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Report Title

Consensus of Multiple Nonholonomic Mechanical Systems

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Technical Report
of
"Consensus of Multiple Nonlinear Systems with Nonholonomic Constraints and Uncertainty"

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Consensus of Multiple Nonholonomic Mechanical Systems

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Abstract

This report considers the consensus problems of multiple nonlinear systems. For different nonlinear systems, distributed controllers are proposed such that the states of a group of systems converge to the same value with the aid of neighbors' information. In order to propose distributed control laws, different techniques are applied. To show the effectiveness of the proposed controllers, simulation has been done.

I. Introduction

Due to increasing task complexity and high performance requirements, distributed cooperative control of multiagent systems has been an active research area in recent years. While such systems have a wide spectrum of military applications there are still various challenging open research issues. Cooperative control of multiagent systems has attracted multi-disciplinary researchers from a wide array of fields: control system theory, physics, biology, applied mathematics, computer science, and robotics. In the past decade, the attention of most researchers has been paid to cooperative control of multiple identical linear systems or multiple mobile robots with specific simplified identical kinematic models without uncertainty. For cooperative control of multiple systems, various control strategies have been proposed, such as behavior-based [1, 12, 30], virtual structure [2, 15, 28], leader-follower [6, 19, 32, 35], artificial potentials [14, 17, 18, 23, 25, 31], and graph theoretical [5, 8, 13, 16] methods, to name a few.

In cooperative control, consensus algorithms play an important role. A major issue in consensus problems is how to achieve an agreement on some quantities among multiple systems, where agreement can mean a collaborative solution to some problem of interest. Many cooperative control problems can be solved with the aid of consensus algorithms or techniques developed for solving consensus problems. These cooperative control problems include collective behavior of flocks and swarms [25, 31], sensor fusion [24], formation control of multiple robot systems [13, 26, 32], synchronization of coupled oscillators [4, 11], etc. Pioneering work on the asynchronous agreement problem for distributed decision-making systems was done in [3, 33]. In [34], alignment of multiple discrete-time agents was discussed and control laws were proposed by using local information. The simulation results in [34] demonstrated that local controllers with neighbors' information can make all agents move in the same direction. In [10], Jadbabaie *et al.* provided a theoretical analysis of the consensus property of the Vicsek model with the aid of results from algebraic graph and matrix theories. For networked continuous-time systems, a theoretical framework for consensus control problems was introduced by Olfati-Saber and Murray in [27]. In [29], Ren extended the results obtained in [10, 27] and presented improved conditions for state agreement under a switching communication case. Paper [20] considered the stability of multiple agents with nonlinear models in discrete time and time-dependent communication links. Necessary and/or sufficient conditions for the convergence of the state of each individual agent to a consensus vector were presented with the aid of graph theory and convexity techniques.

In this report, we consider the consensus problem of multiple nonholonomic mechanical systems with kinematics and dynamics. In solving the consensus problems, different techniques have been applied. For example, algebraic graph theory, backstepping, adaptive control theory, and robust control theory.

II. Distributed Controller Design for Chained Systems

Consider m systems where the j-th system is defined by

$$\dot{x}_{1j} = v_{1j} \tag{1}$$
$$\dot{x}_{2j} = v_{2j} \tag{2}$$
$$\dot{x}_{ij} = v_{1j}x_{i-1,j}, \quad 3 \leq i \leq n \tag{3}$$

The communication between systems is defined by a directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$. We consider the following problem in this section.

Consensus of Multiple Chained Systems: For a group of m systems in (1)-(3), the problem is how to design a distributed control law (v_{1j}, v_{2j}) for system j based on its own information and its neighbors' information such that

$$\lim_{t\to\infty}(x_{*j} - c) = 0, \quad 1 \leq j \leq m, \tag{4}$$

where $x_{*j} = [x_{1j}, \ldots, x_{nj}]^\top$ and c is an unprescribed constant vector which depends on the initial condition of each system and the topology of digraph $\mathcal{G}$.

The system (1)-(3) has a cascade structure. (1) is a linear system and (2)-(3) is a linear time-varying system. We propose the following results for the system in (1).

Lemma 1: For m systems in (1), the distributed control law

$$v_{1j} = \eta_{1j} \tag{5}$$
$$\eta_{1j} = -k_1 x_{1j} + \zeta_{1j} \tag{6}$$
$$\dot{\zeta}_{1j} = -k_2\zeta_{1j} + \zeta_{2j} \tag{7}$$
$$\vdots$$
$$\dot{\zeta}_{n-3,j} = -k_{n-2}\zeta_{n-3,j} + \zeta_{n-2,j} \tag{8}$$
$$\dot{\zeta}_{n-2,j} = -\sum_{i\in\mathcal{N}_j} a_{ji}(\zeta_{n-2,j} - \zeta_{n-2,i}) - b(\zeta_{n-2,j} - \alpha) + \dot{\alpha} \tag{9}$$

ensures that

$$\lim_{t\to\infty}(x_{1j} - x_{1l}) \overset{exp.}{=} 0, \quad 1 \leq j \neq l \leq m \tag{10}$$

where a_{ji} are positive constants, $b > 0$, $k_i > 0$ for $1 \leq i \leq n-2$, and α is a variable. Furthermore, if α is a bounded persistently excited signal and $\dot{\alpha}$ is also bounded, then v_{1j} is a bounded PE signal and $\lim_{t\to\infty}(v_{1j} - v_{1l}) \overset{exp.}{=} 0$ for $1 \leq j \neq l \leq m$.

Proof: Let $\tilde{\zeta}_{n-2,j} = \zeta_{n-2,j} - \alpha$ and $\tilde{\zeta}_{n-2,*} = [\tilde{\zeta}_{n-2,1}, \ldots, \tilde{\zeta}_{n-2,m}]^\top$, with the control laws we have

$$\dot{\tilde{\zeta}}_{n-2,*} = -\mathcal{L}\tilde{\zeta}_{n-2,*} - b\tilde{\zeta}_{n-2,*} = -(\mathcal{L} + bI)\tilde{\zeta}_{n-2,*}. \tag{11}$$

where $\mathcal{L}$ is the Laplacian matrix. Since $b > 0$, $(\mathcal{L}+bI)$ is a Hurwitz matrix. Therefore, $\tilde{\zeta}_{n-2,*}$ exponentially converges to zero, which means that $\zeta_{n-2,j}$ exponentially converges to α.

Let $\tilde{x}_{1,jl} = x_{1j} - x_{1l}$ and $\tilde{\zeta}_{i,jl} = \zeta_{ij} - \zeta_{il}$ for $1 \leq i \leq n-3$ and $1 \leq j \neq l \leq m$, then

$$\dot{\tilde{x}}_{1,jl} = -k_1\tilde{x}_{1,jl} + \tilde{\zeta}_{1,jl}$$
$$\dot{\tilde{\zeta}}_{1,jl} = -k_2\tilde{\zeta}_{1,jl} + \tilde{\zeta}_{2,jl}$$
$$\vdots$$
$$\dot{\tilde{\zeta}}_{n-3,jl} = -k_{n-2}\tilde{\zeta}_{n-3,jl} + \tilde{\zeta}_{n-2,jl}$$

Since k_{n-2} is positive and $\tilde{\zeta}_{n-2,jl}$ exponentially converges to zero, it is obvious that $\tilde{\zeta}_{n-3,jl}$ exponentially converges to zero. Similarly, it can be proved that $\tilde{\zeta}_{n-3,jl}$ exponentially converges to zero. Repeat this procedure, it can be proved that $\tilde{x}_{1,jl}$ exponentially converges to zero.

Since $\lim_{t\to\infty}(x_{1j}-x_{1l}) \overset{exp.}{=} 0$ and $\lim_{t\to\infty}(\zeta_{1j}-\zeta_{1l}) \overset{exp.}{=} 0$, $\lim_{t\to\infty}(v_{1j}-v_{1l}) \overset{exp.}{=} 0$. By (6)-(9), $x_{1j} = H(s)\zeta_{n-2,j}$ where $H(s) = \frac{1}{\Pi_{r=1}^{n-3}(s+k_r)}$ and $H(s)$ is a stable, minimum phase, proper rational transfer function. So, $v_{1j} = sH(s)\zeta_{n-2,j}$. It is obvious that $sH(s)$ is also a stable, minimum phase, proper rational transfer function. By Lemma 4.8.3 in [9], v_{1j} is a bounded PE signal. ■

Remark 1: In Lemma 1, α is a control parameter. The reason for introducing it will be clear later.

Remark 2: In Lemma 1, b is a positive constant. Actually, b can be zero for some systems.

With the aid of the results in Lemma 1, we are ready to design controllers for the systems in (2)-(3). We define the variables

$$z_{ij} = x_{ij} + \beta_{ij}, \quad 2 \le i \le n \tag{12}$$

where

$$\begin{aligned}\beta_{nj} &= 0\\ \beta_{n-1,j} &= v_{1j}^{2n-5} z_{nj}\\ \beta_{lj} &= v_{1j}^{2n-5} z_{l+1,j} + \frac{1}{v_{1j}}\dot{\beta}_{l+1,j}, \quad l = n-2, n-3, \ldots, 2\end{aligned}$$

then we have

$$\dot{z}_{2j} = v_{2j} + \dot{\beta}_{2j} \tag{13}$$
$$\dot{z}_{ij} = -v_{1j}^{2n-4} z_{ij} + v_{1j} z_{i-1,j}, \quad 3 \le i \le n \tag{14}$$

It should be noted that the transform defined in (4) is global since β_{ij} is well-defined because its special form. The following results can be proved.

Lemma 2: By the transform (4), if

$$\lim_{t\to\infty}(z_{ij} - z_{il}) = 0, \quad 2 \le i \le n; 1 \le j \ne l \le m \tag{15}$$

then

$$\lim_{t\to\infty}(x_{ij} - x_{il}) = 0, \quad 2 \le i \le n; 1 \le j \ne l \le m. \tag{16}$$

The system (13)-(14) has a special structure. We will take advantage of this structure and have the following results.

Lemma 3: If v_{1j} is a bounded PE signal, z_{2j} and z_{2l} are bounded, and

$$\lim_{t\to\infty}(z_{2j} - z_{2l}) = 0, \quad 1 \le j \ne l \le m \tag{17}$$

then (15) holds.

Proof: For systems j and l and $j \ne l$, we define $e = z_{3j} - z_{3l}$. Then

$$\dot{e} = -v_{1j}^{2n-4} e + (v_{1l}^{2n-4} - v_{1j}^{2n-4}) z_{2l} + (v_{1j} z_{2j} - v_{2l} z_{2l}) \tag{18}$$

Since v_{1j} is a bounded PE signal and $(z_{2j} - z_{2l})$ converges to zero, it is obvious that e converges to zero, which means that z_{3j} converges to z_{3l}. Similarly, we can prove that z_{ij} converges to z_{il} for $i = 4, 5, \ldots, n$. ■

Thanks to Lemma 3 and Lemma 2, it only needs to design v_{2j} such that (17) holds. We have the following results.

Lemma 4: If the communication digraph has a spanning tree, the control law

$$v_{2j} = \eta_{2j} \tag{19}$$
$$\eta_{2j} = -\sum_{i\in\mathcal{N}_j} a_{ji}(z_{2j} - z_{2i}) - \dot{\beta}_{2j} \tag{20}$$

ensures that (16) holds.

Combine the results in Lemma 1 and Lemma 4, we have the following results.

Theorem 1: For the m systems in (1)-(2), if the communication digraph has a spanning tree and α is a bounded PE signal and $\dot{\alpha}$ is bounded, the control laws (5) and (20) for system j ensure that (4) holds, where the control parameters are defined in Lemmas 1 and 4.

In Theorem 1, $\dot{\alpha}$ is required to be a bounded PE signal. There are many choices of $\dot{\alpha}$. For example, we can choose $\alpha(t) = \sin t$.

There are other methods for designing distributed controllers for the kinematic systems in (1)-(3). For example, the method proposed in our paper [7] can be applied to propose distributed controllers.

III. Distributed Controller Design for Driftless Systems

In this section, we considered the following driftless systems

$$\dot{q}_{*j} = g_{1j}v_{1j} \cdots + g_{sj}v_{sj} = g_{*j}v_{*j} \tag{21}$$

where g_{1j}, …, g_{sj} are smooth functions on R^n such that in a neighborhood of 0 the dimension of the distribution $\Delta_j(q_{*j}) = Span\{g(q_{*j}) : g \in Lie\{g_{1j}, \ldots, g_{sj}\}\}$ is n.

The problem considered in this section is as follows.

Consensus of Multiple Systems: For a group of m systems in (21), the problem is how to design a distributed control law v_{*j} for system j based on its own information and its neighbors' information such that

$$\lim_{t\to\infty}(q_{*j} - c) = 0, \quad 1 \le j \le m, \tag{22}$$

where c is an unprescribed constant vector which depends on the initial condition of each system and the topology of digraph $\mathcal{G}$.

Practical distributed controllers can be proposed with the aid of the transverse function. For simplicity, we assume the communication between systems is bi-directional in this section.

With the aid of the results in [22], we have the following results.

Lemma 5: For the system in (21), if the dimension of the distribution $\mathrm{Span}\{g(x) : g \in Lie\{g_{1j}, \ldots, g_{sj}\}\}$ is n, there exists a function $f_j(\beta_{*j}, \epsilon_j) \in R^n$ such that the matrix

$$H_j(\beta_{*j}) = \left[g_{1j}(f_j), g_{2j}(f_j), \ldots, g_{sj}, \frac{\partial f_j}{\partial \beta_{1j}}, \ldots, \frac{\partial f_j}{\partial \beta_{n-s,j}}\right]$$

is nonsingular for any β_{*j} and $\epsilon_j > 0$, where $\beta_{*j} = [\beta_{1j}, \ldots, \beta_{n-2,j}]^\top$, $\beta_{*j} \in R^{n-s}$ and the function f_j has the following properties:

1) f_j is bounded for any β_{*j};
2) $\lim_{\epsilon_j\to 0} f_j(\beta_{*j}, \epsilon_j) = 0$.

The proof of Lemma 5 can be found in [21, 22]. The construction of f_j can be found also in [21, 22]. The function f_j is called the transverse function.

With the aid of Lemma 5, it can be found the function $f_j(\beta_j, \epsilon_j)$ such that G_j is nonsingular. Define

$$z_{*j} \;=\; q_{*j} f_j(\beta_{*j}, \epsilon_j)^{-1} \tag{23}$$

then, we have

$$\dot{z}_{*j} \;=\; dr_{f_j(\beta_{*j})}(q_{*j}) dl_{z_{*j}}(f_j(\beta_{*j})) H_j(\beta_{*j}) \begin{bmatrix} v_{*j} \\ -\dot{\beta}_{*j} \end{bmatrix} \tag{24}$$

where the meaning of the notations can be found in [22].

We define the neighbors's difference as

$$e_{*j} = [e_{1j}, \ldots, e_{nj}] = \sum_{i\in\mathcal{N}_j} a_{ji}(z_{*j} - z_{*i}). \tag{25}$$

If $\dot{\beta}_{*j}$ is considered as an additional input, we propose the following distributed control law.

Theorem 2: For the m systems in (21), if the communication graph has a spanning tree, the control law

$$\begin{aligned} v_{1j} &= \eta_{1j} && (26)\\ &\vdots \\ v_{sj} &= \eta_{sj} && (27)\\ \begin{bmatrix} \eta_{1j} \\ \vdots \\ \eta_{sj} \\ -\dot{\beta}_{*j} \end{bmatrix} &= -H_j(\beta_{*j})^{-1} dl_{z_{*j}^{-1}}(q_{*j}) dr_{f_j(\beta_{*j})}(z_{*j}) \left[\sum_{i \in \mathcal{N}_j} a_{ji}(z_{*j} - z_{*i}) \right] && (28) \end{aligned}$$

ensures that

$$\lim_{t\to\infty} \|q_{*j} - q_{*i}\| \leq \delta(\epsilon,\epsilon_i). \quad (29)$$

where $\delta(\epsilon,\epsilon_i)$ is a nonnegative continuous function of ϵ_i and ϵ_j and $\lim_{|\epsilon_j|+|\epsilon_i|\to 0} \delta(\epsilon,\epsilon_i) = 0$.

Proof: By Lemma 5, G_j is nonsingular. So, the control law exists. Substitute the control law into the system, we have

$$\begin{aligned} \dot{z}_{1j} &= -e_{1j} && (30)\\ &\vdots && (31)\\ \dot{z}_{nj} &= -e_{nj} && (32) \end{aligned}$$

Choose a function

$$V_i = z_{i*}^\top \mathcal{L} z_{i*}$$

where $z_{i*} = [z_{i1}, \ldots, z_{in}]^\top$ and $\mathcal{L}$ is the Laplacian matrix, we have

$$\dot{V}_i = -z_{i*}^\top \mathcal{L}^2 z_{i*}$$

By integrating both sides of the above inequality, it can be shown that z_{i*} is bounded and $\mathcal{L} z_{i*}$ converges to zero, which means that $(z_{ij} - z_{il})$ converges to zero for $1 \leq j \neq l \leq m$. Therefore, (29) holds. ■

Remark 3: If ϵ_j is chosen to be a small constant, $\|f_j - f_i\|$ is a small constant, which means that $\|x_{*j} - x_{*i}\|$ converges to a small neighborhood of the origin. We say (22) is achieved practically.

Remark 4: In Theorem 2, nothing is said about β_{*j}. So, β_{*j} may be bounded or unbounded. Thanks to the properties of the function f_j, the boundedness of β_{*j} plays no role in the consensus problem.

IV. Distributed Controller Design for Dynamical Systems

We consider m systems. The j-th system is defined as

$$\dot{x}_{1j} = v_{1j}, \quad \dot{x}_{2j} = v_{2j}, \quad \dot{x}_{ij} = v_{1j} x_{i-1,j}, \quad 3 \leq i \leq n \quad (33)$$

$$\tilde{M}_j \dot{v}_{*j} + \tilde{C}_j v_{*j} + \tilde{G}_j + \tilde{D}_j = \tilde{B}_j \tau_j \quad (34)$$

where $v_{*j} = [v_{1j}, v_{2j}]^\top$ and $x_{*j} = [x_{1j}, \ldots, x_{nj}]^\top$. The following properties are satisfied.

Property 1: $\tilde{M}_j$ is bounded and $\dot{\tilde{M}}_j - 2\tilde{C}_j$ is skew-symmetric.

Property 2: For any differentiable vector $\xi \in R^2$,

$$\tilde{M}_j \dot{\xi} + \tilde{C}_j \xi + \tilde{G}_j = \tilde{Y}_(x_{*j}, \dot{x}_{*j}, \xi, \dot{\xi}) a_j$$

where $\tilde{Y}_j$ is a known function of x_{*j}, $\dot{x}_{*j}$, ξ, and $\dot{\xi}$, and a_j is the inertia parameter vector.

The problem considered in this section is defined as follows.

Consensus of Multiple Systems: For a group of m systems in (33)-(34), the problem is how to design a distributed control law τ_j for system j based on its own information and its neighbors' information such that

$$\lim_{t\to\infty}(x_{*j} - c) = 0, \quad 1 \le j \le m, \tag{35}$$

where $x_{*j} = [x_{1j}, \dots, x_{nj}]^\top$ and c is an unprescribed constant vector which depends on the initial condition of each system and the topology of digraph $\mathcal{G}$.

In the dynamics (34), we first assume that the inertia parameter vector a_j is a constant and is unknown.

In order to design controllers for the systems in (33)-(34), we let

$$\tilde{v}_{1j} = v_{1j} - \eta_{1j} \tag{36}$$
$$\tilde{v}_{2j} = v_{2j} - \eta_{2j}, \quad 1 \le j \le m \tag{37}$$

where η_{1j} and η_{2j} are defined in (5) and (20), respectively. Then we have

$$\dot{x}_{1j} = \eta_{1j} + \tilde{v}_{1j} \tag{38}$$
$$\dot{x}_{2j} = \eta_{2j} + \tilde{v}_{2j} \tag{39}$$
$$\dot{x}_{ij} = (\eta_{1j} + \tilde{v}_{1j})x_{i-1,j}, \quad 3 \le i \le n \tag{40}$$
$$\tilde{M}_j\dot{\tilde{v}}_{*j} + \tilde{C}_j\tilde{v}_{*j} = \tilde{B}_j\tau_j - (\tilde{M}_j\dot{\eta}_{*j} + \tilde{C}_j\eta_{*j} + \tilde{G}_j + \tilde{D}_j) \tag{41}$$

where $v_{*j} = [v_{1j}, v_{2j}]^\top$.

For the systems in (38)-(40), we have the following results.

Lemma 6: For the m systems in (38)-(40), if the communication digraph has a spanning tree, then

1) $x_{ij} - x_{il}$ is bounded for $1 \le i \le n$ and $1 \le j \ne l \le m$ if $\tilde{v}_{1j}$ and $\tilde{v}_{2j}$ are bounded for $1 \le j \le m$.
2) $x_{ij} - x_{il}$ converges to zero for $1 \le i \le n$ and $1 \le j \ne l \le m$ if $\tilde{v}_{1j}$ and $\tilde{v}_{2j}$ converge to zero for $1 \le j \le m$.

Proof: Let $\tilde{x}_{1,jl} = x_{1j} - x_{1l}$, we have

$$\dot{\tilde{x}}_{1,jl} = -k_1\tilde{x}_{1,jl} + \tilde{\zeta}_{1,jl} + \tilde{v}_{1j} - \tilde{v}_{1l} \tag{42}$$

Since $\tilde{\zeta}_{1,jl}$ exponentially converges to zero, the system (42) has the input-to-state stability property for input $(\tilde{v}_{1j} - \tilde{v}_{1l})$. This means that $\tilde{x}_{1,jl}$ is bounded if $\tilde{v}_{1j}$ and $\tilde{v}_{1l}$ are bounded and $\tilde{x}_{1,jl}$ converges to zero if $\tilde{v}_{1j}$ and $\tilde{v}_{1l}$ converge to zero.

For the systems in (39)-(40), with the aid of the transform in (4), we have

$$\dot{z}_{2j} = \eta_{2j} + \dot{\beta}_{2j} + \tilde{v}_{2j} \tag{43}$$
$$\dot{z}_{ij} = -v_{1j}^{2n-4}z_{ij} + v_{1j}z_{i-1,j}, \quad 3 \le i \le n \tag{44}$$

It can be proved recursively that z_{ij} is bounded if $z_{i-1,j}$ is bounded and z_{ij} converges to zero if $z_{i-1,j}$ converges to zero. Therefore, $x_{ij} - x_{il}$ converges to zero for $2 \le i \le n$ and $1 \le j \ne l \le m$ if $\tilde{v}_{1j}$ and $\tilde{v}_{2j}$ converge to zero for $1 \le j \le m$. ■

With the aid of the results in Lemma 6, we can design controllers such that $\tilde{v}_{1*}$ and $\tilde{v}_{2*}$ are bounded and converge to zero.

For the dynamics of each system, we have

$$\tilde{M}_j\dot{\eta}_{*j} + \tilde{C}_j\eta_{*j} + \tilde{G}_j = \tilde{Y}_j(x_{*j}, \dot{x}_{*j}, \eta_{*j}, \dot{\eta}_{*j})a_j \tag{45}$$

where a_j is the inertia parameter. For the disturbance $\tilde{D}_j$, it is assumed that

$$\|\tilde{D}_j\| \le \rho_j(x_{*j}) \tag{46}$$

where ρ_j is a known function of x_{*j}. If the inertia parameter a_j is a constant and is unknown, we have the following results.

Theorem 3: For the m systems in (33)-(34), if the communication digraph has a spanning tree, the control law

$$\begin{aligned} \tau_j &= \tilde{B}_j^{-1}\left[-K_j\tilde{v}_{*j} + \tilde{Y}_j\hat{a}_j - \rho_j\text{sign}(\tilde{v}_{*j})\right] && (47)\\ \dot{\hat{a}}_j &= -\Gamma_j\tilde{Y}_j^{\top}\tilde{v}_{*j} && (48)\end{aligned}$$

ensures that (35) holds and $\hat{a}_j$ is bounded, where K_j and Γ_j are positive constant matrices.

Proof: Let

$$V_j = \frac{1}{2}\tilde{v}_{*j}^{\top}\tilde{M}_j\tilde{v}_{*j} + \frac{1}{2}(\hat{a}_j - a_j)\Gamma_j^{-1}(\hat{a}_j - a_j)$$

Differentiating it along the closed-loop system, we have

$$\begin{aligned} \dot{V}_j &= -\tilde{v}_{*j}^{\top}K_j\tilde{v}_{*j} - \rho_j\tilde{v}_{*j}^{\top}\text{sign}(\tilde{v}_{*j}) - \tilde{v}_{*j}^{\top}\tilde{D}_j\\ &\leq -\tilde{v}_{*j}^{\top}K_j\tilde{v}_{*j} \leq 0\end{aligned}$$

Therefore, V_j is bounded, which means that $\tilde{v}_{*j}$ and $\hat{a}_j$ are bounded. By Barbalat's lemma, it can be shown that $\tilde{v}_{*j}$ converges to zero. ■

In Theorem 3, the unknown inertia parameter is estimated by an adaptive control law. If an estimate of a_j is $\bar{a}_j$ and

$$\|a_j - \bar{a}_j\| \leq \gamma_j$$

for $1 \leq j \leq m$ and γ_j is a known constant, we propose the following robust control laws.

Theorem 4: For the m systems in (33)-(34), if the communication digraph has a spanning tree, the control law

$$\tau_j = \tilde{B}_j^{-1}\left[-K_j\tilde{v}_{*j} + \tilde{Y}_j\bar{a}_j - \gamma_j\tilde{Y}_j\text{sign}(\tilde{Y}_j^{\top}\tilde{v}_{*j}) - \rho_j\text{sign}(\tilde{v}_{*j})\right] \quad (49)$$

ensures that (35) holds, where K_j is a positive constant matrix.

Proof: Let

$$V_j = \frac{1}{2}\tilde{v}_{*j}^{\top}\tilde{v}_{*j}$$

Differentiating it along the closed-loop system, we have

$$\begin{aligned} \dot{V}_j &= -\tilde{v}_{*j}^{\top}K_j\tilde{v}_{*j} - \rho_j\tilde{v}_{*j}^{\top}\text{sign}(\tilde{v}_{*j}) - \tilde{v}_{*j}^{\top}\tilde{D}_j\\ &\quad +\tilde{v}_{*j}^{\top}\tilde{Y}_j(\bar{a}_j - a_j) - \gamma_j\tilde{v}_{*j}^{\top}\tilde{Y}_j\text{sign}(\tilde{Y}_j^{\top}\tilde{v}_{*j})\\ &\leq -\tilde{v}_{*j}^{\top}K_j\tilde{v}_{*j} \leq 0\end{aligned}$$

Therefore, V_j is bounded, which means that $\tilde{v}_{*j}$ is bounded. By Barbalat's lemma, it can be shown that $\tilde{v}_{*j}$ converges to zero. ■

In Theorem 4, the unknown inertia parameter a_j is not required to be a constant. In the control laws, γ_j is required to be known. It is possible to estimate it.

V. Distributed Controller Design for General Dynamical Systems

Consider m systems where the j-th system is defined by

$$\begin{aligned} \dot{q}_{*j} &= g_{1j}v_{1j}\cdots + g_{sj}v_{sj} = g_{*j}v_{*j} && (50)\\ \tilde{M}_j\dot{v}_{*j} + \tilde{C}_jv_{*j} + \tilde{G}_j + \tilde{D}_j &= \tilde{B}_j\tau_j && (51)\end{aligned}$$

where $v_{*j} = [v_{1j}, v_{2j}]^{\top}$, g_{1j}, ..., g_{sj} are smooth functions on R^n such that in a neighborhood of 0 the dimension of the distribution $\Delta_j(q_{*j}) = Span\{g(q_{*j}) : g \in Lie\{g_{1j}, \ldots, g_{sj}\}\}$ is n.

The following properties are satisfied.

Property 3: $\tilde{M}_j$ is bounded and $\dot{\tilde{M}}_j - 2\tilde{C}_j$ is skew-symmetric.

Property 4: For any differentiable vector $\xi \in R^2$,

$$\tilde{M}_j\dot{\xi} + \tilde{C}_j\xi + \tilde{G}_j = \tilde{Y}_(x_{*j}, \dot{x}_{*j}, \xi, \dot{\xi})a_j$$

where $\tilde{Y}_j$ is a known function of x_{*j}, $\dot{x}_{*j}$, ξ, and $\dot{\xi}$, and a_j is the inertia parameter vector.

The problem considered in this section is defined as follows.

Consensus of Multiple Systems: For a group of m systems in (50)-(51), the problem is how to design a distributed control law τ_j for system j based on its own information and its neighbors' information such that

$$\lim_{t\to\infty}(q_{*j} - c) = 0, \quad 1 \leq j \leq m, \tag{52}$$

where c is an unprescribed constant vector which depends on the initial condition of each system and the topology of digraph $\mathcal{G}$.

In the dynamics (51), we first assume that the inertia parameter vector a_j is a constant and is unknown.

In order to design controllers, we let

$$\tilde{v}_{ij} \;\; = \;\; v_{ij} - \eta_{ij}, \quad 1 \leq j \leq s \tag{53}$$

where η_{ij} is defined in (26)-(27), respectively. Then we have

$$\dot{z}_{*j} = -e_{*j} + g_{1j}\tilde{v}_{1j} + \cdots + g_{sj}\tilde{v}_{sj} \tag{54}$$

$$\tilde{M}_j\dot{\tilde{v}}_{*j} + \tilde{C}_j\tilde{v}_{*j} = \tilde{B}_j\tau_j - (\tilde{M}_j\dot{\eta}_{*j} + \tilde{C}_j\eta_{*j} + \tilde{G}_j + \tilde{D}_j) \tag{55}$$

For the dynamics of each system, we have

$$\tilde{M}_j\dot{\eta}_{*j} + \tilde{C}_j\eta_{*j} + \tilde{G}_j = \tilde{Y}_j(x_{*j}, \dot{x}_{*j}, \eta_{*j}, \dot{\eta}_{*j})a_j \tag{56}$$

where a_j is the inertia parameter. For the disturbance $\tilde{D}_j$, it is assumed that

$$\|\tilde{D}_j\| \leq \rho_j(x_{*j}) \tag{57}$$

where ρ_j is a known function of q_{*j}. If the inertia parameter a_j is a constant and is unknown, we have the following results.

Theorem 5: For the m systems in (50)-(51), if the communication graph has a spanning tree, the control law

$$\tau_j \;\; = \;\; \tilde{B}_j^{-1}\left[-K_j\tilde{v}_{*j} + \tilde{Y}_j\hat{a}_j - \rho_j\text{sign}(\tilde{v}_{*j}) - \Lambda_j\right] \tag{58}$$

$$\dot{\hat{a}}_j \;\; = \;\; -\Gamma_j\tilde{Y}_j^\top\tilde{v}_{*j} \tag{59}$$

ensures that $\hat{a}_j$ is bounded and (52) holds, where K_j and Γ_j are positive constant matrices, and

$$\Lambda_j \;\; = \;\; \begin{bmatrix} \sum_{i=1}^n e_{ij}[g_{1j}]_i \\ \vdots \\ \sum_{i=1}^n e_{ij}[g_{sj}]_i \end{bmatrix} \tag{60}$$

where $[\cdot]_i$ denotes the i-th element in $[\cdot]$.

Proof: Let

$$V = \sum_{i=1}^n z_{i*}^\top\mathcal{L}z_{i*} + \sum_{j=1}^m \frac{1}{2}\tilde{v}_{*j}^\top\tilde{M}_j\tilde{v}_{*j} + \sum_{j=1}^m \frac{1}{2}(\hat{a}_j - a_j)\Gamma_j^{-1}(\hat{a}_j - a_j)$$

Differentiating it along the closed-loop system, we have

$$\begin{aligned} \dot{V} \;\; &= \;\; -\sum_{i=1}^n z_{i*}^\top\mathcal{L}^2 z_{i*} - \sum_{j=1}^m \tilde{v}_{*j}^\top K_j\tilde{v}_{*j} - \sum_{j=1}^m \rho_j\tilde{v}_{*j}^\top\text{sign}(\tilde{v}_{*j}) - \sum_{j=1}^m \tilde{v}_{*j}^\top\tilde{D}_j \\ &\leq \;\; -\sum_{i=1}^n z_{i*}^\top\mathcal{L}^2 z_{i*} - \sum_{j=1}^m \tilde{v}_{*j}^\top K_j\tilde{v}_{*j} \end{aligned}$$

By integrating both sides of the above inequality, it can be shown that V is bounded, which means that s_{i*}, $\tilde{v}_{*j}$ and $\hat{a}_j$ are bounded. Furthermore, it can be shown that by Barbalat's lemma that z_{i*} and $\tilde{v}_{*j}$ converge to zero. Therefore, (29) holds. ■

In Theorem 5, the unknown inertia parameter is estimated by an adaptive control law. If an estimate of a_j is $\bar{a}_j$ and

$$\|a_j - \bar{a}_j\| \leq \gamma_j$$

for $1 \leq j \leq m$ and γ_j is a known constant, we propose the following robust control laws.

Theorem 6: For the m systems in (50)-(51), if the communication graph has a spanning tree, the control law

$$\tau_j \;=\; \tilde{B}_j^{-1}\left[-K_j\tilde{v}_{*j} + \tilde{Y}_j\bar{a}_j - \gamma_j\tilde{Y}_j\mathrm{sign}(\tilde{Y}_j^\top\tilde{v}_{*j}) - \rho_j\mathrm{sign}(\tilde{v}_{*j}) - \Lambda_j\right] \tag{61}$$

ensures that (52) holds, where K_j is a positive constant matrix and Λ_j is defined in (60).

Proof: Let

$$V = \sum_{i=1}^{n} z_{i*}^\top\mathcal{L}z_{i*} + \sum_{j=1}^{m}\frac{1}{2}\tilde{v}_{*j}^\top\tilde{M}_j\tilde{v}_{*j}$$

Differentiating it along the closed-loop system, we have

$$\begin{aligned}\dot{V}_j \;\leq\; & -\sum_{i=1}^{n} z_{i*}^\top\mathcal{L}^2z_{i*} - \sum_{j=1}^{m}\tilde{v}_{*j}^\top K_j\tilde{v}_{*j} - \sum_{j=1}^{m}\rho_j\tilde{v}_{*j}^\top\mathrm{sign}(\tilde{v}_{*j}) - \sum_{j=1}^{m}\tilde{v}_{*j}^\top\tilde{D}_j \\ & + \sum_{j=1}^{m}\tilde{v}_{*j}^\top\tilde{Y}_j(\bar{a}_j - a_j) - \sum_{j=1}^{m}\gamma_j\tilde{v}_{*j}^\top\tilde{Y}_j\mathrm{sign}(\tilde{Y}_j^\top\tilde{v}_{*j}) \\ \leq\; & -\sum_{i=1}^{n} z_{i*}^\top\mathcal{L}^2z_{i*} - \sum_{j=1}^{m}\tilde{v}_{*j}^\top K_j\tilde{v}_{*j} \leq 0\end{aligned}$$

Therefore, V is bounded, which means that z_{i*} and $\tilde{v}_{*j}$ are bounded. By Barbalat's lemma, it can be shown that z_{i*} and $\tilde{v}_{*j}$ converge to zero. ■

In Theorem 6, the unknown inertia parameter a_j is not required to be a constant. In the control laws, γ_j is required to be known. It is possible to estimate it.

VI. Simulation

To verify the proposed results, simulation has been done for five nonholonomic wheeled mobile robots on a horizontal plane. Robot j is constituted by a rigid trolley equipped with 3 nondeformable wheels. The orientation of the 2 wheels with respect to the trolley is fixed, while the orientation of the third wheel is varying. See Fig. 1 for details. We assume the plane of each wheel remains vertical and the wheel rotates around its (horizontal) axis. The contact between the wheels and the ground satisfies non slipping condition. The mobile robot is driven by 2 motors which provide torques acting on the rotational axes of the 2 wheels whose orientation is fixed.

The constraint of the non slipping condition can be written as

$$\dot{x}_j\sin\theta_j - \dot{y}_j\cos\theta_j = 0 \tag{62}$$

where (x_j, y_j) is the position of robot j and θ_j is the orientation of robot j. The dynamics of robot j are described by the following differential equations

$$\begin{cases} m_j\ddot{x}_j &= \lambda_j\cos\theta_j + \dfrac{1}{R_j}(\tau_{1j} + \tau_{2j})\cos\theta_j \\ m_j\ddot{y}_j &= -\lambda_j\sin\theta_j + \dfrac{1}{R_j}(\tau_{1j} + \tau_{2j})\sin\theta_j \\ I_j\ddot{\theta}_j &= \dfrac{L_j}{R_j}(\tau_{1j} - \tau_{2j}) \end{cases} \tag{63}$$

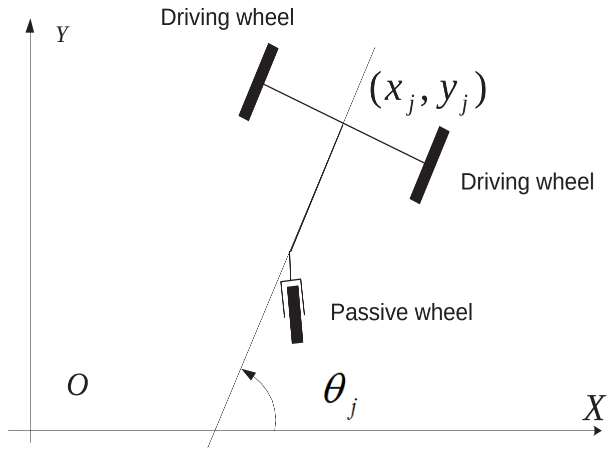

Fig. 1. Configuration of robot j

where m_j is the mass of robot j, and I_j is its inertia moment around the vertical axis at point Q. R_j is the radius of the wheels and $2L_j$ the length of the axis of the front wheels, and τ_{1j} and τ_{2j} are the torques provided by the motors.

Let $q_{*j} = [x_j, y_j, \theta_j]^\top$,

$$M_j(q_{*j}) = \begin{bmatrix} m_j & 0 & 0 \\ 0 & m_j & 0 \\ 0 & 0 & I_j \end{bmatrix}, C_j(q_{*j}, \dot{q}_{*j}) = 0, G_j(q_{*j}) = 0$$

$$B_j(q_{*j}) = \frac{1}{R_j} \begin{bmatrix} \cos\theta_j & \cos\theta_j \\ \sin\theta_j & \sin\theta_j \\ L_j & -L_j \end{bmatrix}, J_j = [\sin\theta_j, -\cos\theta_j, 0]$$

The system (62)-(63) is in the following form

$$J_j(q_{*j})\dot{q}_{*j} = 0 \tag{64}$$

$$\dot{M}_j(q_{*j})\ddot{q}_{*j} + C_j(q_{*j}, \dot{q}_{*j})\dot{q}_{*j} + G_j(q_{*j}) = B_j(q_{*j})\tau_j + J(q_{*j})_j^\top \lambda_j. \tag{65}$$

Let

$$g_{*j} = \begin{bmatrix} \cos\theta_j & 0 \\ \sin\theta_j & 0 \\ 0 & 1 \end{bmatrix}$$

then Equations (62) and (63) are converted into

$$\begin{cases} \dot{x}_j & = & u_{1j}\cos\theta_j \\ \dot{y}_j & = & u_{1j}\sin\theta_J \\ \dot{\theta}_j & = & u_{2j} \\ m_j\dot{u}_{1j} & = & \dfrac{1}{R_j}(\tau_{1j} + \tau_{2j}) \\ I_j\dot{u}_{2j} & = & \dfrac{L_j}{R_j}(\tau_{1j} - \tau_{2j}) \end{cases} \tag{66}$$

With the transformation

$$\begin{cases} x_{1j} & = & -\theta_j \\ x_{2j} & = & x_j\cos\theta_j + y_j\sin\theta_j \\ x_{3j} & = & -x_j\sin\theta_j + y_j\cos\theta_j \\ v_{1j} & = & -u_{2j} \\ v_{2j} & = & u_{1j} - x_{3j}v_{1j} \end{cases}$$

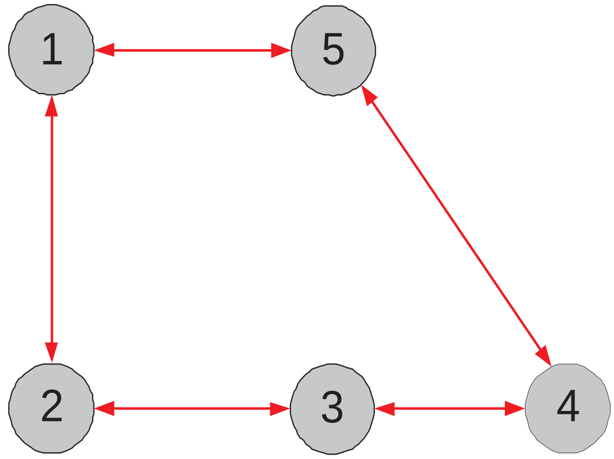

Fig. 2. Communication graph.

Equation (66) can be converted into the following standard form

$$\begin{cases} \dot{x}_{1j} = v_{1j} \\ \dot{x}_{2j} = v_{2j} \\ \dot{x}_{3j} = x_{2j} v_{1j} \\ \tilde{M}_j \dot{v}_{*j} + \tilde{C}_j v_{*j} = \tilde{B}_j \tau_{*j} \end{cases} \tag{67}$$

where

$$\tilde{M}_j = \begin{bmatrix} I_j + m_j x_{3j}^2 & m_j x_{3j} \\ m_j x_{3j} & m_j \end{bmatrix}, \tilde{C}_j = \begin{bmatrix} m_j x_{3j} \dot{x}_{3j} & 0 \\ m_j \dot{x}_{3j} & 0 \end{bmatrix}, \tilde{B}_j = \frac{1}{R_j} \begin{bmatrix} x_{3j} + L_j & x_{3j} - L_j \\ 1 & 1 \end{bmatrix}$$

and

$$\tilde{M}_j(q_{*j})\dot{\xi} + \tilde{C}_j(q_{*j}, \dot{q}_{*j})\xi = \tilde{Y}_j(q_{*j}, \dot{q}_{*j}, \xi, \dot{\xi}) a_j$$

where the inertia parameter vector $a_j = [m_j, I_j]^T$,

$$\tilde{Y}_j(q_{*j}, \dot{q}_{*j}, \xi, \dot{\xi}) = \begin{bmatrix} x_{3j}^2 \dot{\xi}_1 + x_{3j} \dot{\xi}_2 + x_{3j} \dot{x}_{3j} \xi_1 & \dot{\xi}_1 \\ x_{3j} \dot{\xi}_1 + \dot{\xi}_2 + \dot{x}_{3j} \xi_1 & 0 \end{bmatrix}$$

The consensus problem of the kinematics in (67) can be solved with the aid of the results proposed in Theorem 1. If the communication graph is shown in Fig. 2. The controller is proposed as

$$v_{1j} = \eta_{1j} \tag{68}$$

$$\eta_{1j} = -k_1 x_{1j} + \zeta_{1j} \tag{69}$$

$$\dot{\zeta}_{1j} = -\sum_{i \in \mathcal{N}_j} a_{ji}(\zeta_{1j} - \zeta_{1i}) - b(\zeta_{1j} - \alpha) + \dot{\alpha} \tag{70}$$

$$v_{2j} = \eta_{2j} \tag{71}$$

$$\eta_{2j} = -\sum_{i \in \mathcal{N}_j} a_{ji}(z_{2j} - z_{2i}) \tag{72}$$

where $k_1 > 0$, $a_{ji} > 0$, $b > 0$, and $\alpha = sin(14t)$, $z_{2j} = x_{2j} + \beta_{2j}$, $z_{3j} = x_{3j} + \beta_{3j}$, $\beta_{3j} = 0$, and $\beta_{2j} = v_{1j} z_{3j}$. Figs. 3-5 show the time response of x_{1*}, x_{2*}, and x_{3*}, respectively. It is shown that the state of five systems reach consensus.

The consensus problem of the dynamics in (67) can be solved with the aid of the results in Theorem 3. The controller is proposed as in (6)-(48) if the inertia parameter vector a_j is a constant and is unknown. Figs. 6-8 show the time response of x_{1*}, x_{2*}, and x_{3*}, respectively. It is shown that the state of five systems reach consensus.

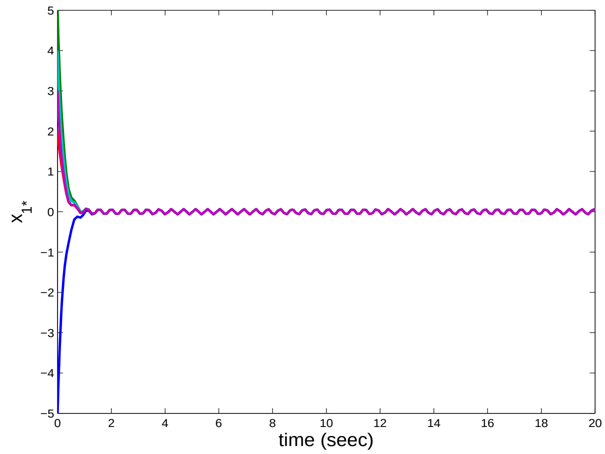

Fig. 3. Response of x_{1*}.

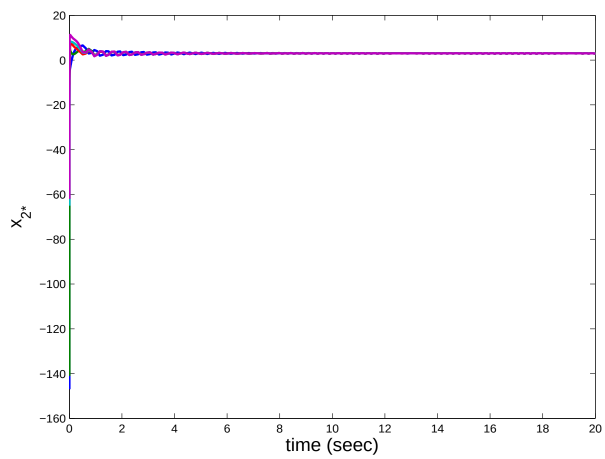

Fig. 4. Response of x_{2*}.

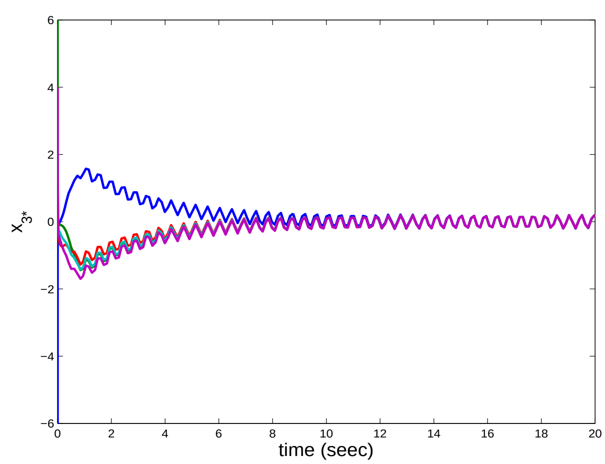

Fig. 5. Response of x_{2*}.

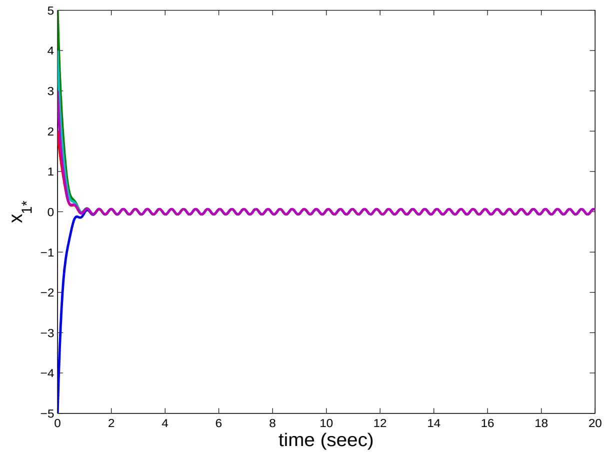

Fig. 6. Response of x_{1*}.

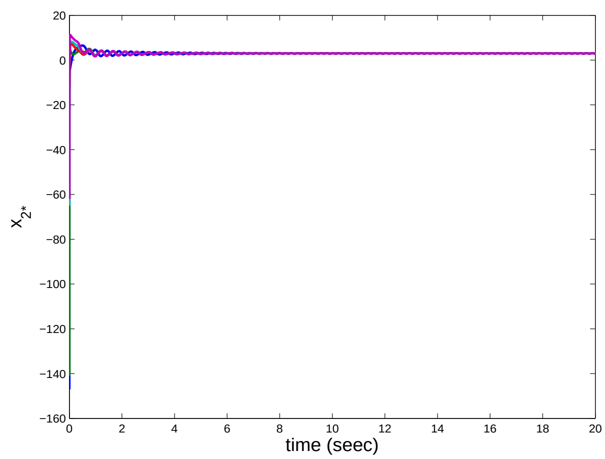

Fig. 7. Response of x_{2*}.

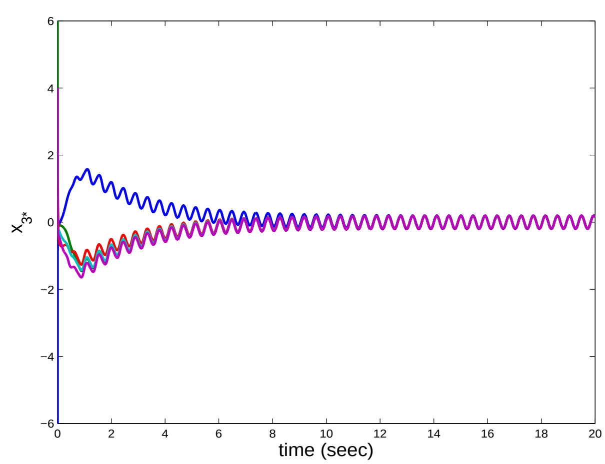

Fig. 8. Response of x_{2*}.

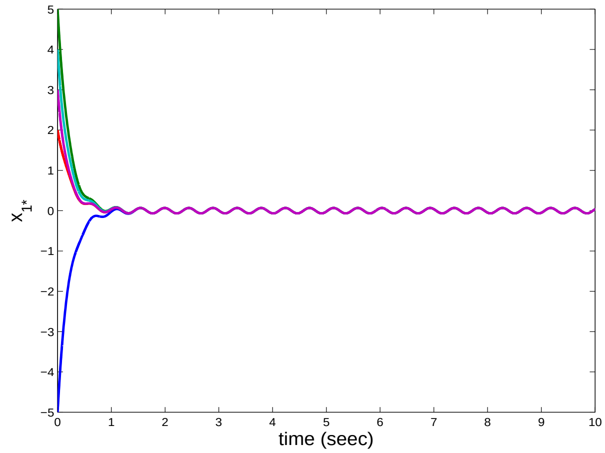

Fig. 9. Response of x_{1*}.

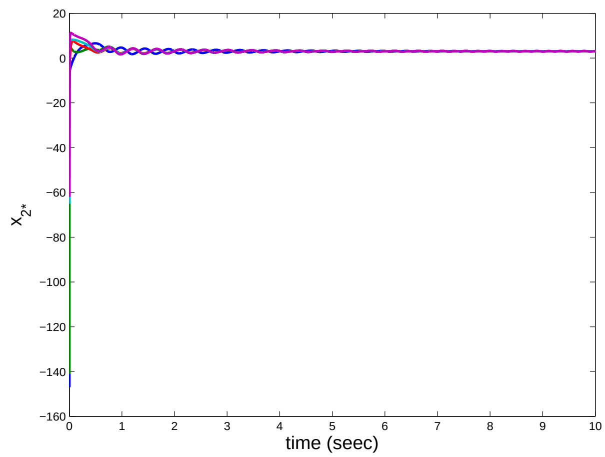

Fig. 10. Response of x_{2*}.

If the inertia parameter vector a_j is a constant and is unknown, we can also solve the consensus problem by the robust control algorithms in Theorem 4. Figs. 9-11 show the time response of x_{1*}, x_{2*}, and x_{3*}, respectively. It is shown that the state of five systems reach consensus.

VII. Conclusion

This report considered the consensus problems of several groups of nonlinear systems. Distributed controllers were proposed. Simulation results show the effectiveness of the proposed results.

References

[1] T. Balch and R. C. Arkin, "Behavior-based formation control for multirobot teams," *IEEE Trans. on Robotics and Automation*, vol. 14, no. 6, pp. 926–939, 1998.

[2] R. W. Beard, J. Lawton, and F. Y. Hadaegh, "A coordination architecture for spacecraft formation control," *IEEE Trans. on Control Systems Technology*, vol. 9, no. 6, pp. 777–790, 2001.

[3] V. Borkar and P. Varaiya, "Asymptotic agreement in distributed estimation," *IEEE Trans. on Automatic Control*, vol. 27, pp. 650–655, 1982.

[4] N. Chopra and M. W. Spong, "On synchronization of kuramoto oscillators," *Proc. of IEEE Conf. on Decision and Control*, vol. 42, no. 7, pp. 3916–3922, 2005.

[5] A. K. Das, R. Fierro, and V. Kumar, *Cooperative Control and Optimization of Applied Optimization, in Control Graphs for Robot Networks*. Norwell, MA: Kluwer, 2002.

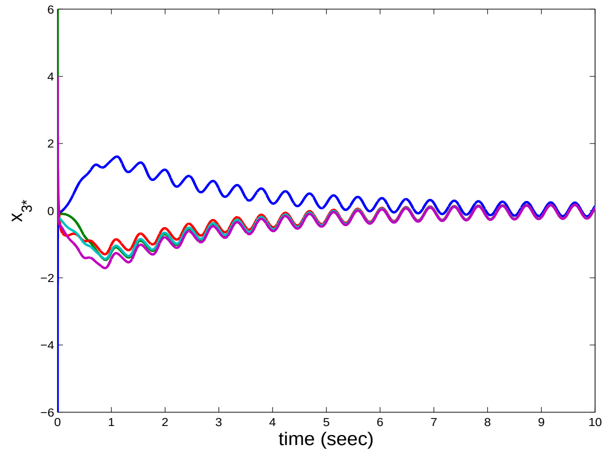

Fig. 11. Response of x_{2*}.

[6] J. P. Desai, J. P. Ostrowski, and V. Kumar, “Modeling and control of formations of nonholonomic mobile robots,” *IEEE Trans. on Robotics and Automation*, vol. 17, pp. 905–908, 2001.

[7] W. Dong and J. A. Farrell, “Cooperative control of multiple nonholonomic mobile agents,” *IEEE Trans Auto Contr*, vol. 53, no. 6, pp. 1434–1448, 2008.

[8] J. A. Fax and R. M. Murray, “Information flow and cooperative control of vehicle formations,” *IEEE Trans. on Auto. Contr.*, vol. 49, pp. 1465–1476, 2004.

[9] P. Ioannou and J. Sun, *Robust Adaptive Control*. Prentice Hall Inc., 1996.

[10] A. Jadbabaie, J. Lin, and A. S. Morse, “Coordination of groups of mobile autonomous agents using nearest neighbor rules,” *IEEE Trans. on Automatic Control*, vol. 48, pp. 988–1001, 2003.

[11] A. Jadbabaie, N. Motee, and M. Barahona, “On the stability of the kuramoto model of coupled nonlinear oscillators,” *Proc. of American Control Conference*, pp. 4296–4301, 2004.

[12] R. T. Jonathan, J. Lawton, R. W. Beard, and B. J. Young, “A decentralized approach to formation maneuvers,” *IEEE Trans. on Robotics and Automation*, vol. 19, pp. 933–941, 2003.

[13] G. Lafferriere, A. Williams, J. Caughman, and J. J. P. Veermany, “Decentralized control of vehicle formations,” *Systems and Control Letters*, vol. 53, pp. 899–910, 2005.

[14] N. E. Leonard and E. Fiorelli, “Virtual leaders, artificial potentials and coordinated control of groups,” *Proc. of the IEEE Conf. on Decision and Control*, pp. 2968–2973, 2001.

[15] M. A. Lewis and K.-H. Tan, “High precision formation control of mobile robots using virtual structures,” *Autonomous Robots*, vol. 4, pp. 387–403, 1997.

[16] Z. Lin, B. Francis, and M. Maggiore, “State agreement for coupled nonlinear systems with time-varying interaction,” *SIAM J. of Control and Optimization*, vol. 46, no. 1, pp. 288–307, 2007.

[17] Y. Liu and K. M. Passino, “Cohesive behaviors of multiagent systems with information flow constraints,” *IEEE Trans. on Automatic Control*, vol. 51, no. 11, pp. 1734–1748, 2006.

[18] Y. Liu, K. M. Passino, and M. M. Polycarpou, “Stable social foraging swarms in a noisy environment,” *IEEE Trans. on Automatic Control*, vol. 49, no. 1, pp. 30–44, 2004.

[19] M. Mesbahi and F. Y. Hadaegh, “Formation flying control of multiple spacecraft via graphs, matrix inequalities, and switching,” *AIAA J. of Guidance, Control, and Dynamics*, vol. 24, pp. 369–377, 2001.

[20] L. Moreau, “Stability of multiagent systems with time-dependent communication links,” *IEEE Trans. on Automatic Control*, vol. 50, no. 2, pp. 169–182, 2005.

[21] P. Morin and C. Samson, “Practical stabilization of a class of nonlinear systems. application to chain systems and mobile robots,” in *Decision and Control, 2000. Proceedings of the 39th IEEE Conference on*, vol. 3, 2000, pp. 2989–2994 vol.3.

[22] ——, “Practical stabilization of driftless systems on lie groups: the transverse function approach,” *Automatic Control, IEEE Transactions on*, vol. 48, no. 9, pp. 1496–1508, Sept 2003.

[23] P. Ogren, E. Fiorelli, and N. E. Leonard, “Cooperative control of mobile sensor networks: adaptive gradient climbing in a distributed environment,” *IEEE Trans. on Automatic Control*, vol. 40, no. 8, pp. 1292–1302, 2004.

[24] R. Olfati-Saber, “Distributed kalman filter with embedded consensus filter,” *Proc. of IEEE Conf. on Decision and Control*, pp. 8179–8185, 2005.

[25] ——, “Flocking for multi-agent dynamic systems: algorithms and theory,” *IEEE Trans. on Automatic Control*, vol. 51, no. 3, pp. 401–420, 2006.

[26] R. Olfati-Saber and R. M. Murray, “Distributed structural stabilization and tracking for formations of dynamic multiagents,” *Proc. IEEE Conf. Decision and Control*, pp. 209–215, 2002.

[27] ——, “Consensus problems in networks of agents with switching topology and time-delays,” *IEEE Trans. on Automatic Control*, vol. 49, pp. 101–115, 2004.

[28] W. Ren and R. W. Beard, "Formation feedback control for multiple spacecraft via virtual structures," *IEE Proceedings - Control Theory and Applications*, vol. 151, no. 3, pp. 357–368, 2004.

[29] ——, "Consensus seeking in multi-agent systems under dynamically changing interaction topologies," *IEEE Trans. on Automatic Control*, vol. 50, no. 5, pp. 655–661, 2005.

[30] K. Sugihara and I. Suzuki, "Distributed algorithms for formation of geometric patterns with many mobile robots," *J. of Robotic Systems*, vol. 13, no. 3, pp. 127–139, 1996.

[31] H. G. Tanner, A. Jadbabaie, and G. J. Pappas, "Flocking in fixed and switching networks," *IEEE Trans. on Automatic Control*, vol. 52, no. 5, pp. 863–868, 2007.

[32] H. G. Tanner, G. J. Pappas, and V. Kumar, "Leader-to-formation stability," *IEEE Trans. on Robotics and Automation*, vol. 20, pp. 443–455, 2004.

[33] J. N. Tsitsiklis and M. Athens, "Convergence and asymptotic agreement in distributed decision problems," *IEEE Trans. on Automatic Control*, vol. 29, pp. 690–698, 1984.

[34] T. Vicsek, A. Czirok, E. B. Jacob, I. Cohen, and O. Schochet, "Novel type of phase transitions in a system of self-driven particles," *Physical Review Letters*, vol. 75, pp. 1226–1229, 1995.

[35] P. K. C. Wang and F. Y. Hadaegh, "Coordination and control of multiple microspacecraft moving in formation," *The J. of the Astronautical Sciences*, vol. 44, no. 3, pp. 315–355, 1996.