



NAVAL POSTGRADUATE SCHOOL

MONTEREY, CALIFORNIA

**PREDICTIVE MODELING FOR NAVY READINESS BASED ON
RESOURCE INVESTMENT IN SUPPLY SUPPORT AND
MAINTENANCE**

by

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November 2022

Distribution Statement A: Approved for public release; distribution is unlimited

Prepared for: N4 – Fleet Readiness and Logistics
This research is supported by funding from the Naval Postgraduate School, Naval
Research Program (PE 0605853N/2098)
NRP Project ID: NPS-22-N080-A

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REPORT DOCUMENTATION PAGE

PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ORGANIZATION.

1. REPORT DATE 30-11-2022	2. REPORT TYPE Technical Report	3. DATES COVERED	
		START DATE Jan 2022	END DATE Dec 2022
4. TITLE AND SUBTITLE Predictive Modeling for Navy Readiness Based on Resource Investment in Supply Support and Maintenance			
5a. CONTRACT NUMBER	5b. GRANT NUMBER	5c. PROGRAM ELEMENT NUMBER 0605853N/2098	
5d. PROJECT NUMBER NPS-22-N080-A; W2223	5e. TASK NUMBER	5f. WORK UNIT NUMBER	
6. AUTHOR(S) Kamel, Magdi N Doerr, Kenneth			
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School 1 University Circle Monterey, CA 93943			8. PERFORMING ORGANIZATION REPORT NUMBER NPS-IS-22-008
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Naval Postgraduate School, Naval Research Program; N4 – Fleet Readiness and Logistics		10. SPONSOR/MONITOR'S ACRONYM(S) NRP; N4	11. SPONSOR/MONITOR'S REPORT NUMBER(S) NPS-IS-22-008; NPS-22-N080-A
12. DISTRIBUTION/AVAILABILITY STATEMENT Distribution Statement A: Approved for public release; distribution is unlimited.			
13. SUPPLEMENTARY NOTES			
14. ABSTRACT The Navy invests substantial resources to fleet maintenance in terms of part supply, corrective maintenance, maintenance availabilities, and overhauls. In order to measure and prioritize weapon systems investment decisions, an endurance supply metric E_s is being developed to ensure these systems are ready for tasking across the full spectrum of operations. This research project will attempt to develop models to determine self-sustaining stock levels of critical parts, for key ship systems, in order to operate for at least T_1 days without resupply, with a risk no greater than β_1 that part shortage will cause system failure. These models are developed for both a single deployed ship and multiple deployed ships.			
15. SUBJECT TERMS Readiness, Logistics, Endurance Supply, Monte Carlo Simulation, Reliability Block Diagrams			
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT
a. REPORT U	b. ABSTRACT U	c. THIS PAGE U	UU
			18. NUMBER OF PAGES 52
19a. NAME OF RESPONSIBLE PERSON Kamel, Magdi, N			19b. PHONE NUMBER (Include area code) 831-656-2494

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The report entitled “Predictive Modeling for Navy Readiness Based on Resource Investment in Supply Support and Maintenance” was prepared for N4 – Fleet Readiness and Logistics and funded by the Naval Postgraduate School, Naval Research Program (PE 0605853N/2098).

Distribution Statement A: Approved for public release; distribution is unlimited.

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ABSTRACT

The Navy invests substantial resources to fleet maintenance in terms of part supply, corrective maintenance, maintenance availabilities, and overhauls. In order to measure and prioritize weapon systems investment decisions, an endurance supply metric E_s is being developed to ensure these systems are ready for tasking across the full spectrum of operations. This research project will attempt to develop models to determine self-sustaining stock levels of critical parts, for key ship systems, in order to operate for at least T_1 days without resupply, with a risk no greater than β_1 that part shortage will cause system failure. These models are developed for both a single deployed ship and multiple deployed ships.

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ACKNOWLEDGEMENTS

The authors acknowledge the enormous assistance received from Raymond Ward, Chief Readiness Engineer, NSWC Corona, in obtaining, understanding, and modeling the data on the SPY-1D radar system. We could not have completed this project in a timely manner (if at all) without that assistance. CAPT Oswald, our sponsor at the beginning of the project, fully engaged with our effort and connected us to Mr. Ward and several other sources of information, and also included us in several of the planning meetings for the overall effort, of which this research was just a small part. In particular, we acknowledge the connections he provided to the work of Institute of Defense Analysis (IDA) and Dr. Eli Alster. We also appreciate the team at the Center for Naval Analysis, and especially Dr. Leon Cui who helped validate our approaches to simulating the endurance of the SPY-1D. They were very generous in sharing their data and code with us, and providing feedback to us on our ideas.

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I. INTRODUCTION

A. BACKGROUND

The CNO's Navigation Plan provides his strategic vision and our Navy's four top priorities. Readiness, the ability to deliver a more-ready fleet, is an important one of these four priorities. Nearly 70% of the Fleet in 2030 is already in service today. Sustaining our ships and aircraft is absolutely critical to meeting future demands. Towards this end, the NIF R3 NAVSUP effort was initiated to develop a predictable supply model that ensures supply issues are not a primary cause of readiness shortfalls. A new metric called E_s or Endurance Supply is being developed to answer the question of how long the onboard inventory should be, to keep a system up and running. Traditional metrics such operational availability do not answer this question. E_s is therefore a measure of readiness in contested environments, without resupply (Alster, Ashwell, & Remley, 2022).

B. RESEARCH/ STUDY AND ANALYSIS OBJECTIVES

The objective of this research is to use the newly developed metric E_s (Endurance Supply) to determine the optimal level of spare parts onboard ships when they are employed beyond the reach of a regular supply chain. Specifically, the research will attempt to answer the following research question: Can an optimal sparing plan be developed to achieve an E_s target for a complex weapon system on a single ship, and on the multiple ships of a battlegroup?

C. RESEARCH METHODOLOGY

The methodology used to guide this research is based on the Cross-Industry Standard Process for Data Mining (CRISP DM), a data science methodology for data-driven projects. Figure 1 depicts the life cycle of the methodology (Vorhies, 2016).

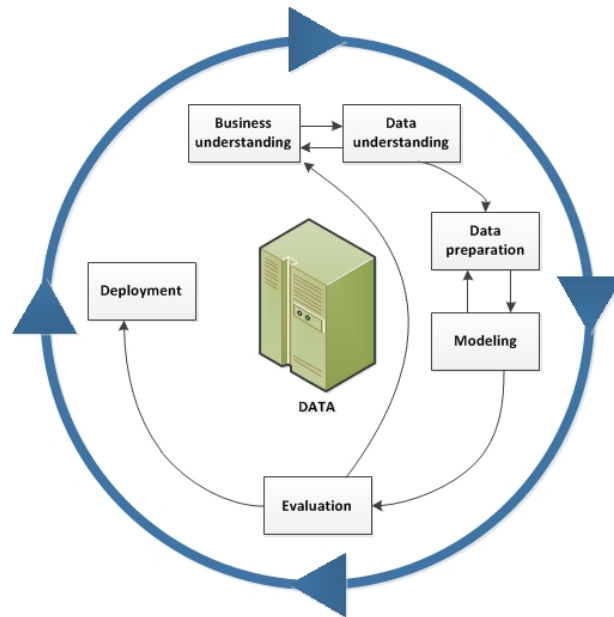


Figure 1. The data science life cycle

The life cycle model consists of six phases: Business understanding, Data understanding, Data preparation, Modeling, Evaluation, and Deployment with arrows indicating the most important and frequent dependencies between phases. The execution of the phases is not strictly sequential; most projects move back and forth between phases as necessary.

1. Business understanding

This is perhaps the most important phase of a data science project. Business understanding includes determining business objectives, assessing the situation, determining data analysis goals to support business objectives, and producing a project plan.

2. Data understanding

This phase addresses the need to understand available data resources and the characteristics of those resources. It includes collecting initial data, describing data, exploring data, and verifying data quality.

3. Data preparation

After cataloging data resources, data need to be prepared for modeling. Preparations include selecting, cleaning, constructing, integrating, and formatting data. These tasks are likely performed multiple times, and not in any prescribed order. These tasks can be very time consuming but are critical for the success of the project.

4. Modeling

This is the most interesting part of the project, where sophisticated analysis methods are used to extract information from the data. This phase involves selecting modeling techniques, generating test designs, and building and assessing models.

5. Evaluation

Before writing final reports and deploying the model, it is important to thoroughly evaluate the model, and review the steps executed to construct the model, to be certain it properly achieves the business objectives. A key aim is to determine if there is some important business issue that has not been sufficiently considered. At the end of this phase, a decision will be made on the use of the project results.

6. Deployment

The deployment phase can be as simple as generating a report or as complex as implementing a repeatable data-mining process. However, the focus of this effort is on the data preparation, data understanding, and modeling phases of the methodology.

For the modeling phase we use both Monte Carlo Simulation (MCS) and Reliability Block Diagrams (RBD) to determine the endurance of the current deployed system.

MCS is used to model the number of days to failure of the major subsystems containing critical parts, and combine the time to failure of these subsystem into an MCS model of time to failure for the entire SPY-1D System.

An RBD displays the reliability relationships among a system's components (Meeker, Escobar, & Pascual, 2021, Tobias & Trindade, 2011). If reliability distributions are assigned to the individual components, the reliability behavior of the entire system can be computed. A Reliability Block Diagram therefore illustrates how component reliability contributes to the reliability of the whole system.

We further use MCS to to determine self-sustaining stock levels of critical parts, for key ship systems, to meet Endurance supply goals.

II. DATA UNDERSTANDING AND PREPARATION

A. DATA UNDERSTANDING

We received four sets of data from the sponsor: 1) National Item Identification Number (NIIN) Failure Metrics, 2) NIIN Impact to System, 3) Reliability Block Data (RBD), and 4) Part Reference Data. We describe the contents of each data set in the following sections.

1. NIIN Failure Metrics

This data set provide parts (NIINs) failure information in different systems during a given period of time when part failure caused a non-critical or critical failure of the reliability block. This information includes System Type, Start Date, End Date, Part MTBF, Part Energized Time, Part Total Failure, Critical Part Failures, Non-Critical Part Failures, and Parts per System.

The data set consisted of 33,903 records with 16 variables each.

2. NIIN Impact to System

This data set describes the impact on the system as a result of a part failure. It provides information on the impact to the reliability block if the part fails and the impact to the system if the block fails.

The data set consisted of 22,934 records with 8 variables each.

3. Reliability Block Data

This data set provides the reliability relationships among a system's components.

The data set consisted of 2,315 records with 14 variables each.

4. Part Reference Data

This data sets provide detailed information on each part. This includes the system and block where the part is installed as well as nomenclature and the number of parts per block.

The data set consisted of 44,796 records with 7 variables each

B. DATA PREPARATION

The following data preparation tasks were undertaken on the initial data sets:

1. Extract NIIN failure metrics for non-overlapping periods for critical part failures for the 421 system variant (D) of the Spy-1 Radar.
2. Compute the average time of critical failures from the data set derived in step one
3. Extract NIIN impact to system data for critical and major mission essential for the 421 Spy-1 system variants and group by NIIN.
4. Perform a 3-way join between NIIN Failure Metrics – Average Time to Critical Failure, NIIN Impact to System – Critical and Major Mission Essential, and RBD. This allowed us to identify all parts that have caused system failures in the historical data, along with (via the RBD) the way those parts are used in the system and the number of redundant parts of that type built into the system.
5. Identify a set of parts that historically have had failure rates likely to cause system failure within the time frame of a post-supply-chain deployment (90 days). Since there are over 300,000 parts on a SPY-1D radar even a small probability a part might fail warrants concern. After discussion with sponsors and POC familiar with reliability engineering on the SPY-1 system, we arbitrarily decided to examine any NIIN whose Mean Days Between Critical Failure (MDBCF) was less than 9,000 days. For the SPY-1D we identified 51 such NIIN, in 4 mission-critical subsystems (See Table 1).
6. Identify via the RBD how each of these parts is used in the design, to facilitate modeling of time to failure. For example, some redundant parts operate continuously, while others are in “stand by” operation until needed.
7. Based upon the amount of built-in redundancy for a part, eliminate from consideration all parts whose built-in reliability creates a MTBCF of more than 9,000 days. Once parts with built-in redundancy were modeled as they were deployed in the SPY-1D system, and with the additional assumption of allowable cannibalization, the number of NIIN that were

predicted to cause a system failure in less than 9,000 days, on average, dropped to 51. These 51 parts were incorporated into our model.

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III. MODELING

In this section it becomes important to distinguish between NIINs (a kind of part, several of which may be installed in the SPY-1D and several of which may be recommended as spares) and the individual copies of that NIIN installed in the system, or recommended as spares. When we refer to the type of part we will use the acronym NIIN. When we say ‘part’ we refer to a single part that is of that type of NIIN.

Once we had selected the critical NIINs to incorporate in our model, our modelling effort proceeded in six stages:

- A. Determine the MDBCFC for each critical NIIN, based on how the NIIN was deployed in the SPY-1D system. These MDBCFC estimates were validated with the Sponsor POC. Using the mean, and assuming the distribution of time to critical failure for each part follows an Exponential distribution, we built a time-to-fail distribution for each NIIN, as deployed according to the RBD. There are four different types of time-to-failure models, as described more fully in the next subsection.
- B. Combine the NIIN-level random variable models into a Monte-Carlo Simulation (MCS) model of the days to failure of the four major subsystems containing those parts, and combine the time to failure of major subsystem models into an MCS model of time to failure for the SPY-1D.

Our model assumes all parts are freely cannibalized as needed. There are several NIINs which appear in multiple reliability blocks within a single major subsystem, and one NIIN which appears in two different major subsystem. Our model is at the NIIN level, meaning that a spare part, or a redundant part can be used where needed to prevent system failure, until the number of working parts falls below the minimum required of that part (across all reliability blocks and subsystems).

- C. Build an RBD model using SAS JMP® software, based on the information in the RBD file, of the (NIINs we examined in the) SPY-1D. Note that the SAS JMP® tool for RBD is meant to support design-for-reliability efforts of engineers, not sparing models. It uses numeric integration to determine time to failure, not Monte Carlo Simulation. Hence, we used this tool to independently calibrate (independent of e.g., sampling error possible in MCS), and further validate our MCS model of the status quo system. But we could not use it to accomplish or validate our sparing plan, as we will explain in subsection C.
- D. Model potential spares for every critical NIIN, and conduct a stochastic search for the optimal number of spares needed to achieve the sponsor’s desired target of 90

days at least 85% of the time. This problem is combinatorial in nature, and we developed upper and lower bounds for the search, as described in the next subsection.

- E. Calibrate the system mean time to failure against the known data, and incorporate an omnibus “other sources of failure” variable so that the mean time to failure of our model matched the mean time to failure of the system. Estimate the percentage of time the parts we modeled cause the system to fail, and the time “other sources of failure” might be causing the system to fail.
- F. Incorporating an estimate of “other sources of failure” to increase the spares of the known critical parts to achieve 90 days of endurance at least 85% of the time.

Each of these stages is described below, and key results are provided.

A. NIIN LEVEL MODELS OF DAYS-TO-FAILURE.

We have already described how we extracted and prepared the part-level data. That effort left us with 155 unique NIINs that had caused critical failures. As described in our methods section, we further narrowed this list focusing on only those parts that failed, on average in less than 9,000 days.

However, critical failures at the part level do not necessarily translate to System failures, because many parts have a great deal of built-in redundancy (Kostanovskii, 2014).

Examining the RBD we found that NIIN fell into four categories:

(1) A stand-alone (serial) NIIN without redundancy that would cause failure as soon as the part failed. These could be modelled as a simple exponential distribution with $MTBCF = 1/\lambda$.

(2) NIIN with N parallel parts which were all required. The failure of any of these N parts would bring the system down as soon as the first copy failed. This we modelled as the minimum time to failure of N independent identically distributed (iid) exponential distributions. The minimum of N iid exponential distributions is known to also follow an exponential distribution as is commonly known, the minimum of N identical exponentials with failure rate λ is itself an exponential with parameter N/λ .

(3) NIIN with N parallel parts, all operating at the same time, only K (<N) of which were required. This we modelled as the $(N-K+1)^{st}$ order statistic of N identical exponential distributions, as the distribution of the Order Statistics of identical

exponentials is itself an exponential (Rényi, 1953) with its parameter a finite sum of decreasing fractions of the underlying part-failure rate. We calculated this parameter for these parts and modelled them as exponentials. Most NIIN fell into this category.

(4) NIIN with N parallel parts, only K of which are required, and only K of which operated at any time, while the remaining $N-K$ parts were idle until one of the operating parts failed. There are four NIIN of this type but it is important because, as we will show, this is how *all* NIIN must be modeled once spare parts are under consideration. So, we will consider how to model this fourth type in some detail by looking at one particular NIIN.

NIIN 12584120 is installed in one block that has 3 redundant parts operating in parallel, 2 of which are required, and in another block (in another subsystem) with 8 parts 6 of which are operating and required, along with 2 redundant stand-by parts which will not begin operating until needed (this type of redundancy is indicated with the notation SEC in the RBD). So, this NIIN will critically fail when $N-K+1=11-8+1 = 4$ parts have failed. But the time to failure of the continuously operating $K = 8$ parts fail according to an order statistic (1st failure, 2nd failure, ..., $N-K+1 = 4^{\text{th}}$ failure) while the remaining parts fail in sequence: that is, the stand-by parts will fail according to an Erlang distribution (i.e., the 2nd part installed when the first part fails, itself fails in a time predicted by the Erlang-2 distribution). Hence, the time to the first part failure can be described by the 1st order statistic of $K=8$ iid exponentials, but the time to the second failure is the maximum of either (1) the 2nd order statistic of 8 iid exponentials, or (2) an exponential which starts its life at the time of the first failure (that is, after the failure time predicted by the first order statistic). Since this NIIN has only two true “spares” and fails at the 4th part-failure, the time to failure of this NIIN is the minimum of

- i. 4th Order statistic (of the 8 continuously operating parts)
- ii. 3rd Order statistic + Erlang-1
- iii. 2nd Order static + Erlang-2

Note that this approach is itself an approximation. This approach is generalized below, to model the time to failure of a NIIN once spares are included. Work has been done on the convolution of Order Statistics of non-identical exponentials, including sums of the Order Statistics, as we have in category four. Asymptotically they appear to

converge to one of the three distributions (the Gumble) predicted by the Extreme Value Theorem (David & Nagaraja, 2003, Nagaraja, 2006). Our simulation table for the two parts was simpler and sufficient to provide a way to numerically estimate the time to failure for these parts. We mention this other research for two reasons. First, if run time of the model were important, it might well be worth implementing a numerical solution rather than the simulation. But second, clearly, the failure time of all sources of failure of the SPY-1D are not distributed exponentially. This of course means that the failure time of the any subset of parts (or subsystems) should not be assumed to be exponential, if it becomes important to model “other sources of failure” beyond the parts we are modeling.

For each of the 51 critical NIINs we identified, Table 1 shows the single-part MTBCF, the total number (N) of parts (across reliability blocks) in the system, the minimum number (K) of parts required (across blocks) for the system to operate, the mean days to the critical (N-K)th failure, and the 15th % of the failure time distribution (the NIIN will last longer than this, 85% of the time).

Table 1. Status quo simulation NIIN models. Mean, std. deviation and 15th % of days to failure for each NIIN.

NIIN	Ttl Parts (N)	Min Req (K)	MDBCF	Std Dev	15%	NIIN	Ttl Parts (N)	Min Req (K)	MDBCF	Std Dev	15%
11365633	2	2	1,374	1,373	223	14531943	2	1	4,250	3,165	1,389
12559540	2	2	654	654	106	16258448	2	1	10,474	7,809	3,416
12756230	2	2	2,614	2,611	424	14872276	4	2	2,667	1,604	1,150
16258874	4	2	1,779	1,071	766	12584120	11	8	2,556	1,438	1,171
9858983	3	2	400	289	134	13926982	99	56	294	45	248
12584119	3	2	1,865	1,347	627	13343887	2	1	9,331	6,964	3,039
12604142	3	2	1,992	1,437	669	13343888	2	1	3,503	2,609	1,147
12615670	3	2	4,054	2,920	1,363	12646948	2	1	4,814	3,593	1,570
12615754	3	2	3,807	2,744	1,281	12584185	2	1	9,122	6,795	2,979
12897908	3	2	2,236	1,612	752	12646946	2	1	9,327	6,965	3,044
13221337	3	2	783	565	264	12584122	1	1	8,868	8,870	1,447
13726234	3	2	570	410	192	12813116	1	1	8,878	8,877	1,442
13892852	3	2	4,904	3,530	1,654	12583671	1	1	1,634	1,632	266
14256506	3	2	4,982	3,587	1,679	12583546	2	1	5,673	4,228	1,853
14596138	3	2	2,235	1,612	752	12583827	2	1	4,705	3,504	1,539
16258869	3	2	3,269	2,356	1,100	12604012	2	1	5,797	4,318	1,897
12583686	4	3	2,806	2,028	943	12583634	2	1	6,947	5,182	2,266
12740647	4	3	3,901	2,812	1,312	14134746	2	2	4,061	4,067	660
14657503	4	3	3,339	2,403	1,124	12584191	4	2	2,639	1,588	1,137
12603988	1	1	5,762	5,771	935	12615764	4	2	5,273	3,161	2,277
12603990	1	1	5,715	5,715	926	13343911	4	2	5,039	3,024	2,176
12604137	1	1	5,761	5,761	933	13152590	2	1	15,305	10,828	5,236
12604143	1	1	5,774	5,766	940	14540088	2	1	4,365	3,087	1,492
12604146	1	1	6,899	6,887	1,123	14540089	3	1	8,950	5,178	3,970
12667170	1	1	8,651	8,663	1,399	12559851	3	2	3,177	2,290	1,070
13202872	1	1	4,597	4,592	746						

B. SUBSYSTEM AND SYSTEM LEVEL MODEL

Any required NIIN might cause a reliability block to fail, and the failure of any required block might cause the failure of a critical subsystem and hence, the failure of the SPY-1D radar. It is important to note that by modeling only 51 NIIN (the 51 NIIN we were able to identify as having a MTBCF of less than 9,000 days) we are necessarily *overestimating* the endurance of the SPY-1D radar. We will return to this point when we calibrate our model against the known time to failure of the SPY-1D, using an estimate of all other sources of failure. But at this point, note that we are trying to model only the time to system failure of the SPY-1D, *given that its failure was caused by one of these 51 NIIN*.

We described in the previous subsection how we built 51 models of the time to failure for each of the NIIN. Since many of the NIIN are installed in multiple reliability blocks, and we are assuming parts will be cannibalized until there are not a sufficient number of parts to keep all of the mission-critical reliability blocks operating, it is not possible for use to predict which of these reliability blocks will critically fail first.

Since every reliability block is used in only one major subsystem, however, we can predict when a subsystem will fail, based on the NIIN failure times. The failure time

of a major subsystem is simply the time that the first critical NIIN in that subsystem fails. Hence our subsystem model is simply the minimum of those random part failure times.

The failure time of the Spy-1D radar is simply the time that the first critical subsystem fails. Table 2 shows the mean days-to-failure and 15% of the days-to-failure distribution for each of the four major subsystems, and the SPY-1D radar. The percent of time each of the subsystems causes failure is also provided.

While we have attempted to make sure we are capturing the NIIN most likely to cause failure, as noted above, there are over 313,116 parts in the SPY-1D, and 7,382 reliability blocks. So again, it is important to interpret Table 2 as a model of a SPY-1D which consists of only its 51 least reliable parts. So, for example, Subsystem 420899 causes 77.5% of the failures of the Spy-1D, *among those failures attributable to these 51 parts.*

Table 2. System and sub-system mean, std. deviation, 15th % of DBCF and % of critical failures attributable to each subsystem

(Sub)System	Nomenclature	MDBCF	Std Dev	15th%	%Crit.Fails
421	Spy-1D	114	82	29	
420786	OE-375 ANTENNA GROUP	1,376	1,377	222	8.3%
420899	OT-146 TRANSMITTER GROUP	137	88	40	77.5%
421491	OL-356 SIGNAL PROCESSOR GROUP	560	430	136	14.0%
421965	AUXILIARY EQUIPMENT	3,518	2,219	1,360	0.2%

C. CALIBRATING THE MCS MODEL WITH RELIABILITY BLOCK DIAGRAMS USING JMP

We developed a Reliability Block Diagram for the Spy-1D System using JMP statistical software (SAS Institute Inc., 2022). Figure 2 is a high-level view of the system diagram. As shown in the diagram, the Spy-1D System we model consists of four main subsystems: Antenna Group, Auxiliary Equipment, Transmitter Group, and Signal Process Group. There are other subsystems in the Spy-1D System, including two other mission critical systems (422082 AN/UYK-43 COMPUTER SET and 4280112 MMSP) but none of those subsystems had parts with CMDBF of less than 9,000 days, so they are not included in our model.

These subsystems are connected in series, but represented using a K out of N components, where $K = N$.

There are two limitations of the SAS JMP® RBD model for our purposes. The first is, because it uses numerical integration, it has a limited set of distributions which can be used to model time-to-failure. If the design objective is to optimize the mean time to critical failure, this is not a significant limitation. But our objective was to model the 15th percentile of the Spy-1D System time to fail, and that percentile might be impacted by the choice of part-level distribution of time to fail. More importantly, we were not designing a system, but instead designing a sparing plan. SAS JMP RBD has no simple feature to search over a number of spares in order to optimize a quantile of the distribution. This is why we chose to conduct our optimization with MCS.

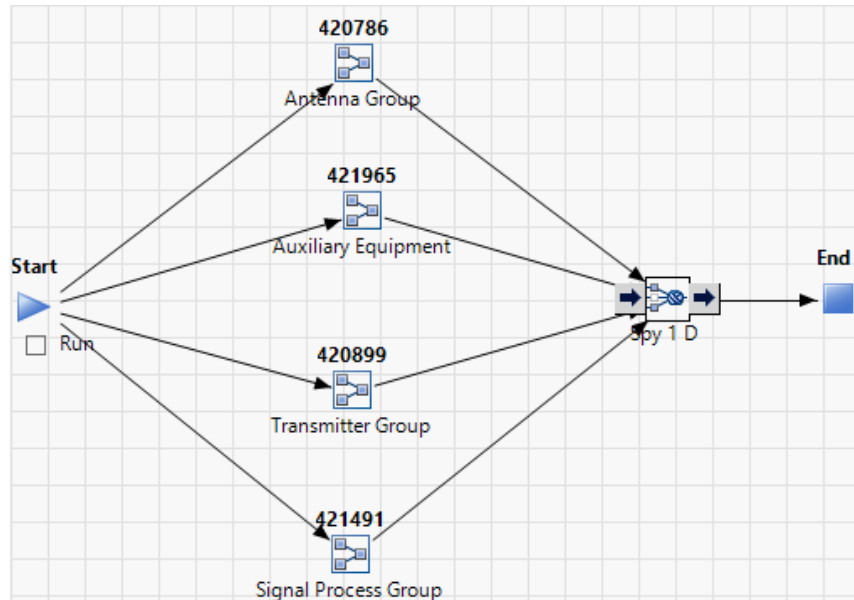


Figure 2. Spy-1D RBD

Figure 3 is the RBD of the Signal Process Group subsystem. Similar to the Spy-1D System RBD, the components of this subsystem are connected in series, but represented using a K out of N components, where $K = N$.

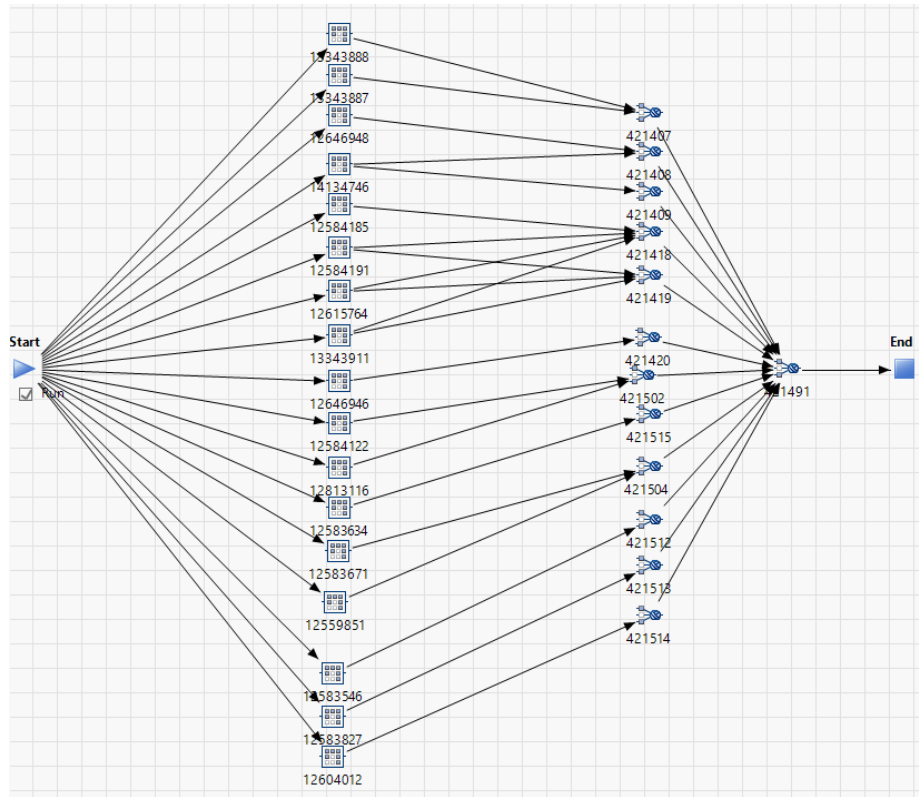


Figure 3. Signal process group subsystem RBD

Figure 4 is the Distribution Profiler of the system while Figure 5 is the Reliability Profiler. The Distribution Profiler displays the probability that the system fails as a function of time while the Reliability profiler shows the probability that the system will operate as a function of time.

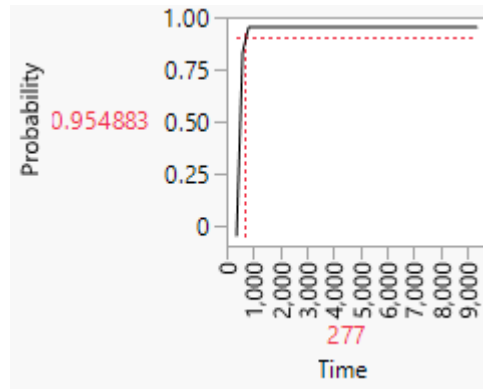


Figure 4. Distribution profiler for Spy-1D system

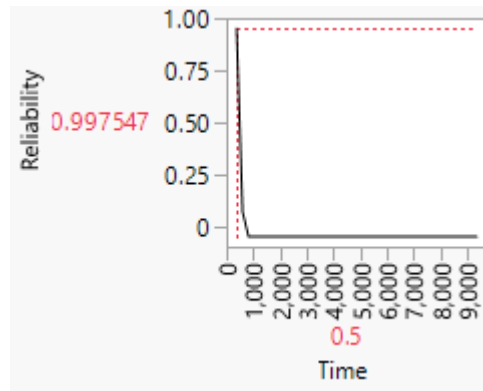


Figure 5. Reliability profiler for Spy-1D system

The Quantile profiler for the system is shown in Figure 6 and displays time as a function of the failure probability. Note that the Quantile function is the inverse of the Distribution function. As shown in the figure, the 0.15 quantile for the system is 28.72 days.

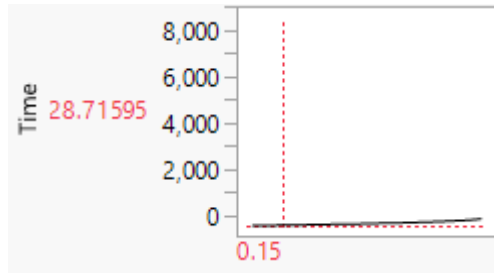


Figure 6. Quantile profiler for Spy-1D system

Figure 7 shows the Component Distribution Functions as an overlay plot of the distribution functions for the components of the Reliability Block Diagram of the system while Figure 8 displays the Component Reliability Functions as an overlay plot of the reliability functions for the components of the Reliability Block Diagram of the system. As shown in the figures, the most critical reliability component of the system is the Transmitter Group, followed by the Signal Process Group, Antenna Group, and Auxiliary Equipment, respectively.

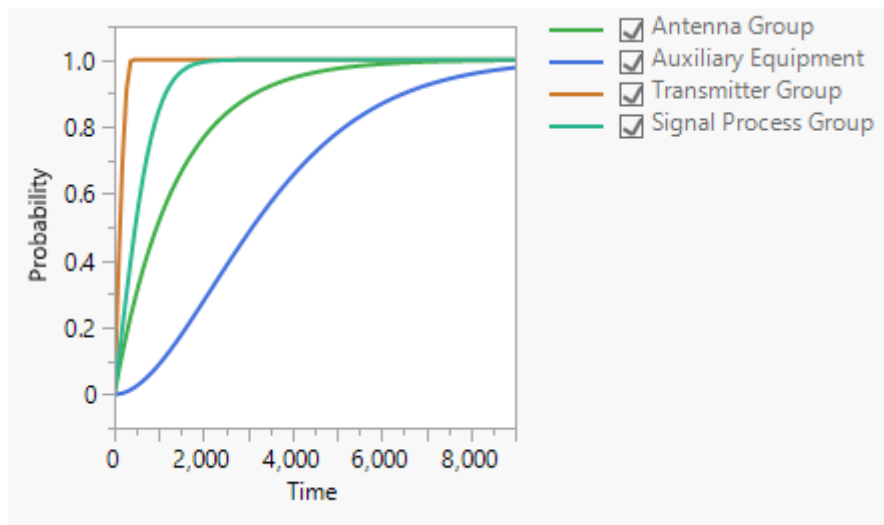


Figure 7. Component distribution functions of Spy-1D system

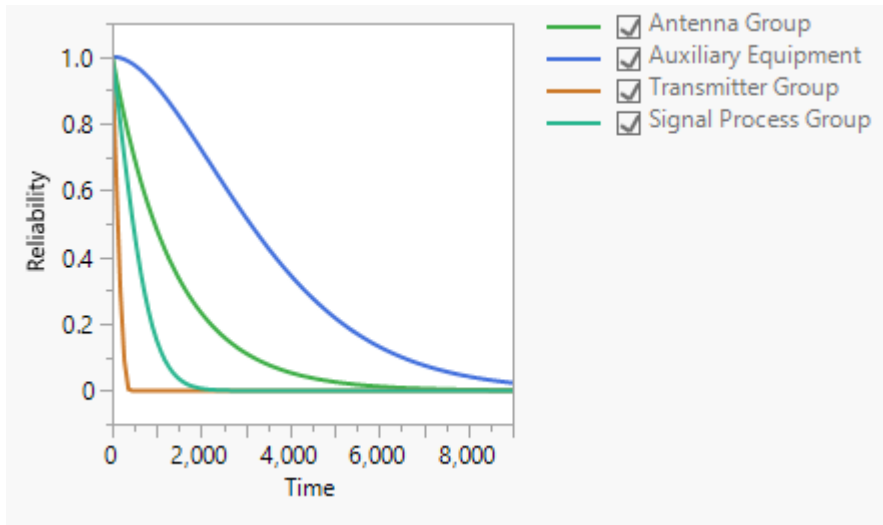


Figure 8. Component reliability functions of Spy-1D system

Figures 9 through 12 compare the results of the MCS with that of the RBD models for the four subsystems we model. As shown in these figures, the results of the RBD models match closely with that of the MCS.

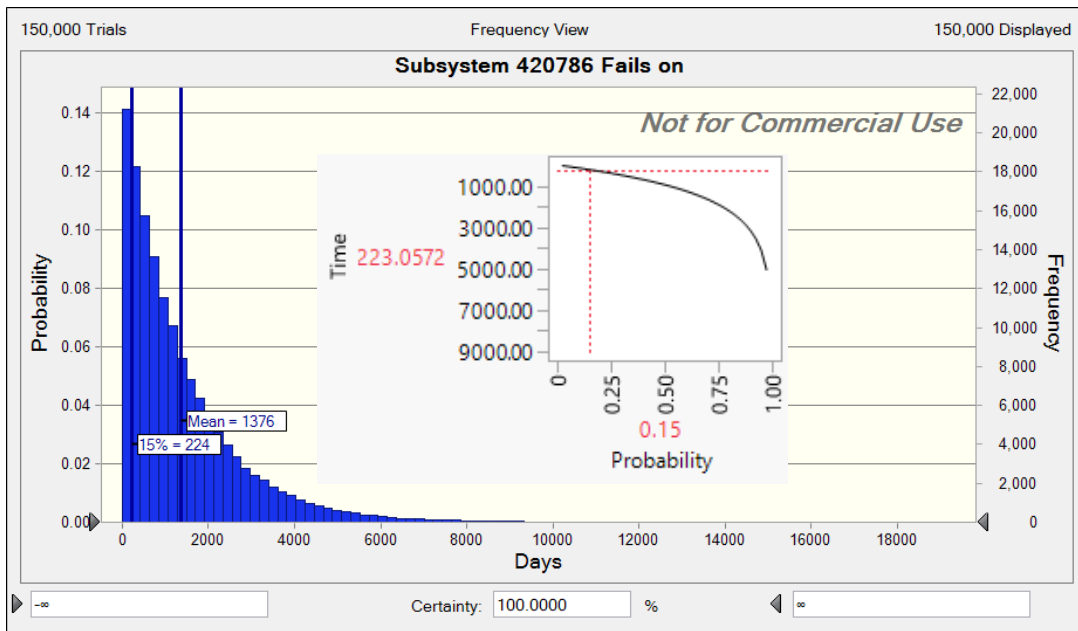


Figure 9. Monte Carlo simulation vs. RBD model for antenna group

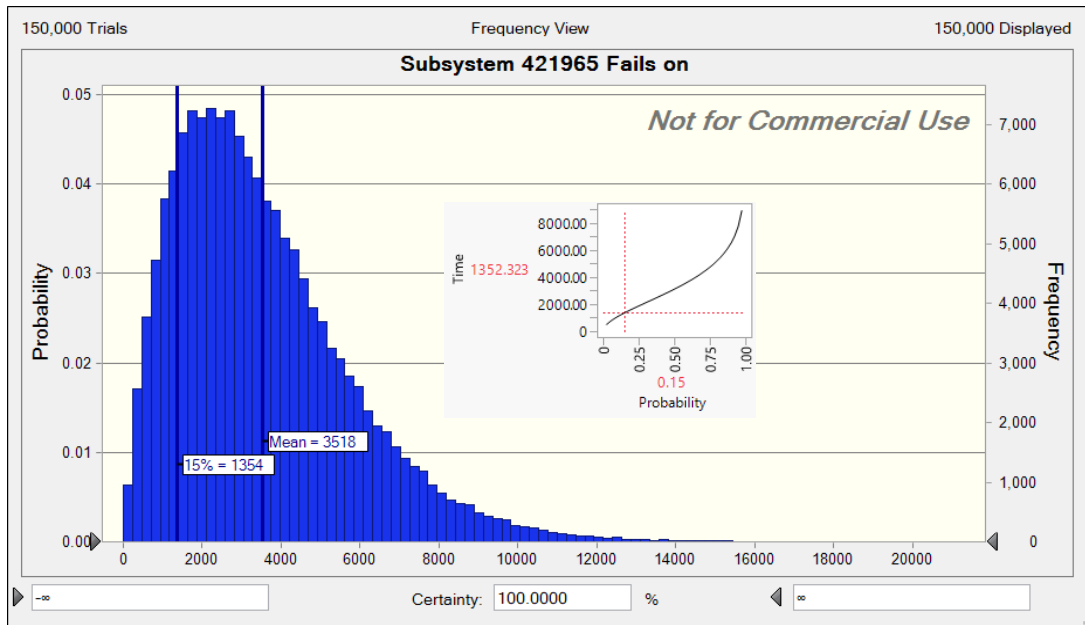


Figure 10. Monte Carlo simulation vs. RBD model for auxiliary group

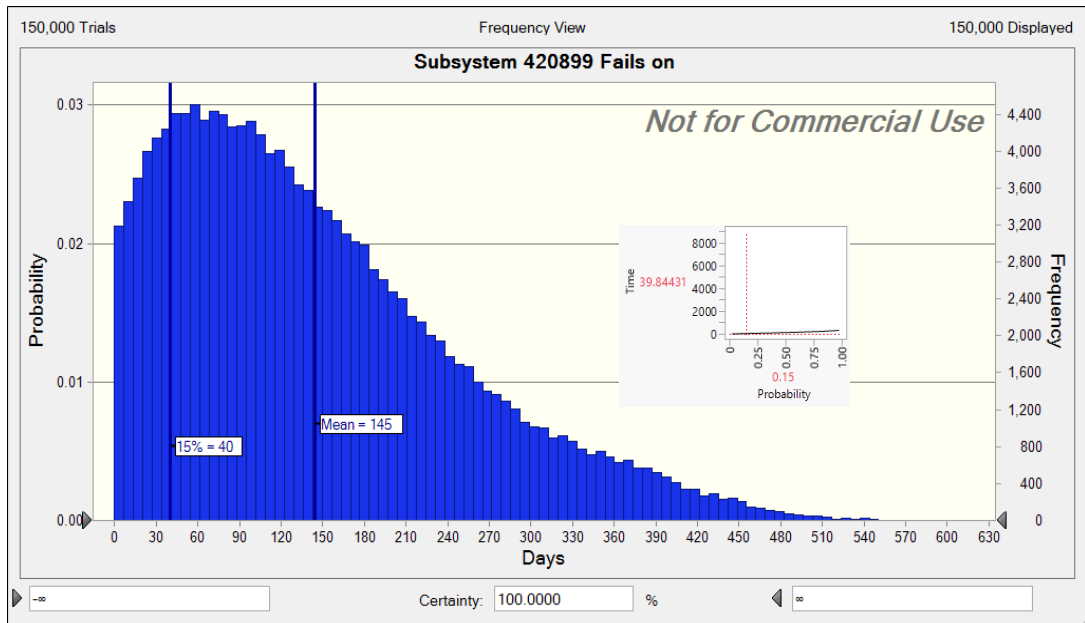


Figure 11. Monte Carlo simulation vs. RBD model for transmitter group

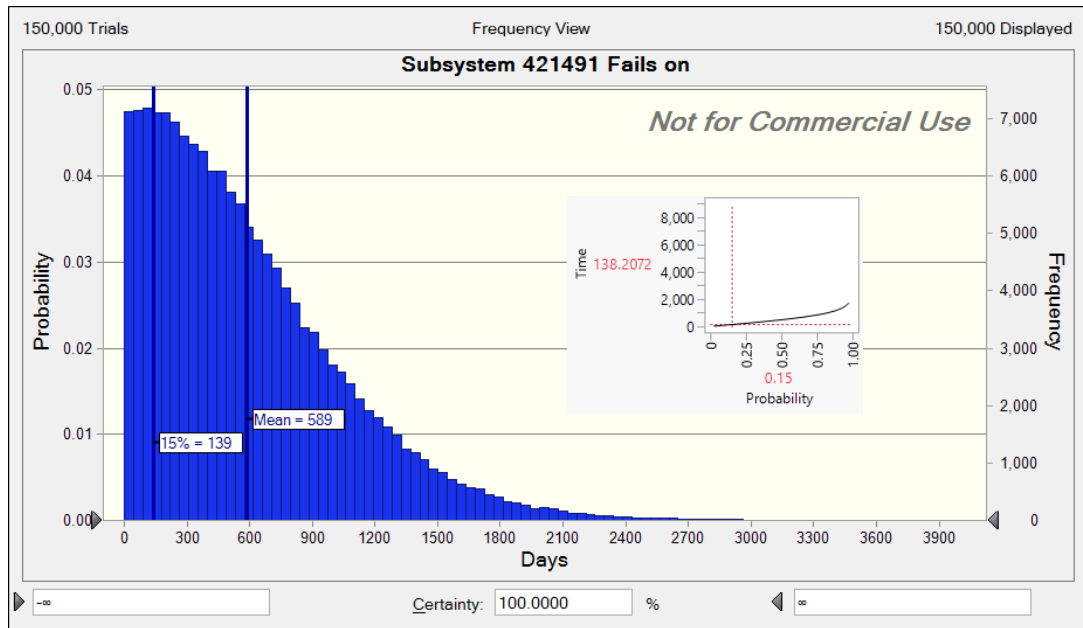


Figure 12. Monte Carlo simulation vs. RBD model for signal process group

D. SPARING FOR 90 DAY ENDURANCE WITH 85% PROBABILITY

Having calibrated the predictions of our model against JMP, we wanted to see if adding additional spare parts could extend the predicted life of the SPY-1D to meet the target of enduring 90 days with at least an 85% probability.

As mentioned above, this was complicated by the fact that for many of the NIIN we were modeling, some of the parts might still be working even though a critical failure had occurred.

For example, NIIN 12583686 is used in reliability block 421006 and has a mean time between critical failures of 3,369 days. There are four of this part in this block, but only 3 are required. So, after the second failure (which we model as the second order statistic of four iid exponentials each with a parameter of $1/3,369$), block 421006 will fail, causing a failure of a critical subsystem. If a single spare is added, the system can continue running, but its failure time now becomes the minimum of:

- (i) The third order statistic
- (ii) The sum of the second order statistic, and an Erlang-1 (or exponential) with parameter 3,369.

And if a second part is added, the failure time becomes the minimum of:

- (i) The fourth order statistic
- (ii) The sum of the third order statistic and an Erlang-1 with parameter 3,369
- (iii) The sum of the second order statistic and an Erlang-2 with parameter 3,369.

In general, if there are K required parts, $N > K$ simultaneously operating parts and $J < K$ spares, the failure time will be modeled as the minimum of the K th order statistic, the $(K-1)^{\text{st}}$ order statistic and an Erlang-1, ..., the $(K-J)$ order statistic and an Erlang- J . If $J \geq K$, then the failure time will simply be extended by an Erlang- $(J-K+1)$.

Programming this into an MCS is relatively simple, but we could see no way to replicate this logic in the JMP reliability module, which is meant to predict the reliability of a system, not build a sparing plan. In the JMP reliability module, reliability blocks must either have redundant parts which are always-operating, or redundant parts which do not operate until needed. There is no simple way to combine the two types of redundant part. It would certainly be possible by coding a script in SAS, but that was beyond the scope of our work. All we could do was to verify that when spare part was added to the "SEC" modules (in which all redundant parts are idle), both JMP and MCS gave the same predictions. We also conducted tests of our model by adding small numbers of spares, and verifying that the results were as predicted.

We used the stochastic search engine in Crystal Ball[®] to determine the number of spares. Our optimization target was to minimize the number of spares, with a constraint that the SPY-1D radar should endure at least 90 days at least 85% of the time (i.e., we constrained the 15th % of the distribution of time to critical failure of the SPY-1D radar to be at least 90 days).

To conduct the search we needed to set upper and lower bounds on the number of spare parts for each NIIN. A lower bound on all NIIN was obtained with the observation that setting the part-service-level spares for every NIIN to 98% with a Poisson-Inverse was insufficient to reach the desired endurance target. The upper bound was set using the Binomial Distribution, based on the observation that the probability a Binomial random variable with failure rate of 15% will have no failures across 51 trials is 0.0025%. So clearly, if all 51 NIIN had at most that failure probability (at 90 days), the system as a whole would fail no more than 15% of the time. We used an Erlang- K inverse to determine the number of spares that would be required, to obtain that failure probability

at 90 days. This is not necessarily an upper bound for those NIIN which have built-in spares that are always operating (because some of the redundant parts for those NIIN will fail in less time, since they are always running). So, to try to ensure our estimate would still be an upper bound for those parts, we simply added 1 more part to the estimate provided by the Erlang-K inverse. The upper bound for two of these parts was tight, but a solution was still obtained. Table 3 contains the upper and lower bound estimates, as well as the solution for the number of spares. A total of 41 spare parts was required. Table 4 contains our prediction of the performance of major subsystems and the SPY-1D radar once those spares are installed.

Table 3. Upper bound, lower bound and recommended endurance spares

NIIN	UB	LB	Spares (J)	NIIN	UB	LB	Spares(J)
11365633	4	0	1	14531943	4	0	1
12559540	4	0	2	16258448	3	0	0
12756230	3	0	1	14872276	4	0	1
16258874	4	0	1	12584120	4	0	2
9858983	5	1	5	13926982	6	1	6
12584119	4	0	1	13343887	3	0	0
12604142	4	0	1	13343888	4	0	1
12615670	3	0	0	12646948	4	0	1
12615754	3	0	0	12584185	3	0	0
12897908	4	0	1	12646946	3	0	0
13221337	4	0	3	12584122	3	0	0
13726234	5	1	4	12813116	3	0	0
13892852	3	0	0	12583671	4	0	2
14256506	3	0	0	12583546	4	0	0
14596138	4	0	1	12583827	4	0	1
16258869	4	0	1	12604012	4	0	0
12583686	4	0	1	12583634	3	0	0
12740647	3	0	0	14134746	3	0	0
14657503	3	0	1	12584191	4	0	1
12603988	3	0	0	12615764	3	0	0
12603990	3	0	0	13343911	3	0	0
12604137	3	0	0	13152590	3	0	0
12604143	3	0	0	14540088	4	0	0
12604146	3	0	0	14540089	4	0	0
12667170	3	0	0	12559851	4	0	1
13202872	3	0	0				

Table 4. Predicted subsystem endurance with spares

(Sub)System	Nomenclature	MDBCF	Std Dev	15th%	%Crit.Fails
421	Spy-1D	371	270	92	
420786	OE-375 ANTENNA GROUP	2,519	1,725	903	4.5%
420899	OT-146 TRANSMITTER GROUP	481	331	128	68.6%
421491	OL-356 SIGNAL PROCESSOR GROUP	972	648	290	24.9%
421965	AUXILIARY EQUIPMENT	3,517	2,223	1,352	2.1%

E. SPARING A BATTLE GROUP OF 3 DDG

To spare a battle group of 3 DDG so that its SPY-1D radar systems will endure 90 days with at least an 85% probability, it would not be sufficient to simply provide each ship with the spares listed in Table 3. This is because each DDG will have a failure rate of about 15% at 90 days, so the chance no ship fails in 90 days is binomially distributed: the chance of having zero failures in three trials when each trial has a probability of failure of 15%. The (binomial) probability of this is 0.614, meaning there is a 39% probability that at least one ship’s radar will fail in less than 90 days. In terms of risk, this is more than double the desired failure probability.

To verify this, we ran a simulation of three identical SPY-1D radar spared at the recommended level, tracking the time to *first* failure of any of three SPY-1D radar, and found that the mean time to first-failure of any (of the three) SPY-1D radar dropped to an average of 160 days, and the 15th % dropped to 32 days (only 3 days better than the single ship endurance, without spares).

So, to investigate whether savings in spares are possible if a battle group pools its spare inventory, one must first determine the level of spares necessary to ensure that *no ship* in the battlegroup has a radar that fails in less than 90 days, at least 85% of the time. To find the level of (un-pooled) spares needed, we increased the level of (identical) spares on each ship until our simulation predicted that the time to the *first ship’s* SPY-1D failure was greater than 90 days, at least 85% of the time. To accomplish this, we used a greedy heuristic, increasing the spare level the NIIN that had the lowest MDBCF one at a time, until the target was met.

We found that each ship required 118 spare parts (compared to 41 for a single ship) are needed to have MDBCF of 936 days (compared to 371 days for a single ship)

with a 15th % = 257 days, in order to obtain a mean time to *first* failure of at least 90 days, with at least an 85% probability.

This is not counter-intuitive. If the failure times were all exponential, sending three ships out for 90 days would be equivalent to sending one ship out for 270 days. Because the distributions are not (quite) exponential, the required additional sparing to insure *all three* ships attain the required endurance target is not (quite) tripled.

So, it is against this target (118x3 = 354 spare parts) that we will seek a reduction in the required spare-parts inventory by pooling spares across the battle group. Our assumption in pooling is that all parts are freely shared and cannibalized as needed across the battle group's three SPY-1D radar systems.

We used the same optimization target – minimize the number of spares, subject to a constraint that the 15th % of the distribution of Days to First Squadron Critical Failure must be greater than or equal to 90. As an upper bound for the optimization, we used the un-pooled solution (354 parts). For a lower bound, we used the number of spare parts required to keep a single SPY-1D enduring for 90 days at least 85% of the time (41 x 3 = 123 parts). We have already explained why this is a lower bound, and indeed, at the 85th %, that solution only endures 33 days.

Assuming pooled spare parts and cannibalization across the battlegroup, we estimate the first SPY-1D radar in the battlegroup fails after 90 days at least 85% of the time if the battlegroup is given 132 spare parts. That is, only 9 additional spare parts are required, above the lower bound (see Table 5).

Table 5. Battle group spares

NIIN	Added Spares
9858983	1
12740647	1
12603988	1
12603990	1
12604137	1
12604143	1
13202872	1
13926982	1
14134746	1

Again, this should be compared to the 354 parts that would be required if spares were not pooled, to ensure that the first-to-fail-of-three radars would endure for 90 days at least 85% of the time. With these additional nine parts, we estimate the mean days to first failure in the battlegroup is 300 days, the standard deviation is 144 days, and the 15th % is 103 days.

This is intuitive. By pooling spares, we are not merely adding 9 spare parts, we are allowing sharing of parts. So, no critical subsystem will fail until the parts needed for that subsystem are exhausted everywhere in the battlegroup.

F. VALIDATING THE MODEL AGAINST HISTORICAL SPY-1D MTBCF

Our base case model without spares predicts the MDBCf will be more than 110 days, with a 15th % of less than 35 days. But historical data suggests the MDBCf for the SPY-1D is about 86 days. Of course, we are only modeling 51 NIIN in a system that is much more complex. Although we tried to capture the parts most likely to bring the system down, undoubtedly, other parts sometimes cause failure. In discussion with our sponsors, it became clear that part failures are not the sole source of system failure. So, to validate the model, we added an additional variable “All other sources of failure” with an exponential distribution.

A mean time between failures of 274 hours for “other sources of failure” validated the model with the historical observations of a MDBCf of 86 days. Failing at that rate, other sources of failure contributed 31.3% of all failures. Incorporating this “other sources of failure” caused the model (without spares) to predict the SPY-1D radar would endure 18 days, at least 85% of the time (compared to 35 days without “other sources of failure”). See Figure 13.

The exponential distribution is a good modeling choice because it has a single parameter, but with a coefficient of variation of 1.0, it probably overstates the degree to which these other sources of failure cause the system to fail in a short period of time (i.e., it probably understates the 15th %).

Still, the model results presented here should be understood to predict the performance of a system that contains only the 51 parts we model. It is likely those parts only cause about 2/3rds of the failures of the SPY-1D radar. The sparing levels we

estimate to be required to provide a 90 day endurance of that simplified system are unlikely to provide 90 days of endurance in the fielded system, and may provide much less.

There are at least three reasons why our model of 51 parts overestimates endurance of the fielded system. 1) Although other parts fail less often, there are very many of them, and they still contribute to the reliability of the SPY-1D radar in an important way, 2) Parts are not the only reason the system fails (O'Haver, Barker, Dockery, & Huffaker, 2018), and 3) We may have overestimated the lifetime of the parts we modeled. We were focused on NIINs that had critical failures. Although our estimates of MDBCFC were vetted by the sponsor, it is possible that not all failures were captured by the system, because of e.g., cannibalization, or a miscoding of failures that were critical as minor.

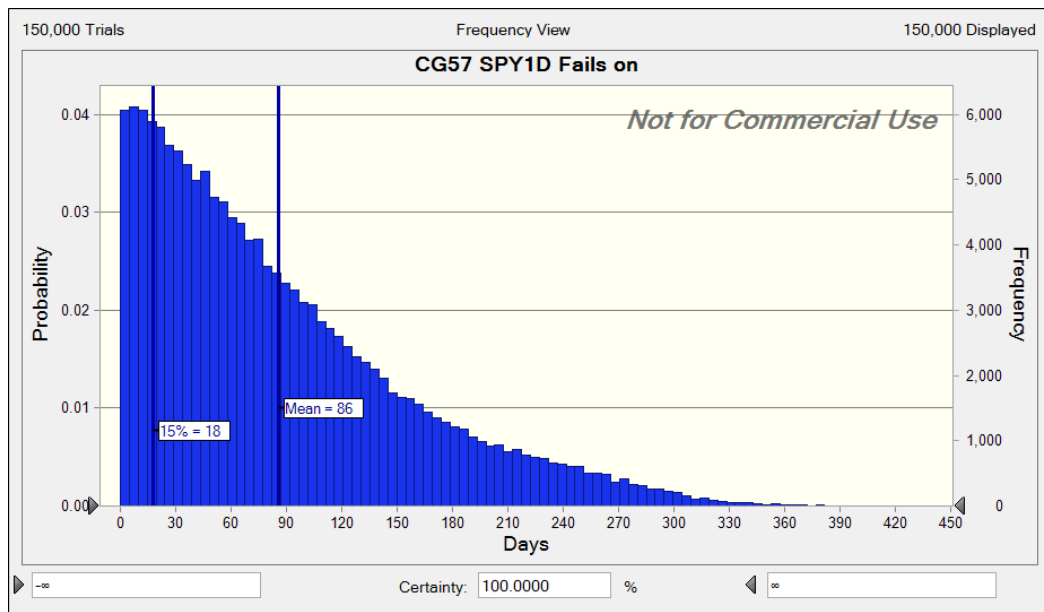


Figure 13. Days to first SPY-1D failure in the battle group, un-pooled spares

Because there is no guarantee that additional spare parts would provide the required endurance, we chose not to model how many additional spare parts would be needed after accounting for “other sources of failure”. Instead, our recommendations will consider the need to validate our current models of NIIN failure, extend the analysis to other NIIN that fail less often, and determine the percentage of failures that are not caused by the lack of availability of spare parts.

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IV. SUMMARY AND CONCLUSIONS

A. SUMMARY

The main goal of this effort was to model the Endurance of the SPY-1D radar for a single ship and for multiple ships in a battlegroup, if spare parts can be shared between ships. The target specified by our sponsor was for the Spy-1D to endure 90 days at least 85% of the time (i.e., the 15th % of the time-to-failure distribution should be at least 90 days).

As the SPY-1D is an enormously complex system with approximately 300,000 parts in over 7,000 reliability blocks, we first had to identify the subset of parts most likely to cause SPY-1D radar failure. After organizing and scrubbing the available data for our purposes, we determined there were 51 NIINs that had caused critical failures, and that also had an average time-to-failure of less than 9,000 days. We found these 51 NIINs were installed in 24 reliability blocks, across four critical subsystems. The SPY-1D has a great deal of built-in redundancy, and these 51 NIINs had a total of 228 parts installed in the SPY-1D (several of these NIINs have only 1 part installed, but the NIIN with the most redundancy has 99 parts, 43 of which are redundant).

We then built a simulation model of the four critical subsystems containing those 51 NIINs, and modeled SPY-1D endurance as the time-to-first-failure of any one of these four critical subsystems. When a NIIN has redundant parts in more than one reliability block, our model assumes free cannibalization of parts between the reliability blocks, as needed. From this model, we estimated that the MDBCf for the SPY-1D should be 114 days, and that 85% of the time, it should endure at least 29 days.

We were able to calibrate these findings using a reliability block diagram (RBD) tool available in JMP©. That RBD tool allows the specification of parts in reliability block diagrams and solves for system reliability numerically. We modeled our 51 NIINs in 24 reliability blocks, and were able to match the system and subsystem MDBCf predicted by our simulation model. But the RBD tool in JMP is meant as a design aid, and we were therefore unable to adapt it to study (or to calibrate our simulation model of) the impact of spare parts.

We next extended the simulation model to assess the impact of additional spare parts across four kinds of reliability blocks: (1) those with no redundancy at all, (2) those with redundant parts that are always operating, (3) those with redundant parts in stand-by mode, and (4) NIINs that are used in blocks of multiple types (i.e. redundant parts in stand-by mode in one block, but always-operating in another block).

After determining upper and lower bounds on the number of spares that would be required for each NIIN in order to obtain a system endurance of 90 days at the 85%, we conducted a simulation search to estimate the smallest number of spares that would be needed to obtain that target. We found that 41 additional spare parts should be sufficient for the SPY-1D to endure more than 90 days at least 85% of the time.

We then used this single-ship model to examine a battlegroup with three DDGs to determine the level of sparing that would allow the battlegroup to reach the same endurance target. Note that the battlegroup target is harder: the sparing level must be such that the first-of-three radars to fail, still fails in more than 90 days at least 85% of the time. Were the time to failure exponentially distributed, this would be equivalent to one ship enduring for 270 days.

We compared battlegroups with 3 SPY-1D radars (i.e., 3 DDGs) that pooled spare-and-redundant parts with battlegroups that did not pool spare-and-redundant parts.

In the case of un-pooled spares, no transshipment of parts was allowed, and every ship had to have sufficient (and equal) number of parts to reach the endurance target. In this case, each ship needs to have 118 spare parts, almost three times the number required for a single-ship expedition. The battlegroup of three DDG required 354 (=118x3) spare parts. This makes sense, because sending three ships that do not share spare parts on an expedition of 90 days is analogous to sending one ship on three such expeditions in a row.

In the case of pooled spares, we assumed ships would share spare and redundant parts as needed, and freely cannibalize to do so. In other words, when a NIIN caused a critical failure, no spare or redundant part of that NIIN was available anywhere in the battlegroup. In this case, only 9 additional spare parts were needed, beyond the 123 (=41x3) required for a single-ship expedition.

Historically, the SPY-1D has MDBCf of 86 days, not the 114 days predicted by our model. To validate our model against the historical data, we added another exponential random variable “all other sources of failure”, and adjusted the parameter of that variable until the simulation predicted a mean time-to-failure of the SPY-1D (without additional spares) that matched the 86 days observed historically. With “all other sources of failure” set to that parameter, we estimate that it causes 31.3% of all failures. This implies our model of just 51 NIINs is capturing about two thirds (68.7%) of the sources of failure of the SPY-1D.

Our study has several limitations. First, since we are only modeling 228 parts (51 NIINs) out of over 300,000 parts, and only 24 reliability blocks out of over 7,000 reliability blocks, it is clear that our model will not explain all part failures. Preliminary validation against historical data, however, suggests we may have captured as much as two-thirds of the sources of failure. Since about a third of the sources of failure remain unexplained, our predictions of endurance probability provided by spare parts need to be understood as conditional. The results are valid for a SPY-1D that only fails because of shortages of one of these 51 NIINs. Nor should one expect that a small number of additional spares of other NIINs will necessarily help reach the endurance target: the other sources of failure may not be due to part-shortage at all. Secondly, we are assuming free sharing of parts across the squadron, and free cannibalization of parts within and across SPY-1D installations. While cannibalization does occur, it is not without cost: cannibalization itself can cause failures. Finally, the high reliability of these NIINs presents a limitation in itself. Because (most of) these parts fail so very infrequently, the samples we used to estimate NIINs MDBCf are necessarily small.

B. CONCLUSIONS

Our model of the SPY-1D, consisting of the 51 NIINs most-likely to cause a critical failure, suggests that a moderate number of spare parts can allow a system to reach the endurance target. In total, these 51 NIINs have 228 parts installed on the SPY-1D, and we are recommending a ship should carry at least 41 spare parts of these NIINs to endure 90 days. While $41/228 = 18\%$ may seem like a high percentage of spares-required, these are the parts that are most likely to fail.

Sparing a battle group with three SPY-1D installations requires additional spares to reach a given endurance target, because the target is essentially different (for the battlegroup to endure, *none* of three installations should fail in less than 90 days). When spare and redundant parts are not shared between ships, sparing each ship to reach this more-difficult target is similar to sparing a single ship to endure for 270 days. And thus requires almost three times more spares (118 versus 41 for each ship, or $118 \times 3 = 357$ for the battlegroup). Nonetheless, if pooling and cannibalization are allowed, our model predicts as few as 9 additional spares ($41 \times 3 + 9 = 132$) could allow the battle group to reach an endurance target of 90 days with an 85% probability.

Without additional spares, we predict the SPY-1D radar will not reach the desired endurance target of 90 days 85% of the time. Current operational plans may require the SPY-1D to operate in a contested environment beyond the reach of a supply chain. Our methodology will help N4 support the fleet when it must endure for longer periods in contested environments.

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