

2 June 1945

DECLASSIFIED

NAVAL RESEARCH LABORATORY
WASHINGTON, 20, D. C.

DECLASSIFIED by NRL Contract
Declassification Team

Date: 9 SEP 2014

Reviewer's name(s): A. THOMPSON,
P. HANNA

Declassification authority: NAVY DECLASS
MANUAL, 11 DEC 2012, OF SERIES

CONVERGENCE EFFECTS IN REFLECTIONS
FROM TROPOSPHERIC LAYERS

by

W. W. Carter, Ensign, USNR

Radio Division - Consultant Group
Report No. R-2546

FR-2546

DISTRIBUTION STATEMENT A APPLIES

Further distribution authorized by _____

UNLIMITED only.

Approved by: Martin Katzin - Coordinator, Wave Propagation

Dr. A. Hoyt Taylor,
Superintendent, Radio Division

A. H. Van Keuren, Rear Admiral, USN (Ret.)
Director, Naval Research Laboratory

Title Page	- 1 sheet (a)
Abstract	- 1 sheet (b)
Table of Contents	- 1 sheet (c)
Text	- 4 sheets
Plates	- 4 sheets

DECLASSIFIED

DECLASSIFIED: By authority of
5000A January 1958

Entered by: E. Bliss Code 2027a -

agm

DECLASSIFIED

DECLASSIFIED

2 June 1945



NAVAL RESEARCH LABORATORY
WASHINGTON 20, D. C.



R-2546

CONSEQUENCE EFFECTS IN REFLECTIONS
FROM TROPOSPHERIC LAYERS

by

W. W. Carter, Ensign, USNR

Radio Division - Consultant Group
Report No. R-2546



Approved by: Martin Katzin - Coordinator, Wave Propagation

Dr. A. Hoyt Taylor,
Superintendent, Radio Division

A. H. Van Keuren, Rear Admiral, USN (Ret.)
Director, Naval Research Laboratory

Title Page	- 1 sheet (a)
Abstract	- 1 sheet (b)
Table of Contents	- 1 sheet (c)
Text	- 4 sheets
Plates	- 4 sheets

agm

DECLASSIFIED

DECLASSIFIED

ABSTRACT

The reflection of radio waves from an elevated duct, or tropospheric layer, is treated like the reflection of light from a small concave spherical mirror in order to calculate the amount of convergence. Ray tracing methods are used. The effect of roughness of the layer in scattering the energy is also considered.

- b -

DECLASSIFIED

DECLASSIFIED

TABLE OF CONTENTS

	<u>Page No.</u>
1. Introduction	1
2. Convergence Factor	1
3. Roughness Effect	2
4. Sample	3
5. Conclusions	3
Plate 1 - Fig. 1	
Plate 2 - Fig. 2	
Plate 3 - Fig. 3	
Plate 4 - Fig. 4	
Distribution List	4

CONFIDENTIAL

DECLASSIFIED

1. Introduction

1-1. An elevated duct is treated like a concave spherical mirror whose radius of curvature is "a", the effective earth's radius. This would include any layer that would act as a reflector if the radiation were incident at a sufficiently small angle. The problem is considered as one of geometrical optics only. Ray tracing methods are used, and the phases are assumed to add randomly. This assumption might introduce an error as large as 3 db in the result, but was necessary to simplify the solution of the problem. If the reflection coefficient is other than unity, it must be multiplied into the general relation which will be given for C, the convergence factor.

2. Convergence Factor

2-1. A bundle of rays leaving a transmitter below the reflecting layer would be converged on reflection from a concave surface. The convergence factor, K, would be the ratio of the power density at the receiving antenna after convergence to the power density at the receiver that would be expected after reflection from a plane surface (essentially free space condition). Referring to Fig. 1, the convergence factor is expressed as

$$K = \frac{(x + y) \delta \theta_1}{x \delta \theta_1 + y \delta \theta_2} \quad (1)$$

or

$$K = \left[1 - \frac{2xy}{aR \sin \phi} \right]^{-1} \quad (2)$$

where
x = distance from transmitter to point of reflection
y = distance from receiver to point of reflection
R = x + y = total range
a = effective earth's radius (usually 4590 n. miles)
 ϕ = angle of incidence of radiation at reflection
Other angles as shown on Fig. 1.

2-2. Equation (2) can be deduced from Equation (1) by remembering that

$$l = a d\alpha = \frac{x \delta \theta_1}{\sin \phi} \quad (3)$$

and

$$\delta \theta_1 - \delta \theta_2 = 2d\alpha = \frac{2x \delta \theta_1}{a \sin \phi} \quad (4)$$

2-3. The form shown in Equation (2) is the more useful and is similar to the divergence factor for reflection at a convex surface that has been in use for some time. Equation (2) shows that K can grow quite large, and even become infinite for certain conditions. Curve 1, Fig. 2, shows a plot of the absolute value of K as a function of b, the height of the layer above the antennas, for a total range of 80 nautical miles. This plot also assumes x = y = 40 miles, which is a necessary condition for a smooth reflector. In

CONFIDENTIAL

- 1 -

DECLASSIFIED

DECLASSIFIED

this case, K becomes infinite for a layer 1100 feet above the antennas. Curve 2, Fig. 2, shows a plot of the layer height, b, necessary to give infinite convergence as a function of the range (plotted on right-hand scale).

3. Roughness Effect

3-1. The most apparent difficulty with the picture presented so far is that the layers actually are not perfectly smooth. In order to try to take that fact into consideration, it was assumed that the layer was composed of a large number of plates set at various small angles about the horizontal according to a Gaussian distribution. As in other parts of this problem, variations are considered only in the plane of transmission, since the effect of sideways deviation would cancel out. This reduces the problem to one of two dimensions only. Each plate is further assumed to retain its original curvature.

3-2. A beam falling on a patch of these plates would be reflected in such a way as to spread the energy at the receiver in a vertical pattern similar to the Gaussian distribution of the plates. This curve is shown on Fig. 3. It is only necessary to integrate this curve over the width of the antenna to find the fraction, L, of the total energy that will be useful. This is illustrated by the shaded area on Fig. 3. Generally, the integration will not come from the exact center of the curve as shown, but will be sufficiently near so that the center value can be used. L will be a function of the probable value of the deviation of the plates, the range, and the antenna width.

3-3. With the rough layer assumption, there will be some plates correctly oriented at each part of the layer to reflect energy into the receiver. Therefore, a third factor, M, must be included that is the ratio of γ/β shown on Fig. 4. γ is the total angle subtended by the layer that can reflect rays to the receiver. γ would be limited by the optical horizons. β is the angle subtended by the receiving antenna when reflection is from a plane surface; i.e., essentially, free space conditions.

3-4. The net convergence factor, C, must be the product of these three quantities, K, L, M. In this case, K must be the mean value of K averaged for various points of reflection. In order to integrate the expression for the mean value of K, it is necessary to substitute for $\sin\theta$ in Equation (2).

$$\sin\theta = \frac{1}{2} \left(\frac{b}{x} + \frac{x}{2a} + \frac{b}{y} + \frac{y}{2a} \right) \quad (5)$$

which gives

$$K = \left[1 - \frac{8(xy)^2}{R^2 (2ab + xy)} \right]^{-1} \quad (6)$$

This expression is easily integrated if the product xy is used for the variable and $x_1 y_1 = x_1 (R - x_1)$

DECLASSIFIED

4. Sample

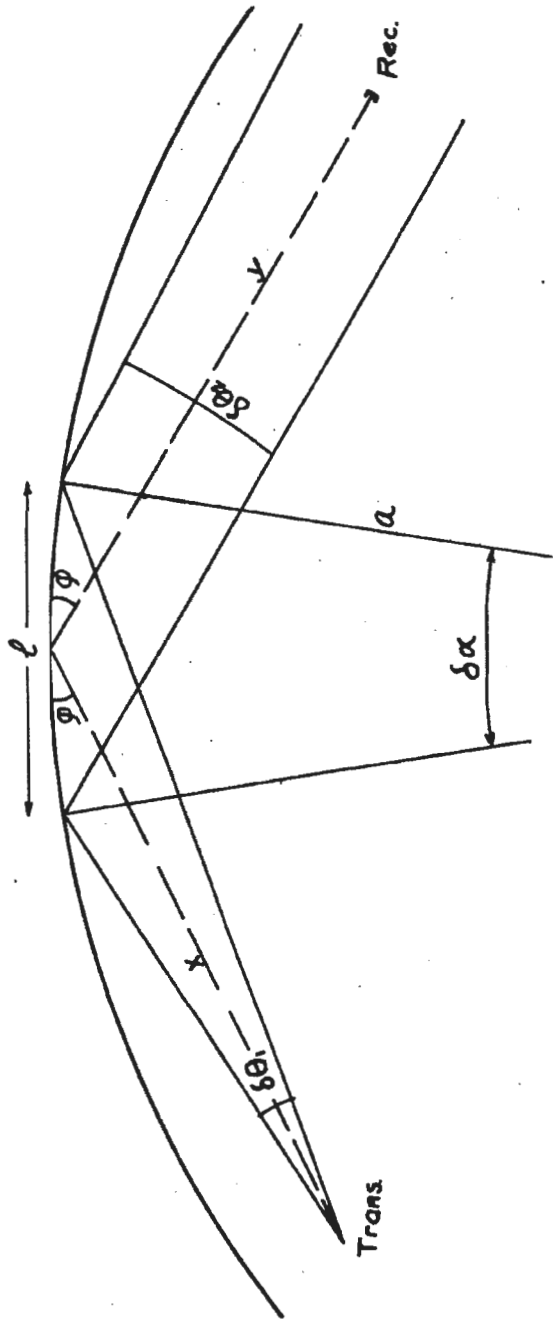
4-1. On Fig. 4 is shown an example. This example is typical of operation of a one-way link of the U. S. Navy Radio & Sound Laboratory at San Diego, that has been extensively studied. High subsidence layers are common for this region. The probable value of the deviation of a reflecting plate from horizontal is taken as 0.1° as an engineering approximation. In this case, C comes out to be 43. This assumes a reflection coefficient of one. If the reflection coefficient is not unity, its value as a function of angle of incidence must be multiplied into the equation.

4-2. Since K, L, and H can each vary through considerable limits, C can vary through a very wide range of values.

5. Conclusions

5-1. The statistical treatment of the roughness is not always applicable, since quite a finite number of plates would actually be engaged in reflecting energy. Hence, the received signal would vary almost randomly with time as the orientation of the plates changed slightly. This could account for marked fading and peaks of large amplitude. Primarily, however, it could explain signals of the magnitude of free space signals or higher.

DECLASSIFIED



$$K = \frac{(x+y)\delta\theta}{x\delta\theta_1 + y\delta\theta_2}$$

$$|K| = \left[1 - \frac{2xy}{a(x+y)\sin\phi} \right]^{-1}$$

Fig. 1

DECLASSIFIED

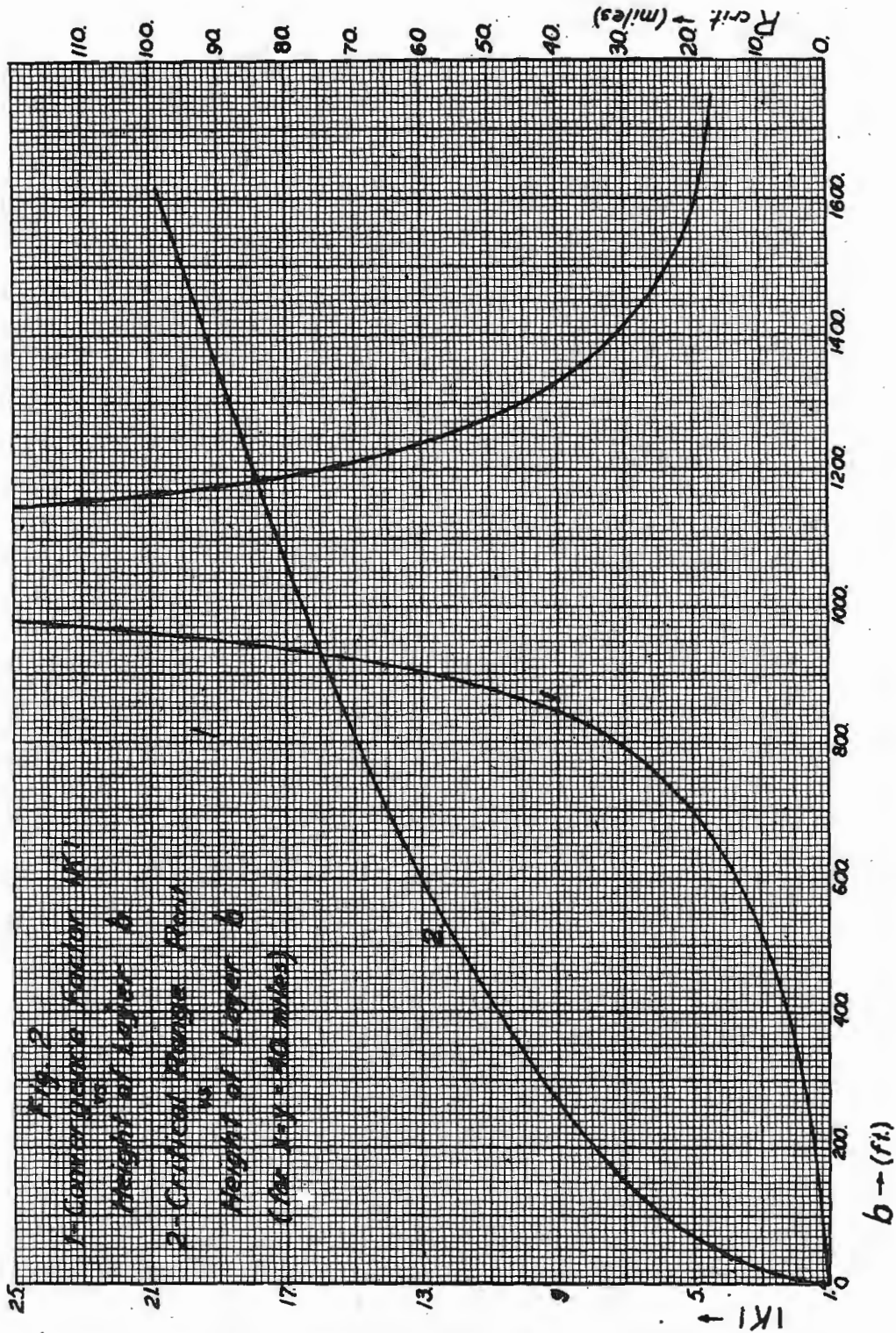
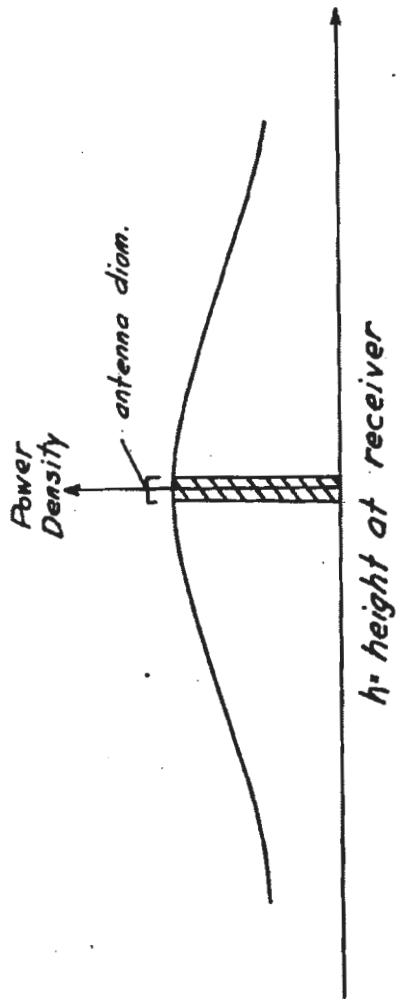


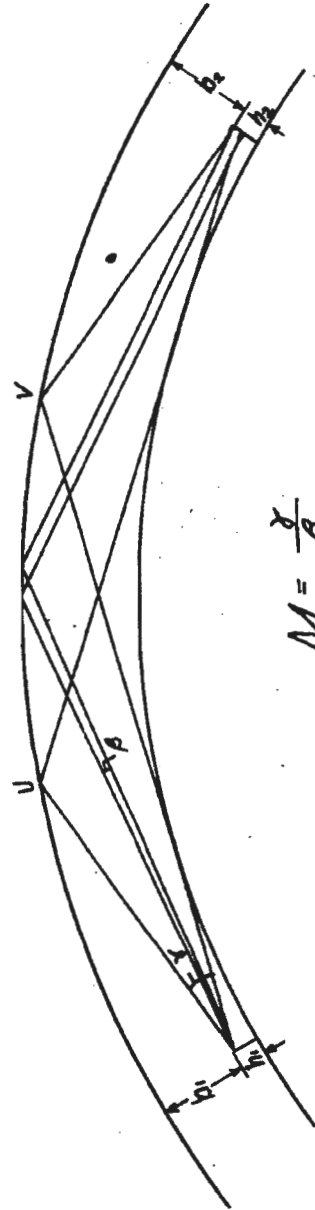
Fig. 3



$$h = (y)(\text{dev. of plate})$$

$$L = \int_{\text{ant. width}} k e^{-h^2} dh$$

Fig. 4



$$M = \frac{\gamma}{\beta}$$

$$C = KLM$$

Example:

$$b_1 = b_2 = .18 \text{ miles} = 1100'$$

$$h_1 = h_2 = 120'$$

$$R = (X+Y) = 80 \text{ miles}$$

Rec. = 5' diam.

Prob. deviation of reflector plate = .1°

$$U = 24 \text{ miles}; v = 56 \text{ miles}$$

$$K = 11.6$$

$$L = .0049$$

$$M = 750.$$

$$C = KLM = 43.$$

DECLASSIFIED

DISTRIBUTION LIST

Copy 2 thru 6 - BuShips, 938
Copy 7 - CominCh, F4
Copy 8 - CominCh, FX-40
Copy 9 - CNO, Op-20F-5
Copy 10 - CNO, Op-20S-4
Copy 11 - CNO, Op-34E
Copy 12 - CNO, Op-25A-2
Copy 13 thru 15 - CNO, Op-35
Copy 16 thru 20 - CNO, Op-16-FA-1 (for transmission 1 copy to ComNavEu,
1 copy to Alusna London, 1 copy to
RNZAF Delegation, 1 copy to Australian
Radio Research Board, 1 copy to retain)

Copy 21 - BuAer, Aer-E-3143
Copy 22 - BuOrd, Re4
Copy 23 - CinCPac
Copy 24 - ComNorPac
Copy 25 - ComThirdFleet
Copy 26 - ComSeventhFleet
Copy 27 - ComEighthFleet
Copy 28 - SONRD
Copy 29 thru 34 - OCSigO, SPSOI-4
Copy 35 thru 37 - Msg Center Branch, G-2 MID
Copy 38 - AAF-ACO
Copy 39 thru 40 - ARL, Wright Field
Copy 41 thru 42 - BAD (for transmission to ASE)
Copy 43 - RAFD (for transmission to TRE)
Copy 44 - BAS (for transmission to ADRDE)
Copy 45 - Joint Intelligence Committee, Canadian Joint Staff
Copy 46 - JEIA
Copy 47 - USNR&SL
Copy 48 - NDRC Committee on Propagation
Copy 49 - NDRC Division 15
Copy 50 - OSRD Liaison Office
Copy 51 - NLO, RRL, Harvard University
Copy 52 thru 53 - NLO, RadLab, MIT
Copy 54 - OSURF
Copy 55 - Department of Physics, Washington State College
Copy 56 - O-in-C Pacific Fleet Radar Center
Copy 57 - USNA, PG School
Copy 58 - Sec'y JCB Wave Propagation Committee
Copy 59 - CNO, Op-20-3 (Lt. Comdr. D. H. Mensel)