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REPORT ON

NOMOGRAMS FOR CALCULATING MINIMUM

JAMMING RANGE FOR SHIPS

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NAVAL RESEARCH LABORATORY
WASHINGTON 20, D. C.

Radio Division - Consultant Group

NOMOGRAMS FOR CALCULATING MINIMUM JAMMING RANGE FOR SHIPS

by

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Title Page	- 1 sheet (a)
Abstract	- 1 sheet (b)
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Text	- 10 sheets
Tables	- 1 sheet
Plates	- 7 sheets

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ABSTRACT

Nomograms have been prepared to facilitate the calculation of the minimum jamming range for self screening of ships from enemy radars. These nomograms are based on theoretical formulas, using parameters which have been found from experimental tests.

Nomograms are included for the far and near zones, and also one determining a limiting range due to the first minimum in the radiation pattern of the jamming antenna.

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Table of Contents

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1. INTRODUCTION

1-1. The vertical radiation pattern of a jamming antenna contains a lobe structure of maximums and minimums due to interference of the rays direct from jammer to receiver and the rays reflected from the sea. As a result, for a given value of jamming antenna height, receiving (radar) antenna height, and wavelength, the received power density will vary as in Curve 1, Fig. 1, as the range is closed.

1-2. The radar transmitter has a similar vertical lobe structure, but a surface target such as a ship reflects the radar signal from each height and hence integrates or "fills in" the vertical lobe structure. The power density of the radar echo returned to the radar receiver is of the form of Curve 2, Fig. 1, the aspect of the ship being assumed to remain unchanged. For small values of the range, R, the returned power of the radar signal is proportional to R^{-4}, while for large R, the power received is proportional to R^{-8}. These conditions respectively define the so-called near and far zones of the radar. The physical difference between the zones is that in the far zone the entire reflecting surface of the ship lies below the first maximum in the vertical lobe pattern of the transmitter, while in the near zone the ship spans one or more maximums.

1-3. Jamming will be considered successful if the radar receives more power from the jammer than from the echo. This has been verified experimentally for random noise jamming. In order to make it apply for other types of jamming modulation, an "effective" value of jamming power is to be used, rather than the total power actually radiated. The part of the total power radiated that is "effective" is a function of the bandwidth over which it is spread, the type of modulation used, the bandwidth of the radar receiver, and various other characteristics of the enemy radar. (For further information on this subject, see, for example, Reference 3.) The minimum jamming range is the least range to which a ship can be taken without increasing the echo power above the effective jammer power. This minimum jamming range is shown by point R_{min} on Fig. 1 for the conditions illustrated there. For all ranges greater than R_{min} the jammer power is greater than the radar power, and for lesser ranges the radar power is greater (see Reference 2 for a fuller discussion of this and related points*). R_{min} is the figure to be determined on the nomograms.

* In order to avoid a possible misunderstanding of the statement made in paragraph 15 of Reference 2, it should be pointed out that simultaneous radiation from two antennas at different heights was not meant to imply the use of a common jamming transmitter feeding these two antennas. Separate jammers would be required for each antenna in order that the corresponding jamming signals may add randomly in the radar receiver.

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1-4. **Curve 2, Fig. 2,** shows a possible situation that illustrates another practical limitation of jamming range. This curve shows how the radar echo power varies with range for a smaller ship than the one for which Fig. 1 was drawn. It can be seen that the ship will be screened in the second, third, and higher lobes of the jammer pattern, but that these are nulls in between where jamming is not successful. Since the enemy could get range information in the intervening nulls, it is not safe to rely on jamming closer than the limit set by the first null of the jammer pattern. Hence the first null sets a limiting range for any amount of jammer power. In order to simplify the determination of this limiting range, without at the same time introducing any significant error, it is arbitrarily defined as the range where the direct and sea-reflected rays are 150° out of phase. At shorter ranges than this, the received power decreases rapidly, while at greater ranges the variation is much slower. This point is shown on Fig. 2 as R_{lim}. R_{lim} is given by the relation

$$R_{lim} = \frac{2.4 h_r h_j}{\lambda} \qquad (1)$$

In this relation, h_r = radar antenna height, h_j = jammer antenna height, and λ = wavelength (all quantities in same units as R_{lim}). It can be seen from this equation that R_{lim} can be decreased by decreasing the height of the jamming antenna.

1-5. Figs. 1 - 3 illustrate four types of situations which may occur, and should make clear the procedure followed in arriving at the proper solution. In Figs. 1 and 3, the minimum jamming range occurs in the far zone, while in Fig. 2 the solutions are in the near zone. For Curve 2, Fig. 2, the solution is determined by the limiting range R_{lim}, rather than by insufficient jammer power.

1-6. Mathematically, it is not possible to solve explicitly the equations determining minimum jamming range. In practice, therefore, it is necessary to resort either to a graphical solution or to introduce approximations, which have varying degrees of accuracy under various jamming situations. Graphical solutions are not very convenient when a large range of variables has to be explored, since new curves have to be drawn whenever a change in made in any one variable, such as radar antenna height, jammer antenna height, or frequency. In the past it has been customary, therefore, to use the first approximation for calculating the jamming signal. If this accuracy is sufficient, **Chart** I, far zone, gives the result with a minimum of labor. However, with jamming being carried into shorter wavelengths (S-band) and under more extreme conditions of range and antenna heights, more accurate values of the minimum jamming range than given by the first approximation are required. To calculate accurate solutions mathematically is a very tedious job. The nomograms included in this report have been devised to reduce the labor of this calculation. In order to handle the various situations illustrated in Figs. 1 - 3, a total of four nomograms is required. It will seldom be necessary to use them all, however, since the first nomogram indicates which of the others should be used. A description of the nomograms is given in Section 2, and their use is illustrated by several examples in Section 5.

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2-6. If $1 < \beta < 3$, the correct solution might be either in the far or near zone, so both solutions have to be found, and then the lesser of the two figures would be the correct value. Curve 1, Fig. 2, illustrates the case when the near zone is proper since R_A, the near zone solution, is less than R_D, the far zone solution. Curve 1, Fig. 3, illustrates the other possibility when R_D, the far zone result, is less than R_A, the near zone result, and hence R_D is the correct value for the minimum jamming range.

2-7. The values of the minimum jamming range found from the nomograms are solutions of certain fixed equations and must be interpreted and used carefully. They are calculated for standard propagation conditions; therefore, no ducts to trap the waves, or subnormal air structures are assumed to be present. Anomalous propagation affects radar and jammer in the same direction, but not to the same degree, since the radar has a two-way path against one way for the jammer.

2-8. The values of σ_n, the radar cross section for the near zone, and the values of K for the far zone, are both tabulated in Table I for the broadside aspect of the ship. At this aspect the radar reflection from the ship is strongest, so if screening is complete for the broadside aspect, it will be screened at any other aspect. Likewise the minimum jamming range computed for the broadside aspect would be conservatively large for another aspect (such as the bow and stern), for which σ_n and K are smaller. σ_n and K are usually 10 db or more higher for the broadside aspect than for other aspects (see reference 1).

3. DERIVATION OF EQUATIONS

3-1. Definitions

3-1-1. The minimum jamming range is that range at which the radar signal returned to its receiver is equal to the signal received from the jammer.

3-1-2. The radar cross section of ships in the near zone is defined as

$$\sigma_n = 4\pi R^2 \frac{W_r}{W_o}$$

where W_r = power density at radar receiving antenna; W_o = free space power density incident on target; and R = range. Experimentally determined values of σ_n are available for various ship types and aspects. See Table I.

3-1-3. The radar cross section of ships in the far zone is defined as

$$\sigma_f = 4\pi R^2 \left(\frac{R}{h_r}\right)^4 \frac{W_r}{W_o}$$

[redacted]

where h_r = height of radar antenna and other symbols are as defined in 3-1-2. Values of σ_f have been reported (see Reference 1), but it has been found convenient to use another quantity K in its place.

3-1-4. It has been found empirically that for most ships $\sigma_f = K \cdot f^4$, where f is the frequency in megacycles, and K a constant typical of the ship. Values of K are given in Table I.

3-2. The power received back from the radar echo when the target is in the far zone is rP_r

$$rP_r = \frac{P_r G_r^2 h_r^4 \lambda^2 \sigma_f}{(4\pi)^3 R^8} \quad . \tag{2}$$

The power received from the jammer is rP_j

$$rP_j = \frac{P_j G_j G_r \lambda^2}{(4\pi)^2 R^2} \; 4 \sin^2 \left(\frac{2\pi h_r h_j}{\lambda R} \right) \quad , \tag{3}$$

$$rP_j = \frac{P_j G_j G_r \lambda^2}{(4\pi)^2 R^2} \; 4 \sin^2 A \quad , \tag{4}$$

where P_r = peak transmitted radar power, P_j = effective jammer power, G_r = gain of radar antenna over isotropic radiator, G_j = gain of jammer antenna over isotropic radiator, h_r = height of radar antenna, h_j = height of jammer antenna, λ = wavelength, R = range to target, σ_f = far-zone radar cross section of target ship, $A = 2\pi h_r h_j / \lambda R$. If we introduce the approximation $\sin A = A$, which is valid for small values of A, we obtain

$$rP_j = \frac{P_j G_j G_r h_r^2 h_j^2}{R^4} \tag{5}$$

3-3. Equating rP_r and rP_j to get approximate minimum jamming range gives

$$R_{approx} = \left(\frac{P_r G_r h_r^2 \lambda^2 \sigma_f}{(4\pi)^3 \; P_j G_j h_j^2} \right)^{\frac{1}{4}} \tag{6}$$

if the solution lies in the far zone. Using the relation $\sigma_f = K \cdot f^4$, this equation can be reduced to

$$R_{approx} = \left[\frac{P_r G_r h_r^2 K c^4}{P_j G_j h_j^2 \lambda^2 (4\pi)^3} \right]^{\frac{1}{4}} \tag{7}$$

where c = 300 m/µsec.

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3-4. The nomogram on Chart I for the far zone (sequence of numbers at top of chart), solves Eq. (7) for R_{approx}. This yields the equivalent of point B in Fig. 1, since the approximation sin A = A was used. This is the approximation most commonly used for calculations. To correct for the error resulting from this approximation, Chart II is used. On Chart II, the far zone operations give A_{approx} in radians on scale (7). On the other side of the same line, the ratio $(A_{min}/\sin A_{min})^{\frac{1}{2}}$ is marked off for corresponding values of A_{approx}. This ratio is the correction factor "C".

3-5. The correction factor, C, is found as follows. Equating (2) and (3),

$$R_{min}^6 = \frac{P_rG_rh_r^4\sigma_f}{16\pi P_jG_j\cdot\sin^2\frac{2\pi h_rh_j}{\lambda R_{min}}}\ . \qquad (8)$$

This can be rearranged to give

$$R_{min}^4 = \frac{P_rG_rh_r^2\lambda^2\sigma_f}{(4\pi)^3P_jG_jh_j^2}\cdot\frac{\left(\frac{2\pi h_rh_j}{\lambda R_{min}}\right)^2}{\sin^2\left(\frac{2\pi h_rh_j}{\lambda R_{min}}\right)} = R_{approx}^4\cdot\left(\frac{A_{min}}{\sin A_{min}}\right)^2 \qquad (9)$$

From (9), the correction factor C is

$$C = \frac{R_{min}}{R_{approx}} = \left(\frac{A_{min}}{\sin A_{min}}\right)^{\frac{1}{2}}\ . \qquad (10)$$

In order to find this ratio, A_{approx} is first found (first 7 steps on Chart II). Then, since

$$C = \frac{R_{min}}{R_{approx}} = \frac{A_{approx}}{A_{min}}\ , \qquad (11)$$

(10) and (11) together give

$$A_{approx} = \left(\frac{A_{min}^3}{\sin A_{min}}\right)^{\frac{1}{2}}\ . \qquad (12)$$

(12) may be solved numerically to give corresponding values of A_{min} and A_{approx}, and then C determined from (11). This was done in order to construct the C scale. The end value of C = 2.2 which appears on the nomogram was chosen to correspond closely to the condition of R_{lim} as defined in paragraph 1-5. If C > 2.2, R_{approx} must be increased until C = 2.2 and then R_{corr} found.

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3-6. For the near zone,

$$rP_r = \frac{P_r G_r^2 \lambda^2 \sigma_n}{(4\pi)^3 R^4} \tag{13}$$

$$rP_j = \frac{P_j G_j G_r \lambda^2}{(4\pi)^2 R^2} \, 4 \sin^2\left(\frac{2\pi h_r h_j}{\lambda R}\right) \tag{14}$$

However, these formulas cannot be equated and R solved for explicitly since sin A cannot here be set equal to A. Recourse is taken to two quantities β and R_{lim}:

$$R_{lim} = \frac{2.4 \, h_r h_j}{\lambda} \qquad \text{(by definition)} \tag{15}$$

$$\beta = \frac{R_{min}}{R_{lim}} \tag{16}$$

Equating (13) and (14) and substituting in (15) and (16), gives

$$\frac{P_j G_j h_r^2 h_j^2 (4\pi)^3}{P_r G_r \sigma_n \lambda^2} = \frac{(6.85)}{\beta^2} \csc^2\left(\frac{5\pi}{6\beta}\right) \tag{17}$$

3-7. The left-hand member of (17) is the quantity Q shown on Figs. 2 and 3. It is the ratio of the received jamming power (with sin A = A) to radar echo power at any range in the near zone. Both these powers are proportional to R^{-4}. On Fig. 2, β is the ratio of R_A to R_{lim}. The right-hand member of (17) gives β as a function of Q. The unlabelled scale on the left side of scale (15), Chart I, near zone, gives Q. Chart I, near zone, solves (17) for β ; Chart II, near zone, uses this value of β and solves (15) and (16) to find R_{min}.

3-8. If $Q \leqq 1$, which would mean $\beta = \infty$, it can be seen from Fig. 2 that the solution would lie in the far zone. If $Q > 27.3$, R_A would be less than R_{lim} and $\beta < 1$; therefore R_{lim} would be used for R_{min}. For $1 < Q < 27$, that is $1 < \beta < 3$, both near and far zone solutions are possible and both must be solved for. Until both have been found and the smaller value selected, it is impossible to tell which will be correct. This amounts to discriminating between the conditions of Curve 1 of Fig. 2 and Curve 1 of Fig. 3. The smaller value of range will always be the correct one.

4. SUMMARY OF PROCEDURE

4-1. In using the nomograms, the following symbols are used with the units indicated:

[redacted]

P_r = Peak power of radar transmitter, in watts.

P_j = Effective power of jammer, in watts.

G_r = Gain of radar antenna over isotropic radiator, in db.

G_j = Gain of jammer antenna over isotropic radiator, in db.

h_r = Height of radar antenna, in feet.

h_j = Height of jammer antenna, in feet.

λ = Wavelength, in meters.

σ_n = Near-zone radar cross section of ship, in $(\text{meters})^2$.

K = Quantity determining far-zone radar cross section of ship obtained from Table I.

R = Range, in yards.

4-2. (a) On Chart I, near zone, find β.

(b) If $\beta > 3.0$, go to far zone, Chart I and II, immediately.

(c) If $\beta < 1$, go to Chart II, near zone, and use R_{lim} on Scale (5) for R_{min}.

(d) For $1 < \beta < 3$, use Chart II, near zone, and find R_{min}. Also find R_{min} from far zone and use the lesser of the two values.

(e) Whenever R_{min} for the far zone is needed, use both Charts I and II to get the correct value.

4-2-1. Large size work sheets are available on request. Their classification is RESTRICTED since the scales are unlabelled.

4-3. <u>Warnings</u>

4-3-1. For all near-zone calculations, use the sequence of operations indicated by the encircled numbers at the bottom of the charts; for far zone, those at the top of the charts. Use the scale on the same side of the line as the number and in the units shown on the sample.

4-3-2. There is a discontinuity in the sequence of Chart II, far zone, between (7) and (8), as explained in paragraph 2-4.

4-3-3. If the correction factor C for the far zone lies in the "Go" region, C may be taken as unity. If C lies in the "No-Go" region, R_{approx} must be increased until C reaches 2.2 and then $R_{corrected}$ found.

[redacted]

5. EXAMPLES

5-1. On the worksheets there are no labels on the scales, in order that the classification of RESTRICTED may be maintained, so there are enclosed two samples showing which quantities belong on the various scales. Two examples are also worked out for the following conditions:

Far Zone:	$G_r = 38$ db	$G_j = 12$ db
	$P_r = 10^6$ watts	$h_j = 30$ feet
	$h_r = 10^2$ ~~watts~~ ft	$K = 10^2$
	$P_j = 5.10^3$ watts	$R_{min} = 9500$ yards (Approx. Chart I)
	$\lambda = 0.2$ meter	$R_{corr} = 10500$ yards (from Chart II)
Near Zone: Chart I	$G_r = 19$ db	$G_j = 8$ db
	$P_r = 10^5$ watts	$h_j = 120$ feet
	$h_r = 400$ feet	$\sigma_n = 10^8$ meters^2
	$P_j = 10^2$ watts	$\beta = 1.55$
	$\lambda = 0.1$ meter	
Chart II	$h_r = 10^2$ feet	$\beta = 1.40$
	$\lambda = 0.2$ meter	$R_{min} = 5000$ yards
	$h_j = 30$ feet	

5-2. On the samples, numbers in squares indicate values for the far zone; numbers in triangles show near zone case.

5-3. When a worksheet is labelled, its classification becomes SECRET.

6. REFERENCES

6-1. NRL Special Reports I-VI, "Radar Cross Section of Ship Targets", RA 3A 213A, R-2232, R-2295, R-2332, R-2466, R-2467.

6-2. NRL Report R-2435 -"Considerations in the Tactical Use of Centimeter Wave Radar Jamming Systems."

6-3. NRL Report R-2498 -"Dependence of the Effectiveness of Clipped Noise Jamming on Receiver Bandwidth." (In preparation).

[redacted]

Table 1

σ_n (meters)^2
(Broadsides)

Type of Ship	400 Mc	560 Mc	700 Mc	970 Mc	3060 Mc	K (far zone) (Broadsides)
BB					2.1×10^8	1.6×10^4
CV				3.1×10^7	1.8×10^8	6.0×10^4
CVE					2.0×10^7	2.7×10^4
CVL	1.2×10^6		8.5×10^6		9.4×10^6	5.0×10^3
CA	4.7×10^6				1.7×10^8	1.2×10^4
CL	7.0×10^3		2.4×10^6	5.0×10^6	1.3×10^7	1.0×10^4
DD		2.5×10^5	1.1×10^6	1.5×10^6	5.0×10^7	7.5×10^3
LST			2.1×10^6		1.7×10^8	1.0×10^3
LCT						0.3
AGC					1.7×10^7	1.9×10^4
AM	1.3×10^4		8.3×10^4	5.4×10^5	9.4×10^5	1.6×10^2
LS (Surf)	1.2×10^2				5.4×10^5	
APA				9.0×10^6	3.0×10^7	9.0×10^3

[redacted]

Table I

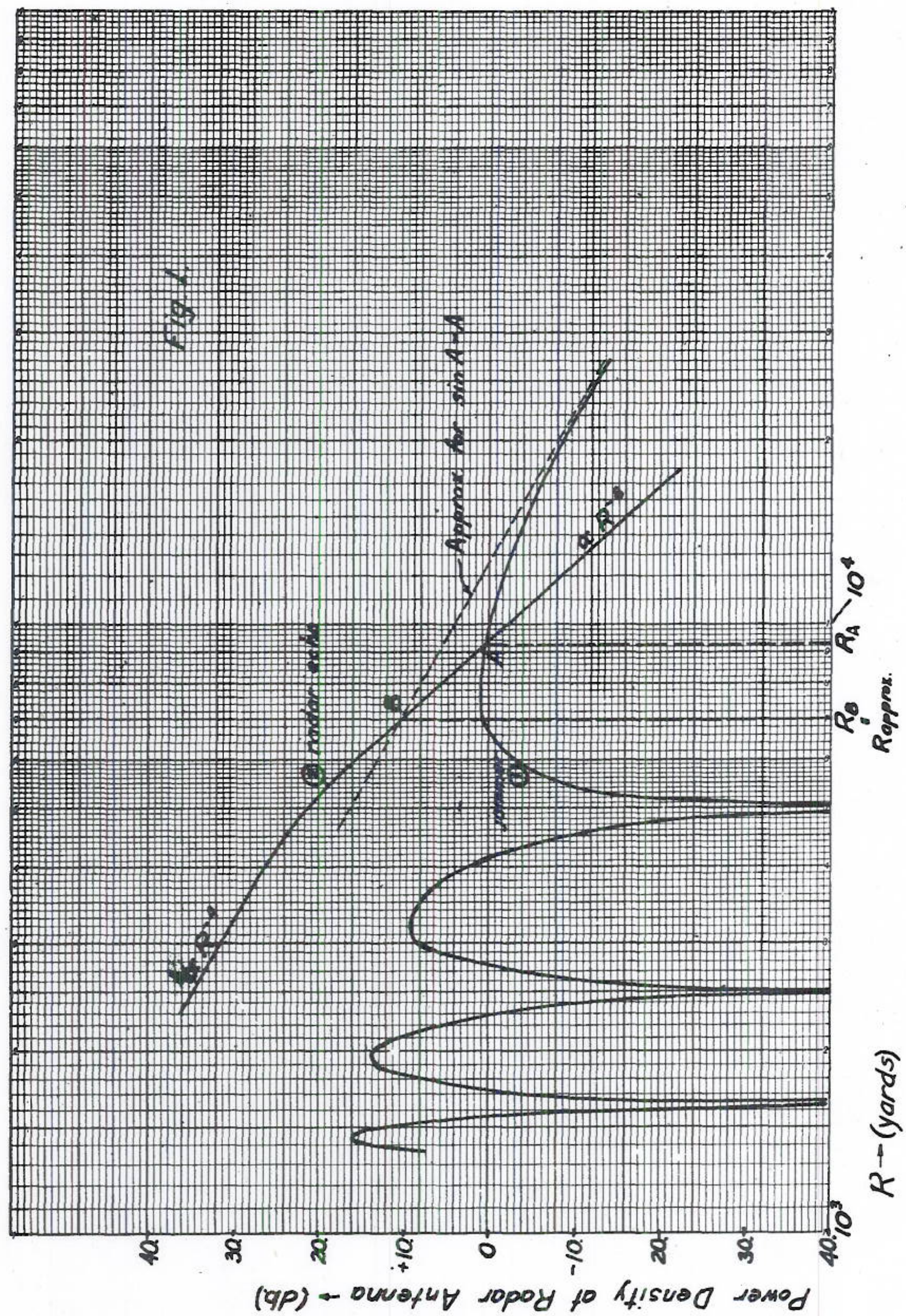

Plate 1.

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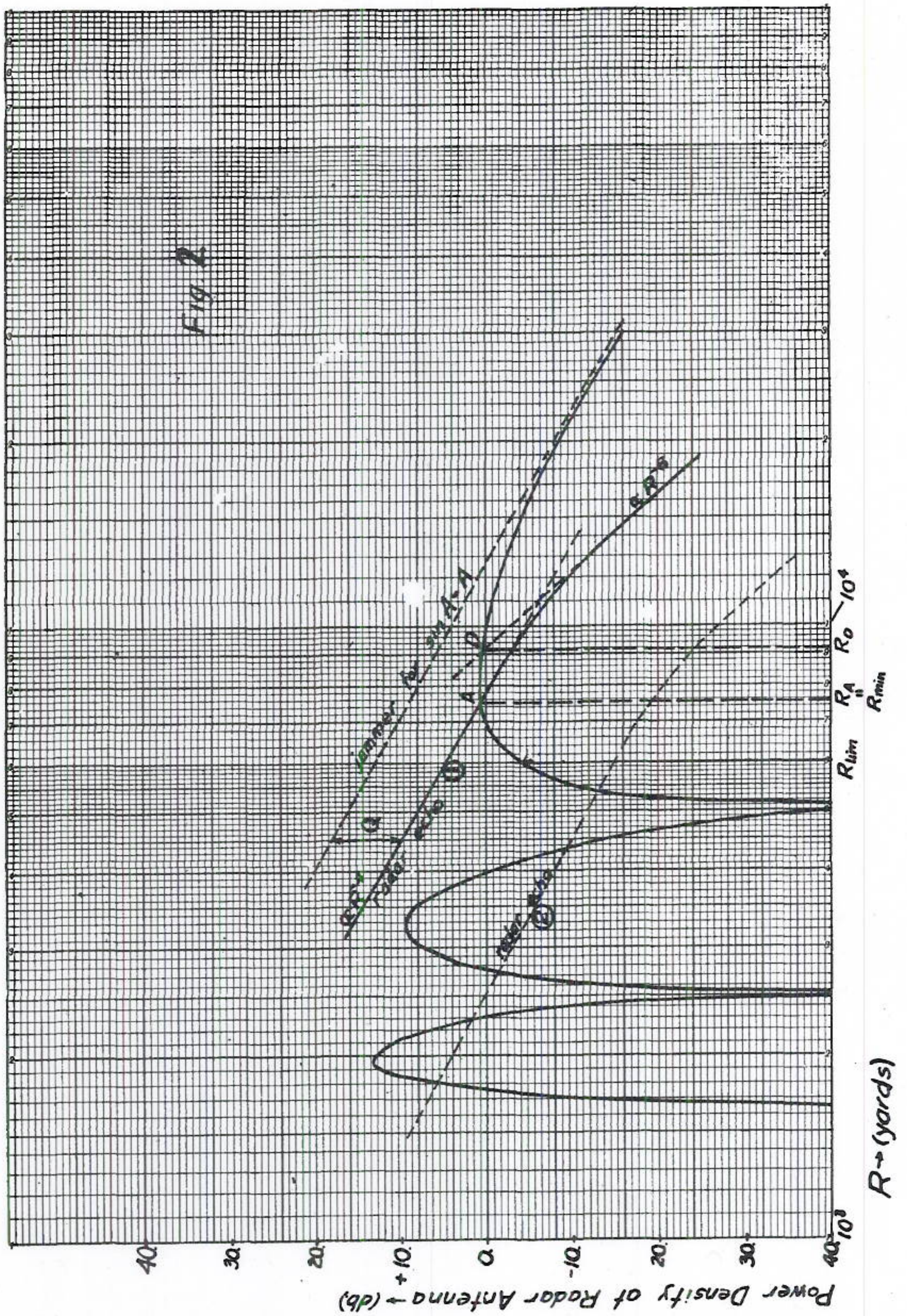
Fig 2
R → (yards)
Power Density of Radar Antenna → (db)
40
30
20
+10
0
-10
20
30
40
10^3
10^4
R_o
$R_A = R_{min}$
R_{lim}
jammer for
radar echo

Plate 2

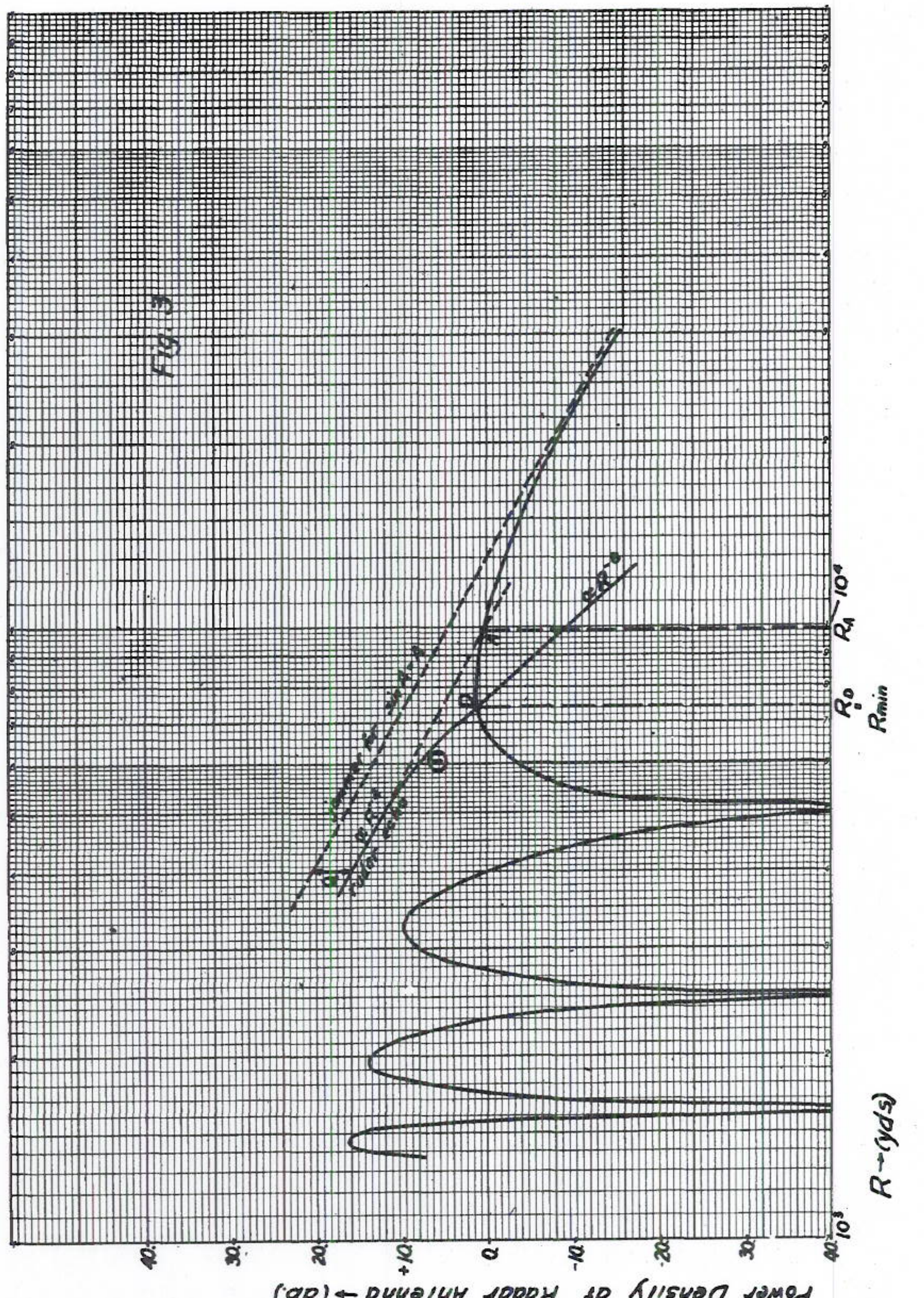

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Plate 3

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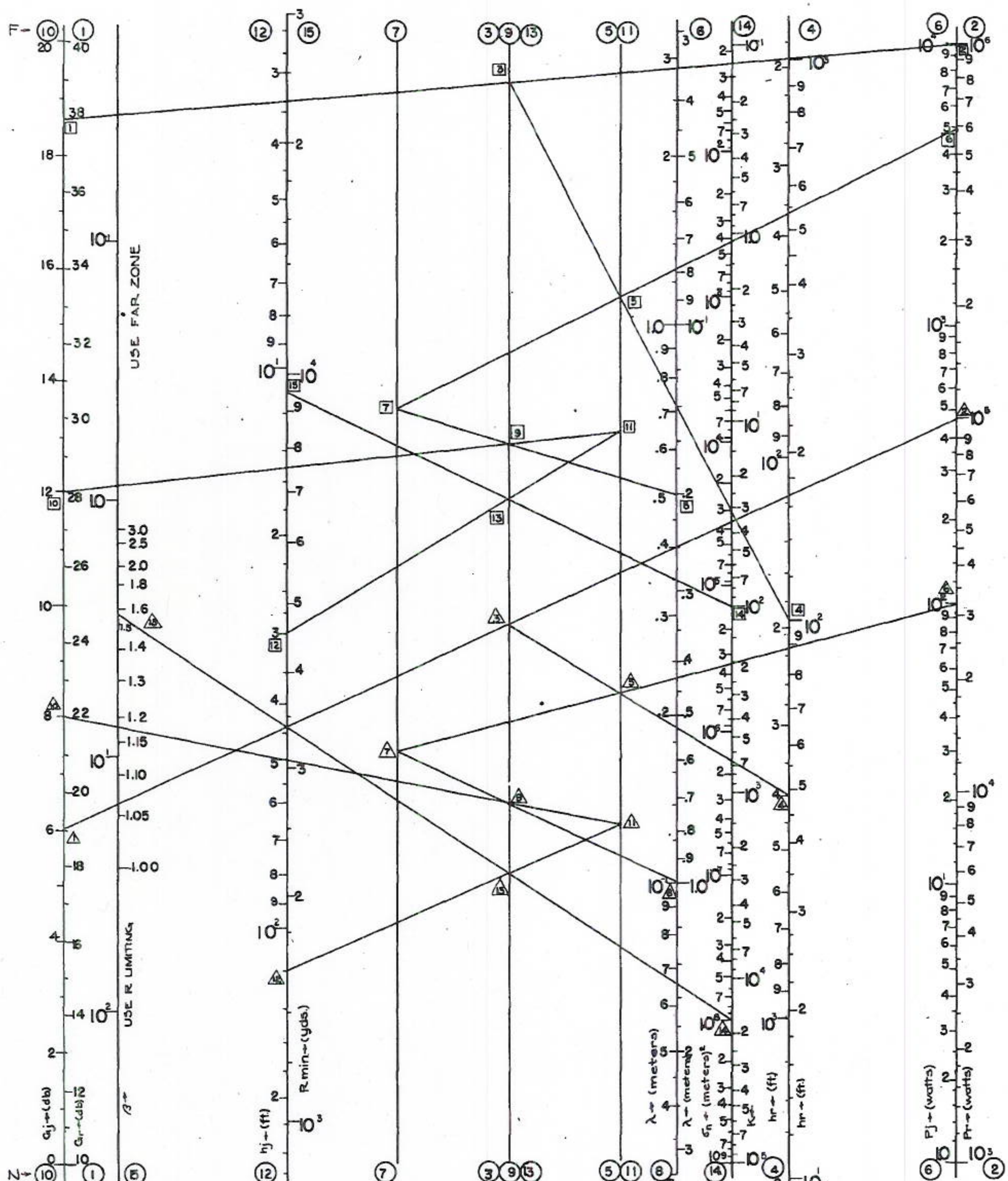

CHART I

PLATE 4

[redacted]

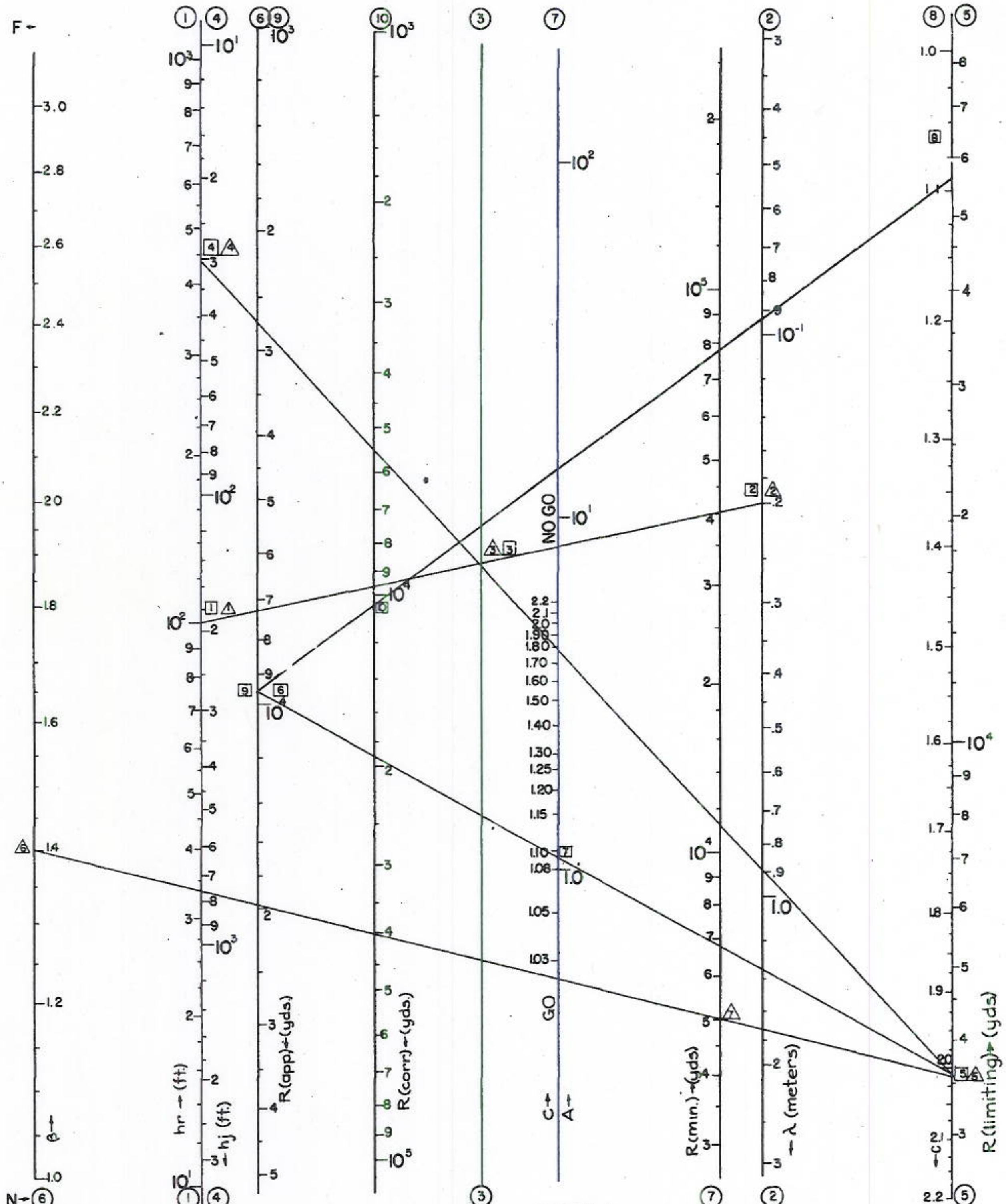

CHART 2
CORRECTION

PLATE 5

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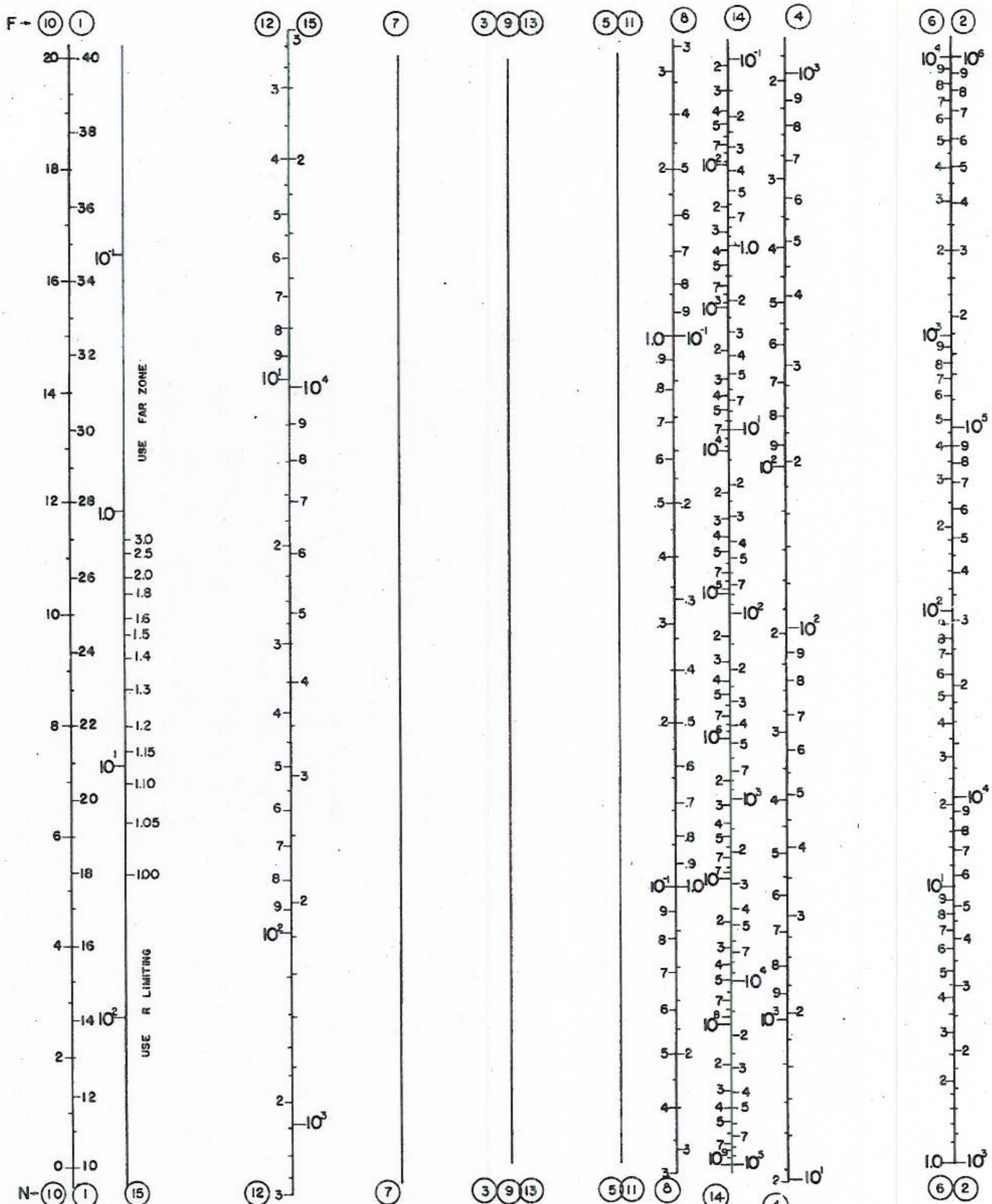

CHART I

PLATE 6

[redacted]

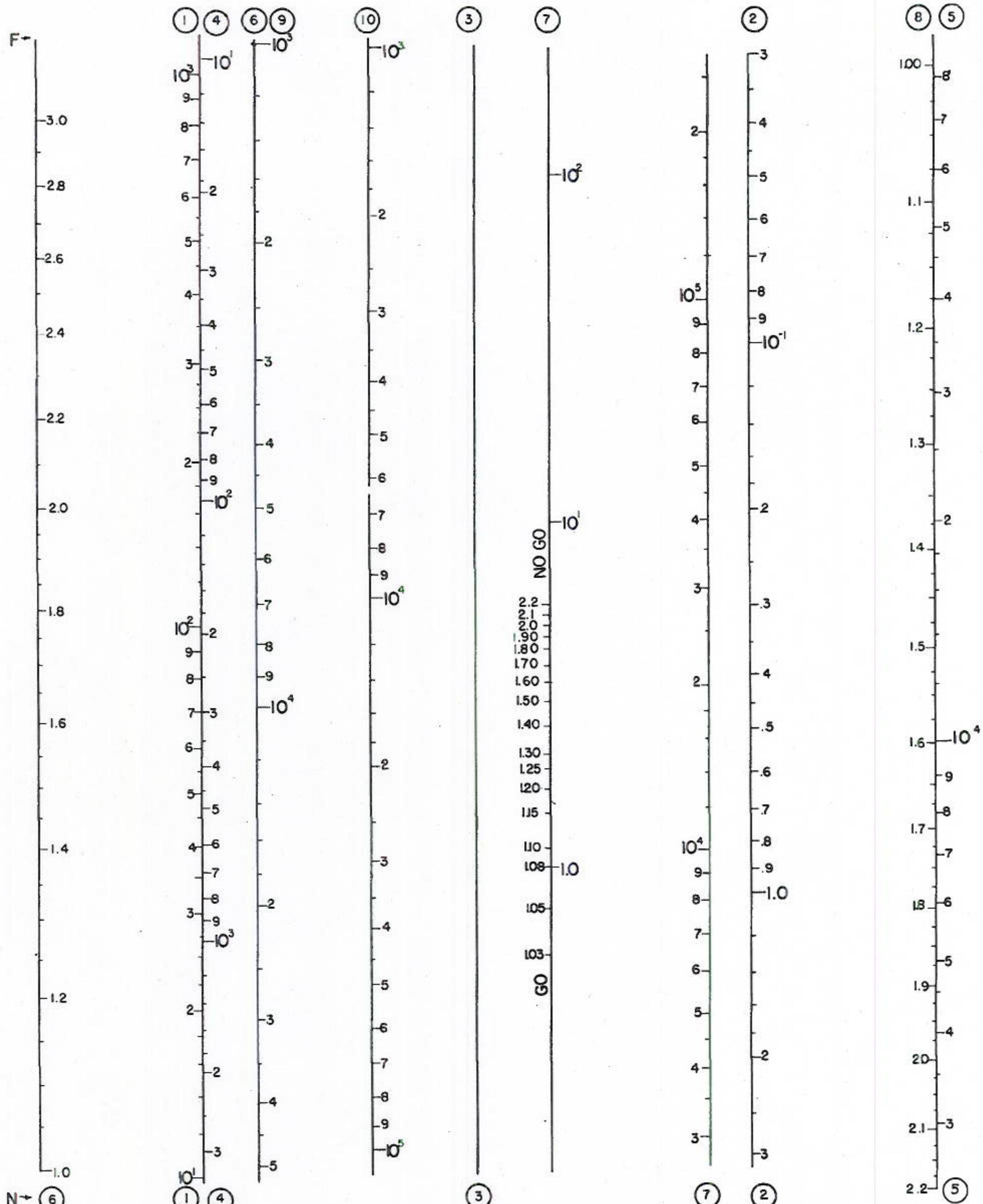

CHART 2

PLATE 7

[redacted]

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Copy #12 - BuAer, Aer-E-3143
Copy #13 - BuOrd, Re4f
Copy #14 - CNO, Op-16-1-V
Copy #15-#17 - CNO, Op-16 (2 for Alusna, London)
Copy #18 - JEIA
Copy #19 - CinCPac
Copy #20-#21 - ComSeventhFleet
Copy #22 - ComTenthFleet
Copy #23 - ComTwelfthFleet
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Copy #29-#31 - Msg Center Branch G-2 MID
Copy #32 - OCSigO SPSOI-4
Copy #33 - AAF-ACO
Copy #34 - ATSC, Wright Field
Copy #35-#39 - BAD (for ASE)
Copy #40-#43 - RAFD (1 for TRE, 3 for RCM Board, London)
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