REPORT DOCUMENTATION PAGE	Form Approved OMB NO. 0704-0188

1. REPORT DATE (DD-MM-YYYY) 10-07-2022	2. REPORT TYPE Final Report	3. DATES COVERED (From - To) 22-Jul-2018 - 31-Oct-2021

4. TITLE AND SUBTITLE Final Report: Next-generation quickest detection	5a. CONTRACT NUMBER W911NF-18-1-0319
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER 611102
6. AUTHORS	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAMES AND ADDRESSES William Marsh Rice University 6100 Main Street Houston, TX 77005 -1827	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS (ES) U.S. Army Research Office P.O. Box 12211 Research Triangle Park, NC 27709-2211	10. SPONSOR/MONITOR'S ACRONYM(S) ARO 11. SPONSOR/MONITOR'S REPORT NUMBER(S) 71636-MA-YIP.10

12. DISTRIBUTION AVAILIBILITY STATEMENT

Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES

The views, opinions and/or findings contained in this report are those of the author(s) and should not contrued as an official Department of the Army position, policy or decision, unless so designated by other documentation.

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	15. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON Philip Ernst
a. REPORT	b. ABSTRACT	c. THIS PAGE			19b. TELEPHONE NUMBER 713-348-2797
UU	UU	UU	UU		

Standard Form 298 (Rev 8/98)
Prescribed by ANSI Std. Z39.18

RPPR Final Report
as of 11-Jul-2022

Agency Code: 21XD

Proposal Number: 71636MAYIP **Agreement Number: W911NF-18-1-0319**

INVESTIGATOR(S):

Name: Philip Andrew Ernst
Email: pe6@rice.edu
Phone Number: 7133482797
Principal: Y

Organization: **William Marsh Rice University**
Address: 6100 Main Street, Houston, TX 770051827
Country: USA
DUNS Number: 050299031 EIN: 741109620
Report Date: 31-Jan-2022 Date Received: 10-Jul-2022
Final Report for Period Beginning 22-Jul-2018 and Ending 31-Oct-2021
Title: Next-generation quickest detection
Begin Performance Period: 22-Jul-2018 **End Performance Period:** 31-Oct-2021
Report Term: 0-Other
Submitted By: Philip Ernst Email: pe6@rice.edu
Phone: (713) 348-2797
Distribution Statement: 1-Approved for public release; distribution is unlimited.

STEM Degrees: 2 **STEM Participants:** 2

Major Goals: Please see attached PDF.

Accomplishments: Please see attached PDF.

Training Opportunities: The PI has actively mentored two Ph.D. students on this project: Dongzhou Huang (graduated May 2022), who will be starting as Assistant Professor at Colorado State University in August 2022, and Yizhou Xia (graduated December 2019, currently Data Strategist at BP). The PI also actively mentored and collaborated with two of his postdoctoral fellows on this project: Quan Zhou (now Assistant Professor of Statistics, Texas A&M University) and Hongwei Mei (starting as Assistant Professor at Texas Tech University in August 2022). The PI is actively engaged in recruiting underrepresented groups in STEM into his research program.

Results Dissemination: The results have been disseminated in multiple conferences and workshops, highlighted by the following key venues: Institute for Mathematics and its Applications (IMA), Conference on Modeling, Stochastic Control, Optimization, and Related Applications (June 2018), 2018 Summer Research Conference of the Southern Regional Council on Statistics (SRCOS) (June 2018), A Symposium on Optimal Stopping in Honor of Larry Shepp (June 2018), Institute of Mathematical Statistics (IMS) New Researcher’s Conference (July 2018), MSU’s Symposium on Mathematical Statistics and its Applications (October 2018), A Workshop in Memory of Larry Brown (November 2018), Probability and Analysis 2019 (Banach Center, Poland, June 2019), The Fifteenth Latin American Congress of Probability and Mathematical Statistics (December 2019), The INFORMS Annual Meeting (November 2020), Canadian Mathematical Society 75th Anniversary Summer Meeting (June 2021), Seminar on Stochastic Hybrid Systems, University of Connecticut (December 2021), and New Advances in Statistics and Data Science, Honolulu, HI. (May 2022). The results have also been disseminated in over 20 departmental seminars in three countries (USA, UK, and Belgium).

Honors and Awards: INTERNATIONAL AND NATIONAL RESEARCH AWARDS

1) Election to 2022 Committee of Presidents of Statistical Societies (COPSS) Leadership Academy. Award Citation: "for significant contributions of extraordinary merit to applied probability and mathematical statistics, particularly the resolution of the longstanding conjecture of Yule's nonsense correlation, outstanding teaching and leadership."

2) Appointed "Chaire Internationale" (visiting international research chair) at Department of Mathematics, University of Brussels, Summer 2022.

3) 2020 INFORMS Donald P. Gaver, Jr. Early Career Award for Excellence in Operations Research. Award Citation: "for outstanding research accomplishments in operations research, probability and statistics, including solving a nearly 100-year old conjecture by Yule on nonsense correlation; for collaborating to solve outstanding problems in queueing, mathematical finance and optimal control; for outstanding teaching; and for mentoring of PhD students and postdoctoral fellows."

4) 2018 Institute of Mathematical Statistics (IMS) Tweedie New Researcher Award. Award Citation: "for his fundamental contributions to exact distribution theory, in particular for his elegant resolution of the Yule's nonsense correlation problem, and for his development of novel stochastic control techniques for computing the value of insider information in mathematical finance problems."

5) 2018 Army Research Office (ARO) Young Investigator Award.

RICE UNIVERSITY RESEARCH AWARDS

6) 2022: Promoted to Full Professor at Rice University (Three Years Early).

7) 2019: Awarded Early Tenure at Rice (Two Years Early).

8) 2019 Rice University School of Engineering Teaching and Research Excellence Award (awarded annually to two professors at Rice University's George R. Brown School of Engineering).

9) 2018 Rice University Dobelman Family Junior Chair of Statistics.

RICE UNIVERSITY TEACHING AWARDS

10) 2022 George R. Brown Award for Excellence in Teaching (This award is considered Rice University's most prestigious teaching award, and is awarded annually to one professor at Rice University).

11) 2021 George R. Brown Award for Superior Teaching (awarded annually to nine professors at Rice University).

Protocol Activity Status:

Technology Transfer: Nothing to Report

PARTICIPANTS:

Participant Type: PD/PI
Participant: Philip Andrew Ernst
Person Months Worked: 1.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: Graduate Student (research assistant)
Participant: Dongzhou Huang

Person Months Worked: 9.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: Graduate Student (research assistant)
Participant: Yixhou Xia
Person Months Worked: 9.00 **Funding Support:**
Project Contribution:
National Academy Member: N

International Collaboration:

GBR

ARTICLES:

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: Queueing Systems
Publication Identifier Type: DOI Publication Identifier: 10.1007/s11134-018-9587-9
Volume: 90 Issue: First Page #: 207
Date Submitted: 8/4/21 12:00AM Date Published: 9/1/18 10:00AM
Publication Location:
Article Title: Stability and busy periods in a multiclass queue with state-dependent arrival rates
Authors: Philip A. Ernst, Søren Asmussen, John J. Hasenbein
Keywords: Busy periods; Fluid models; Multiclass queues; Regular variation; Stability; State-dependent arrival rates
Abstract: We introduce a multiclass single-server queueing system in which the arrival rates depend on the current job in service. The system is characterized by a matrix of arrival rates in lieu of a vector of arrival rates. Our proposed model departs from existing state-dependent queueing models in which the parameters depend primarily on the number of jobs in the system rather than on the job in service. We formulate the queueing model and its corresponding fluid model and proceed to obtain necessary and sufficient conditions for stability via fluid models. Utilizing the natural connection with the multitype Galton–Watson processes, the Laplace–Stieltjes transform of busy periods in the system is given. We conclude with tail asymptotics for the busy period for heavy-tailed service time distributions for the regularly varying case.
Distribution Statement: 2-Distribution Limited to U.S. Government agencies only; report contains proprietary info
Acknowledged Federal Support: **Y**

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: Stochastic Processes and their Applications
Publication Identifier Type: DOI Publication Identifier: 10.1016/j.spa.2021.06.003
Volume: Issue: First Page #:
Date Submitted: 8/5/21 12:00AM Date Published: 6/1/21 10:00AM
Publication Location:
Article Title: Escape and absorption probabilities for obliquely reflected Brownian motion in a quadrant
Authors: Philip A. Ernst, Sandro Franceschi, Dongzhou Huang
Keywords: Escape and absorption probability; Obliquely reflected Brownian motion in a quadrant; Functional equation; Carleman boundary value problem; Neumann's condition; Asymptotics
Abstract: We consider an obliquely reflected Brownian motion Z with positive drift in a quadrant stopped at time T, where $T:=\inf \{ t>0 : Z(t)=(0,0) \}$ is the first hitting time of the origin. Such a process can be defined even in the non-standard case where the reflection matrix is not completely-$\mathcal{S}$. We show that in this case the process has two possible behaviors: either it tends to infinity or it hits the corner (origin) in a finite time. Given an arbitrary starting point (u,v) in the quadrant, we consider the escape (resp. absorption) probabilities $\mathbb{P}_{(u,v)}[T=\infty]$ (resp. $\mathbb{P}_{(u,v)}[T<\infty]$). We establish the partial differential equations and the oblique Neumann boundary conditions which characterize the escape probability and provide a functional equation satisfied by the Laplace transform of the escape probability. We then give asymptotics for the absorption probability in the simpler case where the starting point in the quadrant is $(u,0)$.
Distribution Statement: 2-Distribution Limited to U.S. Government agencies only; report contains proprietary info
Acknowledged Federal Support: **Y**

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: Stochastic Processes and their Applications
Publication Identifier Type: DOI Publication Identifier: 10.1016/j.spa.2021.05.009
Volume: Issue: First Page #:
Date Submitted: 8/5/21 12:00AM Date Published: 6/1/21 5:00AM
Publication Location:
Article Title: Fiscal stimulus as an optimal control problem
Authors: Philip A. Ernst, Michael B. Imerman, Larry Shepp, Quan Zhou
Keywords: Dividend problem; Radner–Shepp model; Fiscal stimulus; Financial strategy; Hamilton–Jacobi–Bellman equation; Stochastic control
Abstract: During the Great Recession, Democrats in the United States argued that government spending could be utilized to "grease the wheels" of the economy in order to create wealth and to increase employment; Republicans, on the other hand, contended that government spending is wasteful and discourages investment, thereby increasing unemployment. This past year we have found ourselves in the midst of another crisis where government spending and fiscal stimulus is again being considered as a solution. In the present paper, we address this question by formulating an optimal control problem generalizing the model of Radner and Shepp (1996). The model allows for the company to borrow continuously from the government. We prove that there exists an optimal strategy; rigorous verification proofs for its optimality are provided. We proceed to prove that government loans increase the expected value of a company.
Distribution Statement: 2-Distribution Limited to U.S. Government agencies only; report contains proprietary info
Acknowledged Federal Support: **Y**

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: Stochastic Processes and their Applications
Publication Identifier Type: DOI Publication Identifier: 10.1016/j.spa.2020.08.010
Volume: Issue: First Page #:
Date Submitted: 8/5/21 12:00AM Date Published: 9/1/20 5:00AM
Publication Location:
Article Title: The rencontre problem
Authors: F. Thomas Bruss, Philip A. Ernst, Dongzhou Huang
Keywords: Hitting times; intersections of random walks; rencontre times.
Abstract: Let $\left\{X^{1}_k\right\}_{k=1}^{\infty}, \left\{X^{2}_k\right\}_{k=1}^{\infty}, \cdots, \left\{X^{d}_k\right\}_{k=1}^{\infty}$ be d independent sequences of Bernoulli random variables with success-parameters $p_1, p_2, \cdots, p_d$ respectively, where $d \geq 2$ is a positive integer, and $ 0<p_j<1$ for all $j=1,2,\cdots,d.$ Let \begin{equation*} S^{j}(n) = \sum_{i=1}^{n} X^{j}_{i} = X^{j}_{1} + X^{j}_{2} + \cdots + X^{j}_{n}, \quad n =1,2 , \cdots. \end{equation*} %denote the corresponding random walk for the $\left\{X^{j}_{k}\right\}_{k=1}^{\infty}$, $j=1,2,\cdots,d$. We declare a ``rencontre'' at time $n$, or, equivalently, say that $n$ is a ``rencontre time,'' if \begin{equation*} S^{1}(n) = S^{2}(n) = \cdots = S^{d}(n). \end{equation*} We motivate and study the distribution of the \textit{first} (provided it is finite) rencontre time.
Distribution Statement: 2-Distribution Limited to U.S. Government agencies only; report contains proprietary info
Acknowledged Federal Support: **Y**

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: Stochastic Processes and their Applications
Publication Identifier Type: DOI Publication Identifier: 10.1016/j.spa.2019.09.017
Volume: 130 Issue: 6 First Page #: 3394
Date Submitted: 8/5/21 12:00AM Date Published: 6/1/20 5:00AM
Publication Location:
Article Title: When is it best to follow the leader?
Authors: Philip A. Ernst, L.C.G. Rogers, Quan Zhou
Keywords: Follow the leader; optimal scanning; quickest search; Tanaka's stochastic differential equation; first exit time.
Abstract: An object is hidden in one of N boxes. Initially, the probability that it is in box i is $\pi_i(0)$. You then search in continuous time, observing box J_t at time t, and receiving a signal as you observe: if the box you are observing does not contain the object, your signal is a Brownian motion, but if it does contain the object your signal is a Brownian motion with positive drift μ. It is straightforward to derive the evolution of the posterior distribution $\pi(t)$ for the location of the object. If T denotes the first time that one of the $\pi_j(t)$ reaches a desired threshold $1-\varepsilon$, then the goal is to find a search policy $(J_t)_{t \geq 0}$ which minimizes the mean of T. This problem was studied by \citet{posner1966continuous} and by \citet{zigangirov1966problem}, who derive an expression for the mean time of a conjectured optimal policy, which we call {\em follow the leader} (FTL); at all times, observe the box with the highest posterior probability.
Distribution Statement: 2-Distribution Limited to U.S. Government agencies only; report contains proprietary info
Acknowledged Federal Support: **Y**

RPPR Final Report
as of 11-Jul-2022

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: Annals of Applied Probability
Publication Identifier Type: Publication Identifier:
Volume: Issue: First Page #:
Date Submitted: 8/5/21 12:00AM Date Published: 12/1/21 6:00AM
Publication Location:
Article Title: Asymptotic behavior of the occupancy density for obliquely reflected Brownian motion in a half-plane and Martin boundary
Authors: Philip A. Ernst and Sandro Franceschi
Keywords: Occupancy density; Green's function; Obliquely reflected Brownian motion in a half-plane; Stationary distribution; Exact Asymptotics; Martin boundary; Laplace transform; Saddle-point method.
Abstract: Let $\pii$ be the occupancy density %(or the density of the Green's function) of an obliquely reflected Brownian motion in the half plane and let (ρ,α) be the polar coordinates of a point in the upper half plane. This work determines the exact asymptotic behavior of $\pii (\rho,\alpha)$ as $\rho\to \infty$ with $\alpha\in (0,\pi)$. We find explicit functions a,b,c such that $$ \pii (\rho,\alpha) \underset{\rho\to\infty}{\sim} a(\alpha) \rho^{b(\alpha)} e^{-c(\alpha)\rho}. $$ This closes an open problem first stated by Professor J. Michael Harrison in August 2013. We also compute the exact asymptotics for the tail distribution of the boundary occupancy measure and we obtain an explicit integral expression for $\pii$. {We conclude by finding the Martin boundary of the process and giving} all of the corresponding harmonic functions satisfying an oblique Neumann boundary problem.
Distribution Statement: 2-Distribution Limited to U.S. Government agencies only; report contains proprietary info
Acknowledged Federal Support: **Y**

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: The Annals of Applied Probability
Publication Identifier Type: DOI Publication Identifier: 10.1214/19-AAP1522
Volume: 30 Issue: 3 First Page #:
Date Submitted: 8/5/21 12:00AM Date Published: 6/1/20 5:00AM
Publication Location:
Article Title: Optimal real-time detection of a drifting Brownian coordinate
Authors: P. A. Ernst, G. Peskir, Q. Zhou
Keywords: Brownian motion, elliptic partial differential equation, free-boundary problem, nonlinear Fredholm integral equation, nonmonotone boundary, Optimal detection, Optimal stopping, sequential testing, smooth fit, the change-of-variable formula with local time on surfaces
Abstract: Consider the motion of a Brownian particle in three dimensions, whose two spatial coordinates are standard Brownian motions with zero drift, and the remaining (unknown) spatial coordinate is a standard Brownian motion with a (known) nonzero drift. Given that the position of the Brownian particle is being observed in real time, the problem is to detect as soon as possible and with minimal probabilities of the wrong terminal decisions, which spatial coordinate has the nonzero drift. We solve this problem in the Bayesian formulation, under any prior probabilities of the nonzero drift being in any of the three spatial coordinates, when the passage of time is penalised linearly. Finding the exact solution to the problem in three dimensions, including a rigorous treatment of its nonmonotone optimal stopping boundaries, is the main contribution of the present paper. To our knowledge this is the first time that such a problem has been solved in the literature.
Distribution Statement: 2-Distribution Limited to U.S. Government agencies only; report contains proprietary info
Acknowledged Federal Support: **Y**

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 3-Accepted
Journal: The Annals of Applied Probability (to appear)
Publication Identifier Type: Publication Identifier:
Volume: Issue: First Page #:
Date Submitted: 7/9/22 12:00AM Date Published:
Publication Location:
Article Title: Quickest real-time detection of a Brownian coordinate drift
Authors: Philip A. Ernst and Goran Peskir
Keywords: Quickest detection, Brownian motion, optimal stopping, elliptic partial differential equation, free-boundary problem, smooth fit, nonlinear Fredholm integral equation, the change-of-variable formula with local time on surfaces

Abstract: Consider the motion of a Brownian particle in two or more dimensions, whose coordinate processes are standard Brownian motions with zero drift initially, and then at some random/unobservable time, one of the coordinate processes gets a (known) non-zero drift permanently. Given that the position of the Brownian particle is being observed in real time, the problem is to detect the time at which a coordinate process gets the drift as accurately as possible. We solve this problem in the most uncertain scenario when the random/unobservable time is (i) exponentially distributed and (ii) independent from the initial motion without drift. The solution is expressed in terms of a stopping time that minimises the probability of a false early detection and the expected delay of a missed late detection. To our knowledge this is the first time that such a problem has been solved exactly in the literature.

Distribution Statement: 4-Distribution authorized to the Department of Defense and U.S. DoD contractors only
Acknowledged Federal Support: **Y**

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 3-Accepted
Journal: Mathematics of Operations Research (to appear)
Publication Identifier Type: Publication Identifier:
Volume: Issue: First Page #:
Date Submitted: 7/9/22 12:00AM Date Published:
Publication Location:
Article Title: Exact optimal stopping for multidimensional linear switching diffusions
Authors: Philip A. Ernst and Hongwei Mei
Keywords: Quickest detection, switching diffusions, optimal stopping, free-boundary problem

Abstract: The paper studies a class of multidimensional optimal stopping problems with infinite horizon for linear switching diffusions. There are two main novelties in the optimal problems considered: the underlying stochastic process has discontinuous paths and the cost function is not necessarily integrable on the entire time horizon, where the latter is often a key assumption in classical optimal stopping theory for diffusions, cf. [22, Corollary 2.9]. Under relatively mild conditions, we show, for the class of multidimensional optimal stopping problems under consideration, that the first entry time of the stopping region is an optimal stopping time. Further, we prove that the corresponding optimal stopping boundaries can be represented as the unique solution to a nonlinear integral equation. We conclude with an application of our results to the problem of quickest real-time detection of a Markovian drift.

Distribution Statement: 4-Distribution authorized to the Department of Defense and U.S. DoD contractors only
Acknowledged Federal Support: **Y**

Partners

,

I certify that the information in the report is complete and accurate:
Signature: Philip A. Ernst
Signature Date: 7/10/22 12:38AM

Final Technical Report
ARO-YIP-71636-MA
“Next-generation quickest detection”

PI: Philip A. Ernst, Ph.D.
Professor of Statistics (with tenure)
Rice University

July 9, 2022

Abstract

This is the final technical report for ARO-YIP-71636-MA “Next-generation quickest detection.”

1 Major goals

The four key objectives of ARO-YIP-71636-MA are as follows:

(I) A solution to Posner and Rumsey’s 1966 optimal search problem ([10]). This problem was first formulated in the context of optimizing engineering systems designed to search for satellite signals. The problem is stated as follows: consider an object to be in one of n boxes. Let the boxes be indexed by J, where $J = 1, ..., n$ with given prior probabilities, $\pi_i = \pi_i(0) = \mathbb{P}\left(J = i\right), i = 1, ..., n$, which may or may not initially all be $1/n$. One may search in any box by any search rule, $I(t) = i$ at any time $t \geq 0$, so long as I is measurable with respect to past observations Time may be considered to be either discrete or continuous; if it is continuous, infinite rapid switching between boxes is permitted. If one searches in the “correct” box, then an observable process $X(t)$ of Brownian motion with drift is observed; otherwise, the observed process $X(t)$ is standard Brownian motion. Let τ^I denote the first time $t < \infty$ at which the posterior probability vector $\vec{\pi}(t) = (\pi_1(t), ..., \pi_n(t))$ has some component $\pi_j(t)$ with $\pi_j(t) \geq 1-\alpha$. Then at time $\tau^I < \infty$, we have that $\mathbb{P}\left(J = j\right) \geq 1 - \alpha$. The task is to find the optimal search rule I which minimizes this expected time to achieve this level of certainty.

(II) The development of an explicit stopping role for the following “optimal real-time detection” problem. Consider the motion of a Brownian particle in three dimensions, whose two spatial coordinates are standard Brownian motions with zero drift, and the remaining (unknown) spatial coordinate is a standard Brownian motion with a non-zero drift. The task is to detect as soon as possible (and with minimal probabilities of the wrong terminal decisions) which spatial coordinate has the non-zero drift. This problem may be viewed as a reformulation of Posner and Rumsey's problem ([10]) from Objective I in which the user receives a signal from each box at every given point in time, regardless of whether the user observes that box.

We provide a visual illustration of Objective II in Figure 1 below.

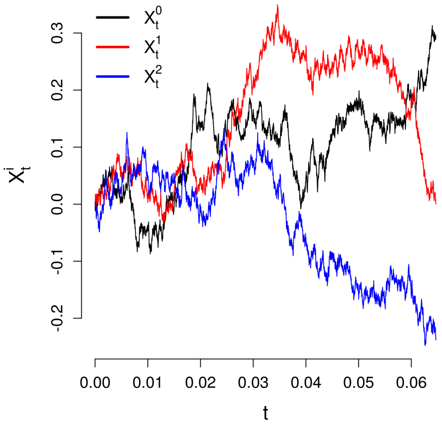

Figure 1: Three coordinates of the Brownian motion (two without drift, and one with drift) begin at time 0 and evolve with time. The evolution of coordinate 0 appears in black, the evolution of coordinate 1 appears in blue, and the evolution of coordinate 2 appears red. Unbeknownst to the viewer, the black coordinate (given by the process X_t^0) has been pre-programmed to be the coordinate with positive drift.

(III) A solution to the following "quickest detection" problem: consider the motion of a Brownian particle in two or more dimensions, whose coordinate processes are standard Brownian motions with zero drift initially, and then at some random/unobservable time, one of the coordinate processes gets a (known) non-zero drift permanently. The problem is to detect the time at which a coordinate process gets the drift as 'accurately' as possible.

Objective III is illustrated by Figure 2 below. The objective of the quickest detection problem in Figure 2 is to determine the time θ as accurately as possible.

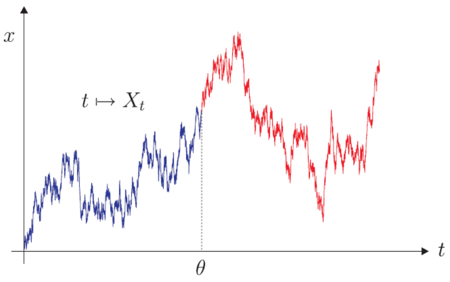

Figure 2: A sample path of one-dimensional Brownian motion, X_t. It begins as Brownian motion without drift (this is the process in blue). At some random time θ (which we assume is exponentially distributed) the process gets a positive drift μ and becomes the process in red.

We also plot Figure 3, which offers a higher-dimensional analog of Figure 2.

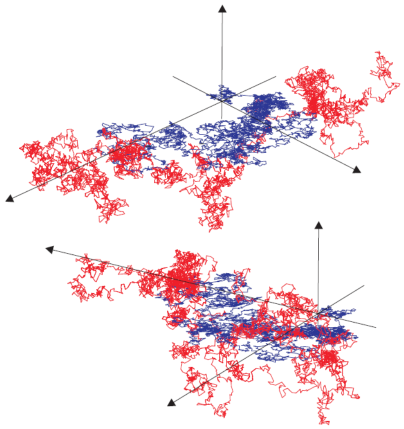

Figure 3: This figure depicts the motion of a Brownian particle (viewed from two angles) that initially takes place in a two-dimensional plane (blue line) and then after some random/unobservable time θ continues in the three-dimensional space (red line). The aim of the present quickest detection problem would be to determine the time θ at which the particle departs from the plane as accurately as possible.

(IV) The purpose of Objective IV is to generalize Objective III. In Objective IV, we consider the motion of a Brownian particle in n dimensions ($n \geq 2$), whose coordinate processes are standard Brownian motions with zero drift initially. At some random/unobservable time, k out of n coordinate processes ($1 \leq k \leq n$) obtain a (known) non-zero drift μ permanently. Given that the position of the Brownian particle is being observed in real time, we seek to detect the time at which the k coordinate processes get the drift as accurately as possible. Objective III therefore solves the special case where $k = 1$.

Applications to U.S. Army

All objectives have been directly motivated by applications of direct relevance to the U.S. Army. Objectives I-III are specifically motivated by the following two applications:

(i) In October 2016, the White House issued an executive order to coordinate efforts to prepare for space weather effects ([18]). The executive order addresses the harm to satellites from space weather effects (in particular, the threat caused by cosmic rays, solar flames, and solar particles). Upon detection of cosmic rays (which we assume may be appropriately modelled by diffusions), satellites are to be covered with a shield to protect them from potential enormous destruction from cosmic rays. The current systems in place that have been designed to protect satellites are noticeably suboptimal, often leading to satellites being shut down for longer periods of time than necessary. The developments proposed in Objectives I-III should prove to be important in optimizing existing navigation algorithms.

(ii) When a plane is in the process of landing, the navigator needs to know its exact position in relation to the ground. This position is determined through communication with satellites. For this determination to be successful, the exact time at each satellite, determined by an atomic clock, must be known. These atomic clocks are extremely precise (in terms of nanoseconds) but they can also cease to function, often spontaneously and unpredictably so. The breakdown point can be considered a change-point that is not directly observable (see [17]). If the change-point is not detected within a short period of time (measured in seconds) after the plane enters a critical height, the plane may end up at a point of no return and potentially crash.

Objectives III and IV are motivated by the potential need for "parallel quickest detection" in army platoons. Specifically: when considering a platoon of soldiers, suppose that we are interested in detecting whether an individual soldier has become "drowsy." One may ask as follows: can we do "better" at detecting this soldier's drowsiness if we monitor his/her colleagues simultaneously? It may very well be that in the past few hours, the soldiers have eaten a contaminated meal that will inevitably lead to food poisoning; alternatively, it may well be that the entire platoon has been exposed to a virus. If so, it may be the case that the soldier of interest becomes drowsy from one of the aforementioned communal occurrences than because of any individual-level occurrence. Following from the setup in Objective IV, suppose that we have 100 platoon members and that we are given a dependence structure for the underlying 100 Brownian motions (each corresponding to an individual soldier). These 100 Brownian motions shall then be assumed to drive the corresponding 100 diffusion processes. Suppose now that the task is to determine when 15 or more of the coordinates (soldiers) change from being "healthy" to "drowsy." Although it is nearly impossible to give closed-form solutions to this problem in dimensions higher than three, numerical methods should prove amenable to obtaining reliably approximate solutions.

2 Accomplished Under Goals

2.1 Objective I

Objective I was fulfilled with our 2020 *Stochastic Processes and their Applications* paper ([6]). This paper studies a classical search problem first considered by [10].

As mentioned above, an object is hidden in one of N boxes; we denote by j^* the index of the true box. Initially,

$$P(j^* = i) = \pi_i(0). \tag{1}$$

We then observe in continuous time, choosing to search box J_t at time t. We see a signal process Y whose dynamics are

$$dY(t) = dW(t) + \mu I_{\{J_t = j^*\}}\, dt, \tag{2}$$

where $\mu > 0$ is a known constant, and W is a Brownian motion. It is straightforward to derive the evolution of $\pi(t)$, the posterior distribution at time t (see equation (10) of [6]). The objective initially proposed by Posner and Rumsey ([10]) is to choose $(J_t)_{t \geq 0}$ to minimize the expected value of T, where

$$T = \inf\{t : \max_j \pi_j(t) \geq 1 - \varepsilon\}, \tag{3}$$

where $\varepsilon \in (\frac{1}{2}, 1)$ is some desired error bound.

Posner and Rumsey ([10]) derived an expression for the mean time of a conjectured optimal policy, which we call *follow the leader* (FTL); at all times, observe the box with the highest posterior probability. Posner and Rumsey asserted without proof that this is optimal, and Zigangirov ([16]) offered a proof that if the prior distribution is uniform then FTL is optimal. The FTL has since been (incorrectly) assumed to be the optimal policy.

Although we were ultimately unable to find the unique optimal policy, our paper [6] disproves the above *fifty year old* conjecture by showing that if the prior is not uniform, then FTL is *not* always optimal; for the case of the uniform prior, the question remains open. Our main result in Section 5 of the paper provides counterexamples that clearly show FTL is not optimal for some specific values of $(\pi_1(0), \ldots, \pi_N(0))$. An additional contribution is the characterization of the solution to a class of stochastic differential equations, which plays a key role in our calculations, and can be considered to be generalizations of Tanaka's SDE.

Our work gives rise to several open problems. First, what is the optimal strategy for this optimal scanning problem for any prior distribution? For decades, it has been (incorrectly) assumed that FTL is optimal. Indeed, as we have shown, FTL is sub-optimal at least for some values of $(\pi_1(0), \ldots, \pi_N(0))$. Another open problem concerns whether FTL is optimal for the case of uniform prior distribution. So the answer to the question in the title of [6] is: 'We don't know, but we know that it is not *always* best to follow the leader!'

2.2 Objective II

Objective II was fulfilled with our 2020 paper entitled "Optimal real-time detection of a drifting Brownian coordinate" ([5]), published in the *The Annals of Applied Probability*. This work begins by mathematizing Objective II above. We assume that one observes a sample path of the three-dimensional Brownian motion $X = (X^0, X^1, X^2)$, whose two coordinates X^j and X^k are standard Brownian motions with zero drift, and the remaining (unknown) coordinate X^i is a standard Brownian motion having a (known) non-zero drift μ with a probability $\pi_i \in [0,1]$ for $i = 0,1,2$ where $\pi_0+\pi_1+\pi_2 = 1$ and $i \neq j \neq k$ belong to $\{0,1,2\}$. The problem is to detect which coordinate is drifting as soon as possible and with minimal probabilities of the wrong terminal decisions. Standard arguments imply that the above setting can be realized on a probability space $(\Omega, \mathcal{F}, \mathsf{P}_\pi)$ with the probability measure P_π decomposed as follows

$$\mathsf{P}_\pi = \pi_0 \mathsf{P}_0 + \pi_1 \mathsf{P}_1 + \pi_2 \mathsf{P}_2 \tag{4}$$

for $\pi = (\pi_0, \pi_1, \pi_2) \in [0,1]^3$ satisfying $\pi_0 + \pi_1 + \pi_2 = 1$, where P_i is the probability measure under which the observed process X has the i-th coordinate equal to a standard Brownian motion with drift μ, and the remaining two coordinates are standard Brownian motions with zero drift for $i = 0,1,2$, with the three coordinates being independent. This can be formally achieved by introducing an unobservable random variable θ taking values $0,1,2$ with probabilities π_0, π_1, π_2 in $[0,1]$ satisfying $\pi_0+\pi_1+\pi_2 = 1$ and being independent from three (mutually independent) standard Brownian motions B^0, B^1, B^2 so that $X = (X^0, X^1, X^2)$ after starting at a point in $\mathbb{R}^3$ solves the system of stochastic differential equations

$$dX_t^i = \mu I(\theta = i)\, dt + dB_t^i \tag{5}$$

for $i = 0,1,2$.

Being based upon the continued observation of X, the problem is to test sequentially the hypotheses $H_0 : \theta = 0$, $H_1 : \theta = 1$, $H_2 : \theta = 2$ with a minimal loss. For this, we are given a sequential decision rule (τ, d_τ), where τ is a stopping time of X (i.e. a stopping time with respect to the natural filtration $\mathcal{F}_t^X = \sigma(X_s \,|\, 0 \leq s \leq t)$ of X for $t \geq 0$), and d_τ is an $\mathcal{F}_\tau^X$-measurable random variable taking values in the set $\{0,1,2\}$. After stopping the observation of X at time τ, the terminal decision function d_τ takes value i if and only if the hypothesis H_i is to be accepted for $i = 0,1,2$. With a constant $c > 0$ given and fixed, our problem then becomes to compute the risk function

$$V(\pi) = \inf_{(\tau, d_\tau)} \mathsf{E}_\pi \Big[\tau + c \Big(I(\theta = 0, d_\tau \neq 0) + I(\theta = 1, d_\tau \neq 1) + I(\theta = 2, d_\tau \neq 2) \Big) \Big] \tag{6}$$

for $\pi = (\pi_0, \pi_1, \pi_2) \in [0,1]^3$ with $\pi_0+\pi_1+\pi_2 = 1$ and find the optimal decision rule $(\tau_*, d^*_{\tau_*})$ at which the infimum in (6) is attained. Note that $\mathsf{E}_\pi(\tau)$ in (6) is the expected waiting time until the terminal decision is made, and $\mathsf{P}_\pi(\theta = i, d_\tau \neq i)$ are probabilities of the wrong terminal decisions for $i = 0,1,2$. Clearly, each probability $\mathsf{P}_\pi(\theta = i, d_\tau \neq i)$ could be further decomposed into the sum of two probabilities $\mathsf{P}_\pi(\theta = i, d_\tau = j)$ and $\mathsf{P}_\pi(\theta = i, d_\tau = k)$ for $i = 0,1,2$ and $i \neq j \neq k$ in $\{0,1,2\}$, and each of the six resulting probabilities could have a different constant/weight placed in front of them. Standard arguments show that the initial optimization problem can be reduced to an optimal stopping problem (see equation (3.8) in [5]) for the posterior probability process $\mathit{\Pi}$ of the non-zero drift being in the spatial coordinates given X. In (12.25) of [5], we expose the exact solution to the optimal stopping problem. Of course, the solution depends strongly on c (see equation (12.4) of [5]). Returning to Figure 1 above, we note that at the present evolution of the coordinates in the figure, the optimal stopping rule in equation (12.25) of [5] does not declare the "pre-programmed" black coordinate to be the coordinate with drift. This is sensible since only .06 units of time (the length of the window in the figure) have elapsed.

In contrast to all the sequential testing problems studied to date, our paper's key contribution is that the two-dimensional Markov diffusion process Π in the sequential testing problem of [5] has the infinitesimal generator of *elliptic* type. Moreover, the optimal stopping boundaries are *non-monotone* as functions of the coordinate variables. To the best of our knowledge, no rigorous treatment of non-monotone optimal stopping boundaries (curves) had been previously studied in the probabilistic literature. Finding the exact solution to the problem for X in three dimensions (exposed in Corollary 20), including a rigorous treatment of its non-monotone optimal stopping boundaries, represents our key contribution.

Our work gives rise to several open problems in four and more dimensions. The analogous problem for X in four/more dimensions introduces substantial challenges for a rigorous treatment of 'non-monotone' optimal stopping boundaries (surfaces).

2.3 Objective III

Objective III was fulfilled with our 2022 paper ([4]), to appear in *The Annals of Applied Probability*. We begin our discussion by mathematizing Objective III. Assume that one observes a sample path of the standard two-dimensional Brownian motion $X = (X^1, X^2)$, whose coordinate processes X^1 and X^2 are standard Brownian motions with zero drift initially, and then at some random/unobservable time θ taking value 0 with probability $\pi \in [0,1]$ and being exponentially distributed with parameter $\lambda > 0$ given that $\theta > 0$, one of the coordinate processes X^1 and X^2 gets a (known) non-zero drift μ permanently.[1] The problem is to detect the time θ at which a coordinate process gets the drift μ as accurately as possible (neither too early nor too late). The observed process $X = (X^1, X^2)$ solves the stochastic differential equations

$$dX^1_t = \mu I(\beta = 1, t \geq \theta)\, dt + dB^1_t \tag{7}$$

$$dX^2_t = \mu I(\beta = 2, t \geq \theta)\, dt + dB^2_t \tag{8}$$

driven by a standard two-dimensional Brownian motion $B = (B^1, B^2)$ under the probability measure P_π specified below, where the random variable β satisfies $\mathsf{P}_\pi(\beta = 1) = p_1$ and $\mathsf{P}_\pi(\beta = 2) = p_2$ for some $p_1, p_2 \in [0,1]$ with $p_1 + p_2 = 1$ given and fixed, meaning that $\beta = i$ if and only if the coordinate process X_i gets drift μ at time θ with probability p_i for $i = 1, 2$. The unobservable time θ, the unknown coordinate β, and the driving Brownian motion B are all assumed to be independent under P_π for $\pi \in [0,1]$ given and fixed.

Standard arguments imply that the previous setting can be realized on a probability space $(\Omega, \mathcal{F}, \mathsf{P}_\pi)$ with the probability measure P_π being decomposable as follows

$$\mathsf{P}_\pi = p_1 \pi\, \mathsf{P}^0_1 + p_2 \pi\, \mathsf{P}^0_2 + p_1 (1-\pi) \int_0^\infty \lambda e^{-\lambda t}\, \mathsf{P}^t_1\, dt + p_2 (1-\pi) \int_0^\infty \lambda e^{-\lambda t}\, \mathsf{P}^t_2\, dt \tag{9}$$

---

[1] For simplicity, we assume that the observed process is two-dimensional. The framework of [4] allows for this assumption to be easily extended to three or more dimensions.

for $\pi \in [0,1]$ where P_i^t is the probability measure under which the coordinate process X^i gets drift μ at time $t \in [0,\infty)$ for $i = 1,2$. The decomposition (9) expresses the fact that the unobservable time θ is a non-negative random variable satisfying $\mathsf{P}_\pi(\theta = 0) = \pi$ and $\mathsf{P}_\pi(\theta > t \,|\, \theta > 0) = e^{-\lambda t}$ for $t > 0$. Thus $\mathsf{P}_i^t(X \in \cdot\,) = \mathsf{P}_\pi(X \in \cdot \,|\, \beta = i, \theta = t)$ is the probability law of the standard two-dimensional Brownian motion process $X = (X^1, X^2)$ whose coordinate process X^i gets drift μ at time $t \in [0,\infty)$ for $i = 1,2$. Moreover, by P_i we denote the probability measure under which the coordinate process X^i gets drift μ at time θ for $i = 1,2$. From (9) we see that

$$\mathsf{P}_\pi = p_1 \mathsf{P}_1 + p_2 \mathsf{P}_2 \tag{10}$$

where $\mathsf{P}_i = \pi \mathsf{P}_i^0 + (1-\pi)\int_0^\infty \lambda e^{-\lambda t}\, \mathsf{P}_i^t \, dt$ for $i = 1,2$ and $\pi \in [0,1]$.

Being based upon continuous observation of $X = (X^1, X^2)$, the problem then becomes to find a stopping time τ_* of X (i.e. a stopping time with respect to the natural filtration $\mathcal{F}_t^X = \sigma(X_s \,|\, 0 \le s \le t)$ of X for $t \ge 0$) that is 'as close as possible' to the unknown time θ. More precisely, we seek to compute the value function

$$V(\pi) = \inf_\tau \Big[\mathsf{P}_\pi(\tau < \theta) + c\, \mathsf{E}_\pi(\tau - \theta)^+ \Big] \tag{11}$$

and find the optimal stopping time τ_* at which the infimum in (11) is attained for $\pi \in [0,1]$ and $c > 0$ given and fixed (recalling also that $p_1, p_2 \in [0,1]$ with $p_1 + p_2 = 1$ are given and fixed). We note in (11) that $\mathsf{P}_\pi(\tau < \theta)$ is the probability of the *false alarm* and $\mathsf{E}_\pi(\tau - \theta)^+$ is the expected *detection delay* associated with a stopping time τ of X for $\pi \in [0,1]$. The linear combination of the probability of the *false alarm* and the expected *detection delay* for quickest detection dates back to [11] and has been extensively studied to date (see [12] and the references therein). We provide the solution for the optimal stopping time τ_* in (8.17) (Corollary 6) of [4].

In contrast to two-dimensional quickest detection problems solved to date (see, i.e. [1, 2, 7, 15] the multi-dimensional Markov diffusion process $\varPhi$ in the quickest detection problem of our work ([4]) has the infinitesimal generator of *elliptic* type. Finding the *exact* solution to the quickest detection problem for the observed process X in *two or more* dimensions is the main contribution of [4].

2.4 Objective IV

Objective IV was fulfilled in the preprint of Ernst, Mei, and Peskir (2022). As we did in Objective III, we continue with a Bayesian formulation of the quickest detection problem. We assume that one observes a sample path of the n-dimensional standard Brownian motion $X = (X^1, \cdots, X^n)$, whose coordinate processes are standard Brownian motions with zero drift initially, and then at some random/unobservable time θ taking value 0 with probability $\pi \in [0,1)$ and being exponentially distributed with parameter $l > 0$ given that $\theta > 0$, $1 \le k \le n$ of the coordinate processes X get a (known) non-zero drift μ permanently. As before, the problem is to detect the time θ at which the two coordinate processes obtain the drift μ as accurately as possible (neither too early nor too late).

To identify all the possible combinations of which k coordinate processes get the drift μ, we consider the set

$$\Upsilon_k^n := \Big\{ \{n_1,\cdots,n_k\} \Big| \{n_1,\cdots,n_k\} \subset \{1,\cdots,n\} \Big\},$$

where $I := \binom{n}{k}$ is the number of elements in Υ_k^n. Let $\beta : \Omega \mapsto \Upsilon_k^n$ be the random variable denoting the coordinate processes which obtain- the drift μ. Then the observed process X solves the following stochastic differential equations

$$dX_t^j = \mu I(j \in \beta, t \geq \theta)dt + dB_t^j, \text{ for } j = 1,\cdots,n\,, \tag{12}$$

where $B = (B^1,\cdots,B^n)$ is a n-dimensional standard Brownian motion under the probability measure P_π specified below. The distribution of β is

$$\mathsf{P}_\pi\big(\beta = \{n_1,\cdots,n_k\}\big) = p_{n_1,\cdots,n_k} \geq 0,$$

with $\sum_{\Upsilon_k^n} p_{n_1,\cdots,n_k} = 1$. We assume that the unobservable time θ, the unknown coordinates β, and the driving Brownian motion B are independent under the measure P_π for $\pi \in [0,1)$ fixed and given.

Standard arguments imply that the previous setting can be realized on a probability space $(\Omega, \mathcal{F}, \mathsf{P}_\pi)$, with the probability measure P_π being decomposable as

$$\mathsf{P}_\pi = \sum_{\Upsilon_k^n} p_{n_1,\cdots,n_k}\Big(\pi \mathsf{P}^0_{n_1,\cdots,n_k} + (1-\pi)\int_0^\infty \lambda e^{-\lambda t}\mathsf{P}^t_{n_1,\cdots,n_k}dt\Big). \tag{13}$$

Here, $\mathsf{P}^t_{n_1,\cdots,n_k}$ is the probability measure conditional on $\beta = \{n_1,\cdots,n_k\}$ and $\theta = t$, i.e.

$$\mathsf{P}^t_{n_1,\cdots,n_k}(\cdot) = \mathsf{P}_\pi(\cdot | \beta = \{n_1,\cdots,n_k\}, \theta = t).$$

Moreover, we shall use the notation $\mathsf{P}_{n_1,\cdots,n_k}$ to denote the probability measure under $\beta = \{n_1,\cdots,n_k\}$ and $\theta < \infty$, i.e. $\mathsf{P}_{n_1,\cdots,n_k}(\cdot) = \mathsf{P}_\pi(\cdot|\beta = i)$. Standard arguments then imply that

$$\mathsf{P}_{n_1,\cdots,n_k}(\cdot) = \pi \mathsf{P}^0_{n_1,\cdots,n_k} + (1-\pi)\int_0^\infty \lambda e^{-\lambda t}\mathsf{P}^t_{n_1,\cdots,n_k}dt.$$

With the above in hand, equation (13) is equivalent to

$$\mathsf{P}_\pi = \sum_{\Upsilon_k^n} p_{n_1,\cdots,n_k}\mathsf{P}_{n_1,\cdots,n_k}. \tag{14}$$

Being based upon continuous observation of X, our optimal stopping problem of interest is to find a stopping time τ^* of X (i.e. a stopping time with respect to the natural filtration of X augmented with all P_π null sets (denoted by $\mathcal{F}_t^X$)) that is 'as close as possible' to the unknown time θ, measured in terms of the cost functional

$$J(\pi;\tau) := \mathsf{P}_\pi(\tau < \theta) + c\mathsf{E}_\pi(\tau - \theta)^+. \tag{15}$$

Therefore, the optimal stopping problem of interest is to find an optimal stopping time τ^* such that the cost function is minimized, i.e.

$$V(\pi) = J(\pi; \tau^*) = \inf_{\tau} J(\pi; \tau). \tag{16}$$

We expose the solution to this optimal stopping problem in Corollary 8.2 of this preprint.

We continue by providing numerical solutions for the value function and optimal stopping boundary in the multidimensional case. First, some necessary notation: for $\binom{n}{k} = I$ dimensions, let us define the function $L(\varphi) := \sum_{i=1}^{I} p_i \big(\varphi_i - \frac{\lambda}{c}\big)$.

(1) For the special case when $n = 2$ and $k = 1$ (with $\lambda = 0.8$, $\mu = 1$, $c = 1$, $p_1 = p_2 = 0.5$), a closed form solution of the value function cannot be obtained. In lieu, we present its numerical solution of a in Figure 1. In Figure 2, the blue line presents the corresponding optimal stopping boundary and the red line is $L(\varphi) = 0$. It is clear that the optimal stopping boundary lies outside of $L = 0$, in concurrence with the paper's results.

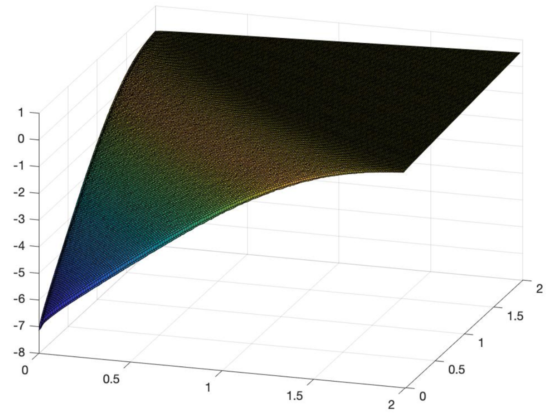

Figure 4: The value function for $n = 2$ and $k = 1$.

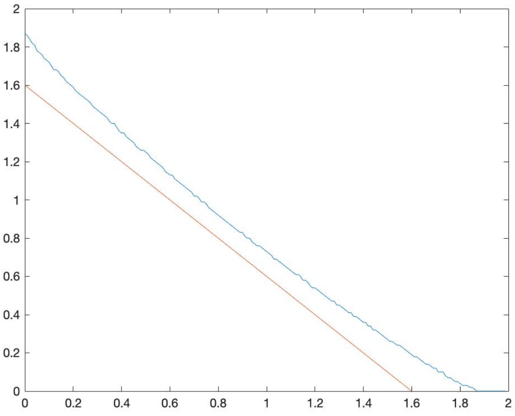

Figure 5: The optimal stopping boundary for $n = 2$ and $k = 1$.

(2). For the special case when $n = 3$ and $k = 2$ (with $\lambda = 0.8$, $\mu = 1$, $c = 1$, $p_1 = p_2 = p_3 = 1/3$), the following picture presents the numerical solution of the optimal stopping boundary and the plane $L(\varphi) = 0$. The picture agrees with the fact that the optimal stopping boundary will be a convex surface lying above the plane $L(\varphi) = 0$.

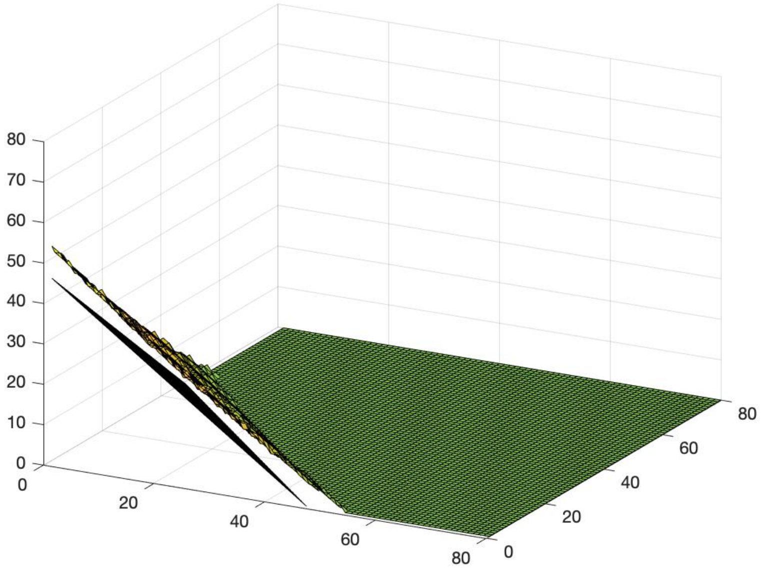

Figure 6: The optimal stopping boundary for $n = 3$ and $k = 2$.

3 Training opportunities

The PI has actively mentored two Ph.D. students on this project: Dongzhou Huang (graduated May 2022), who will be starting as Assistant Professor at Colorado State University in August 2022, and Yizhou Xia (graduated December 2019, currently Data Strategist at BP). The PI also actively mentored and collaborated with two of his postdoctoral fellows on this project: Quan Zhou (now Assistant Professor of Statistics, Texas A&M University) and Hongwei Mei (starting as Assistant Professor at Texas Tech University in August 2022). The PI remains actively engaged in recruiting underrepresented groups in STEM into his research program.

4 Dissemination

The results have been disseminated in multiple conferences and workshops, highlighted by the following key venues: Institute for Mathematics and its Applications Conference on Modeling, Stochastic Control, Optimization, and Related Applications (June 2018), Summer Research Conference of the Southern Regional Council on Statistics (SRCOS) (June 2018), A Symposium on Optimal Stopping in Honor of Larry Shepp (June 2018), Institute of Mathematical Statistics (IMS) New Researcher's Conference (July 2018), MSU's Symposium on Mathematical Statistics and its Applications (October 2018), A Workshop in Memory of Larry Brown (November 2018), Probability and Analysis 2019 (Banach Center, Poland, June 2019), The Fifteenth Latin American Congress of Probability and Mathematical Statistics (December 2019), The INFORMS Annual Meeting (November 2020), Canadian Mathematical Society 75th Anniversary Summer Meeting (June 2021), Seminar on Stochastic Hybrid Systems, University of Connecticut (December 2021), and New Advances in Statistics and Data Science, Honolulu, HI. (May 2022). The results have also been disseminated in over 20 departmental seminars in three countries (USA, UK, and Belgium).

5 Honors

The PI's significant honors during the 2018-2022 project period are listed below.

International and National Research Awards

1. Election to 2022 Committee of Presidents of Statistical Societies (COPSS) Leadership Academy.
   Award Citation: "'for significant contributions of extraordinary merit to applied probability and mathematical statistics, particularly the resolution of the longstanding conjecture of Yule's nonsense correlation, outstanding teaching and leadership."

2. Appointed "Chaire Internationale" (visiting international research chair) at Department of Mathematics, University of Brussels, Summer 2022.

3. 2020 INFORMS Donald P. Gaver, Jr. Early Career Award for Excellence in Operations Research.
   Award Citation: "for outstanding research accomplishments in operations research, probability and statistics, including solving a nearly 100-year old conjecture by Yule on nonsense correlation; for collaborating to solve outstanding problems in queueing, mathematical finance and optimal control; for outstanding teaching; and for mentoring of PhD students and postdoctoral fellows."

4. 2018 Institute of Mathematical Statistics (IMS) Tweedie New Researcher Award.
   Award Citation: "for his fundamental contributions to exact distribution theory, in particular for his elegant resolution of the Yule's nonsense correlation problem, and for his development of novel stochastic control techniques for computing the value of insider information in mathematical finance problems."

5. 2018 Army Research Office (ARO) Young Investigator Award.

Rice University Research Awards

7. 2022: Promoted to Full Professor at Rice University (Three Years Early)

8. 2019: Awarded Early Tenure at Rice (Two Years Early)

9. 2019 Rice University School of Engineering Teaching and Research Excellence Award *(awarded annually to two professors at Rice University's George R. Brown School of Engineering).*

10. 2018 Rice University Dobelman Family Junior Chair of Statistics

Rice University Teaching Awards

12. 2022 George R. Brown Award for Excellence in Teaching *(this award is considered Rice University's most prestigious teaching award, and is awarded annually to one professor at Rice University)*

13. 2021 George R. Brown Award for Superior Teaching *(awarded annually to nine professors at Rice University)*

6 Participants

The personnel supported on this grant at Rice University include the PI Philip A. Ernst, Ph.D. and two Ph.D. students: (i) Dongzhou Huang (graduated May 2022), who will be starting as Assistant Professor at Colorado State University in August 2022, and (ii): Yizhou Xia (graduated December 2019, currently Data Strategist at BP). The PI also actively mentored and collaborated with two of his postdoctoral fellows on this project: Quan Zhou (now Assistant Professor of Statistics, Texas A&M University) and Hongwei Mei (starting as Assistant Professor at Texas A&M University in August 2022).

References

[1] E. Bayraktar and H.V. Poor. Quickest detection of a minimum of two Poisson disorder times. *SIAM Journal of Control and Optimization* **46**: 308–331, 2007.

[2] S. Dayanik, H. V. Poor, and S.O. Sezer. Multisource Bayesian sequential change detection. *The Annals of Applied Probability* **18**: 552–590, 2008.

[3] P. A. Ernst, H. Mei, and G. Peskir. Quickest real-time detection of multiple Brownian drifts Preprint, 2022.

[4] P. A. Ernst and G. Peskir. Quickest real-time detection of a Brownian coordinate drift. *The Annals of Applied Probability*, to appear, 2022.

[5] P. A. Ernst, G. Peskir, and Q. Zhou. Optimal real-time detection of a drifting Brownian coordinate. *The Annals of Applied Probability*, **30**: 1032-1065, 2020.

[6] P. A. Ernst, L. C. G. Rogers, and Q. Zhou. When is it best to follow the leader? *Stochastic Processes and their Applications*, **130**: 3394-3407, 2020.

[7] G. Fellouris and G. Sokolov. Second-order asymptotic optimality in multisensor sequential change detection. *IEEE Transactions on Information Theory*, **62**: 3662–3675, 2016.

[8] V.S. Mikhalevich. A Bayes test of two hypotheses concerning the mean of a normal process. *Visn. Kiiv. Univ.*, **1**: 254–264, 1958.

[9] G. Peskir and A. N. Shiryaev. *Optimal Stopping and Free-boundary Problems.* Springer, 2006.

[10] E. Posner and H. Rumsey. Continuous sequential decision in the presence of a finite number of hypotheses. *IEEE Transactions on Information Theory*, **12**: 248–255, 1966.

[11] A. N. Shiryaev. Two problems of sequential analysis. *Cybernetics*, **3**: 63–69, 1967.

[12] A. N. Shiryaev. *Optimal Stopping Rules.* Springer, 1978.

[13] A. N. Shiryaev. Quickest detection problems: Fifty years later. *Sequential Analysis*, **29**: 345–385, 2010.

[14] A. Wald. *Sequential Analysis.* Chapman & Hall, 1947.

[15] H. Zhang, N. Rodosthenous, and O. Hadjiliadis. Robustness of the *N-CUSUM* stopping rule in a Wiener disorder problem. *The Annals of Applied Probability*, **25**: 3405-3433, 2015.

[16] K. Sh. Zigangirov. On a problem in optimal scanning. *Theory of Probability & Its Applications*, **11**: 294–298,1966.

[17] C. Zucca, P. Tavella, and G. Peskir. Detecting atomic clock frequency trends using an optimal stopping method. *Metrologia*, **53**: S89, 2016.

[18] `https://obamawhitehouse.archives.gov/the-press-office/2016/10/13/executive-order-coordinating-efforts-prepare-nation-space-weather-events`, 2016.