 MEV, or

$$E_f = 1.70 \times 10^8 \times 1.60 \times 10^{-12} \text{ ergs/fission.}$$

(The energy released by the fission products is neglected, as the rocket expends its energy in about 140 seconds). $E_f = 2.72 \times 10^{-4}$ ergs/fission. Therefore,

$$E_g/E_f = 1.447 \times 10^{15} \text{ fissions/gram of fuel.}$$

Thus, the number of U_{235} atoms burned during the flight, per gram of fuel is 1.447×10^{15} and the mass of uranium burned, M_U, when a mass M_F of hydrogen is employed, is

$$\frac{M_{U235}}{M_F} = 1.447 \times 10^{15} \times \frac{235}{6.06 \times 10^{23}} = 5.60 \times 10^{-7} \frac{\text{grams } U^{235}}{\text{grams fuel}} \qquad (25)$$

On page 4.17 the weight of fuel given is

$$M_F = 50.8 \text{ tons} = 4.60 \times 10^7 \text{ grams.}$$

Therefore, to propel a rocket of 83.3 tons a distance of 5000 miles, we need

$$M_U = 5.6 \times 10^{-7} \times 4.60 \times 10^7 = 26 \text{ grams of } U_{235}$$

Therefore, we conclude that the U^{235} added to the reactor to allow for fuel consumption is negligible in view of the inaccuracies of other computations.

7. Additional U^{235} Needed for Control.

We work with the spherical case for simplicity.

In this section, poisoning of the reactor by fission products is neglected. The time dependent diffusion equation is:

$$\nabla^2 n + \frac{K-1}{L_d^2} n = \frac{1}{v N_a \sigma_a L^2} \frac{dn}{dt}, \qquad (26)$$

where v is the velocity of thermal neutrons and $N_a \sigma_a$ is defined below.

We solve by setting $n = F(r)\,T(t)$. For the spherical case,

$$\nabla^2 F + a^2 \mu^2 F = 0; \quad dT/dt = \gamma^2 T.$$

We desire a solution for which $n = 0$ at the boundary R of the sphere, and which can increase exponentially if K is sufficiently large, so

$$n = \sum_{\mu=1}^{\infty} \frac{A\mu}{r} \sin(a\mu r)\, e^{\gamma^2 t}; \quad a = \pi/R. \qquad (27)$$

Substitution in equation (26) shows that

$$\gamma^2 = v N_a \sigma_a L^2 \left(\frac{K-1}{L^2} - \frac{\pi^2 \mu^2}{R^2} \right).$$

Setting $R = R_c$, $\pi^2/R_c^2 = (K_c - 1)/L^2$

and $\gamma^2 = v N_a \sigma_a (K - K_c \mu^2)$,

where K_c is the critical value of K. We shall study only the dominant term, for which $\mu = 1$. Basic considerations show that

$$N_a \sigma_a = \frac{N_U \sigma_{aU} + N_M \sigma_{aM}}{N_M \sigma_{aM}} \qquad N_M \sigma_{aM} = (C+1) N_M \sigma_{aM}$$

and so

$$\gamma^2 = v N_M \sigma_{aM} (C+1)(K - K_c). \qquad (28)$$

Now $1/v N_M \sigma_{aM}$ is the life time of a neutron in the pure moderator. From page 10.22 of "Nuclear Fission and Atomic Energy," we find that an estimation of τ, the lifetime in carbon, is 1.4×10^{-3} second. However, according to page 10.30, the effect of the delayed neutrons is to replace γ^2 by

$$\alpha = \gamma^2 - h\alpha \sum_{i=1}^{4} \frac{B_i}{b_i (b_i + \alpha)} \qquad (29)$$

where $h = K/\tau$; (30)

and B_i is the fractional part of the total neutrons emitted which have a decay time constant of $1/b_i$ seconds.

Now from p. 10.30, we have this table:

B_i	b_i (sec^{-1})
1.1×10^{-3}	0.28
0.37×10^{-3}	0.099
0.09×10^{-3}	0.029
0.005×10^{-3}	0.012

so that the coefficient of h in equation (29) is a slowly varying function of α, particularly in the region of interest.

For α very large compared with the b_i, the summation term is practically constant, so that we have

$$\alpha h \sum \frac{B_i}{b_i(b_i + \alpha)} \longrightarrow h \sum \frac{B_i}{b_i} = h\epsilon$$

where $\epsilon = 0.011$.

Thus, writing t for the desired relaxation time of the pile, we have

$$\alpha = \gamma^2 - h\epsilon \qquad (31)$$
$$= (C+1)(K - K_c)/\tau - K\epsilon/\tau = 1/t$$

and therefore:

$$(C+1)\frac{C - C_c}{(C_c+1)(C+1)} = \frac{C\epsilon}{C+1} = \frac{\tau}{tV} = \beta. \qquad (32)$$

Solving for C, we get the quadratic,

$$C^2 + C\left[1 - C_c - (\epsilon + \beta)(1 + C_c)\right] - C_c - \beta C_c - \beta = 0. \qquad (33)$$

Assuming that a relaxation time of 0.1 second is desirable, and that V = 2, we get

$$\beta = 7 \times 10^{-3}, \text{ and } \epsilon = .011.$$

We compute the following table:

415 054

For C_c =	C must be
2	2.04
3	3.06
4	4.08
6	6.11
8	8.15
10	10.19

Again, our computations of C_c have such inaccuracy that we may neglect the additional uranium needed for control. It is rather striking that the fissionable material needed to maintain the chain reaction at a constant level is so much greater than the amount needed for control and fuel. The nuclear reactor appears to be much better adapted to a vehicle which must be sustained for a long period of time, than to one which needs a great amount of energy in a short time.

Conversely, if C must be increased over C_c for a cold reactor in order to maintain a chain reaction at elevated temperatures, then the relaxation time at room temperature will be exceedingly short. From Equation (32), taking $C = 18$ for a hot and C_c = about 8 for a cold pile (Table 2), we find

$$t \approx \frac{\tau}{V} \left[\frac{C - C_c}{C_c + 1} - \frac{C\epsilon}{C + 1} \right]^{-1}$$

$$\approx 6.4 \times 10^{-4} \text{ sec.}$$

Because the excess of K over K_c at room temperature exceeds the contribution of the delayed neutrons, this is the fast period. When the temperature rises to the design level, the excess of K over K_c falls within the potential contribution of the delayed neutrons, and then the latter will introduce slow controllable fluctuations of the power level. At first sight, it appears that in

before, namely, creation rate = V x rate of absorption in U^{235} whereas the total rate of absorption now becomes: Total absorption rate = rate of absorption in U^{235} + rate of absorption in moderator + rate of absorption in poisons.

We have,

$$K = \frac{V N_U \sigma_{aU}}{N_U \sigma_{aU} + N_M \sigma_{aM} + \sum N_i \sigma_{ai}} \tag{35}$$

where N_i and σ_{ai} are atomic density and absorption cross-section of the ith fission product. These quantities and others in Equation (35) are functions of position. K, for instance, is defined as the average of K(r)

$$\overline{K(r)} = \frac{1}{V} \int K(\underline{r})\, dV.$$

An estimation of an upper limit on the effects of poisoning may be made by setting K = K(0), that is the value of K(r) at the point of highest neutron density, the center of the reactor.

Rearranging, we get

$$K = \frac{V \frac{N_U \sigma_{aU}}{N_M \sigma_{aM}}}{1 + \frac{N_U \sigma_{aU}}{N_M \sigma_{aM}} + \frac{\sum N_i \sigma_{ai}}{N_U \sigma_{aU}} \frac{N_U \sigma_{aU}}{N_M \sigma_{aM}}}$$

$$= \frac{VC}{1 + C\left[1 + \frac{\sum N_i \sigma_{ai}}{N_U \sigma_{aU}}\right]} = \frac{VC}{1 + CX} \tag{36}$$

where X is defined as $\left(1 + \sum N_i \sigma_{ai}/N_U \sigma_{aU}\right) = \left(1 + \sum X_i\right)$. On the other hand, the definition for L^2 is

$$L^2 = D + L^2 \text{ (thermal)},$$

where D is the contribution to L^2 made in slowing down the neutron. Only the thermal term is dependent on C and on poisoning. Furthermore, the absorption cross-section will be expected to fall as energy increases. For this term we have

$$L^2 \text{ (thermal)} = \frac{\lambda_{th}\ \lambda_{th}}{3}$$

where $\lambda_{th} = \frac{1}{N_M\ \sigma_{sM} + N_U\ \sigma_{sU}} \approx \frac{1}{N_M\ \sigma_{sM}} = \lambda_{th}\ (\text{mod.})$

Now,

$$\lambda_{th} = \frac{1}{N_M\ \sigma_{aM} + N_U\ \sigma_{aU} + \Sigma_i\ N_i\ \sigma_{ai}} = \frac{\lambda_{(\text{mod. alone})}}{1 + CX} \qquad (37)$$

and

$$L^2 \text{ (thermal)} = \frac{\lambda_{th}\ (\text{mod.})\ \lambda_{th}\ (\text{mod.})/3}{1 + CX} = \frac{E}{1+CX}. \qquad (38)$$

Equation (15b) becomes:

$$L = D + \frac{E}{1 + CX} \qquad (39)$$

The critical sizes (Equation 8) are altered to

$$\left.\begin{aligned} \frac{(1+\Gamma)^2}{D + \frac{E}{1+CX}} \left(\frac{VC}{1+CX} - 1 \right) &= \left(1 + \frac{0.586}{\mu 2} \right) \frac{\pi^2}{h_c^2} \\ &= \left(1 + \frac{\mu 2}{0.586} \right) \frac{0.586\pi^2}{r_c^2} \end{aligned}\right\} \qquad (40)$$

The critical mass is rewritten as

$$m_c = F \left(\frac{D + \frac{E}{1+CX}}{\frac{VC}{1+CX} - 1} \right)^{3/2} C. \qquad (41)$$

Available Facts about Fission Products.

Data on fission yields have recently been published by the Plutonium Project (Reference 2), and also by Grummitt and Wilkinson of the Canadian National Research Council (Reference 3). The two sets of results are in rough agreement, but the former are much more complete than the latter. Let us review some of the facts.

1. The number of neutrons V may vary from one case of fission to another. Individual values of V have not been published; we know only that

the average value of V for U235 lies between 1 and 3. We have no data on the value of V for plutonium fission. Therefore, the nature of the initial fragments is usually unknown; often they have short lives.

2. The fraction of the fissions which lead directly or indirectly to the formation of a particular isotope is called the fission yield of that isotope. The yield of the final product in a chain is called the yield for that chain.

For example: the yields in the chain for mass 77 are as follows:

Isotope:	Ge_{32}^{77}	As_{33}^{77}	Se_{34}^{77}
Period:	12 hrs.	40 hrs.	Stable
Yield:	.0037	.0091	Unknown

This shows that some As_{33}^{77} is formed directly and that some is formed by decay of Ge_{32}^{77}. On general grounds we expect that any given isotope can be formed in many ways, but reference 2 states that in general the fission yield does not vary much along a chain and that the yields reported for the chains (Table II of the article) are usually measured on isotopes late in the chains.

3. All products produced by the beta-decay of the primary fission products contribute to the poisoning. We must not forget that fission products are decomposed by the neutrons. For each product, (Z,A), Rate of production by fission, by decay of the parent and by neutron absorption on the part of (Z, A-1) equals Rate of spontaneous disintegration plus Rate of destruction by neutrons. The last term in this equation is far from negligible for strong neutron absorbers. Indeed, we shall see that at the power level necessary for a 5000-mile rocket, the time-constant for the build-up or decay of the strongly-absorbing Xe135 is several thousand times shorter than the natural life of this substance. Of course, this is an extreme case. The stable atoms listed as

end-products of the chains are partially destroyed by neutrons so they are not true end-products, while the pile is operating. Thus many beta emitters and stable atoms not listed in the fission chains are produced in the pile. Summarizing, each fission supplies two entities whose descendants continue to absorb, as long as the reactor operates and the products do not escape.

4. It happens that there is one fission product,--Xe135, which has an enormous absorption cross-section, given as 3.5×10^6 barns in "Nuclear Fission and Atomic Energy." The yield for the 135-chain is high, 5.9 per cent. Of course, there might be other such absorbers, not yet announced, but let us suppose that Xe 135 is the only one. For comparison, we note that the cross-section for Cd 113 is only 80,000 barns, this being the second highest value known to us. Indeed, thermal absorption and low-energy resonance cross-sections listed for the stable elements from Z = 30 to Z = 64 run as follows: Twenty-two elements yield an average of 11 barns, none being more than 30 barns. Then we have nine special cases, as follows:

Element	Rough fission yield %	Thermal Barns	Resonance Barns	Comment
Rh	4	125	6000	
Ag	0.06	60	7000	14000 for a 50% isotope
Cd	0.015	2600	10000	80000 for a 12% isotope
In	0.01	85	>1800	1800 for one isotope
I	0.2	9	150	
Xe	2	?	---	($3.5\ 10^6$ for isotope 135, not stable, with yield 5.9%)
Sm	1	4300		
Eu	0.3	3500		
Gd	0.007	30000		

The fission yields given are simply values read off from the fission yield curve at abscissae equal to the atomic weights of these elements. They enable us to judge roughly the fission yields for the elements, and to orient our thoughts in the following way.

If we assume that the fission products, regardless of their age, have an average thermal section comparable to that for the stable isotopes studied above, it is clear that the production per second of Xe135 times its cross-section greatly exceeds the sum of similar products for all other fission products.

This does not mean that the total effect of Xe135 always predominates over that of all the other elements. If Xe135 were produced solely as a primary product, it would predominate in the early stages of operation, but eventually it will come up to quasi-equilibrium, and after that will drop off slowly as the uranium is used up. Meanwhile the horde of isotopes with ordinary cross-sections will be piling up, and will be steadily promoted to higher atomic numbers by neutron-beta processes. They will eventually win out over Xe135 as poisoners.

If Xe135 is a daughter or granddaughter product, then it contributes to the poisoning at a time of the order of its growth period, but eventually is overshadowed by general poisoning, just as in the previous case. We do not have the separate fission yields for production of Xe135 as a parent, a daughter and a granddaughter, and we do not know the cross-sections for its production from the 134 chain by neutron absorption, but the latter effect will be neglected in this report.

We shall show that general poisoning does not matter in the 5000-mile missiles discussed in later chapters. (It will set a limit to the flying life of a nuclear ram jet or turbojet, but that limit is so long that we suspect it exceeds the limit imposed by radiation damage or by corrosion and creep in the reactor). Then we shall consider the xenon poisoning of the 5000 mile missiles in detail.

If all the fission products had the same absorption section, σ, the time-course of the poisoning would be especially simple. We would have

$$X = 1 + \frac{2 N_f t \sigma}{N_U \sigma_{aU}} \qquad (42)$$

where N_f is the number of fissions per cc per sec. For a particular 5000 mile rocket considered in Chapter IV,

$$N_f t = (4.8 \ 10^{14} \text{ fissions/cc sec}) \times 130 \text{ sec.}$$

$$= 6.3 \ 10^{16} \text{ fissions/cc.}$$

$$N_U \sigma_{aU} = 9.6 \ 10^{-3}$$

We shall use for σ a rough value computed from the absorption coefficients of stable atoms in the range of atomic numbers from 30 to 64, so far as we know them. The value employed is the thermal or the resonance cross-section, whichever is larger. As before, we assign to each element a fission yield read from the curve of yields, at an abscissa equal to the atomic weight of the element. The value formed in this rather crude way is 420 barns. Substituting in (42), we get

$$X - 1 = \frac{2 (6.3 \times 10^{16}) (420 \times 10^{-24})}{9.6 \times 10^{-3}} = 0.0055.$$

This value is small compared with the value caused by Xe135. The result for a 5000-mile ram jet or turbojet is similar, though we shall not go into details. In a flight of 50,000 to 100,000 miles the general poisoning would be of the same order as that caused by Xe135, but neither would be large enough to require a large percentage increase in the uranium requirement.

Poisoning by Xe135.

The figures for fission yields in the 135-chain are:

Te135, (?); I135, 5.6%; Xe135, 5.9%.

Since no errors are stated, the genetics of Xe135 are not entirely clear.

If the proton-neutron distribution is constant throughout the U^{236} nucleus, then we should expect to find that the fission products are never any richer in neutrons than the original nucleus. For the Xe^{135} case, this would imply that I^{135} is the direct fission product. However, if the protons are distributed on or near the surface of U^{236} during assymetric fission then because of surface to volume ratios, the heavy fission product will be expected to be richer in nuetrons than U^{236}, while the light fission product will be less rich. Thus the chain proposed by Segre' and Wu, Phys. Rev. 57, 552 (1940) can be understood. With half lives from Reference 2, it is,

$$_{52}Te^{135} \xrightarrow[<2\text{ min}]{} {}_{53}I^{135} \xrightarrow[6.7\text{ hr}]{} {}_{54}Xe^{135} \xrightarrow{9.2\text{ hr}} {}_{55}Cs^{135} \xrightarrow[2.5\times10^{4}\text{ yr}]{} {}_{56}Ba^{135}$$

$$Xe^{135} \xrightarrow[\text{(about 10\%)}]{13\text{ min}}$$

It is logical to suppose that Xe^{135} may be formed in different cases of fission as a parent, daughter, or granddaughter. Accordingly, we consider all three cases.

Xenon as a secondary or tertiary product gives the reactor very interesting characteristics. If the xenon were the direct fission product the reactor would stabilize at a given power level; but, with an appreciable half-life preceding the formation of xenon the reactor may oscillate in energy. If the power is shut down, the xenon concentration will build up for a time because it is no longer being converted by neutron absorption. It may even be impossible to start the reactor again until the xenon decays away. For conditions of very high power output the operation of the reactor may be unstable, and the slightest increase of absorption will at once throttle the chain.

We adopt the following nomenclature:

i, j. k = subscripts referring to the parent, secondary, and tertiary fission products, respectively, in a given chain.

P = average power per unit volume in ergs/cm^3 sec,

p = total power level = P x volume.

d_i = fission yield of the parent of the fission chain.

$\lambda_i, \lambda_j, \lambda_k$ = decay constants in sec^{-1}.

E_f = ergs per fission = 2.72×10^{-4}.

N_f = number of fissions per second per cm^3 = P/E_f.

T = time of operation of the reactor

E = total energy required = pT

We need the abbreviations:

$$I = \frac{\sigma_i}{E_f} \frac{F}{N_U \sigma_{aU}} + \lambda_i \qquad (43)$$

J, K = the same expression with j and k replacing i.

The decay equations are all of the type

$$\frac{dN_i}{dt} = \text{rate of formation of } N_i - \text{rate of destruction by neutron capture} - \text{rate of natural decay.} \qquad (44)$$

$$\text{Rate of formation of } N_i = d_i N_f = \frac{d_i P}{E_f} \qquad (44a)$$

$$\text{Rate of destruction} = \frac{N_i}{N_U} \frac{\sigma_i}{\sigma_{aU}} N_f = \frac{N_i}{N_U} \frac{\sigma_i}{\sigma_{aU}} \frac{P}{E_f} \qquad (44b)$$

(Use of the room-temperature cross-sections in these expressions amounts to assuming that σ_i and σ_{aU} have similar behavior in the temperature range considered, and no pronounced resonances at higher energies),

$$\text{Rate of decay} = \lambda_i N_i$$

Thus we get

$$\frac{dN_i}{dt} = \frac{d_i P}{E_f} - IN_i \qquad (45a)$$

$$\frac{dN_j}{dt} = \lambda_i N_i - JN_j \qquad (45b)$$

$$\frac{dN_k}{dt} = \lambda_j N_j - KN_k \qquad (45c)$$

Remembering that N_i, N_j, and N_k are zero when $t = 0$, we get

$$N_i = \frac{d_i P}{E_f I} (1 - e^{-It}) \qquad (46a)$$

$$N_j = \frac{\lambda_i d_i P}{E_f I J}\left(1 - \frac{J}{J-I} e^{-It} - \frac{I}{I-J} e^{-Jt}\right) \quad (46b)$$

$$N_k = \frac{\lambda_i \lambda_j P d_i}{E_f I J K}\left(1 - \frac{JK}{(J-I)(K-I)} e^{-It} - \frac{KI}{(K-J)(I-J)} e^{-Jt} - \frac{IJ}{(J-K)(I-K)} e^{-Kt}\right) \quad (46c)$$

We must now consider three subchains. Taking the data of Reference 2, quoted above, at face value, the yield of Xe135 as a parent is 0.3%. We assume 0.3% for I 135 and the value 5.3% for Te 135. Letting t approach infinity in Equation (46) we find the following maximum values of $X_{Xe} - 1$ for the three subchains:

Subchain, mass 135	Maximum value of $X_{Xe} - 1$, caused by this subchain
Xe	d_i in 46a = d_{Xe} = 0.003
I, Xe	d_i in 46b = d_{Io} = 0.003
Te, I, Xe	d_i in 46c = d_{Te} = 0.053

It follows that the maximum xenon poisoning, due to all three modes of forming it, is described by the value $X_{Xe} = 1.059$, and this is practically the same as X for any 5000-mile missile.

The effects of such maximum xenon poisoning on K, L, and the uranium mass are as follows:

$K_p/K = 0.954$ (4.6% decrease)

$L_p^2/L^2 = 0.970$ (3% decrease)

$M_{U,p}/M_U = 1.098$ (9.8% increase)

Here the subscript p indicates the value for the poisoned reactor. As a matter of fact, the xenon poisoning in a 5000-mile rocket falls far short of these values. The flight time for a particular case considered on page 4.17 is only 130 seconds,

and since I 135 has a half life of 6.7 hours we need only consider the first subchain, Equation 46a. The data for the case $V = 2$ are:

d_i	0.003
Total energy expended	$1.8\ 10^{19}$ ergs,
Power P, per cc,	$1.3\ 10^{11}$ erg/sec cm^3.
$N_U\ \sigma_{aU}$	.0096 cm^2 ($18.8\ N_M\ \sigma_{aM}$)
E_f	$2.72\ 10^{-4}$ ergs/fission

$$I = \frac{\sigma_{Xe}\ P}{E_f\ N_U\ \sigma_{aU}} + \lambda_{Xe} = 0.174 + 1.91\ 10^{-5}$$

Thus e^{-It} is practically zero, and the first subchain poisoning rises to the value $X_{Xe} = 1.003$. We see from the separate terms in I that the high neutron flux brings the apparent half life of Xe 135 down from 9.2 hours to 3.6 seconds.

Similarly, we can simplify the discussion of poisoning in a 5000-mile ram jet and turbojet. The power level in a typical case will be of the order of 1% of the rocket value, and the flight time will be about 100 times greater, or 13000 seconds say. Assuming destruction cross-sections of only a few barns for Te 135 and I 135, it follows that their effective decay constants are practically unaltered (Equation 43). That for Xe will be .00174, so the subchain contributions at the end of flight boil down to the following:

Subchain	Contribution to $X_{Xe} - 1$
Xe:	0.003
I, Xe:	Equation (46b); $0.003\ (1 - e^{-\lambda t})$
	$= 0.003\ \left[1 - e^{-(2.6\ 10^{-5})(0.13 \times 10^5)}\right]$
	$= 0.0009$
Te, I, Xe:	Equation (46c); $0.053\ (1 - e^{-\lambda t})$
	$= 0.0153$

Thus X_{Xe} = 1.019 at the end of ram jet flight.

Summarizing, the effects of poisoning on the uranium mass in all 5000-mile missiles considered are very slight. The effects on control properties appear to be more important, and will require a separate investigation.

starting the pile control by rods is difficult or unfeasible. However, with a strong negative temperature coefficient of reactivity, combined control by temperature and by rods can become effective. That is, for temperatures less than the design value, the rods will control the temperature only, and the neutron flux level will be determined by the power dissipation. For such a control to be at all practical it is required that the uranium be dispersed homogeneously, and not lumped. The sensitivity of this type of control may be determined by computing the stabilized temperature as a function of, say, the rod-position.

8. Poisoning.

The effect of poisoning of the reactor by its fission products may be analyzed by considering its effects on the reproduction factor K and the diffusion length. The practical situation is that the pile must continue to function for at least a time t, the duration of the powered flight. We assume that the number of fissions per cc per sec. is kept the same by suitable control. Then it is possible to solve the algebraic problem completely, but the numerical problem of getting the optimum concentration of uranium, and the dimensions which will make the pile just critical at the end of flight, cannot be dealt with fully, because of lack of information on the absorption cross-sections of many products in the fission chains.

More General Equations.

To understand the effects of the poisoning, i.e., parasitic absorption of slow neutrons by the fission products, let us reexamine the definition of K and L. (Equations 1 and 15b).

$$K = \frac{\text{rate of creation of neutrons}}{\text{rate of absorption of neutrons}} \qquad (34)$$

In the presence of poisoning the formula for rate of creation is the same as

PART B. DESIGN OF CYLINDRICAL BERYLLIA REACTORS.

By Arthur E. Ruark

In Part A carbon reactors were considered because of their use in nuclear hydrogen rockets. Here we discuss oxide reactors for ram jets and turbo jets. From Table 1 of Chapter VI we find that the four oxides of highest melting point are ThO_2, MgO, ZrO_2, and BeO. It is easy to see that the neutron absorption properties of the first three bar them from consideration (and the same is true of $CaZrO_3$ and $ZrSiO_3$ whose melting points lie close to that of BeO). The field narrows down to BeO and it is useful to compare its thermal and neutronic properties with those of carbon. This is done in Table 3.

Details of Table 3. The cross-section data at our disposal for Be and O are incomplete. We have used the following:

	Be	Ref.	O	Ref.	Sum
Scattering, thermal;	6.1	1	4.1	1	10.1
Absorption, thermal;	0.0085	1	0.0016	1	.0101
Scattering, fast.	1.6	4	1.8	4	----

To calculate thermal cross-sections for 3000°K, we use Equation (23), Part A, based on the 1/v law. Since scattering cross-sections are not known as a function of energy, we have assumed that λ_s^2 is a linear function of log E; here λ_s is defined as the scattering cross-section of a BeO molecule.

Computation of the diffusion length for slowing down, namely, $N\overline{\lambda_s^2}/3$, requires knowledge of the number of collisions, N. Let a fraction a of the collisions be encounters with Be atoms. Then Equation (19) of Part A may be generalized to read

$$E_o\,(1 - f_{Be})^{aN}\,(1 - f_O)^{(1-a)N} = E_N, \qquad (19a)$$

and we can define an effective f, as follows:

$$(1 - f)^N = (1 - f_{Be})^{aN} (1 - f_O)^{(1-a)N}$$

It turns out that f is 0.140, which is practically the same as the value for carbon. This discussion is not a defense of these methods of averaging (which are good enough, in view of the lack of data), but simply an account of what was done.

Beryllia Reactors. The data of Table 3 were used to find the design values for critical enriched beryllia reactors. Table 4 records the results. Figure 2.3 shows how the critical uranium mass, the radius, and the length vary as a function of shape; although the mass-curve is drawn for the particular case in which the optimized uranium mass is 3500 grams, the relative values which can be read from the curve apply to all cylindrical reactors, because the dependence of mass on the ratio radius-over-length is the same for all. Similar remarks apply to the curves showing radii and lengths. The actual data given are for the case V = 2, T = 300°K in Table 4.

Table 3. Comparison of Thermal and Neutronic Properties of Carbon and Beryllia

	Carbon	Beryllia
Density, gr/cc.	2.25	3.02
lb./cu. ft.	138	189
Melting Point: Deg. K.	> 3770	2840
Deg. R.	> 6800	5110
Molecules per cc.	$11.3\ 10^{22}$	$7.33\ 10^{22}$
Cross-sections in barns per molecule, and mean paths:		
σ_a, thermal	.0045	.010
N σ_a, thermal	.000511	.000740
λ_{th}, thermal	1950 cm.	1350 cm.
σ_s, thermal	4.8	10.2
N σ_s, thermal	0.54	0.748
λ_{th}, thermal	1.84 cm.	1.34 cm.
L_{th}^2 ($\equiv$ E)	1200 cm^2	639 cm^2
L_{th}	34.6 cm.	25.3 cm.
$\bar{f}$, average fraction of energy lost in an elastic collision	.142	.140
No. of collisions to slow down:		
to 1/30 volt	114	115
to 1/3 volt	99	100
$\overline{\lambda^2}$, slowing, to 1/30 volt	3.97 cm^2	3.78 cm^2
to 1/3 volt	4.04	4.04
Square of diffusion length, L^2_{sl}:		
to 1/30 volt	151 cm^2	144 cm^2
to 1/3 volt	133	133

Calculation, Ruark, December 1946
Check, Meyer, January 1947

Table 4. Critical Design Values. Cylindrical BeO Reactors.
(Dimensions, cm; masses, grams; C is optimum)

Temperature			300°K			3000°K		
μ	$F(1-\Gamma)^2$		V = 2	2.2	2.4	V = 2	2.2	2.4
0.1	.3062	m_c(gr)	10574	7650	5817	21978	16333	12735
		r_c(cm)	43.5	40.2	37.55	45.4	41.2	38.4
		h_c(cm)	435	402	375.5	454	412	384
0.2	.1648	"	5691	4117	3131	11829	8790	6854
			44.65	41.2	38.50	46.6	42.1	39.4
			222.8	205.7	192.2	232.5	210.0	197
0.3	.1272	"	4393	3178	2417	9130	6785	5508
			46.46	42.9	40.11	47.79	44.02	41.02
			154.9	143.0	133.4	159.3	146.7	136.7
0.542	.1012	"	3495	2528	1923	7264	5398	4209
			53.0	48.95	45.76	54.53	50.22	46.8
			97.8	90.3	84.43	100.6	92.6	86.3
0.75	.1090	"	3764	2723	2071	7824	5814	4588
			60.55	55.92	52.29	62.3	57.38	53.48
			80.73	74.6	69.7	83.1	76.5	71.3
1.0	.1329	"	4590	3320	2525	9539	7089	5527
			70.5	65.3	60.7	72.8	66.6	62.1
			70.5	65.3	60.7	72.8	66.6	62.1
2.0	.3268	"	11286	8164	6209	23457	17432	13592
			120.9	111.7	104.1	126.0	114.3	106.4
			60.3	55.7	52.0	62.9	57.2	53.2
5.0	1.7226	"	59488	43036	32728	123643	91885	71643
			286	264.5	246.7	299	269.5	252.2
			57.1	52.8	49.3	59.7	53.8	50.4
	D			144			133	
	E			603			1905	
	C_{opt}		5.95	5.45	5.08	11.35	10.80	10.40
	L_d^2		230	237	243	287	294	300
	K		1.71	1.86	2.00	1.84	2.02	2.19

Comparison of Carbon and Beryllia Reactors. Comparing Table 2 (page 2.13) and Table 4, we may summarize the essential features as follows:

Table 5. Comparison: Designs of Optimized, Critical, Cylindrical Solid Reactors of Carbon and Beryllia
(Neutrons per fission assumed to be 2, a conservative case)

Item	Carbon	Beryllia
Optimum C, cold	8.1	6.0
Optimum C, hot	18.8	11.4
$m(1-\Gamma)^2$, cold, grams U	3910	3500
hot, grams U	9050	7260
Critical radius, hot, cm	56	55
Critical length, hot, cm	103	101
Total weight for hot operation, kg.	2270	2880
lbs.	5000	6350

Taken at face, this table says: (1) the uranium requirement changes less with temperature in the case of beryllia than in the case of carbon; (2) the hot beryllia reactor requires only 80% as much U235 as the carbon reactor; (3) the dimensions of both are substantially the same; (4) the beryllia reactor weighs 27% more. No emphasis can be laid on these comparisons because of the uncertainties in the data. A reasonable statement would be that in considering uranium costs, the two reactors may be thought of as identical, and that the ratio, beryllia-weight over carbon-weight probably lies between 1.1 and 1.4.

Critique of the Cylindrical Reactor Designs. In these designs fission by neutrons which are slowing down has been neglected. The slowing collisions have been treated as elastic, although it is well known that scattering of fast neutrons in many elements is largely inelastic, resulting in high energy losses.

Simple diffusion theory has been employed. We have assumed that the reactor is not provided with a reflector. Yet, some type of supporting shell will be

essential and this shell will reflect some neutrons, even if it is designed primarily for lightness and strength at high temperatures. Rough theories of neutron reflection are available and slight corrections to diffusion theory are at hand (References 1 and 6).

We have not considered lattice-type piles since we see no reason, at present, for believing that a lattice structure has any advantages for U235 or plutonium reactors. (The device of a lattice was introduced to avoid the consequences of resonance capture in U238 piles).

A defect of our designs with uniform uranium content is the fact that the power level will be high near the center, low near the outer surfaces. Gradation of the uranium content appears to be necessary. An increase in the uranium concentration near the outside will permit some decrease near the center, but from the physics of the situation it appears that a net increase in the amount of uranium will probably result.

Exept for this last point, the several improvements of the theory, listed above, will result in a smaller uranium requirement and a decrease of critical size. Dr. Harvey Hall of BuAer has kindly discussed with us the gross inadequacy of diffusion theory for the parts of a reactor lying within a few mean free paths from the surface; the effect on reactor size is not clear at this writing. Provision of space for controls will increase the size.

We have employed the true densities of C and BeO; in practice the bulk densities will be much lower, and the weight should be corrected by multiplying it by the factor, $(\text{True density/Bulk density})^2$. We shall not attempt refinements here simply because of the uncertainties of available data. It is felt that the designs given are correct in order of magnitude but are definitely on the low side, as regards weight. We cannot set limits on the errors incurred and remind the reader that no emphasis should be placed on the exact numbers given in the tables of reactor-properties, except for comparative purposes.

PART C. CONSIDERATIONS CONCERNING FAST-NEUTRON REACTORS
("Amplification Reactors")

By George Gamow

It is shown in Chapter II, Parts A and B, and in other chapters of this report that slow neutron reactors may be feasible for long range jet propulsion. It is important to analyze the possibility of reactors built from pure fissionable materials (plutonium or U235) which work on the principle of fast-neutron chains. Let us compare reactors of the fast and slow varieties. For the sake of simplicity, we deal with solid reactors, remembering that in actuality the two varieties will operate at different temperatures and may have different relative gas-stream areas when made up in tubular form, for use in a missile.

A solid fast neutron reactor has the advantage of:

(1) Much smaller total weight and size due to the absence of moderating material.

(2) Comparatively small importance of poisoning by fission products because for fast neutrons all nuclei have about equal cross-sections.

(3) The possibility of turning the pure fissionable material used for propulsion, into the explosive at the end of the trajectory instead of uselessly dispersing it as in the case of slow neutron reactors.

(4) Smaller mass of fissionable material, which can be reduced by use of reflectors.

The disadvantages of such reactors lie in:

(1) The danger of a runaway chain reaction which would prematurely turn the reactor into a low order atomic bomb.

(2) Necessity of casings to hold the molten or softened fissionable material, which adds weight and impedes heat transfer.

2. Fast-Neutron Chains.

Let us consider a sphere of pure fissionable material with the radius R and calculate the average effective branching ratio of a fast neutron chain developing in its interior. The effective branching ratio K is evidently given by $K = V(1 - \bar{\omega})$ where V is the number of fast neutrons formed in an individual fission, and $\bar{\omega}$ can be defined as the mean relative number of

neutrons starting in an arbitrary direction from an arbitrary point within the sphere which come out through its surface. Consider a point B (Figure 1) at the distance r from the center of the reactor. The relative number of neutrons ω with the mean free path λ coming from this point and escaping through the surface is approximately given by the fraction of the sphere of radius λ which lies outside the sphere of radius r. In the case shown this is

$$\omega = (1 - \cos\Theta)/2 = \tfrac{1}{2}\left(1 - \frac{R^2 - \lambda^2 - r^2}{2\lambda r}\right). \qquad (1)$$

Summarizing all the four cases which exist, we have these ω- values:

$\lambda \leqq R$: If $r \leqq R - \lambda$, $\omega = 0$

If $r \geqq R - \lambda$, $\omega = (1 - \cos\Theta)/2$

$\lambda \geqq R$: If $r \leqq \lambda - R$, $\omega = 1$

If $r \geqq \lambda - R$, $\omega = (1 - \cos\Theta)/2$.

In the actual case there is always a probability that a neutron will be absorbed. The discontinuity in the slope of ω is a result of approximating the true problem by paths of definite length. As the objective of this computation is an accuracy corresponding to one significant figure, the approximate theory will be followed because of its simplicity. (A closer approximation worked out by Dr. N. M. Smith is available at APL).

Averaging over the volume of the original sphere we obtain:

For $\lambda \leqq R$,

$$\bar{\omega} = \frac{1}{4/3\,\pi R^3} \int_{R-\lambda}^{R} \tfrac{1}{2}\left(1 - \frac{R^2 - \lambda^2 - r^2}{2\lambda r}\right) 4\pi r^2\, dr; \qquad (2a)$$

and for $\lambda \geqq R$,

$$\bar{\omega} = \left(\frac{\lambda - R}{R}\right)^3 + \frac{1}{4/3\,\pi R^3} \int_{\lambda-R}^{R} \tfrac{1}{2}\left(1 - \frac{R^2 - \lambda^2 - r^2}{2\lambda r}\right) 4\pi r^2\, dr \qquad (2b)$$

Introducing $X = r/R$ and $\alpha = \lambda/R$, and integrating we obtain:

For $\alpha \leq 1$,

$$\bar{\omega} = \tfrac{1}{2}\left[1 - (1-\alpha)^3\right] - 3/8\,\frac{(1-\alpha^2)}{\alpha}\left[1 - (1-\alpha)^2\right] + \frac{3}{16\alpha}\left[1 - (1-\alpha)^4\right] \tag{3a}$$

and for $\alpha \geq 1$,

$$\bar{\omega} = (\alpha-1)^3 + \tfrac{1}{2}\left[1 - (\alpha-1)^3\right] + 3/8\,\frac{(\alpha^2-1)}{\alpha}\left[1 - (\alpha-1)^2\right] + \frac{3}{16\alpha}\left[1 - (\alpha-1)^4\right] \tag{3b}$$

The graph of $\bar{\omega}$ is shown in Figure 2. We see from this graph that for V equal to about 2, the critical dimensions of the sphere correspond to $\alpha = 0.69$, i.e., $R_{cr} = 1.45\,\lambda$. Assuming for the free path of fast neutrons in uranium 3.5 cm, we get $R_{cr} = 5$ cm. The corresponding mass is about 10 kg. This critical mass can be reduced by surrounding the reactor with a good reflector. For the variation of K with R near the critical size we obtain

$$K = V(1 - \bar{\omega}) \quad V\,\frac{d(1-\bar{\omega})}{d\alpha}\,\frac{\partial\alpha}{\partial R}\,dR,$$

The first term is 1 by definition; the value of the slope (from Figure 2) is $-0.97/1.44$; also $d\alpha/dR = -\alpha/R$, and it is assumed that V is about 2. Therefore,

$$K = 1 + 0.97\,\frac{R - R_{cr}}{R_{cr}} \tag{4}$$

3. Two Ways of Using Fast Neutron Reactors

One way of using a fast neutron reactor would be to run it slightly above the critical size, preventing the runaway chain reaction (explosion) by checking it periodically through the insertion of a control bar. If T is the vibrational period of the control bar, and τ the time corresponding to one link of a branching chain, the reaction's amplitude will be given by the expression:

$$\left(1 + 0.97\,\frac{R - R_{cr}}{R_{cr}}\right)^{T/\tau} \simeq 1 + 0.97\,\frac{R - R_{cr}}{R_{cr}}\,\frac{T}{\tau}. \qquad (5)$$

For fast neutrons τ is about $\frac{3.5\text{ cm}}{10^9\text{ cm/sec}} = 3.5\cdot10^{-9}$ sec. If we can build a vibrating control rod with a period of one millisecond, the condition that during this period the reaction rate will increase, say, by only a factor of two is:

$$0.97\,\frac{R - R_{cr}}{R_{cr}} \cdot \frac{10^{-3}}{3.5\cdot10^{-9}} = \log_e 2,$$

so that

$$\frac{R - R_{cr}}{R_{cr}} = 2.3\cdot10^{-6}$$

This assumes that the reaction is undercritical in one cycle by such an amount and such a time that the net multiplication factor is one. The calculation is meant only as an illustration. The factor of increase could be any suitable value. This accuracy in the over critical size is a rather hard condition to satisfy. Here lies a serious disadvantage of fast-neutron reactors as compared with slow-neutron reactors, where the time period is considerably longer (of the order of 10^{-4} sec.).

In view of the difficulties of controlling fast-neutron chains even in a reactor of only a slightly overcritical size, we may consider the possibility of using the reactor of slightly subcritical size for the purpose of "amplification" of a neutron beam supplied by some spontaneous decay process. If the effective branching ratio is smaller than unity, being given by (1 - E), the total number of fission processes resulting from one neutron entering the reactor from outside will be, when E is very small compared to 1,

$$f = \sum_{n=0}^{\infty} (1 - E)^n = 1/E \qquad (6)$$

If, for example, the reactor is one-tenth of a per cent smaller than critical size, $E = 0.97 \cdot 10^{-3}$ and the amplification factor becomes $f = 1030$. The (probably unobtainable) accuracy of radius required for control in the above example of the supercritical reactor, with $(R - R_{cr})/R_{cr} = 2.3 \ 10^{-6}$, would correspond in a subcritical reactor to the amplification factor

$$f = 4.5 \cdot 10^{5}!$$

The above described amplifying arrangement represents a hybrid between the methods using the chain reactions and those based on the use of radioactive materials. It has the advantage of both, representing no immediate danger of explosion (in contrast to the case of slightly overcritical size), while on the other hand it can be at least partially controlled. The radioactive neutron source represents here in fact a sort of "pilot-flame" liberating within itself a comparatively small amount of heat before it is pushed into the interior of the fissionable material.

4. Spontaneous Neutron Sources

The best spontaneous source of neutrons known at present is a mixture of beryllium with some alpha-decaying substance as for example polonium. Assume that with a suitable substance we are able to realize one neutron per 4,000 alpha particles. Since the energy liberated per fission is about 40 times larger than the energy of an alpha particle, an amplifier with $f = 1030$ will produce an amount of heat $\frac{40}{4000} \times 1030 \simeq 12$ times larger than the heat liberated in the radioactive "pilot-flame." The use of larger amplification factors, and possibly also more efficient neutron sources will increase this amount many times. It can be suggested, for example, that it may be possible to build an element which is subject to spontaneous fission within a comparatively short period of time. If such an element (a very light isotope of an

element with high atomic number) can be built, we would have a neutron source giving about two neutrons per elementary decay. In this case the heat-ratio between the amplifier and the "pilot-flame" could be easily made equal to several thousands. (Editor's note. Presumably, the spontaneous fisser would be very sensitive to neutrons both slow and fast, and its critical mass would be very small).

5. The Size of "Pilot-Flames" for Ram-Jet Motors

In order to operate a ram-jet of reasonable size we need a total power supply of about 10^{15} erg/sec. (comp. Chapter II, Part A) which means $4 \cdot 10^{18}$ fissions/sec. With an amplification factor of 10^3 we would need a pilot stream of $4 \cdot 10^{15}$ neutrons per second. With the efficiency of one neutron per 4,000 alpha particles we will need about 10^{19} alpha particles per second. The necessary mass of polonium, M, is determined from the equation:

$$M \lambda / m = 10^{19}$$

where m is the mass of the alpha rayer, $3.5 \cdot 10^{-22}$ gm, and λ is 5.2 10^{-8} per second. We get

$$M = 7 \cdot 10^4 \text{ gm} = 70 \text{ kg.}$$

It can be reduced, however, quite considerably by using more efficient neutron sources and high multiplication factors. Thus if one could, for example, build a spontaneously fissioning element with a halflife of one year ($\lambda = 2.2 \cdot 10^{-8}$ per sec.), and if the amplification factor can be increased to 10000, the amount of such material to serve as the "pilot-flame" in a jet motor would be determined by

$$2 \lambda M / m = 4 \cdot 10^{14} \text{ or,}$$

$$M = 3 \text{ gm.} \qquad (7)$$

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It may be remarked here that, according to the theory of the nuclear fission process the possibility of having a short-lived spontaneously fissioning element is not at all excluded. Suppose, for example, that there exists some transuranic element Z which goes through spontaneous fission as soon as it is formed. If this element is obtained from some lighter and more stable element X through the process of two beta emissions (in the same way as Pu^{239} is formed from U^{239}), we can use the element X as the pilot flame. The reaction sequence in this case will be:

$$X \longrightarrow Y + \beta \text{ (long period)},$$
$$Y \longrightarrow \beta + \frac{Z}{2} + \frac{Z}{2} \text{ (shorter period)};$$

This is similar to the familiar reaction: $Li^8 \alpha \longrightarrow \beta + 2_2He^4$. It is not necessary that the element X have its fission barrier equal to zero, since the beta decay leading to its production may produce the necessary excitation.

6. Conclusions

In this study of fast-neutron reactors, the effects of delayed neutrons have been neglected for simplicity. As stated on page 11.12 of "Nuclear Fission and Atomic Energy," it is possible to construct assemblies which are actually overcritical, but which would be subcritical if the delayed neutrons were not present. For such systems the effect of the delayed neutrons is to increase the relaxation time, easing the problem of control. But we shall not pursue the matter here.

It can be concluded from the above discussion that it seems feasible to construct pure reactors which would operate without large danger of explosion, on the amplification principle. Such pure reactors, having considerably smaller geometrical dimensions and total weight, may prove to be more useful for many purposes of atomic propulsion than the ordinary moderated systems. They

can probably be used with advantage for the jet propulsion of airplanes, designed with the hope of long life. Their advantage in the case of bomb-carrying rockets or ram-jets lies in the fact that here one can at least think about the possibility of turning the fissionable material used for propulsion into a bomb at the end of the trajectory, instead of uselessly dispersing it as in the case of enriched pile motors.*

* Editor's note. These conclusions still overlook many practical difficulties.

PART D. SPECIAL METHODS OF HEAT PRODUCTION AND TRANSFER.

By R. B. Roberts

The difficulties connected with the critical size and poisoning of tubular slow-neutron reactors and fast-neutron reactors make it desirable to search for other means of producing heat by nuclear processes and transferring it to a gas stream.

1. Use of Separated Fission Products.

The use of radioactive fission fragments as a source of energy for propulsion has been proposed and various estimates made of the activity available. The recent paper S-5, by Way and Wigner, presented at the Chicago meeting of the American Physical Society, June 20, 1946, gives a formula which permits accurate calculation of the amount available. They give, "Handy rules of thumb giving correct values within a factor of two for times between 10 sec. and 100 days are:

$$\beta + \gamma \text{ Mev/sec./fission} = 2.66t^{-1.2}$$

$$\gamma \text{ Mev/sec./fission} = 1.26t^{-1.2}$$

Calculating with this formula we find that twenty-four hours after fission the activity gives 1.3 kilowatts per gram of uranium fission or per gram of plutonium produced. This includes both gamma rays and beta rays. Taking 10 kilograms as the weight of the bomb, the total power available would be 1.3×10^4 kilowatts (one day after the bomb had been produced instantaneously); and this would be sufficient to power a 16-in ram-jet at sea level (25 per cent thermal efficiency, 2200 ft/sec, 3000 lbs thrust per square foot of combustion chamber). This figure gives the maximum possible available power, as it includes both betas and gammas and makes no allowance for decay during the finite time required to produce the bomb. As most of the gamma rays would escape from the relatively light

structure of a ram-jet, this power should be reduced by a factor of two. Other difficulties arise from radioactivity and from the lack of control over the energy release.

One gram of this material, which gives a total of 1.3 kilowatts, corresponds to 8×10^{15} disintegrations per second, taking an average of one million volts per disintegration. Of these, 3.85×10^{15} per second will be gamma ray disintegrations; the equivalent of 10^5 curies. Since one curie gives approximately one R per hour at one meter, this one gram source could give a lethal dose of 500 R in 18 seconds at one meter. The quantities required for any sort of jet propulsion are 10,000 to 100,000 times as large, with correspondingly increased gamma radiation. As a consequence, the material would have to be handled by remote control through the entire separation process, fabrication, and insertion into the missile. During all the time of fabrication it would have to be cooled sufficiently to dissipate even more than the final desired power.

The combination of difficulties arising from small quantities, radiation and cooling, seem to rule out any possibility of the use of artificial radioactivity in service weapons.

2. Direct Use of Fission Fragments or Evaporated Material in the Gas Stream.

Most of the nuclear energy is initially carried by the fission fragments themselves (170 million volts of 200 million total). The range of the fission fragments in air is less than 5 cm's, so that sheets of any material would have to be extremely thin if the fragment is to dissipate most of its energy in a gas surrounding the sheet. Converting the range for the difference in density between air and a reasonable material of density 3, which might be a mixture of uranium and moderator, we find a range of 2×10^{-3} cm's in the material. Furthermore, the range in the material must be small compared to the

range in air, giving a further reduction in the thickness of the material permissible. Allowing also for the angular distribution of the fragments, we arrive at a maximum thickness of 2×10^{-4} cm's or roughly one-ten thousandth of an inch. These sheets are far too thin and fragile to be practical.

As a second possibility let us consider a special form of pile which would "burn" like a carbon arc. This might conceivably operate at extremely high temperatures and transfer heat to the cooling gas by evaporating some of its material. The basic difficulty here is that the heat cannot be generated on the surface. The range of a neutron is roughly 8 cm's, so that the energy production is distributed throughout a depth of the order of 8 cm.

3. Direct Use of the Momentum of Fission Fragments.

It has been proposed in the "Sunday Astonishers" that rocket ships can be driven by atomic paint which would be applied to the rear of a rocket. Uranium, for example, would be painted on, made to undergo fission by some process not described whereby one fragment shoots out to the rear while the other pushes into the base of the rocket and pushes the rocket along. On calculating the energy and momentum of such a process we find that 3 kilograms thrust would be delivered for 100,000 kilowatts dissipated in the base of the rocket. This, of course, would burn the rocket up long before it moved.

4. Use of Artificial Radioactivity.

Improvement on this idea is to produce polonium artificially and use it as the paint. If the polonium alpha particle could be squirted out to the rear and the recoil heavy atoms caught by the base of the rocket, this would offer an improvement of a factor of 20 in the thrust per kilowatt, or 60 kilograms per 100,000 kilowatts. Unfortunately, there is no conceived mechanism by which the alpha particles could be directed to the rear, as they have a bad habit of coming

off in all directions. Since the alpha particles have roughly the same velocity as fission fragments, they would develop the same number of kilowatts per unit of thrust as do the fission fragments so that only a factor of 2 could be realized from the proportion of the alpha particles that happen to go in the right direction. In addition, a further factor of 2 (unfavorable) comes in because a large proportion of the momentum is directed out to the sides and is not useful. The corresponding energy is not so nicely cancelled out. This matter is considered further in Appendix 2.

5. Use of Accelerated Ions.

A further application of this idea lies in directing a beam of high speed particles to the rear by use of a cyclotron acceleration tube or other mechanisms. This avoids the difficulty of absorbing the energy of the recoil particles but the same or a larger fraction of the energy is still developed in the rocket, as the electrical energy required to accclerate the particles must be derived from heat energy, and the thermal efficiency of the heat engine is unlikely to exceed 50 per cent. These considerations effectively rule out the use of very high speed exhaust gases.

For very large space ships, a low voltage beam of ions can be considered. Such a beam, say 100 volts, directed to the rear would give reasonably high specific impulse, but the thrust would be limited by the power which could be dissipated. This, of course, would only work outside the atmosphere as the low voltage ions would have no penetrating power to emerge through a window.

As an example of this compromise rocket motor, we can consider a 100 volt ion beam of 100,000 kilowatts energy. This beam directed to the rear would give a thrust of 3600 kilograms, a specific impulse of 30,000 and would require 12 grams of material per second. There may be some minor difficulties, of course, as

this beam would require 1 million amperes, and it seems hard to conceive an ion source with this capability, particularly as the efficiency should be very high. It would be necessary to dissipate approximately 200,000 kilowatts if the thermal efficiency can be kept up to 33 per cent. As a final difficulty, the missile would quickly acquire a charge so high that the beam could not escape. The use of two beams, one positive and one negative, would solve this difficulty, but the others remain.

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MULTIPLICATION RATIO OF NEUTRONS
AS A FUNCTION OF REACTOR'S RADIUS

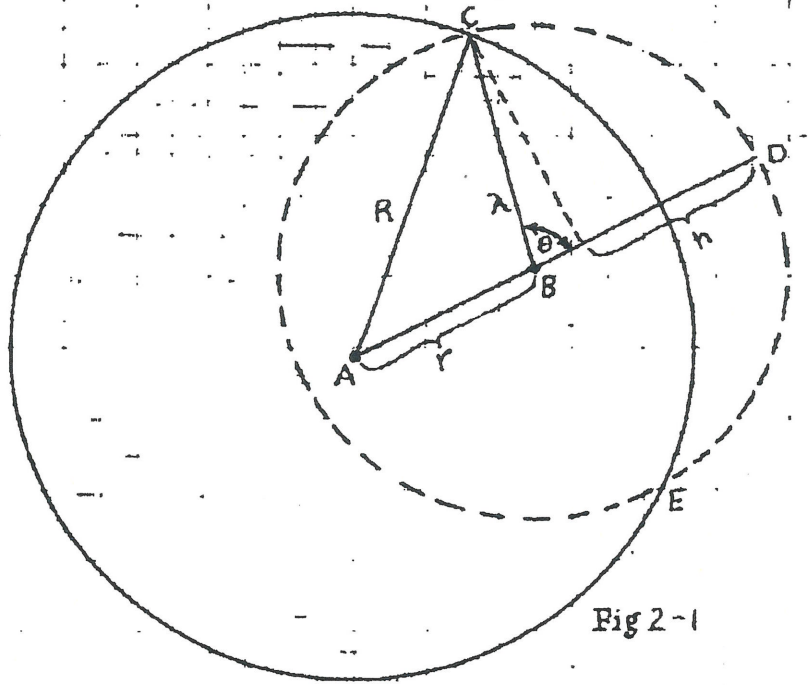

Fig 2-1

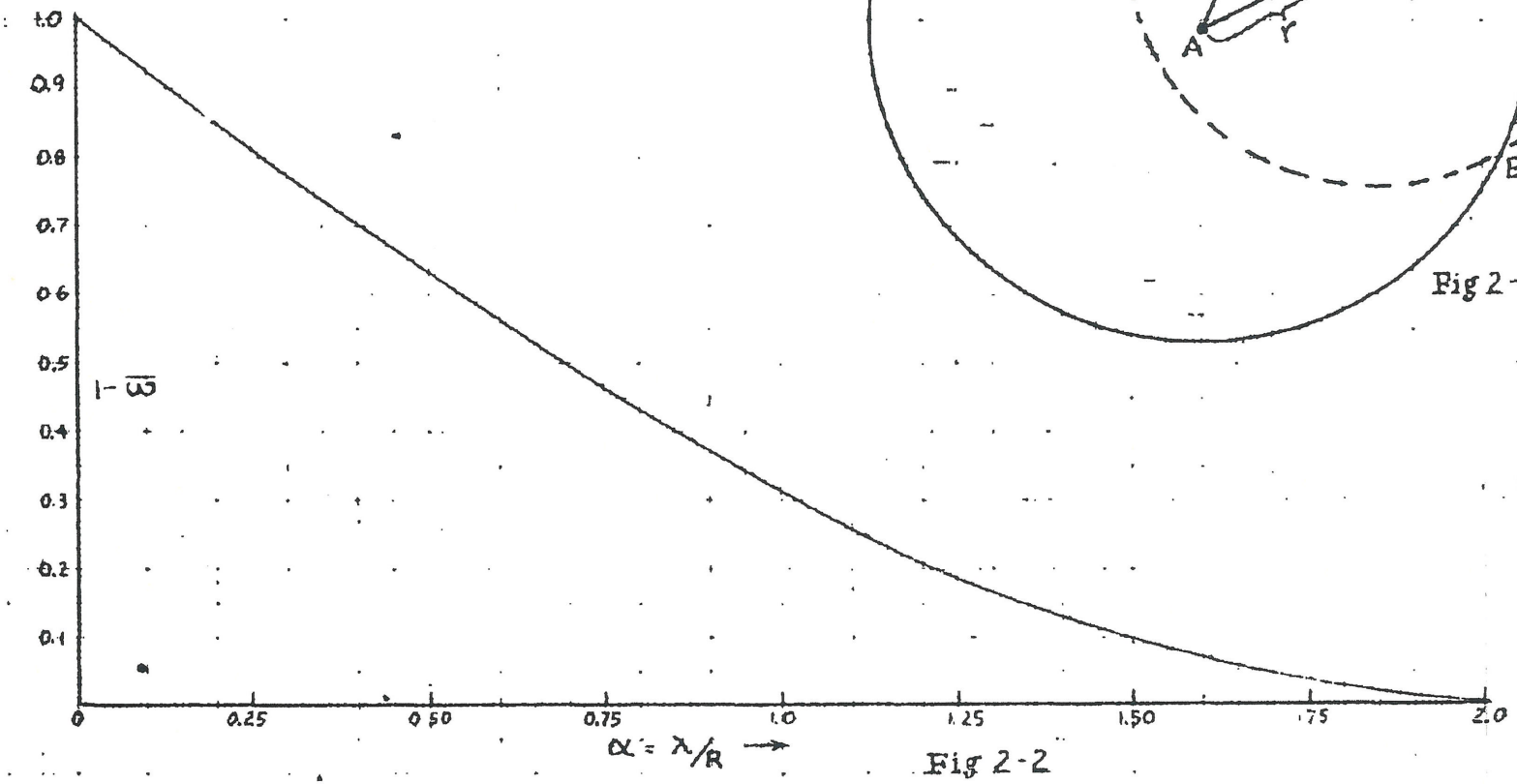

Fig 2-2

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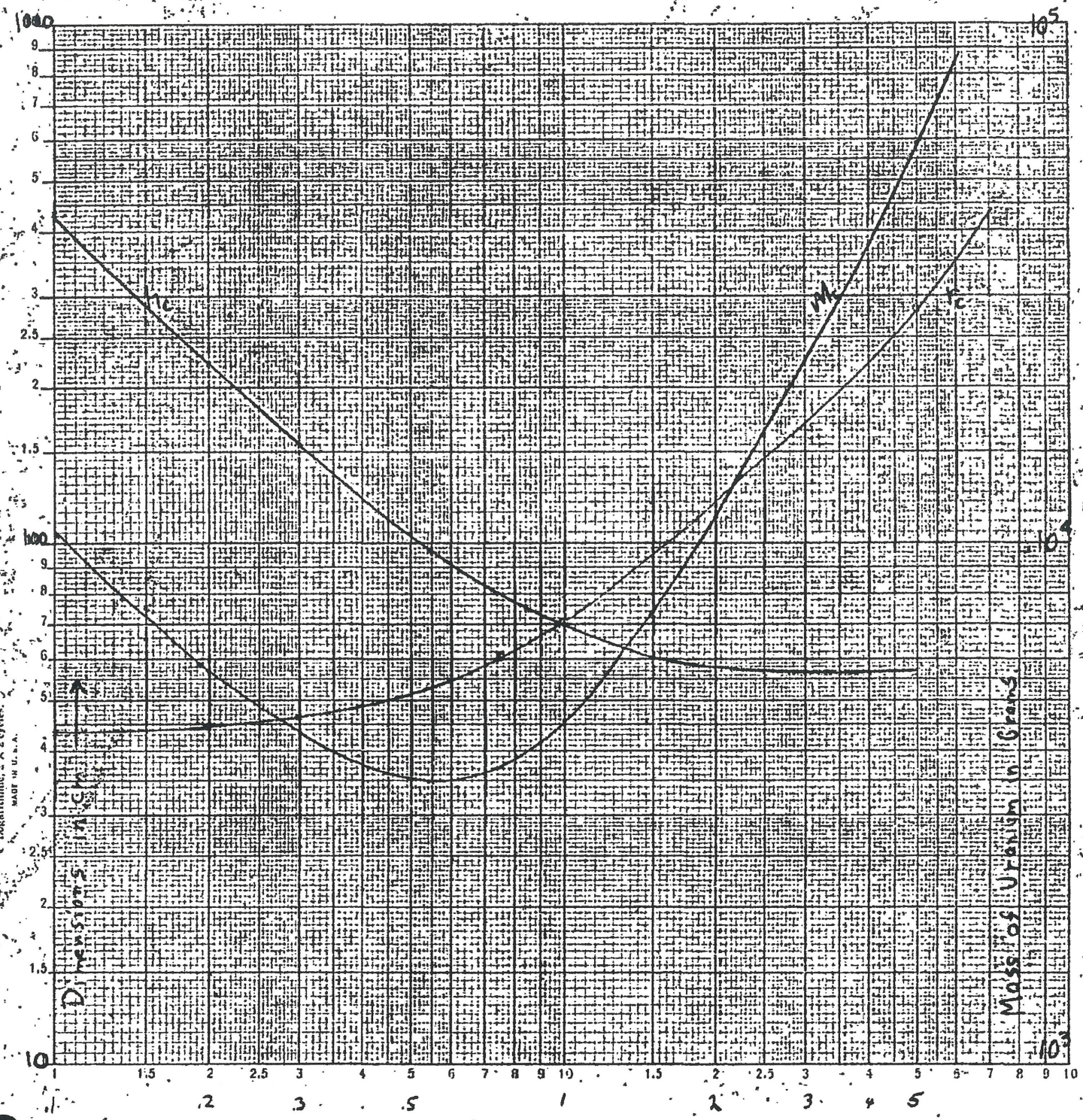
Fig. 2.2 Mass, Radius and Length For Cylindrical Reactors.

(Ordinates refer to BeO reactor, $\nu = 2$; 300°K; but relative ordinates are correct for any homogeneous cylindrical reactors.)



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