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14. ABSTRACT

This report considers the tracking control problems of multiple nonholonomic mechanical systems with nonideal nonholonomic constraints. For different nonlinear systems with uncertainty, distributed tracking controllers are proposed such that the states of a group of systems converge to the state of a leader system with the aid of neighbors' information. In order to propose distributed control laws, different techniques are applied. To show the effectiveness of the proposed controllers, simulation has been done.

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"Distributed Tracking Control of Multiple Nonholonomic Mechanical Systems with Non-ideal Nonholonomic Constraints"

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Distributed Tracking Control of Multiple Nonholonomic Mechanical Systems with Non-ideal Nonholonomic Constraints

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Abstract

This report considers the tracking control problems of multiple nonholonomic mechanical systems with non-ideal nonholonomic constraints. For different nonlinear systems with uncertainty, distributed tracking controllers are proposed such that the states of a group of systems converge to the state of a leader system with the aid of neighbors' information. In order to propose distributed control laws, different techniques are applied. To show the effectiveness of the proposed controllers, simulation has been done.

I. Introduction

Due to increasing task complexity and high performance requirements, distributed cooperative control of multiagent systems has been an active research area in recent years. While such systems have a wide spectrum of military applications there are still various challenging open research issues. Cooperative control of multiagent systems has attracted multi-disciplinary researchers from a wide array of fields: control system theory, physics, biology, applied mathematics, computer science, and robotics. In the past decade, the attention of most researchers has been paid to cooperative control of multiple identical linear systems or multiple mobile robots with specific simplified identical kinematic models without uncertainty. For cooperative control of multiple systems, various control strategies have been proposed, such as behavior-based [5, 18, 42], virtual structure [6, 21, 37], leader-follower [12, 25, 45, 48], artificial potentials [20, 23, 24, 29, 31, 43], and graph theoretical [11, 13, 19, 22] methods, to name a few.

In cooperative control, consensus algorithms play an important role. A major issue in consensus problems is how to achieve an agreement on some quantities among multiple systems, where agreement can mean a collaborative solution to some problem of interest. Many cooperative control problems can be solved with the aid of consensus algorithms or techniques developed for solving consensus problems. These cooperative control problems include collective behavior of flocks and swarms [31, 43], sensor fusion [30], formation control of multiple robot systems [19, 33, 45], synchronization of coupled oscillators [10, 17], etc. Pioneering work on the asynchronous agreement problem for distributed decision-making systems was done in [7, 46]. In [47], alignment of multiple discrete-time agents was discussed and control laws were proposed by using local information. The simulation results in [47] demonstrated that local controllers with neighbors' information can make all agents move in the same direction. In [16], Jadbabaie *et al.* provided a theoretical analysis of the consensus property of the Vicsek model with the aid of results from algebraic graph and matrix theories. For networked continuous-time systems, a theoretical framework for consensus control problems was introduced by Olfati-Saber and Murray in [34]. In [38], Ren extended the results obtained in [16, 34] and presented improved conditions for state agreement under a switching communication case. Paper [26] considered the stability of multiple agents with nonlinear models in discrete time and time-dependent communication links. Necessary and/or sufficient conditions for the convergence of the state of each individual agent to a consensus vector were presented with the aid of graph theory and convexity techniques.

In addition to achieving a constant agreement, there are many papers on designing consensus algorithms such that the state of each system converges to a time-varying trajectory. In [35, 36], for multiple first-order and second-order linear systems, consensus algorithms were proposed such that the state of each system converges to a time-varying reference trajectory under the condition that the reference trajectory is only available to a portion of the systems. In [8, 9], distributed tracking via a variable structure approach was considered for multiple first-order and second-order systems. Distributed discontinuous controllers were proposed such that the state of each system converges to a desired trajectory within finite time. In [14, 15], tracking control for multiple linear agents with an active leader was discussed. Distributed controllers were proposed with the aid of distributed estimators. In [1], a passivity framework was proposed to steer the differences between output variables of a group of systems to a prescribed compact set. The proposed framework in [1] can be applied to solve consensus problems of multiple linear systems. In [3], a motion coordination problem was studied for achieving identical orientation and synchronous rotation for a group of rigid bodies. Distributed adaptive controllers were proposed such that the angular velocity of each rigid body converges to a desired angular velocity under the condition that the desired angular velocity is available only to the leader. In [2], a coordination problem was studied to steer a group of agents to a formation that translates with a prescribed reference velocity. An adaptive design was proposed such that the velocity of each agent converges to a reference velocity and the formation of multiple agents converges to a desired formation. In [4], adaptive motion coordination of multiple systems was studied with the aid of the results in [1]. Distributed adaptive control laws were proposed such that a reference velocity is tracked by each system with the aid of communications between systems. Flocking of multiple systems can be considered as a consensus problem. In [31], flocking of multiple second-order agents was solved with the aid of potential functions under the assumption that a desired trajectory is available to each agent. In [44], flocking of multiple systems was discussed for fixed and switching communication cases such that the velocity of each agent reaches an agreement. In [40, 41], flocking algorithms of multiple second-order linear systems were proposed under the assumptions that the information of a virtual leader is available to a portion of systems with appropriate assumptions on the virtual leader. In papers [32, 39], many results on consensus problems were surveyed.

In this report, we consider the tracking control problem of multiple nonholonomic mechanical systems with non-ideal constraints. A nonholonomic system with non-ideal constraints is defined by a nonlinear system with uncertainty. Therefore, we consider the tracking control problems of multiple nonlinear systems with uncertainty. In solving the tracking cocntrol problems, different techniques have been applied. For example, algebraic graph theory, backstepping, adaptive control theory, and robust control theory.

II. Distributed Controller Design for Kinematic Systems

Consider m systems where the j-th system is defined by

$$\dot{x}_{1j} = v_{1j} + \phi_{1j}(x_{1j}) \quad (1)$$
$$\dot{x}_{2j} = v_{2j} + \phi_{2j}(\bar{x}_{2j}) \quad (2)$$
$$\dot{x}_{ij} = v_{1j}x_{i-1,j} + \phi_{ij}(\bar{x}_{ij}), \quad 3 \le i \le n \quad (3)$$

The communication between systems is defined by a directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$. For simple presentation, it is assumed that the communication between systems is bi-directional in this chapter.

It is given a desired trajectory $x^d = [x_{1d}, \ldots, x_{nd}]^\top$ which is generated by

$$\dot{x}_{1d} = v_{1d} \quad (4)$$
$$\dot{x}_{2d} = v_{2d} \quad (5)$$
$$\dot{x}_{id} = v_{1d}x_{i-1,d}, \quad 3 \le i \le n \quad (6)$$

where v_{1d} and v_{2d} are known functions. It is assumed that

$$\max_{t\in[0,\infty)} |x_{id}| \le \delta_i, \quad 1 \le i \le n.$$

where δ_i is a positive constant.

The problem considered in this chapter is defined as follows.

Tracking Control of Multiple Chained Systems: For a group of m systems in (1)-(3), it is given a desired trajectory x^d, the problem is how to design a distributed control law (v_{1j}, v_{2j}) for system j based on its own information and its neighbors' information such that

$$\lim_{t\to\infty}(x_{*j} - x^d) = 0, \quad 1 \leq j \leq m, \tag{7}$$

where $x_{*j} = [x_{1j}, \ldots, x_{nj}]^\top$.

It is assumed that the uncertain terms in (1)-(3) satisfy the following conditions:

$$|\phi_{1j}(x_{1j})| \leq \gamma_{1j}(x_{1j}), \quad |\phi_{2j}(\bar{x}_{2j})| \leq \gamma_{2j}(\bar{x}_{2j}), \quad |\phi_{ij}(\bar{x}_{ij})| \leq \gamma_{ij}(\bar{x}_{ij}), \ 3 \leq i \leq n \tag{8}$$

where β_{ij} are known positive functions.

With the aid of the results in [28], we have the following results.

Lemma 1: For the system in (1)-(3), there exists a function $f_j(\beta_{*j}, \epsilon_j) \in R^n$ such that the the matrix $G_j(\beta_{*j}) = [g_{1j}(f_j), g_{2j}(f_j), \frac{\partial f_j}{\partial \beta_{1j}}, \ldots, \frac{\partial f_j}{\partial \beta_{n-2,j}}]$ is nonsingular for any β_{*j} and $\epsilon_j > 0$, where $g_{1j} = [1, 0, x_{2j}, \ldots, x_{n-1,j}]^\top$, $g_{2j} = [0, 1, 0, \ldots, 0]^\top$, $\beta_{*j} = [\beta_{1j}, \ldots, \beta_{n-2,j}]^\top$, $\beta \in R^{n-2}$ and the function f_j has the following properties:

1) f_j is bounded for any β_{*j};
2) $\lim_{\epsilon_j \to 0} f_j(\beta_{*j}, \epsilon_j) = 0$.

The proof of Lemma 1 can be found in [27, 28]. The construction of f_j can be found also in [27, 28]. The function f_j is called the transverse function.

With the aid of Lemma 1 and the notations in [28], it can be found the function $f_j(\beta_j, \epsilon_j)$ such that G_j is nonsingular. Let

$$z_{*j} = x_{*j} f_j(\beta_{*j})^{-1}$$

then, we have the augmented system

$$\dot{z}_{*j} \;=\; dr_{f_j(\beta_{*j})^{-1}}(x_{*j}) dl_{z_{*j}}(f_j(\beta_{*j})) G_j(\beta_{*j})[v_{1j}, v_{2j}, -\dot{\beta}_{1j}, \ldots, -\dot{\beta}_{n-2,j}]^\top \tag{9}$$

We define the neighbors's difference as

$$e_{*j} = [e_{1j}, \ldots, e_{nj}] = \sum_{i \in \mathcal{N}_j} a_{ji}(z_{*j} - z_{*i}) + b_j(z_{*j} - x^d). \tag{10}$$

where $b_j = 1$ if the desired trajectory is available to system j and $b_j = 0$ if x^d is not available to system j. If $\dot{\beta}_{*j}$ is considered as an additional input, we propose the following distributed control law.

Theorem 1: For the m systems in (1)-(2), if the communication graph has a spanning tree, the control law

$$v_{1j} \;=\; \eta_{1j} \tag{11}$$

$$v_{2j} \;=\; \eta_{2j} \tag{12}$$

$$\begin{bmatrix} \eta_{1j} \\ \eta_{2j} \\ -\dot{\beta}_{*j} \end{bmatrix} \;=\; -G_j^{-1} dl_{z_{*j}^{-1}}(x_{*j}) dr_{f_j(\beta_{*j})}(z_{*j}) \left[\sum_{i \in \mathcal{N}_j} a_{ji}(z_{*j} - z_{*i}) + b_j(z_{*j} - x^d) - \Delta_{*j}\right] \tag{13}$$

ensures that

$$\lim_{t\to\infty} \|x_{*j} - x^d\| \leq \delta_j(\epsilon_j) \tag{14}$$

where δ_j is a nonnegative continuous function of ϵ_j and δ_j converges to zero when ϵ_j converges to zero, and

$$\Delta_{*j} \;=\; \begin{bmatrix} \frac{(\gamma_{1j}+\delta_1)e_{1j}}{\sqrt{e_{1j}^2 + e^{-t}}} \\ \vdots \\ \frac{(\gamma_{nj}+\delta_n)e_{nj}}{\sqrt{e_{nj}^2 + e^{-t}}} \end{bmatrix} \tag{15}$$

Proof: By Lemma 1, G_j is nonsingular. So, the control law exists. Substitute the control law into the system and define $\tilde{z}_{ij} = z_{ij} - x_{id}$, we have

$$\dot{\tilde{z}}_{1j} = -e_{1j} - \frac{\gamma_{1j} e_{1j}}{\sqrt{e_{1j}^2 + e^{-t}}} + \phi_{1j} - \dot{x}_{1d} \tag{16}$$

$$\vdots \tag{17}$$

$$\dot{\tilde{z}}_{nj} = -e_{nj} - \frac{\gamma_{nj} e_{nj}}{\sqrt{e_{nj}^2 + e^{-t}}} + \phi_{nj} - \dot{x}_{nd} \tag{18}$$

Choose a function

$$V_i = \tilde{z}_{i*}^\top \mathcal{L}^e \tilde{z}_{i*}$$

where $\tilde{z}_{i*} = [\tilde{z}_{i1}, \ldots, \tilde{z}_{in}]^\top$ and $\mathcal{L}$ is the Laplacian matrix, we have

$$\begin{aligned}\dot{V}_i &= -\tilde{z}_{i*}^\top \mathcal{L}^e \mathcal{L}^e \tilde{z}_{i*} - \sum_{j=1}^{m} \frac{\gamma_{ij} e_{ij}^2}{\sqrt{e_{ij}^2 + e^{-t}}} + \sum_{j=1}^{m} e_{ij}(\phi_{ij} - \dot{x}_{id}) \\ &\leq -\tilde{z}_{i*}^\top \mathcal{L}^e \mathcal{L}^e \tilde{z}_{i*} - \sum_{j=1}^{m} \gamma_{ij} \sqrt{e_{ij}^2 + e^{-t}} + \sum_{j=1}^{m} \frac{e^{-t}}{\sqrt{e_{ij}^2 + e^{-t}}} + \sum_{j=1}^{m} \gamma_{ij} |e_{ij}| \\ &\leq -\tilde{z}_{i*}^\top \mathcal{L}^e \mathcal{L}^e \tilde{z}_{i*} + m e^{-t/2}\end{aligned}$$

By integrating both sides of the above inequality, it can be shown that $\tilde{z}_{i*}$ is bounded and $\mathcal{L}^e \tilde{z}_{i*}$ converges to zero, which means that $\tilde{z}_{*j}$ converges to zero. Therefore, (14) holds. ■

Remark 1: If ϵ_j ($1 \leq j \leq m$) are chosen to be small constants, $\delta_{ji}(\epsilon_j, \epsilon_i)$ is a small constant, which means that $\|x_{ij} - x_{id}\|$ converges to a small neighborhood of the origin. We say (7) is achieved practically.

Remark 2: In Theorem 1, nothing is said about β_{*j}. So, β_{*j} may be bounded or unbounded. Thanks to the properties of the function f_j, the boundedness of β_{*j} plays no role in the consensus problem.

III. Distributed Controller Design for Dynamical Systems

We considered m dynamical systems. The j-th system is defined as

$$\dot{x}_{1j} = v_{1j} + \phi_{1j}(x_{1j}) \tag{19}$$

$$\dot{x}_{2j} = v_{2j} + \phi_{2j}(\bar{x}_{2j}) \tag{20}$$

$$\dot{x}_{ij} = v_{1j} x_{i-1,j} + \phi_{ij}(\bar{x}_{ij}), \quad 3 \leq i \leq n \tag{21}$$

$$\tilde{M}_j \dot{v}_{*j} + \tilde{C}_j v_{*j} + \tilde{G}_j + \tilde{D}_j = \tilde{B}_j \tau_j \tag{22}$$

where $v_{*j} = [v_{1j}, v_{2j}]^\top$ and $x_{*j} = [x_{1j}, \ldots, x_{nj}]^\top$. The following properties are satisfied.

Property 1: $\tilde{M}_j$ is bounded and $\dot{\tilde{M}}_j - 2\tilde{C}_j$ is skew-symmetric.

Property 2: For any differentiable vector $\xi \in R^2$,

$$\tilde{M}_j \dot{\xi} + \tilde{C}_j \xi + \tilde{G}_j = \tilde{Y}_(x_{*j}, \dot{x}_{*j}, \xi, \dot{\xi}) a_j$$

where $\tilde{Y}_j$ is a known function of x_{*j}, $\dot{x}_{*j}$, ξ, and $\dot{\xi}$, and a_j is the inertia parameter vector.

The problem considered in this section is defined as follows.

Tracking Control of Multiple Systems: For a group of m systems in (19)-(22), the problem is how to design a distributed control law τ_j for system j based on its own information and its neighbors' information such that

$$\lim_{t \to \infty} (x_{*j} - c) = 0, \quad 1 \leq j \leq m, \tag{23}$$

where $x_{*j} = [x_{1j}, \ldots, x_{nj}]^\top$ and c is an unprescribed constant vector which depends on the initial condition of each system and the topology of digraph $\mathcal{G}$.

In the dynamics (22), we first assume that the inertia parameter vector a_j is a constant and is unknown.

In order to design controllers, we let

$$\begin{aligned} \tilde{v}_{1j} &= v_{1j} - \eta_{1j} \quad (24)\\ \tilde{v}_{2j} &= v_{2j} - \eta_{2j}, \quad 1 \le j \le m \quad (25) \end{aligned}$$

where η_{1j} and η_{2j} are defined in (11) and (12), respectively. Let

$$\chi_j = [\chi_{i,l,j}] = r_{f_j(\beta_{*j})^{-1}}(x_{*j}) dl_{z_{*j}}(f_j(\beta_{*j}))G_j(\beta_{*j})$$

then we have

$$\dot{z}_{1j} = -e_{1j} - \frac{\gamma_{1j}e_{1j}}{\sqrt{e_{1j}^2 + e^{-t}}} + \phi_{1j} + \chi_{1,1,j}\tilde{v}_{1j} + \chi_{1,2,j}\tilde{v}_{2j} \quad (26)$$

$$\dot{z}_{2j} = -e_{2j} - \frac{\gamma_{2j}e_{2j}}{\sqrt{e_{2j}^2 + e^{-t}}} + \phi_{2j} + \chi_{2,1,j}\tilde{v}_{1j} + \chi_{2,2,j}\tilde{v}_{2j} \quad (27)$$

$$\dot{z}_{ij} = -e_{ij} - \frac{\gamma_{ij}e_{ij}}{\sqrt{e_{ij}^2 + e^{-t}}} + \phi_{ij} + +\chi_{i,1,j}\tilde{v}_{1j} + \chi_{i,2,j}\tilde{v}_{2j}, \quad 3 \le i \le n \quad (28)$$

$$\tilde{M}_j \dot{\tilde{v}}_{*j} + \tilde{C}_j \tilde{v}_{*j} = \tilde{B}_j \tau_j - (\tilde{M}_j \dot{\eta}_{*j} + \tilde{C}_j \eta_{*j} + \tilde{G}_j + \tilde{D}_j) \quad (29)$$

where $\tilde{v}_{*j} = [\tilde{v}_{1j}, \tilde{v}_{2j}]^\top$.

For the dynamics of each system, we have

$$\tilde{M}_j \dot{\eta}_{*j} + \tilde{C}_j \eta_{*j} + \tilde{G}_j = \tilde{Y}_j(x_{*j}, \dot{x}_{*j}, \eta_{*j}, \dot{\eta}_{*j}) a_j \quad (30)$$

where a_j is the inertia parameter. For the disturbance $\tilde{D}_j$, it is assumed that

$$\|\tilde{D}_j\| \le \rho_j(x_{*j}) \quad (31)$$

where ρ_j is a known function of x_{*j}. If the inertia parameter a_j is a constant and is unknown, we have the following results.

Theorem 2: For the m systems in (19)-(22), if the communication graph has a spanning tree, the control law

$$\begin{aligned} \tau_j &= \tilde{B}_j^{-1}\left[-K_j\tilde{v}_{*j} + \tilde{Y}_j\hat{a}_j - \rho_j \mathrm{sign}(\tilde{v}_{*j}) - \Lambda_j\right] \quad (32)\\ \dot{\hat{a}}_j &= -\Gamma_j \tilde{Y}_j^\top \tilde{v}_{*j} \quad (33) \end{aligned}$$

ensures that $\hat{a}_j$ is bounded and (14) holds, where K_j and Γ_j are positive constant matrices, and

$$\Lambda_j = \begin{bmatrix} \sum_{i=1}^{n} e_{ij}\chi_{i,1,j} \\ \sum_{i=1}^{n} e_{ij}\chi_{i,2,j} \end{bmatrix} \quad (34)$$

Proof: Let

$$V = \sum_{i=1}^{n} \tilde{z}_{i*}^\top \mathcal{L}^e \tilde{z}_{i*} + \sum_{j=1}^{m} \frac{1}{2}\tilde{v}_{*j}^\top \tilde{M}_j \tilde{v}_{*j} + \sum_{j=1}^{m} \frac{1}{2}(\hat{a}_j - a_j)^\top \Gamma_j^{-1}(\hat{a}_j - a_j)$$

Differentiating it along the closed-loop system, we have

$$\begin{aligned}\dot{V} &\leq -\sum_{i=1}^{n}\tilde{z}_{i*}^{\top}\mathcal{L}^{e}\mathcal{L}^{e}\tilde{z}_{i*}-\sum_{j=1}^{m}\tilde{v}_{*j}^{\top}K_{j}\tilde{v}_{*j}-\sum_{j=1}^{m}\rho_{j}\tilde{v}_{*j}^{\top}\text{sign}(\tilde{v}_{*j})-\sum_{j=1}^{m}\tilde{v}_{*j}^{\top}\tilde{D}_{j}+me^{-t/2}\\ &\leq -\sum_{i=1}^{n}\tilde{z}_{i*}^{\top}\mathcal{L}^{e}\mathcal{L}^{e}\tilde{z}_{i*}-\sum_{j=1}^{m}\tilde{v}_{*j}^{\top}K_{j}\tilde{v}_{*j}+me^{-t/2}\end{aligned}$$

By integrating both sides of the above inequality, it can be shown that V is bounded, which means that $\tilde{z}_{i*}$, $\tilde{v}_{*j}$ and $\hat{a}_j$ are bounded. Furthermore, it can be shown that by Barbalat's lemma that e_{i*} and $\tilde{v}_{*j}$ converge to zero. Therefore, (14) holds. ■

In Theorem 2, the unknown inertia parameter is estimated by an adaptive control law. If an estimate of a_j is $\bar{a}_j$ and

$$\|a_j-\bar{a}_j\|\leq\gamma_j$$

for $1\leq j\leq m$ and γ_j is a known constant, we propose the following robust control laws.

Theorem 3: For the m systems in (19)-(22), if the communication graph has a spanning tree, the control law

$$\tau_j = \tilde{B}_j^{-1}\left[-K_j\tilde{v}_{*j}+\tilde{Y}_j\bar{a}_j-\gamma_j\tilde{Y}_j\text{sign}(\tilde{Y}_j^{\top}\tilde{v}_{*j})-\rho_j\text{sign}(\tilde{v}_{*j})-\Lambda_j\right] \tag{35}$$

ensures that (14) holds, where K_j is a positive constant matrix and Λ_j is defined in (34).

Proof: Let

$$V=\sum_{i=1}^{n}\tilde{z}_{i*}^{\top}\mathcal{L}^{e}\tilde{z}_{i*}+\sum_{j=1}^{m}\frac{1}{2}\tilde{v}_{*j}^{\top}\tilde{M}_j\tilde{v}_{*j}$$

Differentiating it along the closed-loop system, we have

$$\begin{aligned}\dot{V}_j &\leq -\sum_{i=1}^{n}\tilde{z}_{i*}^{\top}\mathcal{L}^{e}\mathcal{L}^{e}\tilde{z}_{i*}-\sum_{j=1}^{m}\tilde{v}_{*j}^{\top}K_{j}\tilde{v}_{*j}-\sum_{j=1}^{m}\rho_{j}\tilde{v}_{*j}^{\top}\text{sign}(\tilde{v}_{*j})-\sum_{j=1}^{m}\tilde{v}_{*j}^{\top}\tilde{D}_{j}\\ &\quad+\sum_{j=1}^{m}\tilde{v}_{*j}^{\top}\tilde{Y}_j(\bar{a}_j-a_j)-\sum_{j=1}^{m}\gamma_j\tilde{v}_{*j}^{\top}\tilde{Y}_j\text{sign}(\tilde{Y}_j^{\top}\tilde{v}_{*j})+me^{-t/2}\\ &\leq -\sum_{i=1}^{n}\tilde{z}_{i*}^{\top}\mathcal{L}^{e}\mathcal{L}^{e}\tilde{z}_{i*}-\sum_{j=1}^{m}\tilde{v}_{*j}^{\top}K_{j}\tilde{v}_{*j}+me^{-t/2}\end{aligned}$$

Therefore, V is bounded, which means that $\tilde{z}_{i*}$ and $\tilde{v}_{*j}$ are bounded. By Barbalat's lemma, it can be shown that e_{i*} and $\tilde{v}_{*j}$ converge to zero. ■

In Theorem 3, the unknown inertia parameter a_j is not required to be a constant. In the control laws, γ_j is required to be known. It is possible to estimate it.

IV. Simulation

We considered five nonholonomic wheeled mobile robots which are moving in a horizon plane. For each robot (see Fig. 1), the constraint on the front wheels can be written as

$$\dot{x}_j\sin\theta_j-\dot{y}_j\cos\theta_j=P_j(x_j,y_j) \tag{36}$$

where (x_j,y_j) is the position of robot j, θ_j is the orientation of robot j, and P_j denotes slight slipping along the axis of the wheels. The dynamics of robot j are described by the following differential equations

$$\begin{cases}m_j\ddot{x}_j = \lambda_j\cos\theta_j+\dfrac{1}{R_j}(\tau_{1j}+\tau_{2j})\cos\theta_j\\ m_j\ddot{y}_j = -\lambda_j\sin\theta_j+\dfrac{1}{R_j}(\tau_{1j}+\tau_{2j})\sin\theta_j\\ I_j\ddot{\theta}_j = \dfrac{L_j}{R_j}(\tau_{1j}-\tau_{2j})\end{cases} \tag{37}$$

where m_j is the mass of robot j, and I_j is its inertia moment around the vertical axis at point Q. R_j is the radius of the wheels and $2L_j$ the length of the axis of the front wheels, and τ_{1j} and τ_{2j} are the torques provided by the motors.

Let $q_{*j} = [x_j, y_j, \theta_j]^\top$, and

$$g_{*j} = \begin{bmatrix} \cos\theta_j & 0 \\ \sin\theta_j & 0 \\ 0 & 1 \end{bmatrix}$$

then Equation (36) and (37) are converted into

$$\begin{cases} \dot{x}_j & = & u_{1j}\cos\theta_j + P_j\sin\theta_j \\ \dot{y}_j & = & u_{1j}\sin\theta_j - P_j\cos\theta_j \\ \dot{\theta}_j & = & u_{2j} \\ m_j\dot{u}_{1j} & = & \dfrac{1}{R_j}(\tau_{1j}+\tau_{2j}) \\ I_j\dot{u}_{2j} & = & \dfrac{L_j}{R_j}(\tau_{1j}-\tau_{2j}) \end{cases} \tag{38}$$

It is given a desired trajectory $q^d = [q_{1d}, q_{2d}, q_{3d}]^\top$ which is generated by

$$\dot{q}_{1d} = v_d\cos q_{3d}, \quad \dot{q}_{2d} = v_d\sin q_{3d}, \quad \dot{q}_{3d} = \omega_d$$

where v_d and ω_d are time-varying functions.

With the transformation

$$\begin{cases} x_{1j} & = & -\theta_j \\ x_{2j} & = & x_j\cos\theta_j + y_j\sin\theta_j \\ x_{3j} & = & -x_j\sin\theta_j + y_j\cos\theta_j \\ v_{1j} & = & -u_{2j} \\ v_{2j} & = & u_{1j} - x_{3j}v_{1j} \end{cases}$$

Equation (38) can be converted into the following standard form

$$\begin{cases} \dot{x}_{1j} = v_{1j} \\ \dot{x}_{2j} = v_{2j} \\ \dot{x}_{3j} = x_{2j}v_{1j} - P_j \\ \tilde{M}_j\dot{v}_{*j} + \tilde{C}_j v_{*j} = \tilde{B}_j\tau_{*j} \end{cases} \tag{39}$$

where

$$\tilde{M}_j = \begin{bmatrix} I_j + m_j x_{3j}^2 & m_j x_{3j} \\ m_j x_{3j} & m_j \end{bmatrix}, \tilde{C}_j = \begin{bmatrix} m_j x_{3j}\dot{x}_{3j} & 0 \\ m_j\dot{x}_{3j} & 0 \end{bmatrix}, \tilde{B}_j = \frac{1}{R_j}\begin{bmatrix} x_{3j}+L_j & x_{3j}-L_j \\ 1 & 1 \end{bmatrix}$$

and

$$\tilde{M}_j(q_{*j})\dot{\xi} + \tilde{C}_j(q_{*j}, \dot{q}_{*j})\xi = \tilde{Y}_j(q_{*j}, \dot{q}_{*j}, \xi, \dot{\xi})a_j$$

where the inertia parameter vector $a_j = [m_j, I_j]^T$,

$$\tilde{Y}_j(q_{*j}, \dot{q}_{*j}, \xi, \dot{\xi}) = \begin{bmatrix} x_{3j}^2\dot{\xi}_1 + x_{3j}\dot{\xi}_2 + x_{3j}\dot{x}_{3j}\xi_1 & \dot{\xi}_1 \\ x_{3j}\dot{\xi}_1 + \dot{\xi}_2 + \dot{x}_{3j}\xi_1 & 0 \end{bmatrix}$$

By the transformation

$$\begin{cases} x_{1d} & = & -q_{3d} \\ x_{2d} & = & q_{1d}\cos q_{3d} + q_{2d}\sin q_{3d} \\ x_{3d} & = & -q_{1d}\sin q_{3d} + q_{2d}\cos q_{3d} \\ v_{1d} & = & -\omega_d \\ v_{2d} & = & v_{1d} - x_{3d}v_{1d} \end{cases}$$

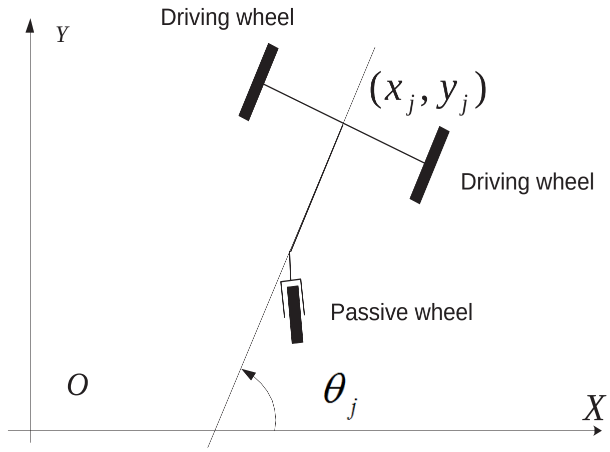

Fig. 1. Configuration of robot j

we have

$$\dot{x}_{1d} = v_{1d}, \quad \dot{x}_{2d} = v_{2d}, \quad \dot{x}_{3d} = v_{1d}x_{2d}.$$

The tracking control problem of the kinematics in (39) can be solved with the aid of the results proposed in Theorem 1. By the results in [28], we choose

$$f_{*j} = \begin{bmatrix} \epsilon_j \sin\beta_j \\ \epsilon_j \cos\beta_j \\ \frac{\epsilon_j^2}{4}\sin 2\beta_j \end{bmatrix}, \quad \epsilon_j > 0 \tag{40}$$

Then

$$\frac{df_{*j}}{d\beta_{1j}} = \begin{bmatrix} \epsilon_j \cos\beta_j \\ -\epsilon_j \sin\beta_j \\ \frac{\epsilon_j^2}{2}\cos 2\beta_j \end{bmatrix} \dot{\beta}_{1j}. \tag{41}$$

It can be verified that f_j satisfies the properties in Lemma 1.

Let

$$z_{*j} = x_{*j} f_{*j}^{-1} = \begin{bmatrix} x_{1j} - f_{1j} \\ x_{2j} - f_{2j} \\ x_{3j} - f_{3j} - f_{1j}(x_{2j} - f_{2j}) \end{bmatrix}$$

the controller is proposed as

$$v_{1j} = \eta_{1j} \tag{42}$$

$$v_{2j} = \eta_{2j} \tag{43}$$

$$\begin{bmatrix} \eta_{1j} \\ \eta_{2j} \\ -\dot{\beta}_j \end{bmatrix} = \begin{bmatrix} 1 & 0 & \epsilon_j \cos\beta_j \\ 0 & 1 & -\epsilon_j \sin\beta_j \\ \epsilon_j \cos\beta_j & 0 & \frac{\epsilon_j^2}{2}\cos 2\beta_j \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ z_{2j} & -\epsilon_j \sin\beta_j & 1 \end{bmatrix}^{-1} \times \left[\sum_{i\in\mathcal{N}_j} a_{ji}(z_{*j} - z_{*i}) + \Delta_{*j} \right] \tag{44}$$

where $a_{ji} > 0$ and Δ_j is defined in (15) with $n = 3$.

In the simulation, we choose $P_j(x_j, y_j) = 0.2\sin t$. The desired trajectory x^d is assumed to be $x^d = [x_{1d}, x_{2d}, x_{3d}]^\top =$. Figs. 3-5 show the time response of x_{1*}, x_{2*}, and x_{3*}, respectively. It is shown that the state of three systems reach consensus.

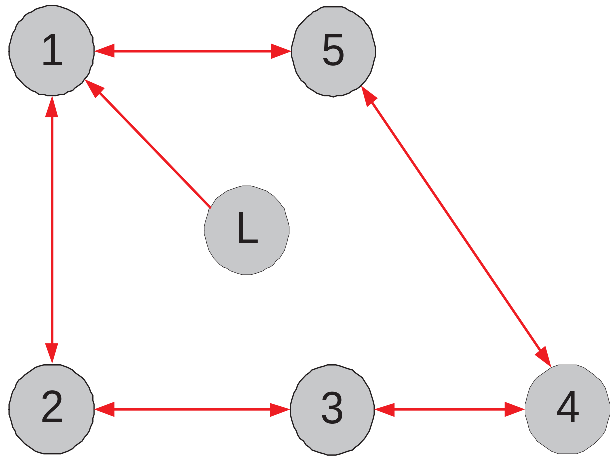

Fig. 2. Communication graph.

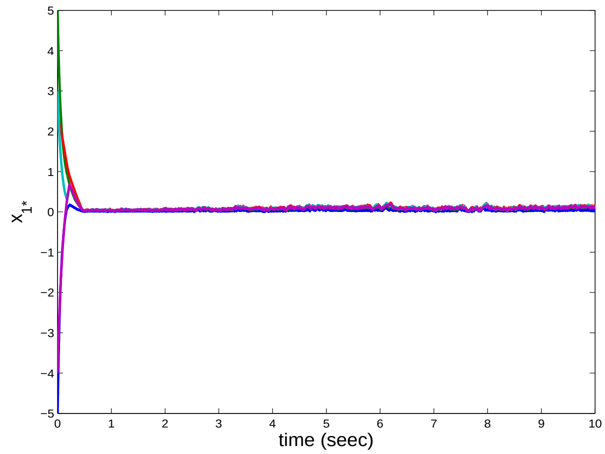

Fig. 3. Response of x_{1*}.

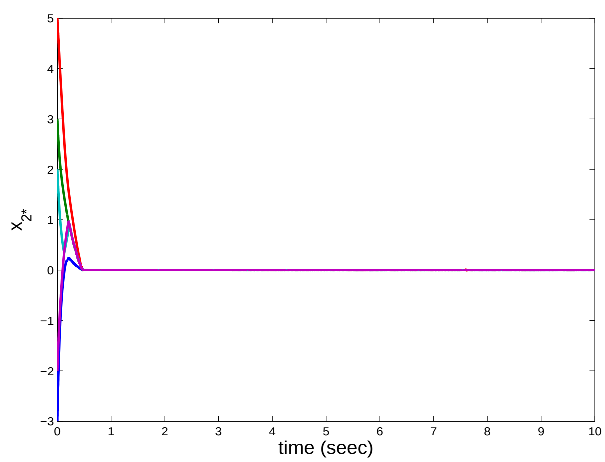

Fig. 4. Response of x_{2*}.

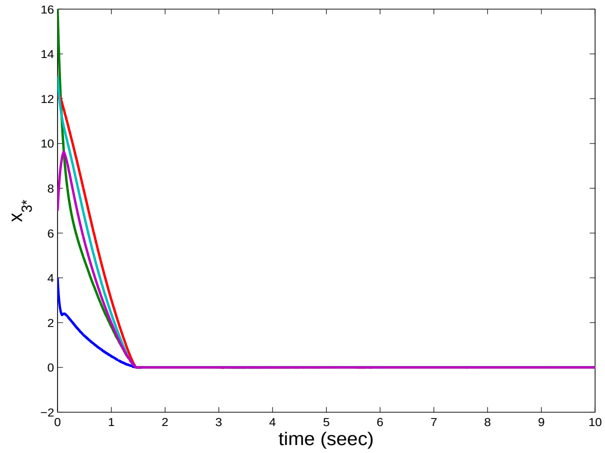

Fig. 5. Response of x_{2*}.

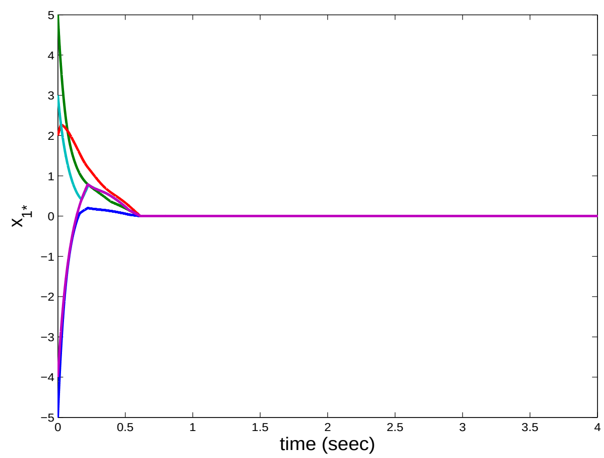

Fig. 6. Response of x_{1*}.

The tracking control problem of the dynamics in (39) can be solved with the aid of the results in Theorem 2. The controller is proposed as in (32)-(33) if the inertia parameter vector a_j is a constant and is unknown. Figs. 6-8 show the time response of x_{1*}, x_{2*}, and x_{3*}, respectively. It is shown that the state of three systems reach consensus.

If the inertia parameter vector a_j is a constant and is unknown, we can also solve the consensus problem by the robust control algorithms in Theorem 3. Figs. 9-11 show the time response of x_{1*}, x_{2*}, and x_{3*}, respectively. It is shown that the state of three systems reach consensus.

V. Conclusion

This report considered the tracking control problems of several groups of nonlinear systems with uncertainty. Distributed tracking controllers were proposed. Simulation results show the effectiveness of the proposed results.

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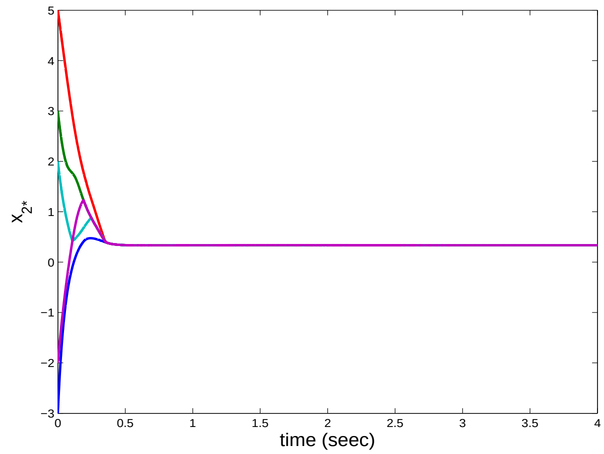

Fig. 7. Response of x_{2*}.

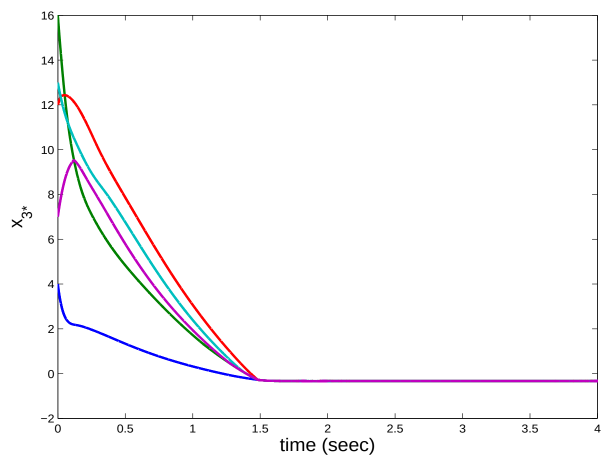

Fig. 8. Response of x_{2*}.

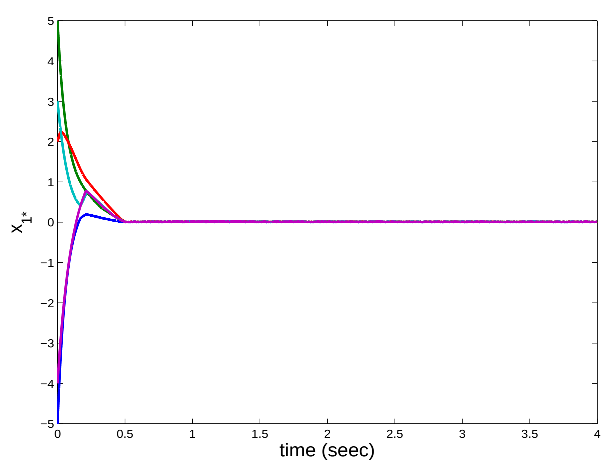

Fig. 9. Response of x_{1*}.

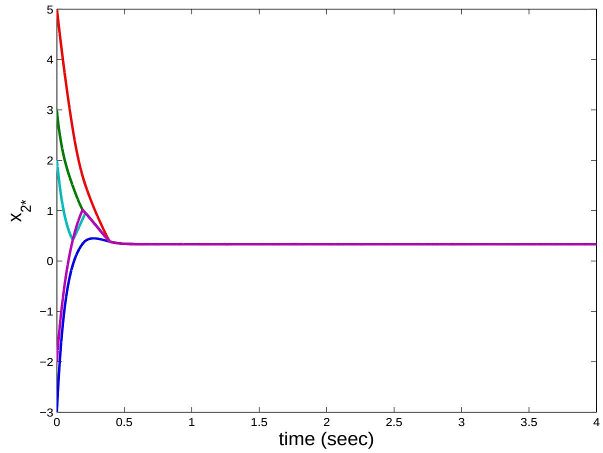

Fig. 10. Response of x_{2*}.

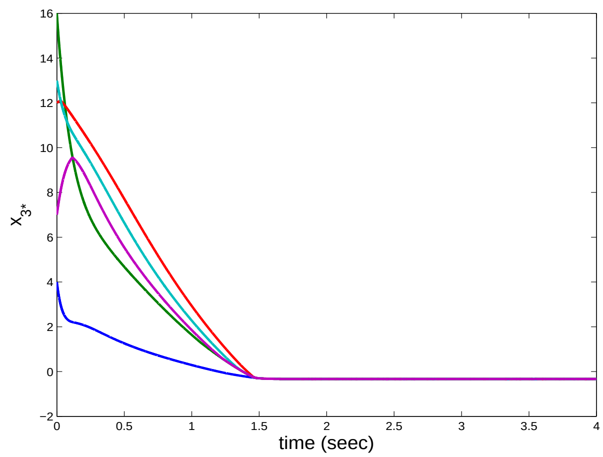

Fig. 11. Response of x_{2*}.

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