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RPPR Final Report
as of 08-Apr-2021

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Title: Multi-Scale Problems in Stochastic Processes
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Major Goals: The main goal of the project is to solve several multi-scale asymptotic problems for stochastic differential equations. The particular objectives include:

- Describing the asymptotic behavior of a population for branching processes and branching diffusions when time goes to infinity and the branching mechanism is time-dependent. Developing an understanding of the critical and near-critical behavior for such processes. For branching diffusions and related processes, describing the growth of the region occupied by the particles and the phenomenon of intermittency, i.e., the appearance of clusters of particles. (Branching processes are widely used in the study of the evolution of various populations such as bacteria, cancer cells, sub-atomic particles, etc., where each member of the population may die (be annihilated) or produce offspring independently of the rest. Among the most important applications, branching processes are used in physics to understand nuclear chain reactions.)

- Describing the limiting behavior of Markov chains (and randomly perturbed dynamical systems that can be eventually reduced to such Markov chains) when time goes to infinity and transition rates go to zero, simultaneously. The main application of such Markov chains is in the study of metastability in complex nonequilibrium systems (exhibited in climate change, genetic mutations, molecular dynamics, etc.).

- Analyzing transport by randomly perturbed deterministic flows (e.g., 2- and 3-d cellular flows). Obtaining the asymptotics of the speed of front propagation in the G-equation (modeling combustion reactions) with periodic coefficients and a large underlying velocity field.

Accomplishments: Excellent progress has been made on all the aspects of the project, with all the major goals accomplished. Moreover, several new ideas emerged while working on the outlined problems that led to progress beyond the scope of the original project. Particular results include:

- I provided a criterion for almost-sure extinction for multi-type branching processes with time-dependent branching rates. I also provided a criterion for the total number of particles (conditioned on survival and divided by the expectation of the resulting random variable) to approach an exponential random variable as time goes to infinity. These two results allow one to classify multi-type branching processes with time-dependent branching rates (under mild non-degeneracy assumptions) into three categories: sub-critical, critical, and super-critical, thus extending earlier classical results available for time-independent branching processes. The paper (joint with D. Dolgopyat, M. Perlman, and my student P. Hebbar) was published in the Journal of Applied Probability.

- In a series of joint papers with M. Freidlin and A. Wentzell, I studied metastable behavior for Markov chains with small transition rates. In particular, we proved a generalization of the ergodic theorem to parameter-dependent Markov chains. We also studied metastability for several physical systems that can be formally reduced to parameter-dependent Markov chains. As a particularly interesting result, we introduced a new class of elliptic operators with non-local boundary conditions. We showed that processes with such generators serve as limits for several classes of parameter-dependent diffusion processes. The papers were published in some of the leading journals (Probability Theory and Related Fields and Journal of Statistical Physics).

- In joint work with M. Hairer, G. Iyer, A. Novikov, and Z. Pajor-Gyulai, we described the asymptotic behavior of particles in cellular flows at those time scales when the particles stay close to the cell boundaries. This work relied on our new results on averaging and is a continuation of our earlier work on the transition between averaging and homogenization regimes in cellular flows. This work was completed soon after the proposal was submitted, even before the term of the proposal started.

- I investigated the behavior of branching diffusion processes in periodic media. For a super-critical branching process, we distinguished two types of behavior for the normalized number of particles in a bounded domain, depending on the distance of the domain from the region where the bulk of the particles is located. At distances that grow linearly in time, we observe intermittency (i.e., the k-th moment dominates the k-th power of the first moment for some k), while, at distances that grow sub-linearly in time, we show that all the moments converge. The analysis relies on the study of the asymptotic behavior of solutions to parabolic partial differential equations with space-periodic diffusion matrix, drift, and potential. This paper with these results, joint with J. Nolen and my student P. Hebbar, was published in the Electronic Journal of Probability.

- In joint work with M. Freidlin, we proved the averaging principle for deterministic and stochastic systems in the presence of multiple invariant measures for the fast process. The limiting motion for the slow component, in this case, is (and needs to be) considered on a graph or an open book (if $d > 1$), i.e., a more sophisticated space than the case of the non-degenerate fast component. This result provides a very interesting generalization of the classical averaging principle. The paper "Averaging in the case of multiple invariant measures for the fast system" was accepted to the Electronic Journal of Probability.

- I studied front propagation in reaction-diffusion equations (that model combustion) in stratified media. We consider asymptotic problems concerning the motion of interface separating the regions of large and small values of the solution in the media consisting of domains with different characteristics (composites). We showed that, under certain conditions, the motion can be described by the Huygens principle in the appropriate Finsler (e.g., Riemannian) metric. In general, the motion of the interface has, in a sense, non-local nature. In particular, the interface may move by jumps. The results, joint with M. Freidlin, were published in the Journal of Statistical Physics.

- One new direction of research that is closely related to the topics of the proposal and that emerged as a result of my work on branching diffusions concerns the limiting distribution of polymers in a mean-field model. In a joint paper with S. Molchanov and B. Vainberg, we consider a mean-field model of a polymer with a spherically-symmetric finitely supported potential. We describe how the typical size of the polymer depends on two parameters: the temperature, which approaches the critical value, and the length of the polymer chain, which goes to infinity. The spectral techniques used in the paper are close to those we employed in the study of branching diffusions. The paper was accepted to the journal Applicable Analysis.

- Under my supervision, my graduate student, P. Hebbar, jointly with another student, K. Fernando, completed two papers on the asymptotic expansions for distributions of sums of random variables in the domain of large deviations. One of the papers was accepted in the journal Asymptotic Analysis, the other was accepted in the journal Stochastics and Dynamics.

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as of 08-Apr-2021

Training Opportunities: Two graduate students who worked under my supervision were supported by the grant. The first student (P. Hebbar) was involved in several parts of the project. She made crucial contributions to our results on multi-type branching processes with time-dependent branching rates and on branching diffusions. She also obtained excellent results on large deviations for sums of weakly-dependent random variables. P. Hebbar defended her Ph.D. in 2019 and accepted the Phillip Griffiths Research Assistant Professorship at the Department of Mathematics of Duke University. The second student, S. Yan, is currently working on problems related to large deviations and metastability. He is a third-year student at UMD. S. Yan is making excellent progress and is about to submit his first research paper for publication.

I am also advising another student, I. Imtiyas, who works on metastability for Markov renewal processes and is making good progress. Two more graduate students (M. Perlman (Stanford) and K. Fernando (UMD)) collaborated with me on problems related to the project.

I also am a co-organizer of the probability seminar and of the colloquium at UMD that are attended by several graduate students and postdocs.

Results Dissemination: Several papers based on the results that I obtained were published (or accepted) in leading
mathematics/physics journals (Journal of Applied Probability, Journal of Statistical Physics, Probability Theory and Related Fields, Electronic Journal of Probability).

I gave a number of talks at conferences and seminars on the results obtained while working on the project (Stochastics meets Statistics Workshop (Moscow, Russia); Conference in Honor of Alexander Wentzel (Tulane University); Courant-Columbia Probability Seminar; Drexel University Colloquium; UNC Charlotte Colloquium; 2018 AIMS Conference (Taipei, Taiwan); Stochastic Analysis International Workshop (Moscow, Russia); Statistical Mechanics Conference (Rutgers University); Probability Seminar (Carnegie-Mellon University); Conference on Perturbation Techniques in Stochastic Analysis and Its Applications (CIRM, France); AMS meeting (Madison); AMS Special Session (Chattanooga)).

Honors and Awards: I was awarded the Simons Fellowship in Mathematics (for the 2020-2021 academic year) by the Simons Foundation.

I was the recipient of the 2019 University of Maryland Summer Research and Scholarship Award.

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Participant Type: PD/PI
Participant: Leonid Koralov
Person Months Worked: 12.00 **Funding Support:**
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Article Title: Multi-type branching processes with time-dependent branching rates

Authors: Dmitry Dolgopyat, Pratima Hebbar, Leonid Koralov, Mark Perlman

Keywords: Multi-type branching, extinction probability, exponential limit law, nonnegative matrix product

Abstract: Under mild non-degeneracy assumptions on branching rates in each generation, we provide a criterion for almost-sure extinction of a multi-type branching process with time-dependent branching rates. We also provide a criterion for the total number of particles (conditioned on survival and divided by the expectation of the resulting random variable) to approach an exponential random variable as time goes to infinity.

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Accomplishments Report (Leonid Koralov)

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