# (U) Intensity Profile of an Ideal Point Source Projected onto a flat screen

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**(U) Abstract:** A closed form expression is derived for the case of an ideal point source that emits total power P, projected onto a flat screen at a distance, R, from the source. The intensity as a function of position on the screen is achieved.

**(U) Research Innovation and Objective(s):** A derivation of this fundamental result is not readily found in the literature. Therefore, the result is derived and checked for qualitative consistency.

**(U) Impacts on Warfighter Mission:** Being able to correct for non-uniform illumination can allow for more objective radiographic disposition. Even if this formula is limited to a research and development context, the ability to correct for nonuniformities from geometrical considerations can prove valuable for analyzing more complex situations. Physically, there is no true point source. However, it is said that isotopes can provide good empirical data for the mathematical theory presented here.

**(U) Keywords:** Non-Destructive Testing (NDT), Radiography, Computer Vision, Machine Learning, Industrial Radiography

## 1. (U) Introduction

(U) Leveraging artificial intelligence with the goal of increasing speed and agility is an army priority research area. Exploring off the shelf tools for object character recognition can greatly aid radiographers throughout the industrial base without the need for years of expensive fundamental research and development. Nonuniform illumination issues can be handled using morphological operations. However, these techniques might not always be successful. Therefore, geometrical techniques should not be underestimated in their ability to correct for nonuniform illumination.

A paper authored by Shree Nayar and Arthur Sanderson from the Carnegie Mellon University titled, "Determining surface ornetiations of specular surfaces by intensity encoded illumination." derives the following expression for point source illumination. Without proof, [1] the author states that "the irradiance E(b) of the surface at the point b is expressed as,

$$E(b) = \frac{Icos\alpha}{r^2}$$

And by using geometry the expression can be written in the following form,

$$E(b) = \frac{Ih}{(h^2 + (a+b)^2)^{\frac{3}{2}}} \qquad (1)$$

It should be noted that the authors describe an intensity encoded line source illumination approach to estimating the surface orientation of specular surfaces. Inspection of solder and machine metal are practical examples of the theory presented by Nayar and Sanderson. [1] Therefore, a derivation of equation one will prove to be a valuable addition to the non-destructive testing literature.

## 2. (U) Theory

(U) By the inverse square law, the intensity anywhere on the surface of a sphere that is tangent to the screen is given by,

$$I_0 = \frac{P_T}{A} = \frac{P_T}{4\pi R^2}$$

Where $P_T$ is the total power emitted by the source, and the flat screen is a distance R from the source.

We begin by finding the fraction of the total power that passes through the surface of the spherical segment shown in orange in figure 1.

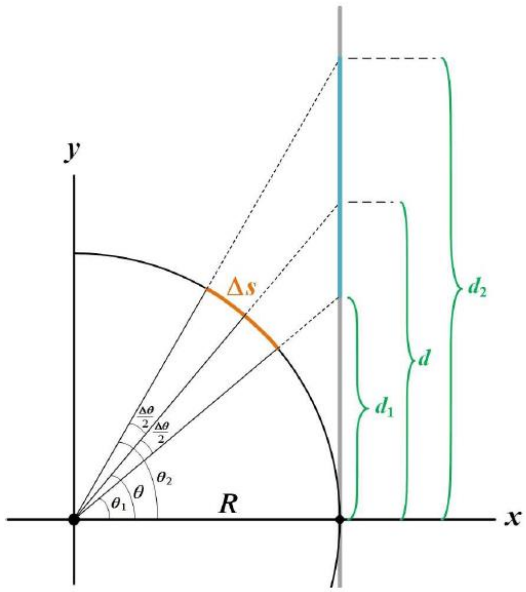


**Figure 1**. A two dimensional view of the spherical source projected onto a flat screen at a distance, R, from the source.

The surface area of the spherical segment, centered at an angle $\theta$, subtends $\Delta\theta$, and goes from $\theta_1$ to $\theta_2$ where,

$$\theta_1 = \theta - \frac{\Delta\theta}{2}$$

$$\theta_2 = \theta + \frac{\Delta\theta}{2}$$

Next, the surface area of a spherical segment rotating $\Delta s$, about the x-axis is given by,

$$A_{segment} = 2\pi R^2 \Delta\theta \sin\theta$$

Therefore, the ratio of the power passing through the surface of the spherical segment to the total power will be proportional to the fraction of the areas,

$$\frac{P_{segment}}{P_{total}} = \frac{2\pi R^2 \Delta\theta \sin\theta}{4\pi R^2} = \frac{1}{2}\Delta\theta \sin\theta$$

$$P_{segment} = \frac{1}{2}\Delta\theta \sin\theta \, P_{total}$$

When the power passing through the surface of this spherical segment gets projected onto the screen, it gets spread out into an annulus shown in figure 2.

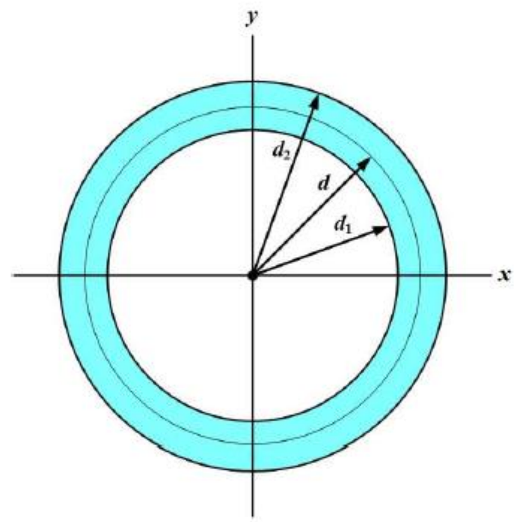


**Figure 2.** Annulus with labelled variables.

The area of the annulus with nominal radius d is given by,

$$A_{annulus} = \pi(d_2^2 - d_1^2)$$

Where

$$d_i = R\tan\theta_i$$

For $i = 1,2$.

Therefore, using expressions for $\theta_1$ and $\theta_2$ we can express the area of the annulus to be,

$$A_{annulus} = \pi(d_2^2 - d_1^2)$$

$$A_{annulus} = \pi R^2 \left[\tan^2\left(\theta + \frac{\Delta\theta}{2}\right) - \tan^2\left(\theta - \frac{\Delta\theta}{2}\right)\right]$$

Recall that we desire an expression for the intensity of the source projected onto the screen which will vary continuously with $\theta$. Therefore, for a given angle, we take $\Delta\theta \sim 0$. We can now expand the expression for the annulus to arrive at the following result.

$$A_{annulus} = 2\pi R^2 \frac{sin\theta}{\cos^3\theta}\Delta\theta$$

The intensity in the annulus at angle $\theta$ is then,

$$I(\theta) = \frac{P_{segment}}{A_{annulus}} = \frac{P_{total}}{4\pi R^2}\cos^3\theta$$

$$I(\theta) = I_0 \cos^3\theta$$

Referring back to figure 1, we have

$$d = R\tan\theta = R\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$$

Rearranging gives,

$$\cos\theta = \frac{1}{\sqrt{1+\left(\frac{d}{R}\right)^2}}$$

Therefore, we arrive at the final result,

$$I(\theta) = \frac{I_0}{\left[1+\left(\frac{d}{R}\right)^2\right]^{\frac{3}{2}}}$$

One can compare and contrast the result presented here with equation (1).

## 3. (U) Results & Discussion

(U) We have successfully derived the intensity profile generated from a point source. A graph of the fractional intensity as a function of the fractional distance from the center is shown in figure 3.

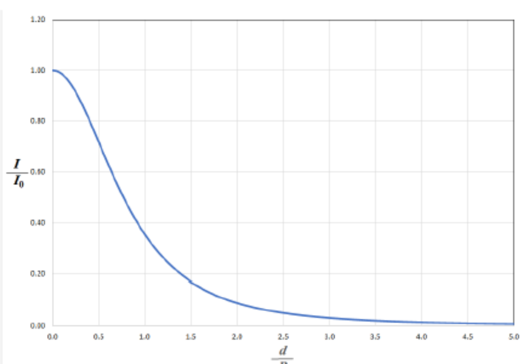


**Figure 3.** A graph of the fractional intensity relative to the intensity at the center, plotted against the fractional distance from the center, relative to the distance of the source from the screen.

One can also produce a three-dimensional plot which is shown in figure 4. One would expect the highest intensity to be at the center of the screen and then decay as one gets further from the center in a symmetrical way. This qualitative behavior is confirmed with the plot shown in figure 4.

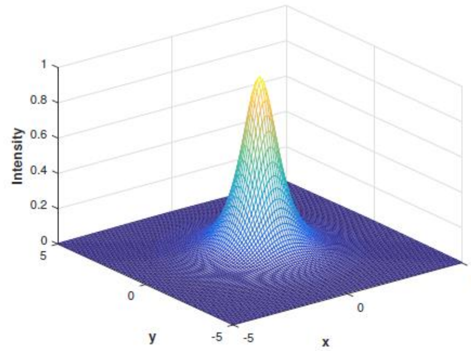


**Figure 4.** A 3-dimensional graph of the intensity on the screen with the origin at the center of the screen. $I_0 = 1$, and $R = 1$.

One can also make the following approximation. In the limit that the screen size is much smaller than the distance of the screen from the source, one can graph the intensity in the region,

$$\left|\frac{x}{R}\right| \leq 1, \left|\frac{y}{R}\right| \leq 1$$

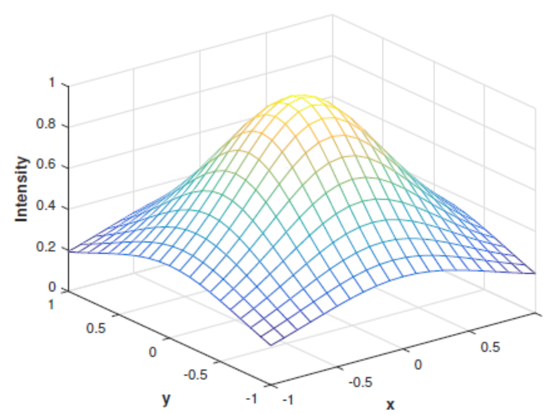


**Figure 5.** A 3 dimensional graph of the intensity on the screen, with the origin plotted in the region $\left|\frac{x}{R}\right| \leq 1, \left|\frac{y}{R}\right| \leq 1$ *and* $I_0 = 1$, and $R = 1$.

## 4. (U) Future Work

(U) In a research and development context the acquisition and subsequent image processing should be achievable to verify the theory presented above. As mentioned earlier, non uniform illumination presents challenges to numerous computer vision tasks. The work herein should inspire additional work in this area to increase the effectiveness of computer vision tools available.

## References

1. Nayar S, Sanderson A. Determining surface orientations of specular surfaces by intensity encoded illumination. International Society for Optical Engineering (SPIE), Vol. 850, pp. 122-127, Nov. 1987