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13. SUPPLEMENTARY NOTES The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.

14. ABSTRACT

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a. REPORT UU	b. ABSTRACT UU	c. THIS PAGE UU	19b. TELEPHONE NUMBER 806-834-2606

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as of 02-Aug-2022

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Proposal Number: 78651MSII

Agreement Number: W911NF-21-1-0267

INVESTIGATOR(S):

Name: Alexander Vilevich Idesman
Email: alexander.idesman@coe.ttu.edu
Phone Number: 8068342606
Principal: Y

Organization: **Texas Technical University**
Address: Box 41035, Lubbock, TX 794091035
Country: USA

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Report Date: 30-Sep-2022

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Final Report for Period Beginning 10-May-2021 and Ending 30-Jun-2022

Title: A new high-order accurate approach for modeling of wave propagation and heat transfer in heterogeneous materials

Begin Performance Period: 10-May-2021

End Performance Period: 30-Jun-2022

Report Term: 0-Other

Submitted By: Alexander Idesman

Email: alexander.idesman@coe.ttu.edu

Phone: (806) 834-2606

Distribution Statement: 1-Approved for public release; distribution is unlimited.

STEM Degrees:

STEM Participants:

Major Goals: The time-independent Poisson equation for heterogeneous materials:

- a) To develop OLTEM for with 9-point and 25-point stencils for curvilinear interfaces and unfitted Cartesian meshes.
- b) To show that the accuracy of OLTEM is practically independent of the location of unfitted Cartesian meshes with respect to interfaces.
- c) To solve benchmark problems showing the advantages of OLTEM compared to linear and high-order finite elements (a significant reduction in the computation time by a factor of 1000 and more for OLTEM at a given accuracy as well as the use of trivial unfitted Cartesian meshes for domains with irregular interfaces).

The time-dependent wave and heat equations for heterogeneous materials:

- a) To develop OLTEM with 9-point stencils for the wave and heat equations with complicated curvilinear interfaces and unfitted Cartesian meshes.
- b) To show that the accuracy of OLTEM is practically independent of the location of unfitted Cartesian meshes with respect to interfaces.
- c) To solve benchmark problems showing the advantages of OLTEM compared to linear and high-order finite elements (a significant reduction in the computation time by a factor of 1000 and more for the new approach at a given accuracy as well as the use of trivial unfitted Cartesian meshes for domains with irregular interfaces).

Accomplishments: We have developed a new numerical method called the Optimal Local Truncation Error Method (OLTEM) for the modeling of wave propagation and heat transfer in heterogeneous materials. OLTEM yields the optimal (maximum possible) accuracy at a given structure of discrete equations and significantly exceeds the accuracy of existing numerical techniques at similar structure of discrete equations. OLTEM with 9-point stencils and unfitted Cartesian meshes provides the third order of accuracy for the 2-D Poisson, heat and wave equations with irregular interfaces (see our papers [2,3,5]); i.e., the increase by one order compared to that for linear finite elements with conformed meshes. OLTEM with 25-point stencils and unfitted Cartesian meshes provides the 11-th order of accuracy for the 2-D Poisson equation with irregular interfaces (see our paper [3]); i.e., a huge increase in accuracy by 8 orders for the new 'quadratic' elements compared to known techniques at similar computational costs. The improvement of accuracy of a numerical technique by one order is usually treated as a significant result. Therefore, the huge improvement of accuracy by OLTEM can be considered as revolutionary. OLTEM with linear elements for the Poisson equation reduces the numbers of the degrees of freedom by million

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times (!) compared to conventional linear finite elements; see our paper [2]. The results for benchmark problems in our six papers [1-6] (see below) show that OLTEM provides a huge reduction in computation time (by 1000-1000000 and more times) compared to existing techniques. This will allow the solution of new real-world problems that currently cannot be solved due to the prohibitively large computation time. Some multiscale problems can be directly solved by the proposed technique without the scale separation. Another advantage of OLTEM is the use of trivial unfitted Cartesian meshes that are totally independent of the complex domain geometry (no need in complicated mesh generators for complex geometry).

Published papers related to the 9-month grant (with no cost extension).
(* indicates my graduate students)

1. The 10-th order of accuracy of 'quadratic' elements for elastic heterogeneous materials with smooth interfaces and unfitted Cartesian meshes. Idesman, A., Dey*, B., Mobin*, M. Engineering with Computers, 2022, pp. 1-25, <https://doi.org/10.1007/s00366-022-01688-5> . IF 8.083
2. Optimal local truncation error method for solution of 3-D Poisson equation with irregular interfaces and unfitted Cartesian meshes as well as for post-processing. Idesman, A., Mobin*, M. Advances in Engineering Software, 2022, 167, 103103, pp. 1-16 (<https://doi.org/10.1016/j.advengsoft.2022.103103>). IF 4.255
3. 3-rd and 11-th orders of accuracy of 'linear' and 'quadratic' elements for the Poisson equation with irregular interfaces on unfitted Cartesian meshes. Idesman, A., Dey*, B. International Journal of Numerical Methods for Heat and Fluid Flow, 2022, Vol. 32, No. 8, pp. 2719-2749. (<https://doi.org/10.1108/HFF-09-2021-0596>). IF 5.181
4. Optimal local truncation error method for solution of elasticity problems for heterogeneous materials with irregular interfaces and unfitted Cartesian meshes. Idesman, A., Dey*, B., Mobin*, M. Mechanics of Advanced Materials and Structures, 2021, pp. 1-17 (<https://doi.org/10.1080/15376494.2021.2014001>). IF 3.338
5. Optimal local truncation error method for solution of wave and heat equations for heterogeneous materials with irregular interfaces and unfitted Cartesian meshes. Idesman, A., Dey*, B. Computer Methods in Applied Mechanics and Engineering, 2021, 384, 113998, pp. 1-32. IF 6.588

Paper under Review:

6. Optimal local truncation error method for solution of 2-D elastodynamics problems with irregular interfaces and unfitted Cartesian meshes as well as for post-processing. Idesman, A., Mobin*, M. Mechanics of Advanced Materials and Structures, 2022, pp. 1-36. IF 3.338

Training Opportunities: One PhD student, Mr. M. Mobin, was trained in the development of the new OLTEM method to the solution of PDEs.

Results Dissemination: 5 published papers in the leading engineering journals with high impact factors, one paper is under review. Two keynote lectures at the 15th World Congress on Computational Mechanics (WCCM-APCOM 2022), Yokohama, Japan, July 31- August 5, 2022 (online) as well as at the 8th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMA2022) June 5-9, 2022, Oslo, Norway.

Honors and Awards: The Edward E. Whitacre Jr. College of Engineering of Texas Tech University has awarded me the Whitacre Engineering Research Award on January 22, 2022.

Protocol Activity Status:

Technology Transfer: Nothing to Report

PARTICIPANTS:

Participant Type: Graduate Student (research assistant)

Participant: Mir Mobin

Person Months Worked: 12.00

Funding Support:

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Project Contribution:
National Academy Member: N

ARTICLES:

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published

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Date Published: 10/1/21 5:00AM

Publication Location:

Article Title: Optimal local truncation error method for solution of wave and heat equations for heterogeneous materials with irregular interfaces and unfitted Cartesian meshes

Authors: A. Idesman, B. Dey

Keywords: Wave and heat equations with discontinuous coefficients; Local truncation error; Irregular interfaces; Cartesian meshes; Optimal accuracy

Abstract: Recently we have developed the optimal local truncation error method (OLTEM) for PDEs with constant coefficients on irregular domains and unfitted Cartesian meshes. However, many important engineering applications include domains with different material properties (e.g., different inclusions, multi-material structural components, etc.) for which this technique cannot be directly applied. In the paper OLTEM is extended to a much more general case of PDEs with discontinuous coefficients and can treat the above-mentioned applications. We show the development of OLTEM for the 1-D and 2-D scalar wave equation as well as the heat equation using compact 3-point (in the 1-D case) and 9-point (in the 2-D case) stencils that are similar to those for linear quadrilateral finite elements. Trivial unfitted Cartesian meshes are used for OLTEM with complex interfaces between different materials. The interface conditions on the interfaces where the jumps in material properties occur are added to the e

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Journal: Mechanics of Advanced Materials and Structures

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Volume:

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Date Published: 12/1/21 6:00AM

Publication Location:

Article Title: Optimal local truncation error method for solution of elasticity problems for heterogeneous materials with irregular interfaces and unfitted Cartesian meshes

Authors: A. Idesman, B. Dey, M. Mobin

Keywords: Elasticity equations for heterogeneous materials; irregular interfaces; local truncation error; unfitted Cartesian meshes; optimal accuracy

Abstract: The optimal local truncation error method (OLTEM) with unfitted Cartesian meshes was recently developed for PDEs with homogeneous materials on regular and irregular domains as well as for the scalar time-dependent wave and heat equations for heterogeneous materials with irregular interfaces. Here, OLTEM is extended to a system of time-independent elastic PDEs for heterogeneous materials with irregular interfaces and unfitted Cartesian meshes. We show the development of OLTEM for the 2D elasticity equations using compact 9-point stencils that are similar to those for linear quadrilateral finite elements. The interface conditions on the interfaces where the jumps in material properties occur are added to the expression for the local truncation error and do not change the width of the stencils. There are no unknowns on interfaces between different materials; the structure of the global discrete equations is the same for homogeneous and heterogeneous materials. The ... (see the paper)

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Acknowledged Federal Support: Y

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Date Published: 12/1/21 6:00AM

Publication Location:

Article Title: 3rd and 11th orders of accuracy of 'linear' and 'quadratic' elements for Poisson equation with irregular interfaces on Cartesian meshes

Authors: Alexander Idesman, Bikash Dey

Keywords: Cartesian meshes, Poisson equation with discontinuous coefficients, Irregular interfaces, Local truncation error, Optimal accuracy

Abstract: The purpose of this paper is as follows: to significantly reduce the computation time (by a factor of 1,000 and more) compared to known numerical techniques for real-world problems with complex interfaces; and to simplify the solution by using trivial unfitted Cartesian meshes (no need in complicated mesh generators for complex geometry). ... (see the paper)

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Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published

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Publication Location:

Article Title: Optimal local truncation error method for solution of 3-D Poisson equation with irregular interfaces and unfitted Cartesian meshes as well as for post-processing

Authors: A. Idesman, M. Mobin

Keywords: Poisson equation with discontinuous coefficients; Irregular interfaces; Unfitted Cartesian meshes; Optimal accuracy; Post-processing; Spatial derivatives;

Abstract: Recently the optimal local truncation error method (OLTEM) has been developed for the 2-D Poisson equation for heterogeneous materials with irregular interfaces and unfitted Cartesian meshes. Here we extend it to the general 3-D case. 27-point stencils that are similar to those for linear finite elements are used with OLTEM. The interface conditions at a small number of selected interface points where the jumps in material properties occur are added to the expression for the local truncation error and do not change the width of the stencils. There are no unknowns on interfaces between different materials; the structure of the global discrete equations is the same for homogeneous and heterogeneous materials. The calculation of the unknown stencil coefficients is based on the minimization of the local truncation error of the stencil equations, includes the entire PDE for the derivations and yields the optimal third order of accuracy of OLTEM with the 27-point stencils ... (see the paper)

Distribution Statement: 1-Approved for public release; distribution is unlimited.

Acknowledged Federal Support: Y

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Partners

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I certify that the information in the report is complete and accurate:

Signature: Alexander Idesman

Signature Date: 7/31/22 5:55PM

Published papers related to the 9-month grant (with no cost extension).
(* indicates my graduate students)

1. The 10-th order of accuracy of 'quadratic' elements for elastic heterogeneous materials with smooth interfaces and unfitted Cartesian meshes. Idesman, A., Dey*, B., Mobin*, M. Engineering with Computers, 2022, pp. 1-25, <https://doi.org/10.1007/s00366-022-01688-5> . IF 8.083
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Paper under Review:

6. Optimal local truncation error method for solution of 2-D elastodynamics problems with irregular interfaces and unfitted Cartesian meshes as well as for post-processing. Idesman, A., Mobin*, M. Mechanics of Advanced Materials and Structures, 2022, pp. 1-36. IF 3.338