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PRACTICAL DESIGN OF TRANSMISSION-LINE NETWORKS FOR BROAD-BAND FILTER APPLICATIONS

Prepared by
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ABSTRACT

This report covers the development of several new types electrical wave filters, employing transmission line elements for use in the Very-High and Ultra-High frequency bands, where ordinary lumped-constant filters, for various reasons, are inconvenient or difficult to realize. The simple concepts and approaches, however, used in lumped-constant network analysis, have been maintained in these new applications in order to facilitate their understanding and efficient use. Comprehensive data and curves are presented for theoretical analysis, design factors, and constructional procedures, along with comparisons between various predicted and experimental results. The practical networks, resulting from the theoretical analyses, may be constructed to provide sharp cut-off, low (pass-band)insertion loss, high (attenuation band) rejection ratios, high power-handling capability, and compact, rugged mechanical form. It is pointed out that the requirements of any one network would not usually include every one of these items, but that each case would have its own peculiar requirement. The great deversity of network configurations available from this report plus the even greater number which have not been thoroughly investigated as yet, provide a wide avenue of solution to network problems within their range of application. A brief review of the theory of the general ladder filter network, on which the analyses of these networks is based, is included for reference.

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INTRODUCTION

1. The design of electric wave filters using ordinary "lumped" inductors and capacitors become increasingly difficult at higher frequencies. The reasons for this are found in the difficulty of obtaining components whose behavoir at ultra-high frequencies will follow, or even approximate the linear frequency characteristic of a pure inductor or capacitor; and in the effects of undesired stray capacitances and/or lead inductances which become more evident with increasing frequency. These effects are so marked in the physically large capacitors necessitated by the voltage and current requirements of high-power installations as to introduce intolerable changes in the electrical performance of the networks. These difficulties can be circumvented by constructing filters with transmission lines as components*. Such constructions have the advantage of not only replacing capacitors of inconveniently small dimensions with elements large enough to be constructed with close electrical tolerances and high power handling capability, but also introduces new types of impedances having new parameters and frequency characteristics, into the network. These new impedances greatly extend the number of filter characteristics available for the designer's use. However, networks composed solely of transmission lines are not without construction limitations. Such networks become too bulky at lower frequencies which are still too high for construction of satisfactory high-power lumped parameter networks**. Moreover, as will be shown in this paper, it is difficult to build an all-transmission line filter, since leads appear in the construction which must be included in the design as lumped parameters.

2. In this report several types of networks will be presented which combine lumped and distributed parameters to obtain the best features of the all-transmission line networks, at considerable economy of space and simplification of construction. The use of lumped and distributed constants in combination results in a greater flexibility of design meeting almost any mechanical and/or electrical requirements. The mechanical design can be extremely simple; several types of the networks here described have been constructed by manufacturers from dimensions and conventional tolerances alone, with no reference to electrical measurements; and have proven on test to work as predicted.

3. The methods of analysis in this report follow the classical lumped circuit network theory and equations, so that the concepts and mathematical techniques are not new. The simple extensions of these methods made and reported herein, to include distributed constants are, therefore, not difficult to understand or use, by a designer familiar with lumped constant networks.

4. The development of electric wave filters to include distributed constants for extending their practical range of application to the microwave region, immediately suggests a further extension of the concept of wave filters, namely; to use in and with waveguides. This problem is barely mentioned in the literature (see references 4 and 5), but has been actively prosecuted during the war at several establishments. It has been indicated that the results of such research carried on at MIT, Radiation Laboratory, will be published in the near future. Although the outward analyses and techniques of waveguide filters appear at wide variance with those followed in lumped constant and transmission-line networks, very fundamental analogies and similarities exist. The impedance concept of electromagnetic fields has

been renewed and studied in recent times and the analysis of transmission lines as a special type of wave guide is well known. This seeming symmetry between the field theory and circuit analysis suggests that future work will result in a unified treatment of electric wave filters which will include, as special cases, lumped constants, two-wire transmission lines, hollow metallic wave guides, etc.

* Reference 5, pp 356-8; Reference 8; Reference 10.

** The seven-foot long transmission line filter described by Salinger (Reference 10), possessing a pass band at 70 mc/s and designed for powers of several kilowatts, weighs 250 pounds.

SECTION I

ULTRA-HIGH-FREQUENCY FILTER DESIGN

Circuit Representation

A filter designed for insertion in a line in which there already exists a close impedance match throughout a desired band of frequencies must be a symmetrical four-terminaled network. In this report each such section is represented as a symmetrical T (Fig. 1-1a) or a symmetrical π (Fig. 1-1b), and the designer's problem reduced to that of devising a Z_1 and Z_2 which will interact to discriminate against undesired frequencies without affecting the impedance match existing in the desired band. The physical form of the networks of this report is also, for the most part, that of several sections of T or π form cascaded to a ladder network (Fig. 1-2). A summary of ladder-network theory apropos to filter design is given in Appendix C, and will be referred to from time to time in this Section.

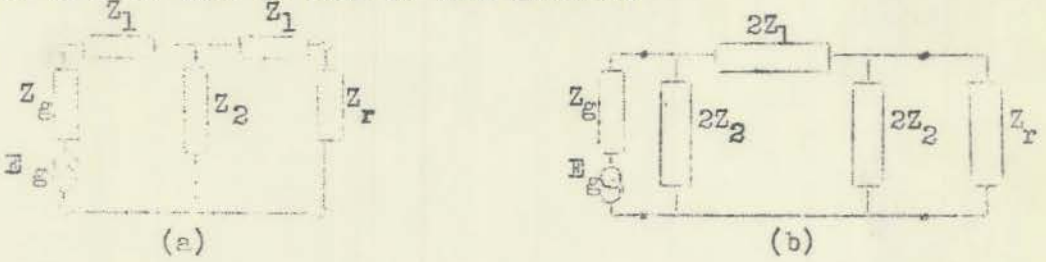


Fig. 1-1. The symmetrical T network (a) and π network (b).

Constructing Prototype Designs

The designer's first problem is that of selecting the type of Z_1 and Z_2 which will meet his requirements. Once this selection has been made, the problem of computing the constants required, though frequently tedious, is not difficult.

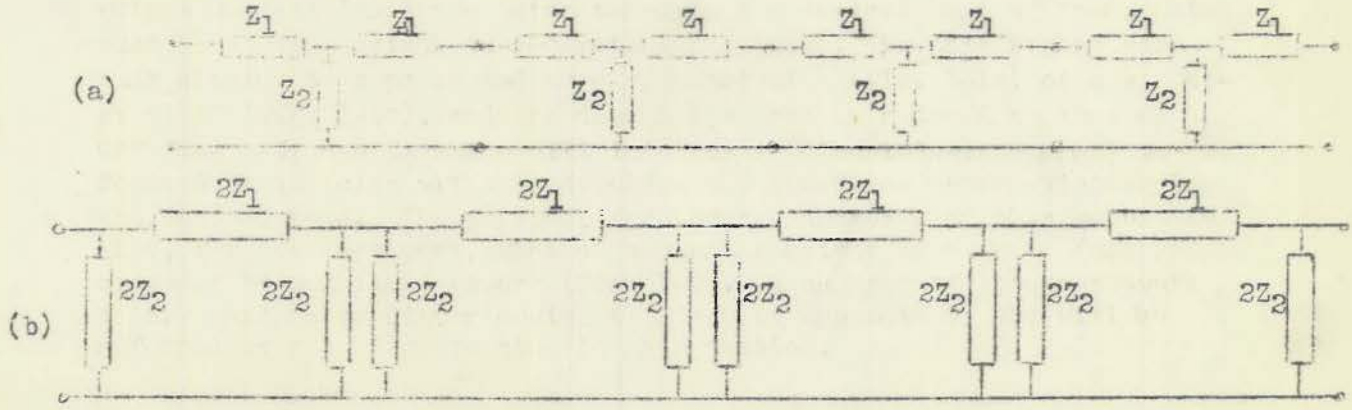


Fig. 1-2 Ladder networks produced by cascading symmetrical T sections (a) and π sections (b).

Attenuation and impedance characteristics for many lumped-constant combinations of Z_1 and Z_2 have been charted* both in summary form and in plots accurate enough for design use. These summaries may profitably be used as guides in selecting circuits for transmission-line networks, by substitution of open- and short-circuited lines for their electrical analogies, the series and parallel resonant circuits.*** If no such substitution results in a configuration suitable for the problem in hand, the designer must devise his own prototype circuits for investigation. No hard and fast rules can be laid down for doing this; however, a few principles may be discussed which have proven useful guides in constructing prototypes for a specific purpose.

For a low-pass network, Z_1 must be inductive and Z_2 capacitive at low frequencies, while for a high-pass network Z_1 must be capacitive and Z_2 inductive at high frequencies. Since most "high-pass" applications do not require a strictly high-pass characteristic at all, but can be satisfied with a low-frequency stop-band followed by a wide pass-band, it is profitable also to consider the conditions for such a "low-stop" characteristic. A low-stop filter is obtained if (1) Z_1 and Z_2 are both inductive or both capacitive at low frequencies, (2) Z_1 is very large at low frequencies, or (3) Z_2 is very small at low frequencies. Conversely, a "high-stop" filter is produced if (1) Z_1 and Z_2 are both inductive or both capacitive at very high frequencies, (2) Z_1 is very large at high frequencies, or (3) Z_2 is very small at high frequencies. In general, high attenuation is obtained when $Z_2 = 0$ or $Z_1 \rightarrow \infty$.

Pass-bands are obtained when $0 \geq \frac{Z_1}{Z_2} \geq -2$, which requires that Z_1 and Z_2 be of opposite sign in this region. If we write $Z_1 = K_1 F(f)$, $Z_2 = -K_2 G(f)$, where $F(f)$ and $G(f)$ are functions of frequency, the pass-band may be widened by increasing the constant K_2 or decreasing the constant K_1. It is also useful to observe that a lumped-constant element in series or in parallel with a transmission line will produce resonance, or anti-resonance, below the half-wave resonant frequency of the line.

Lumped-Constant Designs for Ultra-High Frequencies

Some application has been made of lumped-constant networks for low-power use in the ultra-high frequency band. For applications at the low-frequency end, networks have been constructed using commercially available capacitors and small hand-wound coils as components. Plate I-1 shows an L-C-L** and an L-S-L low-stop network designed to have a

* Reference 2, Figs. 159-63, 166-74, 182-83; Reference 4, pp. 228 ff.

** For an explanation of the nomenclature used in this report, see Appendix A.

*** Strictly, the only analogy between these circuits is their approximate equivalence near resonance.

cut-off at approximately 150 mc/s, and other similar networks have been constructed in this division, not only by the authors of this report,



Figure 1-3. The L-L-C (a) and L-C-L (b) low-pass and low-stop networks.

but also by Messrs. Henry Jasik and Ernest Steele. Use has also been made of the L-L-C type at somewhat higher frequencies by the expedient of utilizing as four-terminaled networks transmission-lines sufficiently short that $\beta l \leq 30^{\circ}$ throughout the pass-band. An excellent filter of this type designed and built by Mr. G. H. Saunders of NRL has a pass-band with less than 0.3 db insertion loss and a sharp cut-off at 400 mc/s, and still another has been built in this division with a cut-off at 3000 mc/s. However, these networks either call for very close machine work or possess relatively high insertion loss and gradual attenuation rise. Other types using very short transmission lines, both as two-terminal and as four-terminal components, can be found in the literature (Reference 3, p. 67ff.).

Design equations for the L-L-C, L-C-L, L-S-L, and other lumped-constant networks are given in a form suggested by Shea*, in Plates B-1, B-2, B-3, and B-7, Appendix B.

Low-Pass Filters

The Y-L-O Network — The Y-L-O network of Figure 1-4 was investigated in response to a need for a low-pass network which would handle several kilowatts of peak power. It may be developed from a series m-derived L-L-C network by replacing the series-resonant circuit of the shunt arm with its electrical analog, the open-circuited transmission line.

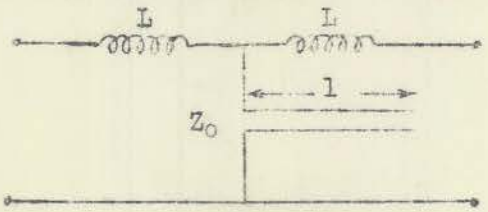


Figure 1-4. The Y-L-O filter circuit.

* Reference 2, loc. cit.

The series and shunt impedances for this network are, respectively:

$$Z_1 = j2\pi fL \qquad (1\text{-}1)$$

$$Z_2 = -jZ_o \cot\left(\frac{\pi}{2}\frac{f}{f_o}\right) \qquad (1\text{-}2)$$

where Z_o is the characteristic impedance of the shunt transmission line and f_o is the frequency for which the shunt line is a quarter-wave long, and is also the lowest frequency of peak attenuation. With the substitution $y = \frac{Z_o}{2\pi f_o L}$, the ratio of the characteristic impedance of the T section to that of the shunt line becomes, by substitution in equation C-2, Appendix C,

$$\frac{Z_{ot}}{Z_o} = \frac{f}{yf_o}\sqrt{2y\frac{f_o}{f}\cot\left(\frac{\pi f}{2f_o}\right) - 1} \qquad (1\text{-}3)$$

and the corresponding ratio for the π section, from equation C-4,

$$\frac{Z_{o\pi}}{Z_o} = \frac{2\frac{f}{f_o}\cot\left(\frac{\pi}{2}\frac{f}{f_o}\right)}{y\frac{Z_{ot}}{Z_o}} . \qquad (1\text{-}4)$$

It can easily be shown that when f = 0

$$\left.\frac{Z_{o\pi}}{Z_o}\right|_o = \left.\frac{Z_{ot}}{Z_o}\right|_o = \sqrt{\frac{2}{\pi y}} . \qquad (1\text{-}5)$$

Plots of equations 1-3 and 1-4 are given in Plates B-8 and B-9. It will be noted that like the lumped-constant m-section to which it is similar, a Y-L-O will yield a flat $Z_{o\pi}$ by a proper choice of parameters.

Cutoffs of the type $\frac{Z_1}{Z_2} = -2$ occur for f_c given by

$$2y = \frac{f_c}{f_o}\tan\left(\frac{\pi}{2}\frac{f_c}{f_o}\right) . \qquad (1\text{-}6)$$

From this expression it can be seen that, with infinite attenuation at f_o, sharper attenuation rises are obtained for sections of larger y, so that increasing y corresponds to decreasing m in the lumped constant network. Cutoffs of the gradual type $\frac{Z_1}{Z_2} = 0$ occur when $f_c' = 2nf_o$, where n is any integer; these cutoffs mark the successive beginnings of higher-frequency pass-bands, the presence of which is a characteristic of all networks

containing transmission lines. The overall attenuation characteristic of the Y-L-0 prototypes in that diagrammed in Figure 1-5 and plotted accurately

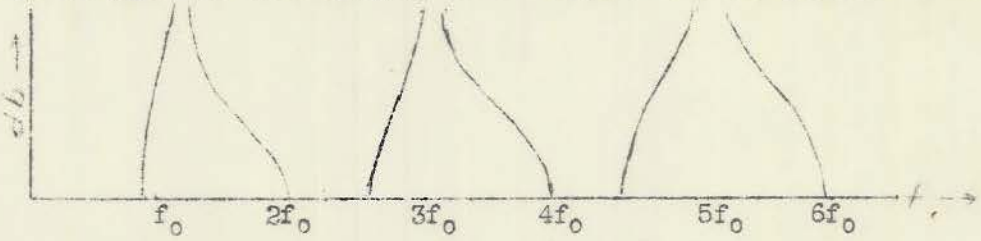


Figure 1-5. Attenuation of the Y-L-0 prototype filter.

in Plates B-10 and B-11. It will be noted from equation 1-6 that the higher order pass-bands become narrower as the frequency increases. It can also be seen from the expression for the stop-band attenuation,

$$\cosh \alpha = \left| 1 - \frac{f}{yf_o} \tan\left(\frac{\pi f}{2f_o}\right) \right| \qquad (1\text{-}7)$$

that the attenuation for a given value of $\tan\left(\frac{\pi}{2}\frac{f}{f_o}\right)$ is greater in the higher-order stop-bands; this implies that the sharpness of the attenuation rise increases with n. This sharpening of attenuation rise and narrowing of pass region for the higher-order bands is another characteristic of combined lumped- and distributed-parameter filters. Attempts are being made to utilize these regions for narrow-pass-band filters; however these higher pass-bands are more difficult to use in design, as the impedance characteristics become less and less flat and their levels change with increasing order, as may be seen by combining equations 1-3 and 1-4.

Measurements of a cascade of five y = 1.7 sections, f_o = 265 mc/s, are plotted in Plate I-2, and a photograph of the filter itself is found in Plate I-3. The transmission lines are made of brass rods supported inside the outer conductors by polystyrene washers. To obtain the proper f_o, empirical correction must be made for the decreased velocity of a wave passing through the washers, as well as for the slight capacitive loading of the open end of the line by its housing.

If m-derived sections are to be used, the lengths of the transmission-line elements for a given cut-off steepness will be reduced, although at the expense of greater difficulty in resonating the shunt element. If a great reduction in bulk is to be effected, the line becomes so short that its pass-band behavior is very nearly that of a lumped element, and L-L-C design equations can be used, a capacity C being approximated through the pass-band by a transmission line whose length l, characteristic impedance Z_o, and velocity of propagation c satisfy the relation $C = \frac{l}{Z_o c}$. This approximation is accurate within 10% as long as $2\pi l/\lambda < 30^\circ$. For longer lines, the lumped-constant design may still be used and the capacitors approximated by the formula $C = \tan\left(\frac{2\pi l f_r}{c}\right) / 2\pi f_r Z_o$ where f_r is a

frequency near the middle of the portion of the pass-band it is desired to use.

Plate I-4 illustrates a network designed on this basis, and its characteristics are plotted in Plate I-5. The transmission lines are used to realize capacitances of 7$\mu\mu$f and 3.5$\mu\mu$f. The network is capable of handling several tens of kilowatts peak power.

Cascading Sections of Different f_o. — Just as L-L-C sections of varying m-values may be connected together to produce a low-pass filter with several attenuation peaks and a broad band of high attenuation, so several Y-L-C prototypes may be connected in tandem for the same purpose. Two methods can be used for doing this.

The first makes use of the fact that the characteristic impedance of either a T or π section is relatively constant over the low frequency portions of the pass-band. If two such sections, #1 possessing a cutoff

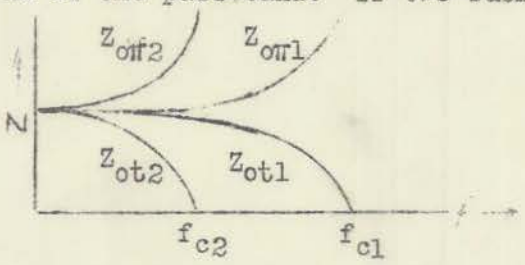


Figure 1-6. Impedance curves of Y-L-C networks possessing different cutoffs.

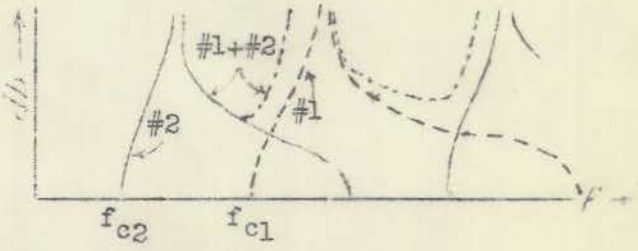


Figure 1-7. Attenuation characteristics of two Y-L-C networks, showing elimination of higher-order pass bands.

f_{c1} and #2 possessing a cutoff f_{c2}, are operated in cascade, and if f_{c1} is greater than f_{c2} by a considerable amount (at least 60%, for example), it will be noted (see Fig. 1-6) that either the π or T impedance characteristic of section #1 is relatively flat when $f \leq f_{c2}$, and will cause little reflection in this band if connected to a constant-impedance line. Hence only filter #2 need have a sharp cutoff in order to impart a sharp characteristic to the entire network. Moreover, this arrangement will provide an overlapping of attenuation characteristics (see Fig. 1-7) which can be used to eliminate any desired fraction of the higher-order pass-bands or to cancel them entirely.

A network constructed on this basis is shown in Plate I-6 and its characteristics are plotted in Plate I-7. It consists of four y = 1.3 sections, f_o = 520 mc/s, and two "booster" sections of y = 1.3, f_o = 780 mc/s. A time-saving construction feature of this network is the use of standard polyethylene-dielectric cable for the transmission-line elements.

The second method depends upon the similarity in shape of the various impedance curves below the first cutoff. To provide maximum steepness of attenuation rise combined with broadest overall attenuation, it is

desirable that all sections possess the same initial cutoff. Sections having the same cutoffs, but different frequencies of infinite attenuation, or different y, may be made to match closely over the pass-band because of the similar shape of all the curves of $\frac{Z_{ot}}{Z_o}$.

Let it be desired to produce an approximate pass-band impedance match between two T sections having frequencies of infinite attenuation of f_o and f_o', y values of y and y', and shunt transmission-line characteristic impedances of Z_o and Z_o'. If the filter impedances are matched for f = 0 and the two cutoff frequencies are identical, there will be a close match over the entire pass-band due to the similar shapes of all the characteristic T impedance frequency characteristics.

Theoretical impedance and attenuation characteristics of such a filter are shown in Plate B-12, in terms of two parameters $r = \frac{f_o'}{f_o}$ and $a = \frac{Z_o'}{Z_o}$. Matching the filter impedances at zero frequencies requires, from equation 1-5,

$$a = \sqrt{\frac{y'}{y}} \qquad (1\text{-}8)$$

If the cutoff frequency of each section is f_c, application of equation 1-6 gives

$$a^2 r = \frac{\cot\left(\frac{\pi}{2}\frac{f_c}{f_o}\right)}{\cot\left(\frac{\pi}{2r}\frac{f_c}{f_o}\right)} \qquad (1\text{-}9)$$

from which r may be computed. Cascading sections of different y, f_o or Z_o can be done with relatively small mismatches if the conditions in equations 1-8 and 1-9 are fulfilled. Since there are three variables in constructing the additional section and only two conditions to be met, it follows that a third condition may be set arbitrarily. For instance, f_o' may be set at a predetermined value, y' may be chosen for desirable termination characteristics or for a broad band of attenuation, or Z_o' may be chosen for easy construction.

It may be preferable under certain circumstances for one of the cutoffs to be chosen slightly higher than the other, so that the two Z_{ot} curves intersect just below cutoff, with the result that the mismatch is held within a given tolerance over a broader band.

Tolerances required in Y-L-O — Some conclusions may be drawn as to the effect upon the filter characteristics of errors occurring in manufacture. Consider a filter designed with y so chosen as to present a flat $Z_{o\pi}$ over most of the pass-band. This characteristic impedance is nearly that

of the filter for direct current (equation 1-5), which can be written

$$Z_{ot}\Big|_{o} = Z_{o\pi}\Big|_{o} = 2\sqrt{2Lf_oZ_o} \qquad (1\text{-}10)$$

Since this impedance is jointly proportional to the square root of all three of the filter parameters, it follows that a small percentage error in any of these is reflected as an error of only half this percentage in the characteristic impedance of the filter.

If a filter is designed to tolerate a ± 10% pass-band mismatch, and if f_o can be realized within 2% and Z_o within 5%, realizing the inductance within 15% should produce a filter with a maximum pass-band insertion loss (equation C-14) of only 0.2 db. This broad tolerance leads to a simple method of adjusting a Y-L-O experimental model for proper initial characteristic; it consists in building the network first with Z_o and f_o as nearly accurate as mechanically realizable, and obtaining the proper inductances by adjusting for proper cutoff frequency, combined with minimum insertion loss or minimum standing-wave-ratio over the pass-band. Once the proper inductances for the experimental model have been obtained by this method, accurate mechanical copies will usually serve in production-type filters. It is fortunate for the designer that this is true, as accurate measurements of small inductances at high frequencies are difficult to carry out.

From equation 1-6, the effects of manufacturing errors on cutoff frequency may be predicted. Rewriting equation 1-6

$$f_c = \frac{Z_o}{\pi L}\cot\left(\frac{\pi}{2}\frac{f_c}{f_o}\right). \qquad (1\text{-}11)$$

Differentiating with respect to f_o, we obtain the approximation

$$\frac{\Delta f_c}{f_c} = \frac{\Delta f_o}{f_o}\,\frac{\pi y}{\pi y + \sin^2\left(\frac{\pi}{2}\frac{f_c}{f_o}\right)} \qquad (1\text{-}12)$$

which holds if we may neglect second- and higher-order terms in $\frac{\Delta f_o}{f_o}$. The percentage error in cutoff frequency is then no more than the percentage error in f_o; and for values of y normally used, it will be less. For y = 2.0, the percentage error in f_c is 0.865 of that in f_o, and as y becomes smaller, the ratio approaches a limiting value of 0.5.

If the differentiation be carried out in terms of the variable $\frac{Z_o}{L}$, we obtain the interesting result

$$\frac{\Delta f_c}{f_c} = \frac{\sin^2 \left(\frac{\pi}{2} \frac{f_c}{f_o}\right)}{\sin^2 \left(\frac{\pi}{2} \frac{f_c}{f_o}\right) + \pi y} \frac{\Delta\left(\frac{Z_o}{L}\right)}{\left(\frac{Z_o}{L}\right)} \qquad (1\text{-}13)$$

As the ratio $\frac{\pi y}{\sin^2 \left(\frac{\pi}{2} \frac{f_c}{f_o}\right)}$ approaches a limiting value of unity as y becomes small, a small percentage error in either Z_o or L is always reflected as an error in f_c, at most, of half that percentage. If y is at all large, L and Z_o exert an even smaller effect on the cutoff -- for y = 2, evaluation of equation 1-13 gives $\frac{\Delta f_c}{f_c} = 0.136 \frac{\Delta\left(\frac{Z_o}{L}\right)}{\left(\frac{Z_o}{L}\right)}$. The cutoff frequency of Y-L-O thus depends principally on the quarter-wave frequency of the shunt line.

The Y-L-OS Filter -- An interesting modification of the Y-L-O configuration is the network obtained (Fig. 1-8) by substituting a series combination of an open- and a short-circuited line for the open-circuited line of the Y-L-O. This new shunt element has an impedance characteristic similar to that of the single open-circuited line, but is much shorter mechanically, while simply constructed as two concentric coaxial lines. If we define $k = \frac{Z_{o1}}{Z_{o2}}$, $d = \frac{l_1}{l_2}$, and set $f_o = \frac{c}{4l_2}$, the impedance of the
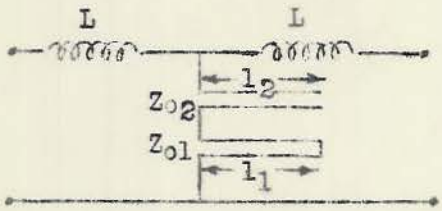


Figure 1-8. The Y-L-OS prototype circuit.

two transmission lines in series is

$$Z_2 = jZ_{o2} \left[k \tan \left(d \frac{\pi}{2} \frac{f}{f_o} \right) - \cot \left(\frac{\pi}{2} \frac{f}{f_o} \right) \right] \qquad (1\text{-}14)$$

If d is unity, resonance, and hence infinite attenuation in the Y-L-OS, occurs when

$$\tan^2 \left(\frac{\pi}{2} \frac{f}{f_o}\right) = \frac{1}{k} \qquad (1\text{-}15)$$

For all values of k, this will be at a frequency less than f_o, so that the lengths of line required to produce resonance at a given frequency are decreased. For high values of k, resonance occurs at lower and lower values of $\frac{f}{f_o}$, and the behavior of the network gradually approaches that of a lumped series-resonant circuit. When d = k ≠ unity, equation 1-14 reduces to

$$Z = -j2Z_o \cot \left(\frac{\pi}{4} \frac{f}{f_o}\right) . \qquad (1\text{-}16)$$

This is the impedance of an open-circuited line of length twice l and characteristic impedance $2Z_o$. The Y-L-OS under these conditions becomes identical electrically with a Y-L-O; the resulting network is, however, less bulky physically. The experimental attenuation curve of such a network is shown in Plate I-8; while a good filter, it was more difficult to adjust then Y-L-O. Design equations for Y-L-OS are given in Plate B-7.

Another method of shortening the shunt lines is by use of the folded line sketched in Figure 1-9. There is somewhat greater difficulty in predicting the behavior of this arrangement due to the gap at the "fold".



Figure 1-9. Short-circuited line
folded to decrease its physical length

The T-S-O Configuration -- An all-transmission-line network which can be used in many low-pass applications is the T-S-O configuration illustrated in Figure 1-10. This network is analogous to the lumped constant L-P-S band elimination filter diagrammed in Figure 1-11. If we define

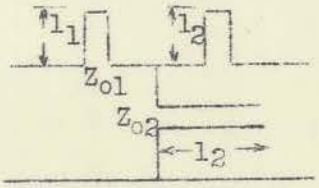


Figure 1-10. The T-S-O network.

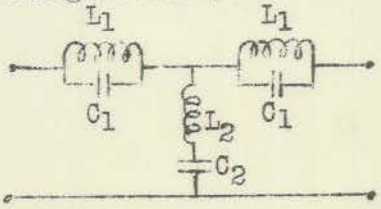


Figure 1-11. The L-P-S band-elimination filter.

$f_o = \frac{c}{4l_1}$, $d = \frac{l_2}{l_1}$, $k = \frac{Z_{o2}}{Z_{o1}}$, the impedance and attenuation functions for the T-S-O prototype become

$$\frac{Z_{ot}}{Z_{o1}} = \sqrt{\tan\left(\frac{\pi}{2}\frac{f}{f_o}\right)\left[2k \cot\left(\frac{\pi}{2}\frac{d\,f}{f_o}\right) - \tan\left(\frac{\pi}{2}\frac{f}{f_o}\right)\right]} \qquad (1\text{-}17)$$

$$\frac{Z_{or}}{Z_{o1}} = \frac{2k \tan\left(\frac{\pi}{2}\frac{f}{f_c}\right) \cot\left(d\frac{\pi}{2}\frac{f}{f_o}\right)}{\frac{Z_{ot}}{Z_{o1}}} \qquad (1\text{-}18)$$

$$\cosh \alpha = \left| 1 - \frac{\tan\left(\frac{\pi}{2}\frac{f}{f_o}\right) \tan\left(d\frac{\pi}{2}\frac{f}{f_o}\right)}{k} \right| \qquad (1\text{-}19)$$

It will be noted that when d is unity, this network becomes constant-k, and standard constant-k design curves and analyses*, particularly those pertaining to m-sections, become applicable if the frequency scales are modified or properly interpreted.

Just as there are four variable quantities in the L-P-S prototype — two inductances and two capacitances — so there are four variables in designing the T-S-O prototype — two characteristic impedances and two lengths. Including m, there are five degrees of freedom available in the design. There are only three quantities ordinarily specified in the filter design problem: cutoff frequency, peak attenuation frequency, and pass-band characteristic impedance; hence two degrees of freedom are available for flattening pass-band impedance, providing additional attenuation peaks, or for other purposes. A few of the extremely flat pass-band impedance curves obtainable are illustrated in the T-S-O design curves of Plate B-14; note that it is possible to obtain impedance curves (m = 0.5, d = 0.7, k = 1.0), similar to those given by double m-derived networks. (See Fig. C-7, Appendix C, for comparison.)

As with most transmission-line designs at ultra-high frequencies, there is great difficulty in constructing the network without introducing some elements which behave as lumped constants. The graph of Plate I-9, comparing the experimental and theoretical results for a network consisting of a prototype T-S-O terminated in m = 0.6 half-sections, is an example. It will be noted that the introduction of undesired inductance in leads connecting to the shunt transmission line causes the attenuation to fall earlier than predicted.

* Reference 2, p. 299, p. 360-61; Reference 1, Chap. IX; Reference 4, p. 233.

The difficulties are increased if it is desired to construct a network of more sections in a compact form, as the lead lengths become greater and the possibilities of inter-section coupling increase. Plate I-10 shows experimental results for a four-section T-S-O network in which compact construction was attemped by telescoping the series elements within one another and connecting the shunt elements with leads as short as possible. The departure from theory at high frequencies is evident, while the slightly premature cutoff indicates probable coupling between leads of adjacent shunt arms.

An approximate correction for this effect may be made by shortening the transmission line, providing the lead is not too long and the input impedance of the line is not great. It is well known (Reference 6, p. 173) that a dissipationless transmission line may be represented by a π section like that of Figure 1-12, where l is the length of the line, f the frequency, Z_o the characteristic impedance, and c the velocity of propagation. When $\frac{2\pi fl}{c} \leq 30^\circ$, the shunt impedance of the line's distributed capacitance becomes high enough to be neglected if the impedance of the termination is not high also, and the equivalent circuit of the line becomes approximately that of Figure 1-13. At these low frequencies $\sin(\frac{2\pi fl}{c}) = \frac{2\pi fl}{c}$ within 10%. Hence a short length of transmission line is approximately equivalent to an inductance whose value is given by

$$L = \frac{Z_o l}{c} \qquad (1\text{-}20)$$

If, then, it is required to place a lead of inductance*L in series with a transmission line, equation 1-20 may be used to compute the length l by which the line must be shortened to compensate for the presence of

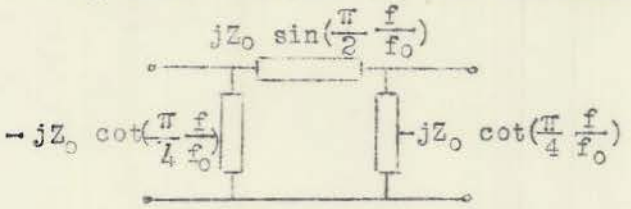


Figure 1-12. Equivalent π section of transmission line

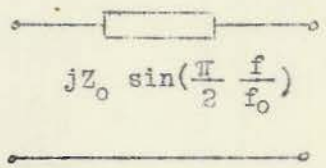


Figure 1-13. Approximate equivalent circuit of transmission line, $\frac{2\pi fl}{c} \leq 30^\circ$, terminating impedances small compared to $j Z_o \cot(\frac{\pi fl}{c})$.

the lead, providing $l \leq \frac{c}{4\pi f}$ where f is the highest frequency at which accurate approximation of the transmission line's frequency characteristic is required. In filter work, f is usually the highest pass-band operating frequency.

*Formulae for L are given in Reference 4, p. 48.

It must be noted that this correction does not hold if the line in series with the lead is in the neighborhood of anti-resonance, where its impedance is large, as the shunt impedances which are present in a short additional line length but absent in the lead are essential to the anti-resonance. Anti-resonance for the lead-and-line combination will thus appear at the frequency at which it occurs for the line alone. If it is necessary to make a lead correction in this vicinity, or to use leads possessing larger inductance, the entire network may be re-designed to include such inductance. This gives rise to configurations such as T-S-LO, T-LS-O, and T-LS-LO (see Fig. 1-14), which are obtained from T-S-O by the insertion of inductances at points where leads inevitably appear in the construction.

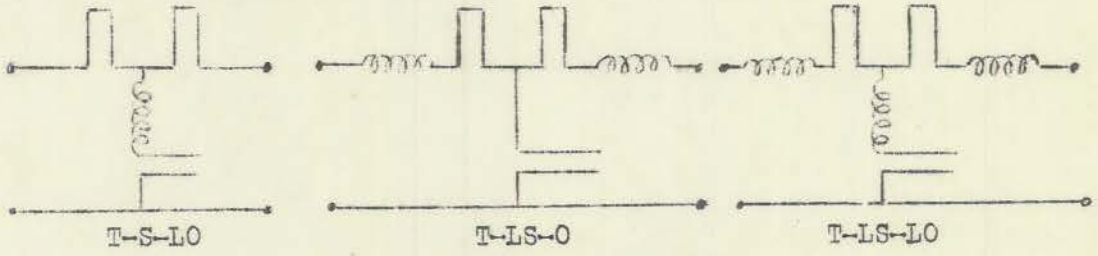


Figure 1-14. Networks obtained by including lead inductances in T-S-O prototypes.

Because the Y-L-O network proved suitable for so many ultra-high frequency applications requiring a low-pass filter, it was given considerably more experimental attention than other low-pass prototypes. However, Fig. 1-15 illustrates a few other likely prospects for these applications. The Q-S-C is the inverse of the Y-L-O, while the Q-LS-C is a modification of Q-S-C to utilize leads. Q-LS-S is a further modification presenting the possibility of two infinite attenuation frequencies and a pass-band approaching double m-derived flatness. Y-P-O and its inverse Q-S-S also present the possibility of double attenuation peaks. Design computations for some of these networks are given in Plates B-1, B-2, B-8 to B-12 and B-18 to 19.

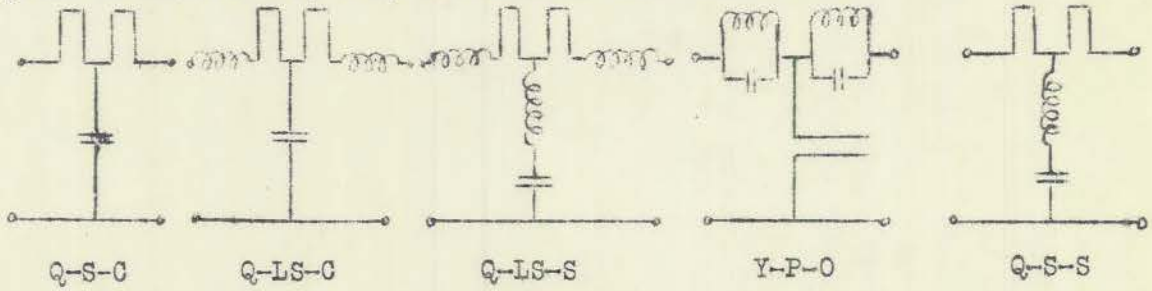


Figure 1-15. Low-pass networks containing transmission lines.

The networks of Figure 1-15 may prove extremely useful at frequencies high enough so that a coaxial line may be folded within a center conductor without inconveniently increasing the length of the filter.

Networks With Low-Stop Characteristics

As we remarked in a previous paragraph of this section (page 3), a true "high-pass" network is not required in most high-pass applications; a band-pass filter passing the lowest and highest frequencies of desired operation, and attenuating sharply all frequencies below this, is adequate. It has been possible to construct low-stop networks designed on the conventional L-C-L basis (Figure 1-16) for cut-offs in the vicinity of 200 mc/s, providing the highest pass-band frequency to be used is not too great. At frequencies greater than 250 mc/s, and in many cases at lower frequencies, the distributed

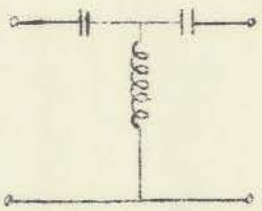


Figure 1-16. The L-C-L low stop network

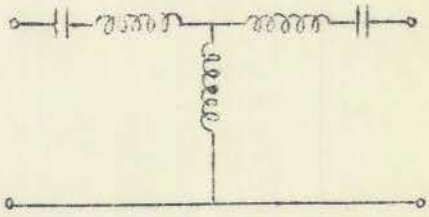


Figure 1-17. The L-S-L configuration.

capacitance of shunt coils to ground exercises a sufficiently great effect to require analysis as a line, while residual inductance of the series capacitors produces resonance effects. The L-S-L network (see Fig. 1-17) is a logical outgrowth of the L-C-L which includes the series resonances. Characteristics of the filter of Plate I-1 designed from this prototype are shown in Plate I-11. It will be noted that the insertion loss of this network is very low at low pass-band frequencies, but rapidly increases above 250 mc/s to the order of three or four db, due to the coil capacitance to ground.

Several prototype circuits which make use of this distributed capacitance by replacing the shunt inductances with transmission lines are shown in Figure 1-18. The Y-C-O may be obtained by replacing the shunt element of a series-derived L-C-L network by its analog, the

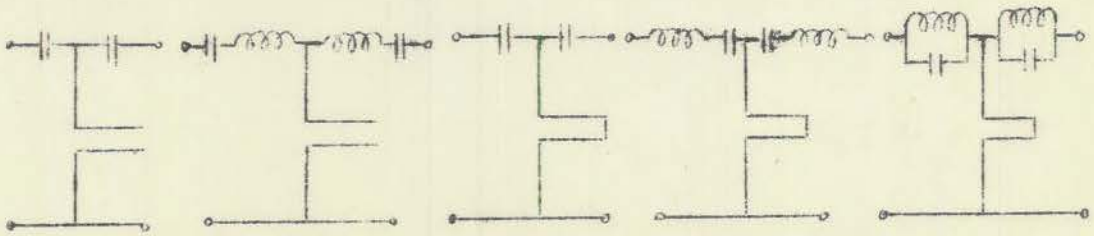


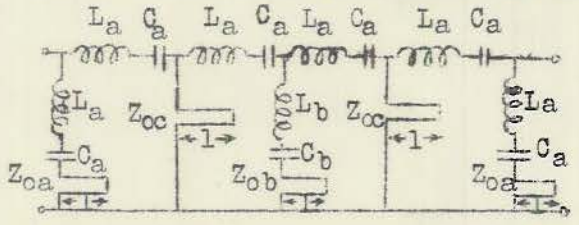
Figure 1-18. High-pass networks for ultra-high-frequency use at low powers.

open-circuited transmission line. The resonance of the line, just below cutoff, produces a sharp attenuation rise without the need for m-sections, but the shunt element is bulky if constructed of a conventional coaxial line for operation below a few hundred megacycles, while the requirement that the residual inductance of the capacitors be negligible tends to limit its use to pass-band frequencies in just this range. The Y-S-O network provides for residual inductance in the capacitors.

The Y-C-S and Y-S-S networks replace the shunt inductance of L-C-L and L-S-L with a short-circuited line, sharp attenuation rise being provided by m-derivation. Since the resonant frequency of this line may be made well above cutoff frequency, it is much less bulky than the lines required in Y-C-O and Y-S-O. If a shorted line is substituted for the shunt inductance in a shunt-derived L-C-L network, the Y-P-S circuit is obtained; it has a sharp attenuation rise but the use of relatively large coils in the series arm re-introduces the problem of distributed capacitance. Design equations, data, and curves for these networks are given in Plates B-3 to B-6, and B-20 to B-34.

Because it best combines utilization of residual parameters with small bulk, the Y-S-S of the above networks was the one chosen for experimental investigation. The analysis follows the method used for the Y-L-O and T-S-O, and will not be repeated here. The results are summarized in Plate B-4. As in the analogous L-S-P network, there are four design parameters, exclusive of m, leaving two degrees of freedom in the prototype for production of flat pass-band. If the resonant frequency for the series arm is chosen equal to the anti-resonant frequency of the shunt arm (a = unity, in the terminology of Plate B-4), Y-S-S becomes a confluent band-pass filter*. This is the type which was constructed, as it possesses the broadest and flattest pass-band for a given ratio of cutoff frequency to f_o.

Experimental results on the Y-S-S filter whose circuit is shown in Fig. 1-19 are plotted in Plate I-12. The network itself is shown in Plate I-13. Erie Ceramicon capacitors were used for the condensers, with the inductances realized by leads of 10 mm on each end of the 5$\mu\mu$f capacitors and 5 mm on each end of the 10$\mu\mu$f capacitors. This network has also been combined with a Y-L-O to produce a band-pass filter whose characteristics are shown in Plate I-14.



C_a = 5.0$\mu\mu$f Z_{oa} = 129.8 Ω
C_b = 10.0$\mu\mu$f Z_{ob} = 64.4 Ω
L_a = 0.0118μh Z_{oc} = 32.2 Ω
L_b = 0.0059μh l = 4.5 in (air)

Figure 1-19. A composite Y-S-S filter designed to attenuate frequencies below 295 mc/s. Two prototype and two m = 0.5 sections are used.
a = 1, y = 1, f_o = 650 mc/s.
Note the number of identical components obtained by cascading alternate prototype and m = 0.5 sections.

* See page 25.

Low-Stop Networks for Power Applications -- For applications where power is a consideration, the circuits of Fig. 1-18 may be unsatisfactory, as they require lumped capacitors. If these elements are replaced by transmission lines, the T networks of Fig. 1-20 result. The first two of these networks require at least one of the line elements to be resonant below the pass-band, a disadvantage for operation at v-h-f and u-h-f if lines are to be kept non-bulky. The other three require the construction of two concentric coaxial lines, or even more complicated structures,

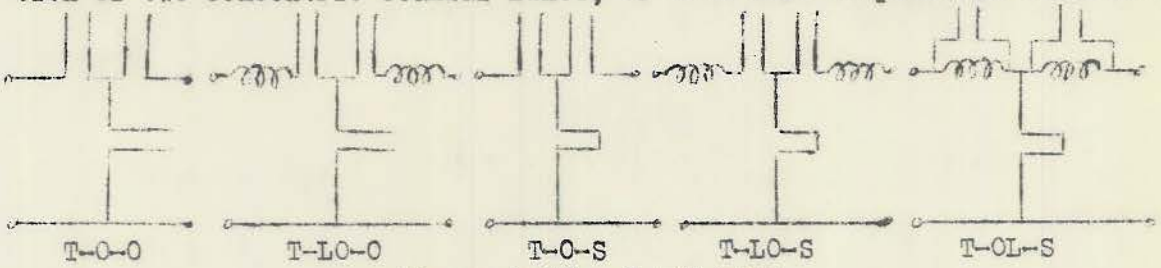


Figure 1-20. T networks which can be utilized in low-stop high-power applications.

if the attenuation-rise is to be sharpened by m-derivation. In addition, only in T-L0-0 and T-L0-S can provision be made for the inductance that must appear in the physical construction of the shunt element, although the method of page 13 can, of course, be applied to correct for the presence of relatively short leads.

The Q-L0-L Prototype -- A prototype which overcomes these difficulties is the Q-L0-L network diagrammed in Figure 1-21. Clearly inductance is present where it will be needed in the construction of the filter, and in an m-derived section any distributed capacitance to ground of the shunt element can be thrown into the line which appears in the shunt

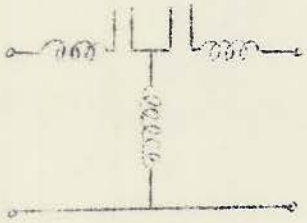


Figure 1-21. The Q-L0-L circuit.

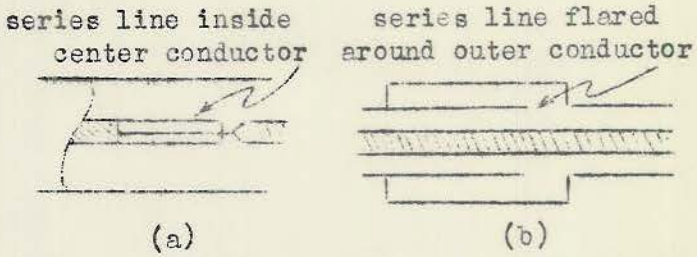


Figure 1-22. Two methods of placing a length of transmission-line in series with a coaxial line.

arm. It is not so clear that coil capacitances to ground are accounted for in the prototype, but it is found that the inductance required at frequencies well into the UHF band can be made as small as those obtained by a single short wire lead, which thus minimizes the distributed capacitance.

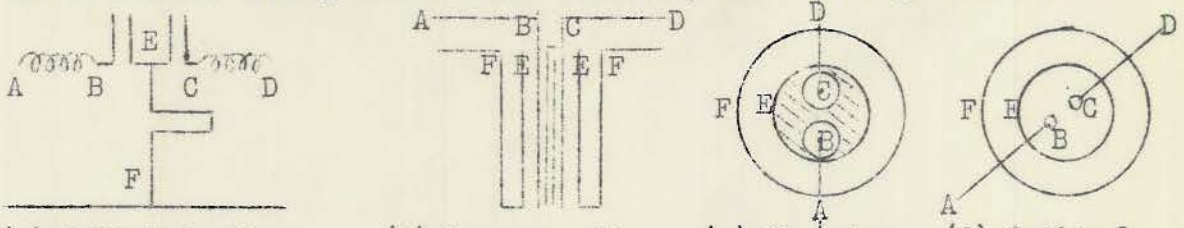
The analytical results for Q-LO-L are summarized in Plate B-4, and design information is plotted in Plates B-40 to 43.

The mechanical problem of placing a transmission line in the series arm of the network was solved by folding it inside the center conductor as may be seen in the sketch of Fig. 1-22(a) and the closeup of the network in Plate I-16. By proper choice of dimensions for the outside line, the series line may be made of standard cable with or without the braid removed.

An alternate method of placing a line in series is the flared construction suggested by Salinger (Reference 10) sketched in Figure 1-22(b). This method is best applied only to short-circuited lines if the possibility of current flow on the outside of the shield is to be avoided.

Experimental results are given in Plate I-15 on a Q-LO-L network of three m = 0.58 and three m = 0.4 sections. The network itself is shown in Plate I-16, each tube and box assembly containing one m = 0.58 and one m = 0.4 section.

The T-LO-S Network.— Another low-stop network of which an experimental model has been built is the T-LO-S configuration sketched in Figure 1-23. Experimental data on a cascade of five prototypes is given in Plate I-17. This network does not lend itself to sharp attenuation rises without m-derivation, and it will be noted that the attenuation rise of the prototype cascade is slow. This particular network is most interesting for the mechanical arrangement of its elements, shown in Fig. 1-23(b).



(a) T-LO-S Section (b) Cross-section (c) Top view (d) Optional arrangement (top view)

Figure 1-23. Mechanical arrangement of T-LO-S section

The inductance is realized by the leads AB and CD, while the series transmission lines BE and CE are folded inside the center conductor of the shunt line EF. This method has some advantage in compactness, as it is not necessary to use long inter-section connecting lines to house the series transmission-line elements. Theoretically the outer jackets of the series lines are not necessary, as shown in the optional arrangement of Fig. 1-23 (d), but the arrangement of (c) is easier to construct in practice.

Networks for Very High Power Applications -- For applications in which extremely high peak voltages appear, networks using open-circuited transmission lines are poor, as there is great risk of arcing at the open end. In the absence of dissipation, the voltage at this point rises to V_0 sec $(\frac{\pi}{2} \frac{f}{f_0})$, where V_0 is the voltage applied to the line. In a high-powered pulse generator, the output voltage may reach a peak of several thousand volts, and if the line is the shunt element of a filter meeting a maximum 2:1 standing-wave-ratio specification, V_0 may easily be as high as 15,000 volts. If then the frequency of the generator is as much as 90% of the resonant frequency of the line, potentials as great as 100 kilovolts may appear across the end of an open line. In a practical line, only dissipation prevents this voltage from becoming infinite as the line approaches nearer to resonance.

This high voltage limits the sharpness of attenuation rise that can be obtained with any single section of a high-powered network obtaining peak attenuation by resonance of any kind of shunt element.

Experimentation with filters designed for high-power networks quickly revealed that an open-circuited air-dielectric line is likely to arc over in these applications even when the end of the center conductor is machined to a smooth hemisphere. The higher voltage gradients required to cause breakdown in solid dielectrics suggested replacing the air with some high polymer, as has already been done in high-voltage polyethylene-dielectric coaxial cables such as RG-17u and RG-19u. However, an open-circuited cable end has no better breakdown characteristic than an open-circuited air-dielectric line unless the cable dielectric can be formed to cover completely the open end. Through the cooperation of Messrs. Ralph Taylor and J.J. Goulka of the NRL Chemistry Division, and of the MIT Radiation Laboratory, techniques were developed for molding a cylinder of polyethylene around the center conductor (Figure 1-24), inserting the whole into a brass tube to complete the line. These techniques involved extrusion molding under high pressures, a costly and, at the time, slow process. A second, quicker method involved drilling holes into polystyrene cylinders and inserting the brass center conductor into the hole along with a heavy coating of a gelatinous ignition sealing compound (Dow-Corning #5). This expedient procedure worked, but was never used because of its obvious mechanical disadvantages and the fact that the sealing compound never dried, but attracted dirt, metal particles, etc. In both these methods care had to be taken to prevent the forming of a galaxy of small air bubbles about the end (A, Figure 1-24) of the center conductor. These air pockets generally refract the electric field to breakdown strength at some point.

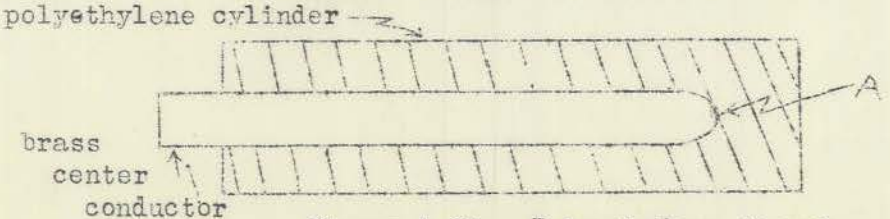


Figure 1-24. Polyethylene insert for open-circuited high-voltage line.

The design of networks using only inductors and short-circuited transmission lines eliminated breakdown difficulties to a great extent. Use of a shorted line element places the voltage peak somewhere in the center of the line, where there are no discontinuities to produce such intense fields as occur at the end of an open-circuited center conductor. As the short-circuit can be constructed without exposing a high-voltage point to the air, it is possible to use the center conductor and dielectric from standard RG/19U transmission line, an assembly relatively free from air pockets and other discontinuities. Such a network is discussed below under Q-LS-L.

The Q-LS-L Network -- Several examples of low-stop filters which use only inductors and shorted lines are shown in Figure 1-25. Because it best promised mechanical simplicity combined with good electrical characteristics, the Q-LS-L was chosen for experimental construction. As

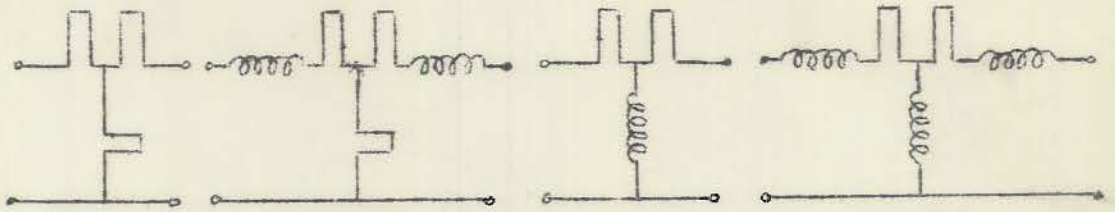


Figure 1-25. "High-pass" high power filter configurations.

there are already four degrees of freedom and a finite frequency of infinite attenuation in the prototype, it is possible to obtain sharp attenuation rises and fairly flat impedance characteristics without resorting to m-derived sections. The absence of shunt transmission lines in this case provides a possibility of constructing the filters completely inside a coaxial line, with no protruding tubes or boxes. This was realized in practice: a photograph of a network consisting essentially of three prototypes ($y_1 = 3$, $y_2 = 1.85$) is shown in plate I-18 and the experimental attenuation characteristic and a sketch of a cross section through the axis given in Plate I-19. It will be noted that both the shunt and series inductances have proven to be so small as to be realizable by short straight lengths of wire. The one-turn coils visible in the sketch of Plate I-19 together with the lines across which they are shunted, comprise four W-N-L sections whose design and function will be described below.

Design curves for the Q-LS-L type are given in Plates B-4 and B-45 to B-49. Note that infinite attenuation occurs at the resonant frequency of the line and is always below the pass-band frequencies. This makes the network long if the pass-band is in the v-h-f range; nevertheless the fact that the filter may be constructed in such a form that a single section will replace a coaxial line makes it worthy of consideration even here. This same feature is of course an advantage at frequencies approaching s-h-f, where the designer's problem is to provide himself with elements small enough to resonate at the desired frequencies and yet large enough to build accurately.

Stop-band Attenuation Decrease Through Conjugate Match Between Sections: Use of booster sections. -- It can be deduced (see Appendix C, equation C-8) that a filter network terminated in the conjugate of its stop-band characteristic impedance will act like a matching transformer rather than an attenuator, producing a net insertion gain which may be as large as its theoretical attenuation. There are two ways in which this might occur in a practical composite filter. As the input impedance of a resistance-terminated network with a high nominal attenuation is very nearly its reactive characteristic impedance, cascading two sections whose characteristic impedances become conjugate at some frequency may result in very low attenuation at that frequency. The effect can also be produced by connecting two identical networks by a length of transmission line. It can easily be shown that the characteristic impedance of the terminating filter will be reflected as its conjugate to the sending-end filter when

$$j \tan\left(\frac{\pi}{2} \frac{f}{f_o}\right) = \frac{2}{\frac{Z_{of}}{Z_o} - \frac{Z_o}{Z_{of}}} \qquad (1\text{-}21)$$

where Z_{of} represents the characteristic impedance of each filter section, and Z_o and f_o are the characteristic impedance and quarter-wave frequency, respectively, of the connecting line.

An illustration of this effect is given in plate I-20 which shows the experimental attenuation of a network consisting of three Q-LS-L prototypes arranged as shown in the cross-section sketch of Figure 1-26. It will be noted that there is a deep dip in attenuation

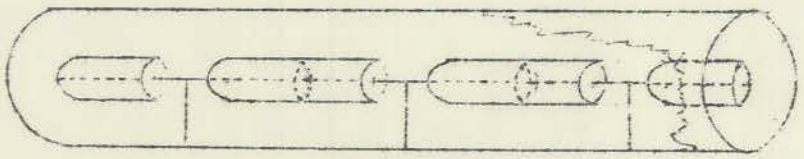


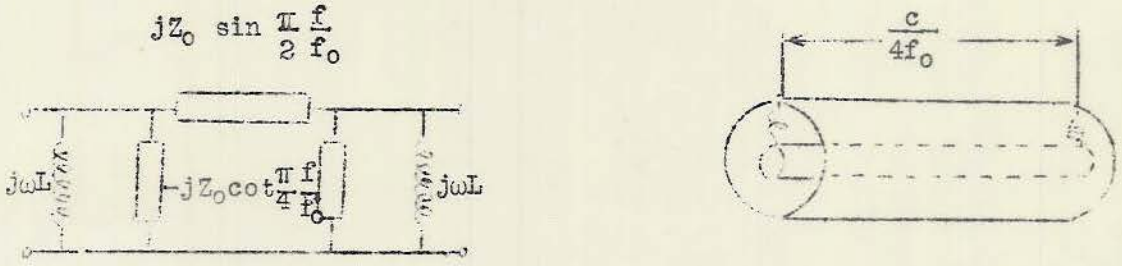
Figure 1-26. Three-section Q-LS-L Network.

at 320 mc/s. Evaluation of equation 1-21 predicts this effect at 315 mc/s, with satisfactory experimental confirmation. It will be noted that there is another, less deep but no less sharp, dip in attenuation at 720 mc/s.

The condition can be eliminated if the lines connecting the three sections are replaced with networks whose characteristic impedances are a reactance of the same sign as that of the Q-LS-L network. The shunt inductances previously referred to (Plate I-19) together with the connecting lines themselves, form such a network. Its prototype is shown in Figure 1-27. The characteristic impedance

of this "booster" network, like that of the Q-LS-L, is inductive at lower frequencies, and the values of the shunt inductances, L, are so chosen that the characteristic impedance of the booster changes from inductive to resistive at about the anti-resonant frequency of the Q-LS-L series line. Above this frequency the characteristic impedance of the Q-LS-L becomes capacitive, providing another solution of equation 1-21. This explains the attenuation dip at 720 mc/s. It also indicates that this attenuation "hole" cannot be eliminated by the interposition of the W-N-L network, whose characteristic impedance cannot be made capacitive.

The curves of Plate I-18 bear out these conclusions. Addition of the booster section eliminates the "hole" at 320 mc/s completely, while the dip at 720 mc/s is not eliminated, merely being shifted to 810 mc/s. It will be noted that at frequencies not in this vicinity, the attenuation closely follows perfect-termination theory.



(a) Circuit of W-N-L filter (b) Sketch of W-N-L filter

Figure 1-27. W-N-L filter.

Addition of the booster sections, incidentally, increases the theoretical attenuation of the network at low frequencies by fifteen db.

As can be seen on Plate I-16 the experimental Q-LO-L network also required coupling lines and hence exhibits this effect. It was found that the attenuation dip in this case occurred in the relatively high-attenuation region between the attenuation peaks. This makes it possible to keep the minimum attenuation in the "hole" greater than 60 db merely by detuning the shunt section of one of the m = 0.58 sections slightly, so that the matching phenomenon was not at maximum in both coupling lines simultaneously.

The W-N-L and W-C-N Networks — The W-N-L network used for the "booster" sections of Q-LS-L, whose circuit is sketched in Figure 1-27, forms a gradual cut-off high-pass filter useful for high power applications.

Using $Z_1 = j\frac{Z_0}{2}\sin\frac{\pi}{2}\frac{f}{f_0}$ and $Z_2 = \dfrac{-j\frac{Z_0}{2}}{1-\frac{f_0}{f}\,y\cot\frac{\pi}{4}\frac{f}{f_0}}$, where $y = \dfrac{Z_0}{\omega_0 L}$,

the attenuation and impedance functions may be written

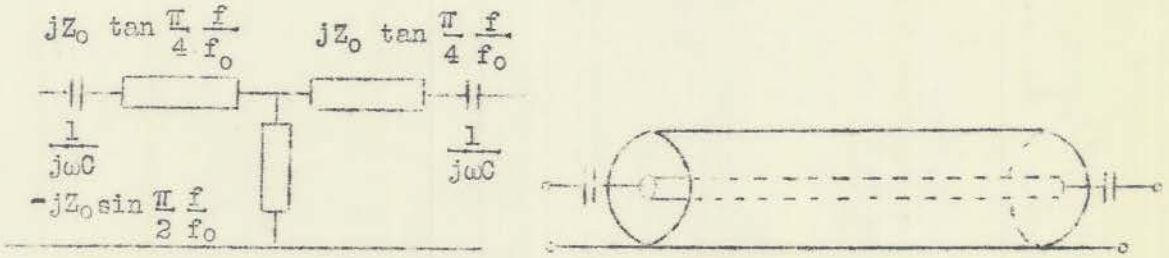
$$\cosh \alpha = \left| \left(1 + y \frac{f_o}{f} \tan \frac{\pi}{2} \frac{f}{f_o}\right) \cos \frac{\pi}{2} \frac{f}{f_o} \right| \qquad (1\text{-}22)$$

$$\frac{Z_{o\pi}}{Z_o} = \frac{1}{\sqrt{\left(1 - y \frac{f_o}{f} \cot \frac{\pi}{4} \frac{f}{f_o}\right)\left(1 + y \frac{f_o}{f} \cot \frac{\pi}{4} \frac{f}{f_o}\right)}} \qquad (1\text{-}23)$$

Plots of these functions are found in Plate B-50. The $\frac{Z_{ot}}{Z_o}$ function is, of course, not of interest, and the m-derivation of a network containing a four-terminal transmission line is not physically realizable.

The inverse of the W-N-L network is the W-C-N network sketched in Figure 1-28. If we define $y' = Z_o \omega_o C$, the exact inverse W-N-L is that for which $y = \frac{1}{y'}$.

The W-C-N network is not as applicable to high-power work as the W-N-L, because of the presence of the capacitor. It can be useful, however, for relatively low-power applications as a "booster" section in lines connecting high-pass networks of capacitive characteristic impedance.



(a) Equivalent circuit of W-C-N network.

(b) Sketch of W-C-N section built in coaxial line.

Figure 1-28. The W-C-N filter.

Each of these networks has also been used alone in applications where sharpness of cut-off was not an important factor. Here again, the fact that they may be constructed within short lengths of coaxial line proves advantageous.

Band-pass applications

It is frequently required to produce a network which will pass a given band of frequencies and attenuate frequencies both higher and lower than the assigned pass-band. There are in general two methods

for doing this: (1) to connect in cascade a high-pass and a low-pass network, and (2) to construct a prototype which itself provides two attenuation bands separated by a pass-band. The further requirement is usually made that one or both sides of the pass-band have a reasonably sharp attenuation rise, which is seldom obtained unless the cut-off in question is one of the $\frac{Z_1}{Z_2} = -2$ type.

Because of the recurrent impedance characteristic of a transmission line, any filter containing a line as an element will in general possess a multi-band-pass characteristic, as is illustrated again and again in the plates of appendix B.c . It is therefore possible to utilize most of the networks previously discussed in band-pass applications.

The Y-L-S Network - A band-pass network constructed for high-power application with a sharp cut-off on the high frequency side only is the Y-L-S diagrammed in Figure 1-29. This network has one stop-band of relatively low attenuation and gradual cut-off up to the quarter-wave resonant frequency of the shunt line, and another stop-band with a sharp cut-off occurring shortly below the half-wave frequency of this line. The attenuation curve (Figure 1-30) looks very much like that of the Y-L-O network displaced along the frequency scale by a distance of f_o. The cutoffs at $(2n-1)f_o$ are of the gradual $\frac{Z_1}{Z_2} = 0$ type, while those between $(2n-1)f_o$ and $2nf_o$ are of the sharp $\frac{Z_1}{Z_2} = -2$ variety.

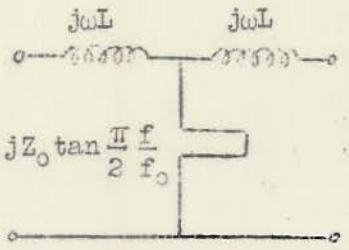


Figure 1-29. Prototype Y-L-S circuit.

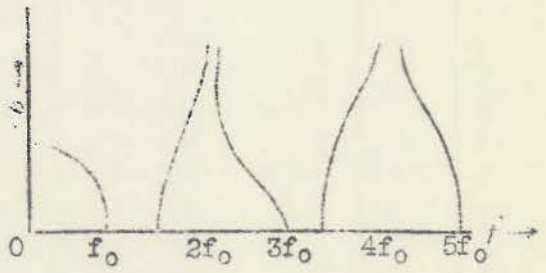


Figure 1-30. Sketch of Y-L-S attenuation curve.

Application has been made of this network to replace a Y-L-O designed to eliminate interference produced above its operating frequency by a high-powered pulse transmitter.

Since the transmission lines are short-circuited, the Y-L-S can be used at powers under which a Y-L-O would break down. Because of its band-pass characteristic, however, the Y-L-S network cannot be applied to any equipment covering more than an octave, and in practice its pass band is restricted to band-widths considerably less. Alone, it is not satisfactory for applications requiring sharp attenuation rise both above and below the pass band.

Design equations for the Y-L-S are summarized in Plate B-7 and plotted in detail in Plates B-53 to 55. A photograph of a production-type Y-L-S for a 50-ohm line is given in Plate I-21. It will be noted that the inductance is reduced to a single straight wire while the shunt lines are again made of standard solid dielectric cable stripped of its braid and inserted in the frame. An unusual feature of this network is the use of two 50-ohm transmission lines in parallel to realize a line of half that impedance, at the same time eliminating sharp corners, prone to arc over, from the top of the line. It was necessary to spread a silicone compound, Dow-Corning compound #4, over all surfaces near the top of the transmission line to eliminate the possibility of arcing. When so treated the filter consistently withstood peak powers of 800 kw.

A prototype network to provide a sharp attenuation rise on both sides of the pass-band should have a $\frac{Z_1}{Z_2} = -2$ cut-off on both sides. As Foster's reactance theorem requires that $\frac{Z_1}{Z_2} = -2$ and $\frac{Z_1}{Z_2} = 0$ cutoffs occur in pairs, this implies that two $\frac{Z_1}{Z_2} = 0$ cutoffs must occur together producing a "confluent" filter. This condition can be obtained with any prototype network which includes a resonant circuit in the series arm and an anti-resonant circuit in the shunt arm, by arranging that the resonance and anti-resonance occur at the same frequency. Examples of networks in which this effect is possible include L-S-P, Y-S-S, Y-S-O, all T-type configurations, and their inverses.

Design computations for confluent band-pass Y-S-S, T-O-O and T-O-S networks are given in Plates B-29 to 32, B-35, and B-36 to 38 respectively.

SECTION II

FILTER MEASUREMENTS

Two sets of measurements are required in network construction - the first to measure components, and the second to measure characteristics of the completed network. In transmission-line networks the component measurements can be carried out partly in terms of electrical and partly in terms of mechanical quantities; the network characteristics, of course, must be measured electrically.

Component Measurements: Substitution of transmission lines for elements containing lumped inductance and capacity considerably reduces the complexity of component measurements, since fixing the linear dimensions of transmission lines establishes their electrical characteristics. The reduction in the number of time-consuming electrical measurements required on the assembly line greatly decreases both the unit production time and the unit cost of the filters. Electrical measurements of components have been found necessary only on elements containing a capacitor, or on elements both containing a lumped parameter and requiring resonance at an accurately predetermined frequency.

It has been found feasible to construct the transmission line lengths to tolerances of $\pm$ 1/32 inch, which holds the resonant frequency within 2% of its assigned value for the shortest line used in any production-type network discussed in this report. Choosing any combination of standard tube and rod diameters which results in a line whose characteristic impedance is within 5% of design requirements yields adequate electrical performance. Inductances not part of resonant circuits have been constructed as mechanical copies of those in the experimental models with gratifying results. It is also possible to construct capacitors as lines within required tolerances, but this procedure leads at low vhf frequencies to inconveniently thin dielectrics or to lines electrically too long to be considered lumped circuits.

The electrical characteristics of filters constructed only of inductors and transmission lines, including several types of Y-L-O, Q-LS-L, Y-L-S, Y-L-OS, and T-IO-S, have proven repeatable using dimensionally measured components alone, providing an attenuation rise of only moderate steepness is required. As yet no filter with an attenuation rise from 2 db. to 60 db. occurring in less than 5% frequency change has been built without resorting to some electrical measurements of its components.

Electrical Measurements: The small capacitors necessary in some of the circuits (c.f. Y-S-S, L-S-L), may be measured on a low-frequency instrument such as the General Electric capacitometer, a "Q" meter operated below 1 megacycle, or an alternating-current bridge.

Two methods have been devised for adjusting an impedance arm to its frequency of resonance. The first, most applicable when the impedance contains an inductor and is to be adjusted to anti-resonance,

requires that all elements of the arm, except the inductor, be first adjusted to their correct values by other methods. The inductor is then loosely coupled to a "Q" meter whose oscillator and tank circuit are then tuned together over the frequency band. At the resonant frequency of the unknown circuit, the meter of the "Q" indicator will dip as the unknown circuit draws energy from the "Q" meter tank. The inductor of the unknown is then adjusted until this occurs at the desired frequency. It is necessary that the unknown be placed in a box simulating as nearly as possible the electrical surfaces which will surround it in the filter installation. The method can also be applied, less successfully, to arms which are to be operated as series resonant circuits, providing the circuit can be closed without changing its effective inductance too greatly.

The second method, particularly useful for adjusting series-resonant arms, uses an oscillator which is adjusted to the desired resonant frequency and connected to a slotted line. This method also requires that all parameters of the unknown, save one, have been previously adjusted. The line is terminated in a short-circuited fitting and the position of the nodes recorded as reference points. The shorted fitting is replaced by an identical open-circuited one, the unknown connected across it and adjustments made to the remaining parameter of the unknown until the nodes are restored to their previous position. Precaution must be taken to have the unknown in the same surroundings as it will occupy in the network, to take into account proximity effects.

The shorted-fitting-and-line techniques have been applied to resonate the shunt elements of the filters illustrated in Plates I-1, 4 and 16. This method can also be used on anti-resonant elements, with somewhat less accuracy, by obtaining the reference node positions with the open-circuited termination in place. The principal source of error is the reactive load produced by the field end-effects at the open fitting, rendering the exact position of the end of the line somewhat uncertain. A somewhat more accurate procedure assumes the proper position of the nodes to be one-quarter wavelength distant from those obtained with the short-circuited termination, but this fails to consider the actual increase in the physical length of the line caused by open-circuiting some types of fittings. Experiment indicates that a procedure using the mean between the two reference nodes is a good approximation for adjusting anti-resonant elements, providing the reference nodes determined by the two methods are not too far apart.

An important advantage of the slotted-line technique is the absence of the need for a coil or lead for inductive coupling to a "Q" meter.

Network Characteristic Measurements: The two quantities measured to describe the performance of experimental networks are the attenuation and the standing wave ratio produced by insertion of the network into a matched line of its nominal pass-band characteristic impedance. The attenuation function is, of course, part of the propagation function; the standing-wave ratio is a function of the actual characteristic of the filter and the phase function β. Because the effective electrical length of the filter was in most cases unimportant, no separate measurement of β was undertaken. Measurements of SWR were also omitted on most filters designed for relatively low-power applications.

Attenuation Measurements: A block diagram of the insertion-loss measuring circuit is shown in Plate II-1. The procedure is to measure the decibel difference between the two generator voltages required to produce the same receiver output indication when the filter is between the generator and receiver and when the filter is replaced by a short length of line. The LAF-2 and Model A signal generators used have an accurately calibrated attenuator admirably suited to this purpose. Measurements of insertion losses as great at 70 db. could be made at all frequencies between 100 mc/s and 900 mc/s.

As may be seen from equation C-6, Appendix C, the overall attenuation of a network has meaning only with reference to a specific set of terminating impedances Z_g and Z_r. The most convenient reference terminations are those between which the filter is designed to work in the pass-band. For the networks discussed in this report, these were 50 or 70 ohms resistance on each end.

The impedances of generator or receiver alone fall far short of being purely resistive, let alone being the proper magnitude. Therefore, attenuating pads of some sort must be inserted between filter and generator, and filter and receiver, to insure that the proper impedance is presented to each end of the network. Most satisfactory for this use is a long line whose characteristic impedance is the desired resistance, although the resistance pads illustrated in Plates II-1 and 3 are also fairly good, and require less space. If 20 db. of the proper impedance isolation cable is used in the pass band on each side of the filter, attenuations of less than 30 db. can be measured with less than 3% error.

Removal of the attenuation pads in the stop-band, except for a small amount (approximately 6 db.) of attenuating cable on the receiver end, is justified by Guillemin's conclusion, referred to on Page C-3 that the attenuation of a network terminated resistively should not be more than 6 db. below its perfect termination attenuation. Experience has shown it good practice to remove the isolation between the generator and filter for insertion loss measurements between 30 and 50 db., and to remove all but 6 db. isolation from between the filter and receiver at higher attenuation values.

Receiver-Indicator: Two methods of obtaining a receiver-input indication have been successfully used:

(a) Heterodyne Level - This method requires a receiver (such as the AN/APR) which contains a heterodyne oscillator to produce an audio-output voltage proportional to the carrier level. This voltage may be measured with a suitable audio-frequency voltmeter. A photograph of an actual set-up using this method is shown in Plate II-3. The limiting factor in sensitivity of this arrangement is the residual audio noise from either internal or external sources.

(b) Carrier Level - A meter is connected into the detector circuit of the receiver to indicate the rectified i-f voltage, which is proportional to carrier level. Such a meter is contained in the AN/APR receiver. A similar meter may be connected in RDO receiver circuit.

Attenuation Test Procedure: Following are detailed instructions for using the equipment shown in Plate II-3. All control adjustments must be made in the same direction to eliminate back-lash effects.

(a) Connect pads or attenuation cables which match nominal characteristic impedance of filter as described above.

(b) Insert filter between pads.

(c) Set generator frequency and plate-voltage to desired values.

(d) Set generator-attenuator dial to a suitable reference value. As the accuracy of the attenuator calibration is greater at higher db. readings, it is advisable to use 10 db. as reference throughout as large a band as possible.

(e) Tune receiver for peak reading of the output indicator and set receiver i-f gain control to give a convenient output reading. Note this reading.

(f) Replace filter with connector or short piece of cable.

(g) Leaving receiver controls set, adjust generator-attenuator dial to give same receiver output voltage reading as in (e). If attenuator reference setting is less than 10 db., be sure generator frequency and output level have not changed.

(h) Filter attenuation in db. is the difference between the attenuator setting with filter out (g) and the setting with filter in (d).

Standing-Wave-Ratio Measurements: For transmitting applications, where it is particularly important that impedance match in the pass-band be as close as possible, it is frequently required that the insertion of the filter between the generator and a matched load shall not produce more than a specified standing wave ratio. A block diagram of a method for measuring this quantity is shown in Plate II-2 and a photograph of the apparatus in Plate II-4. The procedure is to insert the filter between the slotted line and the load, and to measure the difference between the two generator voltages required to produce the same receiver output indication for both the maximum and minimum positions of the standing wave. Coupling of the slotted-line probe which feeds the receiver should be loose enough to produce negligible field distortion, as indicated by minimum change in receiver output when the probe is moved past an insulating spacer.

The following steps are given for measuring filter SWR. All control adjustments must be made in the same direction to eliminate back-lash effects.

(a) Insert filter between slotted line and load.
(b) Set generator to desired frequency and convenient output level.
(c) Tune receiver for peak reading of its output indicator.
(d) Move slotted line slider to position of minimum voltage on standing wave.
(e) Set receiver i-f gain control to give a convenient reading on its output indicator. (Be sure receiver is not overloaded!) Note attenuator reading of signal generator.
(f) Move slotted-line slider to position of maximum voltage on standing wave.
(g) Leaving receiver controls set, adjust generator attenuator dial to give same receiver output voltage reading as in (e).
(h) From the difference D in decibels between the generator attenuator setting (g) at maximum standing wave and the setting (e) at minimum, the standing wave ratio may be computed by the formula

$$SWR = \text{antilog}_{10} \frac{D}{20}$$

General Considerations: In order to reduce the errors to a practical minimum in measurement of filter insertion loss and SWR, certain precautions must be employed. Instrument errors such as receiver overloading, back-lash effects, fluctuating output power and frequency, etc., were discussed above. Extraneous interference can be eliminated by making the measurements in an adequately shielded room with sufficient incoming power-line filtering. As an extra precaution a filter may be

connected in the receiver power line. To insure stability a voltage regulator should be placed in the common power line to the equipment. It is important to be sure that all coaxial connectors are properly assembled to their cables so that a minimum of mismatch results. The measuring instruments shown in Plates II-3 and II-4, were selected because of their high degree of stability and accuracy of calibration. The slotted line shown has a nominal impedance of 51.5 ohms and the concentricity of inner with outer conductor was accurate within ± .0015 inches. When terminated in 1,000 ft of RG-8u plus 250 ft. of RG-21u, this line gave a maximum standing-wave ratio of 1.04 throughout the band 100 - 300 mc/s. For most purposes, an attenuating cable sufficiently long to provide 20 db. of attenuation is an adequate load. Care must be taken to match the signal generator, as well as the test load, to the line. If measurements are to be made over a wide frequency range, this is best done by feeding the line through a short length - approximately 5 db. - of attenuating cable.

ACKNOWLEDGMENTS

The work done in this report was done under the active supervision of Dr. M. K. Goldstein. Mr. R.S. Timm, Mr. A. Brodzinsky and Dr. Goldstein initiated the development of the basic ideas and methods, and are responsible for their successful utilization in practise. Messrs. Jacob Benveniste, Murray Simpson and J. E. Mekota, made valuable contributions in applying and extending the fundamental approach. Further acknowledgment is made to Mr. Mekota for his assistance in writing the report.

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APPENDIX A

TABLE OF POSSIBLE TRANSMISSION-LINE FILTER NETWORKS

In Tables 1 to 11, the filter sections possible using no more than two components per circuit arm are arranged in the form of a framework of "periodic table", intended to illustrate the wide variety of circuits possible with a few transmission line and lumped parameter combinations. It is hoped that these also serve as guides for future developments in this field. Networks obtainable by m-derivation of others have been omitted. Where special cases of an otherwise independent type may reduce to m-derived section of another, this is indicated in parentheses.

Nomenclature of Filters: The following system for uniquely identifying the various kinds of networks is used in this report:

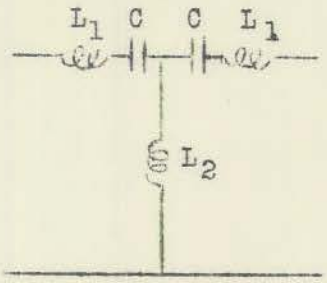
a. The first letter of a designation indicates the general classification by means of the following symbols:
   1. L- denotes filter made up of lumped parameters only
   2. Q- denotes filters containing (1) an open- or short-circuited transmission line in the series arm and (2) only lumped parameters in the shunt arm.
   3. Y- denotes filters containing (1) only lumped parameters in the series arm and (2) an open- or short-circuited transmission line in the shunt arm.
   4. T- denotes filters containing an open- or short-circuited transmission line in each arm.
   5. W- denotes filters* consisting of a four-terminal transmission line with two-terminal impedances placed in series or in shunt at disrete intervals along the line.

b. The second group of letters indicates the components in the series arm of the prototype section in the following manner:
   1. -L- denotes an inductance
   2. -C- denotes a capacitance
   3. -P- denotes a parallel resonant lumped parameter (LC) circuit.
   4. -S- denotes a series parallel lumped parameter (LC) circuit, or a short-circuited transmission line.
   5. -O- denotes an open-circuited transmission line.
   6. -N- denotes no parameter (used only in W-type networks)

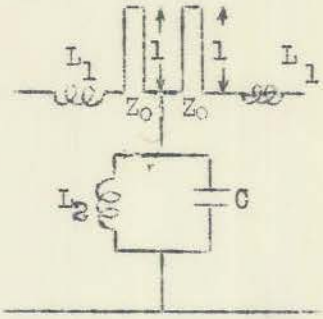
c. A third group of letters identical to th second group is used to indicate the components in the shunt arm of the prototype section, and in the same manner. Combinations of two letters in either group may be used to indicate a combination of a lumped parameter and a transmission line in series of parallel. If the two are in series, the first letter is that of the lumped constant; if the two are in parallel, the first letter is that of the transmission line.

* The circuits shown in Tables 1 to 11 do not include W -type filters

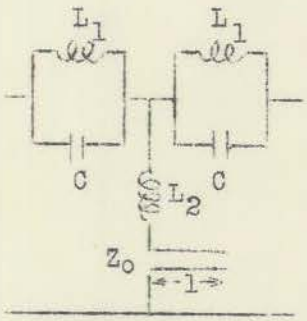
Examples of Filter Configurations: To illustrate the use of this nomenclature, a few examples are shown in Figure A-1. The first letter in each case refers to the basic type of filter, i.e., the "L", "Q", "Y", "T", or "W"; the second letter or pair of letters indicates the configuration of the series arm; the last letter or pair of letters indicates the configuration of the shunt arm. Thus, in Figure A-1(a), the first "L" specifies an L-type filter, the second letter, "S", specifies a series-resonant circuit in the series arm, and the third letter, "L", specifies an inductance in the shunt arm. "Q" of Figure A-1(b) specifies a circuit containing a line in the series arm only; "LS" indicates that it is a shorted line in series with an inductance, and "P" describes the shunt arm as a parallel-resonant lumped circuit. "Y" of Figure A-1(c) indicates a transmission line in the shunt arm only, the final "LO" indicates that it is an open-circuited line in series with an inductance, and the central "P" establishes the series arm as a lumped, anti-resonant circuit. In Figure A-1(d), "T" refers to a T-type filter, "O" to the open-circuited line in the series arm, and "SC" to a short-circuited line in parallel with a capacitance in the shunt arm. Figure A-1(e) shows a W-N-L filter in which "N" indicates there is no parameter in series with each end of the line, and "L" indicates that the line is shunted at each end by an inductance.



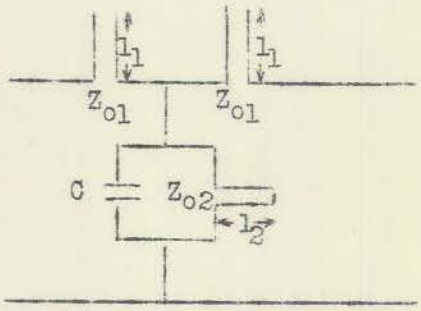
(a) L-S-L



(b) Q-LS-P



(c) Y-P-LO



(d) T-O-SC



(e) W-N-L

Figure A-1.

APPENDIX B - Filter Characteristic Summaries and Design Curves

Plates B-1 to B-56 inclusive give in symbolic form the characteristics of a selection of the networks considered in this investigation, as well as summarizing the design equations for each type.

Due to the large number of design variables available in a prototype network containing both transmission lines and lumped constants, the possible arrangement of stop- and pass-bands, attenuation and impedance characteristics in such a network is very large. If $n = 2(T-1) + L$ where T is the number of transmission lines in the circuit of the prototype half-section, and L the number of lumped constants, there are $n!$ different types of characteristics available using only the first two quadrants of the transmission lines, each type possessing a different arrangement of the attenuation bands and/or impedance characteristics from the others. If arrangements producing confluences between adjacent bands are considered, the number of possibilities is increased; for one confluence only per section, the number of circuit possibilities is increased to $\frac{(n+1)!}{2}$. It cannot be emphasized too strongly, therefore, that the design curves here plotted include only a fraction of the possibilities even for these prototypes.

A given set of curves, in general, gives information on two inverse networks. The attenuations are, of course, identical, while the impedance curves represent either $\frac{Z_{0\pi}}{Z_0}$ or $\frac{Y_{0T}}{Y_0}$, where $Z_0 = \frac{1}{Y_0}$ is some constant-impedance reference level, chosen arbitrarily for each network. The $\frac{Z_{0\pi}}{Z_0}$ curves for the T (series) m-derived version of the network is also given, as the series m-derived network is in general most easy to construct at ultra-high frequencies. The curves apply, of course, to the π m-derived network of the inverse.

The design constants are defined first for the topmost network of a given pair, and then for the inverse.

For brevity, the $\frac{Z_{0T}}{Z_0}$ and $\frac{Y_{0\pi}}{Y_0}$ curves have been indicated as $\frac{Z_{0\pi}}{Z_0}\Big|_{m=0}$ and $\frac{Y_{0T}}{Y_0}\Big|_{m=0}$. This procedure is justifiable on a mnemonic basis from equation C-13, Appendix C, although its physical implication is that neither the shunt nor the series impedance of the network in question exists.

APPENDIX C

Summary of Theory Useful to the Design of Symmetrical Ladder Filters

The performance of a symmetrical network may be described in terms of two quantities, the characteristic impedance and the propagation function. This Appendix summarizes for easy reference expressions for these two quantities in terms of two other parameters -- the series and shunt impedances of a ladder network. Certain other relations particularly useful in uhf filter design are also presented. More detailed discussions of the ladder network theory may be found in numerous sources*.

As any summetrical four-terminaled network is representable by a T or π section (Figure C-1), which may be considered one "rung" of a ladder network (Figure C-2), these expressions may conceivably be applied to the design of a network of any type, by first obtaining its equivalent T or π. However networks for which the T or π equivalent is not easily obtainable may profitably be analyzed from other standpoints (c.f. Reference 9).

The Impedance Functions The T equivalent to any symmetrical network is formed by Z_1 and Z_2 of Figure C-1(a), where Z_g is the internal impedance of the generator, and Z_r the impedance of the load. The input impedance,



Figure C-1. Symmetrical T- and π-Networks.

Z_s, is given by

$$Z_s = Z_1 + \frac{(Z_1 + Z_r)(Z_2)}{Z_1 + Z_2 + Z_r} \qquad (C\text{-}1)$$

At some value of Z_r, the network will transmit energy as though the generator were connected directly to Z_r. The value of Z_r at which this occurs is termed the "iterative impedance" of the network. For symmetrical networks this is also termed the "characteristic impedance", Z_o.

* Reference 1, Chps VIII, IX. Reference 2; Reference 6, Chps, IV, - VI; Reference 7, Chps IV, VI, IX and X.

Setting $Z_s = Z_r = Z_o$ in equation C-1, this quantity for the network of Figure C-1(a) is found to be

$$Z_{ot} = \sqrt{Z_1 (Z_1 + 2Z_2)} \qquad (C-2)$$

where Z_{ot} represents Z_o for a T network.

If several T networks are connected in cascade, as in Figure C-2(a), a ladder configuration is obtained. A similar structure may also be made up of a series of π networks arranged as in Figure C-2(b). In each of these ladders the total series impedance per section is $2Z_1$ and the total

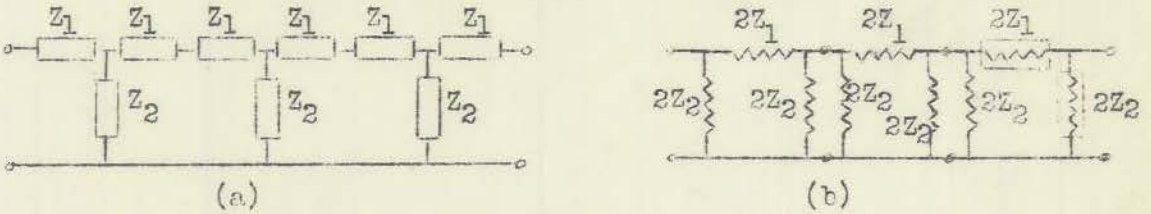


Figure C-2. T and π Networks in Tandem

shunt impedance per section is Z_2. The characteristic impedance $Z_{o\pi}$ for the π network shown in Figure C-1(b) is

$$Z_{o\pi} = \frac{2 Z_1 Z_2}{\sqrt{Z_1 (Z_1 + 2Z_2)}} \qquad (C-3)$$

or, substituting from equation C-2

$$Z_{o\pi} = \frac{2 Z_1 Z_2}{Z_{ot}} \qquad (C-4)$$

If the impedance arms Z_1 and Z_2 are pure reactances of opposite sign such that $Z_1 Z_2 = K^2$, where K is a real quantity, the network is known as a constant-K filter. Several very important filter configurations are of this type, particularly the lumped constant L-L-C* low- and L-C-L high-pass configurations, and two of the simplest forms, T-S-O and T-O-S, of transmission-line filters.

* For an explanation of the nomenclature used to describe the filter circuits of this report, see Appendix A.

Propagation Functions: - The propagation function, $\gamma = \alpha + j\beta$, of a single symmetrical T-section is given by

$$\cosh \gamma = 1 + \frac{Z_1}{Z_2} \qquad (C-5)$$

where α is the attenuation in nepers, β the phase shift in radians. The propagation function of a composite network of several sections may be obtained by computing γ from equation C-5 for each section and adding the results, providing each section is impedance-matched to its neighbors. If the network is terminated in other than its characteristic impedance, additional reflections and phase shifts occur at the junctions, which, added with γ, make the total propagation function of the system. Under the usual operating conditions, in which terminations are primarily resistive, the γ of equation C-5 usually forms the major portion, from the design standpoint, of the overall propagation function — for instance, if the terminations are pure resistances of any value, the total attenuation of the network will not be more than 6 db below that computed from equation C-5.* However, it is possible for attenuation of a filter whose termination is highly reactive to differ widely from that predicted by this expression. The insertion loss in nepers of a symmetrical network, allowing for the effect of termination impedances, may be written as**

$$\alpha_T = \log_e \left| \frac{(Z_o + Z_g)(Z_o + Z_r) - (Z_o - Z_g)(Z_o - Z_r)e^{-2\gamma}}{2Z_o(Z_g + Z_r)} \right| + \gamma \qquad (C-6)$$

where Z_o is the characteristic impedance of the network, Z_g the generator impedance, Z_r the load impedance and $\gamma = \alpha + j\beta$ the propagation function computed from C-5. In the worst possible case $Z_o + Z_r = 0$ or $Z_o + Z_g = 0$, and the effective attenuation can be shown to become $-\alpha$, or the insertion of the filter can produce a net gain as large as its nominal attenuation. Under these conditions the filter is acting as a matching transformer, and a condition of anti-resonance occurs between the network and one terminating impedance. This situation is sometimes met with in the stop-bands of practical filter installations, where the terminations - a transmitter and an antenna, for instance - may become highly reactive. If an approximate conjugate match occurs in the system attenuation will be greatly reduced from that for which the network was designed.

* Reference 1 p. 292

** Reference 1 p. 290, eq. 658, Reference 2 p. 117, eq. 46.

If the network is not to attenuate desired frequencies, the elements must be non-dissipative, with the result that $\frac{Z_1}{Z_2}$ is real. For the nearly dissipationless elements used at ultra-high frequencies, this approximation holds quite closely, and we may re-write equation C-5 as

$$\cos \beta = 1 + \frac{Z_1}{Z_2}, \qquad \alpha = 0 \qquad \text{(C-7a)}$$

when $-2 \leq \frac{Z_1}{Z_2} \leq 0$, while for other values of $\frac{Z_1}{Z_2}$

$$\cosh \alpha = \left|1 + \frac{Z_1}{Z_2}\right|, \qquad \beta = n\pi \qquad \text{(C-7b)}$$

From the conditions $\frac{Z_1}{Z_2} = 0$, and $\frac{Z_1}{Z_2} = -2$, representing the boundaries of the pass-band, the cutoff frequencies may be computed. It can also be seen that when $\alpha = 0$ the characteristic impedance is purely resistive for all values of β, while in the attenuation band it is a pure reactance.

M-Derived Filters: The constant-K and other prototype filters discussed thus far frequently have two disadvantages, namely:

(a) The characteristic impedance varies with frequency so that, under the usual condition of constant-resistance termination, impedance match cannot be maintained over the entire pass-band.

(b) For many applications the attenuation just outside the pass-band does not rise rapidly enough as the frequency changes from cutoff.

These difficulties may be overcome for networks in which Z_1 and Z_2 can be adjusted independently by the use of the so-called "m-derived" filters*, which may be made to have a characteristic impedance varying much less with frequency than that of the original network and an attenuation peak at a selected point in the stop-band. When a sharp change from low to high attenuation is desired, the peak may be set very close to cut-off frequency. A composite network meeting any sort of attenuation requirement is easily obtained by cascading several suitable m-sections.

An m-derived T-section (also known as a "series-derived" section) is obtained from the prototype T-section when Z_1 is replaced by a Z_1' and Z_2 by a Z_2' such that

$$Z_1' = mZ_1 \qquad \text{(C-8)}$$

* Reference 11 is Zobel's original discussion of this subject.

$$Z_1 (Z_1 + 2Z_2) = Z_{ot}^2 = Z'_{ot}{}^2 = Z_1' (Z_1' + 2Z_2') \qquad (C-9)$$

which determines Z_2' to be

$$Z_2' = \frac{Z_2}{m} + \frac{1 - m^2}{2m} Z_1 \qquad (C-10)$$

This can be constructed as shown in Figure C-3. Clearly, if Z_2' is to be physically realized as a combination of two positive impedances, we require that $0 \leq m \leq 1$. Perfect impedance match may be obtained between



(a) Prototype (b) m-derived

Figure C-3. Development of m-derived T section from the Prototype.

various m-derived sections since the characteristic impedance of the m-derived T section is by hypothesis the same as that of the prototype T section regardless of the value of m. However, the characteristic impedance of the π section constructed from an m-derived T section, shown in Figure C-4, depends on the value of m, as indicated by equation C-11. This equation, obtained by substitution of Z_1' and

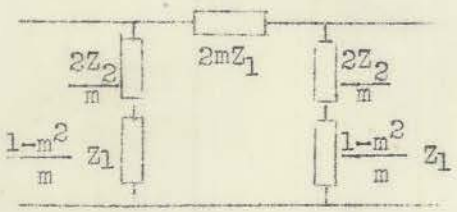


Figure C-4. π-section constructed from m-derived T section.

Z_2' respectively for Z_1 and Z_2 in equation C-4 is

$$Z_{o\pi}' = \frac{2Z_1Z_2 + (1 - m^2) Z_1^2}{Z_{ot}} \qquad (C-11)$$

$$= Z_{o\pi} + (1 - m^2) \frac{Z_1^2}{Z_{ot}} \qquad (C-12)$$

where $Z_{o\pi}'$ is the characteristic impedance of a π-section constructed from an m-derived T section. When m=1, the filter reduces to the prototype π-section. As m becomes smaller and smaller,

$$\lim_{m \doteq 0} Z'_{o\pi} = Z_{ot} \qquad (C-13)$$

For intermediate values of m, $Z_{0\pi}$ must be between Z_{0T} and $Z_{0\pi}$, and some m-value can be found to give a $Z_{0\pi}'$ which is relatively constant over a large portion of the pass-band. Such a section, split in half, with the remaining T-sections of the filter inserted in the gap, is useful to terminate the network.

The attenuation produced by a symmetrical network of resistive characteristic impedance Z_0' and phase function β inserted in a line of resistive characteristic impedance Z_0 is given* in db by

$$\alpha_{db} = 10 \log_{10} \left[1 + \frac{\sin^2 \beta}{4} \left(\frac{Z_0'}{Z_0} - \frac{Z_0}{Z_0'} \right)^2 \right] \qquad (C-14)$$

This result is useful in formulating criteria for the amount of "flatness" required in the impedance characteristic of the terminating section. It can be seen that if an insertion loss of 1 db can be tolerated, while dissipation and manufacturing errors are negligible, ratios of $\frac{Z_0'}{Z_0}$ as high as 1.62 and as low as 0.62 may be tolerated. If $1.25 \geq \frac{Z_0'}{Z_0} \geq 0.8$, the maximum insertion loss is 0.2 db. For closer matches, the maximum insertion loss is nearly proportional to the percentage of mismatch, $(\frac{Z_0'}{Z_0} - 1)$.

A fuller discussion of matching of constant-K m-sections to lines is given by Guillemin, in Reference 1, pp 531 ff.

The stop-band attenuation of the dissipationless m-derived section is, by substitution in equation C-7(b)

$$\cosh' \alpha = \frac{2Z_2 + (1 + m^2) Z_1}{2Z_2 + (1 - m^2) Z_1} \qquad (C-15)$$

For infinite attenuation the denominator of equation C-15 must be zero** or

$$\frac{Z_1}{Z_2} = \frac{-2}{1-m^2} \qquad (C-16)$$

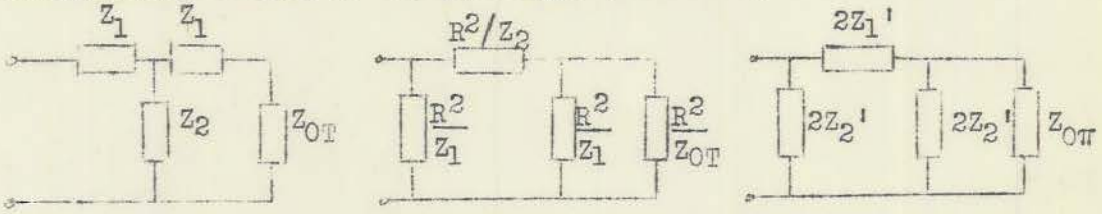
* Richards, Reference 9, p. 149, eq. 26. The same result in different form is given by Shea, Reference 2, p. 119, eq. (50).

** If a network is terminated in finite, non-zero impedances, infinite attenuation will also occur, due to interaction between the filter and its termination, at any other points in the stop-band at which the characteristic impedance of the network becomes zero or infinite. C. f. Richards, Reference 9, p. 150. The same result may be derived from Guillemin, Reference 1, p. 282, eq. (639a). This result is of considerable practical importance, in particular, it implies infinite zero-frequency attenuation for any dissipationless network possessing a stop-band at zero frequency, providing the impedances of the termination contain resistance.

This attenuation peak may be moved as closely as desired to the cut-off point, $\frac{Z_1}{Z_2} = -2$, by choosing "m" sufficiently small. It will be noted that equation C-16 is simply the condition that the shunt impedance Z_2' (equation C-10) be in resonance, thereby introducing a short circuit across the line.

Inverse Filter Networks It can be shown* that for any network there exists a set of "inverse" networks whose admittances are identically a constant multiplied by the impedance of the original. This property can be utilized in filter design to obtain characteristics of both a given network and its inverse from one set of computations.

Figure C-5 shows (a) a T-section terminated in its characteristic impedance, with (b) its inverse, and (c) the inverse with nomenclature corresponding to that of our prototype π section.



(a) T Section (b) Inverse of T Section (c) Inverse with nomenclature of prototype π section.

Figure C-5. Inverse Filter Networks

Then

$$Z_1 = \frac{R^2}{2Z_2} \qquad (C-17)$$

and

$$Z_2' = \frac{R^2}{2Z_1} \qquad (C-18)$$

where R is an arbitrary constant. The characteristic admittance, $Y_{0\pi}'$, of the inverse network is

$$Y_{0\pi}' = \frac{Z_{0T}}{R^2} \qquad (C-19)$$

which may be written as

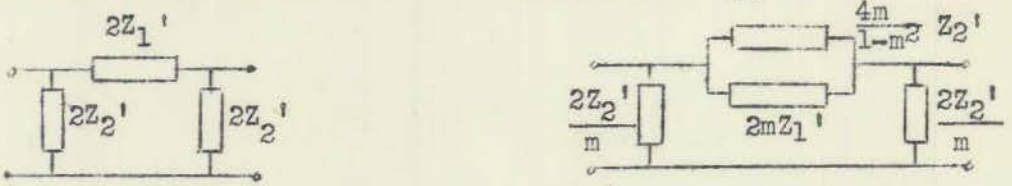
$$\frac{Z_{0\pi'}}{R} = \frac{R}{Z_{0T}} \qquad (C-20)$$

The characteristic admittance $Y'_{0\pi}$ of the π-section of Figure C-5(b)

* Shea, Reference 2, p.124 ff; Guillemin, Reference 1, p.204 ff, P.252 ff.

follows the same law of variation with frequency as the characteristic impedance of the original T section of Figure C-5(a). If equations C-17 and C-18 for Z_1' and Z_2' are substituted in equation C-5, the result shows the propagation function for the inverse network to be the same as that for the original. It will be noted that the T and π arrangements of a constant-K network are inverse with respect to one another, with $R = \sqrt{2K}$.

The M-Derived π Section: If the inversion process be applied, with R = 1 ohm, to the T sections of Figure C-3, the resulting networks have the form of Figure C-6. The network of C-6b may be considered as derived from that of C-6a, subject to the restriction that $Z_{O\pi}$ remain constant.



(a) π Prototype (b) Shunt-derived Section

Figure C-6. The π or shunt-derived m-section and its prototype.

It is known as the π or shunt-derived m-section. Of course this relationship between C-6a and C-6b holds if the prime marks are dropped.

By applying the inversion process to equations C-2, C-4, and C-11, and writing the results in terms of admittances, the following are obtained:

$$Y_{O\pi} = \sqrt{\frac{Y_2}{2}\left(\frac{Y_2}{2} + Y_1\right)} \qquad \text{(C-21)}$$

$$Y_{OT} = \frac{Y_1 Y_2}{2Y_{O\pi}} \qquad \text{(C-22)}$$

$$Y_{OT}' = \frac{\frac{Y_1 Y_2}{2} + \frac{1-m^2}{4} Y_2^2}{Y_{O\pi}} \qquad \text{(C-23)}$$

where $Y_{OT} = \frac{1}{Z_{OT}}$, $Y_{O\pi} = \frac{1}{Z_{O\pi}}$.

A split shunt-derived section will thus provide a T-input characteristic impedance which varies with m, while the π-impedance is that of the prototype. Infinite attenuation is obtained when the new series arm is anti-resonant. The attenuation is, of course, the same as that of its inverse, the series-derived section.

It is possible also to carry out several derivations on one section of a filter - to shunt m-derive the prototype, rearrange the result to a "T", series m-derive this network, etc. * Configurations obtained by this system can be matched at all frequencies to an m-derived section and thence to the prototype, while the match to a constant-impedance pass-band termination is improved greatly over that obtained with a single derivation as indicated in Figure C-7. The propagation

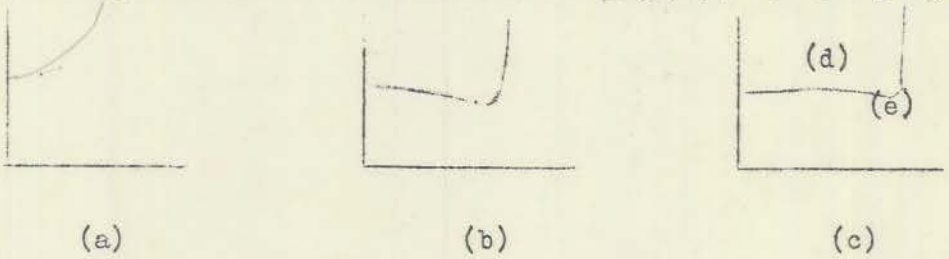


Figure C-7. Comparison of the Characteristic Impedance Functions of prototype (a), single m-derived (b), and double m-derived (c) networks. Note maximum (d) and minimum (e) of Z_0 functions of double m-derived network.

function is that of two m-sections in cascade. However, the tolerances required for the components of such repeated-derivation sections become rapidly more critical as the number of derivations are increased. For this reason construction of such networks at ultra-high frequencies has not yet been attemped. It is, however, possible to get results quite similar to those of a double m-derived section from networks containing transmission lines. (See Page 12 of section I.) The essential requirement in obtaining such extremely flat pass-band characteristic impedances is that there be several parameters available in the circuit for varying the impedance without pushing the attenuation outside of rather wide limits. The additional parameters can frequently be obtained by addition of elements other than those introduced by m-derivation, or by making use of the additional parameters available when lines are introduced into the circuit.

Equivalence of Dissipationless Lines to Resonant Circuits: It is well known that at radio and ultra-high frequencies the attenuation of short transmission lines is negligible. Such dissipationless lines may be used as ultra-high frequency resonators to good advantage. An open-circuited, dissipationless line has an impedance characteristic similar to that of a series-resonant circuit, provided the frequency is considerably less than the half-wave frequency of the line, while a short-circuited, dissipationless line has an impedance characteristic similar to that of an anti-resonant circuit. Although parallel-wire lines

* Reference 12. A further discussion is given in Reference 1, p.341 ff.

may be used, coaxial lines are more suitable because of their mechanical ruggedness and minimum risk of undesired coupling with neighboring circuits.

The input impedance Z_{so} of an open-circuited dissipationless transmission line to a signal of wavelength λ is given by

$$Z_{so} = -j\, Z_o \cot \beta l \qquad \text{(C-24)}$$

where Z_o is the characteristic impedance, $\beta = \frac{2\pi}{\lambda}$ is the phase shift per unit length, and l is the length of the line. The input impedance of a short-circuited, dissipationless line is given by

$$Z_{ss} = j\, Z_o \tan \beta l \qquad \text{(C-25)}$$

Writing $l = \frac{\lambda_o}{4}$ and using $\frac{\lambda_o}{\lambda} = \frac{f}{f_o}$, we may rewrite equations (C-24) and (C-25)

$$Z_{so} = -j\, Z_o \cot \left(\frac{\pi}{2} \frac{f}{f_o}\right) \qquad \text{(C-26)}$$

and

$$Z_{ss} = j\, Z_o \tan \left(\frac{\pi}{2} \frac{f}{f_o}\right) \qquad \text{(C-27)}$$

These lines thus provide us with two-terminal networks whose impedances and admittances are tangent junctions of frequency, augmenting the linear functions furnished by lumped capacitors and inductors. The additional flexibility given network design by the use of these parameters has been discussed in Section II.

APPENDIX A

TABLE I. L-TYPE FILTERS

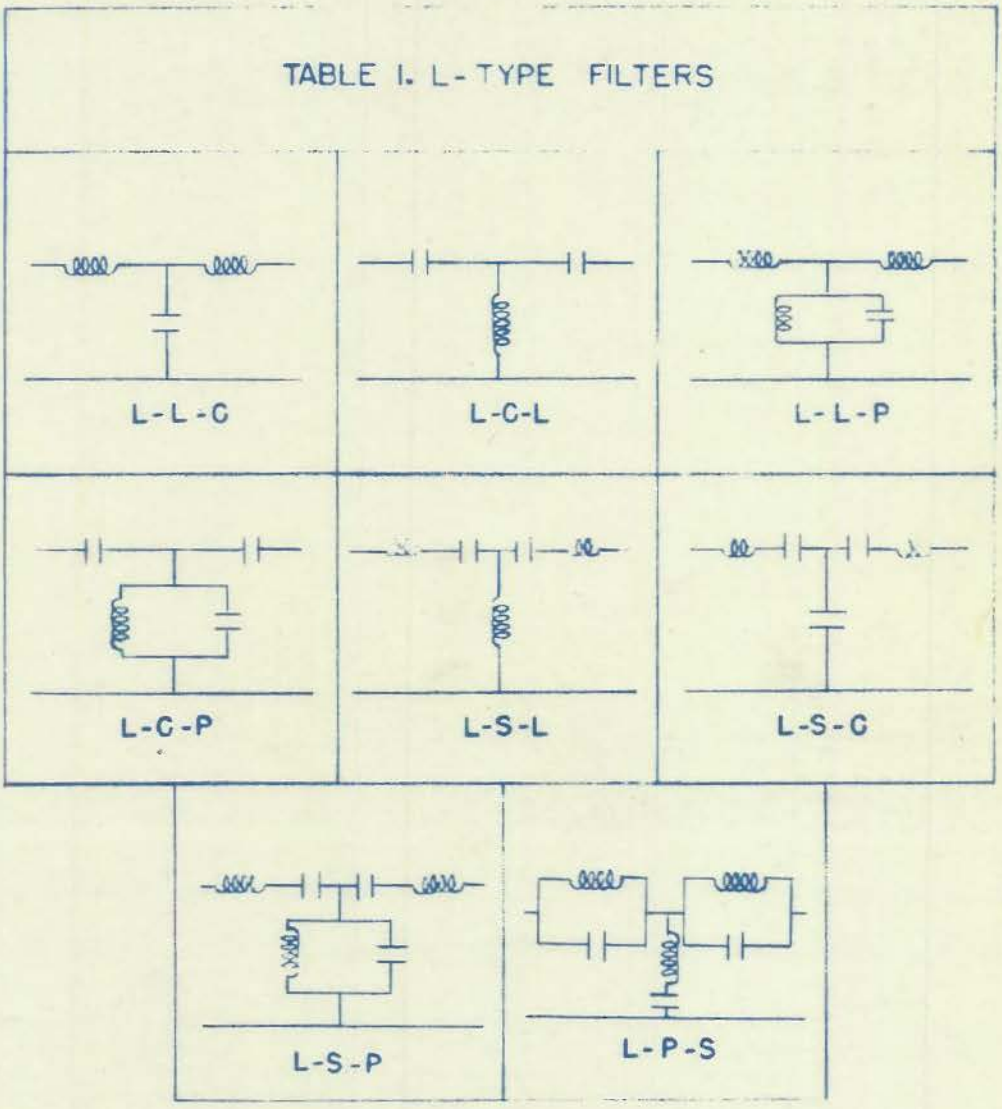


TABLE 2. Q-TYPE FILTERS

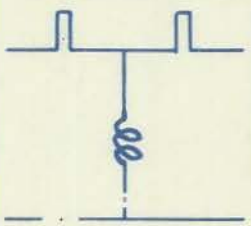
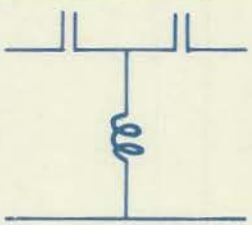
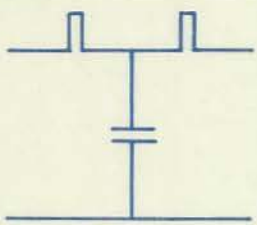
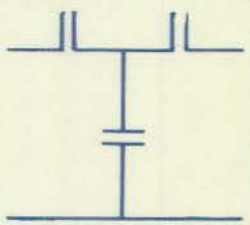
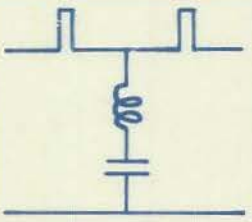
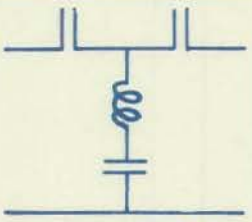
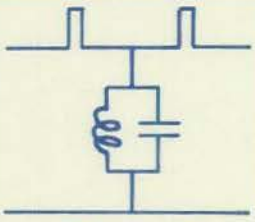
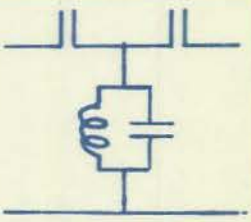
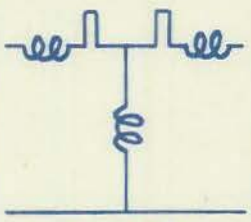
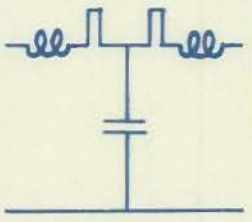
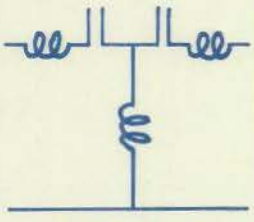
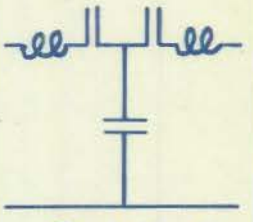
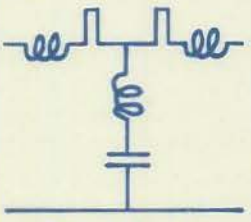
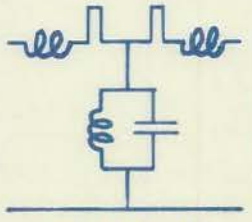
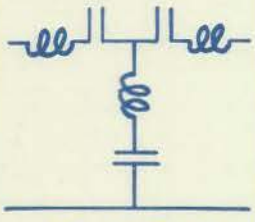
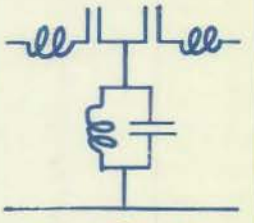
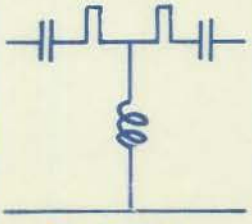
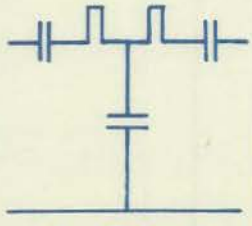
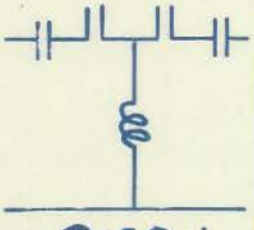
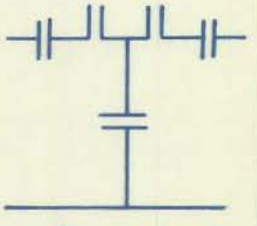
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 Q-S-S	 Q-O-S	 Q-S-P	 Q-O-P
 Q-LS-L	 Q-LS-C	 Q-LO-L	 Q-LO-C
 Q-LS-S	 Q-LS-P	 Q-LO-S	 Q-LO-P
 Q-CS-L	 Q-CS-C	 Q-CO-L	 Q-CO-C

TABLE 3. Q- TYPE FILTERS

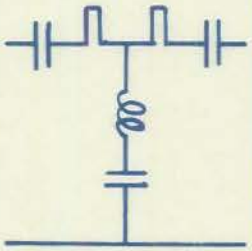
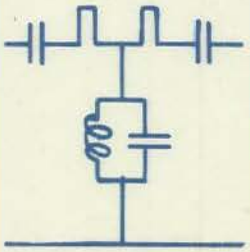
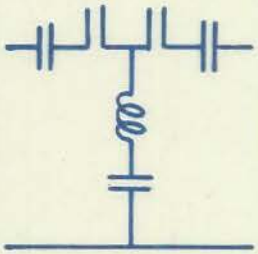
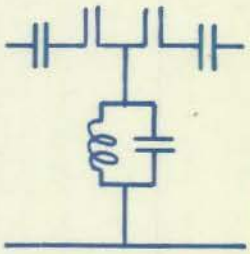
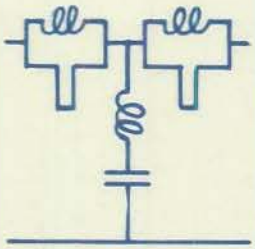
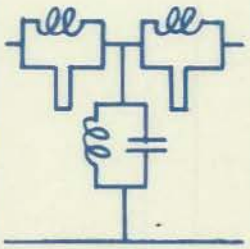
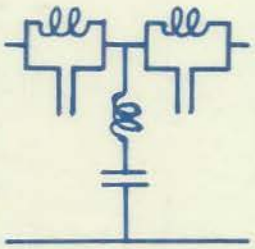
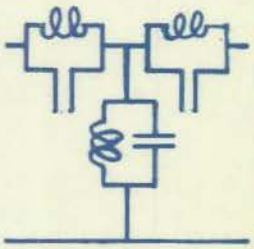
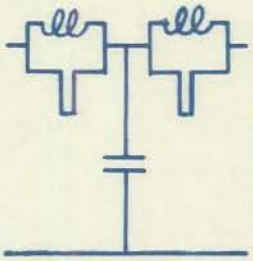
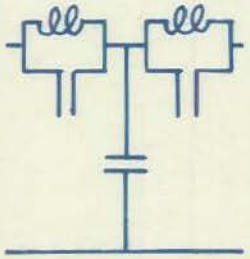
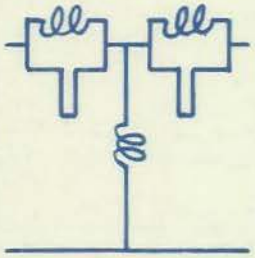
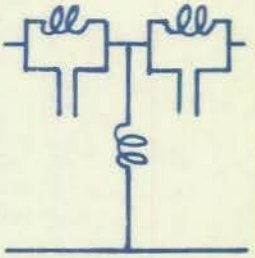
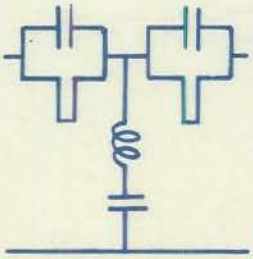
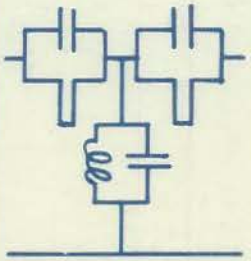
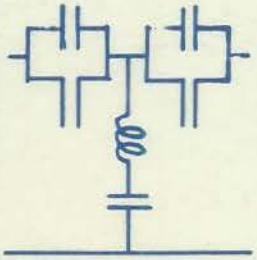
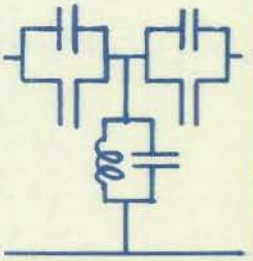
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 Q-SL-C	 Q-OL-C	 Q-SL-L	 Q-OL-L
 Q-SC-S	 Q-SC-P	 Q-OC-S	 Q-OC-P

TABLE 4. Y-TYPE FILTERS

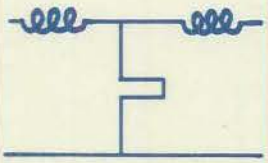
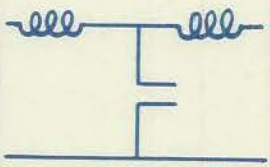
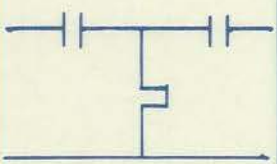
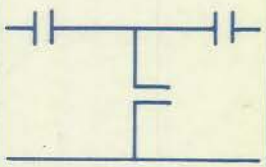
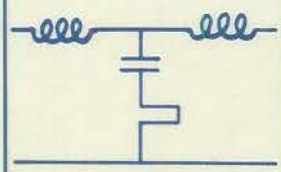
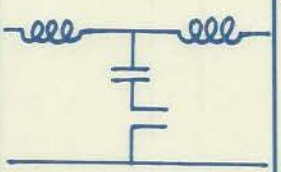
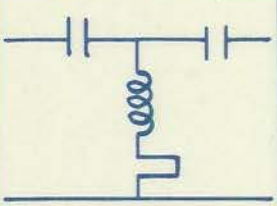
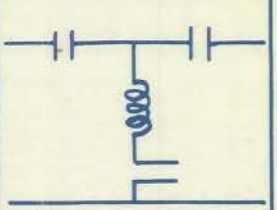
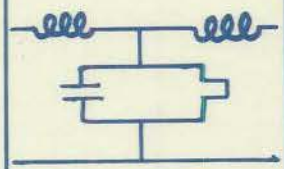
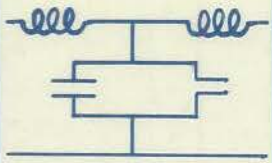
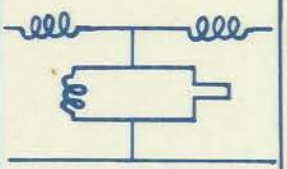
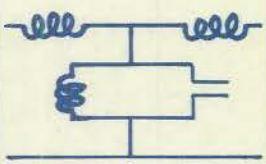
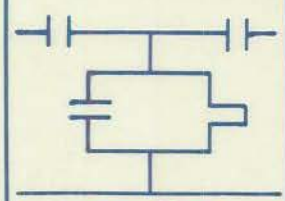
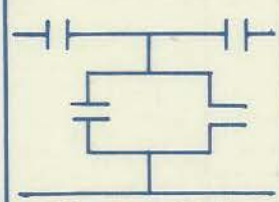
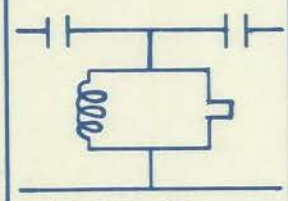
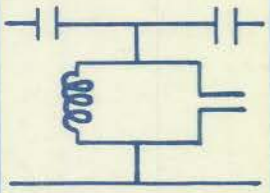
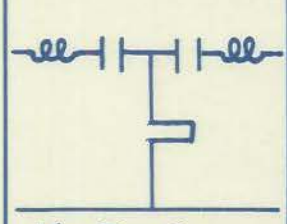
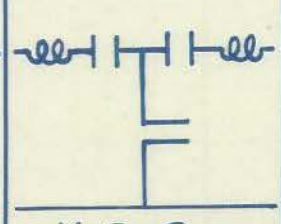
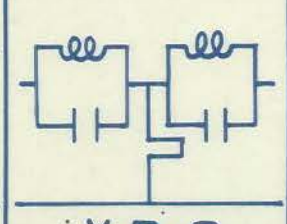
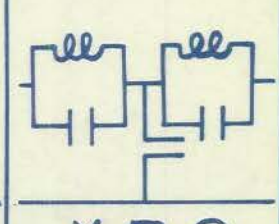
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 Y-L-CS	 Y-L-CO	 Y-C-LS	 Y-C-LO
 Y-L-SC	 Y-L-OC	 Y-L-SL	 Y-L-OL
 Y-C-SC	 Y-C-OC	 Y-C-SL	 Y-C-OL
 Y-S-S	 Y-S-O	 Y-P-S	 Y-P-O

TABLE 5. Y- TYPE FILTERS

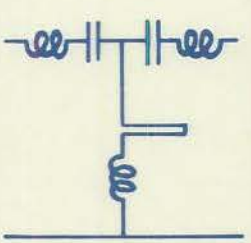
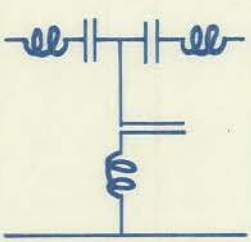
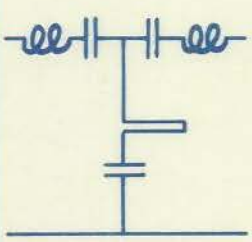
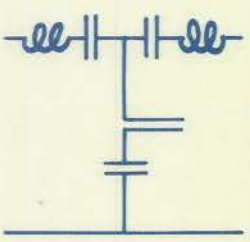
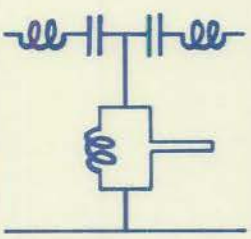
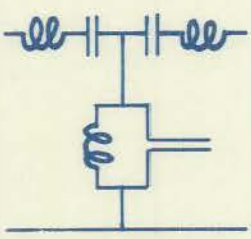
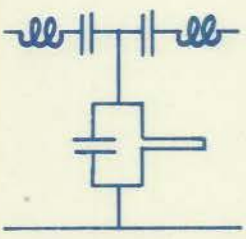
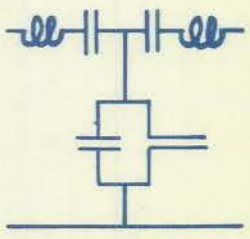
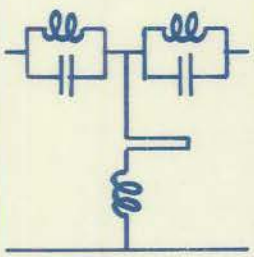
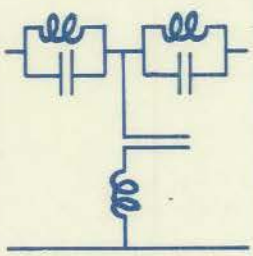
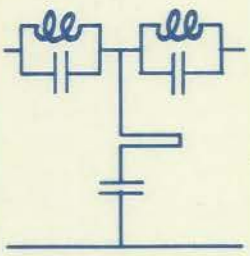
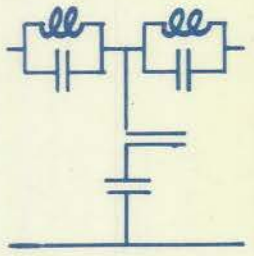
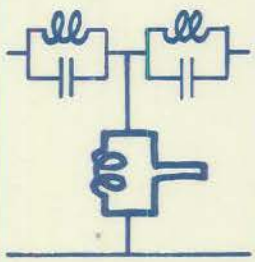
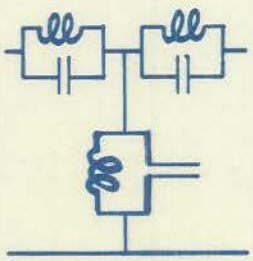
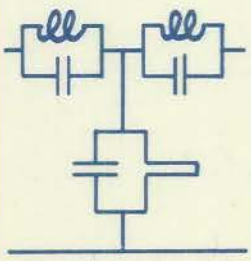
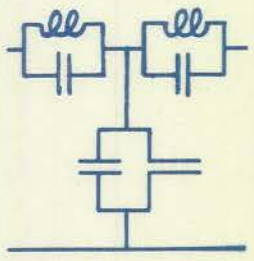
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 Y-S-SL	 Y-S-OL	 Y-S-SC	 Y-S-OC
 Y-P-LS	 Y-P-LO	 Y-P-CS	 Y-P-CO
 Y-P-SL	 Y-P-OL	 Y-P-SC	 Y-P-OC

TABLE 6. T-TYPE FILTERS

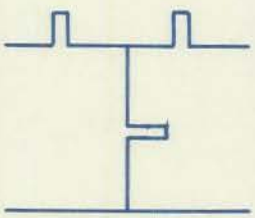
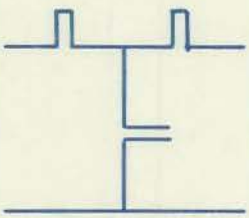
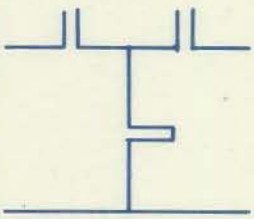
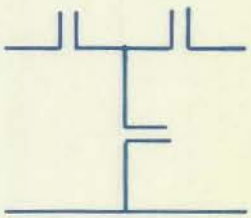
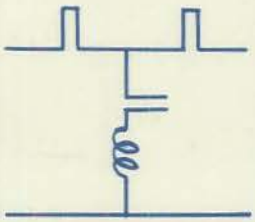
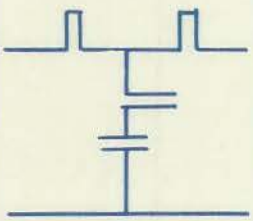
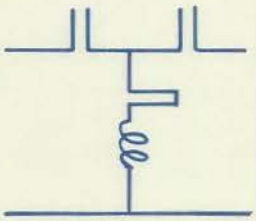
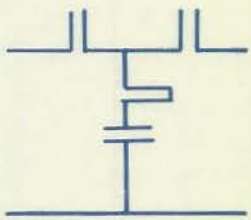
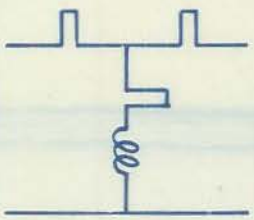
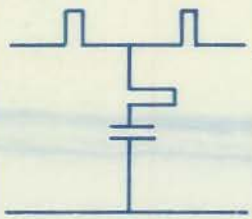
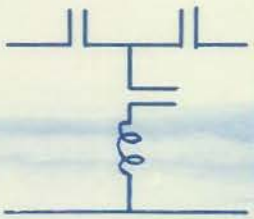
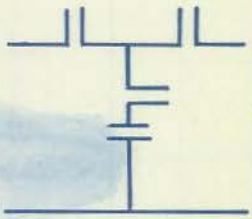
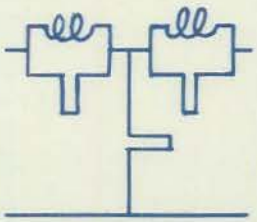
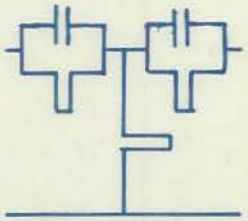
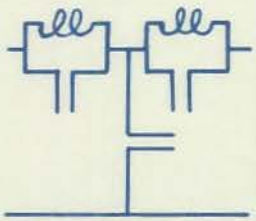
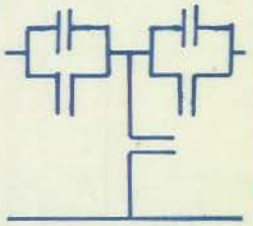
 T-S-S	 T-S-O	 T-O-S	 T-O-O
 T-S-LO	 T-S-CO	 T-O-LS	 T-O-CS
 T-S-LS (POT. TM-DER. Q-S-L)	 T-S-CS (POT. TM-DER. Q-S-C)	 T-O-LO (POT. TM-DER. Q-O-L)	 T-O-CO (POT. TM. DER. Q-O-C)
 T-SL-S (POT. π M-DER. Y-L-S)	 T-SC-S (POT. π M-DER. Y-C-S)	 T-OL-O (POT. π M-DER. Y-L-O)	 T-OC-O (POT. π M-DER. Y-C-O)

TABLE 7. T-TYPE FILTERS

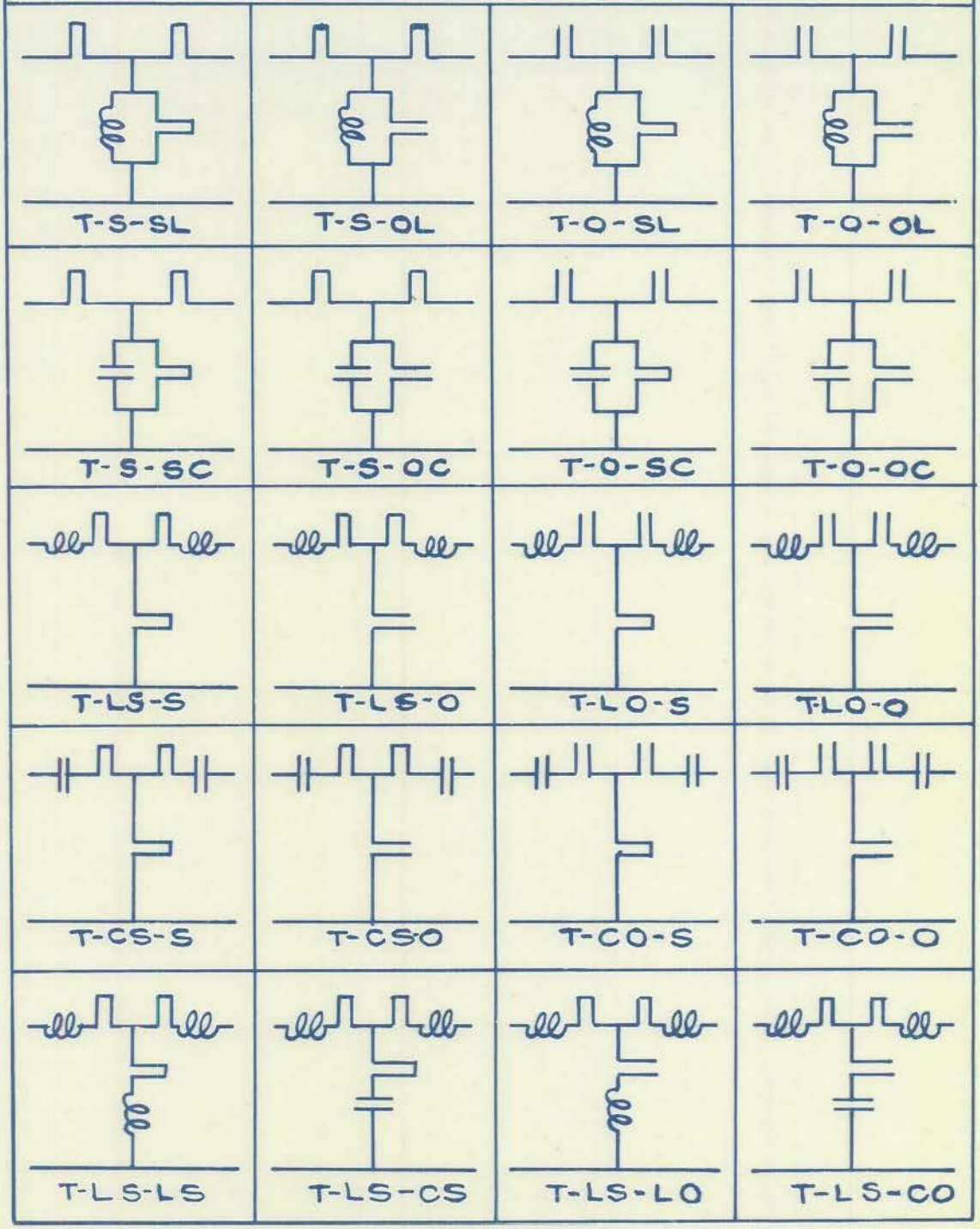


TABLE 8. T-TYPE FILTERS

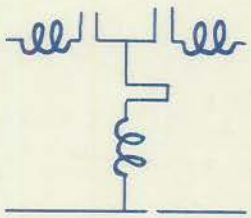
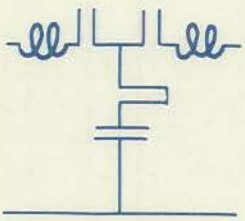
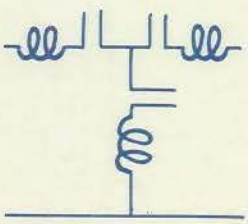
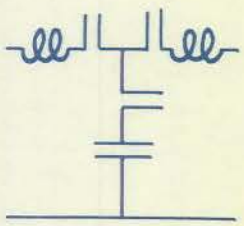
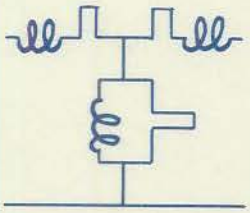
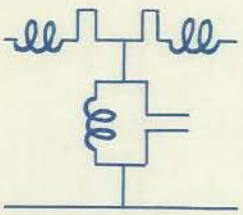
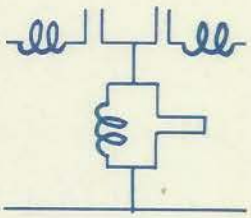
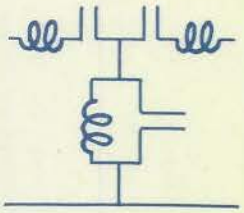
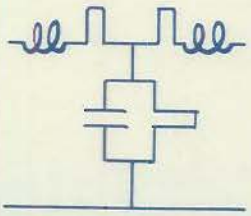
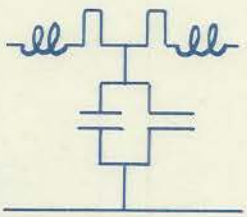
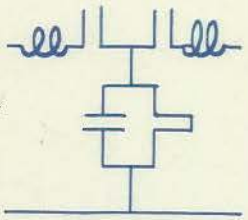
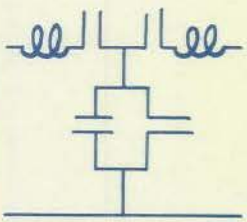
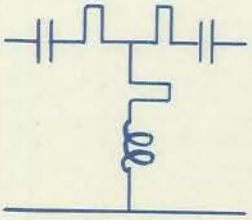
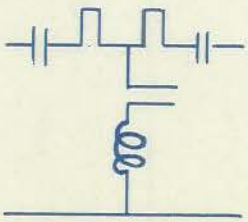
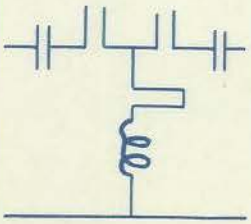
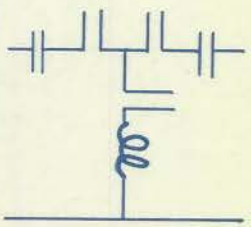
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 T-LS-SL	 T-LS-OL	 T-LO-SL	 T-LO-OL
 T-LS-SC	 T-LS-OC	 T-LO-SC	 T-LO-OC
 T-CS-LS	 T-CS-LO	 T-CO-LS	 T-CO-LO

TABLE 9. T-TYPE FILTERS

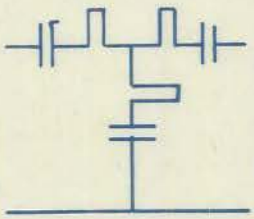
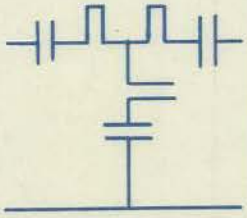
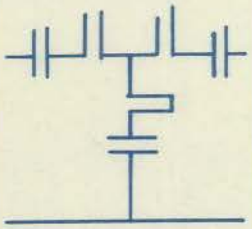
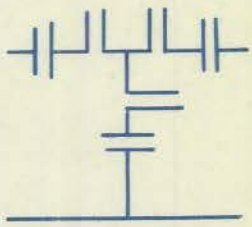
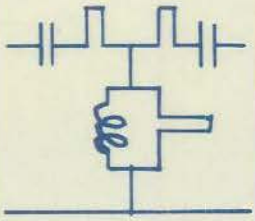
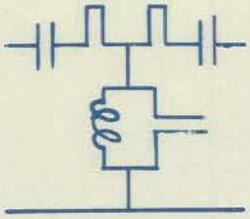
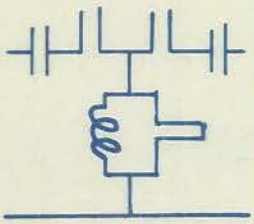
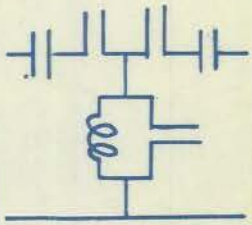
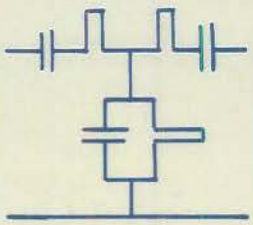
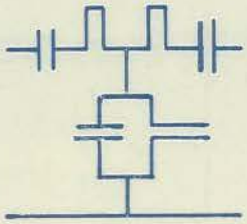
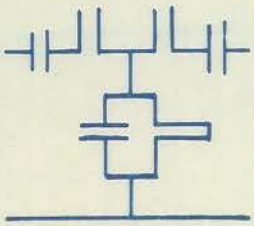
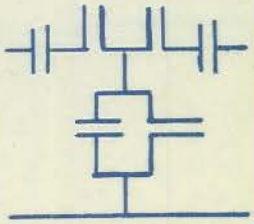
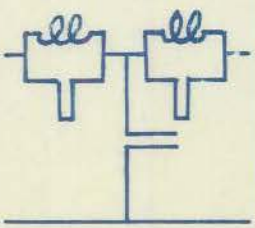
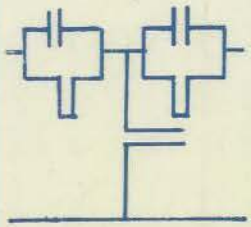
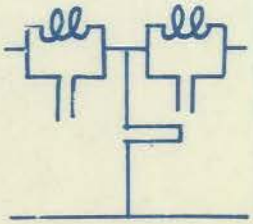
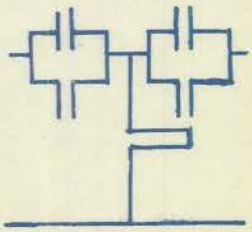
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 T-CS-SL	 T-CS-OL	 T-CO-SL	 T-CO-OL
 T-CS-SC	 T-CS-OC	 T-CO-SC	 T-CO-OC
 T-SL-O	 T-SC-O	 T-OL-S	 T-OC-S

TABLE 10. T-TYPE FILTERS

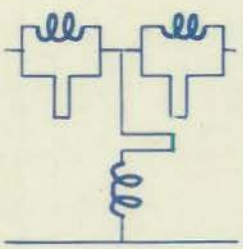
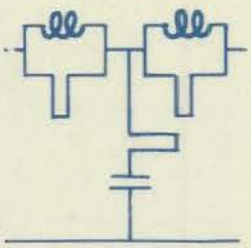
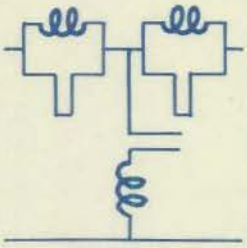
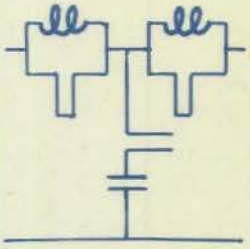
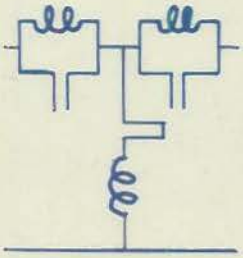
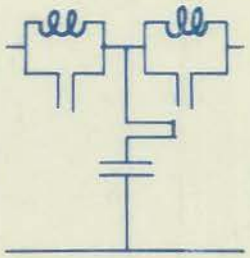
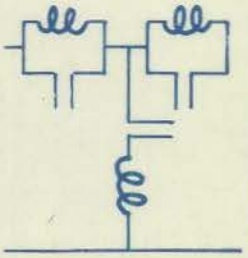
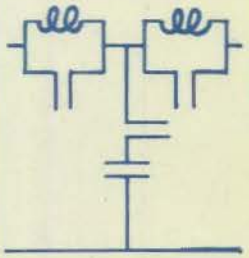
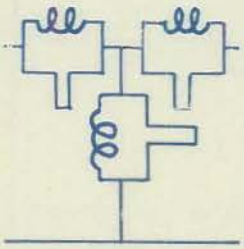
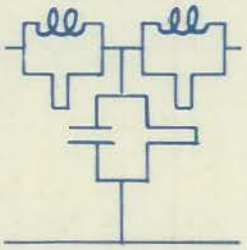
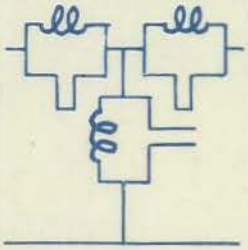
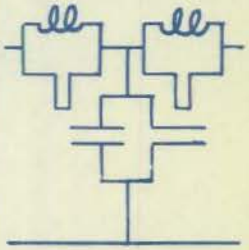
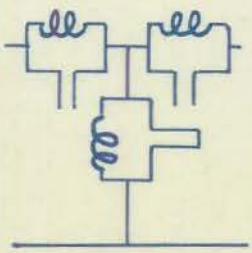
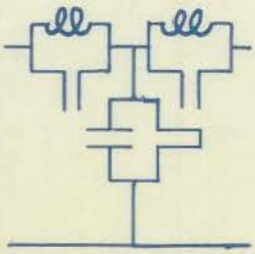
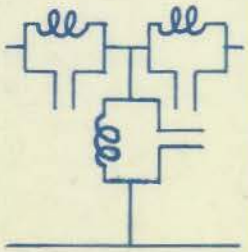
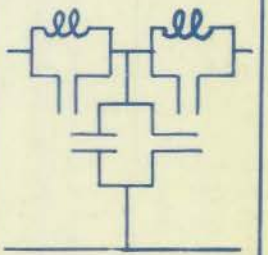
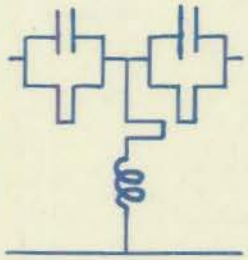
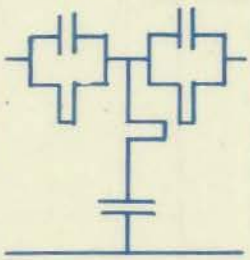
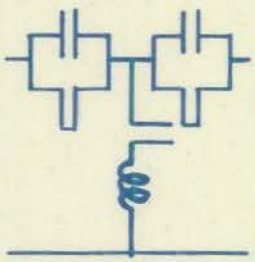
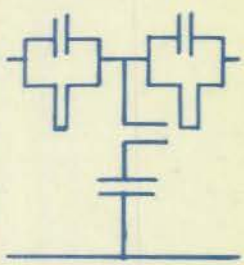
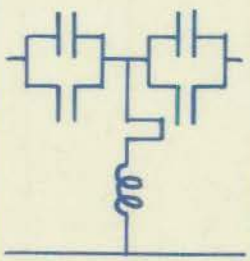
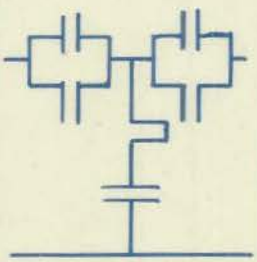
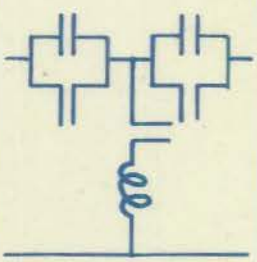
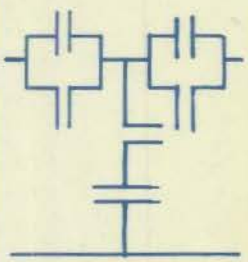
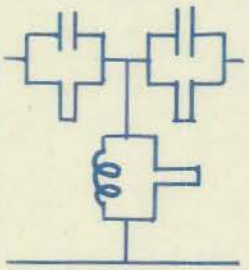
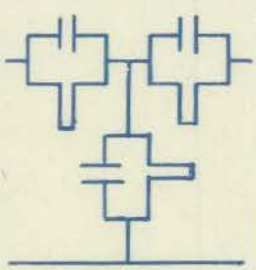
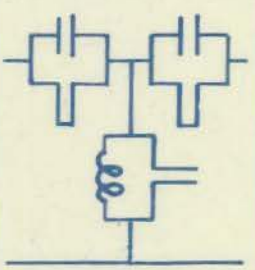
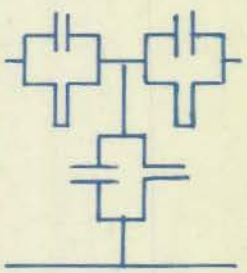
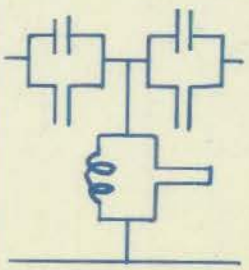
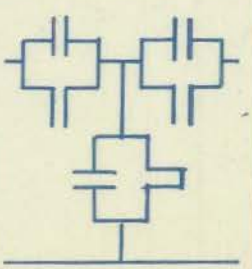
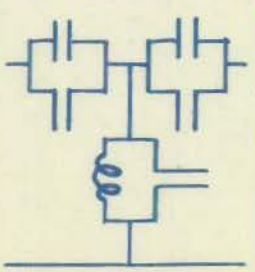
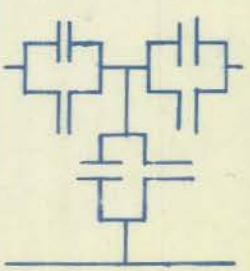
 T-SL-LS	 T-SL-CS	 T-SL-LO	 T-SL-CO
 T-OL-LS	 T-OL-CS	 T-OL-LO	 T-OL-CO
 T-SL-SL	 T-SL-SC	 T-SL-OL	 T-SL-OC
 T-OL-SL	 T-OL-SC	 T-OL-OL	 T-OL-OC

TABLE II T-TYPE FILTERS

 T-SC-LS	 T-SC-CS	 T-SC-LO	 T-SC-CO
 T-OC-LS	 T-OC-CS	 T-OC-LO	 T-OC-CO
 T-SC-SL	 T-SC-SC	 T-SC-OL	 T-SC-OC
 T-OC-SL	 T-OC-SC	 T-OC-OL	 T-OC-OC

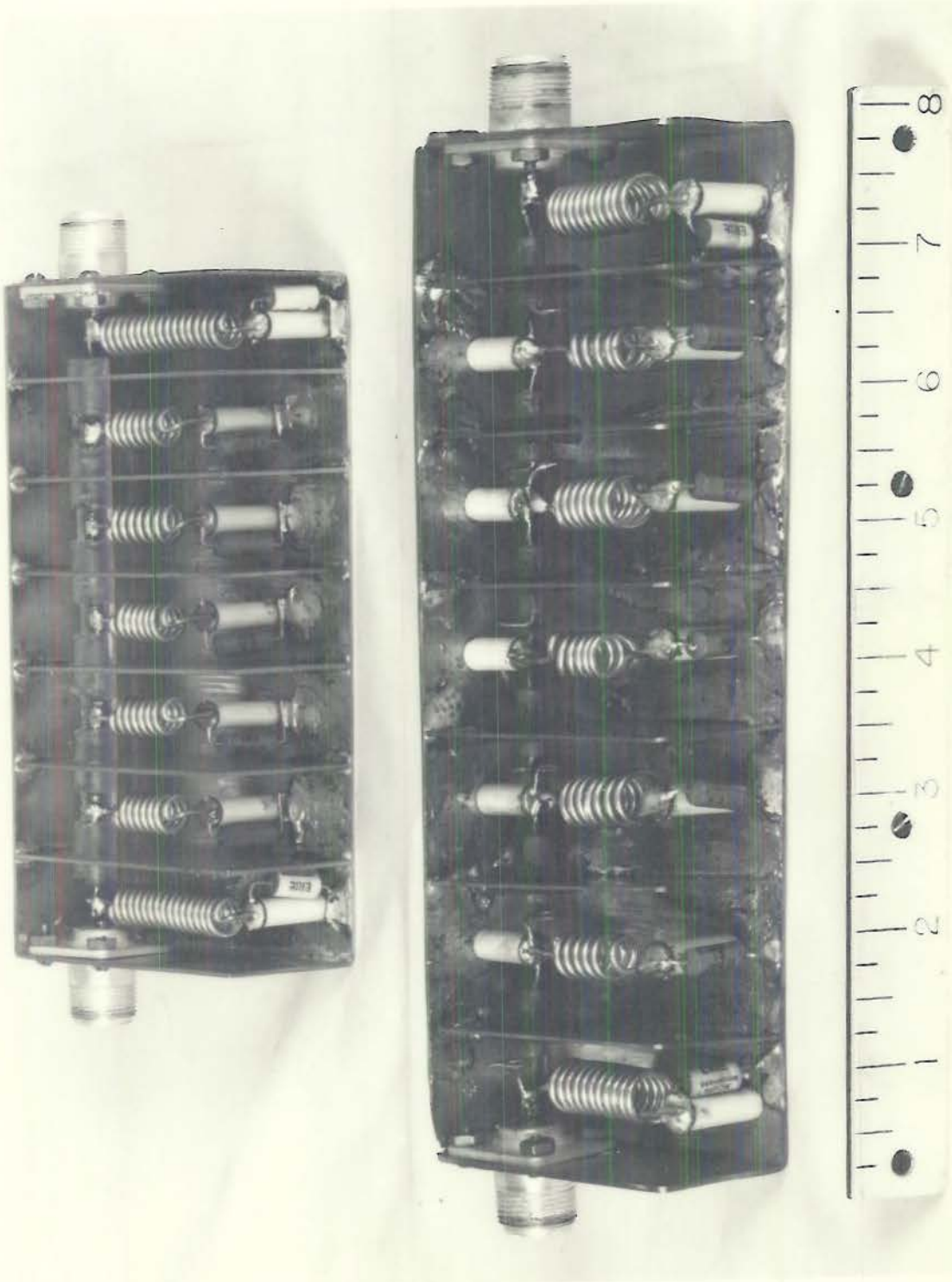


PLATE I-1 SECTION I

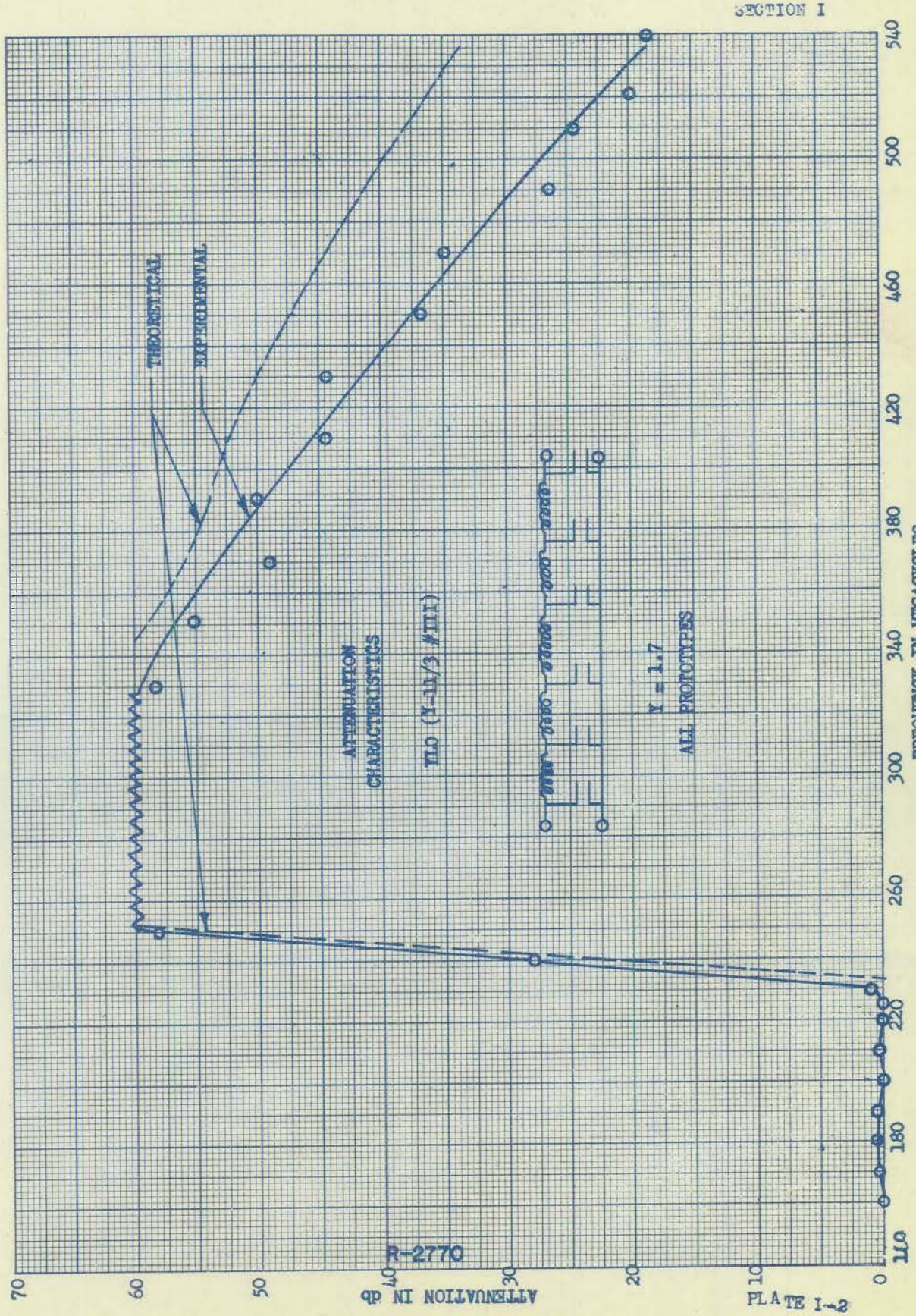
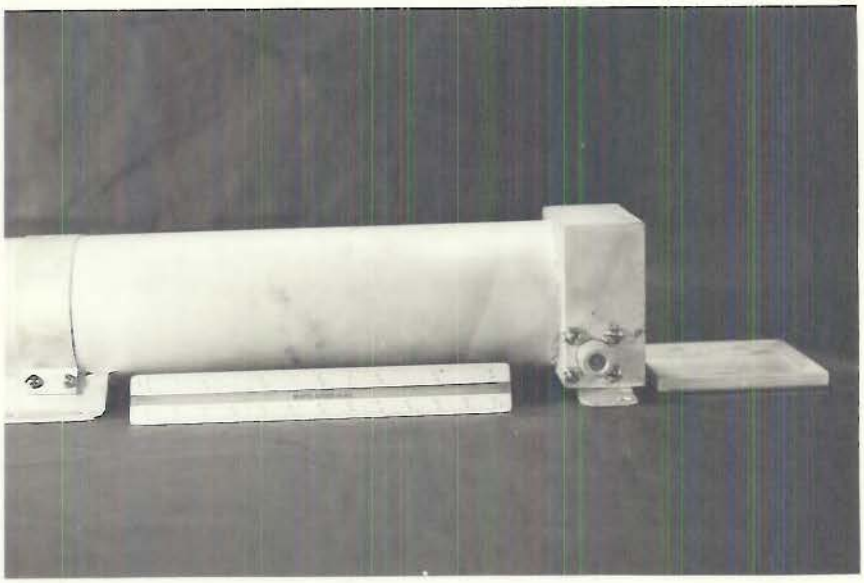
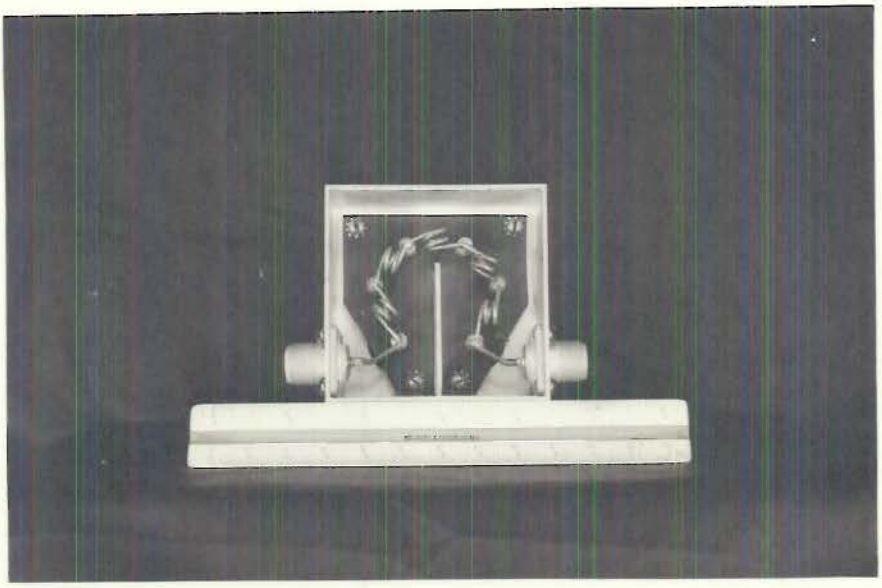
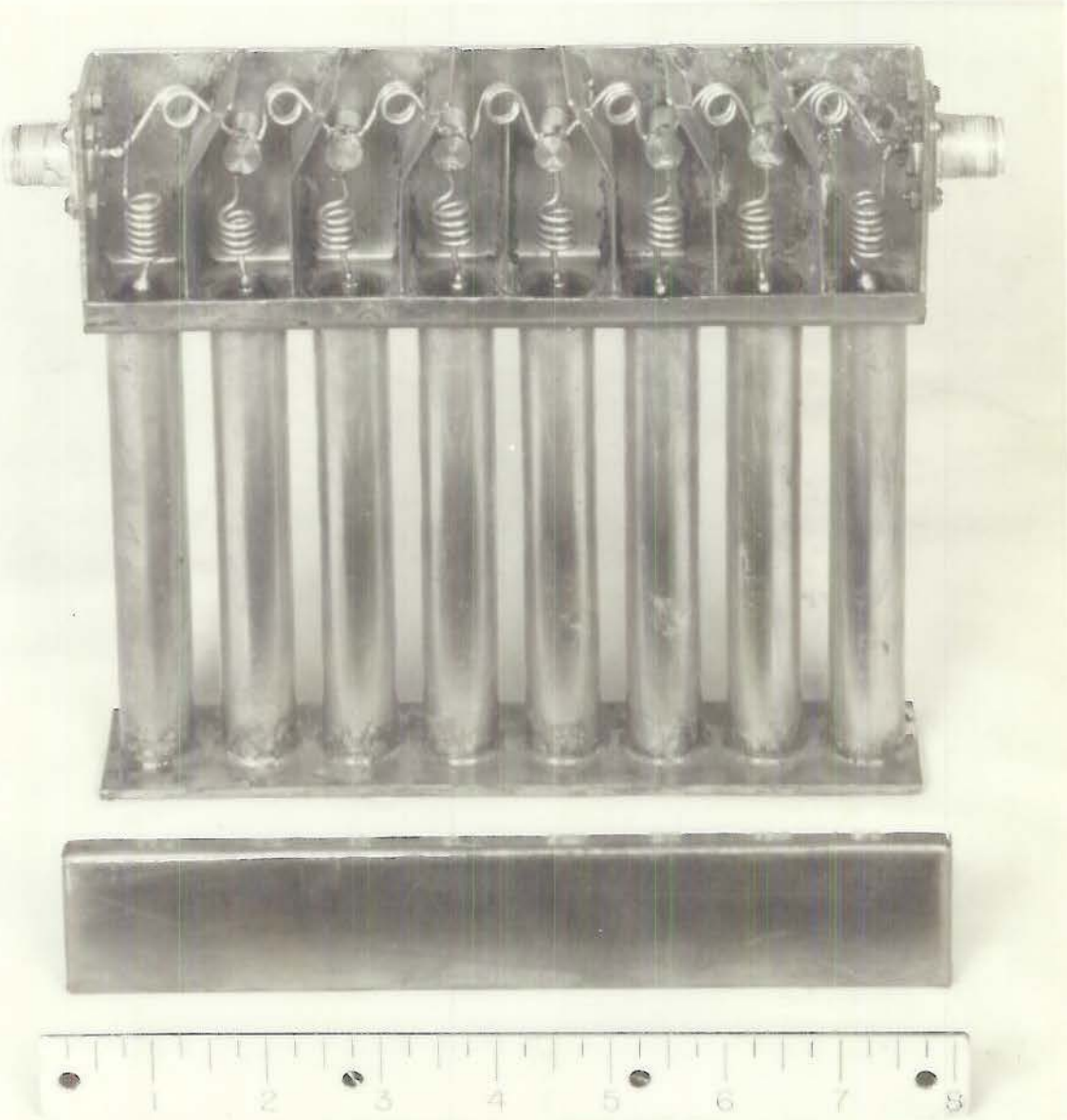

THEORETICAL
EXPERIMENTAL
ATTENUATION CHARACTERISTICS
TLD (T-11/3 #111)
X = 1.7
ALL PROTOTYPES
ATTENUATION IN db
FREQUENCY IN MEGACYCLES
70
60
50
40
30
20
10
0
110
180
220
260
300
340
380
420
460
500
540

PLATE I-2





Y-L-O FILTER NETWORK (YL-11)



YL-10

PLATE I-4 SECTION I

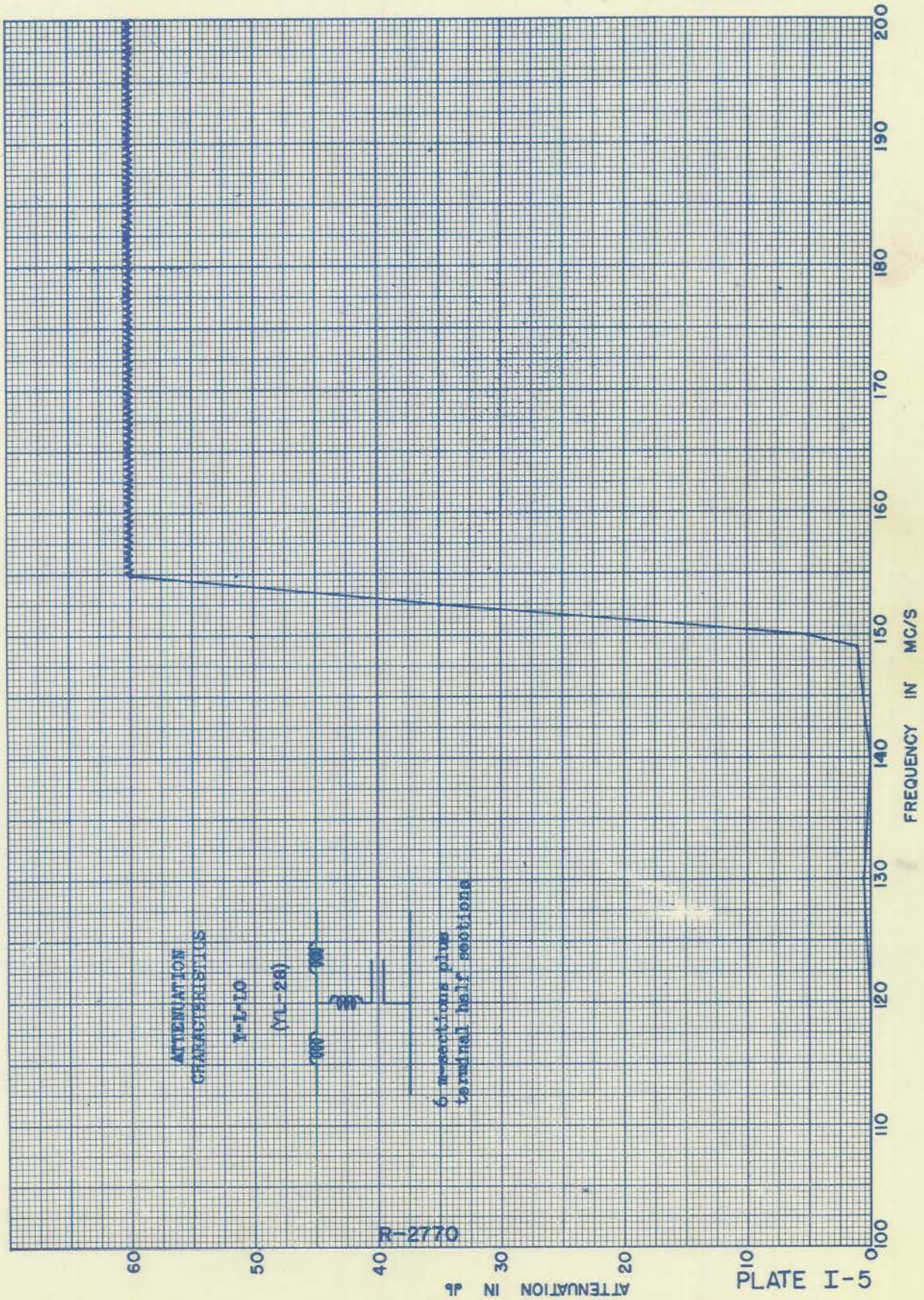
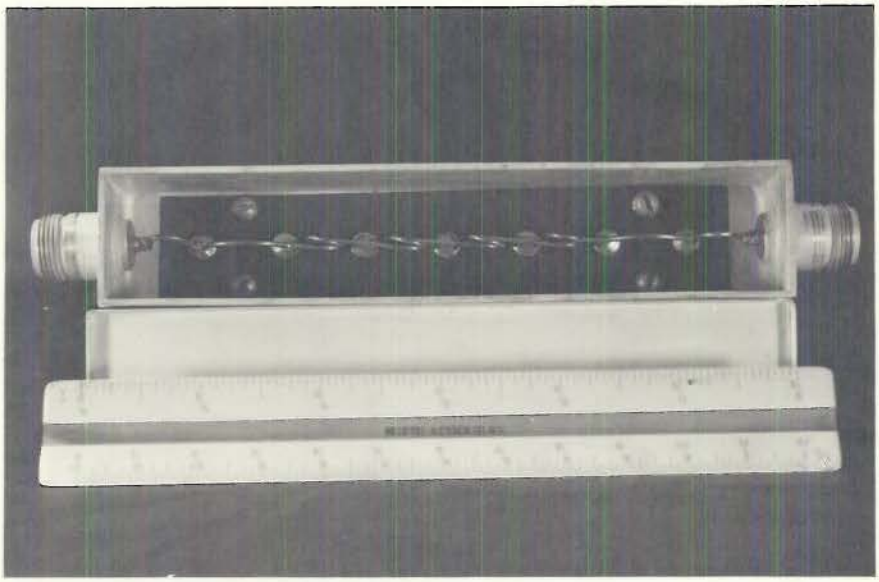
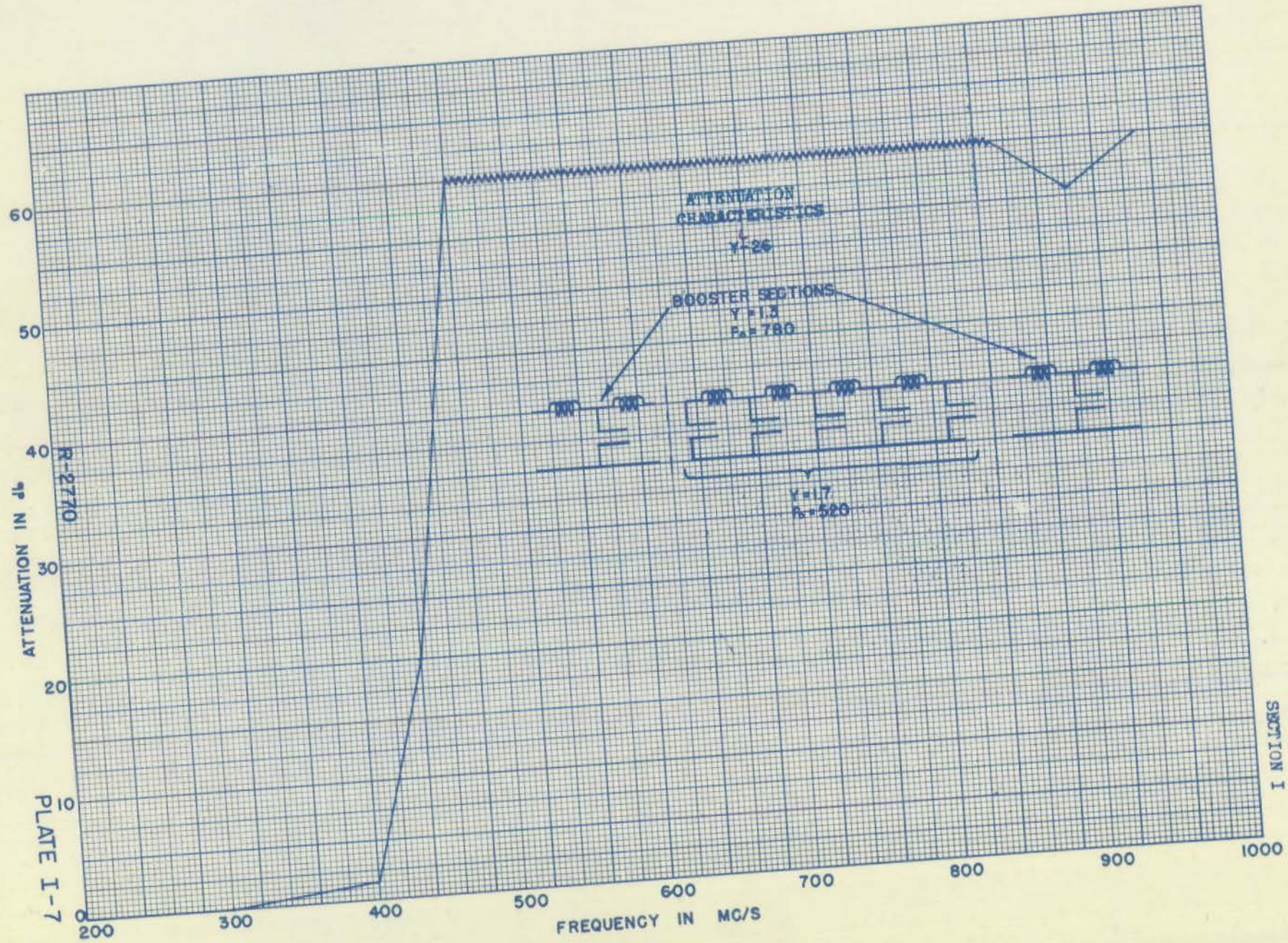

ATTENUATION CHARACTERISTICS
M-L-10
(XL-28)
6 m-sections plus
terminal half sections
R-2770
ATTENUATION IN db
0
10
20
30
40
50
60
FREQUENCY IN MC/S
100
110
120
130
140
150
160
170
180
190
200

PLATE I-5





Y-L-O FILTER NETWORK (YL-26)



R-2270

PLATE I-7

SECTION I

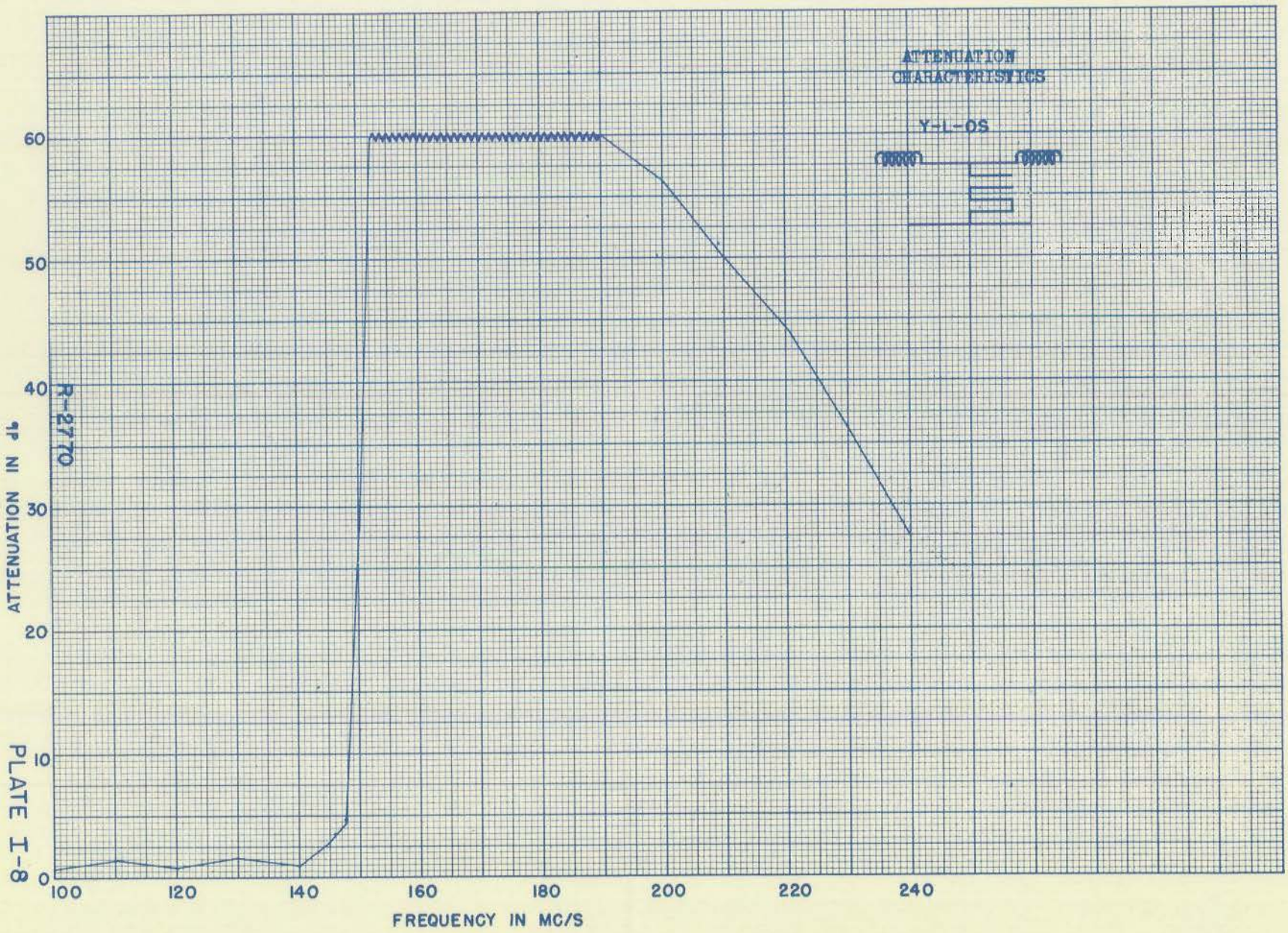
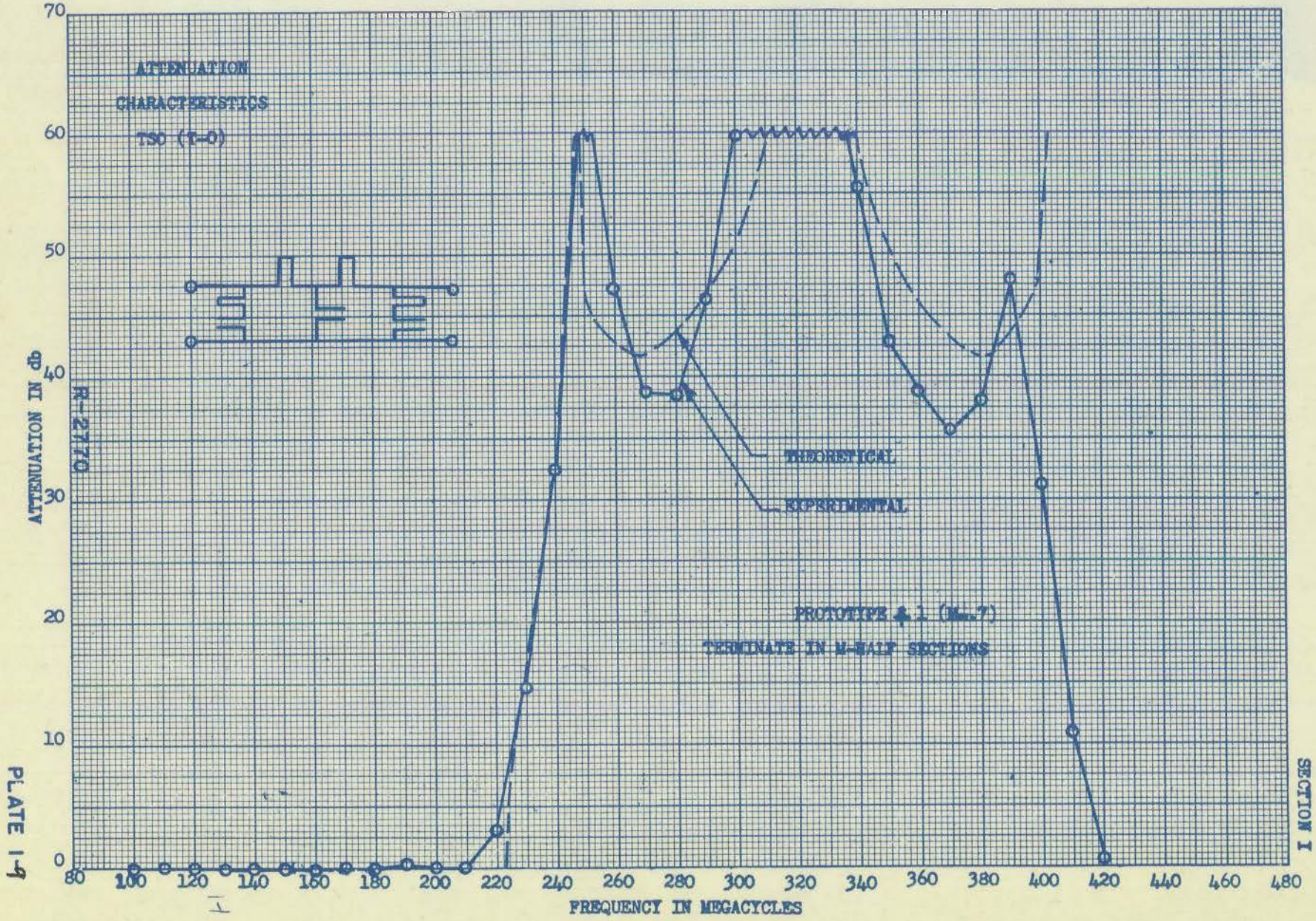


PLATE I-8

SECTION 1



R-2770

PLATE I-4

SECTION I

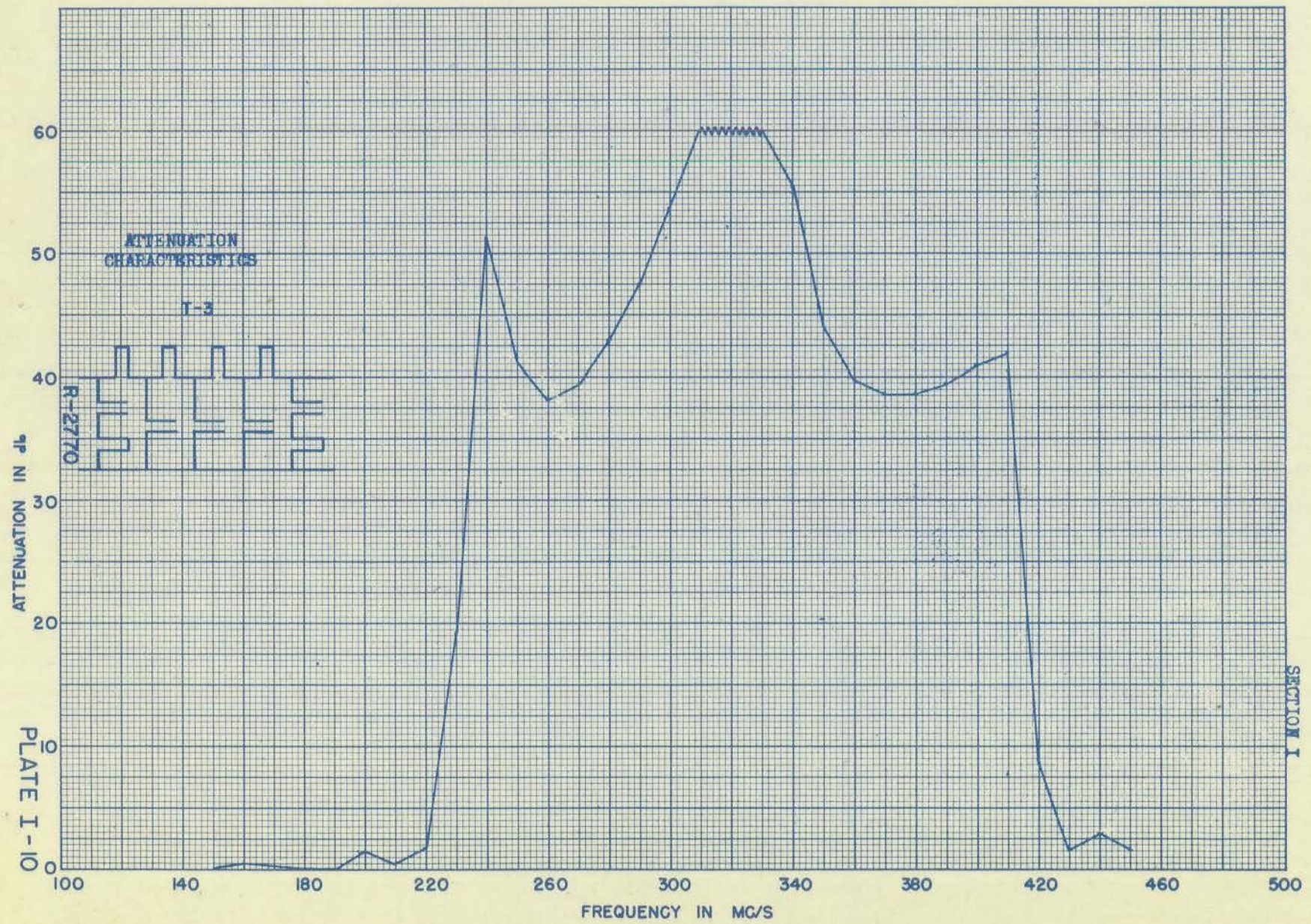


PLATE I-10

SECTION I

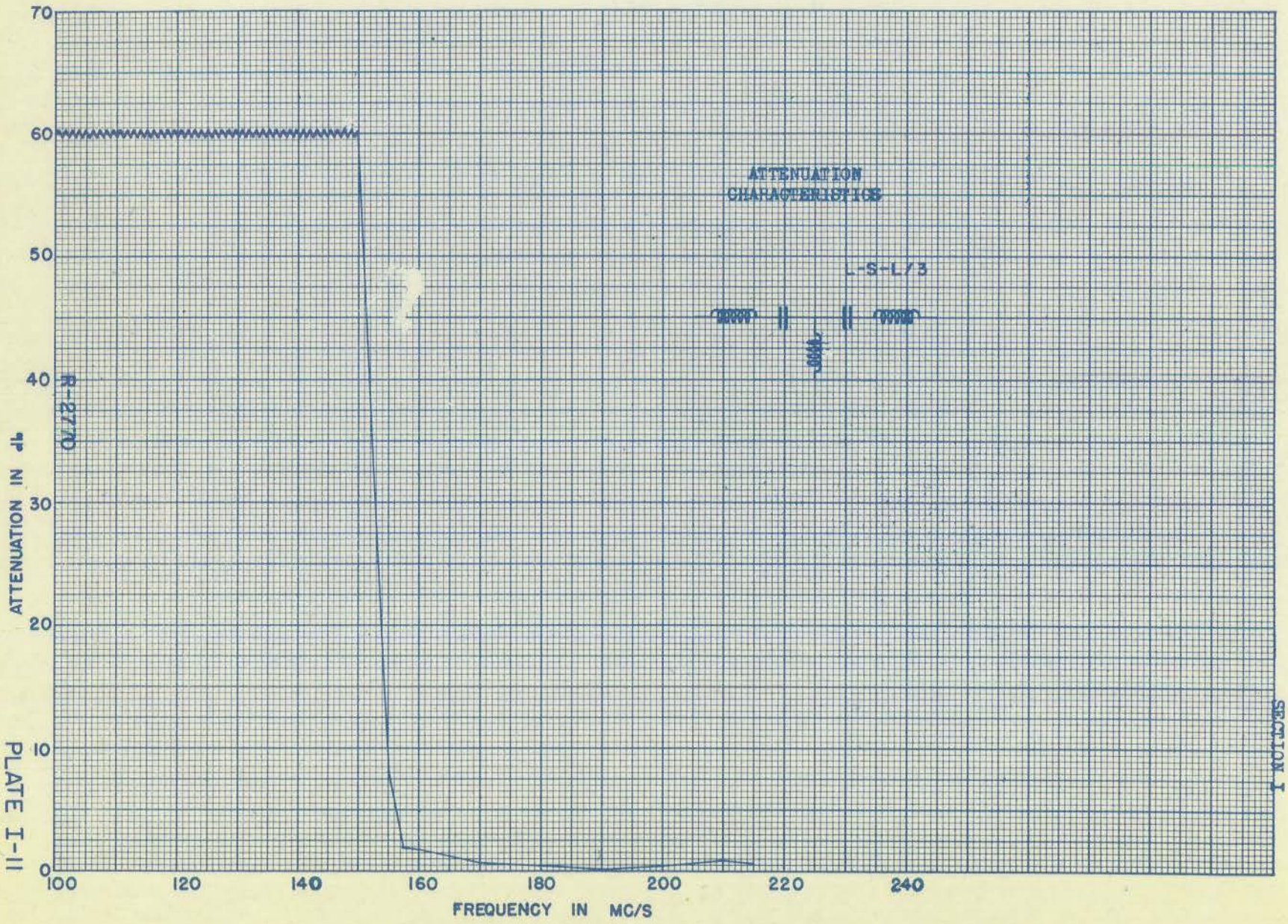


PLATE I-11

SECTION I

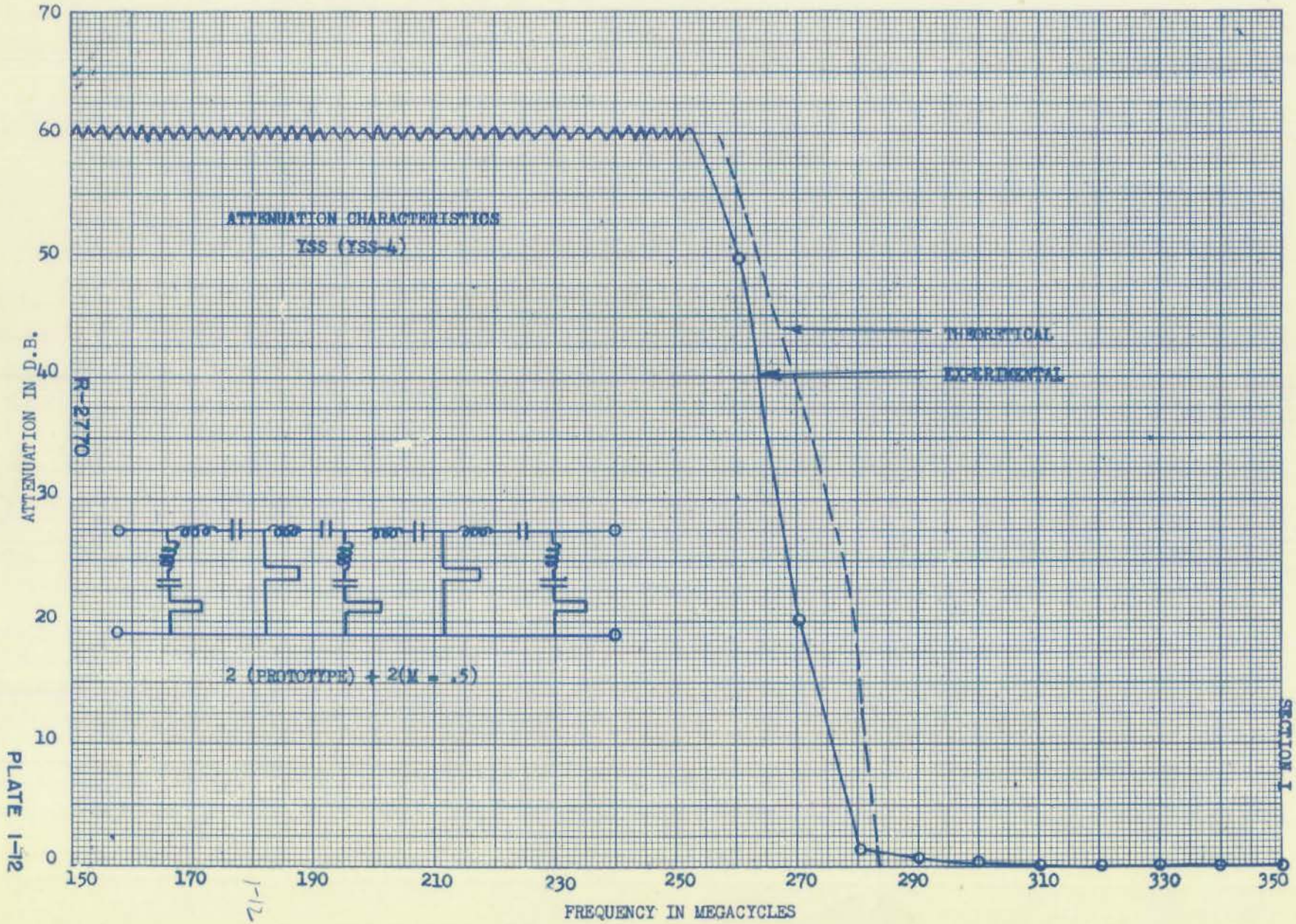


PLATE I-12

SECTION I



PLATE I-13

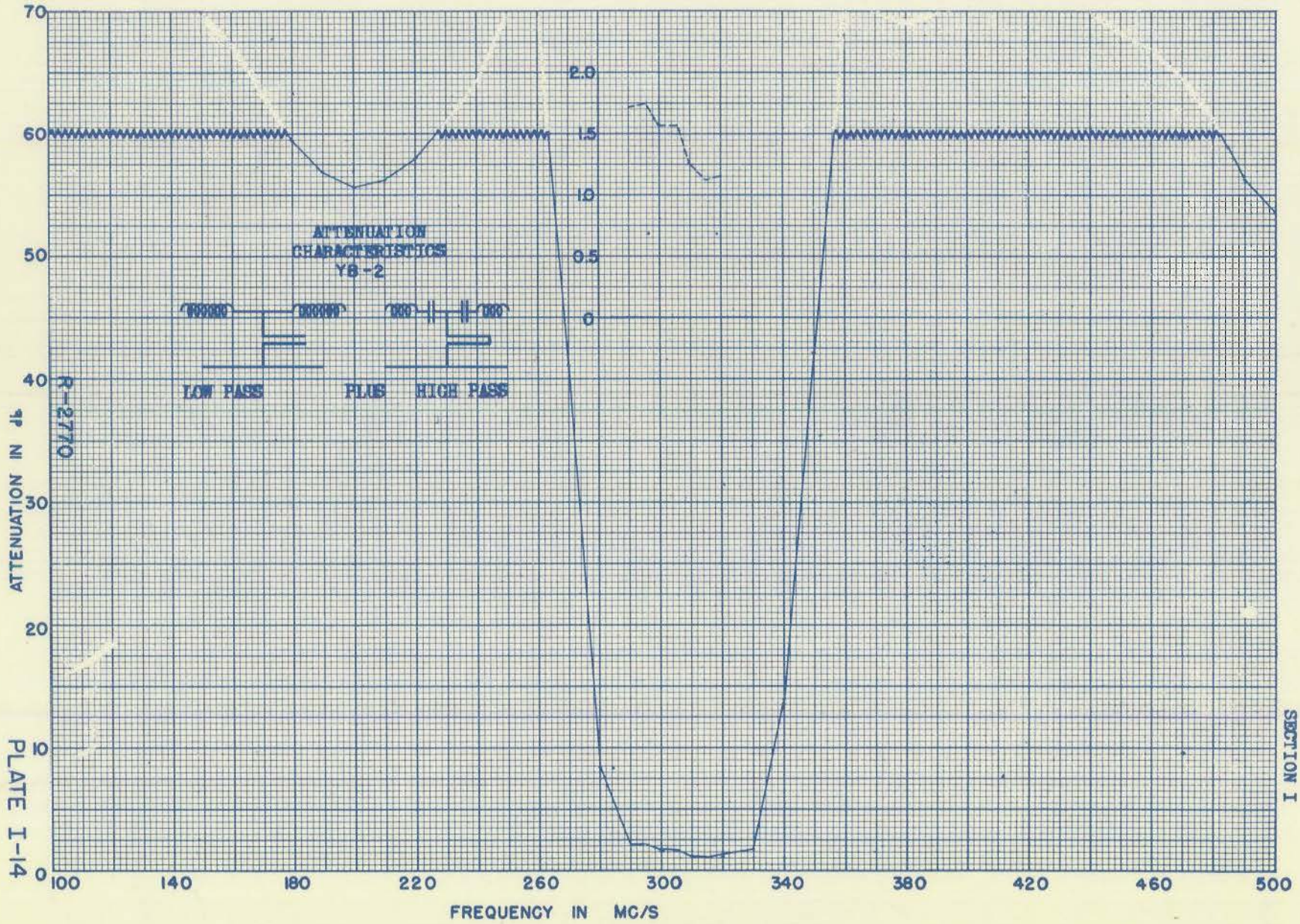


PLATE I-14

SECTION I

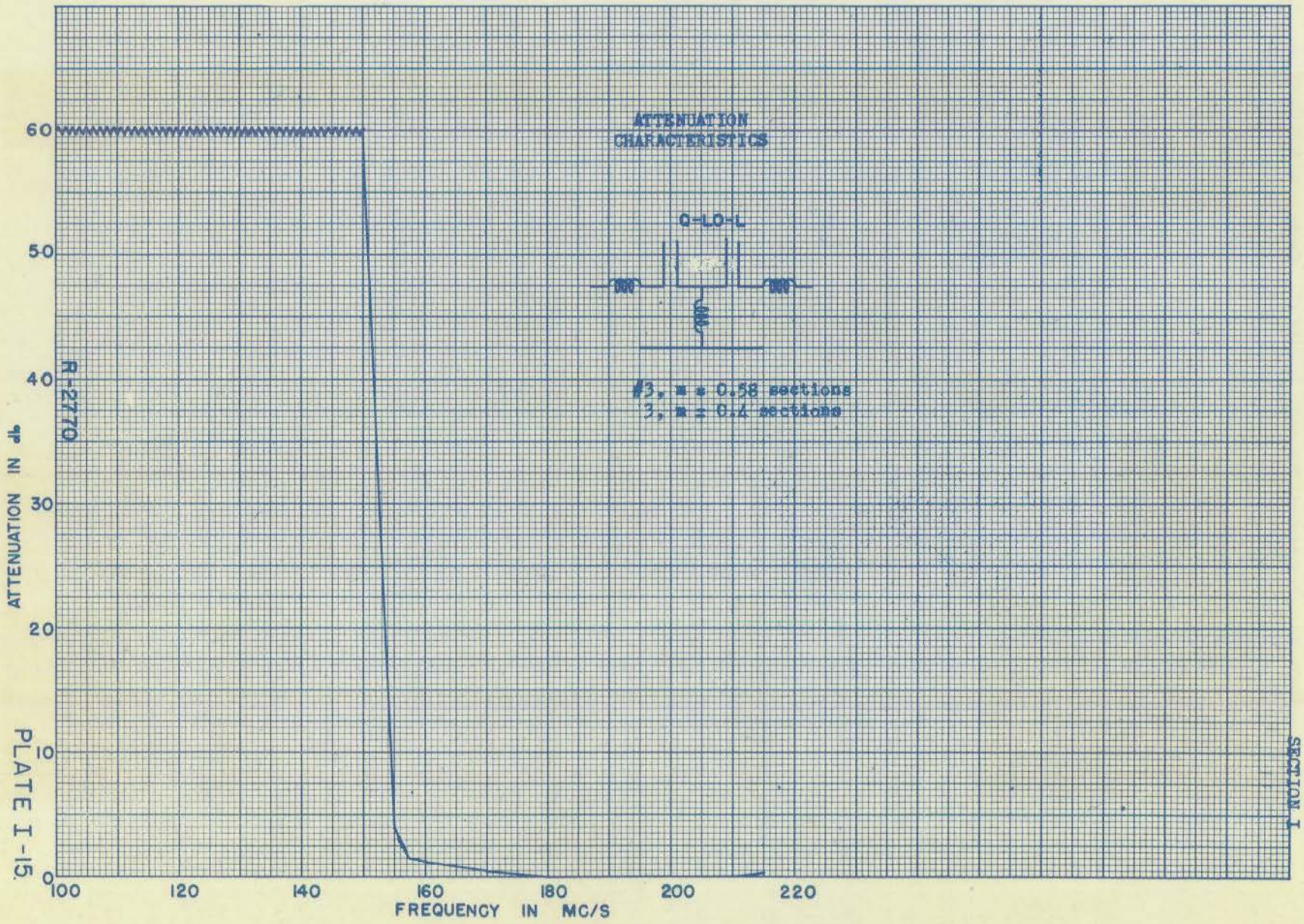


PLATE I-15.

SECTION I

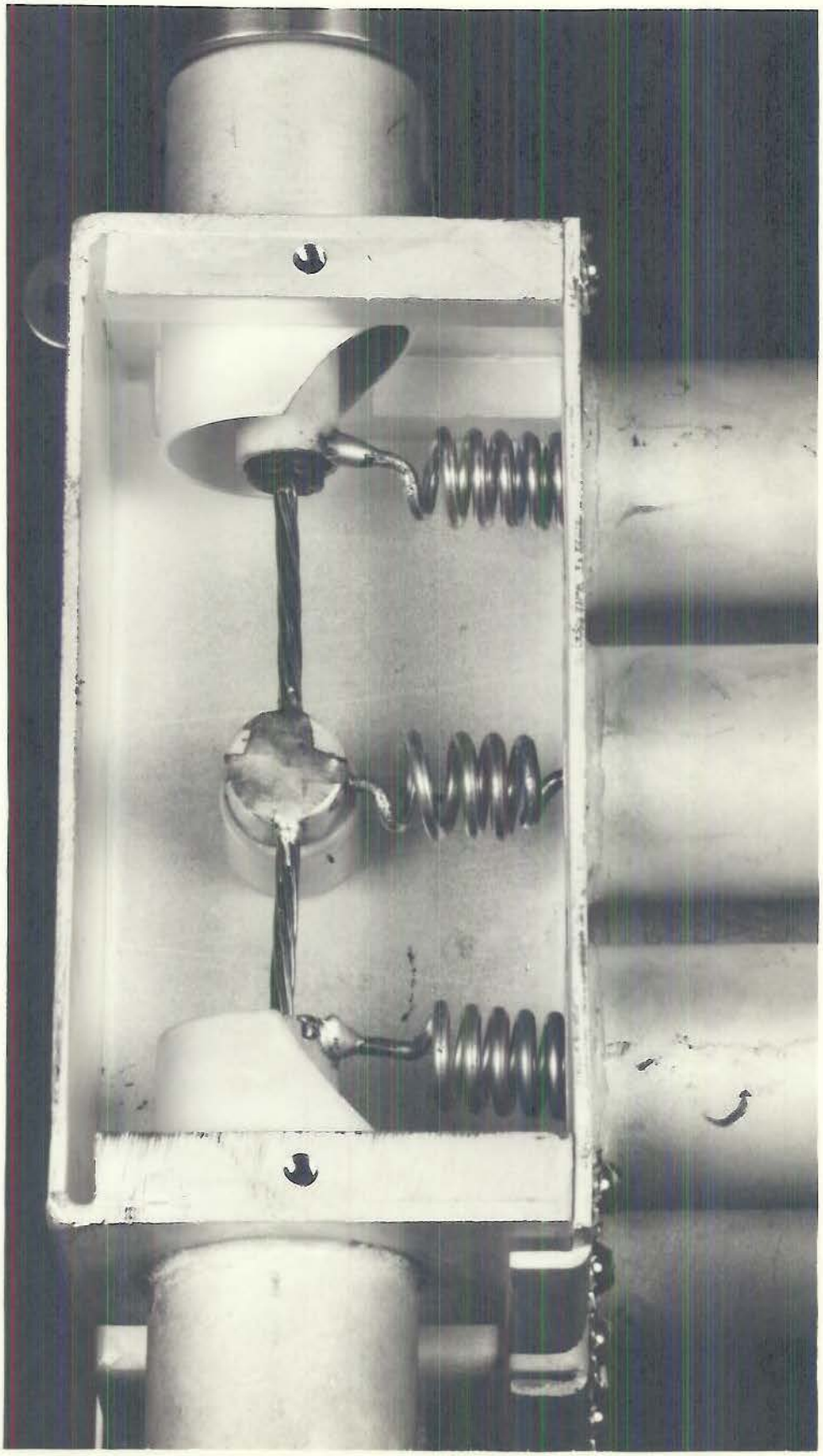


PLATE I-16a

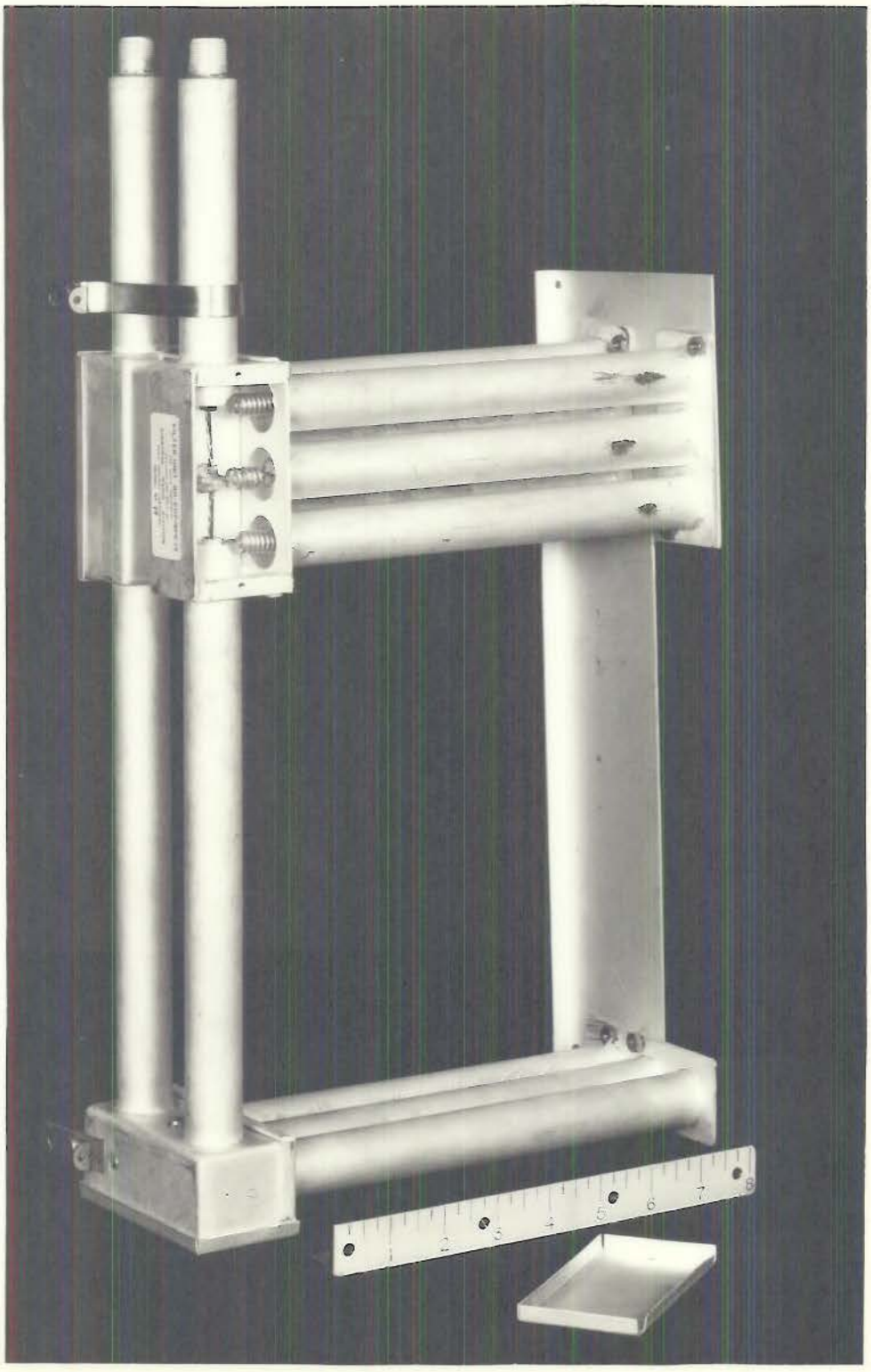


PLATE I-16b SECTION I

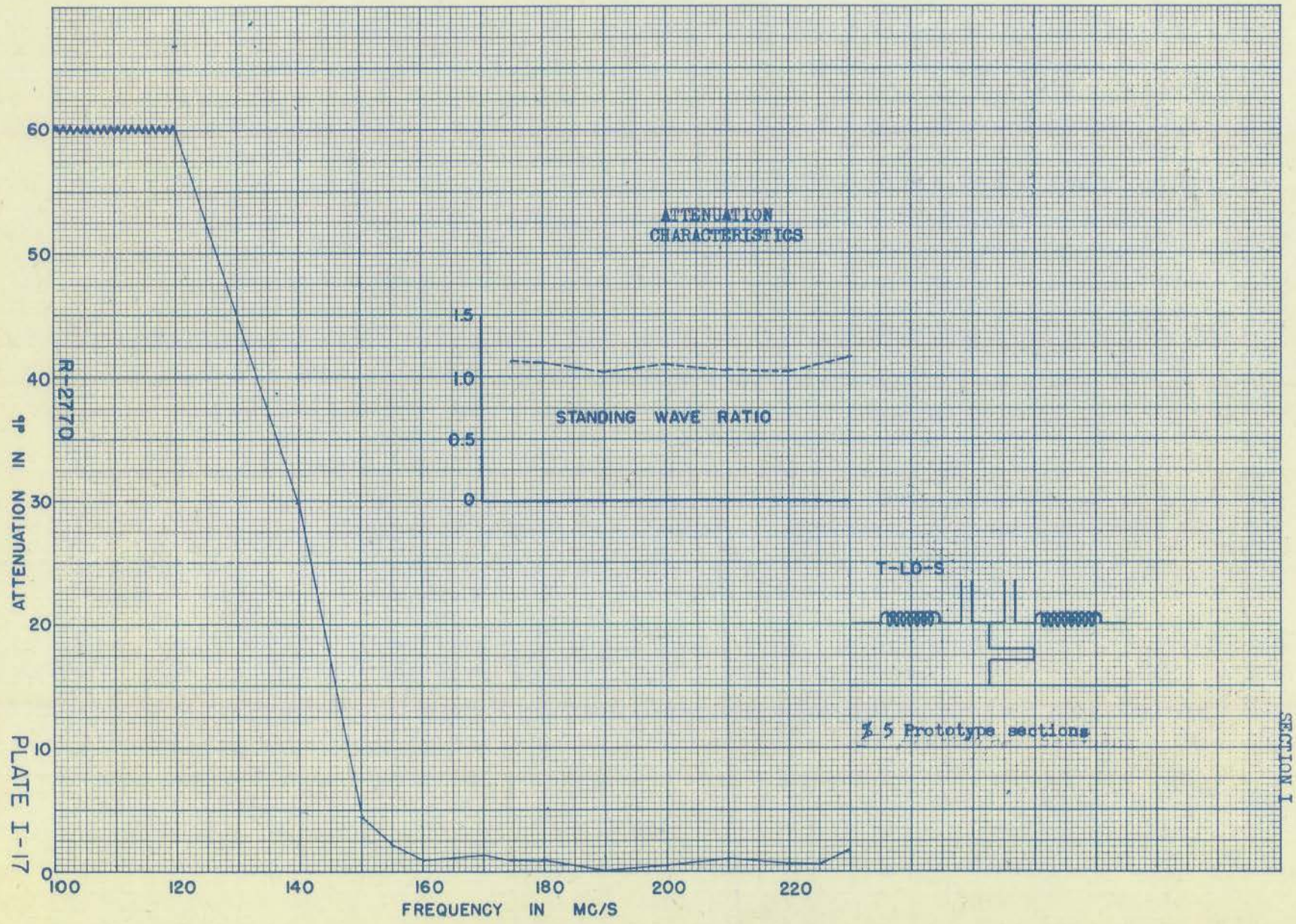


PLATE I-17

SECTION I

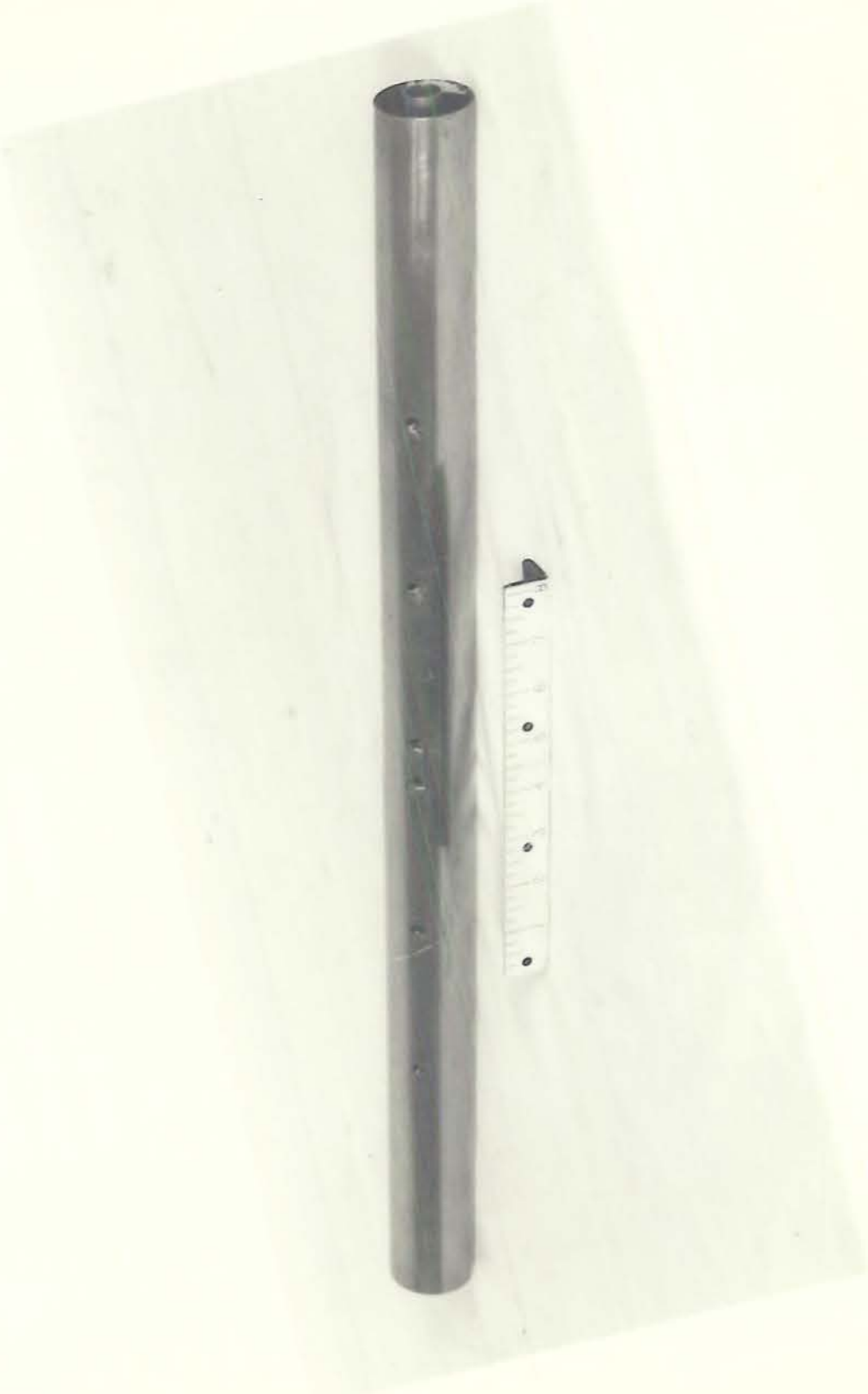
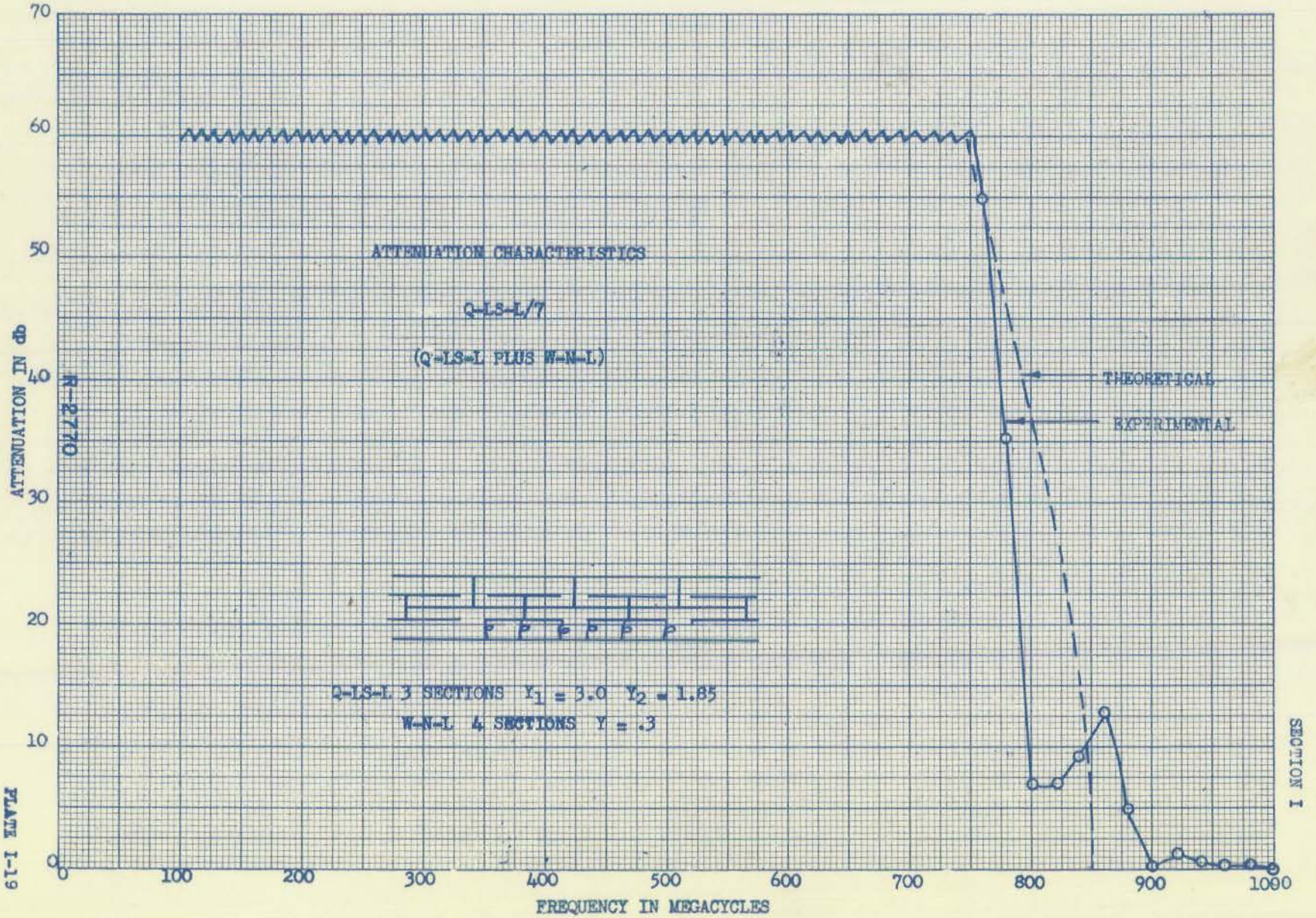
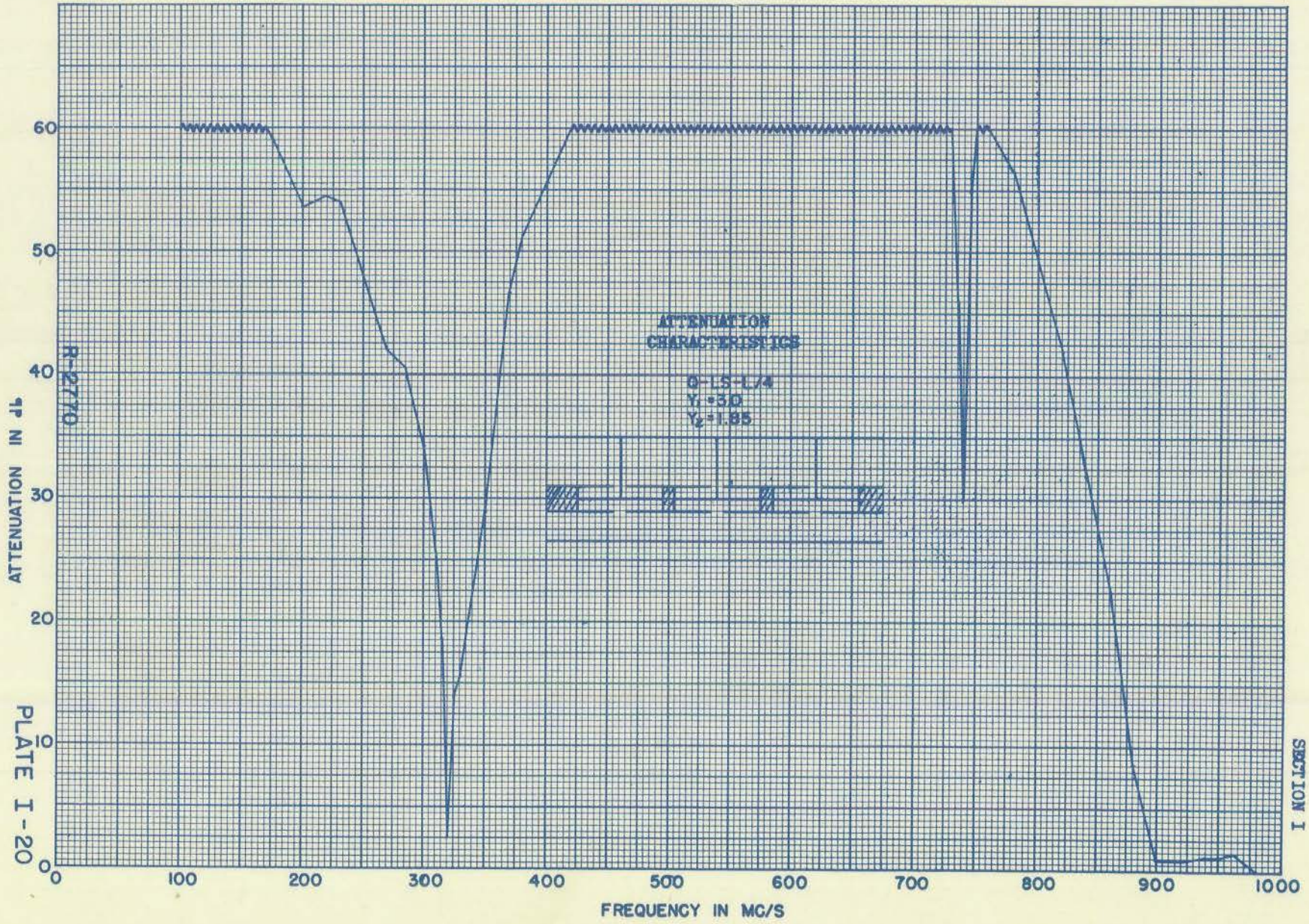


PLATE I-18

SECTIO

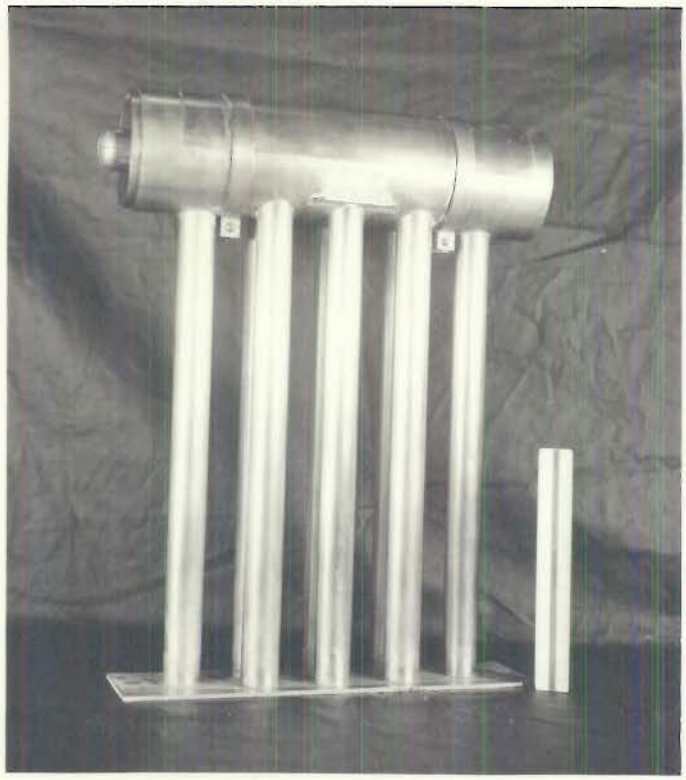

ATTENUATION CHARACTERISTICS
Q-LS-L/7
(Q-LS-L PLUS W-N-L)
R-2770
Q-LS-L 3 SECTIONS $Y_1 = 3.0$ $Y_2 = 1.85$
W-N-L 4 SECTIONS Y = .3
THEORETICAL
EXPERIMENTAL
ATTENUATION IN db
FREQUENCY IN MEGACYCLES
PLATE I-19
SECTION I
0
10
20
30
40
50
60
70
100
200
300
400
500
600
700
800
900
1000



R-2770

PLATE I-20

SECTION I





HIGH POWER Y-L-S FILTER NETWORK

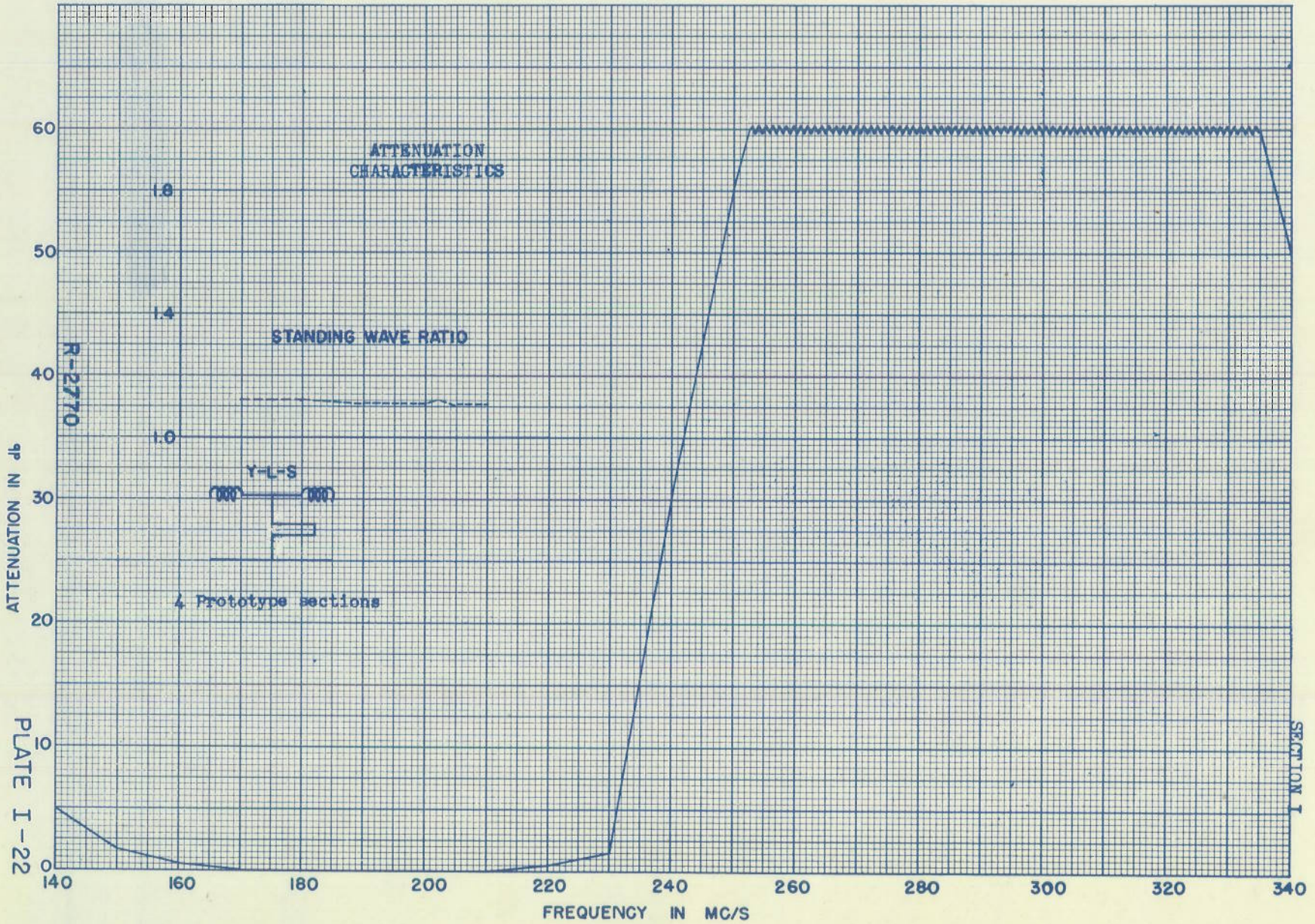
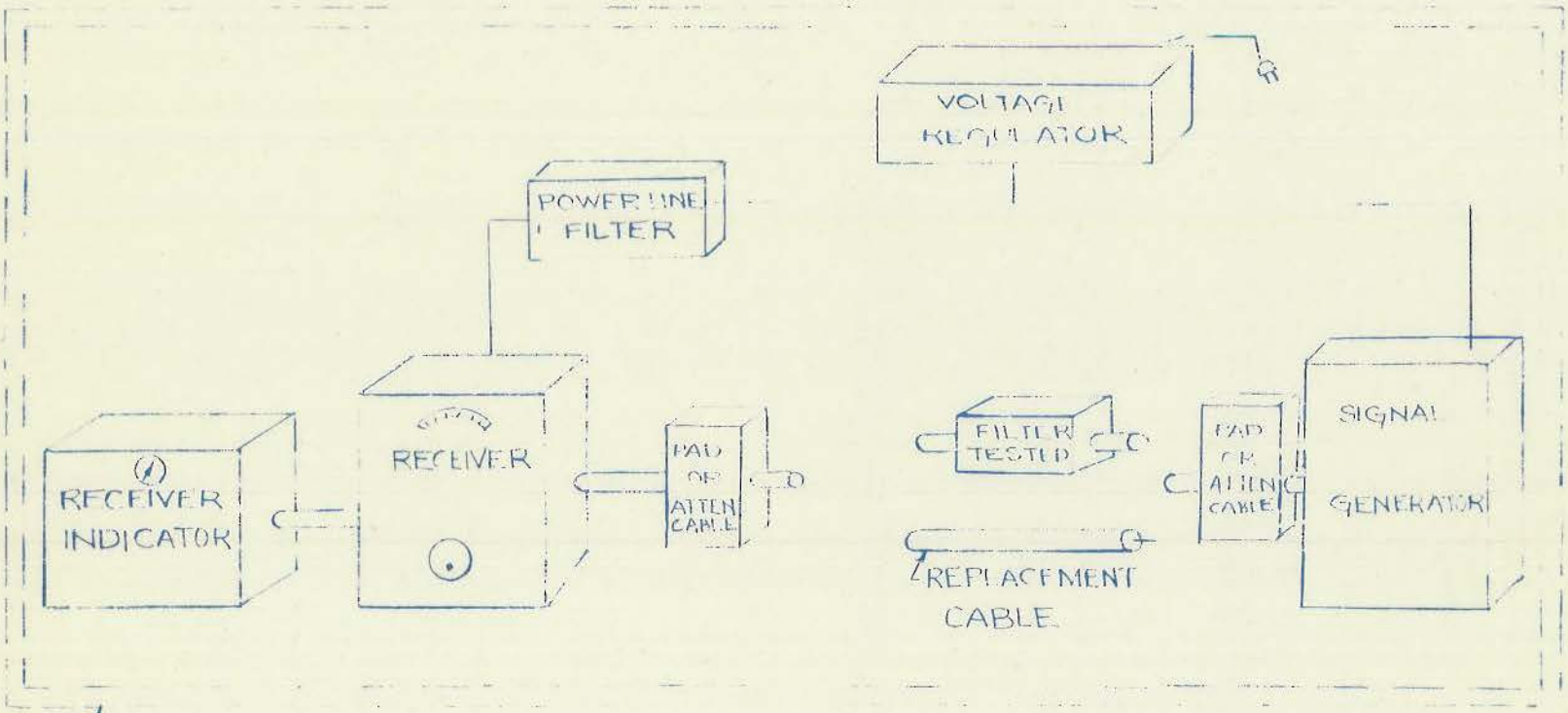


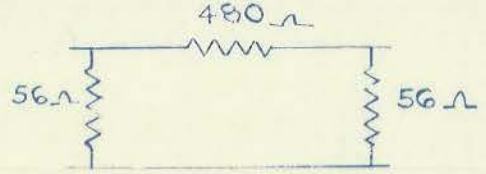
PLATE I-22

SECTION I



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PADS:-



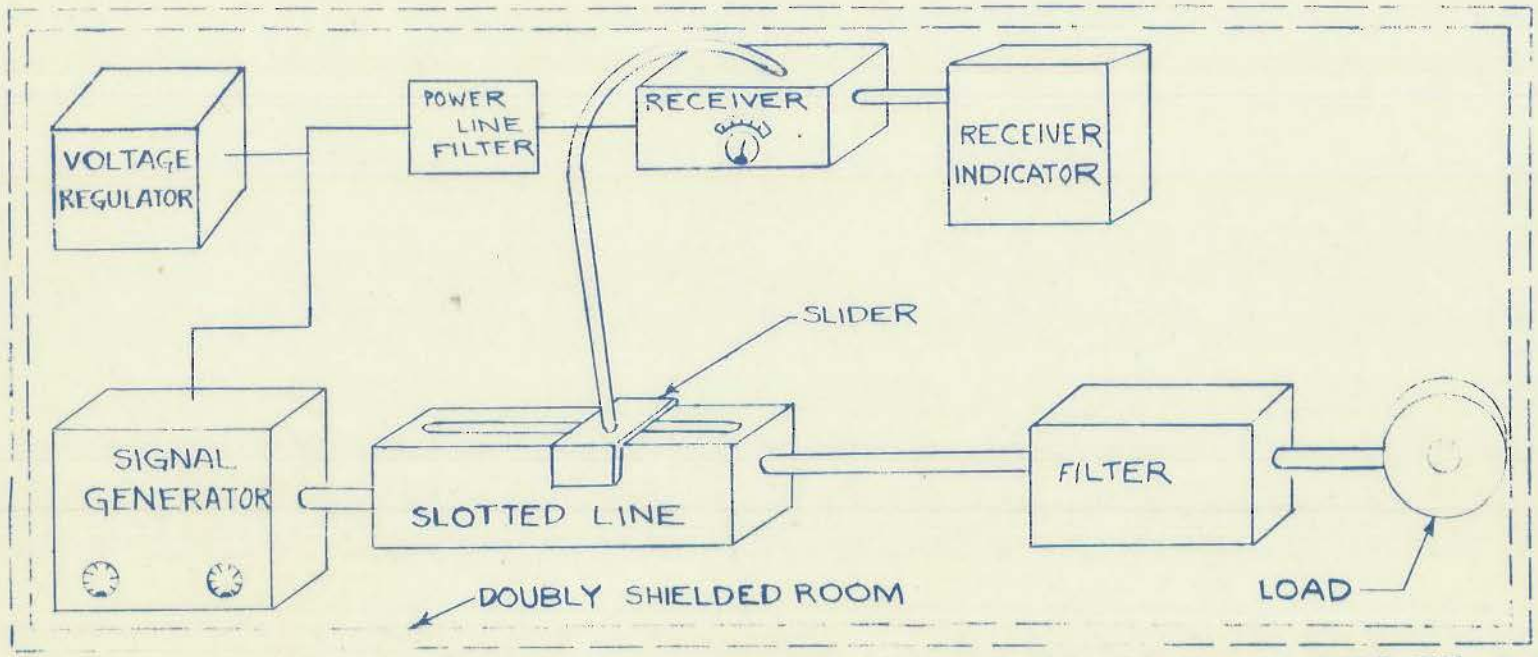


ATTEN. CABLE: (50Ω) RG-21/U } SUFFICIENT LENGTH TO GIVE MAX. ISOLATION
(70Ω) RG-35/U } WITH READABLE RECEIVER INDICATION

VHF AND UHF FILTER INSERTION LOSS MEASUREMENTS

SECTION H

PAGE 11



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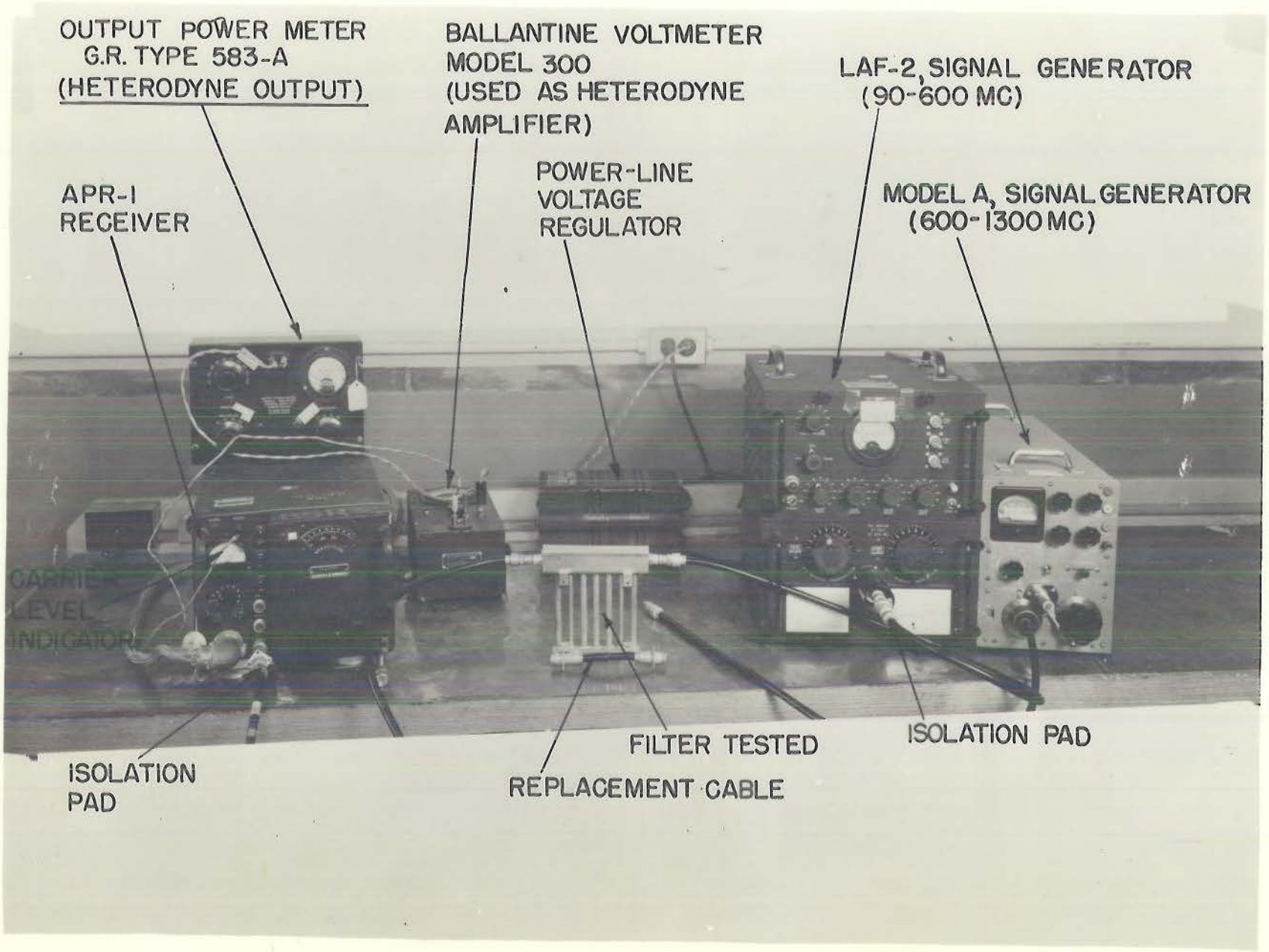
50 Ω LOAD CONSISTS OF CABLE: 1000 FT OF RG 8/U PLUS 250 FT OF RG-21/U

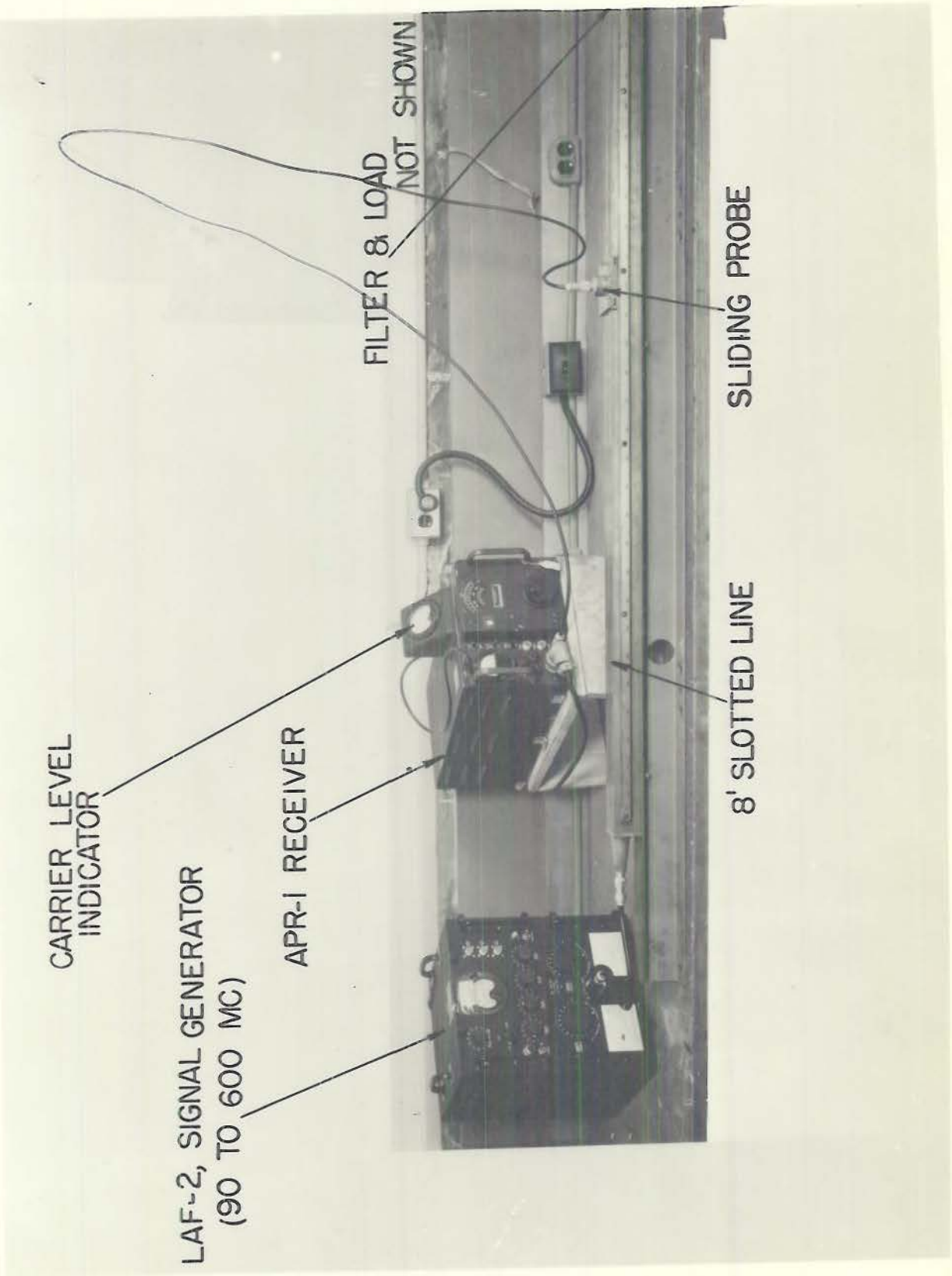
70 Ω LOAD CONSISTS OF CABLE: 1000 FT. OF RG-11/U PLUS 250 FT. OF RG-35/U

VHF AND UHF FILTER SWR MEASUREMENTS

PLATE II-2

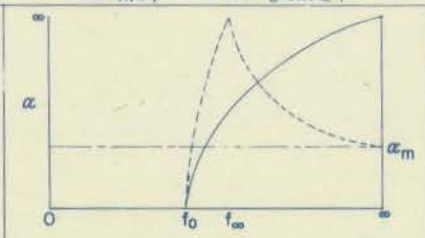
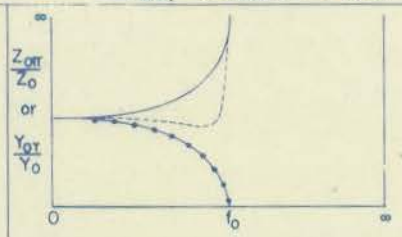
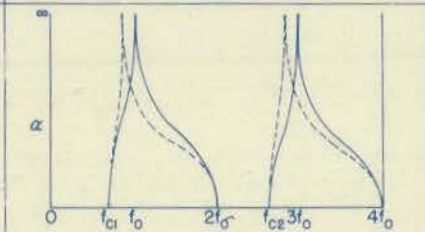
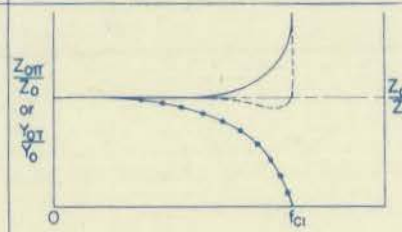
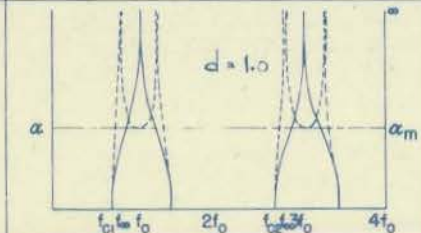
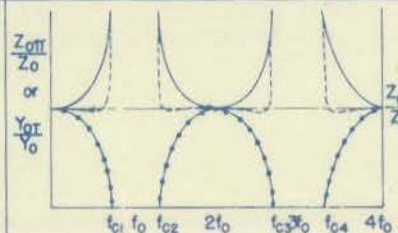
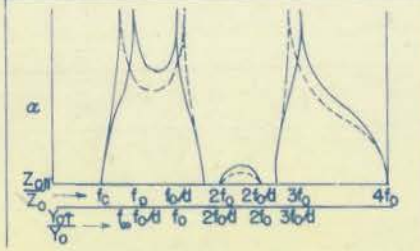
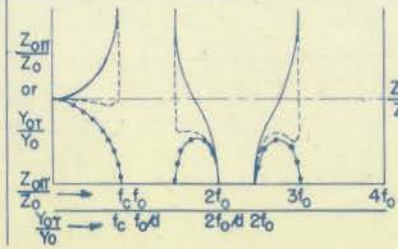
SECTION II


OUTPUT POWER METER
G.R. TYPE 583-A
(HETERODYNE OUTPUT)
BALLANTINE VOLTMETER
MODEL 300
(USED AS HETERODYNE
AMPLIFIER)
LAF-2, SIGNAL GENERATOR
(90-600 MC)
APR-1
RECEIVER
POWER-LINE
VOLTAGE
REGULATOR
MODEL A, SIGNAL GENERATOR
(600-1300 MC)
ISOLATION
PAD
REPLACEMENT CABLE
FILTER TESTED
ISOLATION PAD



VHF AND UHF FILTER STANDING WAVE-MEASUREMENTS

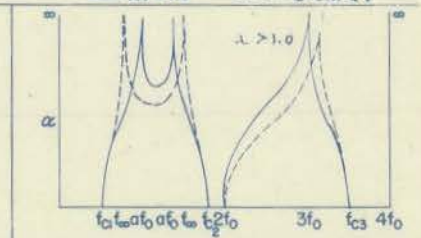
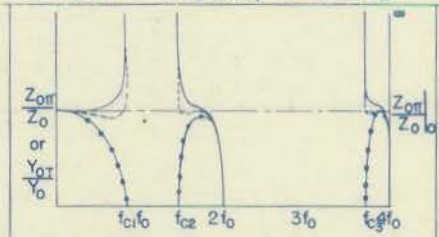
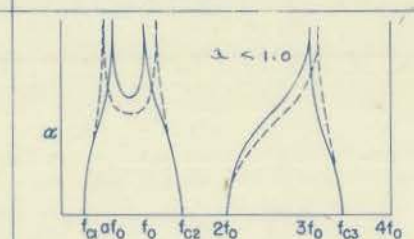
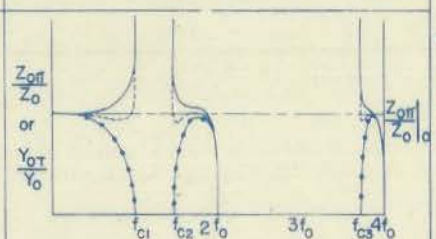
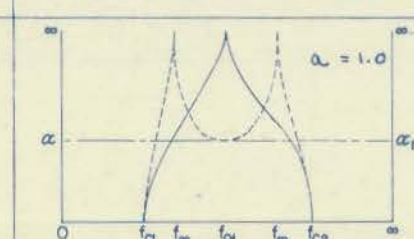
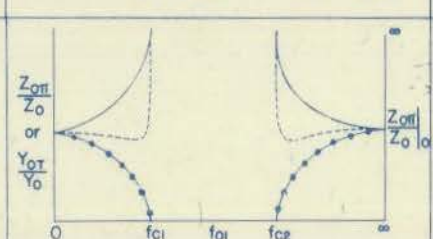
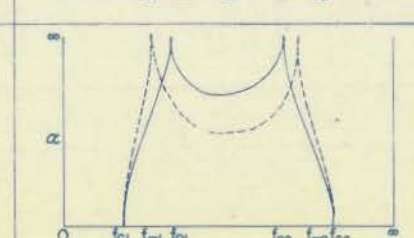
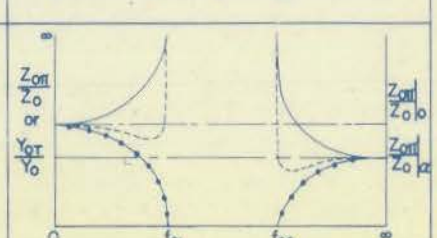
SUMMARY OF FILTER CHARACTERISTICS

CIRCUIT AND DESIGNATION	SYMBOLIC ATTENUATION CHARACTERISTIC — $m=1$ ---- $0<m<1$	CHARACTERISTIC IMPEDANCE CURVES — $m=1$ ---- $0<m<1$ ——— $m=0$	DESIGN EQUATIONS AND PARAMETERS
L L C L-L-C			$\frac{1}{Y_0}=Z_0=\sqrt{\frac{2L}{C}}$ $f_0=\frac{1}{\pi\sqrt{2LC}}$ $\alpha_m=\ln\frac{1+m}{1-m}$ $\cosh\alpha=\frac{(1+m^2)\left(\frac{f}{f_0}\right)^2-1}{(1-m^2)\left(\frac{f}{f_0}\right)^2-1}$ $\frac{f_\infty}{f_0}=\frac{1}{\sqrt{1-m^2}}$ $\frac{Z_{0T}}{Z_0}=\sqrt{1-\frac{f^2}{f_0^2}}$ $\frac{Z_{0\pi}}{Z_0}=\left(1-(1-m^2)\frac{f^2}{f_0^2}\right)\left(1-\frac{f^2}{f_0^2}\right)^{-\frac{1}{2}}=\frac{Y_{0T}}{Y_0}$
L L Z_0 Y-L-O; Z_0 C Q-S-C			$f_0=\frac{c}{4l}$ $y=\frac{Z_0}{2\pi f_0 L}$ $2y=\frac{f_c}{f_0}\tan\frac{\pi}{2}\frac{f_c}{f_0}$ $\frac{Z_{0T}}{Z_0}=\frac{1}{y}\sqrt{\frac{f}{f_0}\left(2y\cot\frac{\pi}{2}\frac{f}{f_0}-\frac{f}{f_0}\right)}$ $\frac{Z_{0\pi}}{Z_0}\Big\vert_0=\frac{2}{\sqrt{\pi y}}$ $\frac{Z_{0\pi}}{Z_0}=\frac{\frac{f}{f_0}\left(2\cot\frac{\pi}{2}\frac{f}{f_0}-\frac{1}{y}(1-m)\frac{f}{f_0}\right)}{\frac{f}{f_0}\left(2y\cot\frac{\pi}{2}\frac{f}{f_0}-\frac{f}{f_0}\right)}$ $\frac{Y_{0\pi}}{Y_0}=\frac{1}{2}\frac{Z_{0\pi}}{Z_0}$ $\cosh\alpha=\left[2y-(1+m^2)\frac{f}{f_0}\tan\left(\frac{\pi}{2}\frac{f}{f_0}\right)\right]\left[2y-(1-m^2)\frac{f}{f_0}\tan\frac{\pi}{2}\frac{f}{f_0}\right]^{-1}$
Z_{01} T-S-O; Z_{02}			$\frac{Z_{0T}}{Z_0}=\sqrt{\tan\frac{\pi}{2}\frac{f}{f_0}\left(2k\cot\left(d\frac{\pi}{2}\frac{f}{f_0}\right)-\tan\left(\frac{\pi}{2}\frac{f}{f_0}\right)\right)}$ $\frac{Z_{0\pi}}{Z_{01}}=\frac{2k\cot\left(d\frac{\pi}{2}\frac{f}{f_0}\right)-(1-m^2)\tan\left(d\frac{\pi}{2}\frac{f}{f_0}\right)}{\sqrt{2k\cot\left(d\frac{\pi}{2}\frac{f}{f_0}\right)\cot\frac{\pi}{2}\frac{f}{f_0}-1}}$ $\cosh\alpha=\frac{2k-(1+m^2)\left(\tan\frac{\pi}{2}\frac{f}{f_0}\right)\tan\left(d\frac{\pi}{2}\frac{f}{f_0}\right)}{2k-(1-m^2)\left(\tan\frac{\pi}{2}\frac{f}{f_0}\right)\tan\left(d\frac{\pi}{2}\frac{f}{f_0}\right)}$ $d=\frac{l_2}{l_1}$ $K=\frac{Z_{02}}{Z_{01}}$
Z_{01} T-S-O; Z_{02}			$\alpha_m=\ln\frac{1+m}{1-m}$ $2k\cot d\frac{\pi}{2}\frac{f_c}{f_0}\cot\frac{\pi}{2}\frac{f_c}{f_0}=1$; $2k\cot d\frac{\pi}{2}\frac{f_\infty}{f_0}\cot\frac{\pi}{2}\frac{f_\infty}{f_0}=1-m^2$ $\frac{Z_{0\pi}}{Z_0}\Big\vert_0=\sqrt{\frac{2k}{d}}=\frac{1}{2}\frac{Y_{0T}}{Y_0}\Big\vert_0$

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PLATE B-1

SUMMARY OF FILTER CHARACTERISTICS

CIRCUIT AND DESIGNATION	SYMBOLIC ATTENUATION CHARACTERISTIC — $m=1.0$ ----- $0<m<1$	CHARACTERISTIC IMPEDANCE CURVES — $m=1$ ----- $0<m<1$ ——— $m=0$	DESIGN EQUATIONS AND PARAMETERS
Y-P-O Q-S-S			$y = 2\pi f_0 Z_0 L \qquad f_1 = \frac{1}{2\pi\sqrt{LC}} \qquad a = \frac{f_1}{f_0} \qquad f_0 = \frac{c}{4\ell}$ $\frac{Z_{0T}}{Z_0} = \frac{f\,\frac{1}{f_0 y}}{1-\frac{f}{af_0}} \sqrt{2y\left(1-\left(\frac{f}{af_0}\right)^2\right)\frac{f_0}{f}\cot\frac{\pi}{2}\frac{f}{f_0}-1}$ $\frac{Z_{0\pi}}{Z_0} = \frac{2\cot\frac{\pi}{2}\frac{f}{f_0}}{\sqrt{2y\left(1-\left(\frac{f}{af_0}\right)^2\right)\frac{f_0}{f}\cot\frac{\pi}{2}\frac{f}{f}-1}}$ $\cosh\alpha = \frac{2y\left(1-\left(\frac{f}{af_0}\right)^2\right)+\frac{f_0}{f}(1+m^2)\tan\frac{\pi}{2}\frac{f}{f_0}}{2y\left(1-\left(\frac{f}{af_0}\right)^2\right)+(1-m^2)\frac{f}{f_0}\tan\frac{\pi}{2}\frac{f}{f_0}}$ $\left.\frac{Z_{0\pi}}{Z_0}\right\|_0 = \frac{2}{\sqrt{\pi y}} \qquad \frac{f_c}{f_0} = 2y\left(1-\left(\frac{f}{af_0}\right)^2\right)\cot\frac{\pi}{2}\frac{f}{f_0}$ For Q-S-S $\frac{Y_{0T}}{Y_0} = \frac{1}{2}\frac{Z_{0\pi}}{Z_0}$; $y = \frac{1}{Z_0\omega_0 C}$; $Z_0 = \frac{1}{Y_0}$; $af_0 = f_0'$; $f_0' = \frac{1}{2\pi\sqrt{LC}}$
Y-P-O Q-S-S			
L-P-S			$f_{01} = 2\pi\sqrt{L_1 C_1}^{-1} \qquad f_{02} = 2\pi\sqrt{L_2 C_2}^{-1} \qquad a = f_{02}/f_{01}$ $\frac{Z_{0T}}{Z_0} = \frac{2L_1}{C_2} \qquad k = \frac{C_2}{C_1}$ $\frac{Z_{0T}}{Z_0} = \frac{1}{\frac{f_{01}}{f}-\frac{f}{f_{01}}}\sqrt{\frac{1}{a}\left(\frac{f}{af_{01}}-\frac{af_{01}}{f}\right)\left(\frac{f_{01}}{f}-\frac{f}{f_{01}}\right)-k^2}$ $\frac{Z_{0\pi}}{Z_0} = \frac{2\left(\frac{f_{01}}{f}-\frac{f}{f_{01}}\right)\left(\frac{f}{af_{01}}-\frac{af_{01}}{f}\right)-(1-m^2)ak^2}{k\,\frac{f_{01}}{f}-\frac{f}{f_{01}}\sqrt{a\left[\left(\frac{f}{af_{01}}-\frac{af_{01}}{f}\right)\left(\frac{f_{01}}{f}-\frac{f}{f_{01}}\right)-ak^2\right]}}$
L-P-S			$\cosh\alpha = \left\|\frac{2\left(1-\frac{f^2}{f_{01}^2}\right)\left(\frac{f^2}{af_{01}^2}-a^2\right)+a^2k^2\frac{f^2}{f_{01}^2}(1+m^2)}{2\left(1-\frac{f^2}{f_{01}^2}\right)\left(\frac{f^2}{af_{01}^2}-a^2\right)+a^2k^2\frac{f^2}{f_{01}^2}(1-m^2)}\right\|$ $f_c/f_0 = 1/2\sqrt{2a^2+a^2k^2+2 \pm \sqrt{(2a^2+a^2k^2+2)^2-16a^2}}$ $\frac{f_\infty}{f_0} = \sqrt{\frac{2(1-m^2)(a^2+1)+a^2k^2 \pm \sqrt{(2(a^2+1)(1-m^2)+a^2k^2)^2-16a^2(1-m^2)^2}}{4(1-m^2)}}$ $\alpha_m = \ln\frac{1+m}{1-m} \qquad \left.\frac{Z_{0\pi}}{Z_0}\right\|_0 = 1 \qquad \left.\frac{Z_{0\pi}}{Z_0}\right\|_\infty = a \qquad \frac{Y_{0T}}{Y_0} = \frac{1}{2}\frac{Z_{0\pi}}{Z_0}$

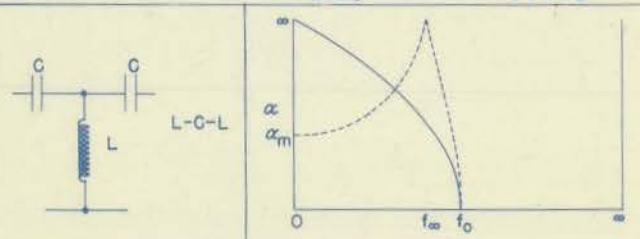
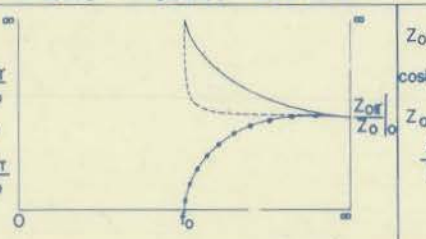
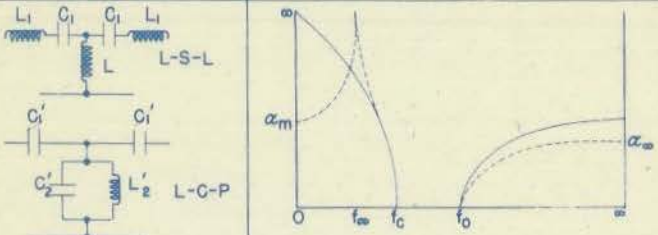
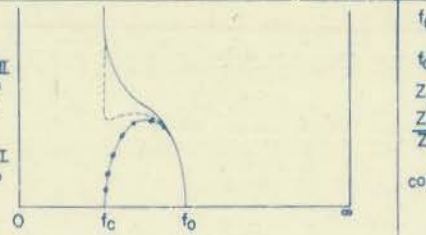
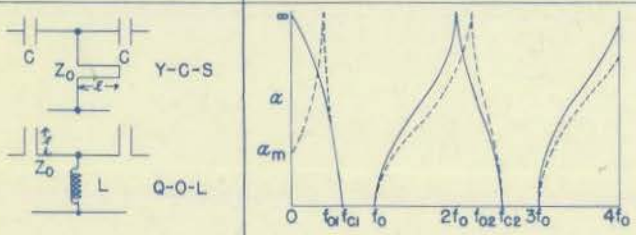
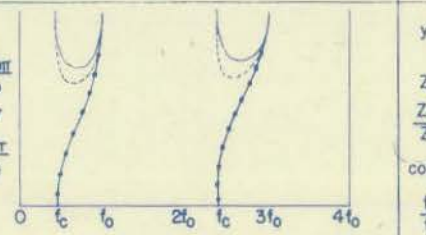
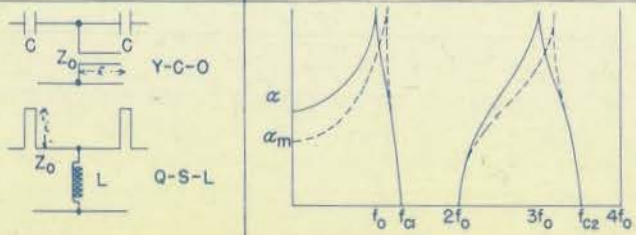
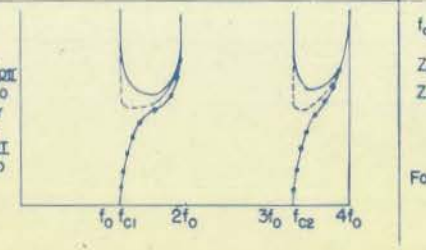
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PLATE B-2

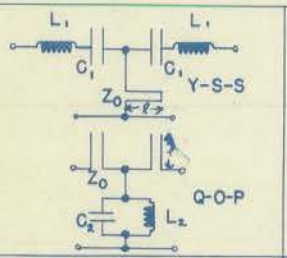
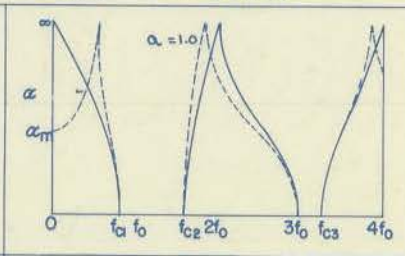
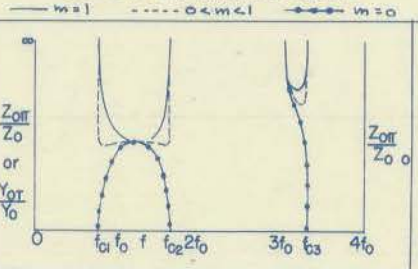
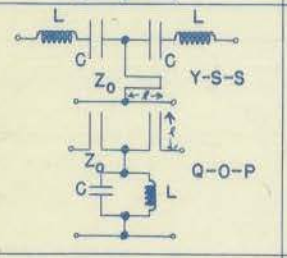
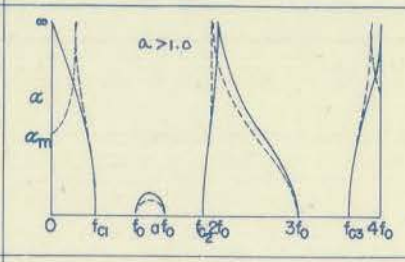
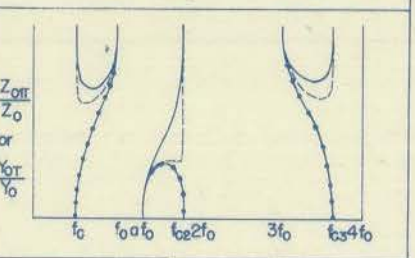
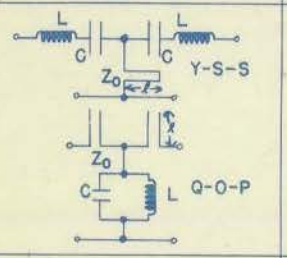
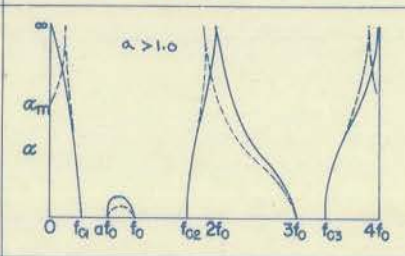
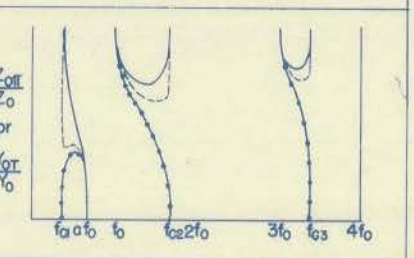
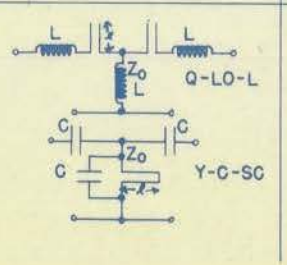
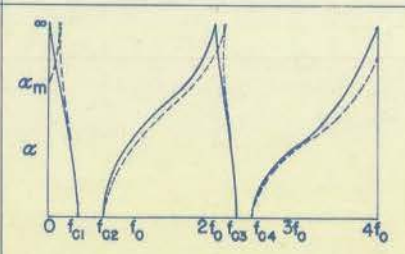
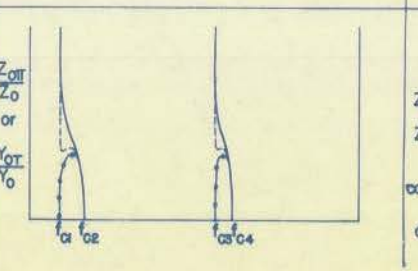
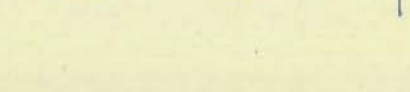
SUMMARY OF FILTER CHARACTERISTICS

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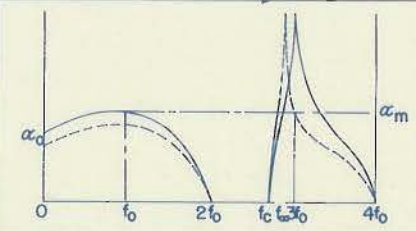
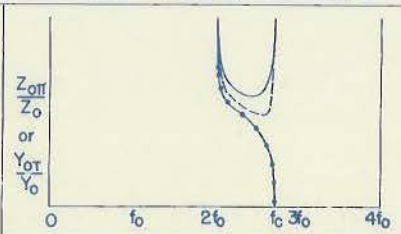
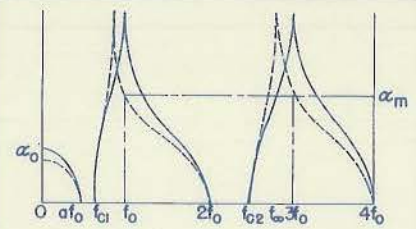
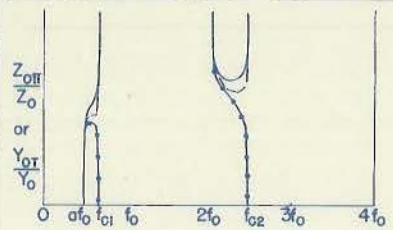
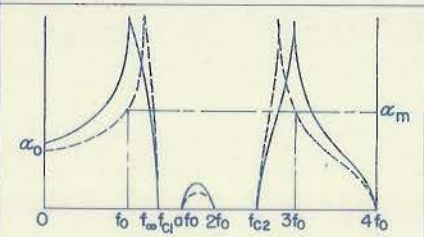
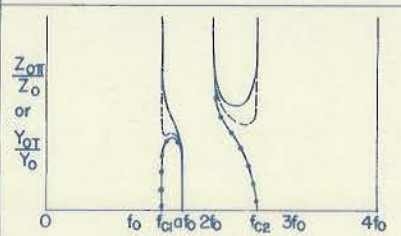
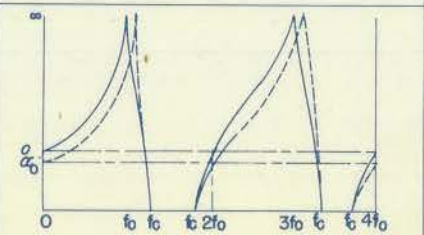
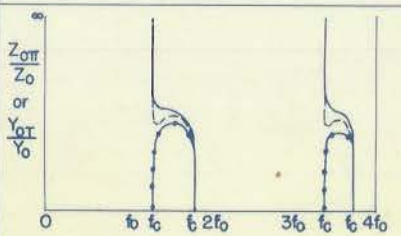
PLATE B-3

CIRCUIT AND DESIGNATION	SYMBOLIC ATTENUATION CHARACTERISTIC — $m=1.0$ ----- $0<m<1$	CHARACTERISTIC IMPEDANCE CURVES — $m=1.0$ ---- $0<m<1$ ●●● $m=0$	DESIGN EQUATIONS AND PARAMETERS
L-C-L			$Z_0=\sqrt{\frac{2L}{C}}$ $f_0=\frac{1}{\pi\sqrt{2LC}}$ $\alpha_m=\ln\frac{1+m}{1-m}$ $\cosh\alpha=\frac{1+m^2-x^2}{1-m^2-x^2}$ $\frac{f_\infty}{f_0}=\sqrt{1-m^2}$ $Z_{0T}/Z_0=\sqrt{1-v x^2}$ $X=\frac{f}{f_0}$ $\frac{Z_{0\pi}}{Z_0}=\frac{x^2-1+m^2}{x\sqrt{x^2-1}}$
L-S-L			$f_0=\frac{1}{2\pi\sqrt{L_1C_1}}$ $Z_0=\frac{L_1}{C_1}$ $k=\frac{L_2}{L_1}$ $f_0=\frac{1}{2\pi}\sqrt{L_2C_2'}$ $Z_0=\frac{L_2}{C_2}$ $k=\frac{C_1'}{C_2'}$ $Z_{0T}/Z_0=\sqrt{(1-x^2)(1-1/x^2+2k)}$ $\frac{Z_{0\pi}}{Z_0}=\frac{x^2-1\ 2kx^2+m^2-1}{x\sqrt{(1-x^2)(2k+1)x^2-1}}$ $\cosh\alpha=\frac{(2k+1+m^2)x^2-1-m^2}{(2k+1-m^2)x^2+m^2-1}$ $\frac{Y_{0T}}{Y_0}=\frac{Z_{0\pi}}{2Z_0}$ $f_c/f_0=1+2k^{-\frac{1}{2}}$ $\frac{f_\infty}{f_0}=\sqrt{\frac{1-m^2}{1-m^2+2k}}$ $x=\frac{f}{f_0}$
L-C-P			
Y-C-S			$y=\begin{cases}2\pi Z_0 f_0 C & \text{Y-C-S}\\ \frac{2\pi f_0 L}{Z_0} & \text{Q-O-L}\end{cases}$ $f_0=\frac{f_c}{4\ell}$ $x=\frac{f}{f_0}$ $Z_{0T}/Z_0=1/xy\sqrt{1-2xy\tan x\pi/2}$ $\alpha_m=\ln\frac{1+m}{1-m}$ $\frac{Z_{0\pi}}{Z_0}=-\frac{2xy\tan x\pi/2+1-m^2}{\sqrt{1-2xy\tan(x\pi/2)}}$ $\cosh\alpha=\frac{2xy-(1+m^2)\cot(x\pi/2)}{2xy-(1-m^2)\cot(x\pi/2)}$ $\frac{f_c}{f_0}\tan\frac{\pi}{2}\frac{f_c}{f_0}=\frac{1}{2y}$ $\frac{f_0}{f_0}\tan\frac{\pi}{2}\frac{f_\infty}{f_0}=\frac{1-m^2}{2y}$
Q-O-L			
Y-C-O			$f_0=\frac{f_c}{4\ell}$ $y=2\pi f_0 Z_0 C$ $\cosh\alpha=\frac{4y+(1+m^2)\pi}{4y+(1-m^2)\pi}$ $x=\frac{f}{f_0}$ $Z_{0T}/Z_0=(1/xy)\sqrt{-(1+xy\cot(x\pi/2))}$ $Z_{0\pi}/Z_0=-\frac{2\cot(x\pi/2)+(1-m^2)/xy}{\sqrt{-(1+xy\cot(x\pi/2))}}$ $\alpha_m=\ln\frac{1+m}{1-m}$ $\cosh=\left\lvert\frac{2xy\cot(x\pi/2)+1+m^2}{2xy\cot(x\pi/2)+1-m^2}\right\rvert$ $\cosh\alpha_\infty=\frac{2k+1+m}{2k+1-m}$ For Q-S-L: $y=\frac{2\pi f_0 L}{Z_0}$ $\frac{Y_{0T}}{Y_0}=2\frac{Z_{0\pi}}{Z_0}$
Q-S-L			

SUMMARY OF FILTER CHARACTERISTICS

CIRCUIT AND DESIGNATION	SYMBOLIC ATTENUATION CHARACTERISTICS — m=1 ---- 0<m<1	CHARACTERISTIC IMPEDANCE CURVES — m=1 ----- 0<m<1 ●—● m=0	DESIGN EQUATIONS AND PARAMETERS
			$f_0=\frac{c}{4\ell}$ $y=\begin{cases}2\pi f_0 Z_0 C_1 & \text{Y-S-S}\\ \frac{2\pi f_0 L_2}{Z_0} & \text{Q-O-P}\end{cases}$ $f_1=\begin{cases}\frac{1}{2\pi\sqrt{L_1C_1}} & \\ \frac{1}{2\pi\sqrt{L_2C_2}} & \end{cases}$ $a=\frac{f_1}{f_0}$
			$\frac{Z_{0T}}{Z_0}=\frac{1}{y}\frac{f_0}{f}\sqrt{\left[1-\left(\frac{f}{af_0}\right)^2\right]\left[2y\frac{f}{f_0}\tan\frac{\pi}{2}\frac{f}{f_0}-1-\left(\frac{f}{af_0}\right)^2\right]}$
			$\frac{Z_{0\Pi}}{Z_0}=\frac{\left[1-\left(\frac{f}{af_0}\right)^2\right]\left[2\tan\frac{\pi}{2}\frac{f}{f_0}-\frac{1}{y}\left(1-m^2\right)\left(1-\left(\frac{f}{af_0}\right)^2\right)\right]}{\sqrt{\left[1-\left(\frac{f}{af_0}\right)^2\right]\left[2y\frac{f}{f_0}\tan\frac{\pi}{2}\frac{f}{f_0}-1+\left(\frac{f}{af_0}\right)^2\right]}}$
R-2770			$\cosh\alpha=\left\lvert\frac{\left(1+m^2\right)\left(\left(\frac{f}{af_0}\right)^2-1\right)+2y\frac{f}{f_0}\tan\frac{\pi}{2}\frac{f}{f_0}}{\left(1+m^2\right)\left(\left(\frac{f}{af_0}\right)^2-1\right)+2y\frac{f}{f_0}\tan\frac{\pi}{2}\frac{f}{f_0}}\right\rvert$
			$\alpha_m=\ln\frac{1+m}{1-m}$ $\left.\frac{Z_{0\Pi}}{Z_0}\right\rvert_0=2\sqrt{\frac{2}{\pi y}}$ $\frac{Y_{0T}}{Y_0}=\frac{1}{2}\frac{Z_{0\Pi}}{Z_0}$
			$\left(\frac{f_c}{af_0}\right)^2+2\frac{f_c}{f_0}y\tan\frac{\pi}{2}\frac{f_c}{f_0}+1=0$
			$z_1=\begin{cases}\frac{Z_0}{2\pi f_0 L_1} & \\ (2\pi f_0 C_2 Z_0) & \end{cases}$ $z_2=\begin{cases}\frac{Z_0}{2\pi f_0 L_2} & \\ (2\pi f_0 C_1 Z_0)^{-1} & \end{cases}$ $f_0=\frac{c}{4\ell}$ $\alpha_m=\ln\frac{1+m}{1-m}$
PLATE B-4			$Z_{0T}/Z_0=\sqrt{[\cot x\pi/2-xz_1]\left[(z_1+2z_2)x-\cot x\pi/2\right]}$ $\frac{Y_{0T}}{Y_0}=\frac{1}{2}\frac{Z_{0\Pi}}{Z_0}$
			$Z_{0\Pi}/Z_0=\left[Z_{0T}/Z_0\right]^{-1}\left[\cot x\pi/2-xz_1\left[2z_2+z_1(m^2-1)\right]x+(1-m^2)\cot x\pi/2\right]$
			$\cosh\alpha=\left\lvert\frac{x\left[(1+m^2)z_1+2z_2-(1+m^2)\cot x\pi/2\right]}{x\left[(1-m^2)z_1+2z_2-(1-m^2)\cot x\pi/2\right]}\right\rvert$ $p=2,4,\ldots$ $q=1,3,\ldots$
			$\cot\frac{\pi}{2}\frac{f_{cp}}{f_0}=z_1\frac{f_{cp}}{f_c}$ $(z_1+2z_2)\frac{f_{cq}}{f_0}=\cot\frac{\pi}{2}\frac{f_{cq}}{f_0}$ $x=\frac{f}{f_0}$

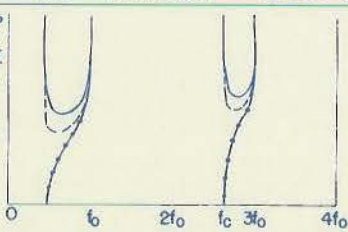
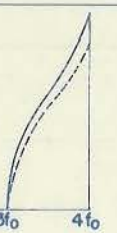
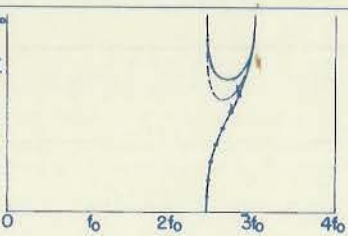
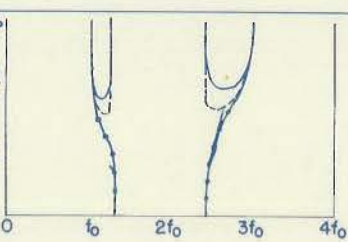
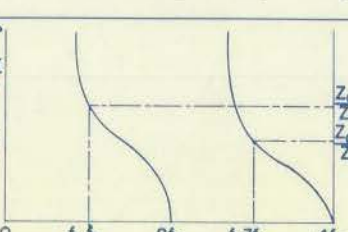
SUMMARY OF FILTER CHARACTERISTICS

CIRCUIT AND DESIGNATION	SYMBOLIC ATTENUATION CHARACTERISTIC — $m=1$ ---- $0<m<1$	CHARACTERISTIC IMPEDANCE CURVES — $m=1$ --- $0<m<1$ ●●● $m=0$	DESIGN EQUATIONS AND PARAMETERS
Y-S-O Q-S-P			$f_c=\frac{c}{4\ell}$ $a=\frac{1}{2\pi f_0\sqrt{L_1C_1}}=\frac{1}{2\pi f_0\sqrt{L_2C_2}}$ $y=2\pi f_0 Z_0 C_1=\frac{2\pi f_0 L_2}{Z_0}$ $\frac{Z_{0T}}{Z_0}=\frac{1}{ay}\sqrt{\left[\frac{f}{af_0}-\frac{af_0}{f}\right]\left[\frac{af_0}{f}-\frac{f}{af_0}+2ay\cot\left(\frac{\pi}{2}\frac{f}{f_0}\right)\right]}$ $\frac{Z_{0\pi}}{Z_0}=\frac{\frac{1}{ay}\left[\frac{f}{af_0}-\frac{af_0}{f}\right]\left[2\cot\left(\frac{\pi}{2}\frac{f}{f_0}\right)-\frac{1-m}{ay}\left(\frac{f}{af_0}-\frac{af_0}{f}\right)\right]}{\frac{Z_{0t}}{Z_0}}$
Y-S-O Q-S-P			$\cosh\alpha=\left\lvert\frac{2ay\cot\left(\frac{\pi}{2}\frac{f}{f_0}\right)+\left(1+m^2\right)\left(\frac{af_0}{f}-\frac{f}{af_0}\right)}{2ay\cot\left(\frac{\pi}{2}\frac{f}{f_0}\right)+\left(1-m^2\right)\left(\frac{af_0}{f}-\frac{f}{af_0}\right)}\right\rvert$ $\cosh\alpha_m=\frac{1+m}{1-m}$ $\cosh\alpha_1=\frac{2}{2}\frac{1+m}{1-m}+\pi y$ $\cosh\alpha_0=\frac{4y+1+m\ \pi}{4y+1-m\ \pi}$
Y-S-O Q-S-P			$\frac{f_c}{a\,f_0}+2\frac{f_c}{af_0}ay\cot\frac{\pi}{2}\frac{f_c}{f_0}-1=0$ $\frac{f_\infty}{a^2f_0^2}-\frac{f_\infty}{af_0}\frac{2ay\cot\frac{\pi}{2}\frac{f_\infty}{f_0}}{1-m^2}-1=0$
Q-LS-L Y-C-OC			$z_1=\frac{2\pi f_0L_1}{Z_0}=2\pi f_0Z_0C_2$ $z_2=\frac{2\pi f_0L_2}{Z_0}=2\pi f_0Z_0C_1$ $f_0=\frac{c}{4\ell}$ $Z_{0T}/Z_0=\left[\tan x\pi/2-xz_1\right]\left[(z_1+2z_2)x-\tan x\pi/2\right]$ $Z_{0\pi}/Z_0=\left(Z_{0T}/Z_0\right)^{-1}\left[\tan x\pi/2-xz_1\right]\left[2z_2+z_1(m^2-1)x+(1-m^2)\tan x\pi/2\right]$ $\cosh\alpha=\frac{x\left[(1+m^2)z_1+2z_2-(1+m^2)\tan x\pi/2\right]}{x\left[(1+m^2)z_1+2z_2-(1-m^2)\tan x\pi/2\right]}$

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PLATE B-5

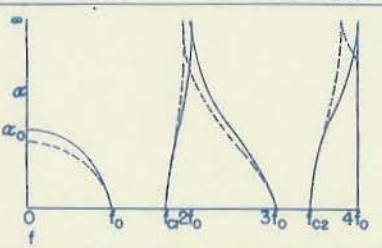
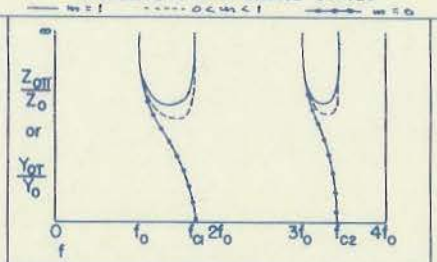
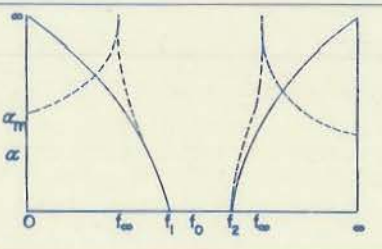
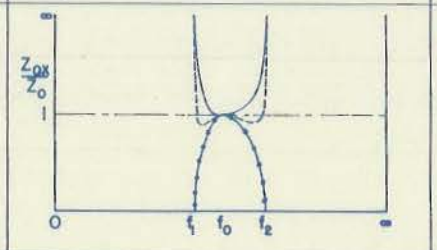
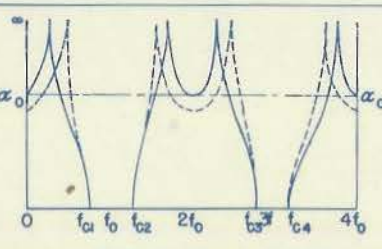
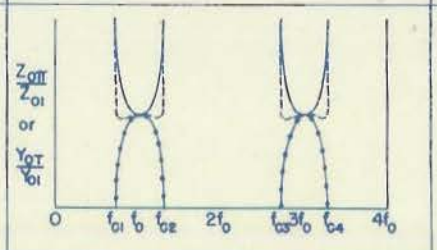
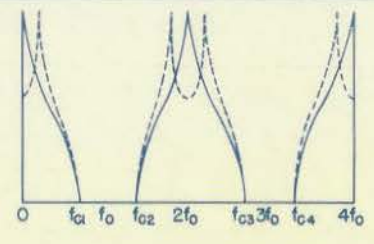
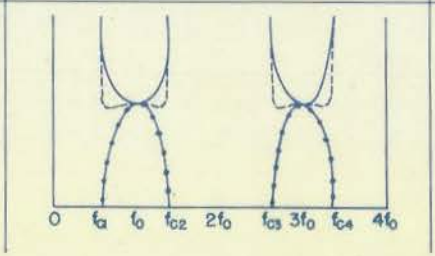
SUMMARY OF FILTER CHARACTERISTICS

CIRCUIT AND DESIGNATION	SYMBOLIC ATTENUATION CHARACTERISTIC — m=1 ------ 0<m<1	CHARACTERISTIC IMPEDANCE CURVES — m=1 ---- 0<m<1 ●—● m=0	DESIGN EQUATIONS AND PARAMETERS
Y-P-S Q-0-S			$f_0=\frac{c}{4\ell}$ $y=\frac{Z_0}{2\pi f_0 L}$ $a=\frac{1}{2\pi f_0\sqrt{LC}}=\frac{f_1}{f_0}$ $f_\infty=f_1$ $\frac{Z_{0T}}{Z_0}=\frac{a}{y}\frac{1}{\frac{af_0}{f}-\frac{f}{af_0}}\sqrt{-\left[1+\frac{2y}{a}\left(\frac{af_0}{f}-\frac{f}{af_0}\right)\tan\frac{\pi}{2}\frac{f}{f_0}\right]}$ $\frac{Z_{0\pi}}{Z_0}=\frac{2\tan\frac{\pi}{2}\frac{f}{f_0}+(1-m^2)\frac{a}{y}\left(\frac{af_0}{f}-\frac{f}{af_0}\right)^{-1}}{1+\frac{2y}{a}\left(\frac{af_0}{f}-\frac{f}{af_0}\right)\tan\frac{\pi}{2}\frac{f}{f_0}}$ $\cosh\alpha=\frac{2+(1+m^2)\frac{a}{y}\left[\cot\frac{\pi}{2}\frac{f}{f_0}\left(\frac{af_0}{f}-\frac{f}{af_0}\right)^{-1}\right]}{2+(1-m^2)\frac{a}{y}\left[\cot\frac{\pi}{2}\frac{f}{f_0}\left(\frac{af_0}{f}-\frac{f}{af_0}\right)^{-1}\right]}$ $\cosh\alpha_0=\frac{1+\frac{1+m^2}{\pi y}}{1+\frac{1-m^2}{\pi y}}$ $\cosh\alpha_m=\frac{1+\frac{1+m}{8}\frac{\pi}{y}}{1+\frac{1-m}{8}\frac{\pi}{y}}$ $\frac{2y}{a}\left(\frac{f_c}{af_0}-\frac{af_0}{f_c}\right)=\cot\frac{\pi}{2}\frac{f_c}{af_0}$
Y-P-S Q-0-S			
Y-P-S Q-0-S			
W-N-L W-C-N			$y=\frac{Z_0}{2\pi f_0 L}=\frac{1}{2Z_0\pi f_0 C}$ $Y_0=\frac{1}{Z_0}$ $\frac{Z_{0\pi}}{Z_0}=\frac{1}{\sqrt{\left(1-y\frac{f_0}{f}\cot\frac{\pi}{4}\frac{f}{f_0}\right)\left(1+y\frac{f_0}{f}\tan\frac{\pi}{4}\frac{f}{f_0}\right)}}$ $\left.\frac{Z_{0\pi}}{Z_0}\right\|_n=\frac{1}{\sqrt{1-\frac{y^2}{n^2}}}$ $\cosh\alpha_0=1+\frac{\pi}{2}y$ $\frac{f_c}{f_0}\tan\frac{\pi}{2}\frac{f_c}{f_0}=y$ $\cosh\alpha=\left\|\cos\frac{\pi}{2}\frac{f}{f_0}\left(1+y\frac{f_0}{f}\tan\frac{\pi}{2}\frac{f}{f_0}\right)\right\|$ $Y_{0T}=\frac{1}{2}\frac{Z_{0\pi}}{Z_0}$

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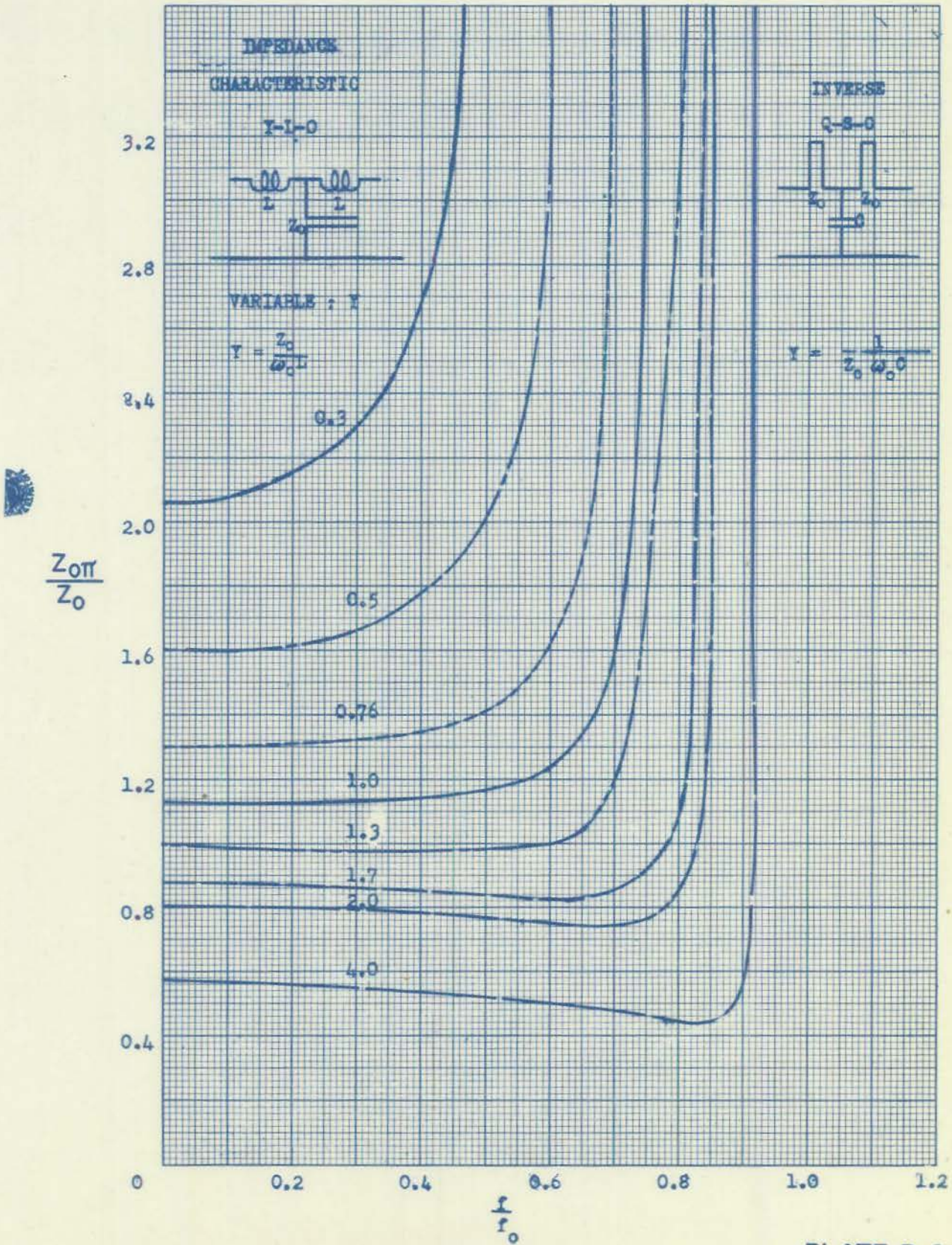
PLATE B-6

SUMMARY OF FILTER CHARACTERISTICS

CIRCUIT AND DESIGNATION	SYMBOLIC ATTENUATION CHARACTERISTIC (—— m=1 ------ 0<m<1)	CHARACTERISTIC IMPEDANCE CURVES (—— m=1 ------ 0<m<1 —•— m=0)	DESIGN EQUATIONS AND PARAMETERS
Y-L-S Q-0-C			$f_0=\frac{c}{4\ell}$ $y=\frac{Z_0}{2\pi f_0 L}=\frac{1}{2\pi f_0 Z_{0l}}$ $2y=\frac{f_c}{f_0}\cot\frac{\pi}{2}\frac{f_c}{f_0}$ $\frac{Z_{0T}}{Z_0}=\frac{1}{y}\frac{f}{f_0}\sqrt{-\left(1+\frac{2y\tan(x\pi/2)}{x}\right)}$ $x=\frac{f}{f_0}$ $\frac{Z_{0\pi}}{Z_0}=\frac{-2y\tan(x\pi/2)+x/y\,(1-m^2)}{\sqrt{-(1+2y\tan(x\pi/2))}}$ $\cosh\alpha=\frac{1+((1+m^2)/2y)\,x\cot(x\pi/2)}{1+((1-m^2)/2y)\,x\cot(x\pi/2)}$ $\cosh\alpha_0=\frac{1+(1+m^2)/4ry}{1+(1-m^2)/4ry}$
L-S-P			$Z_0=\sqrt{\frac{L_2}{C_1}}=\sqrt{\frac{L_1}{C_2}}$ $f_0=\frac{1}{2\pi\sqrt{L_2C_1}}=\frac{1}{2\pi\sqrt{L_1C_2}}$ $x=\frac{f}{f_0}-\frac{f_0}{f}$ $\cosh\alpha=\frac{1+(1+m^2)x^2}{1+(1-m^2)x^2}$ $\alpha_m=\ln\frac{1+m}{1-m}$ $\frac{f_2}{f_0}=\sqrt{1+\frac{C_1}{2C_2}}+\sqrt{\frac{C_1}{2C_2}}$ $\frac{Z_{0T}}{Z_0}=\sqrt{1-\frac{2}{x^2}}$ $\frac{f_1}{f_0}=\sqrt{1+\frac{C_1}{2C_2}}-\sqrt{\frac{C_1}{2C_2}}$ $\frac{Z_{0\pi}}{Z_0}=\frac{1-(1-m^2)x^2}{\frac{Z_{0T}}{Z_0}}$ $\frac{f_{\infty1}}{f_1}=\frac{f_2}{f_{\infty2}}=\frac{\frac{f_2}{f_1}-1-\sqrt{\left(\frac{f_2}{f_1}\right)^2-1+4\frac{f_2}{f_1}-4m^2\frac{f_2}{f_1}}}{2\sqrt{1-m^2}}$
T-0-0 T-S-S			$k=\frac{Z_{02}}{Z_{01}}$ $d=\frac{\ell_2}{\ell_1}$ $f=\frac{c}{4\ell_2}$ $Y_{01}=(2Z_{01})^{-1}$ $Z_{0T}/Z_{01}=\sqrt{-\cot(x\pi/2)\left[\cot(x\pi/2)+2k\cot(dx\pi/2)\right]}$ $Z_{0\pi}/Z_{01}=-2k\cot(dx\pi/2)\,\cot(x\pi/2)/(Z_{0T}/Z_0)$ $\cosh\alpha=\frac{2k\cot(dx\pi/2)+(1+m^2)\cot(x\pi/2)}{2k\cot(dx\pi/2)+(1-m^2)\cot(x\pi/2)}$ $\cosh\alpha_0=\frac{2k+(1+m^2)d}{2k+(1-m^2)d}$ $2k=-\cot\frac{\pi}{2}\frac{f_c}{f_0}\tan d\frac{\pi}{2}\frac{f_c}{f_0}$ $\frac{-2k}{1-m^2}=\cot\frac{\pi}{2}\frac{f_\infty}{f_0}\tan\frac{\pi}{2}\frac{f_\infty}{f_0}$ $x=\frac{f}{f_0}$
T-0-S			$k=\frac{Z_{02}}{Z_{01}}$ $d=\frac{\ell_2}{\ell_1}$ $f_0=\frac{c}{4\ell_2}$ $Y_{01}=2Z_{01}$ $Z_{0T}/Z_{01}=\sqrt{-\cot(x\pi/2)\left[\cot(x\pi/2)+2k\tan(dx\pi/2)\right]}$ $Z_{0\pi}/Z_{01}=-2k\tan(dx\pi/2)\cot(x\pi/2)/(Z_{0T}/Z_0)$ $\cosh\alpha=\frac{2k\tan(dx\pi/2)+(1+m^2)\cot(x\pi/2)}{2k\tan(dx\pi/2)+(1-m^2)\cot(x\pi/2)}$

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PLATE B-7



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PLATE B-8

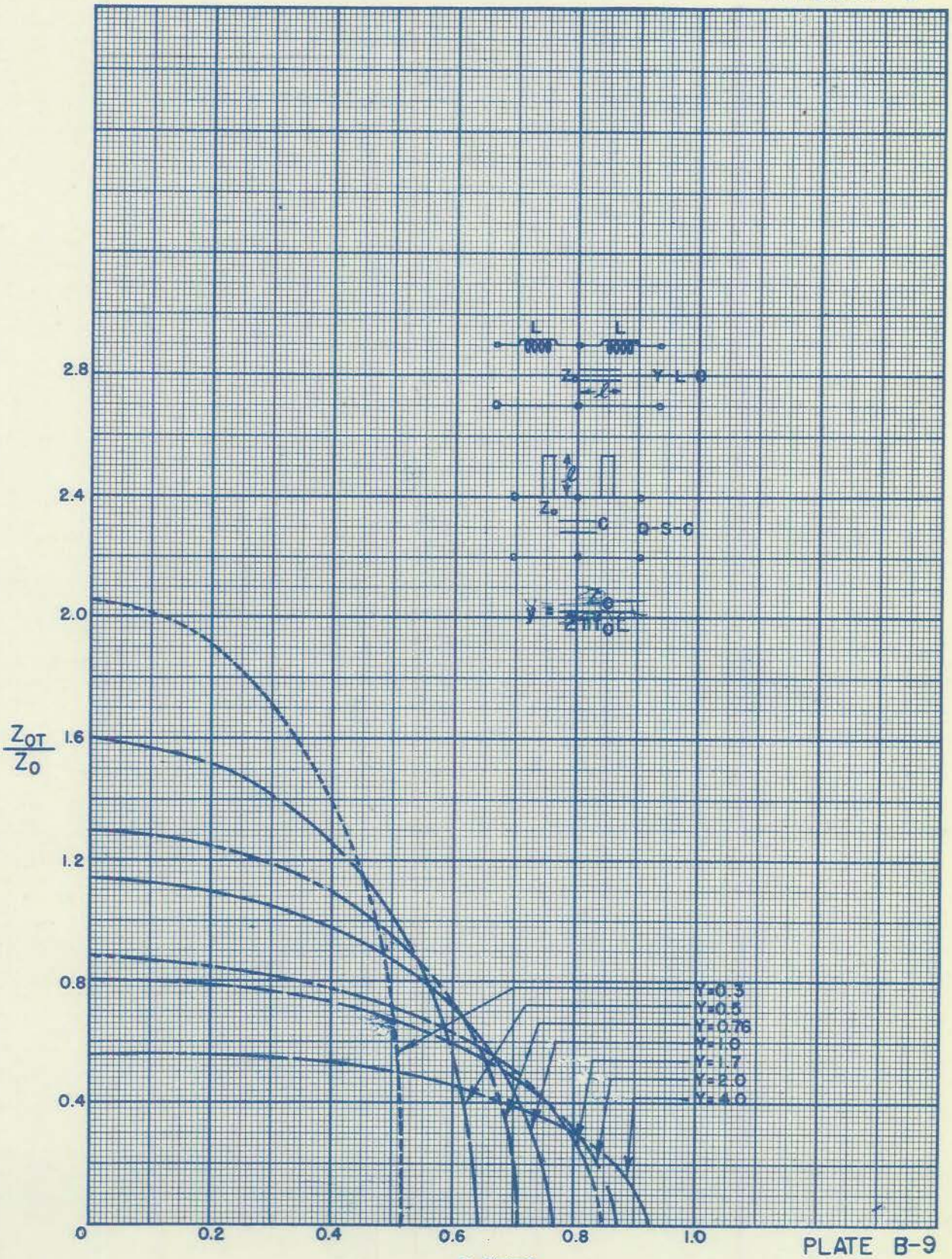
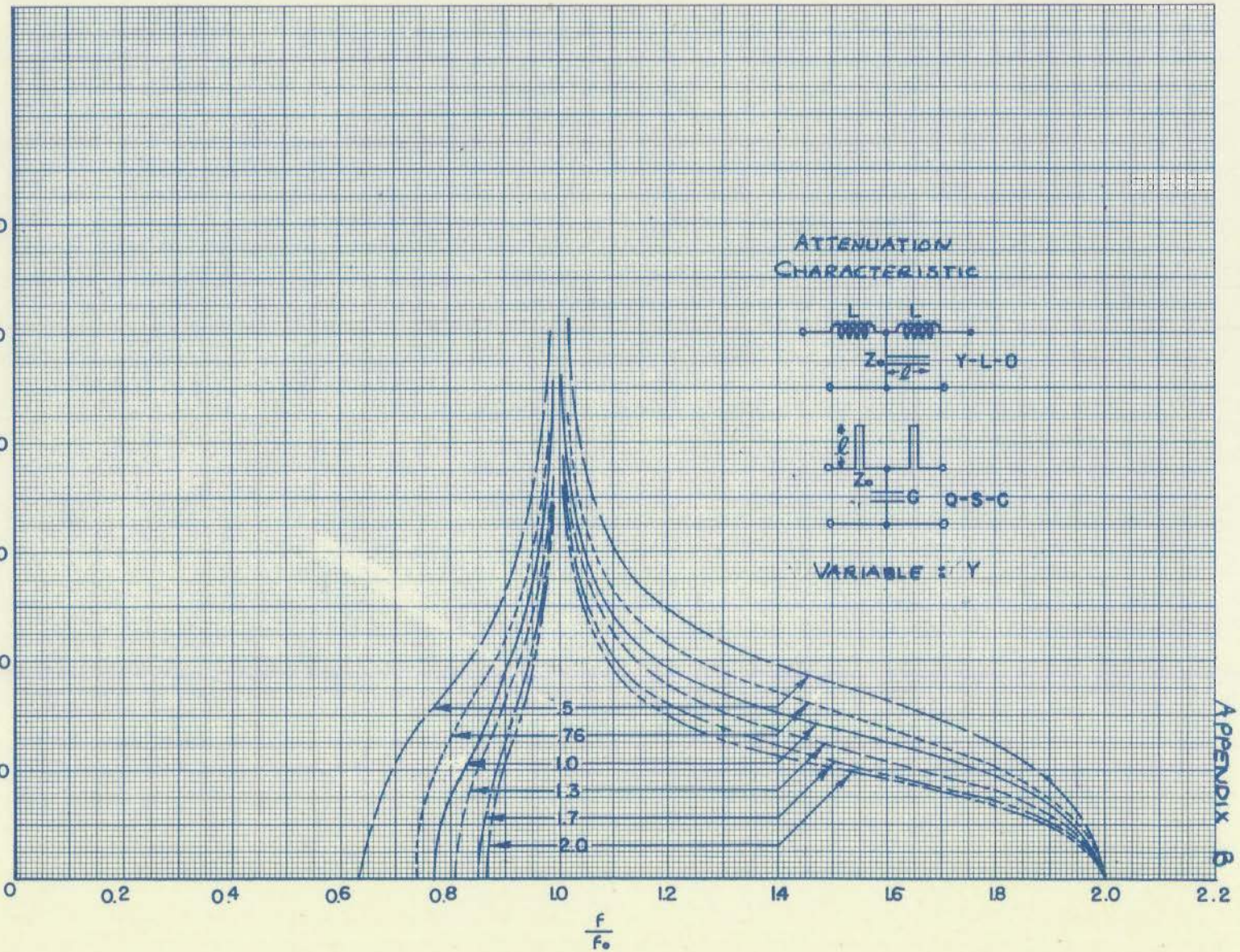


PLATE B-9

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PLATE B-10

APPENDIX B

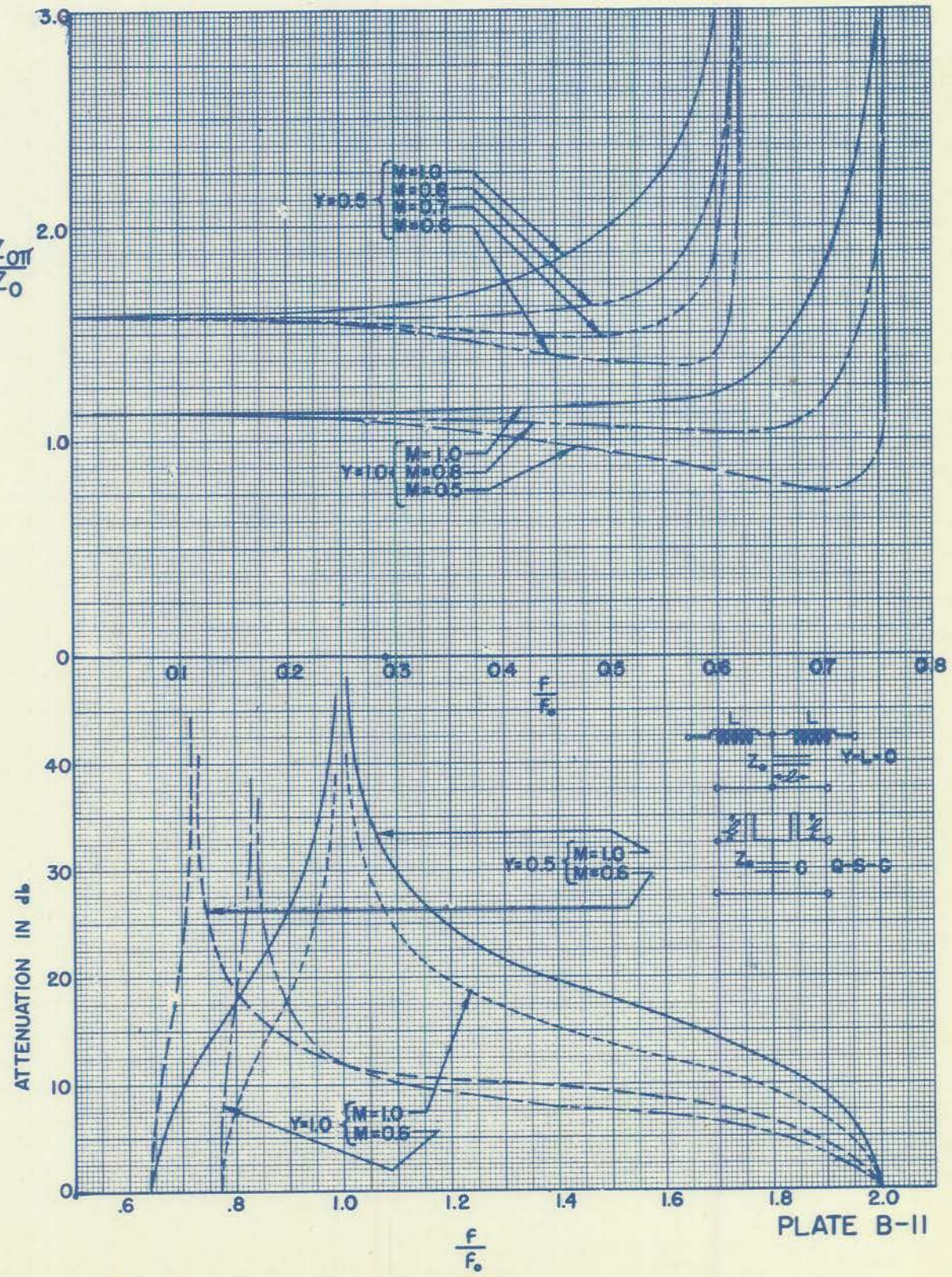
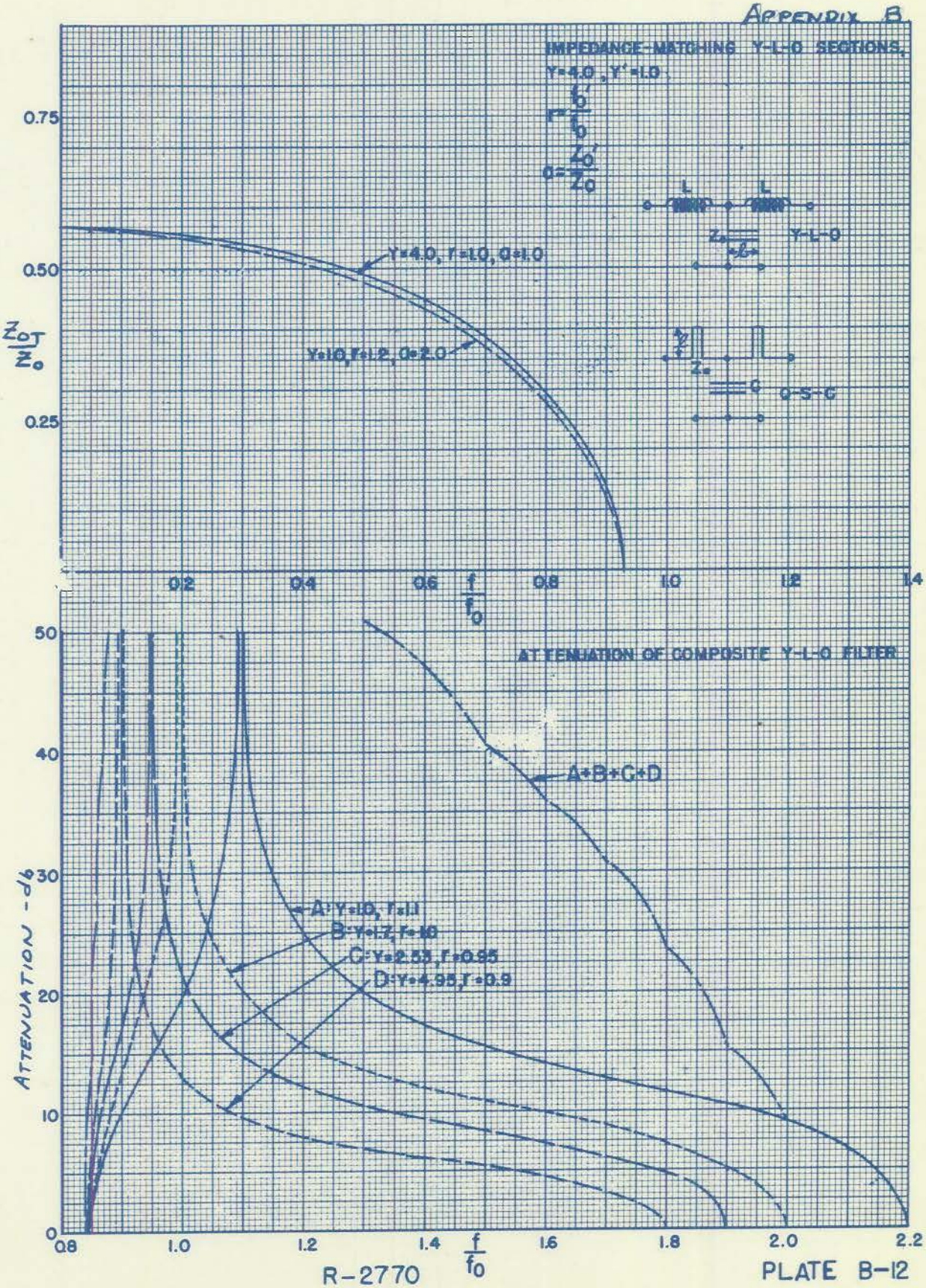
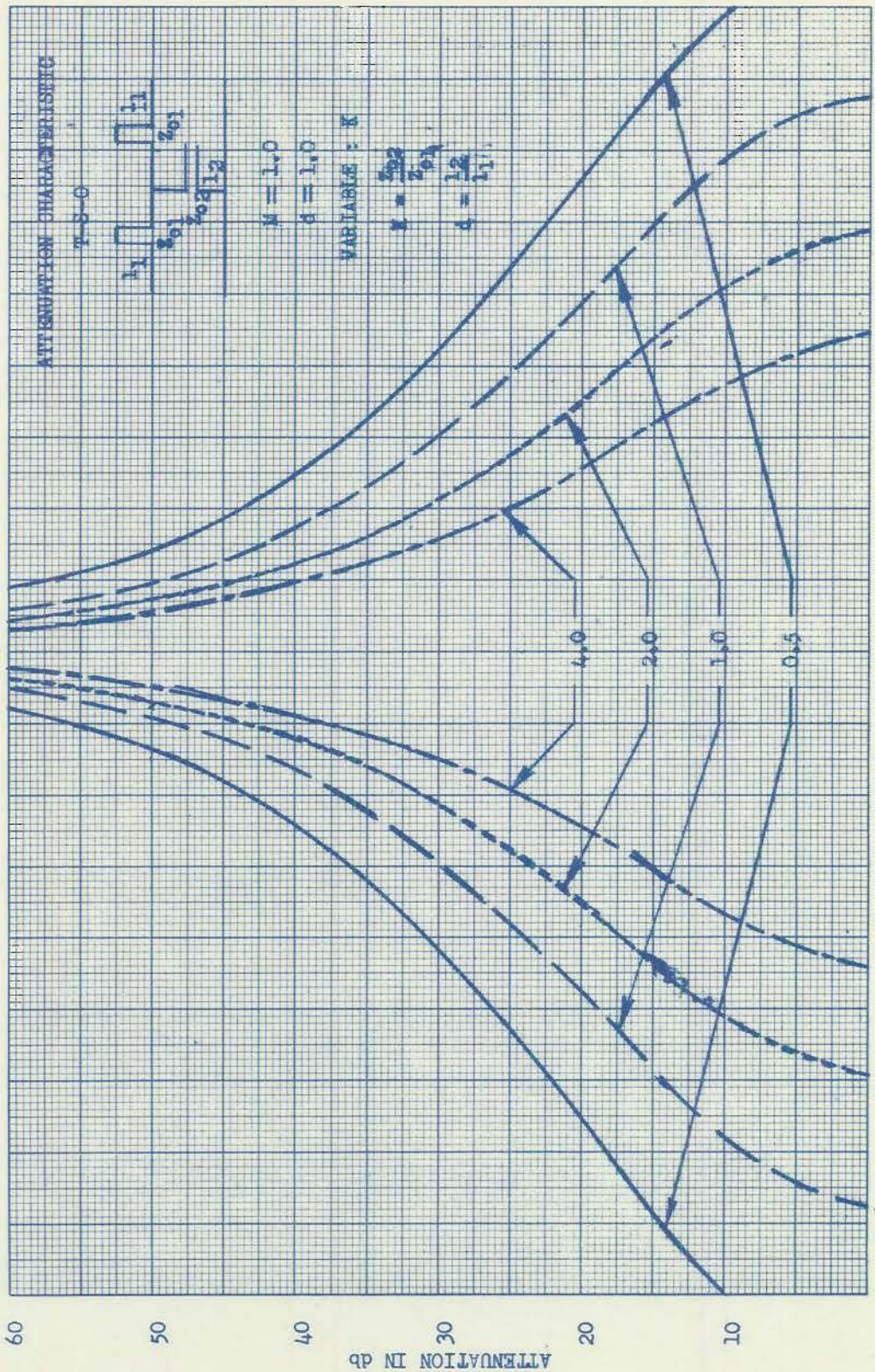

3.0
2.0
1.0
0
$\frac{Z_{0m}}{Z_0}$
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Y=1.0 { M=1.0 M=0.8 M=0.5
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
$\frac{f}{f_0}$
40
30
20
10
0
ATTENUATION IN db
Y=0.5 { M=1.0 M=0.6
Y=1.0 { M=1.0 M=0.5
.6 .8 1.0 1.2 1.4 1.6 1.8 2.0
$\frac{f}{f_0}$
L L
Z_0 Z_0
Y=L=0
Z_0 = 0 Q-S-C

PLATE B-II



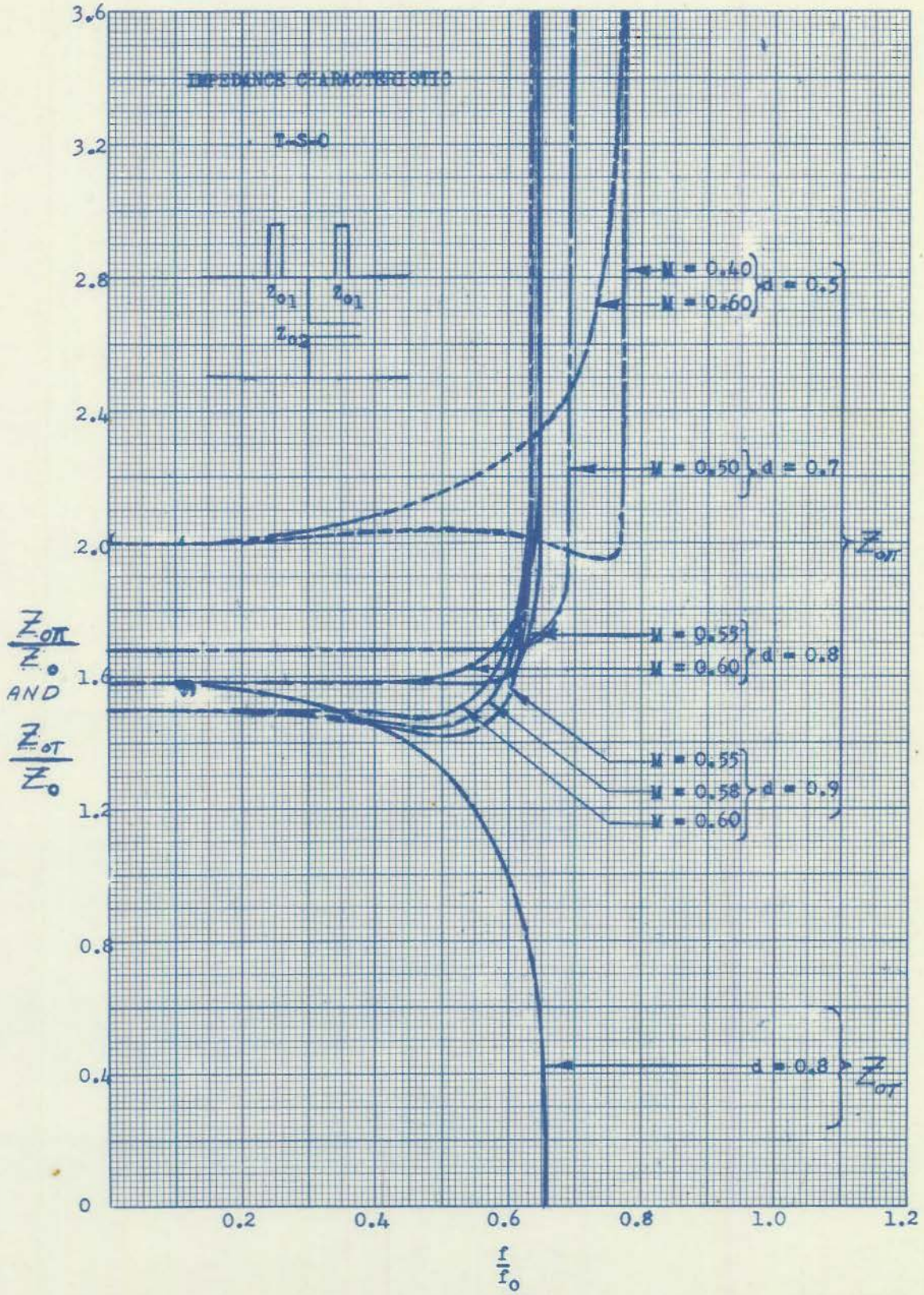
R-2770

PLATE B-12



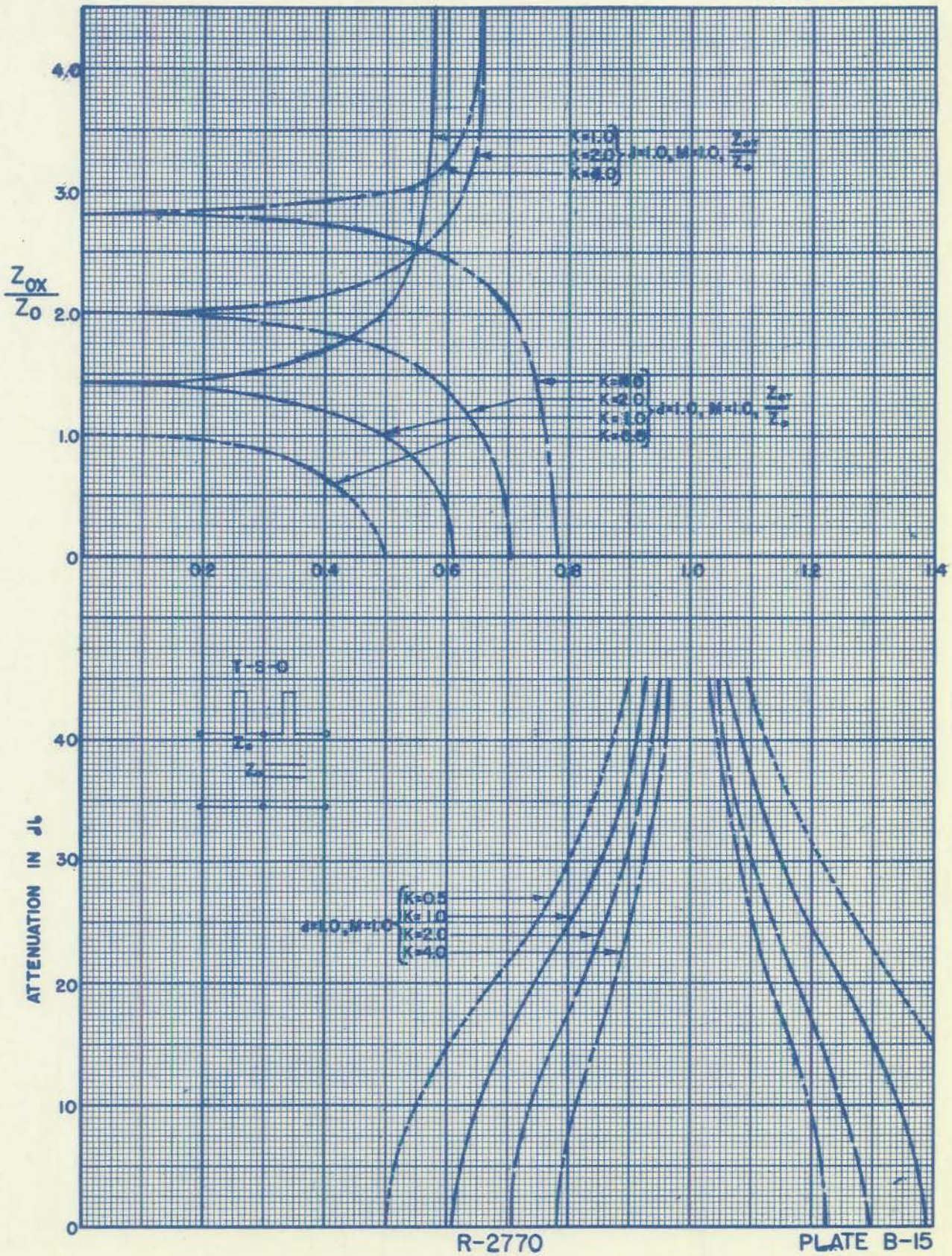
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PLATE B-13



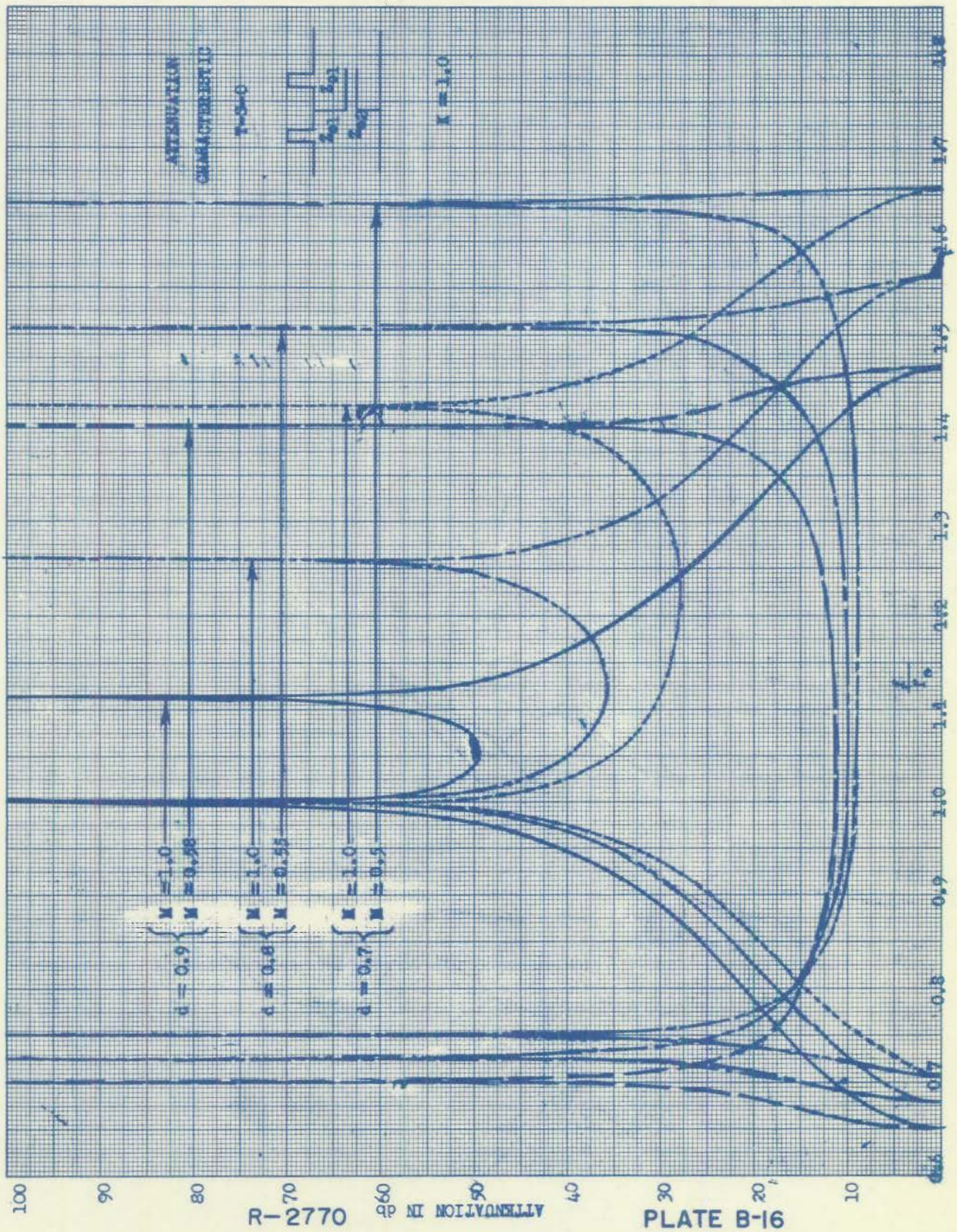
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PLATE B-14



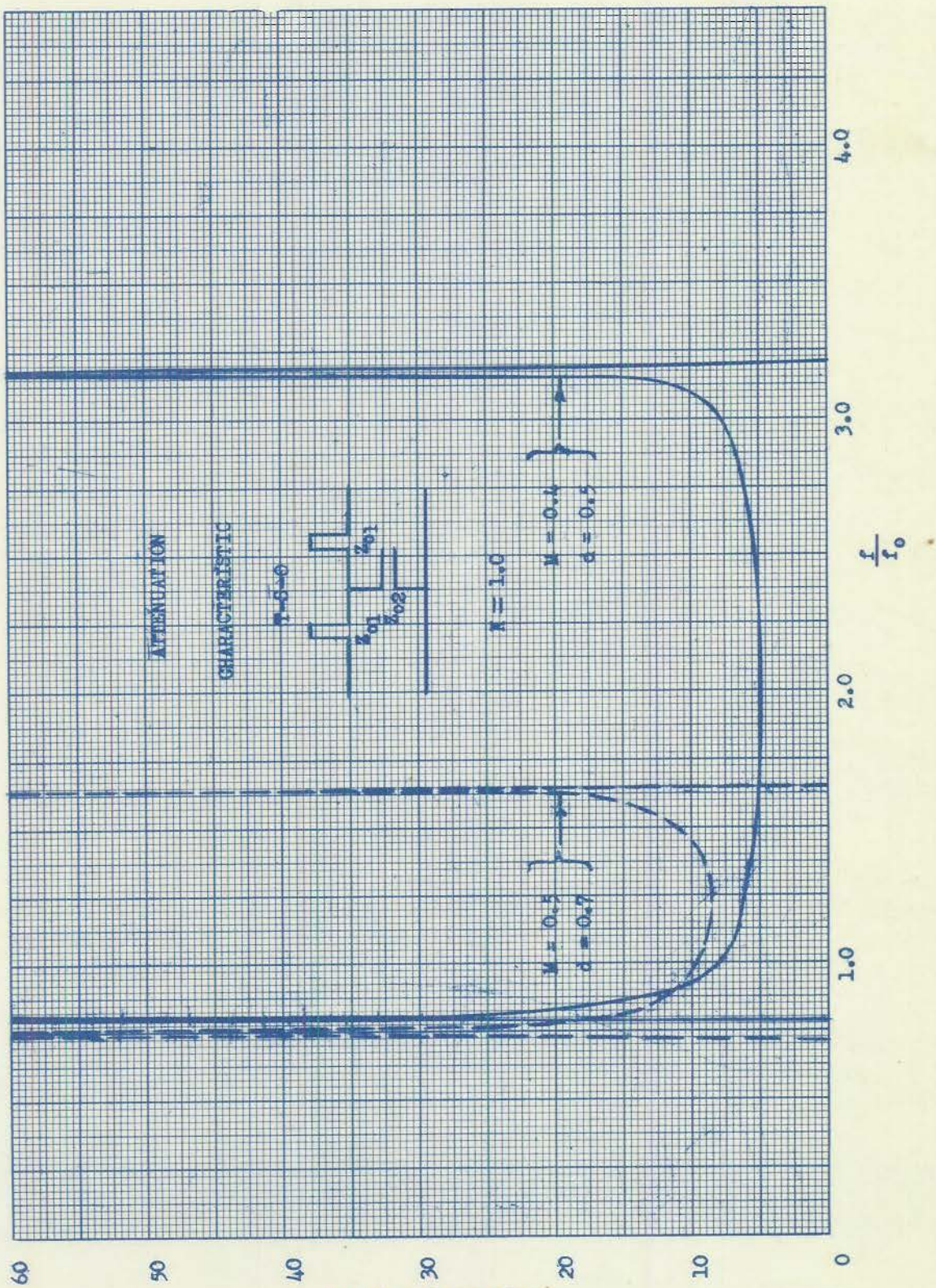
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PLATE B-15



R-2770

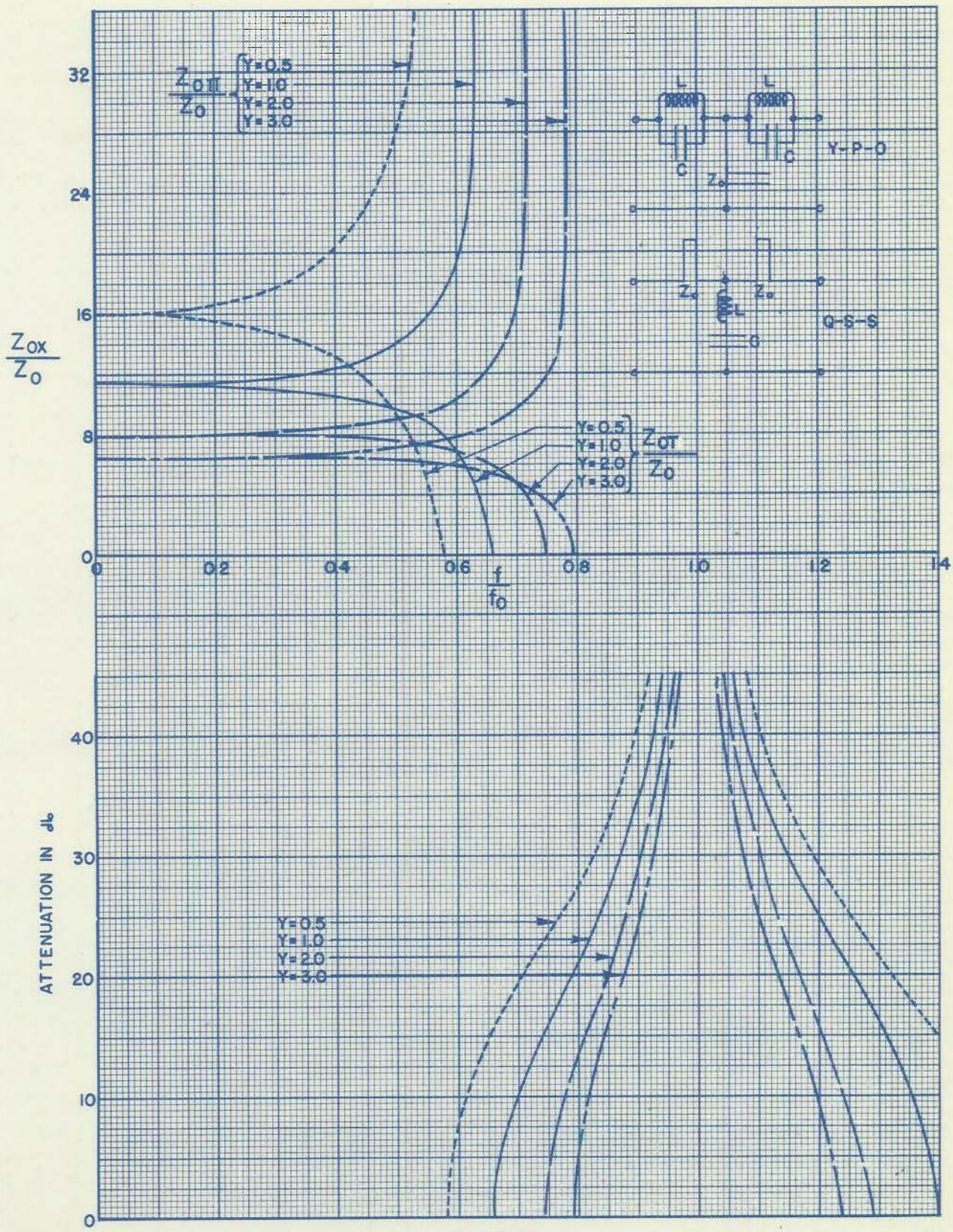
PLATE B-16



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PLATE B-17

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PLATE B-18A

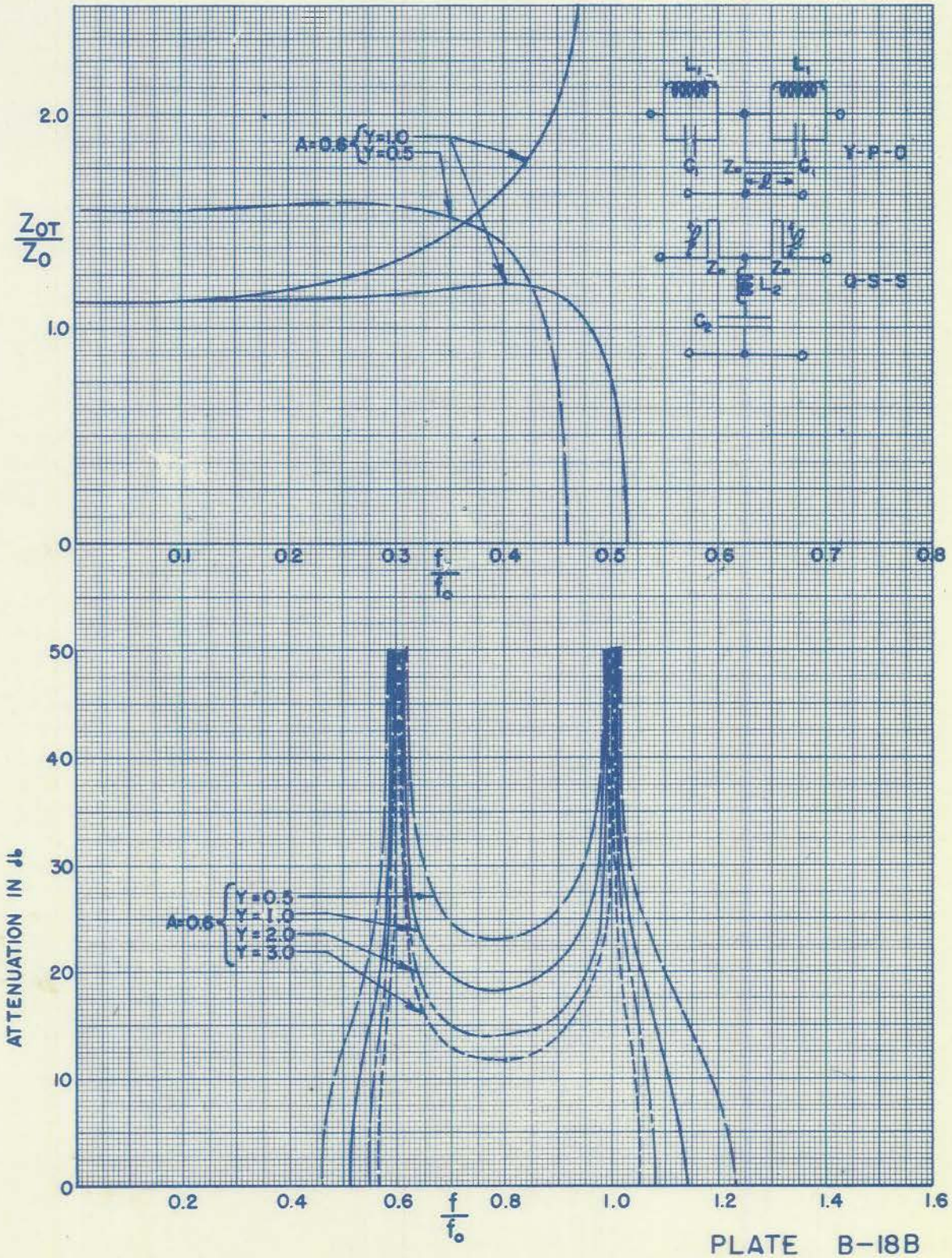


PLATE B-18B

R-2770

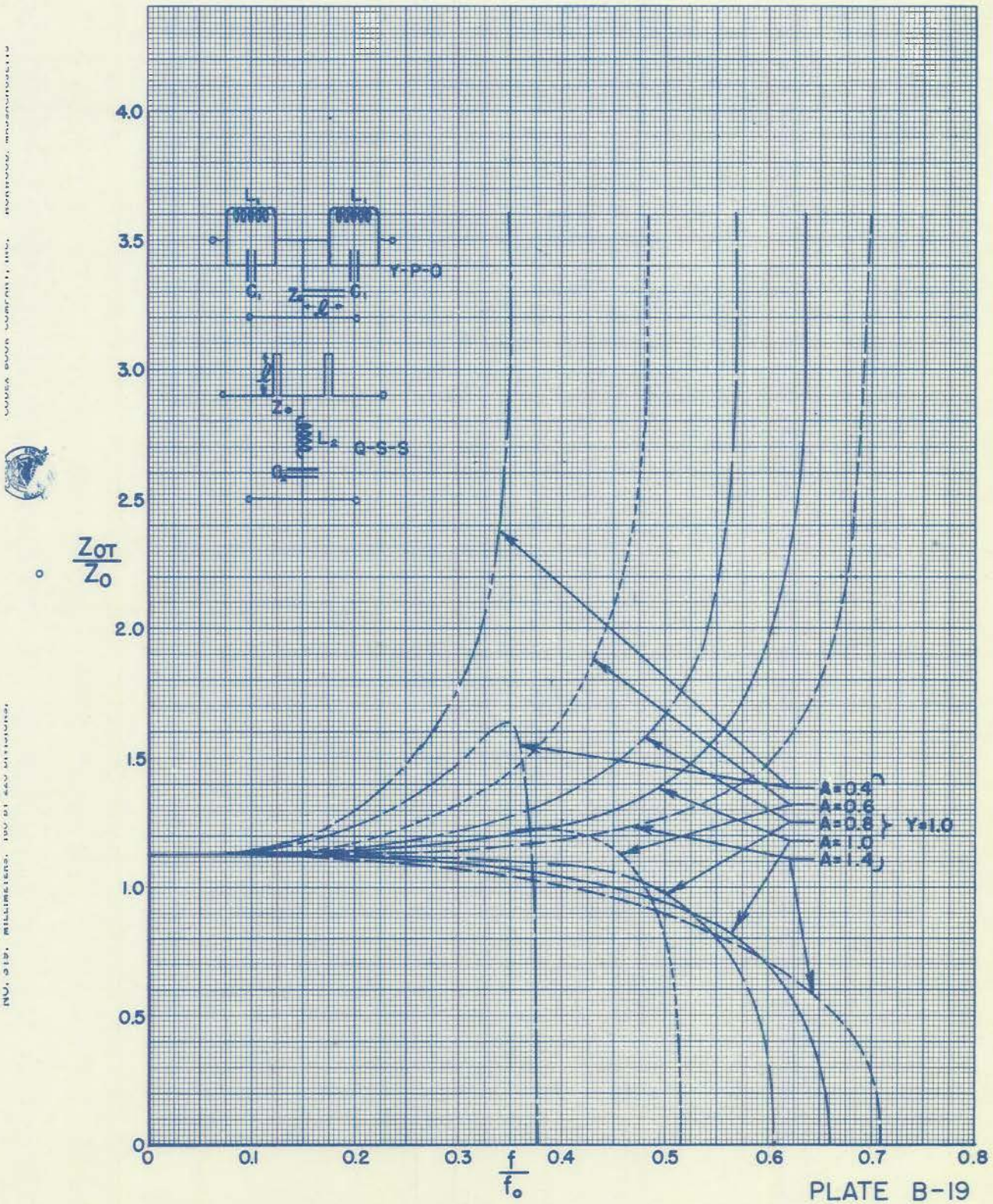
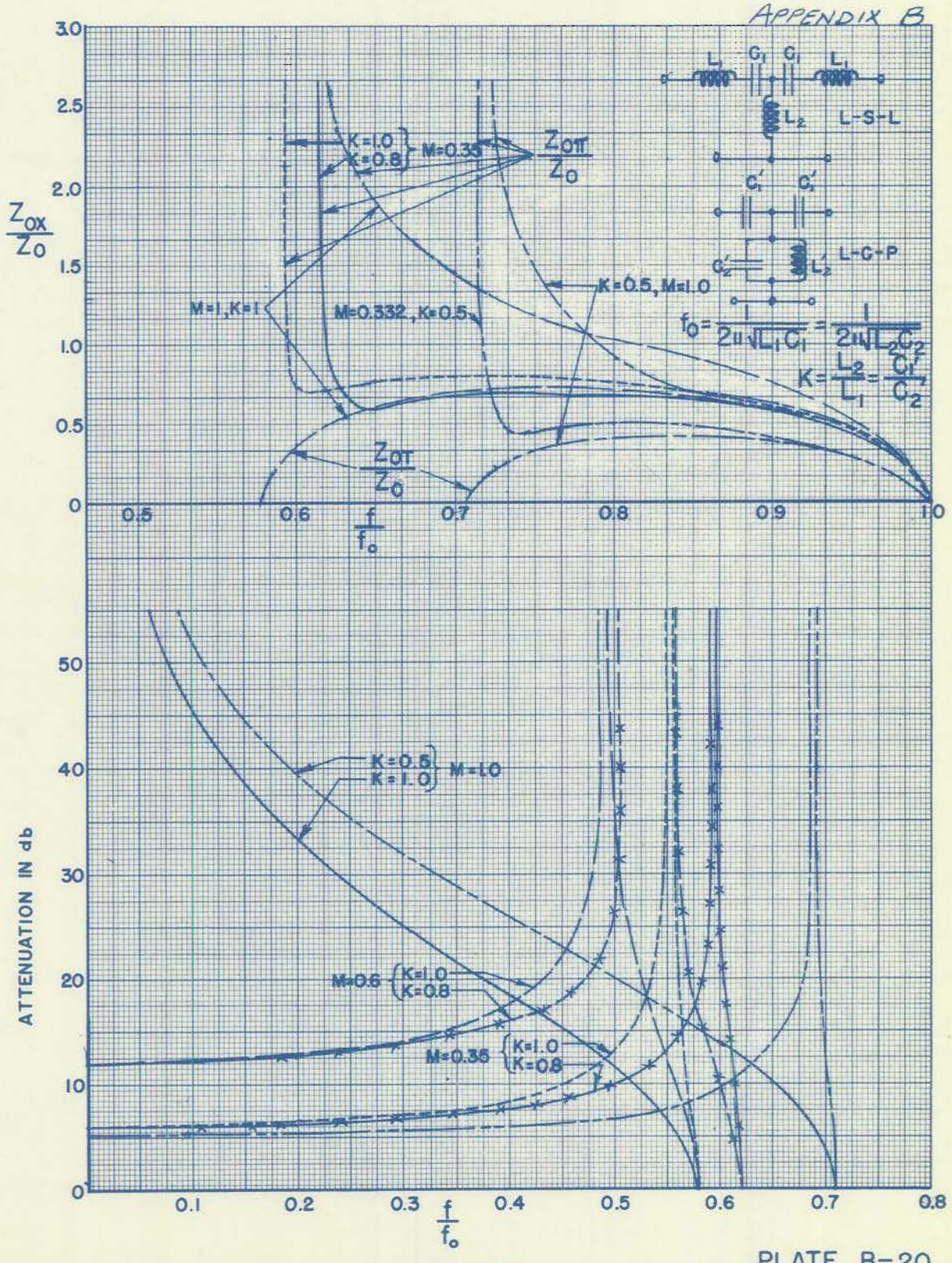


PLATE B-19

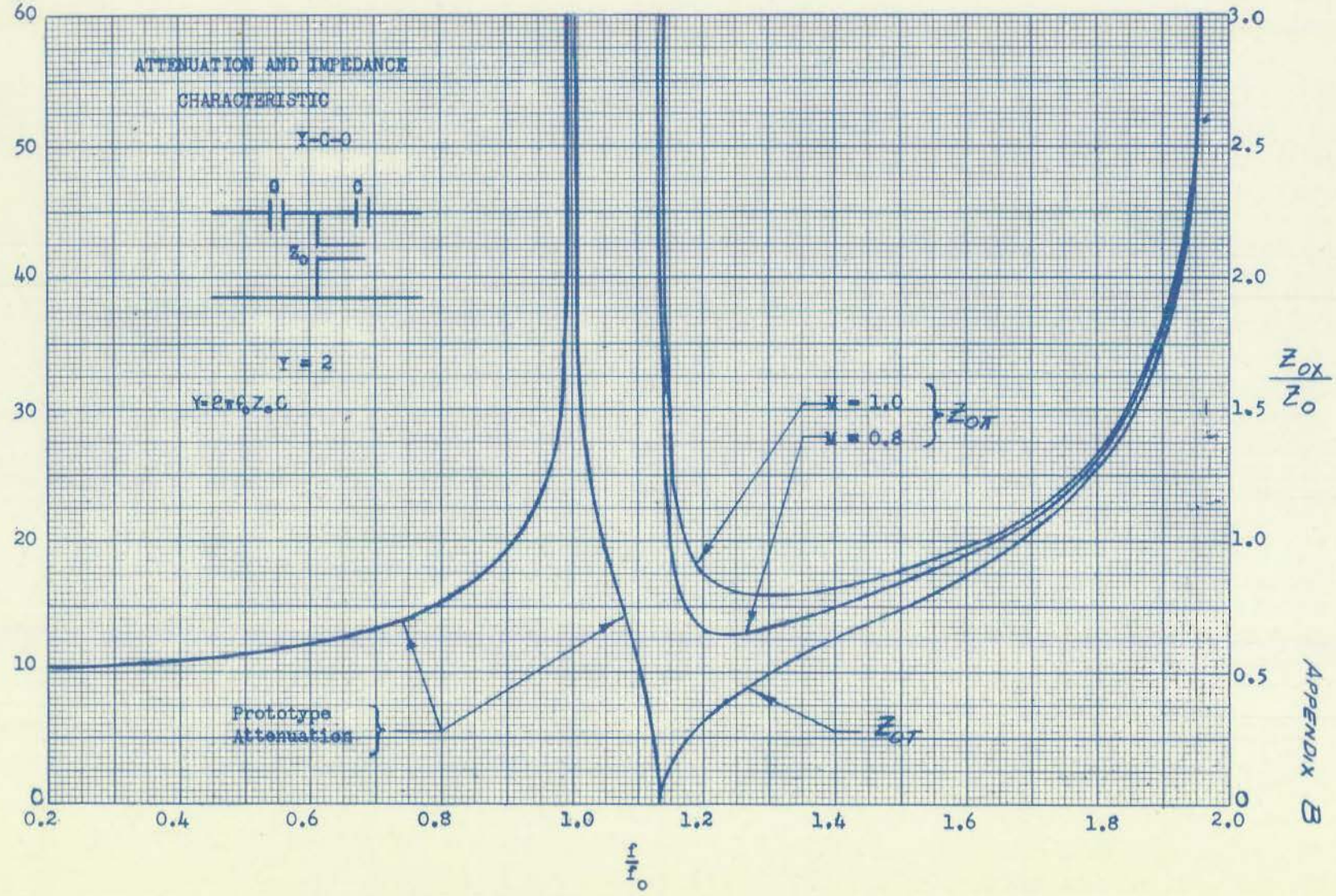
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PLATE B-20

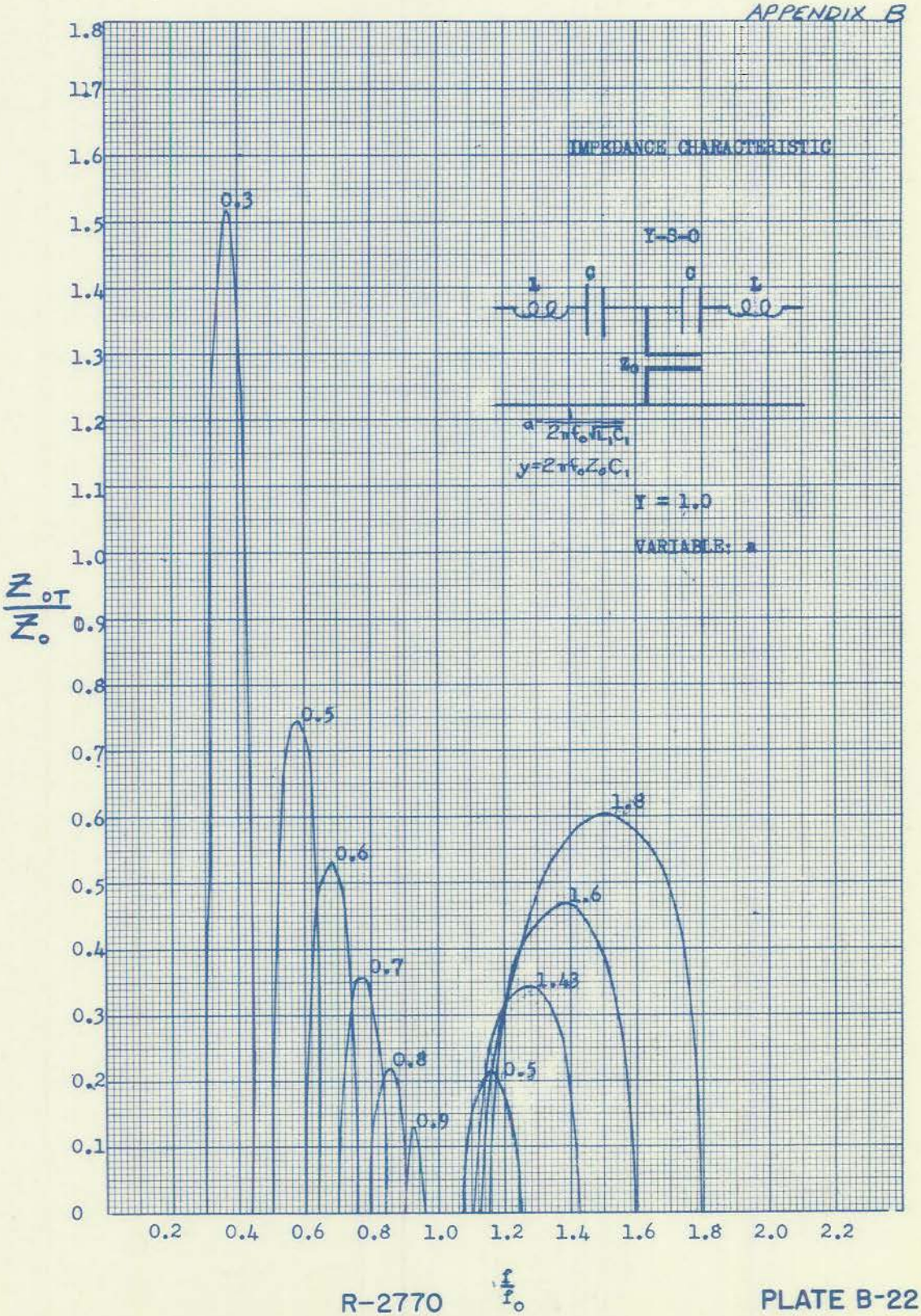
NO. 3110. 20 DIVISIONS PER INCH BOTH WAYS. 120 BY 180 DIVISIONS. CODEX BOOK COMPANY, INC. NORWOOD MASSACHUSETTS.



R-2770

PLATE B-21

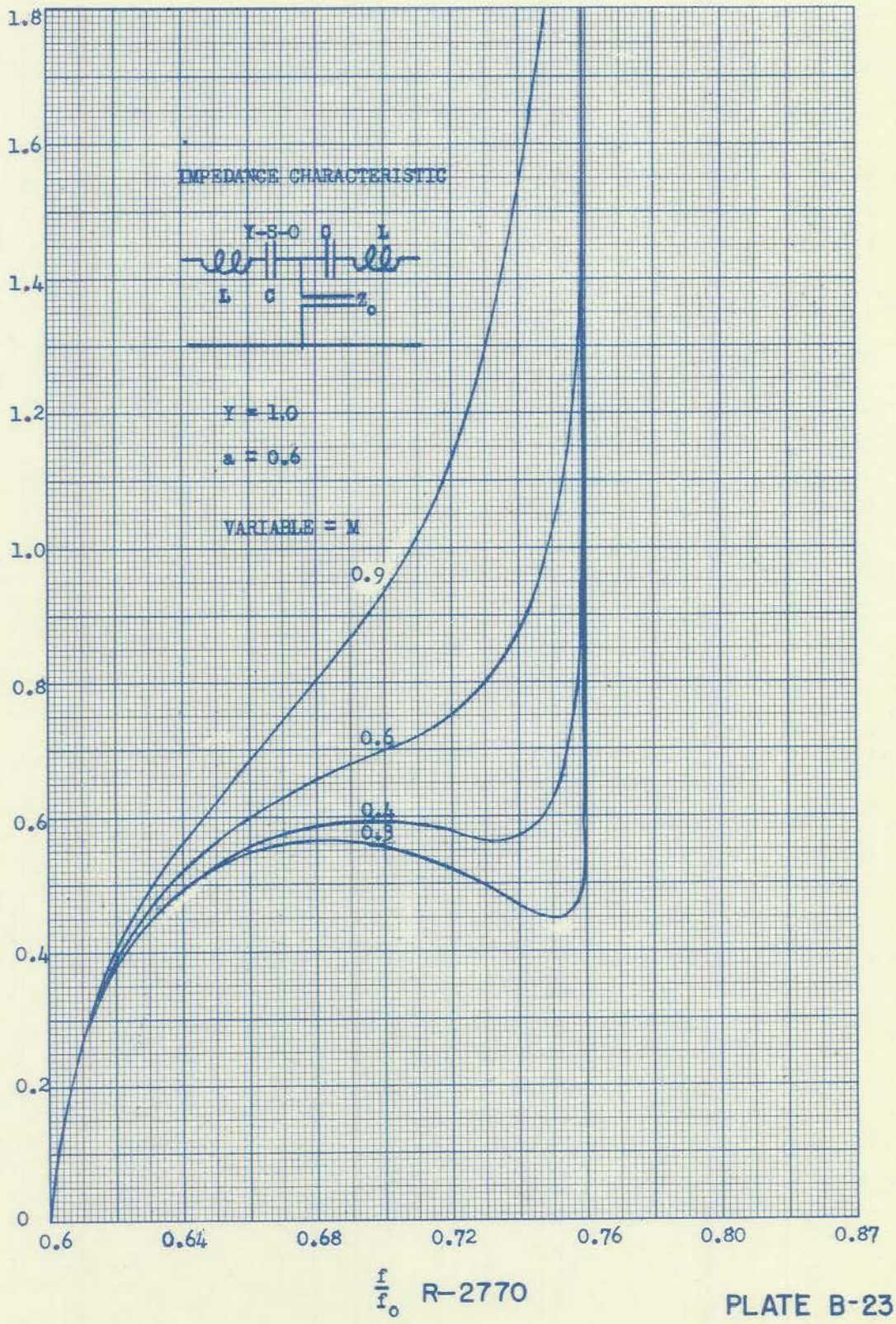
APPENDIX B



R-2770

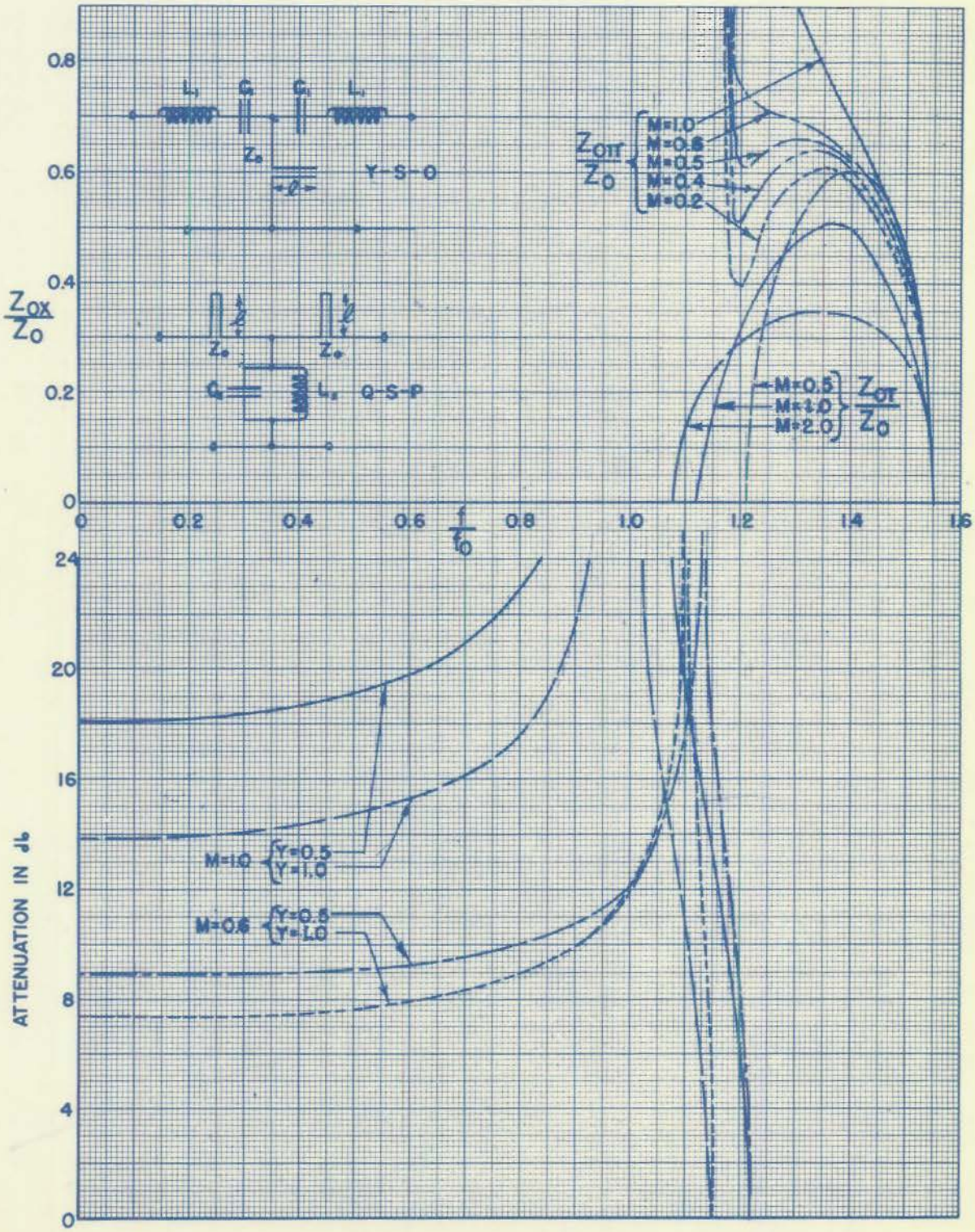
PLATE B-22

CODEX BOOK COMPANY, INC., NORWOOD, MASSACHUSETTS
PRINTED IN U.S.A.



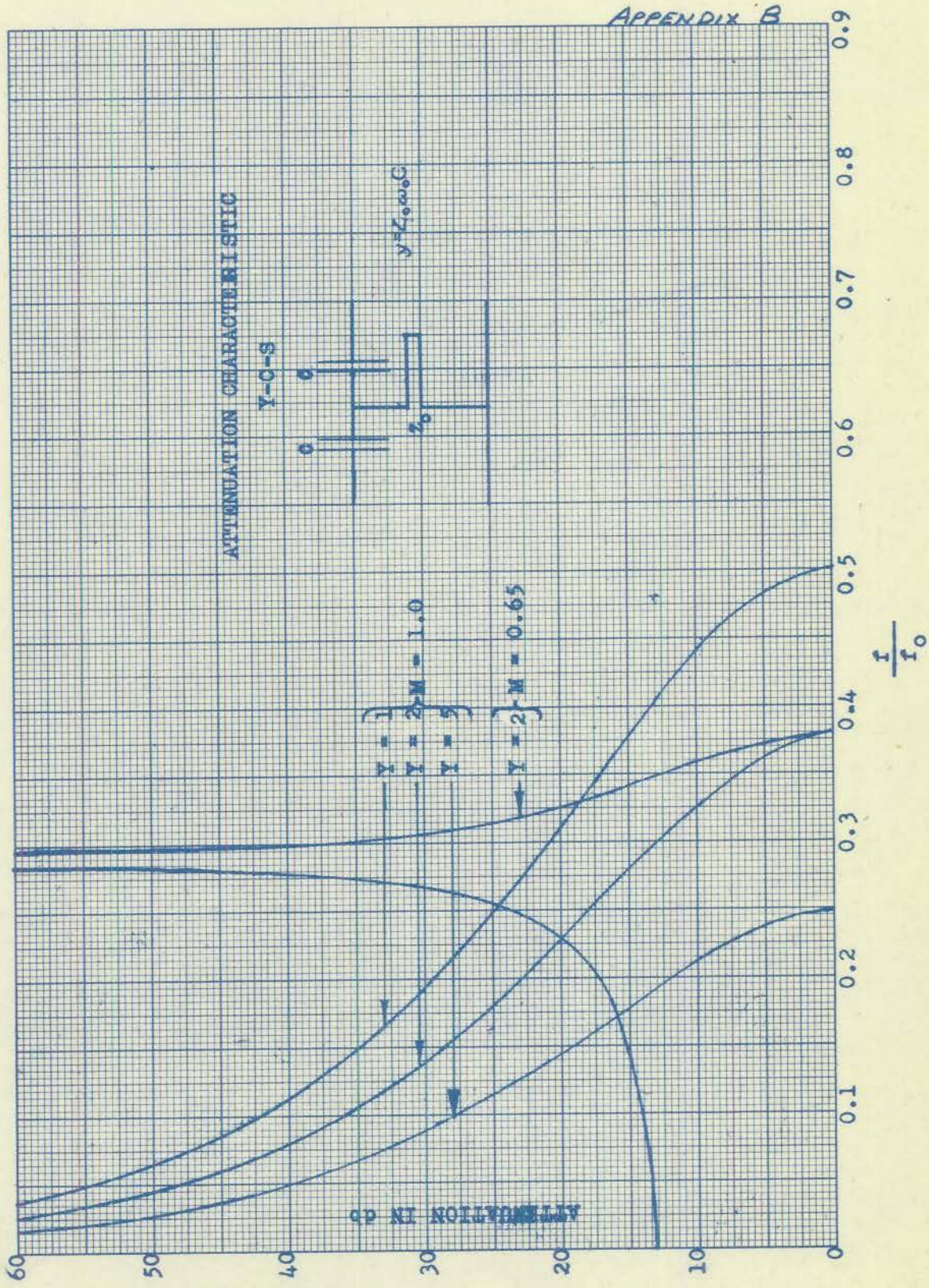
R-2770

PLATE B-23



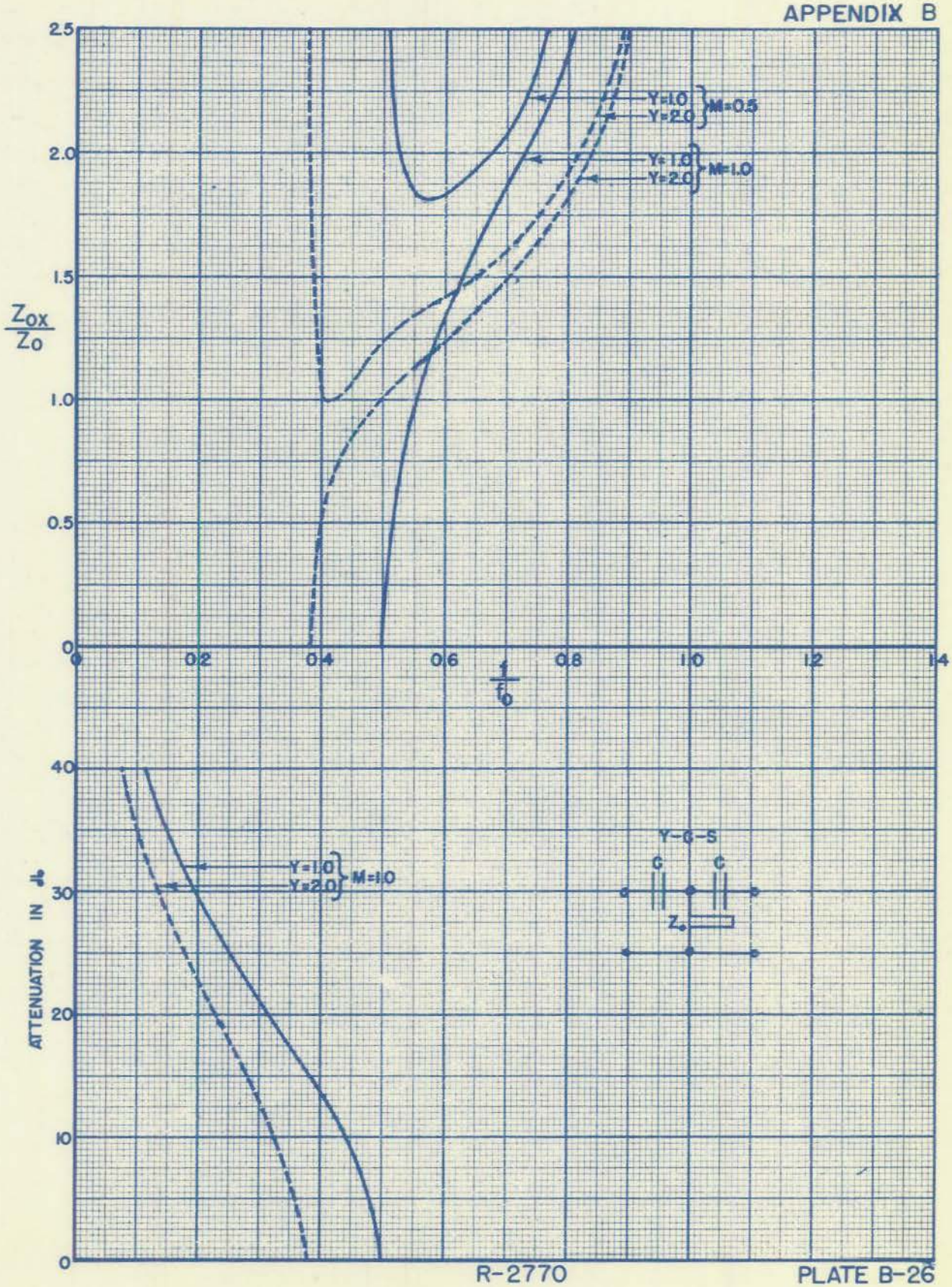
R-2770

PLATE B-24



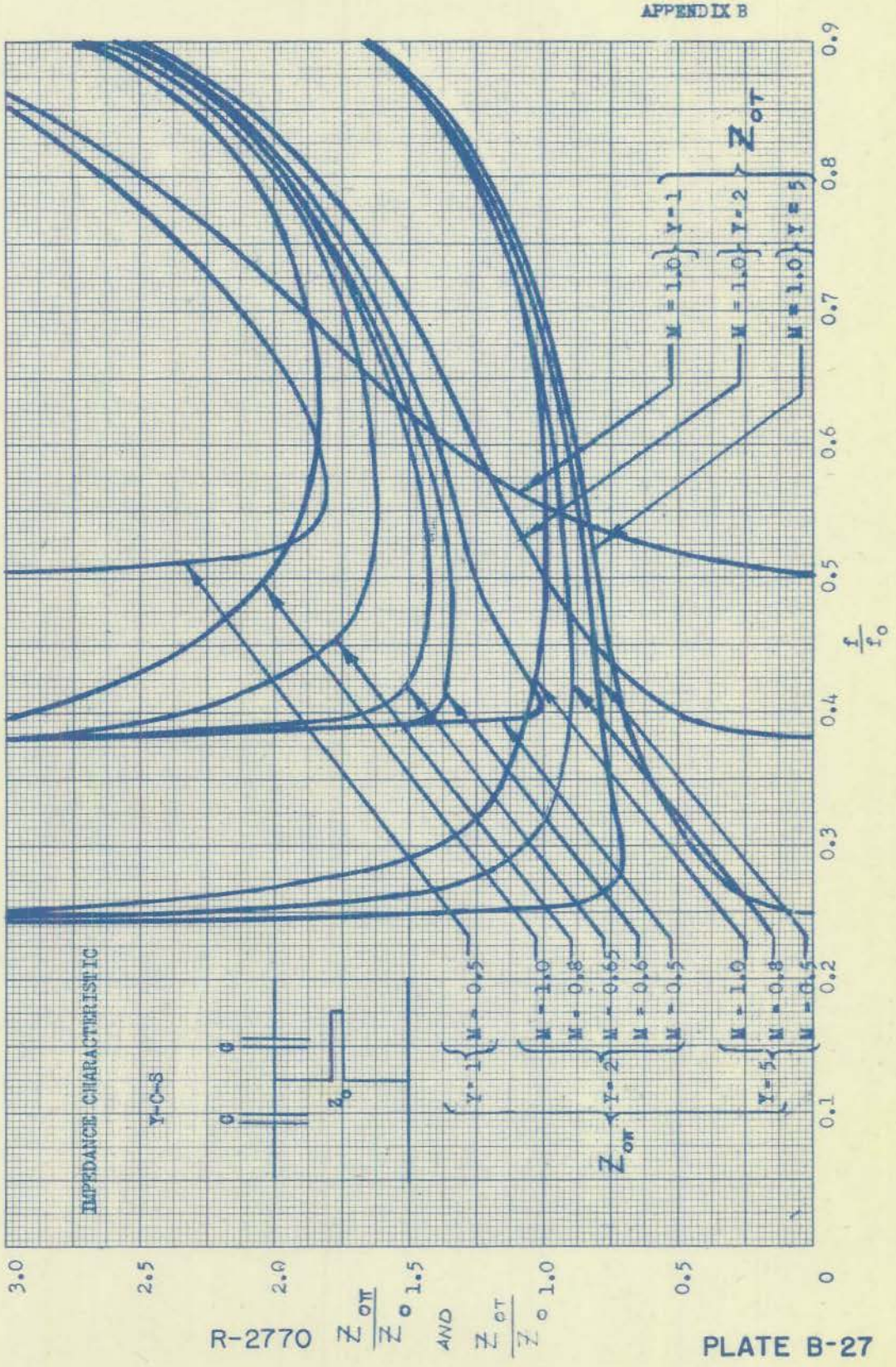
R-2770

PLATE B-25



R-2770

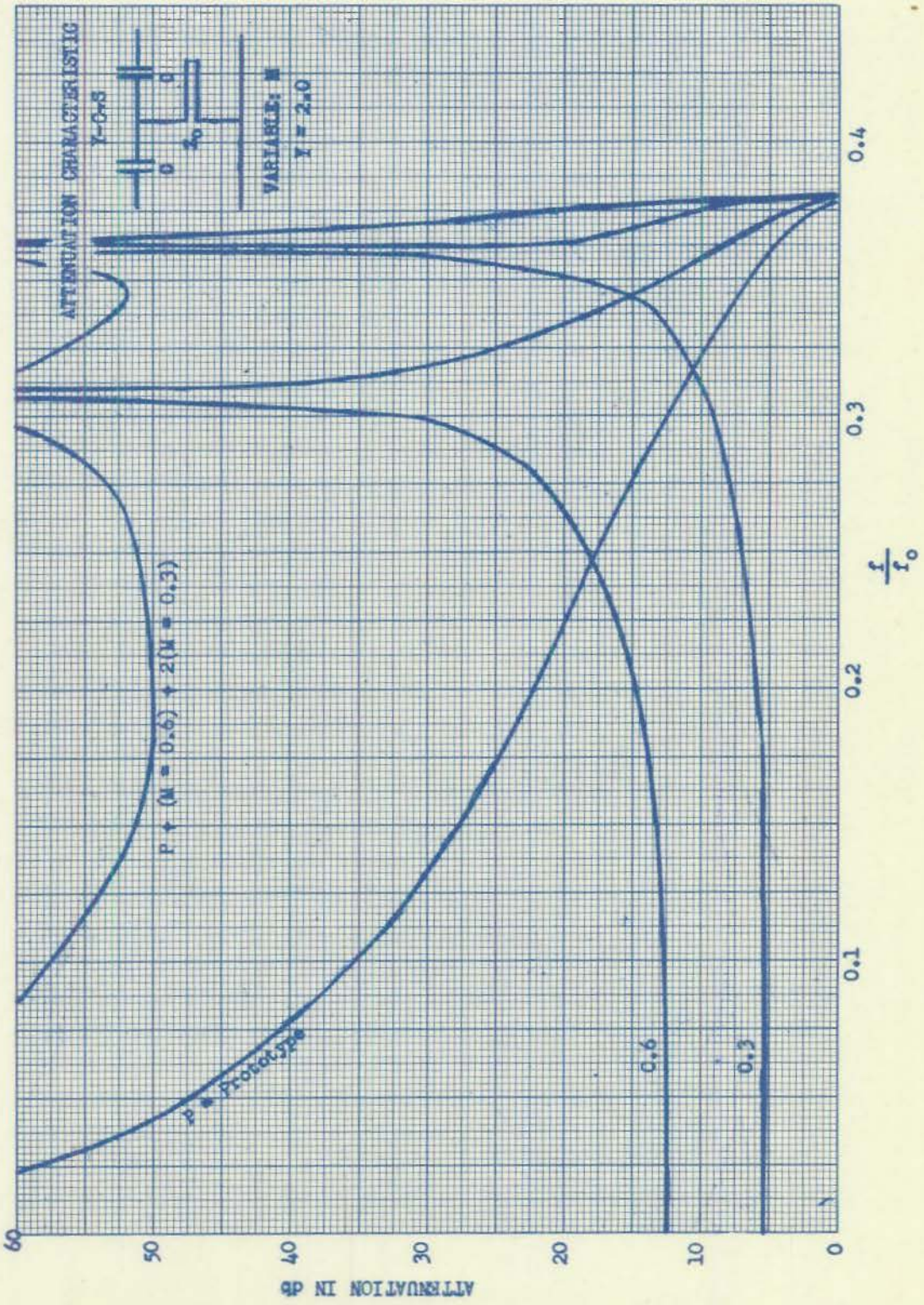
PLATE B-26

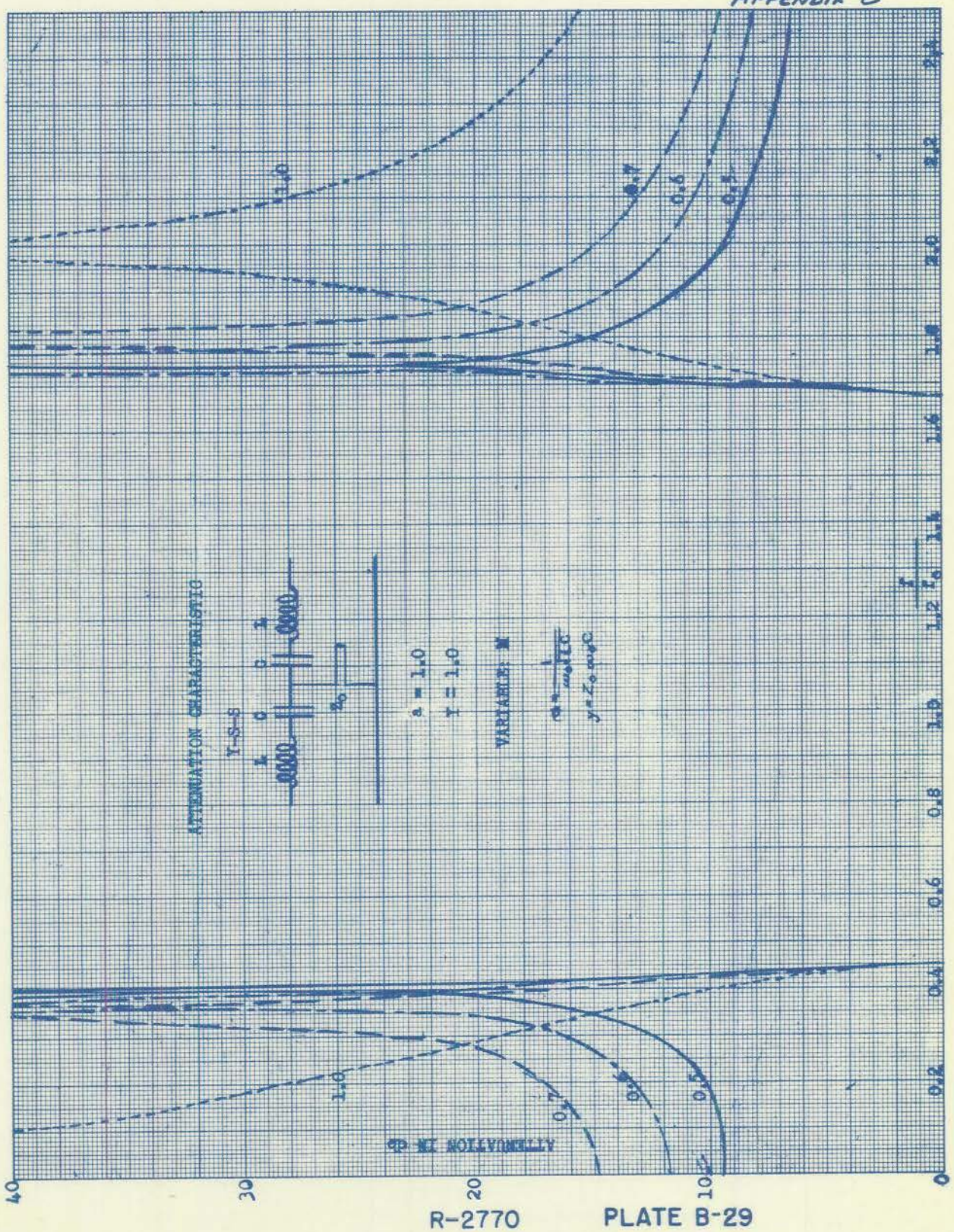


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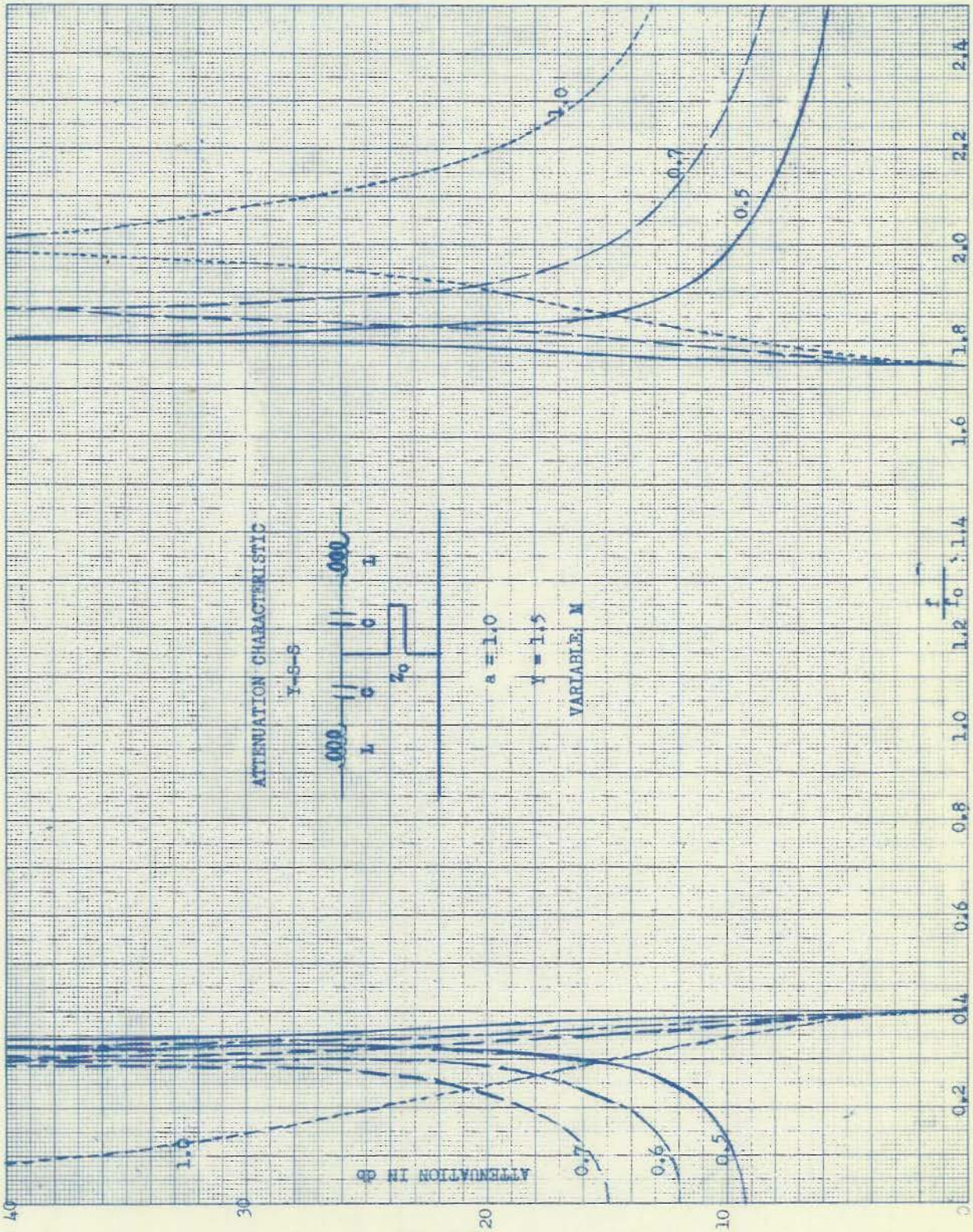
PLATE B-27

CODEX BOOK COMPANY, INC., NORWOOD, MASSACHUSETTS
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ATTENUATION CHARACTERISTIC
Y=0.5
VARIABLE: M
X = 2.0
P + (M = 0.6) + 2(M = 0.3)
P = Prototype
0.6
0.3
0.4
0.3
0.2
0.1
0
f/f₀
60
50
40
30
20
10
0
ATTENUATION IN db



R-2770 PLATE B-29



R-2770 PLATE B-30

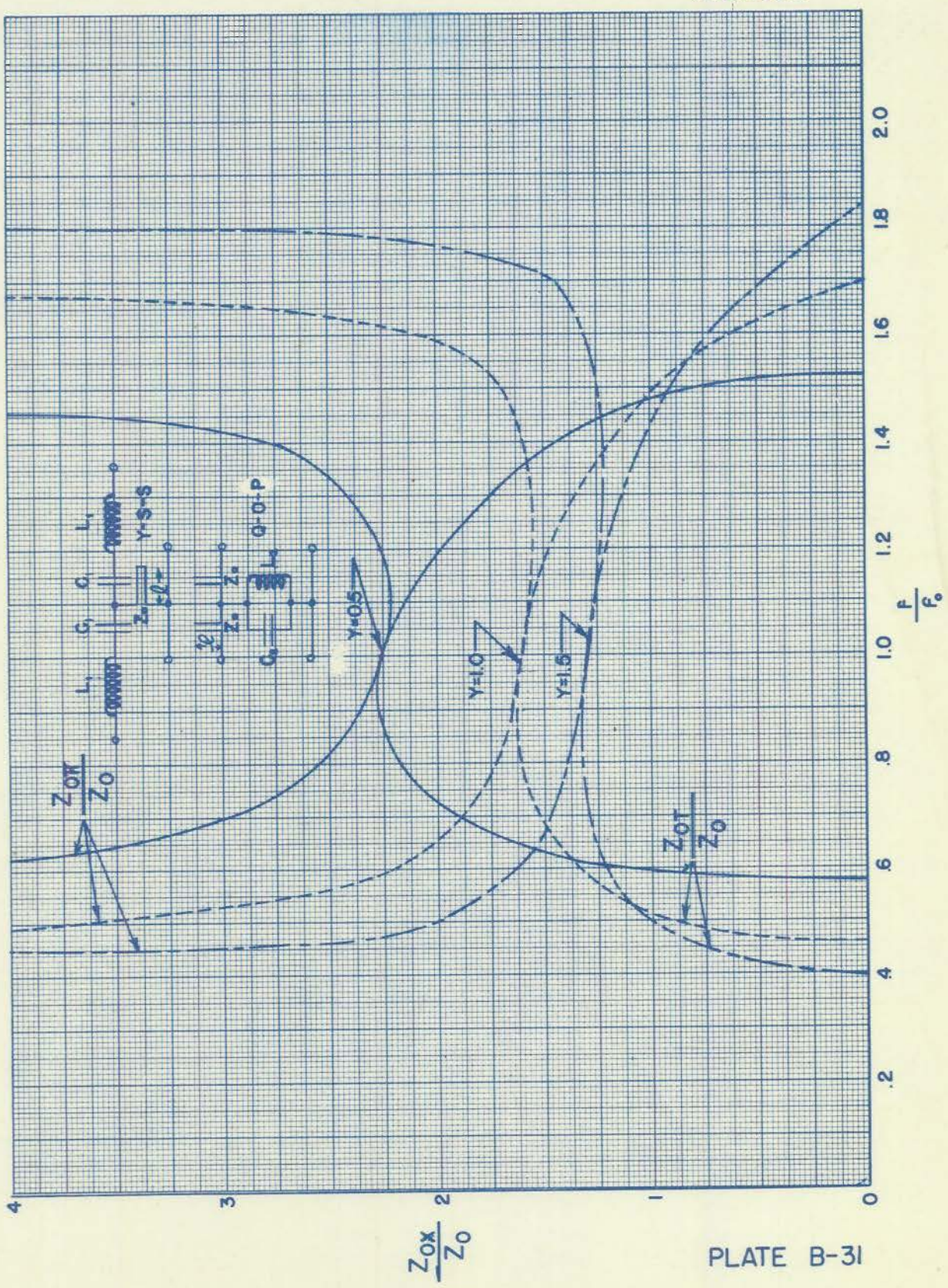
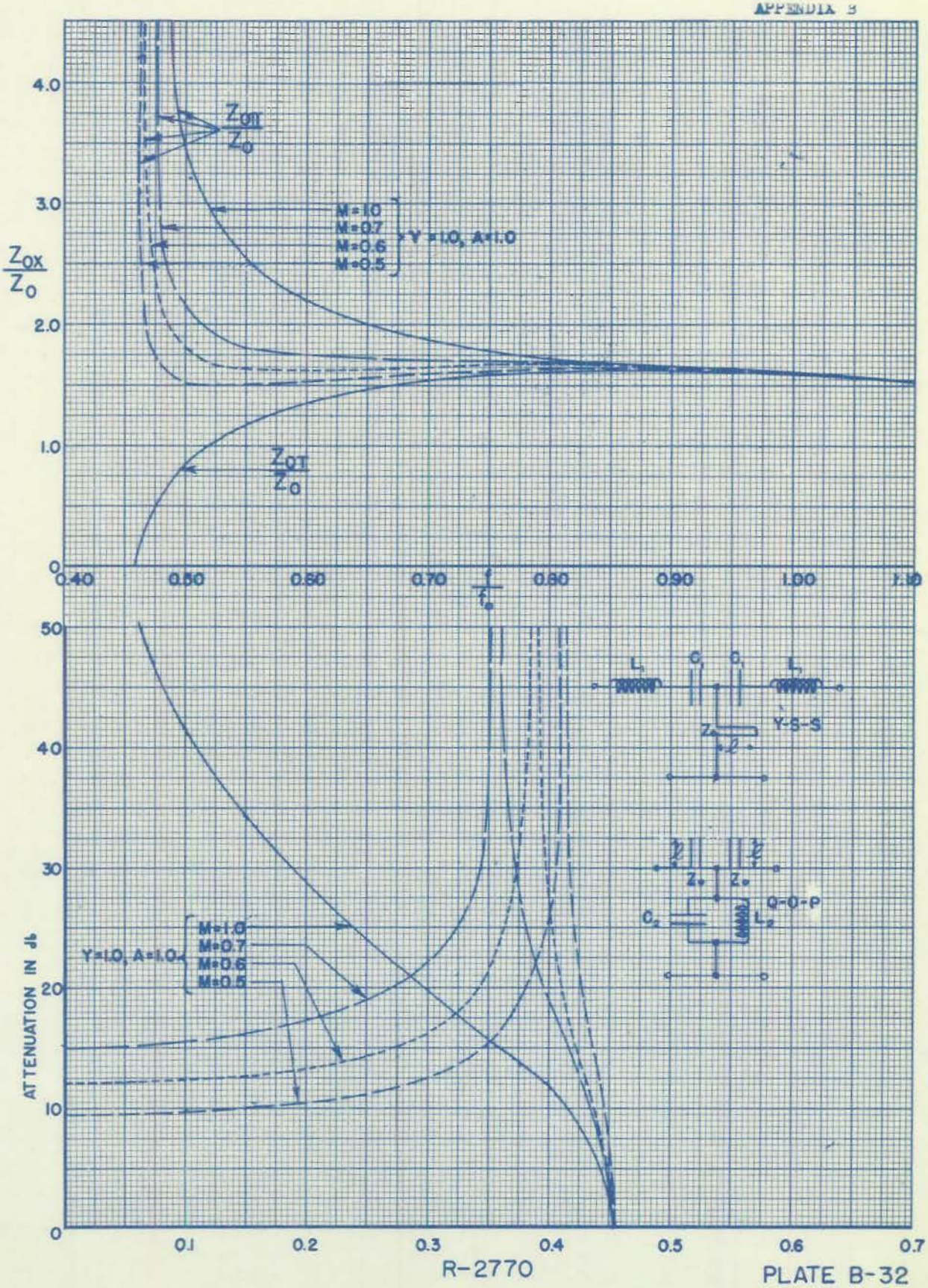


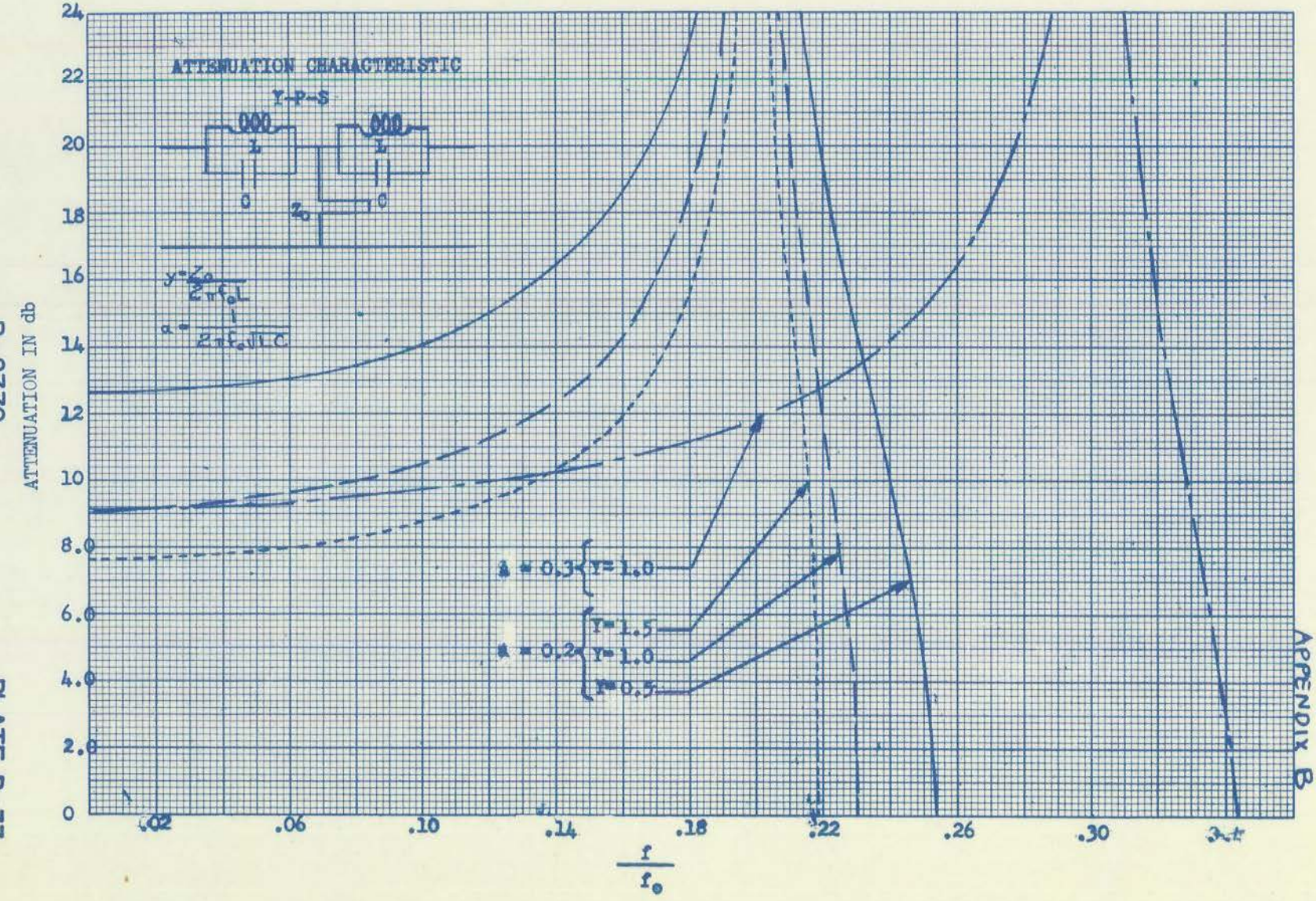
PLATE B-31

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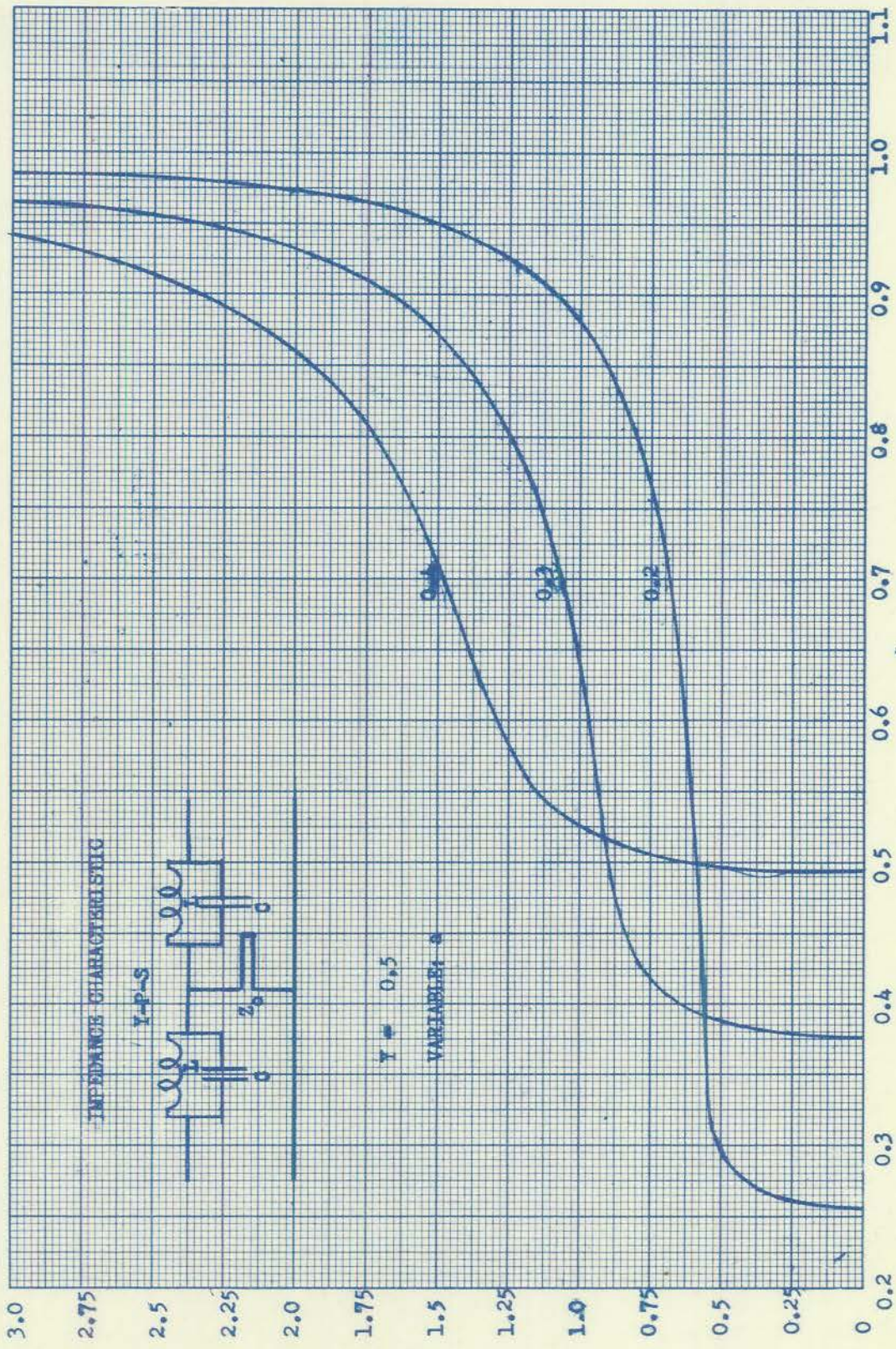
PLATE B-32



R-2770

PLATE B-33

APPENDIX B



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PLATE B-34

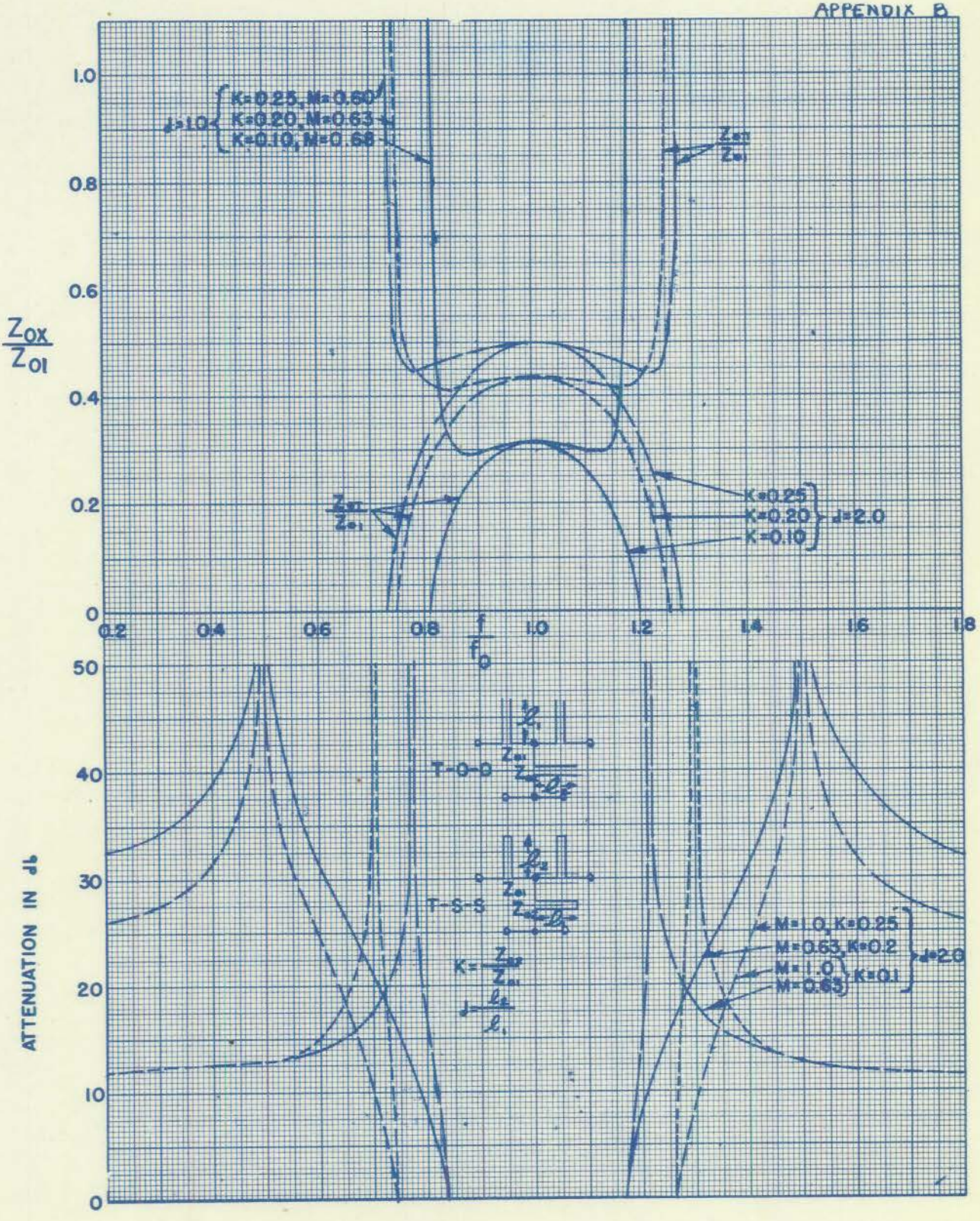
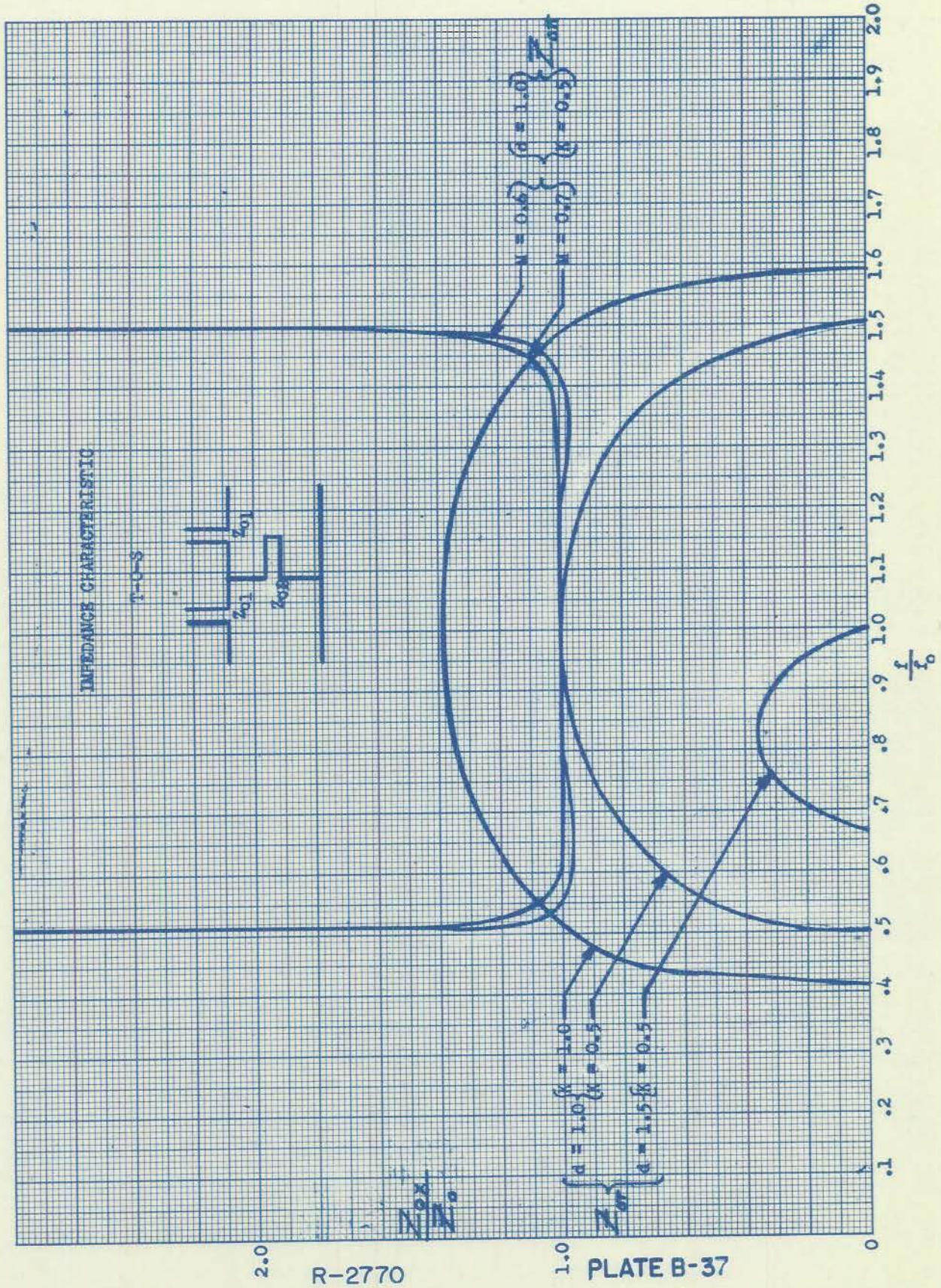

J=1.0
K=0.25, M=0.60
K=0.20, M=0.63
K=0.10, M=0.68
K=0.25
K=0.20
K=0.10
J=2.0
T-O-O
T-S-S
M=1.0, K=0.25
M=0.63, K=0.2
M=1.0
M=0.63
K=0.1
J=2.0
ATTENUATION IN dB

PLATE B-35

ATTENUATION CHARACTERISTICS
T-O-S
K = Z02 / Z01
d = 0.5
k = 0.5
d = 1.0
k = 1.0
f / fo
ATTENUATION IN db
0
10
20
30
40
50
60
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0



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PLATE B-37

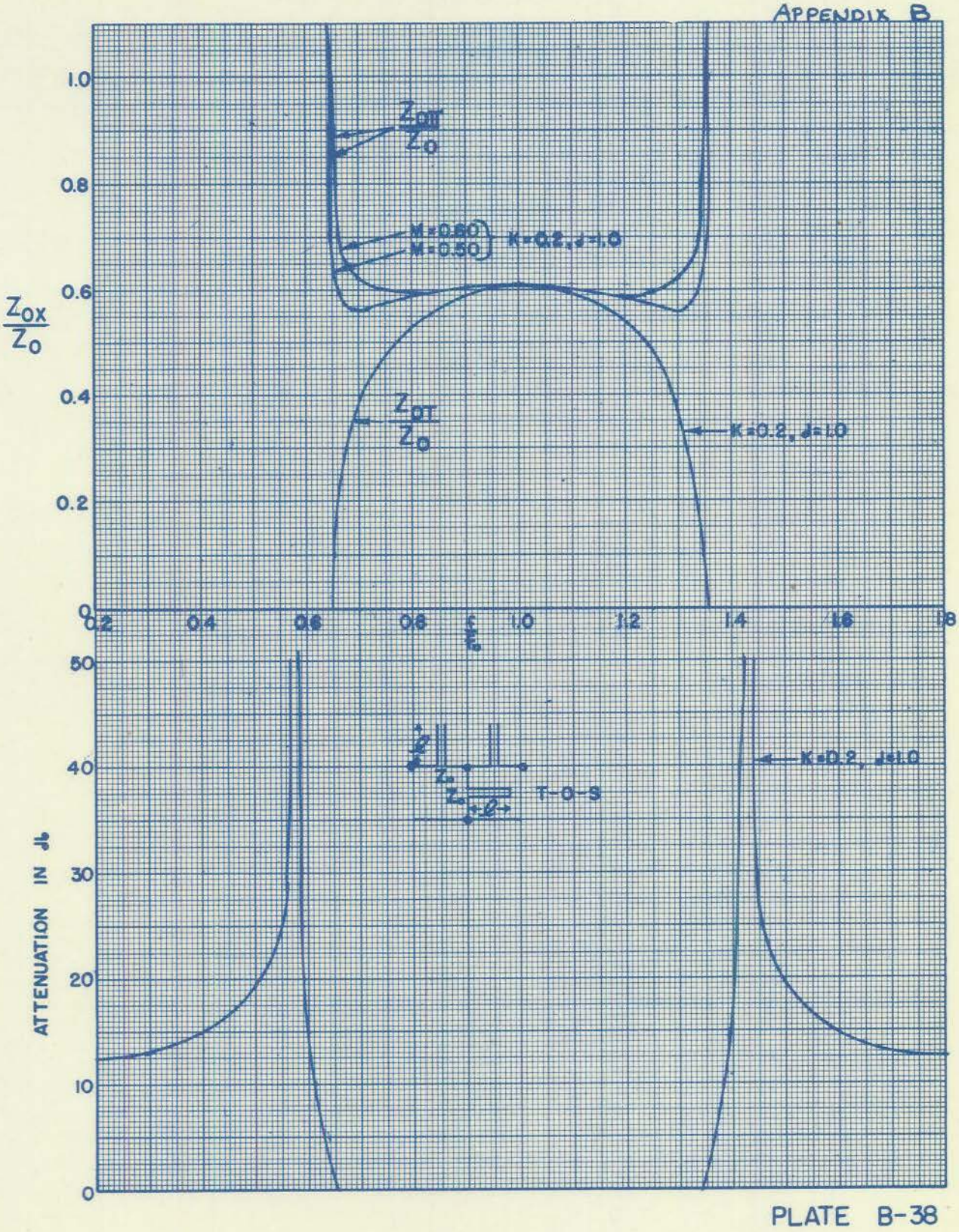

1.0
0.8
0.6
0.4
0.2
0
$\frac{Z_{0X}}{Z_0}$
$\frac{Z_{0T}}{Z_0}$
M=0.80
M=0.50
K=0.2, d=1.0
K=0.2, d=1.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
$\frac{\xi}{\xi_0}$
50
40
30
20
10
0
ATTENUATION IN db
K=0.2, d=1.0
T-0-8

PLATE B-38

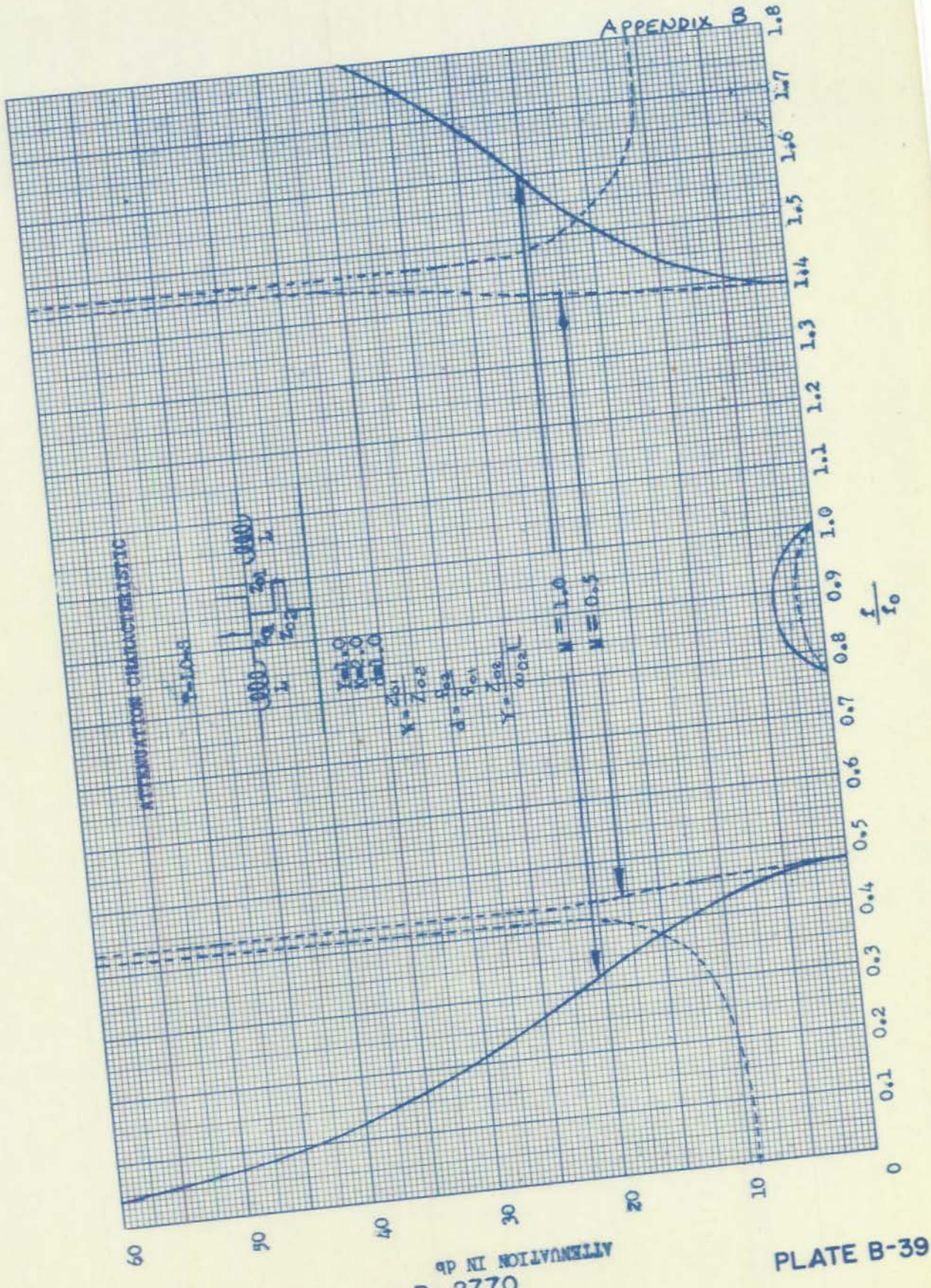


PLATE B-39

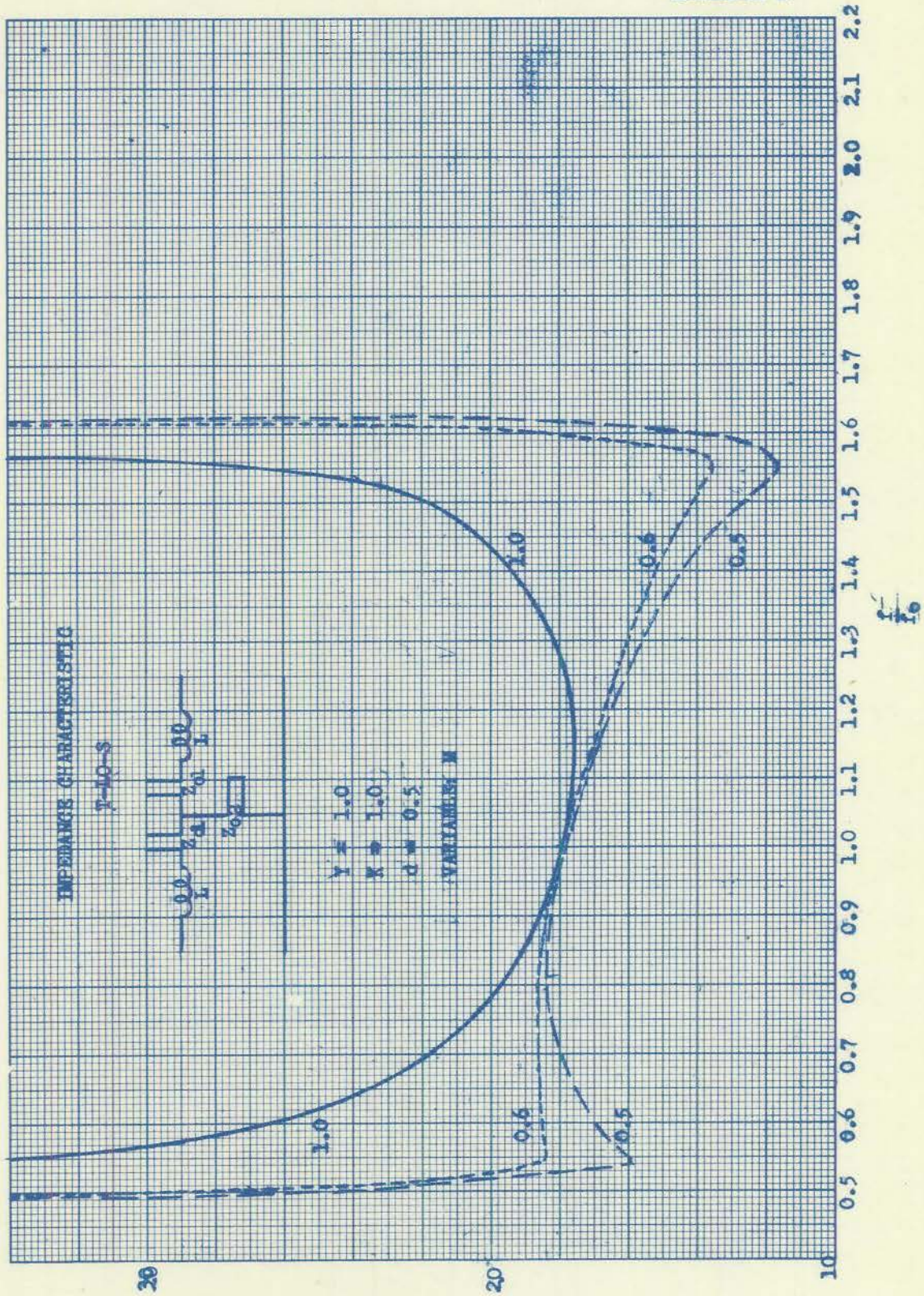

IMPEDANCE CHARACTERISTICS
T-40-S
Y = 1.0
K = 1.0
d = 0.5
VARIABLES M
1.0
0.6
0.5
2.0
0.6
0.5
30
20
10
0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0 2.1 2.2
f/f0
Z0 II / Z0
R-2770

PLATE B-40

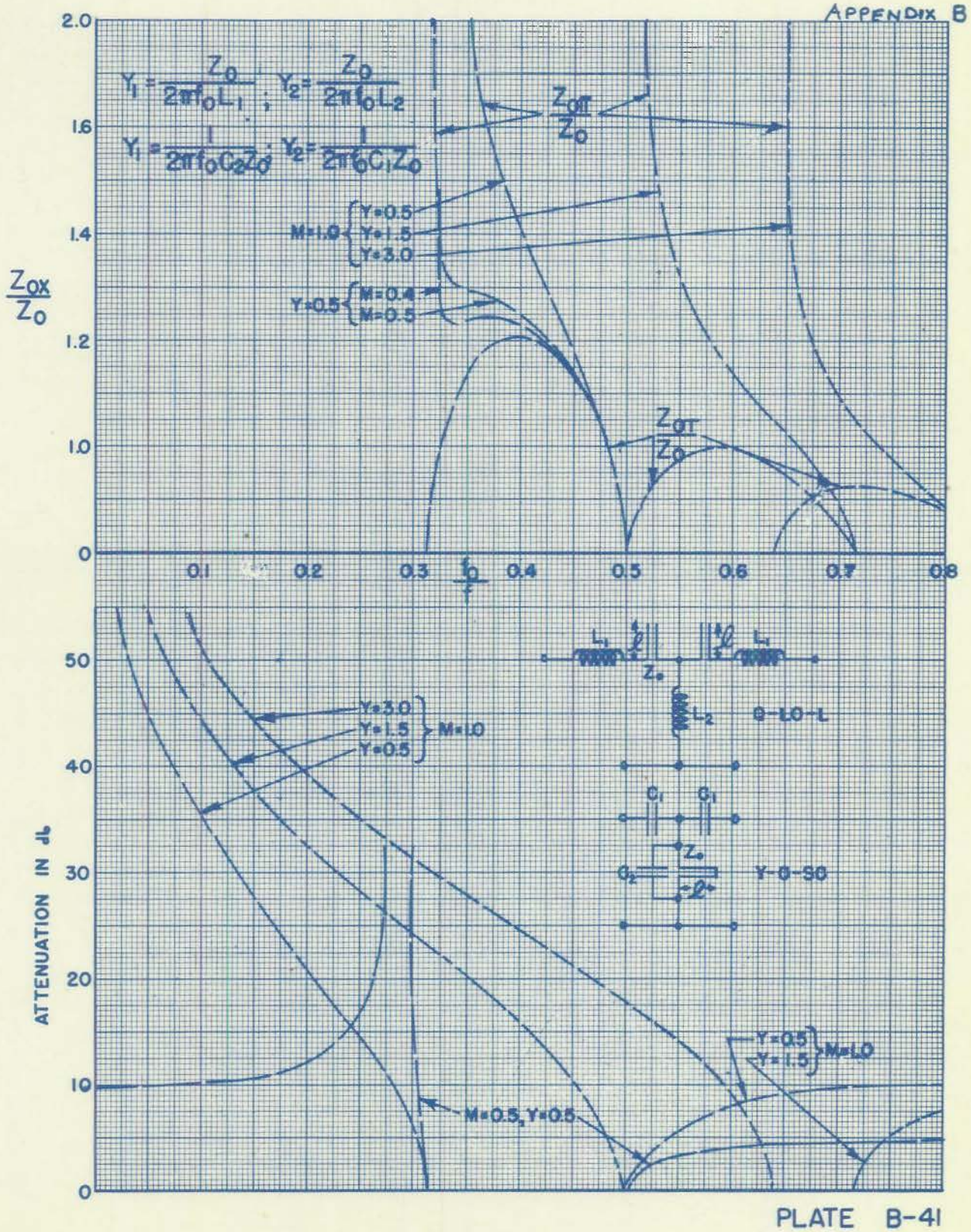
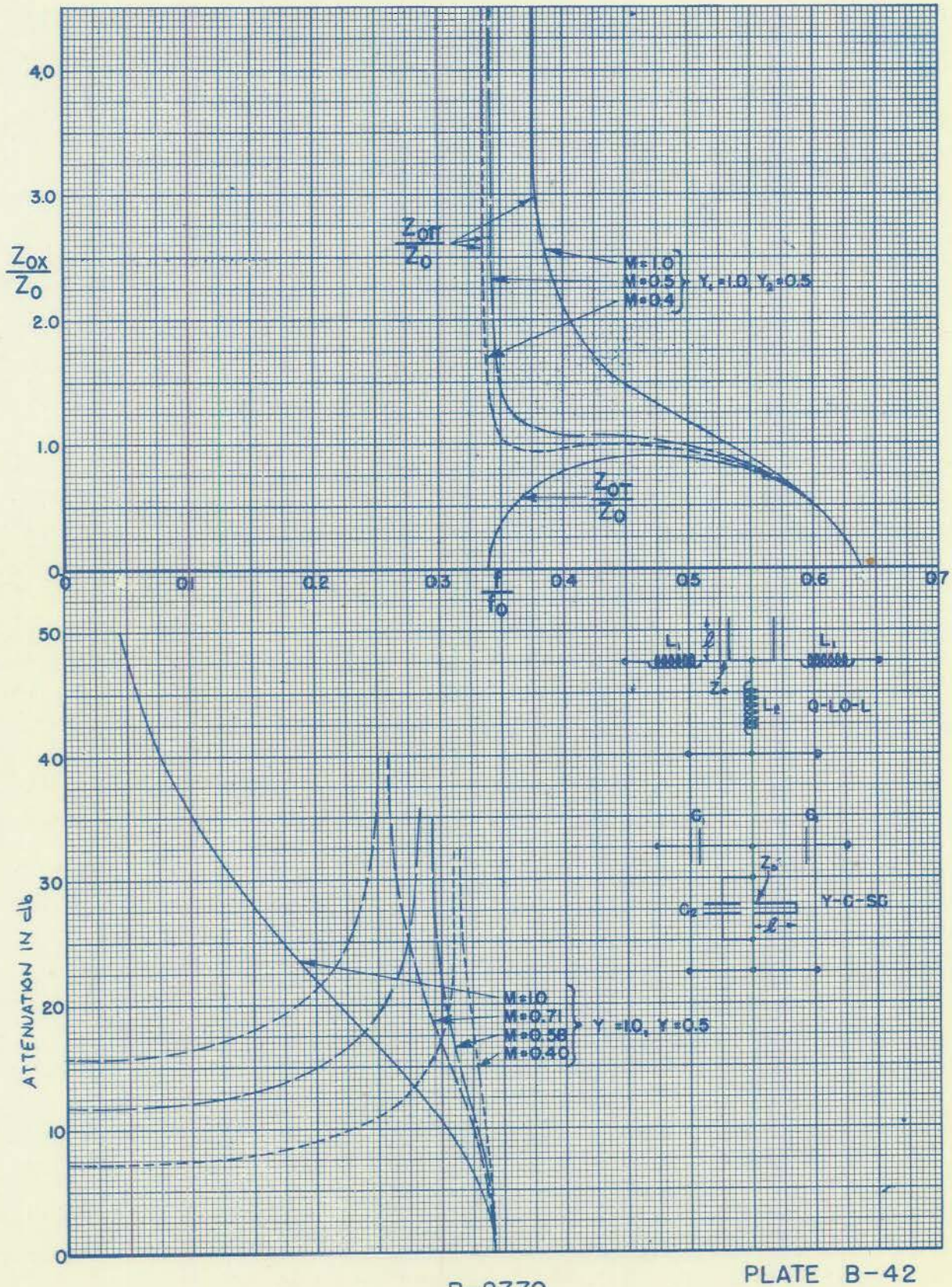

$Y_1 = \frac{Z_0}{2\pi f_0 L_1}$; $Y_2 = \frac{Z_0}{2\pi f_0 L_2}$
$Y_1 = \frac{1}{2\pi f_0 C_2 Z_0}$; $Y_2 = \frac{1}{2\pi f_0 C_1 Z_0}$
$\frac{Z_{0x}}{Z_0}$
$\frac{Z_{0\pi}}{Z_0}$
$\frac{Z_{0T}}{Z_0}$
M=1.0 { Y=0.5, Y=1.5, Y=3.0
Y=0.5 { M=0.4, M=0.5
2.0
1.6
1.4
1.2
1.0
0
0.1
0.2
0.3
$\frac{f_0}{f}$
0.4
0.5
0.6
0.7
0.8
ATTENUATION IN db
50
40
30
20
10
0
Y=3.0, Y=1.5, Y=0.5 } M=1.0
M=0.5, Y=0.5
Y=0.5, Y=1.5 } M=1.0
L_1
L_2
Z_0
C_1
C_2
θ-1.0-L
Y-0-S8

PLATE B-41



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PLATE B-42

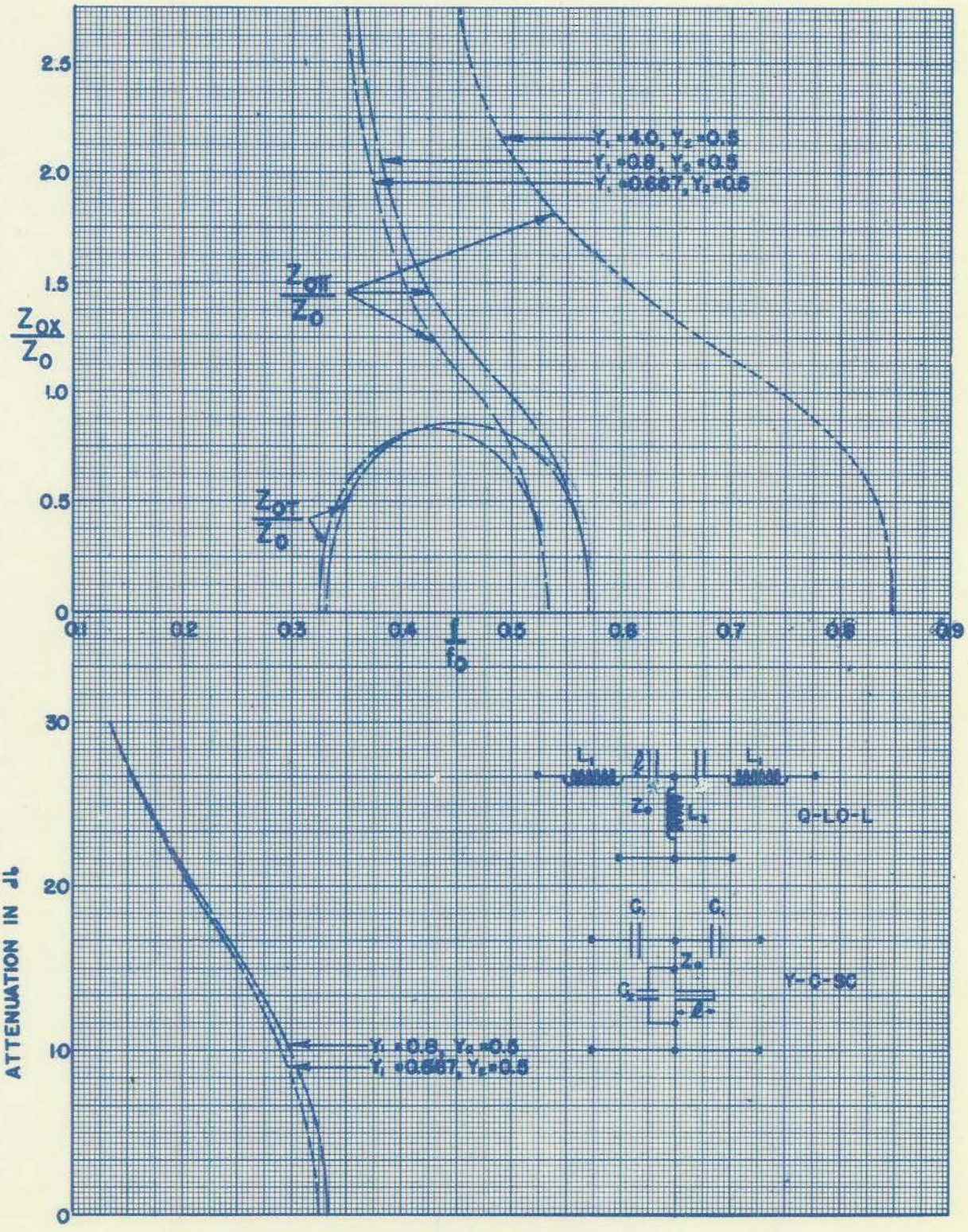

2.5
2.0
1.5
1.0
0.5
0
$\frac{Z_{OX}}{Z_0}$
$Y_1 = 4.0, Y_2 = 0.5$
$Y_1 = 0.8, Y_2 = 0.5$
$Y_1 = 0.667, Y_2 = 0.5$
$\frac{Z_{OX}}{Z_0}$
$\frac{Z_{OY}}{Z_0}$
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
$\frac{f}{f_0}$
30
20
10
0
ATTENUATION IN dB
$Y_1 = 0.8, Y_2 = 0.5$
$Y_1 = 0.667, Y_2 = 0.5$
L_1
Z_0
L_1
0-L0-L
C_1
C_1
Z_0
C_2
$2Z_0$
Y-C-3C

PLATE B-43

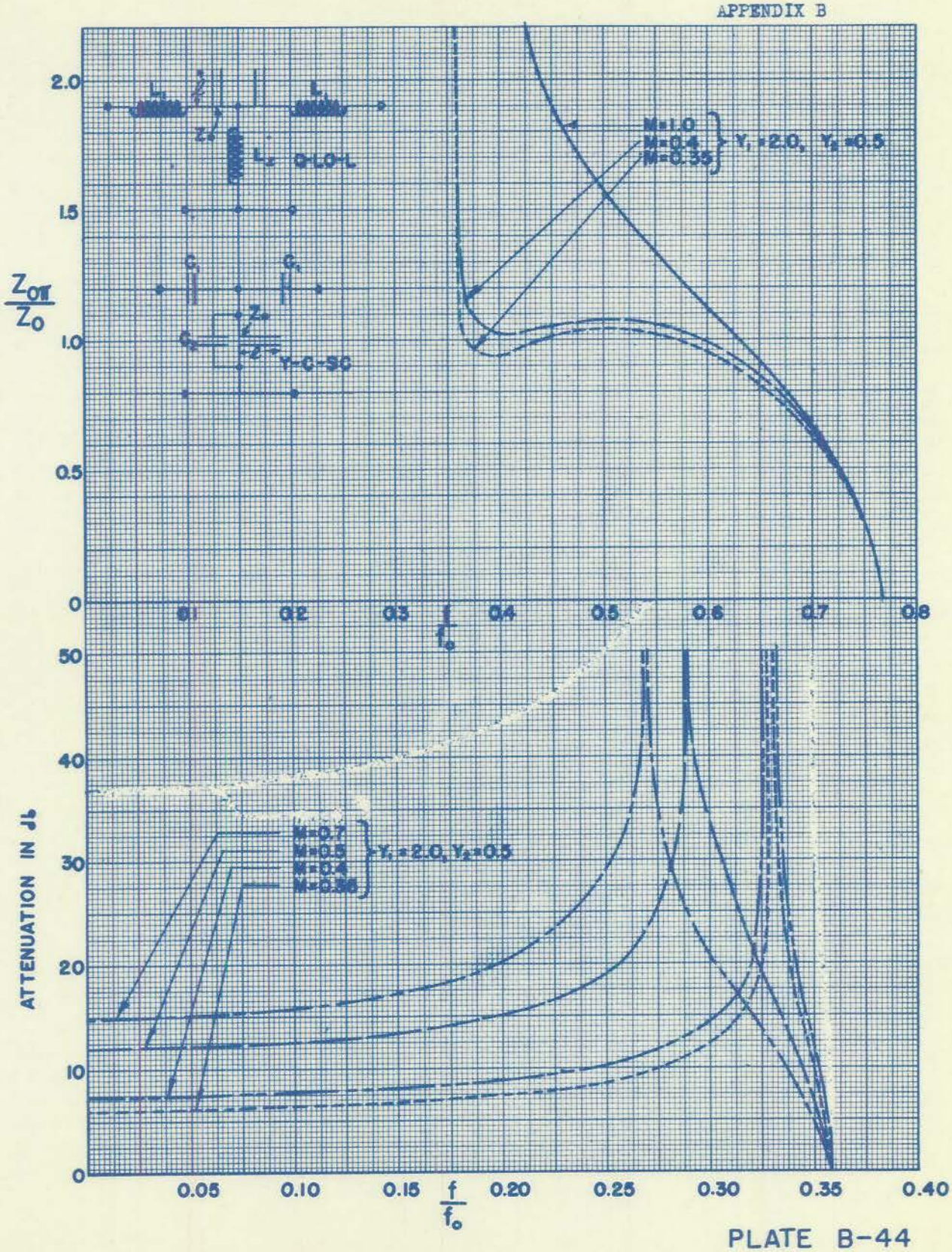
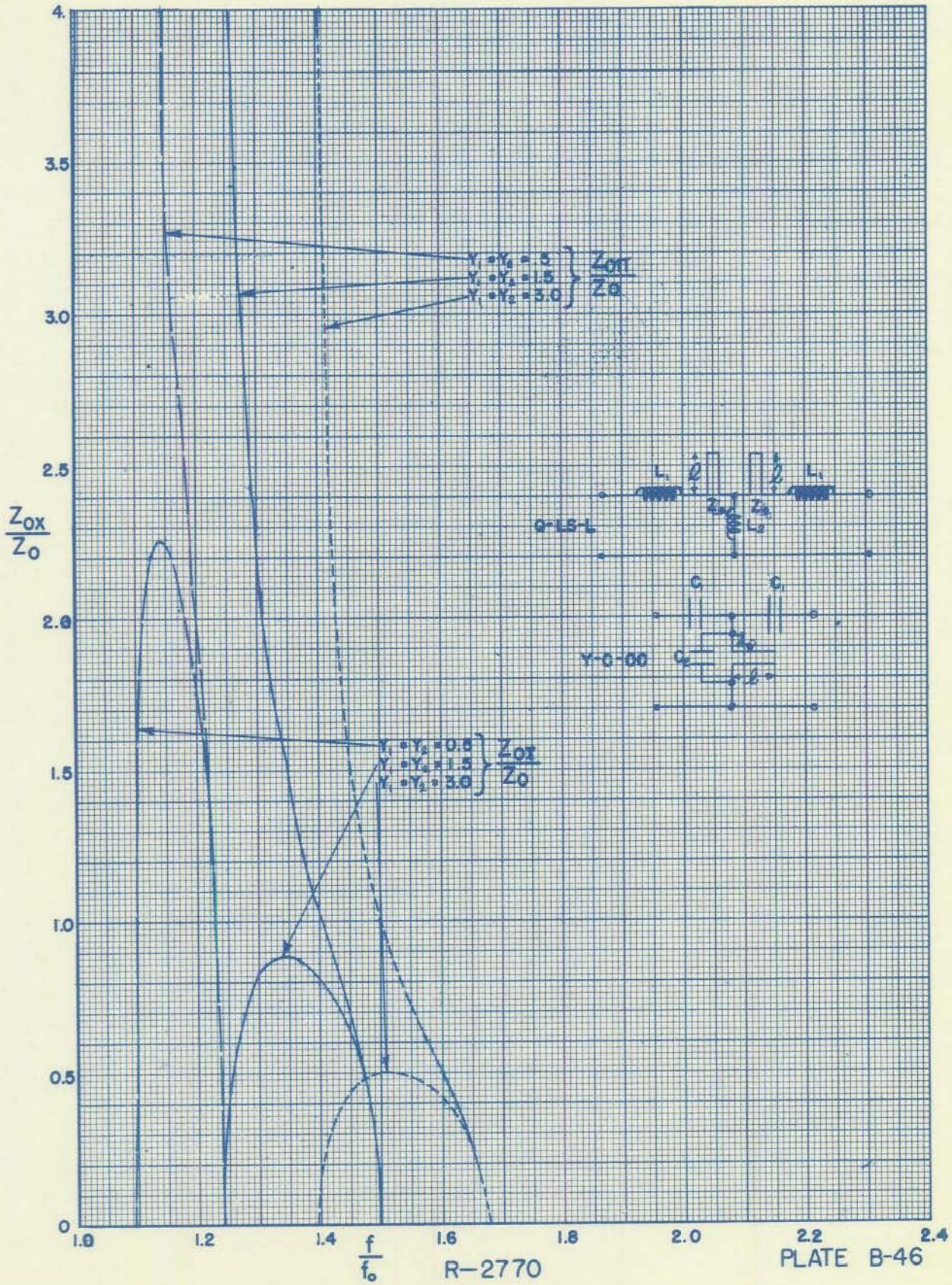
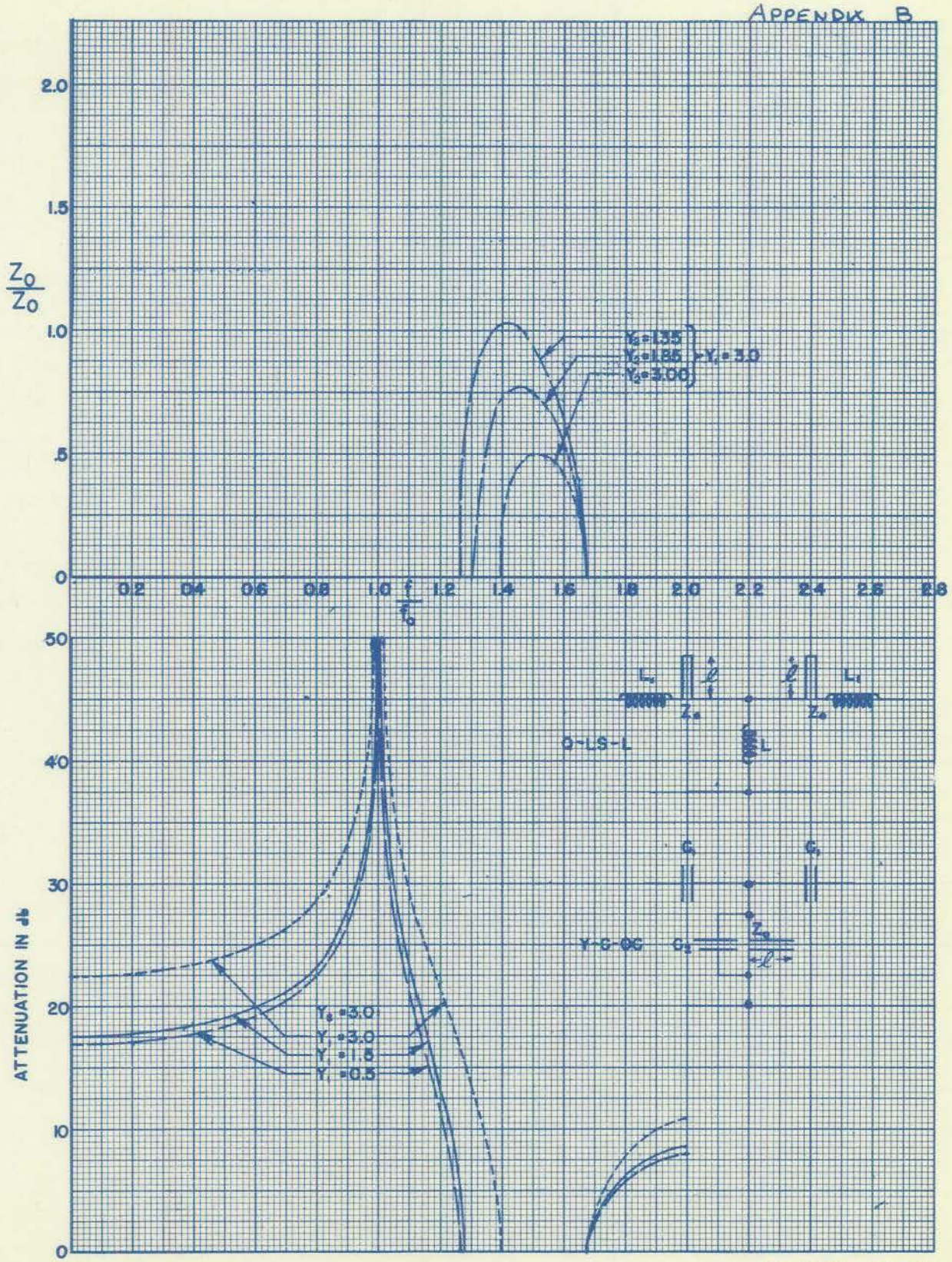

$\frac{Z_{OR}}{Z_O}$
2.0
1.5
1.0
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$\frac{f}{f_o}$
M=1.0
M=0.4
M=0.35
$Y_1=2.0, Y_2=0.5$
$Y=C=SC$
ATTENUATION IN db
50
40
30
20
10
0
M=0.7
M=0.5
M=0.4
M=0.35
$Y_1=2.0, Y_2=0.5$
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
$\frac{f}{f_o}$

PLATE B-44



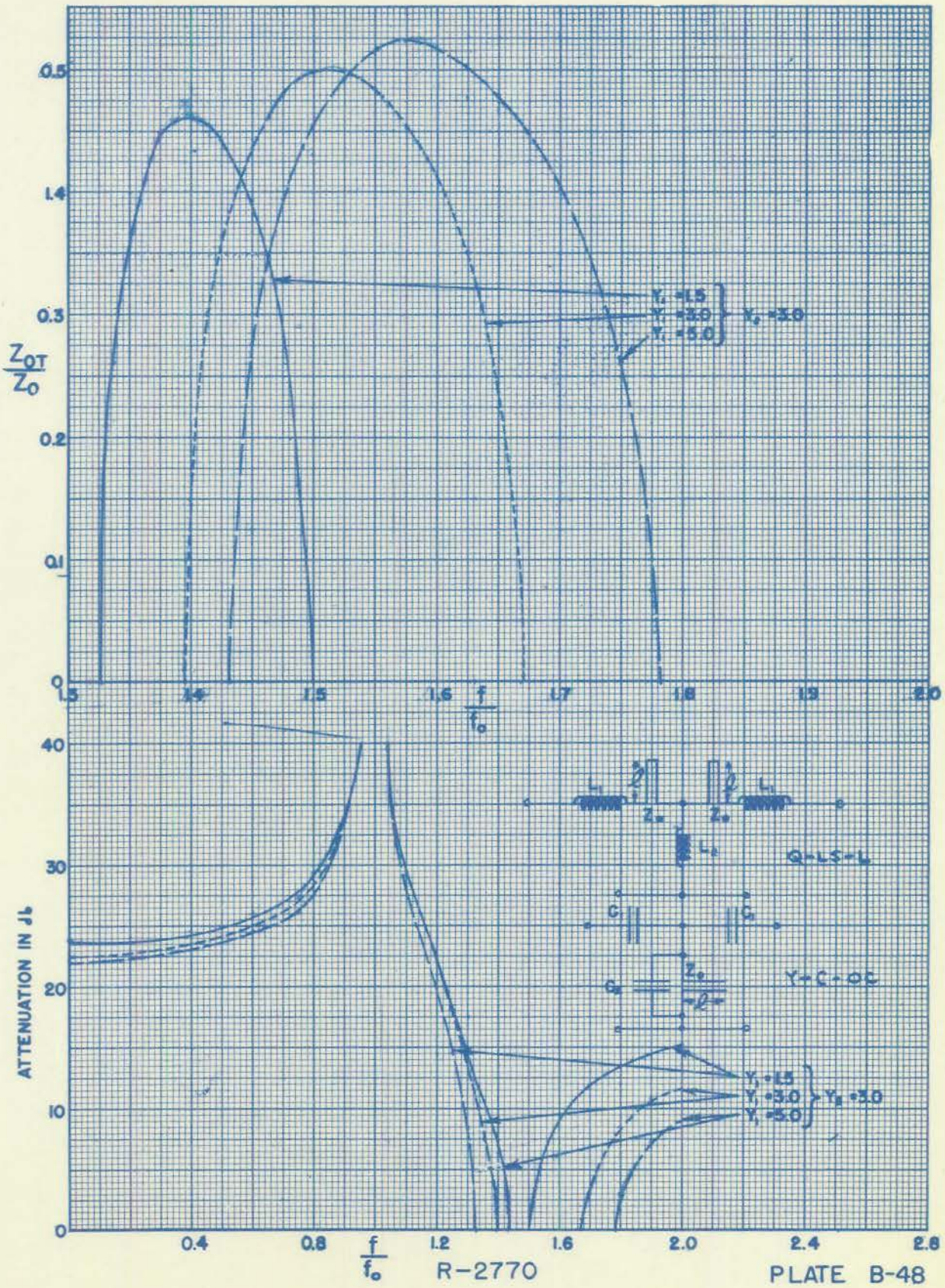
R-2770

PLATE B-46



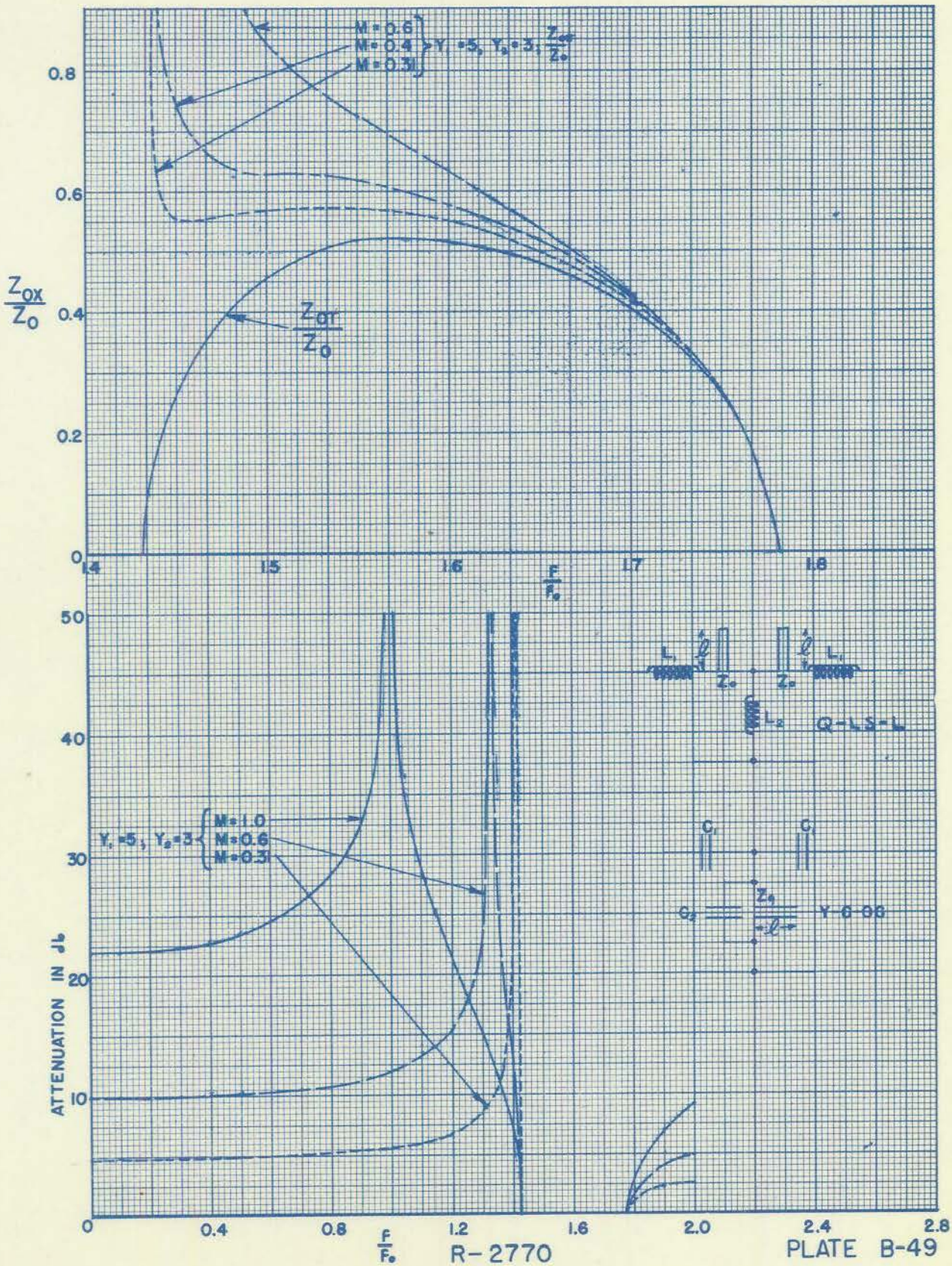
R—2770

PLATE B-47



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PLATE B-48



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PLATE B-49

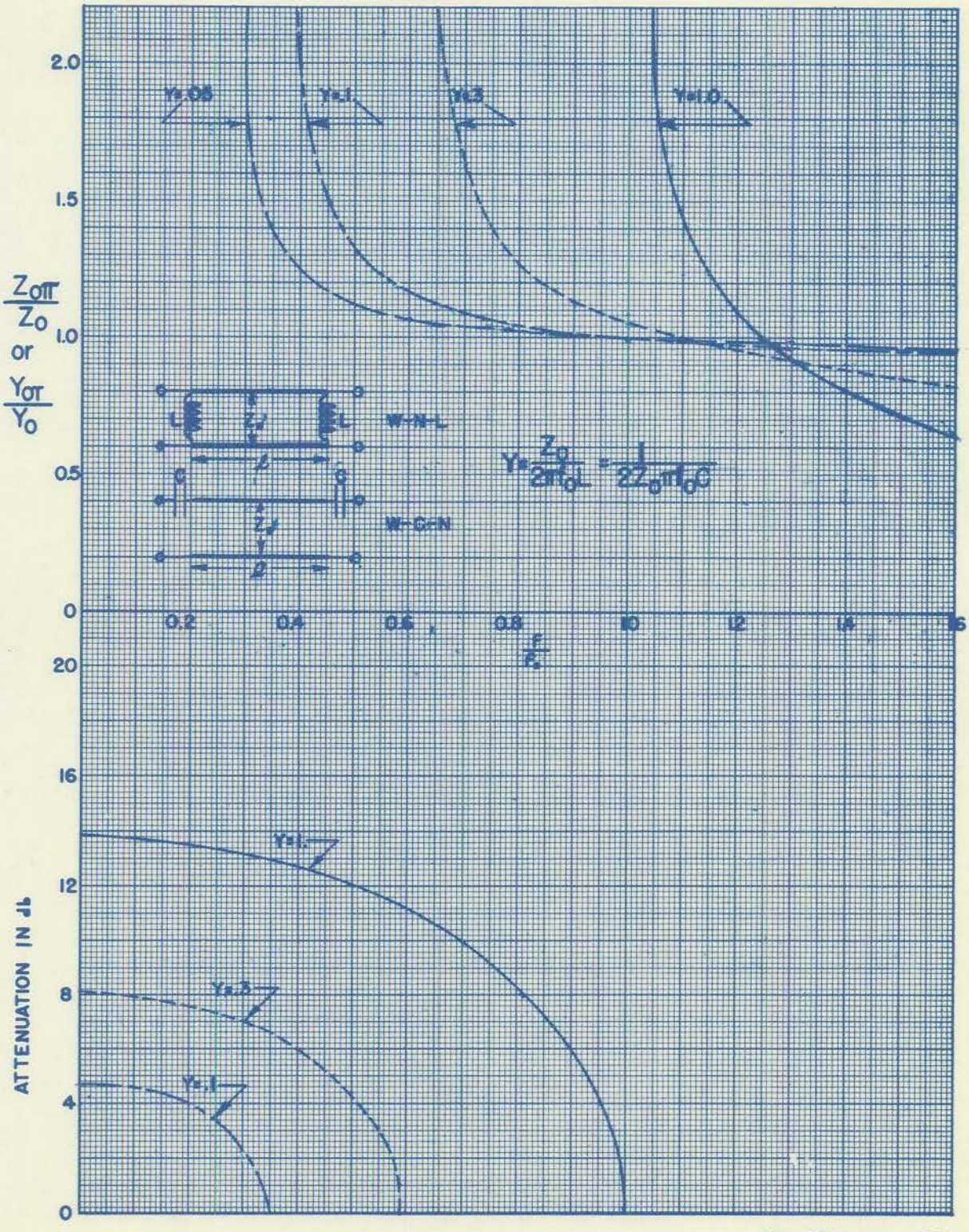
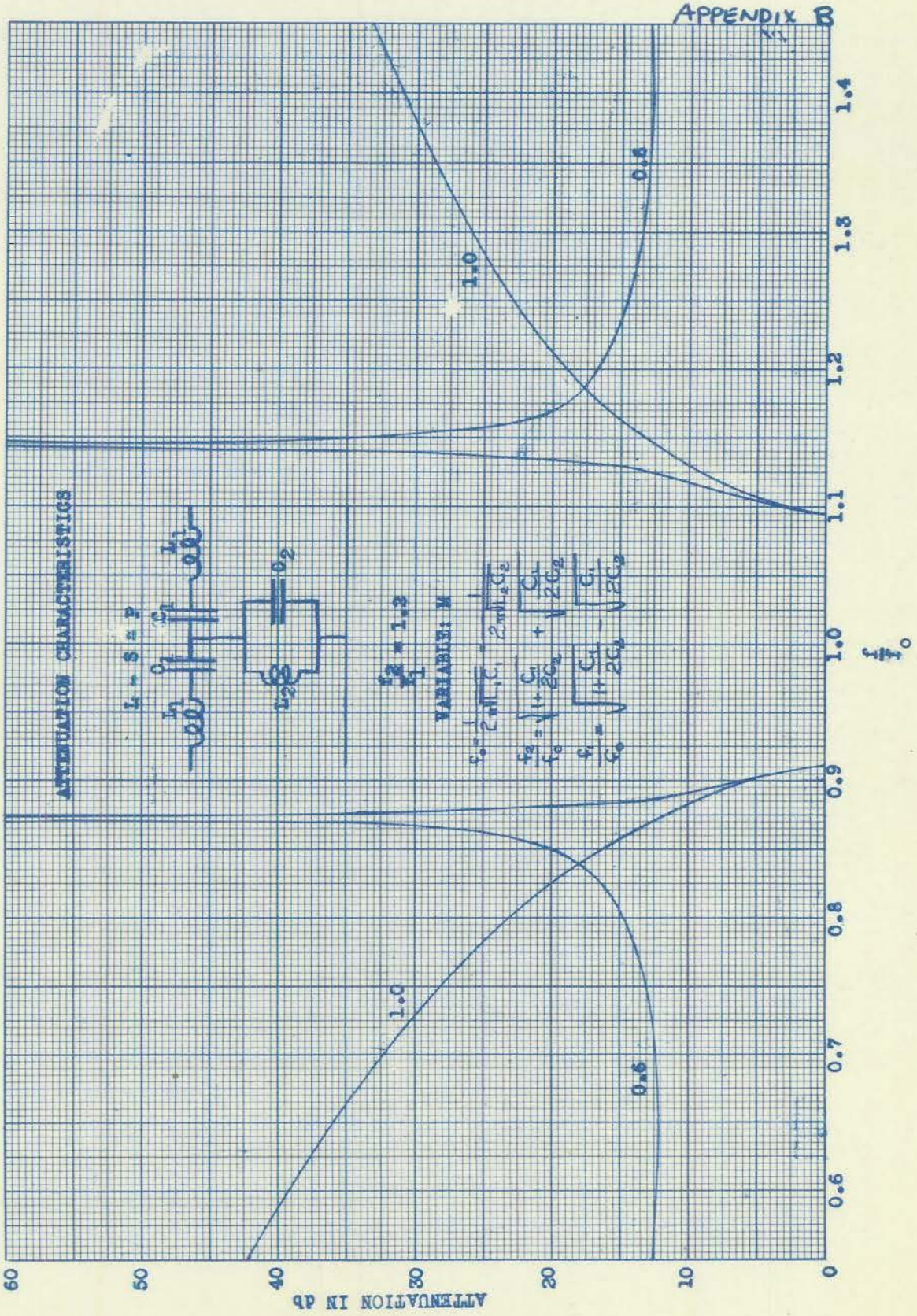
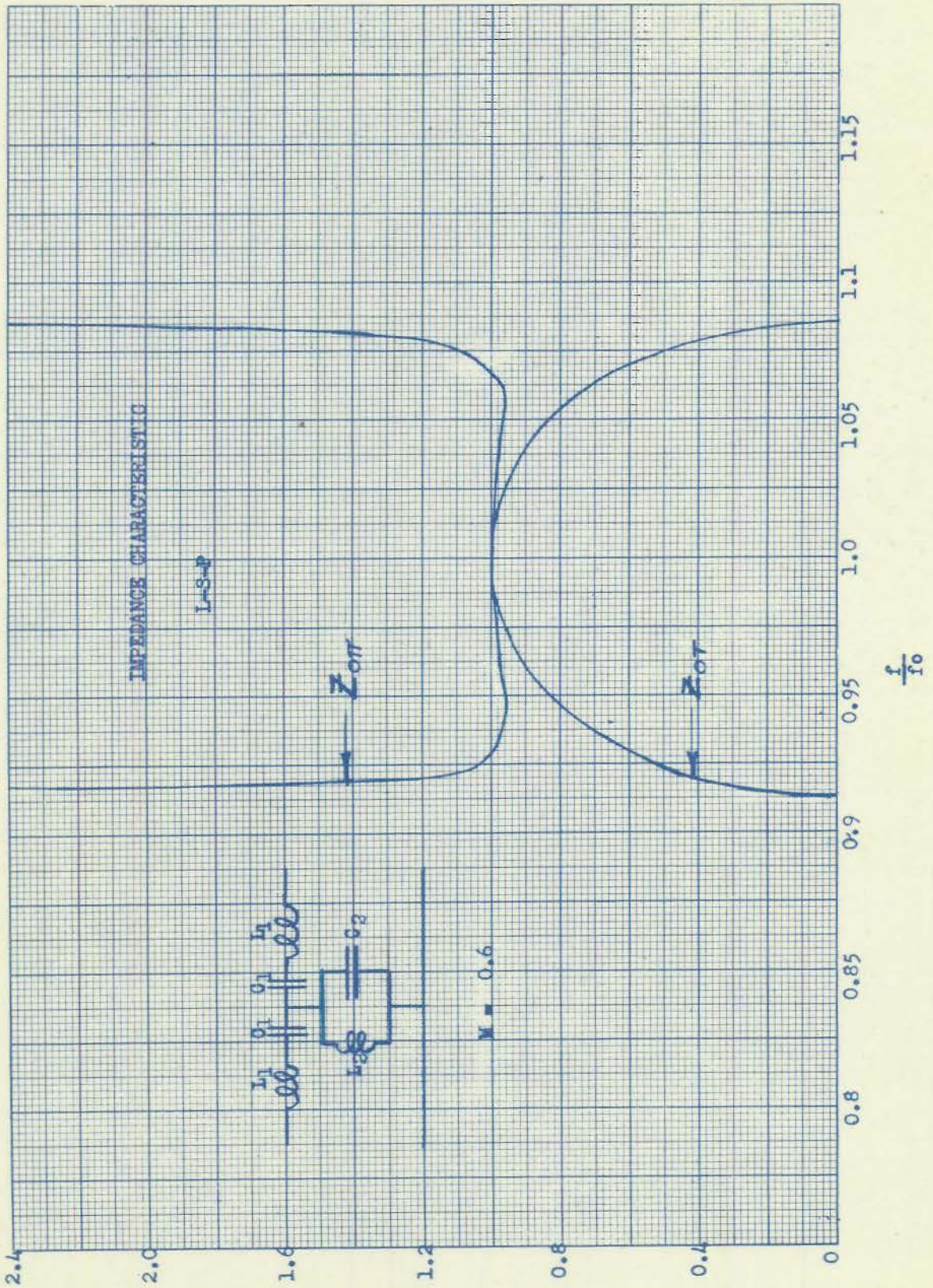

2.0
1.5
1.0
0.5
0
Z_OπT / Z_O or Y_OT / Y_O
Y=0.5
Y=1
Y=2
Y=10
W-N-L
W-C-N
Y = Z_O / 2πf_O L = 1 / 2Z_O πf_O C
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
f / f_c
20
16
12
8
4
0
ATTENUATION IN db
Y=1
Y=.5
Y=.1

PLATE B-50



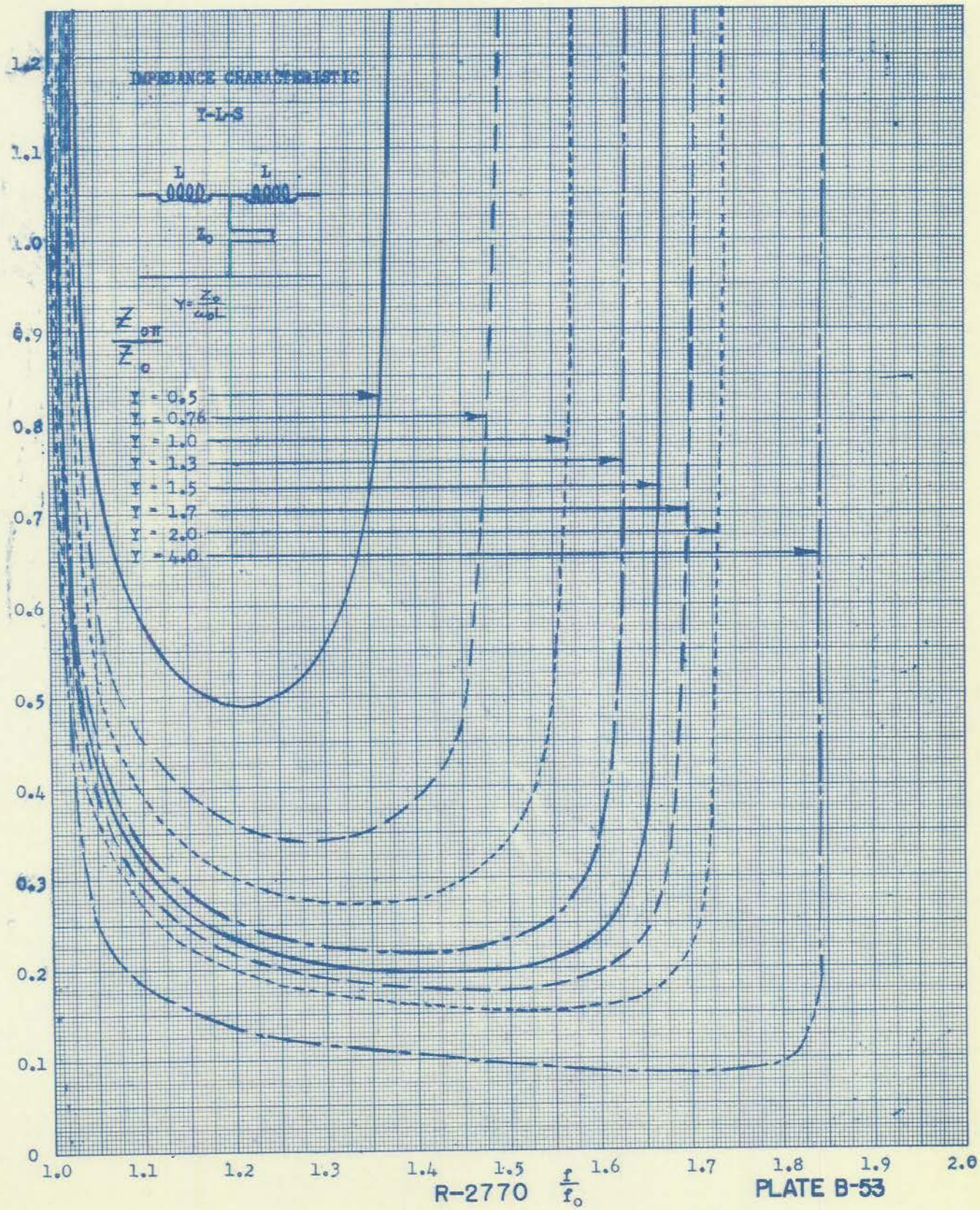
R-2770

PLATE B-51



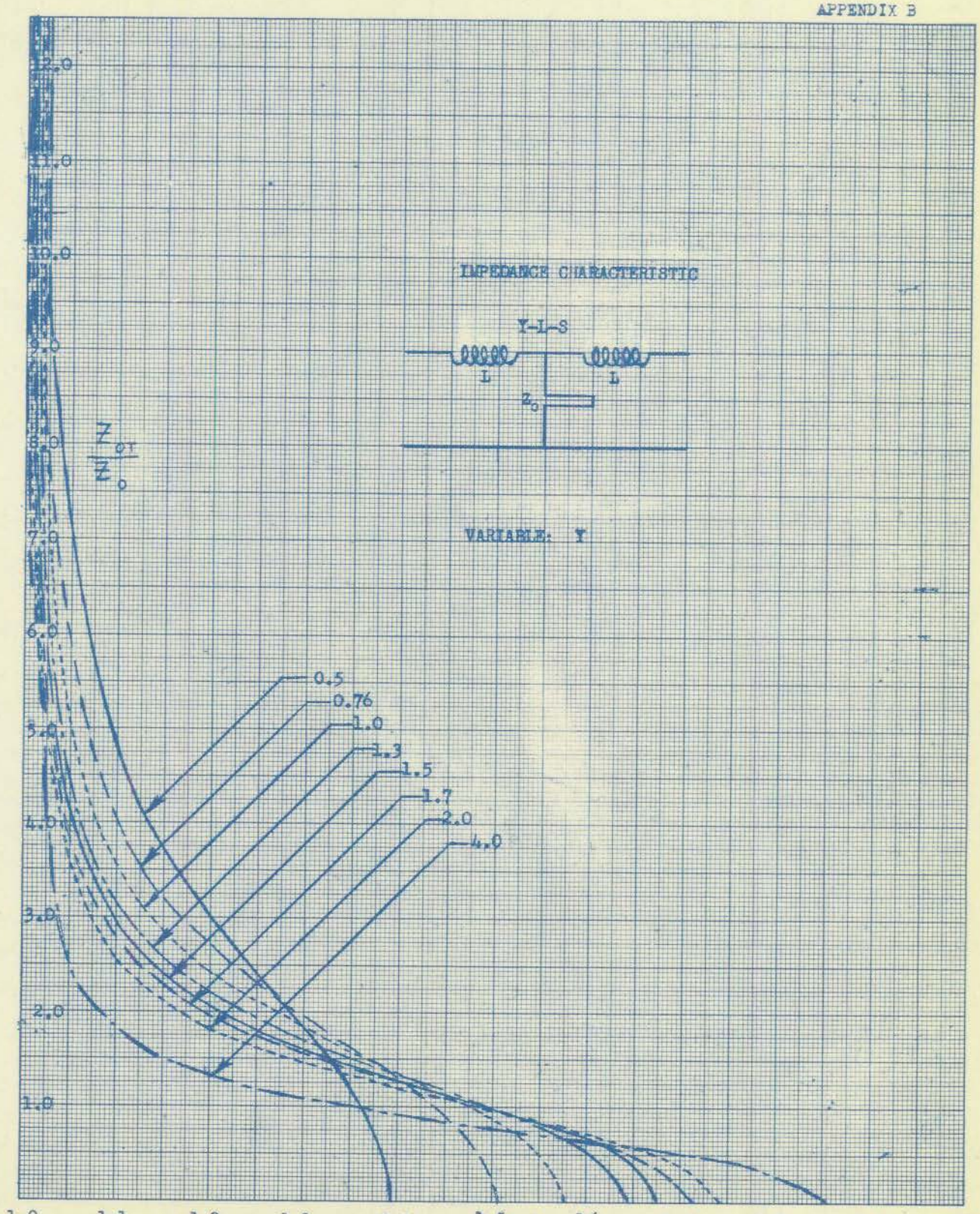
R-2770 $\frac{Z_{OT}}{Z_0}$ and $\frac{Z_{OT}}{Z_0}$

PLATE B-52



R-2770

PLATE B-53



R-2770

PLATE B-54

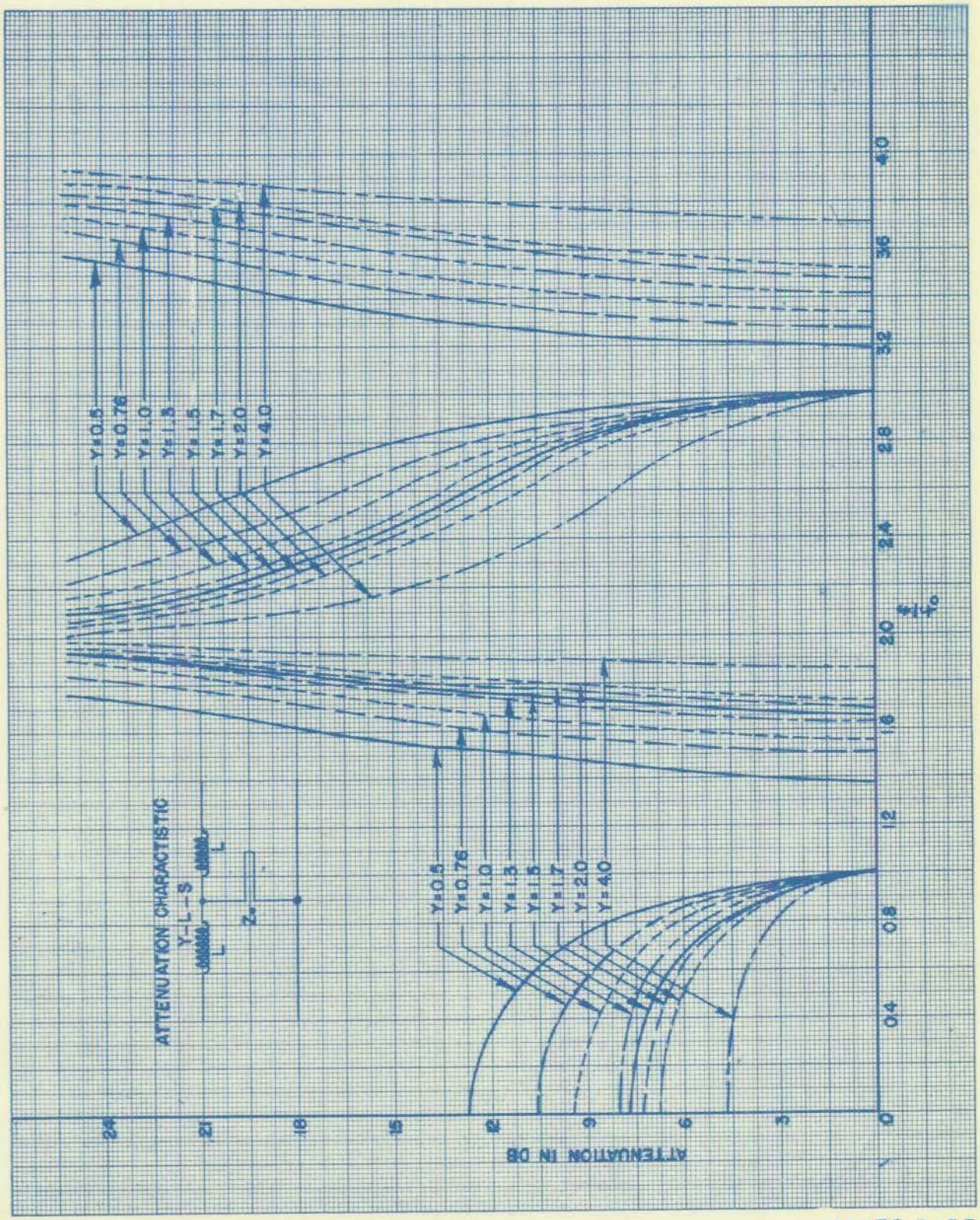

Y=0.5
Y=0.76
Y=1.0
Y=1.3
Y=1.5
Y=1.7
Y=2.0
Y=4.0
ATTENUATION CHARACTERISTIC
Y=L-S
L
S
L
2a
ATTENUATION IN DB
24
21
18
15
12
9
6
3
0
4.0
3.6
3.2
2.8
2.4
2.0
1.6
1.2
0.8
0.4
0
Y

PLATE B-55

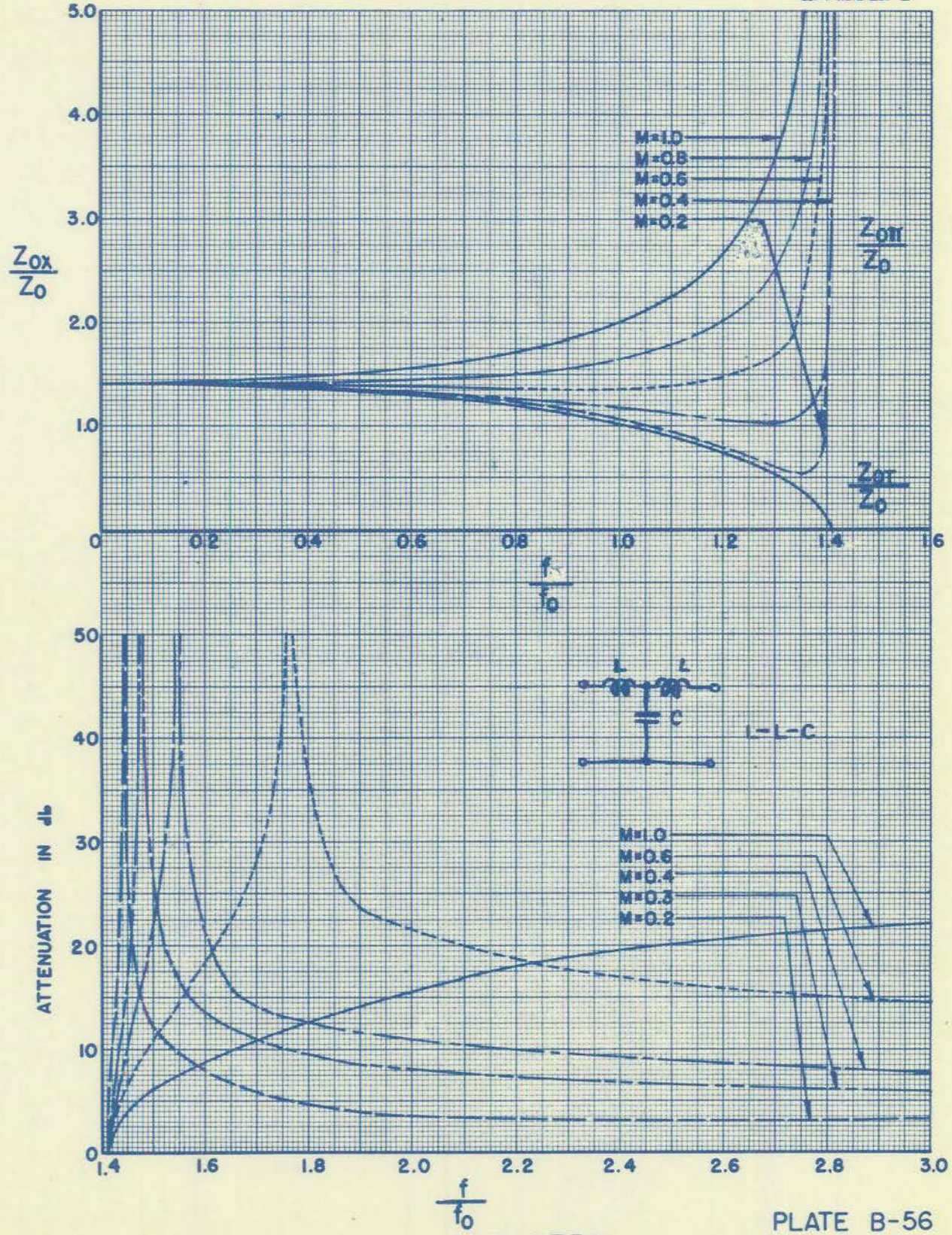


PLATE B-56

R-2770