AFRL-AFOSR-UK-TR-2023-0036

Dynamic scheduling of interacting automated VTOLs

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01/05/2023
Final Technical Report

DISTRIBUTION A: Distribution approved for public release.

Air Force Research Laboratory
Air Force Office of Scientific Research
European Office of Aerospace Research and Development
Unit 4515 Box 14, APO AE 09421

REPORT DOCUMENTATION PAGE

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1. REPORT DATE	**2. REPORT TYPE**	**3. DATES COVERED START DATE**	**3. DATES COVERED END DATE**
20230105	Final	20200801	20220731

4. TITLE AND SUBTITLE
Dynamic scheduling of interacting automated VTOLs

5a. CONTRACT NUMBER	**5b. GRANT NUMBER**	**5c. PROGRAM ELEMENT NUMBER**
	FA8655-20-1-7026	61102F
5d. PROJECT NUMBER	**5e. TASK NUMBER**	**5f. WORK UNIT NUMBER**

6. AUTHOR(S)
Matthias Gerdts

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
UNIVERSITAET DER BUNDESWEHR MUENCHEN
WERNER-HEISENBERG-WEG 39
NEUBIBERG 85579
DEU

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	**10. SPONSOR/MONITOR'S ACRONYM(S)**	**11. SPONSOR/MONITOR'S REPORT NUMBER(S)**
EOARD UNIT 4515 APO AE 09421-4515	AFRL/AFOSR IOE	AFRL-AFOSR-UK- TR-2023-0036

12. DISTRIBUTION/AVAILABILITY STATEMENT
A Distribution Unlimited: PB Public Release

13. SUPPLEMENTARY NOTES

14. ABSTRACT

The f nal performance report summarizes the activities and achievements of the project entitled "Dynamic scheduling of interacting automated VTOLs".
The project aims at the development of an efficient numerical framework for dynamic scheduling of a small fleet of vertical take-of and landing vehicles (VTOLs). The goal is to couple scheduling problems and dynamic path planning in a bi-level optimization approach. Applications arise at busy terminals, during long-term observation missions, or in logistics and transportation.
In the f rst year our focus was on modelling of an eVTOL and developing a solution framework for the envisaged problem class. We implemented a f rst version of an algorithm and applied it to a scheduling problem for automated vehicles at intersections. This scenario already contains a major challenge, which is the coupling between scheduling and optimal control, which will later occur in a similar way for airborne vehicles. The method performed very well for this problem setting (even in realtime) and we extended it to scenarios in
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15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF: a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**	**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**
U	U	U	SAR	25

19a. NAME OF RESPONSIBLE PERSON	**19b. PHONE NUMBER** *(Include area code)*
BARRETT FLAKE	314 235 6115

Final Performance Report
Version: October 29, 2022

Dynamic Scheduling of Interacting Automated VTOLs

Grant Number:	**FA8655-20-1-7026**
Period of Performace:	**08/01/2020 – 07/31/2022**
Principal Investigator:	**Prof. Dr. Matthias Gerdts***

This material is based upon work supported by the Air Force Office of Scientific Research, Air Force Materiel Command, USAF, under Award No, FA8655-20-1-7026.
Any opinions, findings, and conclusions or recommendations expressed in this publication are those of the author(s) and do not necessarily reflect the views of the Air Force Office of Scientific Research, Air Force Materiel Command, USAF.

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List of Symbols, Abbreviations and Acronyms

VTOL	vertical take-off and landing vehicle
eVTOL	electrical vertical take-off and landing vehicle
OCP	optimal control problem
SP	scheduling problem
MIP	mixed-integer linear program
MINLP	mixed-integer nonlinear program

Summary

The final performance report summarizes the activities and achievements of the project entitled “Dynamic scheduling of interacting automated VTOLs”.
The project aims at the development of an efficient numerical framework for dynamic scheduling of a small fleet of vertical take-off and landing vehicles (VTOLs). The goal is to couple scheduling problems and dynamic path planning in a bi-level optimization approach. Applications arise at busy terminals, during long-term observation missions, or in logistics and transportation.
In the first year our focus was on modelling of an eVTOL and developing a solution framework for the envisaged problem class. We implemented a first version of an algorithm and applied it to a scheduling problem for automated vehicles at intersections. This scenario already contains a major challenge, which is the coupling between scheduling and optimal control, which will later occur in a similar way for airborne vehicles. The method performed very well for this problem setting (even in realtime) and we extended it to scenarios in flight path optimization with moving obstacles. Currently the numerical results are restricted to the 2d plane, but the method can be applied in 3d as well. In addition we modeled a scheduling scenario for the distribution of aircrafts arriving at an airport to multiple runways.

Acknowledgment

The working group of Prof. Gerdts sincerely thanks the Air Force Office of Scientific Research and especially the program officers Shad Reed, Mark Friend, and Barrett Flake for the on-going support and the opportunity to work on the exciting topic of dynamic scheduling problems of automated VTOLs.

Chapter 1

Introduction

The idea behind the project is to combine path planning for automated/autonomous vertical take-off and landing vehicles (VTOLs) and scheduling of tasks for a (small) fleet of such VTOLs. From a mathematical viewpoint this setting can be formulated as a bi-level optimization problem with a finite dimensional nonlinear mixed-integer optimization problem at the upper level and optimal control problems at the lower level. The upper level problem aims at finding the optimal starting times and the optimal sequence of tasks for the VTOL agents, while the lower level problem aims at finding optimal trajectories for the assigned VTOL tasks and starting times. Since we assume throughout that the durations of the missions depend on the starting times and the sequence of tasks, we obtain a strong coupling of the two levels in the bi-level optimization problem, compare [8, 9, 11].

A mathematical model for such dynamic scheduling problems reads as follows. Let N automated/autonomous VTOLs with states x_i of dimension n_i, controls u_i of dimension m_i, and equations of motion

$$x_i'(t) = f_i(x_i(t), u_i(t)), \qquad i = 1, \ldots, N,$$

be given. For a given initial time t_i and a given initial state $\bar{x}_i$ the i-th VTOL has to complete a task (or mission), which can be formulated as a parametric optimal control problem of the following type.

$OCP_i(t_i, \bar{x}_i)$: Minimize

$$\int_{t_i}^{T_i} f_{0,i}(x_i(t), u_i(t))dt$$

with respect to (x_i, u_i, T_i) *subject to the constraints*

$$x_i'(t) = f_i(x_i(t), u_i(t)), \qquad \textit{for almost every (a.e.) } t \in [t_i, T_i], \tag{1.1}$$

$$x_i(t) \in \Omega_i, \qquad \textit{for every } t \in [t_i, T_i], \tag{1.2}$$

$$u_i(t) \in U_i, \qquad \textit{for a.e. } t \in [t_i, T_i], \tag{1.3}$$

$$x_i(t_i) = \bar{x}_i, \tag{1.4}$$

$$x_i(T_i) \in S_i. \tag{1.5}$$

Herein, the final time T_i is free and $\Omega_i \subset \mathbb{R}^{n_i}$, $U_i \subset \mathbb{R}^{m_i}$, $S_i \subset \mathbb{R}^{n_i}$ are appropriate sets, which define state-, control-, and terminal constraints. The function $f_{0,i}$ defines appropriate Lagrange costs.

In order to coordinate the VTOLS, we introduce a superordinate scheduling problem (or job-shop problem) with the aim to find the optimal sequence and starting times t_i, $i = 1, \ldots, N$, of the individual VTOLs. We assume that a task has to be completed before another task commences. In its most basic version this scheduling task uses the binary variables $w_{ij} \in \{0, 1\}$ with $i, j \in \{1, \ldots, N\}$, $i < j$, where $w_{ij} = 1$ indicates that the i-th VTOL commences before the j-th one and vice versa for $w_{ij} = 0$. Let

$$d_i(t_i, \bar{x}_i) = T_i(t_i, \bar{x}_i) - t_i, \qquad i = 1, \ldots, N, \tag{1.6}$$

denote the task durations, which are given by the solution of the optimal control problems $OCP(t_i, \bar{x}_i)$ and depend implicitly on the starting time t_i and the initial state $\bar{x}_i$. Then a prototype scheduling problem reads as follows:
SP: Minimize

$$\max_{1 \le i \le N} \{t_i + d_i(t_i, \bar{x}_i)\}$$

with respect to t_i, $\bar{x}_i$, $w_{ij} \in \{0, 1\}$, $i, j \in \{1, \ldots, N\}$, $i < j$, *subject to the constraints*

$$t_i + d_i(t_i, \bar{x}_i) - t_j \le M \cdot (1 - w_{ij}) \tag{1.7}$$

$$t_j + d_j(t_j, \bar{x}_j) - t_i \le M \cdot w_{ij} \tag{1.8}$$

$$t_i \ge 0 \tag{1.9}$$

for $i, j = 1, \ldots, N$, $i < j$.

In constraints (1.7)-(1.8) the constant M is supposed to be sufficiently large in order to ensure that exactly one of the two constraints becomes active for a given w_{ij}. A typical choice is to choose M greater than the sum of all starting times t_i and all durations d_i. Note that $w_{ij} = 1$ implies $t_j \ge t_i + d_i$ by (1.7), i.e. VTOL j starts after VTOL i. Likewise, $w_{ij} = 0$ implies $t_i \ge t_j + d_j$ by constraint (1.8).

Since the scheduling problem SP depends on the solutions of the optimal control problems $OCP(t_i, \bar{x}_i)$, the coupled problem is a bi-level problem, where SP is called the *upper-level problem* and $OCP(t_i, \bar{x}_i)$ the *lower-level problem*. We call this problem a *dynamic scheduling problem*.

Solving the dynamic scheduling problem is a difficult task in general and often heuristic methods, e.g. genetic algorithms, are applied to find solutions. In contrast our focus is on deterministic methods, which allow for a certifcation of solutions.

Chapter 2

Results and Discussions

The purpose of this chapter is to summarize the achievements and advances regarding the treatment of dynamic scheduling problems. To this end we decided to increase the complexity step by step. Firstly we investigated static scheduling problems in order to derive realistic problem formulations and to validate solvers. To this end we used GUROBI `https://www.gurobi.com` for solving mixed-integer linear optimization problems throughout, compare Section 2.1. Secondly, we investigated an intersection management problem in autonomous driving, see Section 2.2. Mathematically the problem leads to a dynamic scheduling problem and it exposes exactly the difficulties as the envisaged class of dynamic scheduling problems with VTOLs. Hence it turned out to be a good test example and a first step towards the development of a suitable numerical method. First attempts towards bi-level aircraft scheduling problems at busy terminals can be found in [3] and [11]. In Section 2.3 we finally extended the methodology from the intersection management problem towards the scheduling of multiple VTOLs. Herein, the aim of the VTOLs is to reach a target area in shortest distance and minimal total time while avoiding moving obstacles.

2.1 Scheduling of Aircrafts

Motivation: The goal of Aircraft Landing Problem (or Aircraft Schedulung Problem) is to find an optimal aircraft sequence on the available runways, and to schedule their landing times taking into account some operational constraints. This problem can be modeled as a mixed-integer linear program (MIP), see [6]. However, the MIP formulation in that reference is not optimal, due to a large number of binary variables, and hence it can be improved. This improvement is essential if one tends to solve such a problem in real-time (in this case in appprox. 1 min), since the introduction of each additional binary variable in the worst case doubles the runtime of the underlying solver.

Problem setting:

- N aircrafts are approaching an airport, $\mathcal{A} = \{1, ..., N\}$
- An airport has K runways, $\mathcal{K} = \{1, ..., K\}$
- c_j^-, c_j^+, $j \in \mathcal{A}$ are weights for arriving too early and too late, respectively
- T_i, $i \in \mathcal{A}$ are target landing times, e.g. 8:00, 8:05, 8:10 and so on
- $[L_i, U_i]$, $i \in \mathcal{A}$ are landing time windows ($L_i < U_i$) based on fuel availability or on possible speed-ups. Usually, an aircraft can be tracked by the airport radar about 45-60 minutes from the airport. A decision-support tool computes an Estimated-Time of Arrival (ETA) at the runway threshold. If the aircraft speeds up, the Actual Landing Time (ALT) might be earlier and vice versa later if the aircraft slows down [2].
- S_{ij}, $i, j \in \mathcal{A}$ is a minimum separation time between aircraft i and j, where i lands before j, which guarantees that no aircraft is affected by the wake-vortex turbulence generated by a leading aircraft
- t_i, $i \in \mathcal{A}$ are the optimal landing times
- t_i^-, t_i^+, $i \in \mathcal{A}$ are deviations from target landing time T_i
- m is Constrained-Position Shifting (CPS). This constraint limits the deviation from the First Come First Serve (FCFS) sequence. This quantity is usually small (≤ 4) [1].

Our MIP formulation:

$$\min \sum_{i=1}^{N} \left[\underbrace{c_j^- \max(0, T_i - t_i)}_{t_i^-} + \underbrace{c_j^+ \max(0, t_i - T_i)}_{t_i^+} \right] \tag{2.1}$$

subject to

$$t_i = T_i - t_i^- + t_i^+ \qquad i \in \mathcal{A}, \tag{2.2}$$

$$t_i + S_{ij} - t_j \le M(1 - w_{ij}) + M(1 - x_{ik}) + M(1 - x_{jk}) \qquad i < j, i, j \in \mathcal{A}, k \in \mathcal{K},$$

$$t_j + S_{ji} - t_i \le M w_{ij} + M(1 - x_{ik}) + M(1 - x_{jk}) \qquad i < j, i, j \in \mathcal{A}, k \in \mathcal{K}, \tag{2.3}$$

$$w_{ij} \in \{0, 1\} \qquad i < j, i, j \in \mathcal{A}, \tag{2.4}$$

$$\sum_{k=1}^{K} x_{ik} = 1 \qquad i \in \mathcal{A}, \tag{2.5}$$

$$x_{ik} \in \{0, 1\} \qquad i \in \mathcal{A}, k \in \mathcal{K}, \tag{2.6}$$

$$L_i \le t_i \le U_i \qquad i \in \mathcal{A}, \tag{2.7}$$

$$t_i \le t_j \qquad i + m + 1 < j, \quad i \in \mathcal{A} \setminus \{N - m - 1, N\}. \tag{2.8}$$

Let us shortly comment on the problem formulation above. Objective function (2.1) minimizes the total deviation cost from target times T_i, $i \in \mathcal{A}$. Constraints (2.2) are introduced to liearize the objective function. Constraints (2.3) are the scheduling constrants where the w-variables determine the aircraft landing sequence on a particular runway, which in turn is indicated by the x-variables. Constraints (2.4) and (2.6) enforce that the w- and x-variables are binary. Constraints (2.5) are indicate that one aircraft can land only on one runway. Constraints (2.7) are the kanding time windows, and constraints (2.8) are the CPS constraints.

Advantages of our formulation: As already mentined above the main concern when dealing with problems containing integer variables is the number of these variables in the problem formulation. Hence, our goal was to make this number as small as possible to get solutions in real-time. In our model there are $(N^2 - N)/2 + N \cdot K$ binary variables which is considerably less than $(N^2 - N) \cdot K/2 + N \cdot K + N^2 - N$ in [6]. Moreover, the CPS constraints are modeled with the continious variables an not with the binary as in [6], which contributes to a faster solution.

Numerical results: For numerical experiments we investigated several busy airports (Paris-Orly, Munich, Frankfurt, Atlanta). Details can be found in [10].

2.2 Dynamic Scheduling in Intersection Management

We propose a bi-level optimization algorithm for an intersection management problem, see Figure 2.1.

The task is to coordinate autonomous vehicles arriving at an intersection such that the overall crossing time becomes minimal subject to collision avoidance constraints. Each

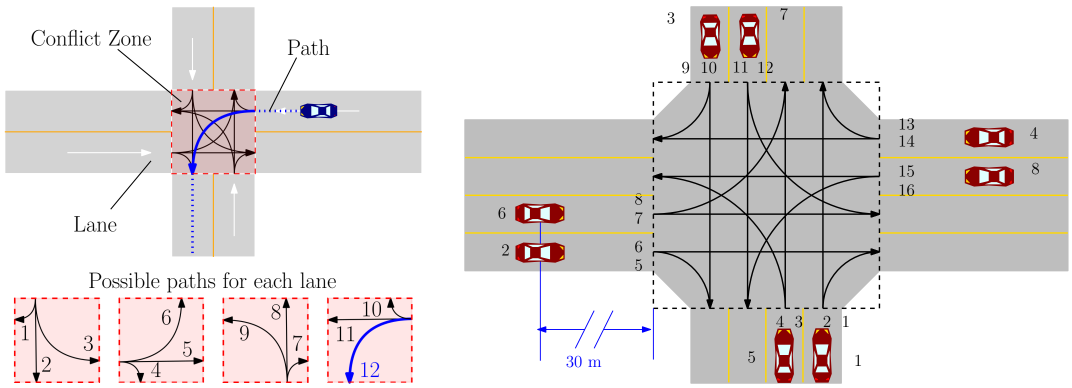

Figure 2.1: Automatic intersection management as a dynamic scheduling problem (from [4]).

car can control its velocity profile in an optimal way, but the arrival time and sequence of arrivals at the intersection are subject to optimization. This task fits into the general class of dynamic scheduling problems in Section 1.

On the lower level, for each vehicle we consider a simple optimal control problem, which provides us optimal final velocities as nonlinear functions of final times. On the upper level we have a scheduling problem which depends on these nonlinear functions. We solve the combined dynamic scheduling problem by approximating the nonlinearities with piecewise linear functions. To this end we adopted a method proposed in [12]. This allows to derive a certain MIP formulation for the whole dynamic scheduling problem. The MIP can be conveniently solved by GUROBI, even in real-time. In addition, the approximation techniques allows us to estimate and control the error of the approximation. Technical details can be found in [4, 5]. An animation is available at `https://youtu.be/uzZGm8WOFRA`.

2.3 Dynamic Scheduling of VTOLs

The overall goal is to determine an optimal schedule for a small fleet of autonomous VTOLs and to find the respective trajectories. Hereby, their dynamics and the condition that while one VTOL executes its operation, all others have to idle are taken into account. In our case, the mission for all VTOLs is to reach a static target from some starting points while avoiding moving obstacles such that the total mission time, i.e. the sum of the respective flight durations, is minimal.
The number of VTOLs is denoted by $N \in \mathbb{N}$ with the corresponding index set $\mathcal{N} := \{1, \ldots, N\}$. Moreover, $P \in \mathbb{N}_0$ and $\mathcal{P} := \{1, \ldots, P\}$ indicate the number of (moving) obstacles, whose dynamics is supposed to be known, and its index set, respectively. In

addition, the fixed starting positions of all aerial vehicles are assumed to be given. The overall goal is to schedule VTOLs and to simultaneously compute their optimal trajectories in some time dependent domain $\Omega(t) \subset \mathbb{R}^n$ with $t \in \mathbb{R}_{\geq 0}$ and $n \in \{2, 3\}$ such that all of them reach a predefined static target set $\Omega_T \subset \Omega(t),\ t \in \mathbb{R}_{\geq 0}$ while avoiding possible collisions with any of the obstacles. Note that the latter form the time-dependent set $\Omega_O(t)$ with $\Omega_O(t) = \bigcup_{i \in \mathcal{P}} \Omega_{O,i}(t) \subset \Omega(t),\ t \in \mathbb{R}_{\geq 0}$. The set, which the VTOLs are allowed to move across at any time point is denoted by $\Omega_S(t)$. The whole domain $\Omega(t)$ can be represented as $\Omega(t) = \Omega_T \cup \Omega_O(t) \cup \Omega_S(t),\ t \in \mathbb{R}_{\geq 0}$. An illustrative example of $\Omega(t_\star)$ with two obstacles is given in Figure 2.2 for a specific time point $t_\star \in \mathbb{R}_{\geq 0}$.

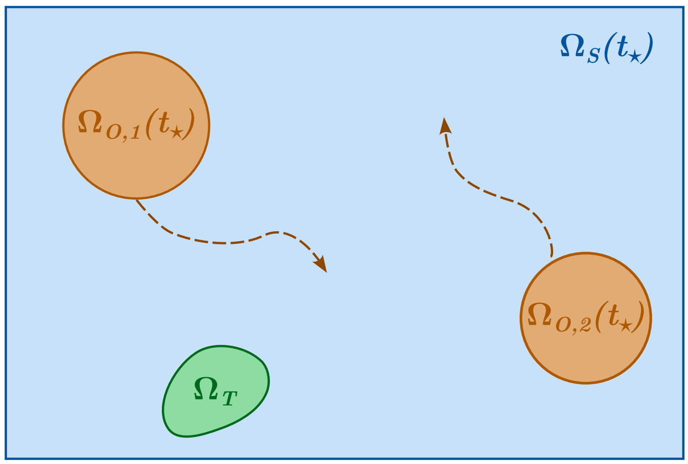

Figure 2.2: Example of $\Omega(t_\star),\ t_\star \in \mathbb{R}_{\geq 0}$. In green: static target set Ω_T, in orange: dynamic obstacles with $\Omega_O(t_\star) = \Omega_{O,1}(t_\star) \cup \Omega_{O,2}(t_\star)$, in blue: time-dependent set $\Omega_S(t_\star)$, across which VTOLs can move.

We assume the following:

- The target region Ω_T can be reached by every VTOL in finite time.
- The velocity of all VTOLs is constant.
- All obstacles stop their motion at some time point $t^\star$, which is bounded from below by the time point, at which the last VTOL in the sequence reaches Ω_T.

The dynamic scheduling problem is a bi-level problem with a coupling between the upper level scheduling problem and the lower level optimal control problem. The upper level problem reads as follows:

Upper Level: Scheduling of VTOLs

$$\min_{t_i, w_{ij}} \sum_{i=1}^{N} \left[t_i + \alpha D_i(t_i)\right] \tag{2.9a}$$

subject to

$$
\begin{aligned}
t_i + D_i(t_i) - t_j &\leq M(1 - w_{ij}), && (2.9b)\\
t_j + D_i(t_i) - t_i &\leq M w_{ij}, && (2.9c)\\
t_i &\in [t_{i,min}, t_{i,max}], && (2.9d)\\
w_{ij} &\in \{0, 1\} && (2.9e)
\end{aligned}
$$

for $i, j \in \mathcal{N}, i < j$.

Herein, t_i denotes the starting time of VTOL i, $i \in \mathcal{N}$, α is a flight duration penalization term, M is some sufficiently large constant, and the binary variables $w_{i,j}$, $i, j \in \mathcal{N}$ have the following meaning:

$$
w_{ij} = \begin{cases} 1 & \text{if VTOL } i \text{ starts before VTOL } j, \\ 0 & \text{otherwise.} \end{cases}
$$

Moreover, $D_i(t_i)$, $i \in \mathcal{N}$ is a nonlinear function and denotes the flight's duration of the corresponding VTOL if it starts at time t_i, which in turn is a continuous variable bounded by $t_{i,min}$ and $t_{i,max}$ from below and above, respectively. Since the velocity is supposed to be constant, we have the relation

$$
D_i(t_i) = d(t_i)/v_i, \quad i \in \mathcal{N},
$$

where v_i is the average velocity of the VTOL i. For simplicity reasons, we assume that $v_i = v$, $\forall i \in \mathcal{N}$. The numerator in the previous equality is a parametric function, which denotes the distance to the target at time t_i, $i \in \mathcal{N}$, and can be computed from the following parametric optimal control problem:

Lower Level: Minimum Distance

$$
d(t_i) := \min_{x_i, u_i} \int_{t_i}^{\infty} \chi_{\Omega_S(t)}(x_i(t)) dt \tag{2.10a}
$$

subject to

$$
\begin{aligned}
x_i'(t) &= u_i(t) && \textit{for a.e. } t \in [t_i, \infty), && (2.10b)\\
x_i(t) &\in \Omega_S(t) && \textit{for every } t \in [t_i, \infty), && (2.10c)\\
u_i(t) &\in U_i = \{u \in \mathbb{R}^n \mid ||u|| = 1\} && \textit{for a.e. } t \in [t_i, \infty), && (2.10d)\\
x_i(t_i) &= \overline{x}_i. && && (2.10e)
\end{aligned}
$$

Herein, $\overline{x}_i$ is the initial position of VTOL i, and χ_A is the indicator function defined as

$$
\chi_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{otherwise.} \end{cases}
$$

Note that in case that $n = 2$, the control sets $U_i,\ i \in \mathcal{N}$ can be represented in the following way using polar coordinates:

$$U_i = \left\{ \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \;\middle|\; u_1 = \cos\alpha_i,\ u_2 = \sin\alpha_i,\ \alpha_i \in [0, 2\pi) \right\}.$$

Similarly, the spherical coordinate system can be exploited for describing $U_i\ i \in \mathcal{N}$ if $n = 3$:

$$U_i = \left\{ \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \;\middle|\; u_1 = \cos\alpha_i \sin\beta_i,\ u_2 = \sin\alpha_i \sin\beta_i,\ u_3 = \cos\beta_i\, \alpha_i \in [0, 2\pi),\ \beta_i \in [0, \pi] \right\}.$$

The lower level problem yields the shortest distance to the target set, compare Figure 2.3.

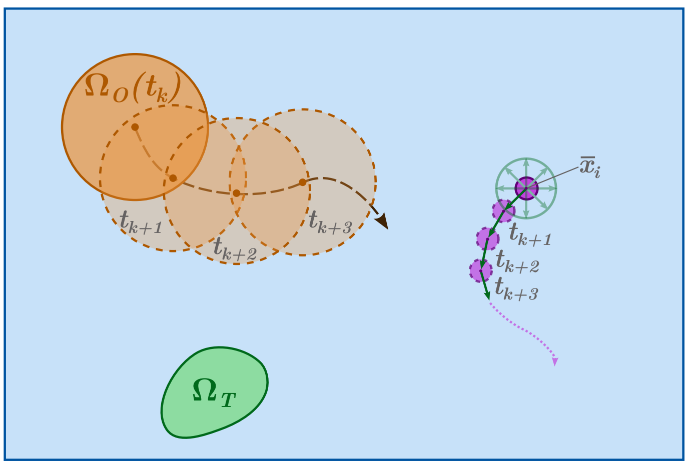

Figure 2.3: Approach for the minimum distance computation. In green: static target set Ω_T, in orange: a dynamic obstacle $\Omega_O(t)$, in purple: a moving VTOL

The solution procedure for the bi-level problem can be divided into three main steps indicated below:

(1) Solve the lower level problems by computing the corresponding value function $V(t, x)$, which depends on the starting time t and the starting position x.

(2) Insert all given VTOLs' starting positions $\overline{x}_i,\ i \in \mathcal{N}$ into the value function V obtained in step (1) and set $d_i(t_i) := V(t_i, \overline{x}_i)$ to obtain the time-dependent minimum distance functions. The bi-level problem is hence recast into a single-level mixed-integer nonlinear program (MINLP). The resulting MINLP is then piecewise linearized and solved by a mixed-integer linear programming solver (GUROBI in our case). A solution to that problem is an optimal sequence of VTOLs provided by the optimal starting times $t_i^*,\ i \in \mathcal{N}$.

(3) Compute the optimal trajectories of every VTOL via the forward recursion using the value function and its associated feedback control law.

Further technical details and results can be found in [7] (the paper is attached to this report as it is not published as yet). Animations are available at `https://youtu.be/QgMEM1QQP9M` and `https://youtu.be/BQ39fRMV-ew`.

Chapter 3

Publications

The following publications have been prepared within the project:

- M. Gerdts, S. Rogovs, G. Valenti: *A Piecewise Linearization Algorithm for Solving MINLP in Intersection Management*, In Proceedings of the 7th International Conference on Vehicle Technology and Intelligent Transport Systems (VEHITS 2021), pages 438-445, 2021, DOI: 10.5220/0010437104380445, [4].

  In this paper, we propose a linearization algorithm for solving a Mixed Integer Non-Linear Problem for Intersection Management of Connected Autonomous Vehicles. The objective of such problem is to minimize the time it takes to clear a given arbitrary intersection for all vehicles in the consideration. We treat the IM problem as a bi-level optimization problem. On the lower level we solve an Optimal Control Problem for each individual vehicle, whereas on the higher level we deal with an optimization problem of finding the optimal sequence and starting times for every car, which essentially yields a MINLP. An intuitive linearization technique is presented to solve the emerging MINLP in a reasonable time. The actual controls, if necessary, are computed a posteriori by minimizing the L2-norm of control variables. The algorithm is tested in different intersection scenarios. Numerical results show that it is suitable for real-time applications.

- M. Gerdts, S. Rogovs, G. Valenti: *Solving Complex Intersection Management Problems Using Bi-level MINLPs and Piecewise Linearization Techniques*, Springer Nature Switzerland AG 2022, C. Klein et al. (Eds.): SMARTGREENS 2021/ VEHITS 2021, CCIS 1612, pp. 255–273, 2022, https://doi.org/10.1007/978-3-031-17098-0_13.

  We investigate a complex Intersection Management Problem (IMP) for automated vehicles and introduce a method for the automated coordination of vehicles with the aim to minimize total clearance time for the intersection. The method is capable of handling multiple vehicles (per lane or distributed over sev- eral lanes), multiple lanes, variable arrival times and velocities, and multiple turn options in non-symmetric intersection scenarios. In order to optimally coordinate the vehicles we employ a bi-level optimization formulation coupling a schedul- ing problem at the upper level with an optimal control problem at the lower level. The latter takes into account the dynamics of the vehicles and the former aims to find an optimal sequence of arrivals at the intersection. Collision avoidance on the complete driving paths, i.e., at, before and after the intersection, is ensured by the problem formulation. In order to solve the resulting mixed-integer nonlinear bi-level optimization problem we develop suitable piecewise linearization tech- niques for the value function of the optimal control problems which eventually yields a large-scale mixed-integer linear problem. Numerical examples show the efficiency of the proposed approach.

- S. Rogovs, V. Nikitina, M. Gerdts: *A novel mixed-integer programming approach for the aircraft landing problem*, Frontiers in Future Transportation, pp. 957–968, 2022, https://doi.org/10.3389/ffutr.2022.968957.

  This paper deals with the aircraft landing problem, which consists of determining a landing time for each aircraft within the radar range of an airport and allocating it to a runway. We propose an exact solution approach that involves mixed-integer linear programming. The objective is hereby to minimize the sum of weighted deviations from the target landing times under consideration of different safety, efficiency and fairness constraints. Despite of the problem's NP-hardness, our method exhibits low execution times thanks to a modified modeling strategy and provides near-optimal results. Numerical experiments prove efficiency of the approach for different large airports.

- M. Gerdts, S. Rogovs, V. Nikitina: *Piecewise linear value function approximations in nonlinear dynamic scheduling problems with VTOLs*, Technical Report, Universität der Bundeswehr München, submitted, 2022.

  This article presents an efficient numerical framework for dynamic scheduling of multiple VTOLs. It combines a scheduling problem with optimal trajectory planning. The whole setting is formulated as a mixed- integer bilevel optimization problem. At the upper level, VTOLs are scheduled, and their starting times are computed. The solution of the lower level problem involves the computation of a value function and yields optimal trajectories for every aerial vehicle. In order to solve the bilevel problem, it is recast into a single-level one. The resulting mixed-integer nonlinear program (MINLP) is then piecewise linearized and solved numerically with the help of an efficient and robust linear solver based on the Branch-and-Bound algorithm.

The following publications contain material that is related to the project:

- H. Hong, P. Piprek, M. Gerdts, F. Holzapfel: *Computationally Efficient Trajectory Generation for Smooth Aircraft Flight Level Changes*, Journal of Guidance, Control, and Dynamics, Vol. 44, No. 8, pp. 1532–1540, 2021, https://doi.org/10.2514/1.G005529
- A. Britzelmeier, M. Gerdts, O. Moslehi Rad, S. Rani, T. Rottmann: *ROCS: A Real-time Optimization and Control Simulator*, Proceedings ASIM SST 2020, ARGESIM Report AR 59, DOI 10.11128/arep.59
- M. Zwick, M. Gerdts, P. Stütz: *Sensor Model-Based Trajectory Optimization for UAVs Using Nonlinear Model Predictive Control*, AIAA SciTech Forum, AIAA Science and Technology Forum (2022, San Diego, Calif.)
- M. Zwick, M. Gerdts, P. Stütz: *Enhancing Detection Performance through Sensor Model-based Trajectory Optimization for UAVs*, 2021 IEEE/AIAA 40th Digital Avionics Systems Conference (DASC), San Antonio, TX, USA, 2021.

Chapter 4

Talks and Presentations

The work was presented at the following conferences and seminars:

- M. Gerdts: *Path planning optimal control*, Colloquium, TU Wien, December 2020.
- M. Gerdts, S. Rogovs, G. Valenti: *A Piecewise Linearization Algorithm for Solving MINLP in Intersection Management*, VEHITS 2021 – 7th International Conference on Vehicle Technology and Intelligent Transport Systems, 2021.
- M. Gerdts: *Numerical Methods for ODE Optimal Control Problems*, MINOA Summer School, Heidelberg, June 21-25, 2021.
- M. Gerdts: *Path planning techniques for interacting systems*, Rutgers, Mechanical & Aerospace Engineering Colloquium Series, 2021.
- M. Gerdts: *Control of interacting systems using model-predictive control and generalized Nash equilibrium problems*, Joint Workshop of GAMM Activity Groups Dynamics and Control Theory and Optimization with Partial Differential Equations, Bayreuth, September 2021.
- M. Gerdts: *Coordination of interacting systems using optimal control techniques*, plenary talk, French German Portuguese Conference on Optimization, Porto, May 3-6, 2022.
- M. Gerdts: *Applied Optimal Control*, lecture series for PhD students at Politecnico di Bari, Italy, during a visiting professorship, April-July 2022.

Chapter 5

Contributions by Student Projects, Bachelor's and Master's Theses, and Phd Theses

The following student theses and student projects contributed to the project by exploring ideas and solution approaches:

- *Modellierung und Simulation eines eVTOLs (Modeling and simulation of an eVTOL)*, Bachelor thesis, Universität der Bundeswehr München, 2021.

  In this thesis a mathematical model for the dynamics of an eVTOL has been developped. The configuration of the eVTOL consists of three rotors with adjustable direction and a fixed wing. Standard maneuvers have been simulated to validate the model.

- *Ein projiziertes Gradientenverfahren in der Bahnfolgeregelung (A projected gradient method for path tracking tasks)*, Bachelor thesis, Universität der Bundeswehr München, 2021.

  In this thesis a projected gradient method was combined with a linear model-predictive control method for a path tracking task. Numerical tests have been performed to evaluate the approach.

- *Reihenfolgenoptimierung von ankommenden Flugzeugen bei mehreren Landebahnen (Scheduling of arriving aircrafts for multiple runways)*, Master thesis, Universität der Bundeswehr München, 2021.

  In the thesis a mixed-integer linear programming formulation for aircraft scheduling was investigated numerically using GUROBI software. The task was to schedule arriving aircrafts and distribute them on different runways.

- *Entwicklung eines Bahnfolgereglers mittels dynamischer Inversion für ein eVTOL (Design of a path tracking controller for an eVTOL using dynamic inversion)*, Student project, Universität der Bundeswehr München, 2022.

  The aim of the project was to develop a tracking controller for an eVTOL model using dynamic inversion. This tracking controller is supposed to track a geometric reference path in 3D, which is obtained from a guidance method.

- *Development of a feedback-law to control a docking-maneuver using Reinforcement Learning*, Master thesis, Universität der Bundeswehr München and Rutgers University, 2022.

  The goal of this thesis is to develop a feedback-law to control a docking-maneuver in space using Reinforcement Learning.

- *Implementation of a solver for structured linear equation systems in optimal control*, Master thesis, Universität der Bundeswehr München and Rutgers University, 2022.

  The thesis addresses a numerical method for solving optimal control problems in discrete time. The particular structure of the optimal control problem is exploited on linear algebra level. To this end, solvers for block banded matrices are used to achieve low computation times, which enables the use of the method in realtime optimal control tasks.

- *Variation der Historie beim gewöhnlichen Lösen eines Steuerungsproblems mit POMDP und LSTMs (Variation of the history during the solution of a control problem with POMDP and LSTMs)*, Master thesis, Universität der Bundeswehr München, 2022.

  The thesis uses recurrent neural networks (LSTMs) and a suitable learning algorithm (deep recurrent Q-learning) to solve a control problem. The method was tested for the Cartpole-v1 example of the OpenAI-Gym. The goal is to take as input a sequence of environment observations at different points in time in order to achieve better solutions than with ordinary feedforward networks. It is investigated to what extent the optimization of the network structure or of the parameters lead to a better learning success.

- *Approximation von Lösungen parametrischer Optimierungsprobleme in der Echtzeitoptimierung (Approximation of solutions of parametric optimization problems in real-time optimization)*, Master thesis, Universität der Bundeswehr München, 2022

  The thesis explores the use of machine learning methods to approximate the solution map of paramatric optimization methods. The approximation can then be used in realtime optimization tasks within model-predictive control.

- *Vehicle routing problems*, Bachlor thesis, Universität der Bundeswehr München, in preparation, finishes in 2022.

  The thesis summarizes various formulations of vehicle routing problems and applies a Branch & Bound method to solve instances of the problems.

- *Maschinelles Lernen von Wertefunktionen durch Defektminimierung (Machine learning of the value function by defect minimization)*, Bachlor thesis, Universität der Bundeswehr München, in preparation, finishes in 2022.

  A method for the approximation of value functions of optimal control problems is developed in the thesis. To this end neural networks are used to approximate the value function and are trained such that the error of a certain fixed-point equation is minimized.

Chapter 6

Conclusions and Outlook

The first year of the project was devoted to define model scenarios that serve to validate numerical methods. This was done successfully for a large-scale scheduling problem and for a dynamic scheduling problem. In the latter, the linearization strategy, which was used to approximate the value function of the underlying optimal control problem, turned out to be very promising. In the second year of the project the linearization technique was further improved and applied to a dynamic scheduling problem with multiple UAVs in the presence of moving obstacles. The method was able to provide accurate solutions for minimum distance problems. Details of the methods, which were developed within the project, are provided in four publications, which are attached to this report.

Future work will further extend the method for dynamic scheduling problems and couple it to model-predictive control techniques in observation and exploration tasks.

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