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THE CONDITIONS FOR CERTAINTY OF INTERCEPTION OF AN INTERMITTENT SIGNAL BY A ROTATING DIRECTIONAL ANTENNA

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ABSTRACT

The various factors affecting the interception of a rotating beam signal by a rotating directional antenna (such as is often employed for radio direction finding purposes), assuming that the signal is "on" for only a limited number of rotations of the respective beams, are considered. Equations are derived for determining the minimum number of beam rotations and the "Speed Ratios" which produce "Certainty of Interception". The "Speed Ratio" is the ratio of the angular speed of the directional antenna beam to that of the signal beam. Also an equation is derived (Appendix 1) for calculating the "Probability of Interception" within a given number of rotations of the signal beam as a function of the "Speed Ratio" for given beam widths of the two beams. It is shown that for many of the more practical and currently employed rotational speeds and beam widths, "Certainty of Interception" exists only within a number of very narrow ranges of "Speed Ratios". It is also seen from Plates 1 - 6 that the "Probability of Interception" shows a general trend of increasing probability in the direction of increasing "Speed Ratios" with points of minimum probability at the "Speed Ratio" corresponding to synchronization of the two beams and at "Speed Ratios" which are simple rational multiples thereof. Hence a slowly rotating direction finder is all but useless in intercepting a rotating beam signal if the signal is to be "on" for only a few antenna rotations. It is further shown that the "Probability of Interception" increases with increasing beam widths but that this is of doubtful significance since larger bearing errors will result from increasing the beam width of the direction finder. The derived equations are generalized in an Appendix to be useful in applications to a wide range of intercept problems, such as the "Certainty" and "Probability" of intercepting an intermittent signal by a frequency scanning receiver.

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INTRODUCTION

1. Direction finders using a rotating antenna pattern and the information from the variation in receiver output obtained thereby are subject to certain limitations in their capabilities not attendant on an instantaneous or "ideal" direction finder. On a steady signal, the time required to assure taking a bearing is limited to a minimum of one rotation of the antenna pattern. As long as the magnitude and direction of the received energy are relatively constant, the rotational speed of the rotating antenna pattern is not important provided the signal persists for a sufficiently long period of time to obtain a satisfactory bearing.

2. If, however, the signal, whose direction is being sought, varies periodically in magnitude by a great amount, as in the case of a rotating radar beam, the time required for intercepting the signal depends very markedly upon the rotational speeds of the antenna patterns and also their beam widths. It is possible that the receiving antenna pattern be so synchronized to the rotational speed of the radar beam, and so phased with respect to it, that no interception will ever occur. Synchronization of the two rotational speeds may be so nearly approached that an excessive length of time is required for an interception.

3. As the rotational speed of the receiving antenna is made progressively slower, a critical speed is reached below which an interception results for each rotation of the receiving antenna. As the rotational speed of the receiving antenna is progressively increased, another critical speed is reached above which an interception results each time the signal illuminates the receiving antenna. Between these critical speeds there exist rotational speeds of the receiving antenna for which an interception will occur with certainty within a given number or rotations of the receiving antenna. There exist, also, some rotational speeds for which there is no certainty of an interception ever occurring. It is the purpose of this report to show how these rotational speeds can be calculated; i.e., the above mentioned critical speeds, the speeds for which an interception is certain to occur within a given number of rotations, and the speeds for which an interception is never certain to occur.

ASSUMPTIONS

4. The intermittent signal may be considered to be a rotating beam, which is the actual case in rotating radar beams. Reference will be made to Figure 1. At O' is located the source of a rotating signal beam of beam width θ_S and assumed to be rotating with a uniform angular speed ω_S. A directional receiving antenna of beam width θ_D rotates with uniform angular speed ω_D about the point O. Each of the positions O and O' is considered to be stationary.

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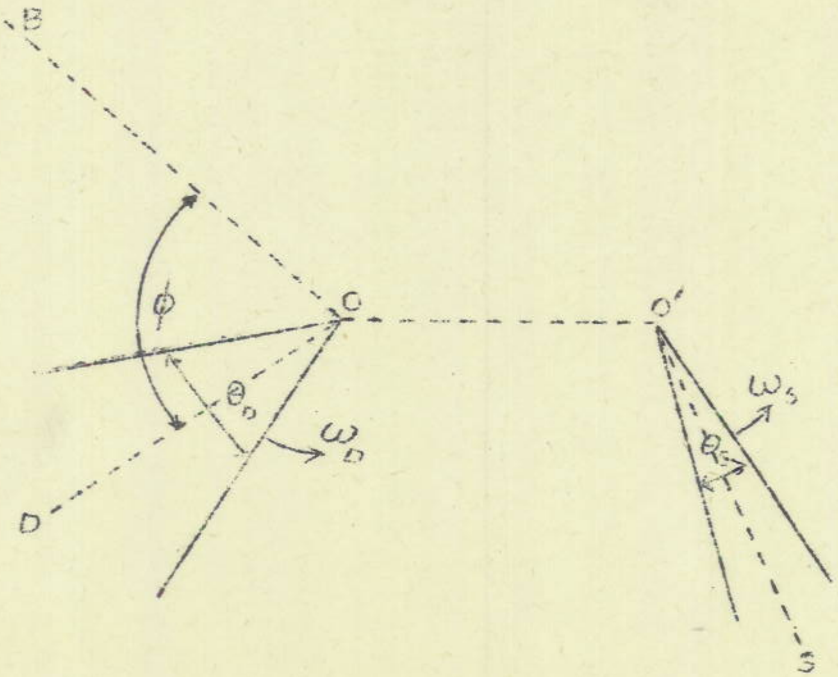

Figure 1.

5. It is assumed that the signal is of constant maximum intensity over the angle θ_S and zero over the remaining $360^\circ - \theta_S$. Likewise it is assumed that the receiving antenna is of constant maximum sensitivity over the angle θ_D and zero over the remaining $360^\circ - \theta_D$. Furthermore it is assumed that any reception of energy from the signal by the receiving antenna is sufficient for obtaining a satisfactory interception. Actually a bearing error may result from making use of a single interception; the maximum value of such a bearing error is $\pm \theta_D/2$. Hence if a single interception is to be of any use, the beam width of the receiving antenna must not be allowed to be excessively large.

6. In Figure 1 the line $\overline{O'S}$ is the center line of the signal beam, and $\overline{OD}$ is the center line of the receiving antenna beam. The line $\overline{OB}$ represents the orientation of the line $\overline{OD}$ extrapolated backwards to the first instant before the initial instant when $\overline{O'S}$ would have lain along the line $\overline{O'\phi}$. ϕ is the angle which $\overline{OD}$ makes with respect to $\overline{OB}$ at any instant.

DETERMINATION OF INTERCEPT REGIONS

7. The orientation of the line $\overline{OB}$ is arbitrary; certainty of interception results only after a sufficient number of rotations of the signal and receiving antenna beams have been made to cause an interception from any initial orientation. Of course, it is possible that an interception will occur in less time than that required for certainty.

8. During the time that the signal illuminates the receiving antenna, this antenna covers an angular region of width $\theta_D + \theta_S A$, where

$$A = \omega_D/\omega_S \qquad (1)$$

(The ratio A will in the future be referred to as the speed ratio.) The angle θ_D is covered by virtue of the beam width of the receiving antenna; the additional angle $\theta_S A$ is covered by virtue of the motion of the receiving antenna during the time the signal illuminates this antenna.

9. During successive illuminations of the receiving antenna by the signal, different angular regions of width $\theta_D + \theta_S A$ are covered, until when the full 360° have been covered, an interception is certain to have occurred. As will be shown in a later section, this condition cannot be satisfied for numerous speed ratios; the method for calculating these speed ratios will be developed.

10. At the instant when $\overline{O'S}$ is in the position $\overline{O'O}$, ϕ assumes a particular value. As $\overline{O'S}$ successively reaches the position $\overline{O'O}$, ϕ assumes a discrete set of values. Let these discrete values be denoted by the symbol ψ in order to distinguish them from the continuously varying values of ϕ. These discrete values are given by

$$\psi = 360°(n_S A - n_D), \qquad (2)$$

where n_S is the number of times the signal has illuminated the receiving antenna, and n_D is the number of rotations which have been made by the receiving antenna with respect to the reference position $\overline{OB}$.

11. In order to insure an interception it is necessary and sufficient that ψ assume a number of positions so distributed that the separation, $\Delta\psi$, between adjacent values of ψ be equal to or less than $\theta_D + \theta_S A$. $\Delta\psi$ is given by

$$\Delta\psi = 360°(A\Delta n_S - \Delta n_D), \qquad (3)$$

or

$$\Delta\psi = 360°(A\Delta n_S - \Delta n_D - 1), \qquad (3')$$

if the region between adjacent values of ψ contains the line $\overline{OB}$.

12. Let N_S represent the total number of illuminations of the receiving antenna by the signal for certainty of interception. Let N_D be the corresponding number of rotations of the receiving antenna. How these "permissable" values of N_S and N_D can be calculated will be shown in a later section.

13. For given beam widths of signal and receiving antennas, N_D and N_S determine a region of speed ratios within which an interception is certain to occur for these numbers of rotations of receiving antenna and illuminations of the receiving antenna. Such a region of speed ratios will be called an intercept region. The notation $(N_D; N_S)$ will be used to denote the intercept region determined by N_D and N_S.

[redacted]

14. For the intercept region ($N_D; N_S$) to be a unique intercept region, N_D and N_S must be numbers which are relatively prime with respect to each other. That this must be so is easily seen from the fact that the critical $\Delta\psi$ is that one containing the line $\overline{OB}$. Hence for the N_Sth value of ψ to be required in determining an intercept region no other previous value of ψ must lie between it and the line $\overline{OB}$. Now if N_D is not relatively prime with respect to N_S an earlier value of ψ will lie between $\overline{OB}$ and the N_Sth value of ψ.

15. The limits to the intercept region ($N_D; N_S$) are readily established by considering that speed ratio within the intercept region given by $A = N_D/N_S$. In this case the N_S values of ψ are evenly distributed with the N_Sth value corresponding to the line $\overline{OB}$, as shown in Figure 2.

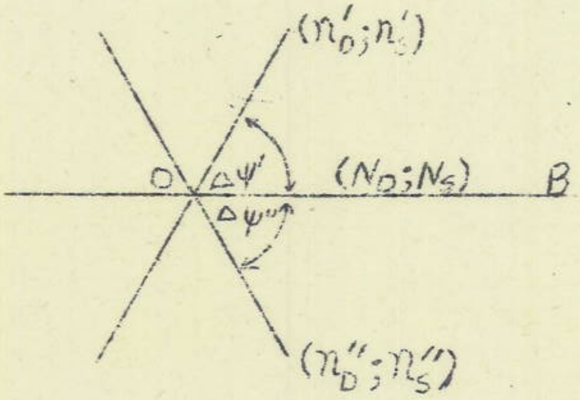

Figure 2.

16. From Figure 2 and equation (2) it is seen that $n_S' N_D - n_D' N_S = 1$ (4)

Setting $N_D - n_D' = m = -\Delta n'_D$ (5)

gives $N_S - n_S' = \dfrac{mN_S - 1}{N_D} = -\Delta n'_S$ (6)

Putting the values of $\Delta n_D'$ and $\Delta n_S'$ from (5) and (6) into equation (4), using the equality sign, and solving for A gives the lower limit to the intercept region ($N_D; N_S$). This gives

$$A_{min} = \frac{360^\circ m - \theta_D}{360^\circ r + \theta_S} \qquad (7)$$

where

$$r = \frac{mN_S - 1}{N_D} \qquad (8)$$

[redacted]

17. The upper limit to the intercept region (N_D; N_S) is obtained in a similar fashion from the region $\Delta\psi''$ of Figure 2. The result is

$$A_{max} = \frac{360^\circ(N_D - m) + \theta_D}{360^\circ(N_S - r) - \theta_S} \tag{9}$$

CALCULATION OF m

18. Before equations (7) and (9) can be used to yield numerical results for the limits of the intercept region (N_D; N_S) for given values of θ_D, θ_S, N_D, and N_S, the number m must be calculated. An examination of equations (5) and (6) shows how this number is to be computed. Equation (5) reveals that m must be a positive integer equal to or less than N_D. Equation (6) further reveals that m must be such an integer that $mN_S - 1$ is divisible by N_D. Hence m must be the smallest positive integer for which $mN_S - 1$ is divisible by N_D; i.e., m is the root of the linear congruence

$$mN_S \equiv 1 (\text{mod } N_D) \tag{10}$$

19. If the modulus N_D is a small number, then m can be quickly calculated by a process of trial and error, trying $m = 1, 2, 3$, etc. in order until an integer is found which will make $mN_S - 1$ divisible by N_D. If the modulus is large, this method becomes laborious; a less time-consuming method must then be developed.

20. The congruence $mN_S \equiv 1(\text{mod } N_D)$ is equivalent to the indeterminant equation $mN_S - xN_D = 1$. This can be solved by reducing N_D/N_S to a continued fraction. If p/q be the convergent immediately preceding N_D/N_S, $N_Sp - N_Dq = \pm 1$. Thus the congruence $mN_S \equiv 1(\text{mod } N_D)$ is reduced to the congruence

$$m \equiv \pm p (\text{mod } N_D) \tag{11}$$

If the plus sign holds, then $m = p$; if the minus sign holds $m = N_D - p$.

21. An example will make clear how p is found and how the proper sign is determined. Suppose the congruence

$$467m \equiv 1(\text{mod } 823)$$

is to be solved. A convenient arrangement of the work is the following:

823		0
467	1	1
356	1	1
111	3	2
23	4	7
19	1	30
4	4	37
3	1	178
1	3	215
0		823

[redacted]

This table is constructed in the following manner: (1) N_D is written at the top of the left-hand column with N_S written immediately beneath N_D; (2) the remaining numbers in the left-hand column and the middle column are obtained by dividing each number in the left-hand column by the number immediately beneath it, writing the quotient in the middle column beside the divisor and the remainder beneath the divisor in the left-hand column. This process is continued until a zero remainder results. The numbers in the right-hand column are obtained by writing zero and one at the top of this column in all examples; then the remaining numbers in this column are written by multiplying the number in the right-hand column immediately above the number to be calculated by the number beside it in the middle column and adding to this product the second number above the number to be calculated; e.g., the number 178 is obtained by multiplying 37 by 4 and adding 30 to the resulting product. The numbers appearing in the right-hand column beneath the original zero and one are the numerators of the convergents of the partial fraction of N_D/N_S. The number at the bottom of the right-hand column is always N_D. The number immediately above it is p(in this case 215). If p is in an even place the plus sign holds in equation (11); if p is in an odd place the minus sign holds. In this case p is in the 7th place which is odd and hence the minus sign holds. Thus $m = N_D - p = 823 - 215 = 608$.

PERMISSIBLE VALUES OF N_D AND N_S

22. For a given value of N_S and given beam widths there exists a minimum value of N_D for which certainty of interception results. Any number equal to or greater than this minimum value and which is relatively prime with respect to N_S yields certainty of interception. These numbers are called permissible values of N_D.

23. Similarly, for a given value of N_D and given beam widths there exists a minimum value of N_S which yields certainty of interception. Any number equal to or greater than this minimum value and which is relatively prime with respect to N_D is a permissible value of N_S.

24. These minimum values are determined by setting $A_{min} = A_{max}$ in equations (7) and (9). When this is done, the number m drops out of the resulting equation, and when solved for N_D, the result is the exceedingly simple form

$$N_D = \frac{360^\circ - N_S\theta_D}{\theta_S} \qquad (12)$$

Equation (12) determines the minimum permissible value of N_D. This minimum permissible value of N_D is the smallest positive integer which is equal to or greater than (12) and which is relatively prime with respect to N_S.

25. For example, suppose that $\theta_S = 10^\circ$, $\theta_D = 45^\circ$, and it is desired to know the minimum number of rotations of the receiving antenna required to assure an interception within 3 illuminations of the receiving antenna

[redacted]

by the signal. Substitution into equation (12) gives

$$N_D = \frac{360^\circ - 3 \times 45^\circ}{10^\circ} = 22.5 \text{ rotations.}$$

Since N_D must be an integer, relatively prime with respect to 3, the minimum permissible value of N_D in this case is 23 rotations.

26. The minimum permissible value of N_S for a given value of N_D and given beam widths is found by solving equation (12) for N_S; the result is

$$N_S = \frac{360^\circ - N_D \theta_S}{\theta_D} \tag{13}$$

27. Suppose that for the same beam widths used in the above example it is desired to determine the minimum number of illuminations by the signal beam required for certainty of interception in one rotation of the receiving antenna. Then

$$N_S = \frac{360^\circ - 1 \times 10^\circ}{45^\circ} = 7.78 \text{ rotations.}$$

Hence the minimum number of illuminations of the receiving antenna by the signal for certainty of interception in one rotation of the receiving antenna for these beam widths is 8 illuminations.

SPEED RATIOS FOR WHICH INTERCEPTION IF NEVER CERTAIN

28. Obviously if the two beams are rotating with the same speed and initially oriented as shown in Figure 3, an interception will never occur.

Figure 3

29. There are numerous other speed ratios for which an interception will never occur if the two beams are so phased initially. These speed ratios are easily calculated from equation(s) (12) and/or (13) which give the minimum permissible values of N_D and N_S respectively. For a given value of N_S, the ratio of any integer less than the corresponding minimum permissible value of N_D to N_S is a speed ratio for which interception cannot be assured. Likewise for a given value of N_D, the ratio of N_D to any integer less than the corresponding minimum permissible value of N_S is a speed ratio for which interception cannot be assured.

30. If all such ratios are calculated, then all the speed ratios for which certainty of interception cannot be assured will have been calculated, but there will have been considerable duplication of effort. In order to calculate all such speed ratios without duplication of effort either equation (12) or (13) may be used alone. If θ_D is larger than θ_S, it is more convenient to use equation (12); if θ_D is less than θ_S, it is more convenient to use equation (13); if θ_D is equal to θ_S, either equation may be used equally well. In any case all speed ratios for which interception can never be assured are obtained by using only those integers which are less than the minimum permissible value of N_S or N_D which are relatively prime with respect to N_D or N_S.

31. For example, suppose that $\theta_D = 45^\circ$ and $\theta_S = 10^\circ$ and it is desired to determine the speed ratios for which interception can never be assured. In this case θ_D is larger than θ_S, and hence it is more convenient to use equation (12). For $N_S = 1$, the minimum permissible value of N_D is 32. Hence speed ratios for which interception can never be assured are

1, 2, 3, ..., 31.

For $N_S = 2$, the minimum permissible value of N_D is 27. Hence additional speed ratios for which interception can never be assured are

1/2, 3/2, 5/2, ..., 25/2.

This process is continued with $N_S = 3, 4$, etc. until the minimum permissible value of $N_D = 1$, whereupon all the speed ratios for which interception can never be assured will have been calculated. Tabulated below are all the speed ratios for these beam widths for which interception can never be assured.

N_S	SPEED RATIOS FOR WHICH INTERCEPTION CAN NEVER BE ASSURED
1	1, 2, 3, ..., 31
2	1/2, 3/2, 5/2, ..., 25/2
3	1/3, 2/3, 4/3, ..., 22/3
4	1/4, 3/4, 5/4, ..., 17/4
5	1/5, 2/5, 3/5, ..., 13/5
6	1/6, 5/6, and 7/6
7	1/7, 2/7, 3/7, and 4/7
8 and larger	None

EXAMPLE

32. Suppose that a directional antenna of beam width 45° is to be used to intercept the signal of a rotating radar of beam width 10° and which will be switched off after making 10 rotations. Further suppose that the receiving antenna has a maximum rotational speed of 6 rotations per minute and an interception is required in 1 minute.

[redacted]

33. Since the maximum rotational speed of the receiving antenna is 6 rotations per minute and an interception is required in 1 minute, the maximum value which N_D can have is 6. Hence the first step is to calculate from equation (13) the permissible values of N_S corresponding to $N_D = 1, 2, 3, 4, 5,$ and 6. Tabulated below are these permissible values of N_S.

N_D	Permissible Values of N_S
1	8, 9, 10, 11, etc.
2	9; 11, 13, 15, etc.
3	8, 10, 11; 13, etc.
4	9; 11. 13, 15, etc.
5	7; 8, 9, 11, etc.
6	7, 11, 13, 17, etc.

34. In this case the minimum permissible value of N_S is always greater than the corresponding value of N_D; hence all of the intercept regions will lie below unity. The table also shows that there are 11 possible intercept regions corresponding to N_S equal to or less than 10; actually there are only 9 such regions since the regions (1; 8), (1; 9), and (1; 10) overlap giving a single intercept region.

35. For this example N_D is sufficiently small that the trial and error method for computing m is quite practical. For example, in determining m for the intercept region (5; 7) it is found after trying

$m = 1$ and $m = 2$, that $m = 3$ and

$$r = \frac{mN_S - 1}{N_D} = 4.$$

36. To calculate the limits to the intercept regions equations (7) and (9) are applied after calculating m. Thus for the (5; 7) region

$$A_{min} = \frac{360^\circ \times 3 - 45^\circ}{360^\circ \times 4 + 10^\circ} = 0.7138,$$

and

$$A_{max} = \frac{360^\circ(5 - 3) + 45^\circ}{360^\circ(7 - 4) - 10^\circ} = 0.7150.$$

Thus interception is certain to occur, for these beam widths, within 7 rotations of the radar beam and 5 rotations of the receiving antenna if the speed ratio lies between 0.7138 and 0.7150.

37. The remaining 8 intercept regions are calculated in exactly the same fashion, and all are tabulated below.

[redacted]

INTERCEPT REGIONS

Values of A	$(N_D; N_S)$
0.0969 to 0.1286	(1; 8, 9, 10)
0.2172 to 0.2262	(2; 9)
0.2890 to 0.3048	(3; 10)
0.3729 to 0.3785	(3; 8)
0.4315 to 0.4482	(4; 9)
0.5514 to 0.5704	(5; 9)
0.6193 to 0.6285	(5; 8)
0.7138 to 0.7150	(5; 7)
0.8514 to 0.8581	(6; 7)

38. Depending upon the rotational speed of the radar beam, the speed ratios corresponding to some of these intercept regions could not be realized. For example, with a radar making 10 rotations per minute the maximum speed ratio which could be reached by a receiving antenna having the above limitations would be 0.6; hence the intercept regions (5; 8), (5; 7), and (6; 7) could not be used. Likewise for a slowly rotating radar beam some of the lower speed ratios are unusable, since, although an interception is insured in 10 or fewer rotations of the radar beam, more than 1 minute may be required.

CONCLUSIONS

39. The equations derived in this report are applicable to a wide range of intercept problems. They have so far been applied only to the problem of the interception of a rotating beam signal, such as that employed in radar, by a rotating direction finder. Specific conclusions will be drawn from that one application, but the same conclusions with only a slightly different wording will also often apply in some other application.

40. It is shown that for many of the more practical and currently employed rotational speeds and beam widths certainty of interception exists only within a number of relatively narrow ranges of speed ratios if the signal is to be on for only a limited number of rotations. These ranges can be widened by increasing the direction finder beam width but only if bearing accuracy is sacrificed.

41. The probability of interception within a limited number of rotations of the signal beam shows a general trend of increasing probability in the direction of increasing speed ratios. Hence a slowly rotating direction finder is all but useless in intercepting a rotating beam signal if the signal is to be on for only a few rotations.

42. There exist numerous speed ratios for which interception cannot be assured given even an infinite length of time. To insure that such a speed ratio is not being used, it might be advisable (though this has not been established) to rotate the direction finder at a variable speed.

[redacted]

43. Usually not only the direction but also the frequency of the source will be unknown. Hence the difficulty of obtaining a bearing is very considerably increased. In this case it would seem that the only hope for consistently obtaining a bearing within the limited number of rotations to be expected of radar signals is to employ one of the instantaneous types of direction finders together with a frequency scanning receiver capable of producing an interception within a few rotations of the signal beam.

ACKNOWLEDGEMENTS

44. An acknowledgement is made to Mr. James O. Spriggs of the Naval Research Laboratory at whose suggestion this work was undertaken and who rendered invaluable assistance during its development.

45. A further acknowledgement is made to Mr. John M. Hollywood of the Naval Research Laboratory who reviewed a rough draft of the manuscript and made numerous valuable suggestions, particularly the inclusion of the material in the second appendix.

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[redacted]

APPENDIX 1

THE PROBABILITY OF OBTAINING AN INTERCEPTION WITHIN A GIVEN NUMBER OF ROTATIONS OF THE SIGNAL BEAM

1. The method given above enables one to calculate for which speed ratios interception of a rotating beam signal by a rotating directional antenna is certain to occur within a specified number of rotations of signal and directional receiving antennas. It should not be inferred that between successive intercept regions there exists no probability of obtaining an interception.

2. For given beam widths the probability of obtaining an interception within N_S rotations of the signal beam is obviously given by

$$P = 1 + 1/360^\circ \left[k(\theta_D + \theta_S A) - \sum^{k} (\Delta \psi) \right], \qquad (14)$$

where k is the number of values of $\Delta\psi$, as given by equations (3) and (3'), which are larger than $\theta_D + \theta_S A$ and the $\Delta\psi$'s to be summed are those which are larger than $\theta_D + \theta_S A$. The curve of probability vs A is readily calculated from equation (14) since the curve is made up of segments of straight lines. In constructing the curve it is necessary only to calculate the values of speed ratios at which there is a discontinuity of slope and to calculate the probability of an interception at these speed ratios. The curve is then plotted by locating these points on the graph and connecting them by straight lines; cf. Plates 1-6.

3. Some of these slope discontinuities occur at the speed ratios for which an interception can never be assured. For these speed ratios equation (14) reduces to a simpler form, since all N_S of the $\Delta\psi$'s are then of equal size and all are larger than $\theta_D + \theta_S A$. Then equation (14) reduces to

$$P = \frac{N_S\theta_D + N_D\theta_S}{360^\circ} \qquad (15)$$

4. It might be pointed out that when N_S is set equal to one, equation (14) reduces to the well-known formula for the probability of interception within one rotation of the signal beam. Then $k = 1$, $\Delta\psi = 360^\circ$, and equation (14) reduces to

$$P = \frac{\theta_D + \theta_S A}{360^\circ}$$

[redacted]

GENERALIZATION OF INTERCEPT EQUATIONS

1. The equations derived above can be generalized so as to be applicable to a wider range of intercept problems such as the interception of an intermittent signal by a frequency scanning receiver. In making this generalization the receiving antenna beam of Figure 1, having a beam width of θ_D and rotating through 360° of azimuth, is replaced by a beam of beam width h_D and rotating at a uniform rate in generalized angular space of extent H_D. Thus for a frequency scanning receiver h_D is the bandwidth of the receiver and H_D is the frequency range to be covered by the intercept receiver. Similarly the signal beam of Figure 1 is replaced by a beam of beam width h_S which rotates at a uniform rate in angular space of extent H_S. Thus if a pulsed signal is to be intercepted, h_S is the pulse width and H_S is the pulse repetition time.

2. The method for making this generalization is obvious. Hence only the final forms will be written. For the limits to the intercept regions the results are

$$A_{min} = \frac{m - h_D/H_D}{r + h_S/H_S} \tag{16}$$

and

$$A_{max} = \frac{N_D - m + h_D/H_D}{N_S - r - h_S/H_S} \tag{17}$$

3. The quantities m and r are determined in precisely the same fashion as before. The minimum permissible values of N_D and N_S are again found by setting A_{min} equal to A_{max}; the results are

$$N_D = \frac{1 - N_S h_D/H_D}{h_S/H_S} \tag{18}$$

and

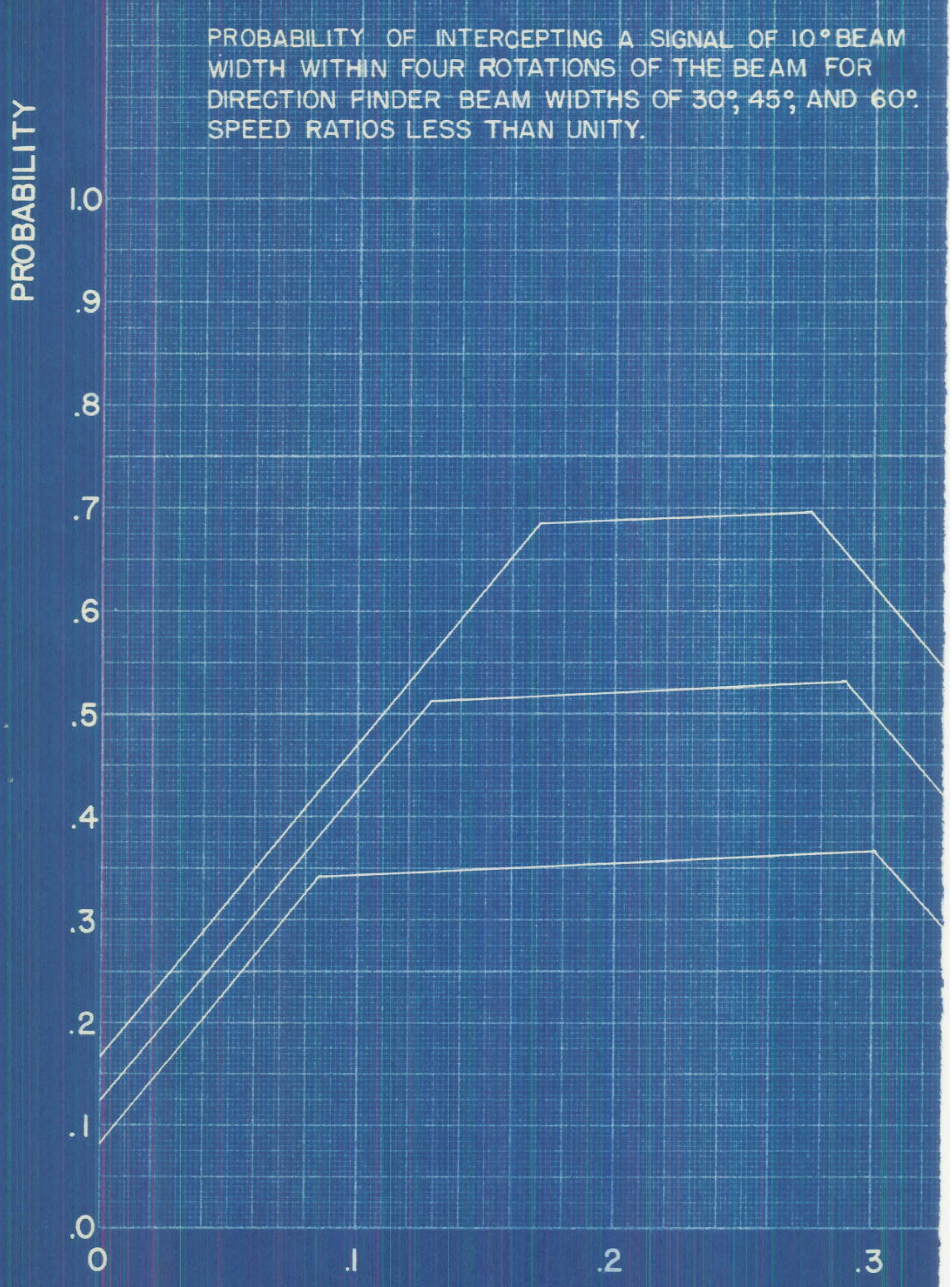
$$N_S = \frac{1 - N_D h_S/H_S}{h_D/H_D} \tag{19}$$

4. The generalized form of Equation (14), giving the probability of interception within N_S illuminations of the detector by the source, is

$$P = 1 + 1/H_D \left[k(h_D + H_D A h_S/H_S) - \sum^{k} \Delta\psi \right] , \tag{20}$$

where ψ is now given by $\psi = H_D(n_S A - n_D)$.

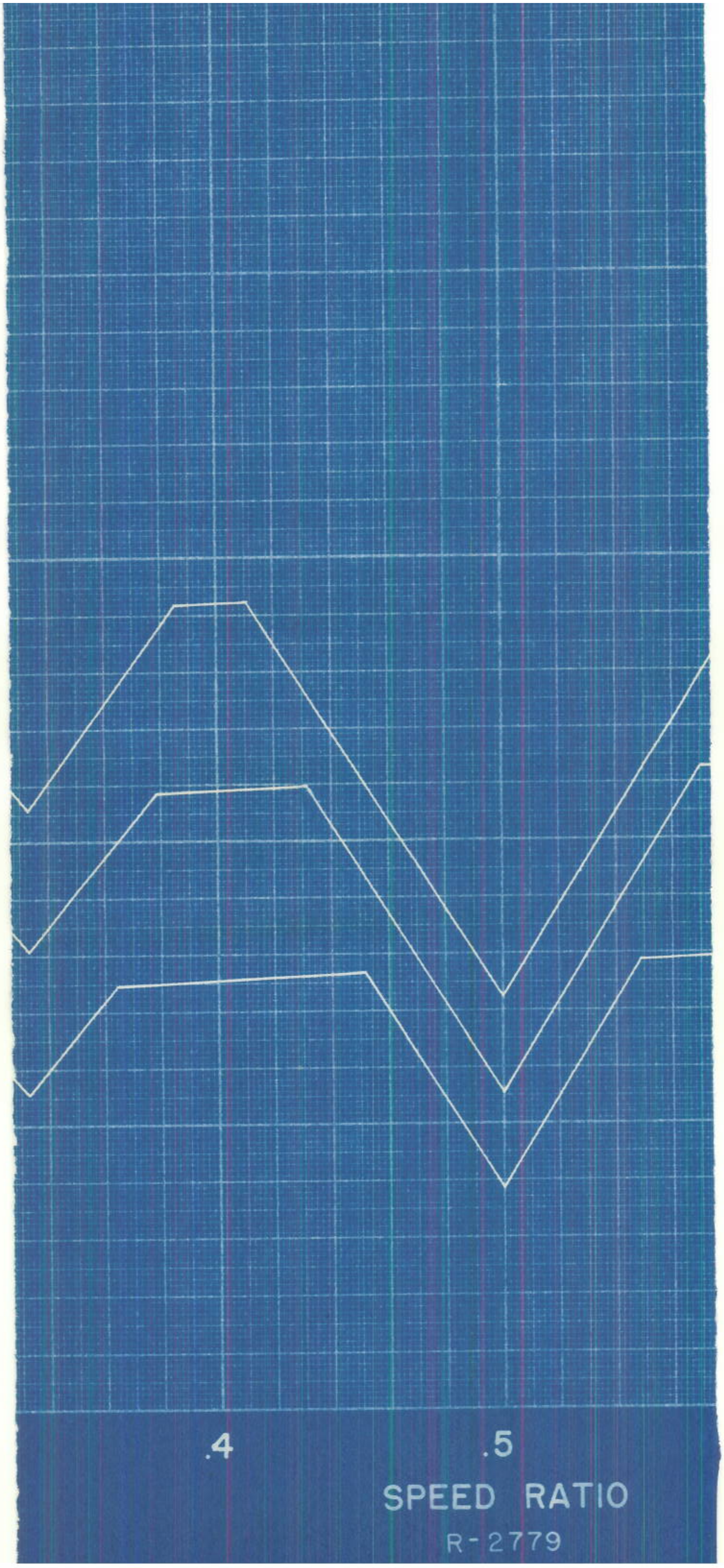
[redacted]

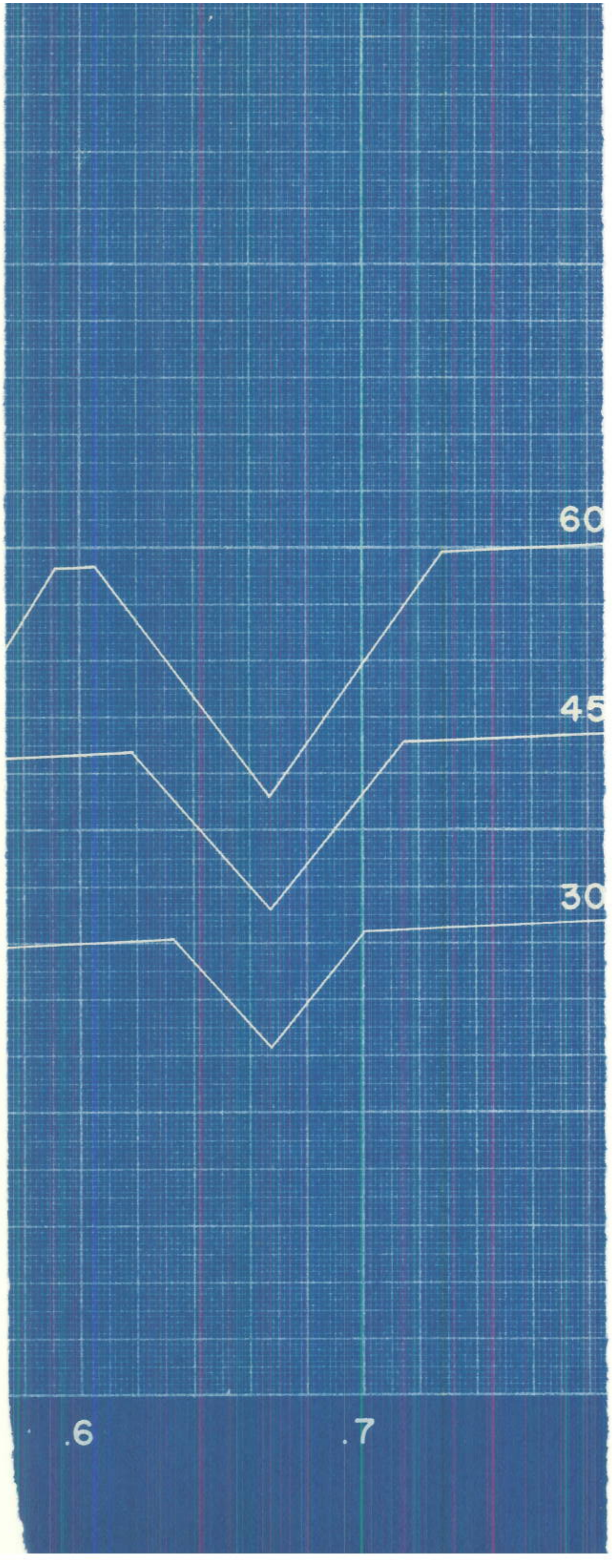


PROBABILITY OF INTERCEPTING A SIGNAL OF 10° BEAM WIDTH WITHIN FOUR ROTATIONS OF THE BEAM FOR DIRECTION FINDER BEAM WIDTHS OF 30°, 45°, AND 60°. SPEED RATIOS LESS THAN UNITY.

[redacted]

DECLASSIFIED


.4
.5
SPEED RATIO
R-2779


60
45
30
.6
.7

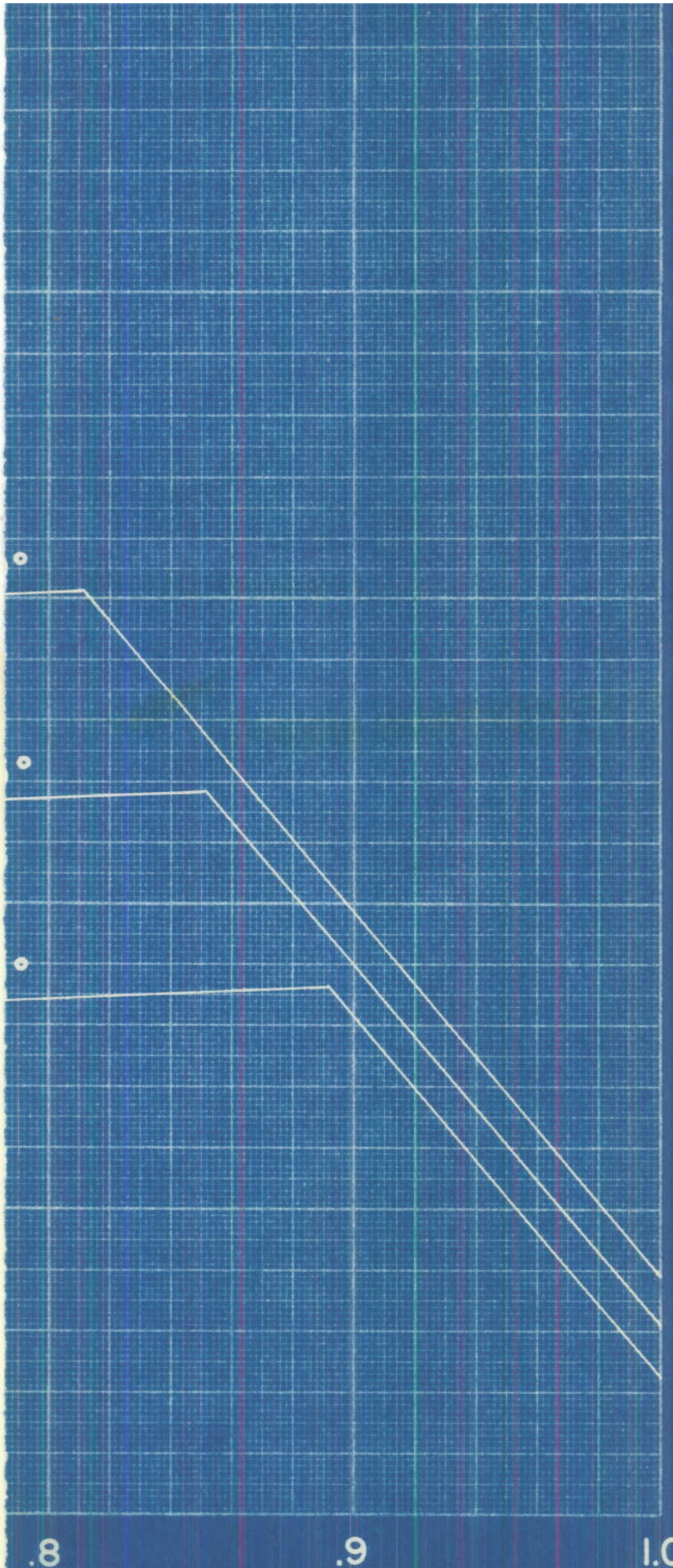
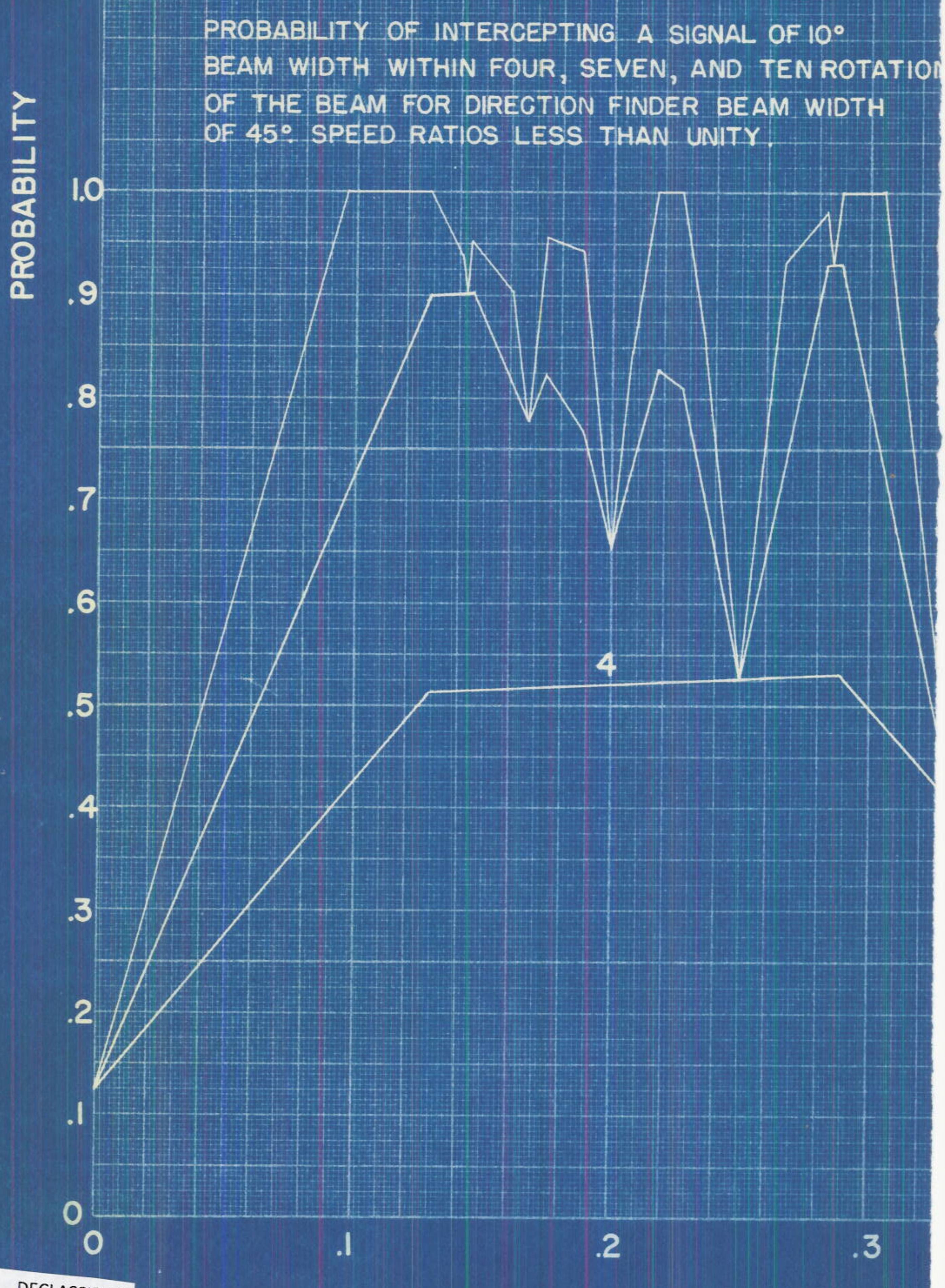


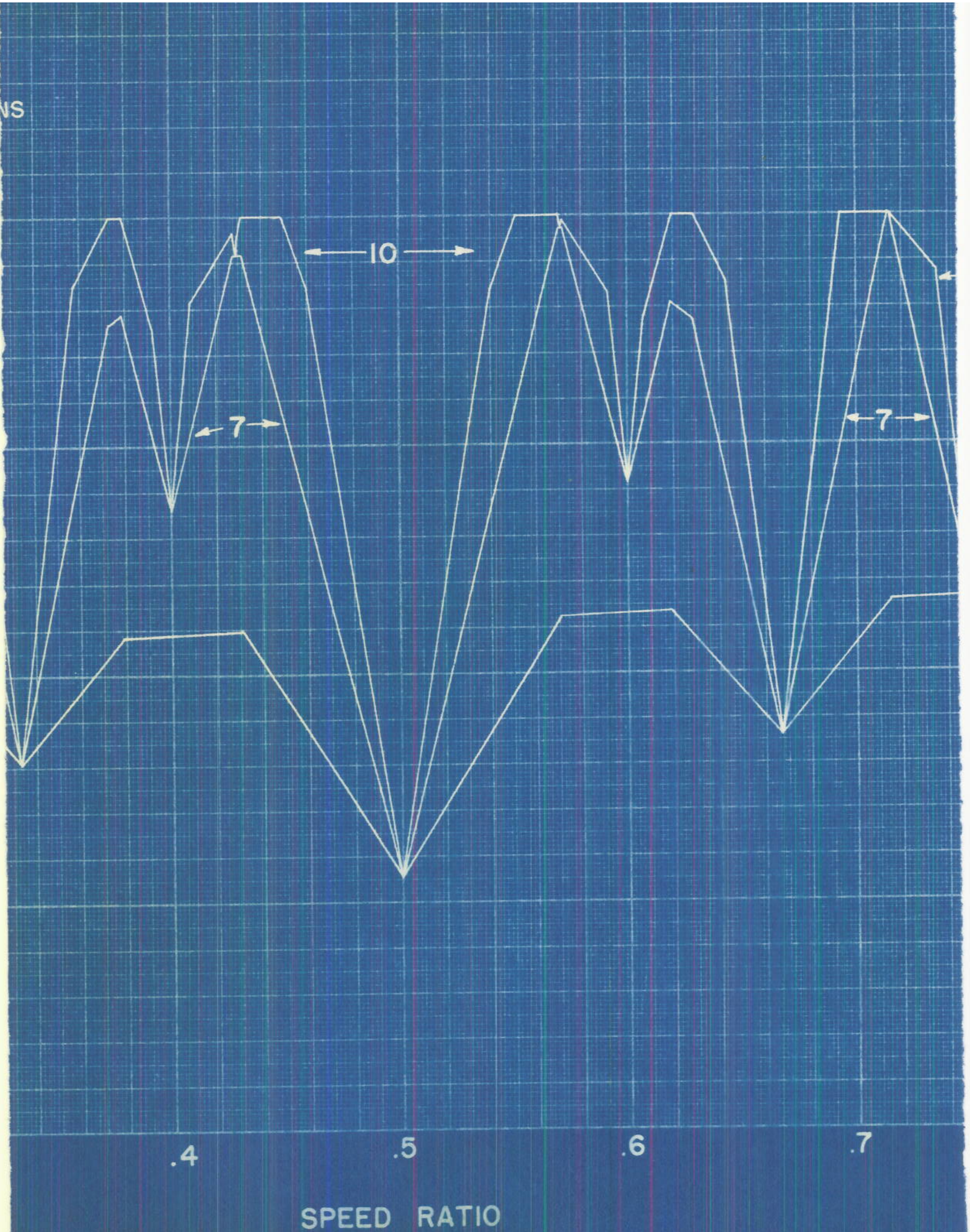
PLATE I



PROBABILITY OF INTERCEPTING A SIGNAL OF 10° BEAM WIDTH WITHIN FOUR, SEVEN, AND TEN ROTATIO OF THE BEAM FOR DIRECTION FINDER BEAM WIDTH OF 45°. SPEED RATIOS LESS THAN UNITY.

DECLASSIFIED

[redacted]



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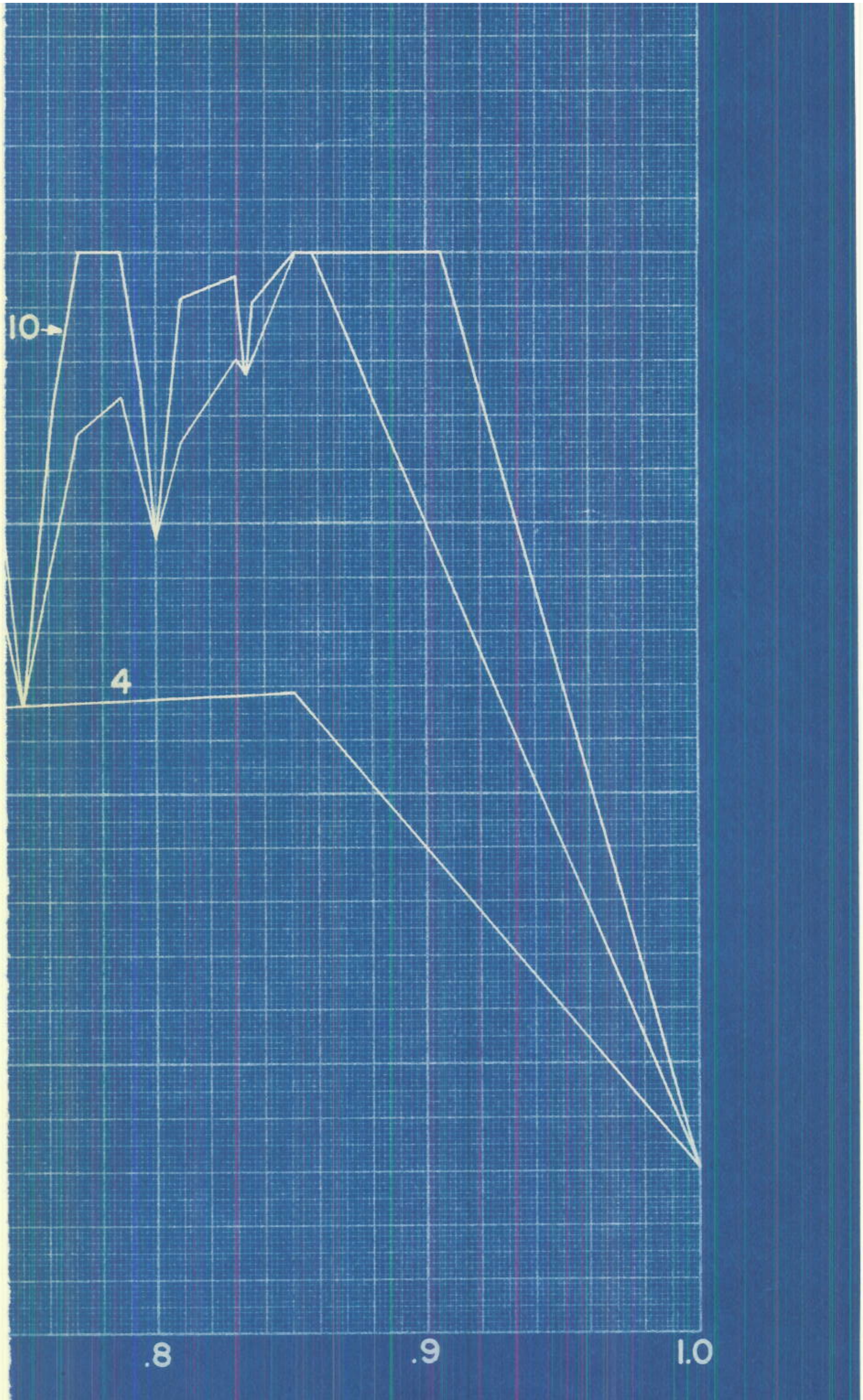
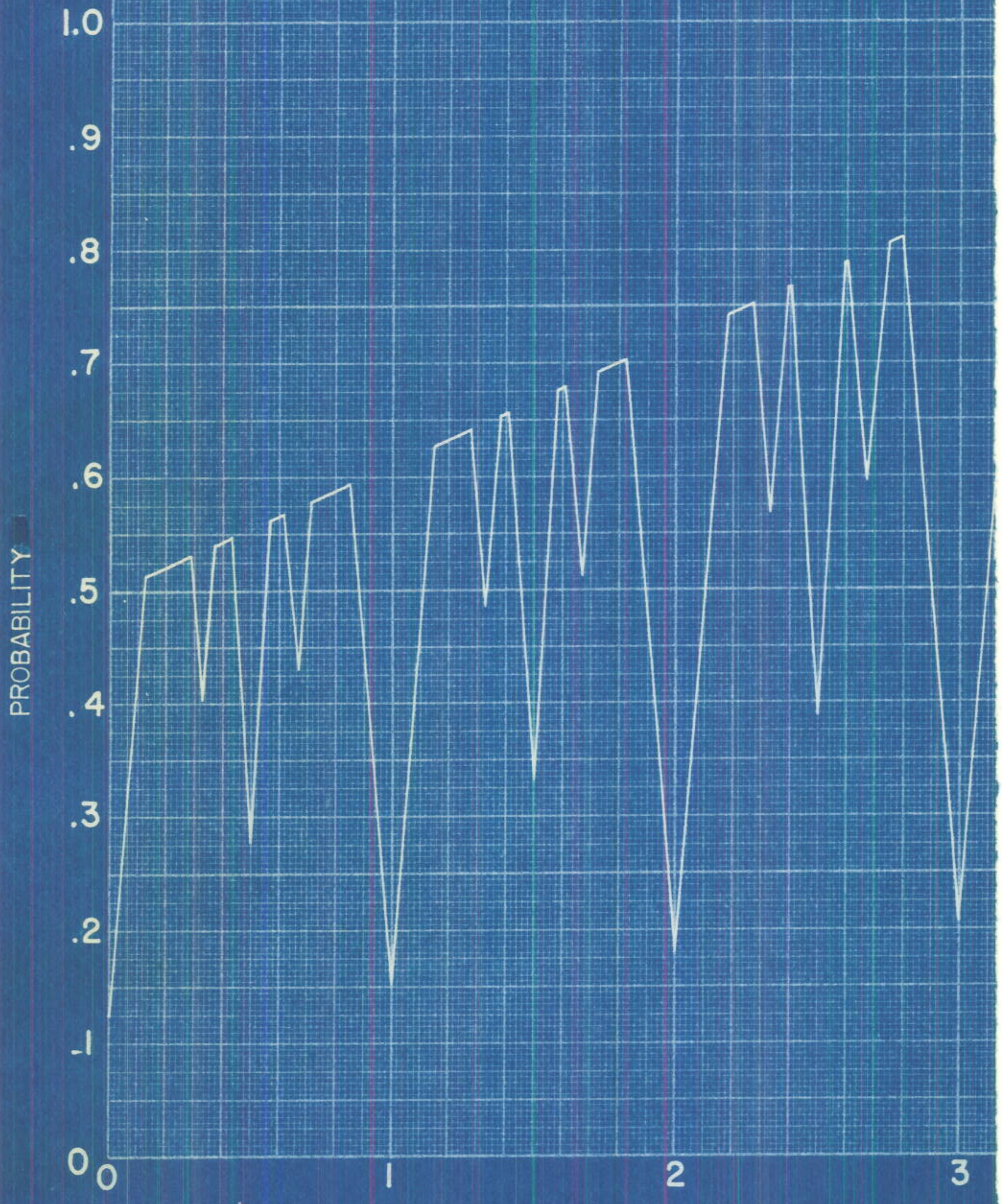
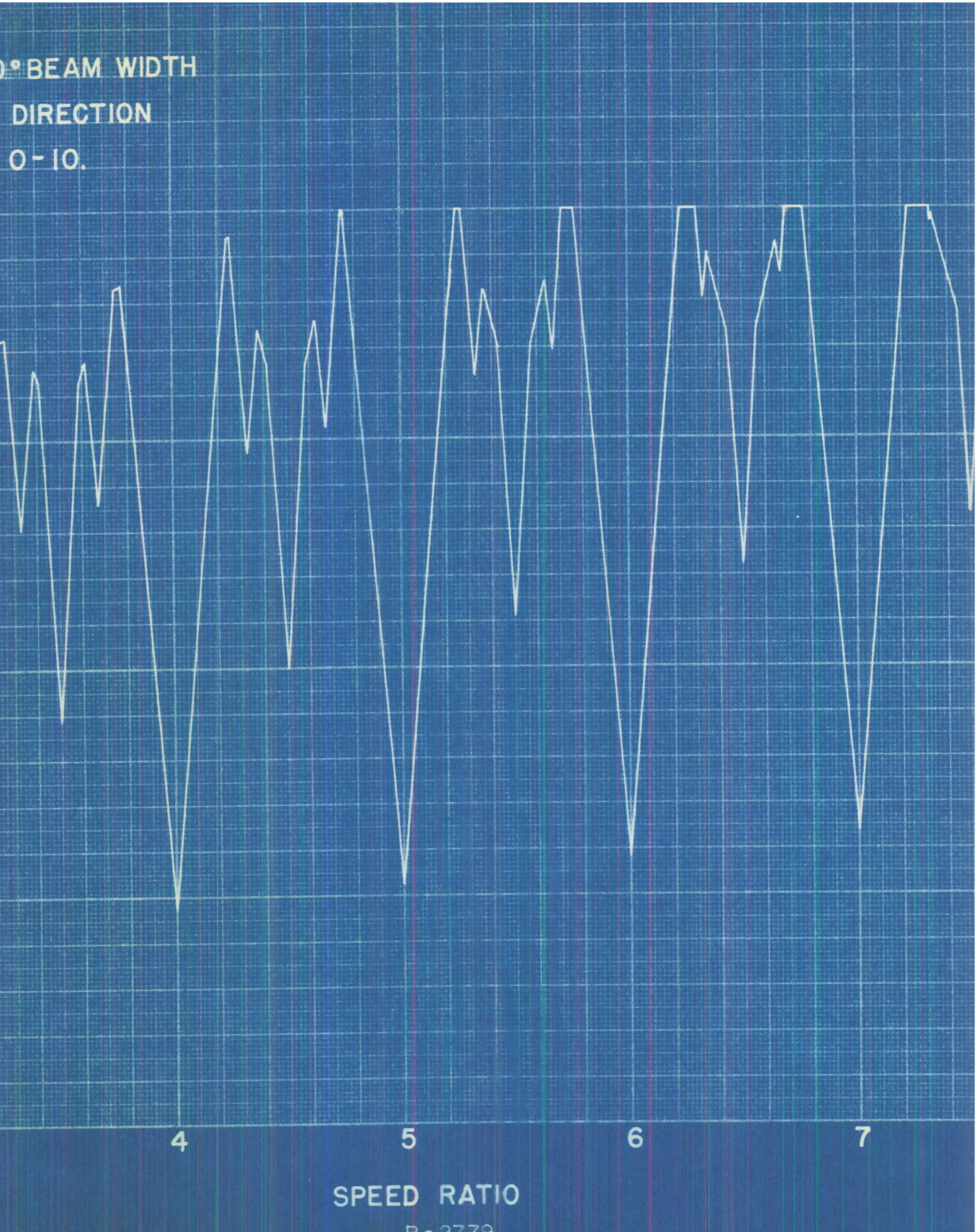


PLATE 2

PROBABILITY OF INTERCEPTING A SIGNAL OF
WITHIN FOUR ROTATIONS OF THE BEAM FO
FINDER BEAM WIDTH OF 45° SPEED RATIO





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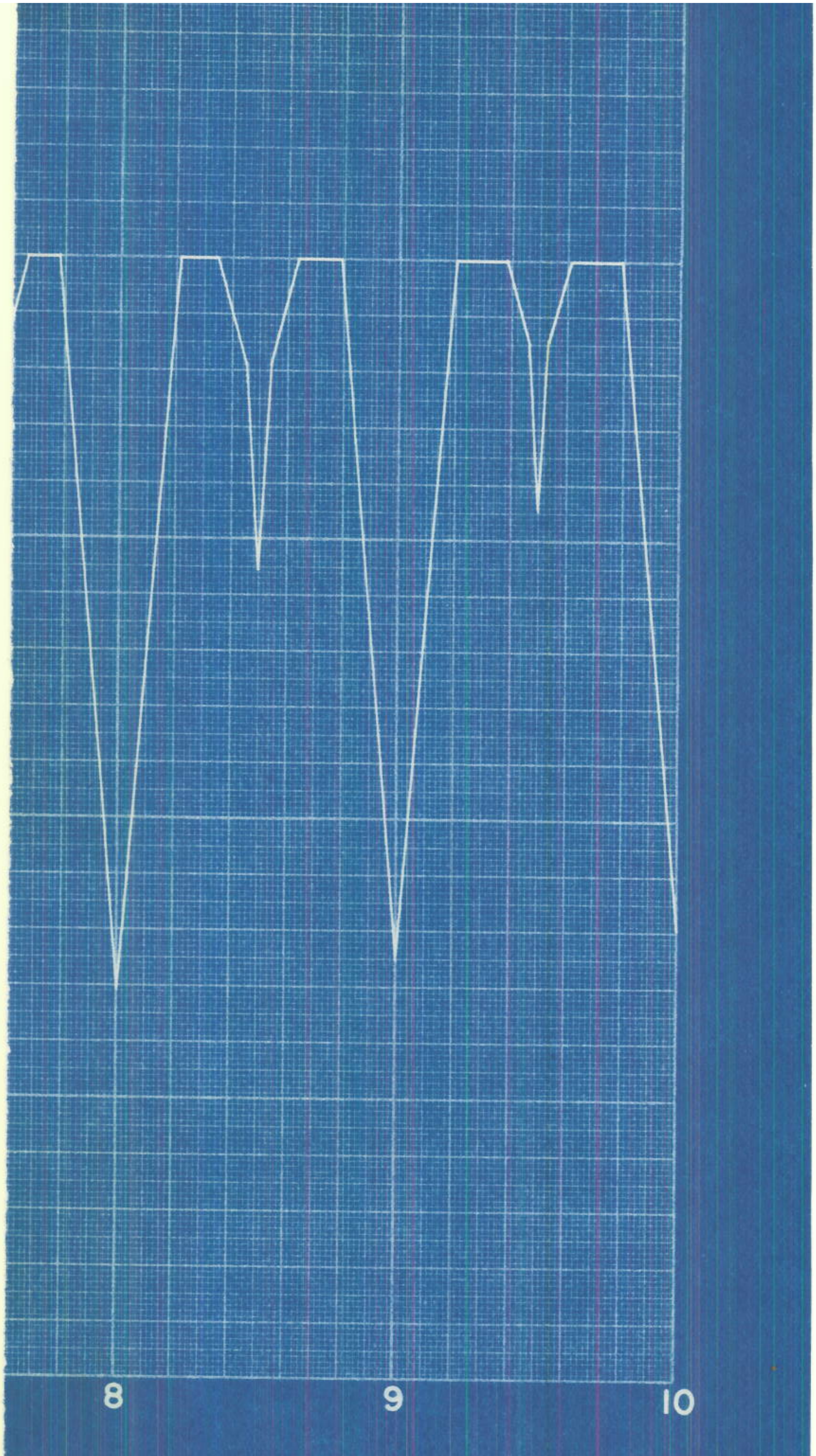
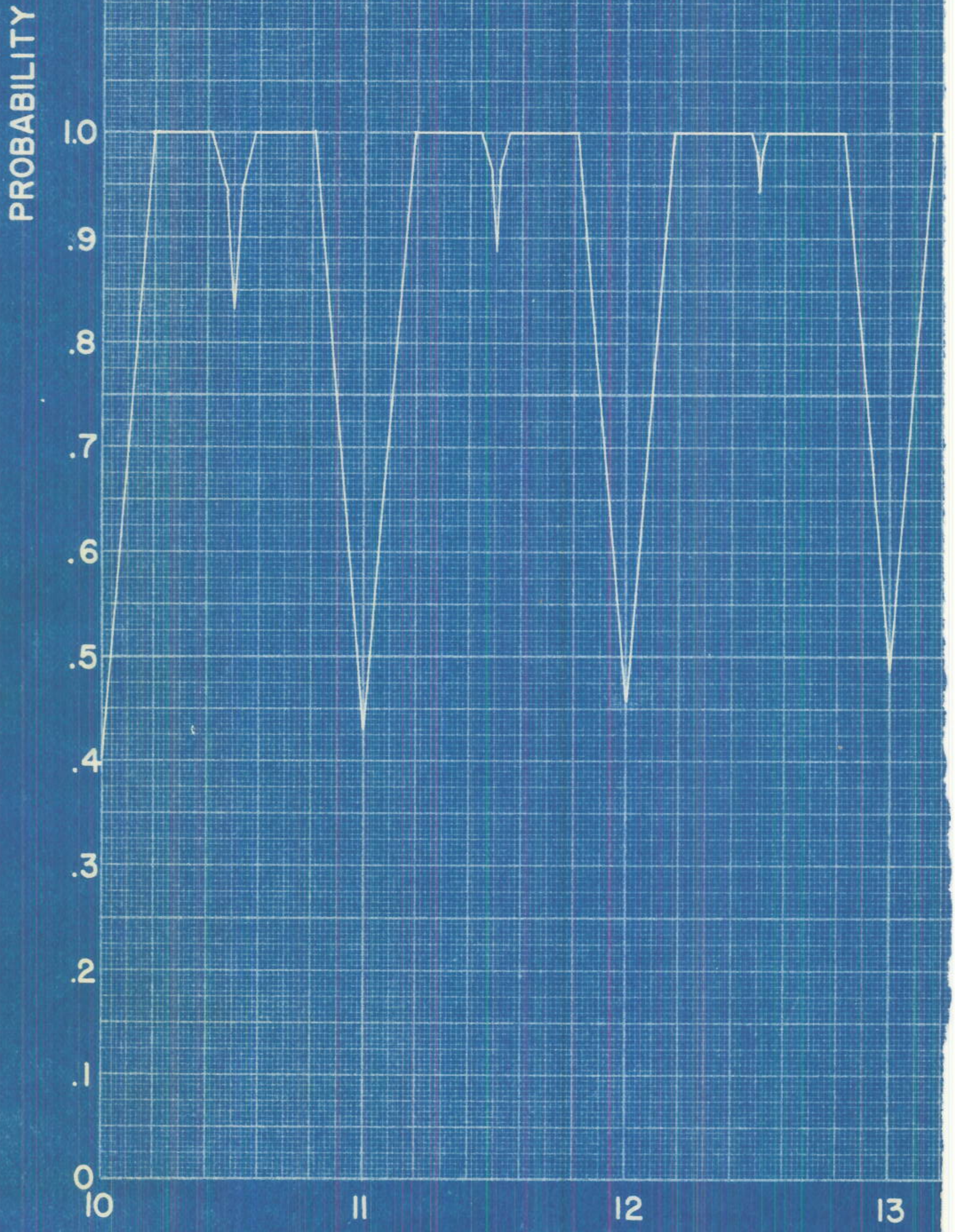
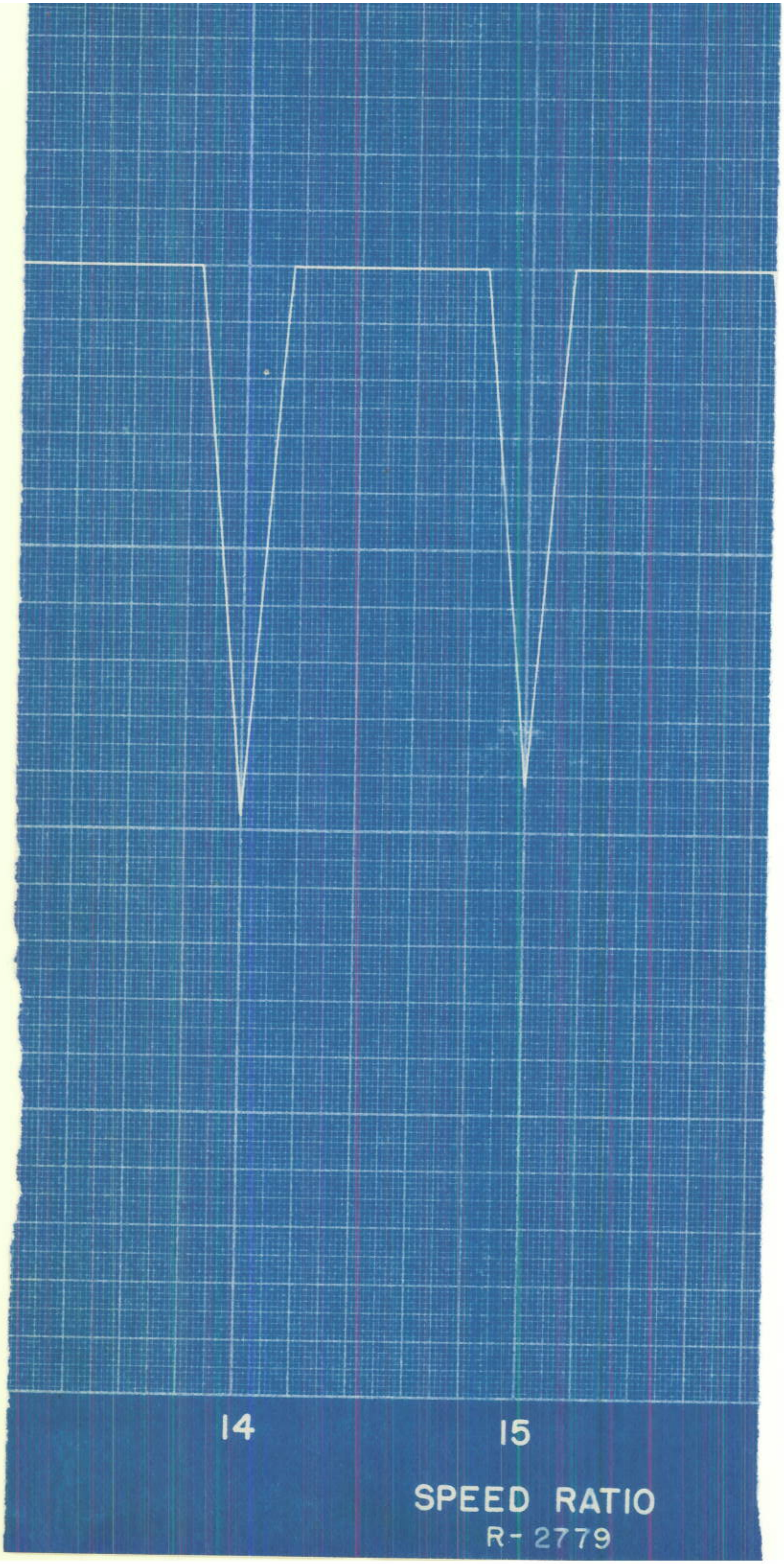


PLATE 3



DECLASSIFIED

[redacted]



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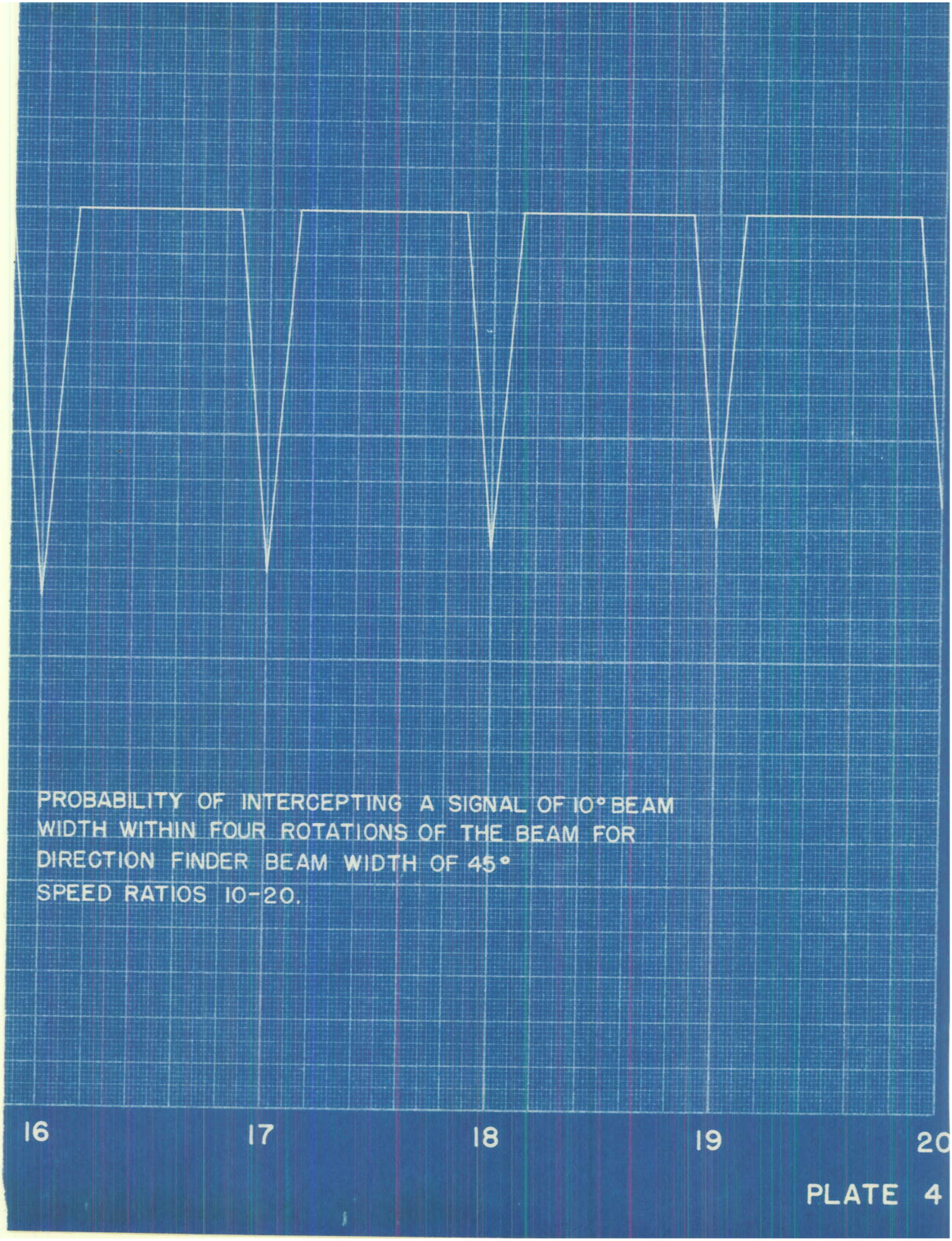
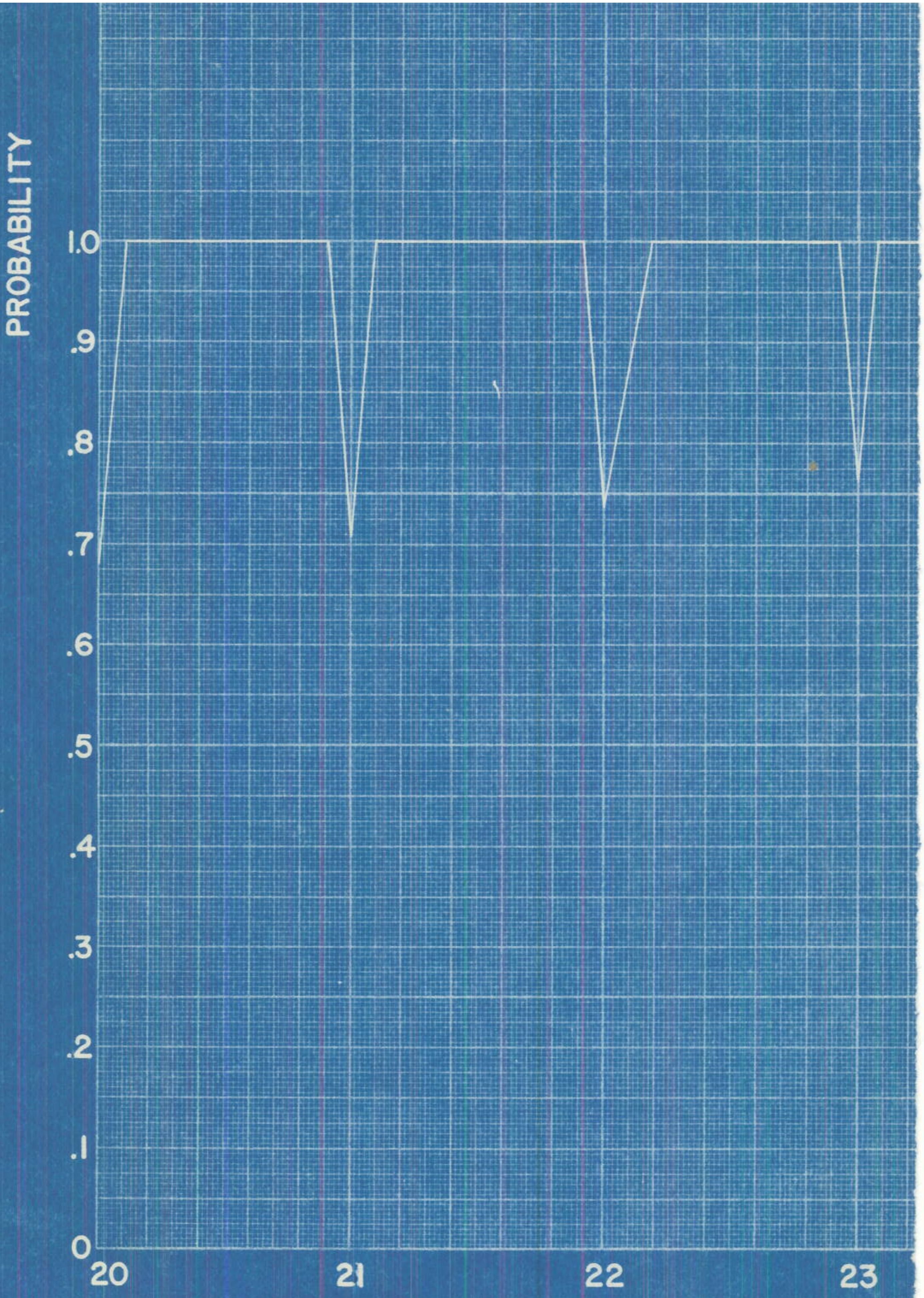
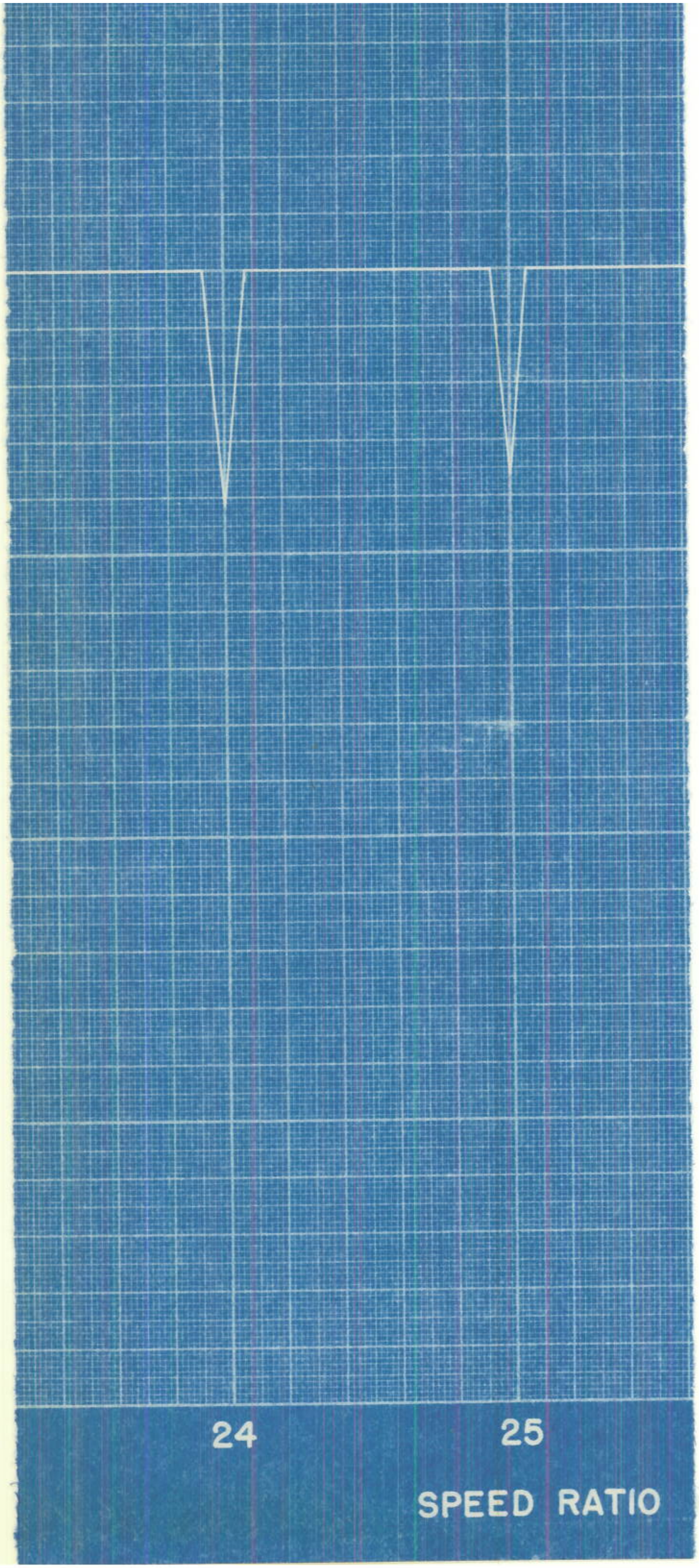
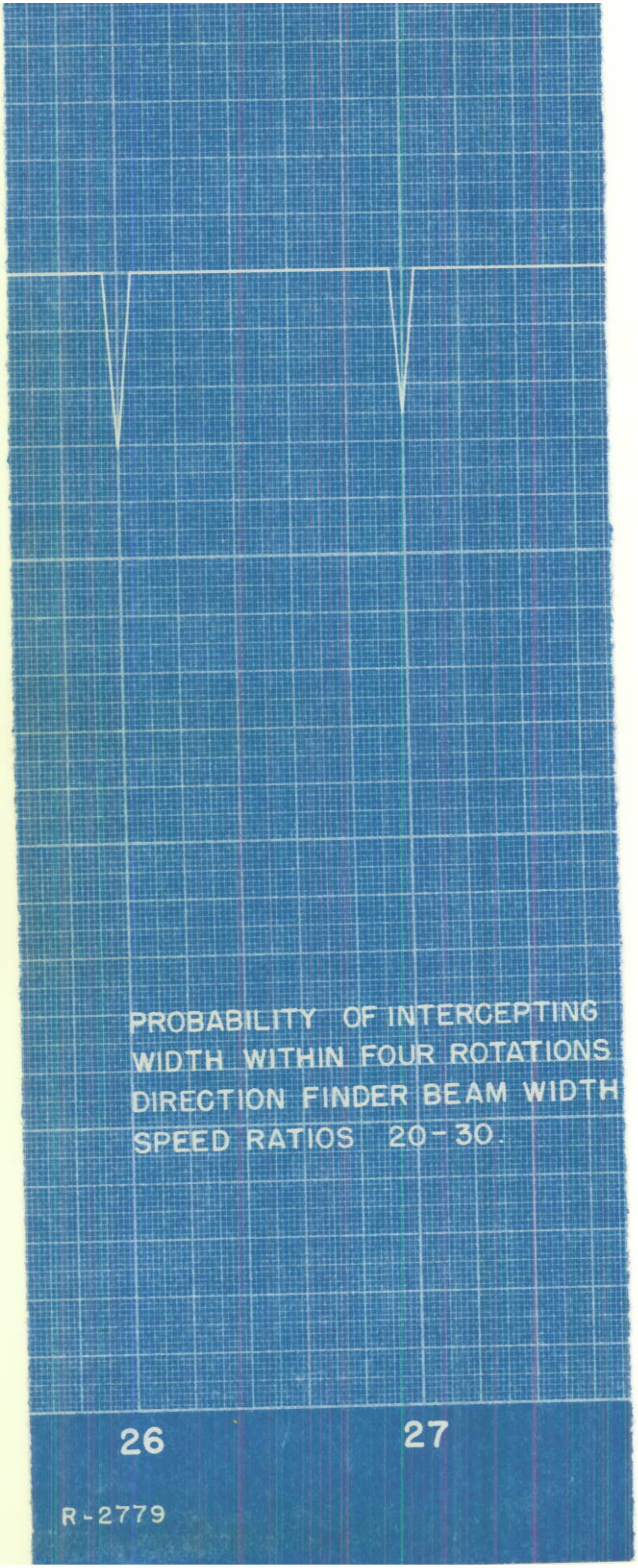

PROBABILITY OF INTERCEPTING A SIGNAL OF 10° BEAM
WIDTH WITHIN FOUR ROTATIONS OF THE BEAM FOR
DIRECTION FINDER BEAM WIDTH OF 45°
SPEED RATIOS 10-20.
16
17
18
19
20

PLATE 4

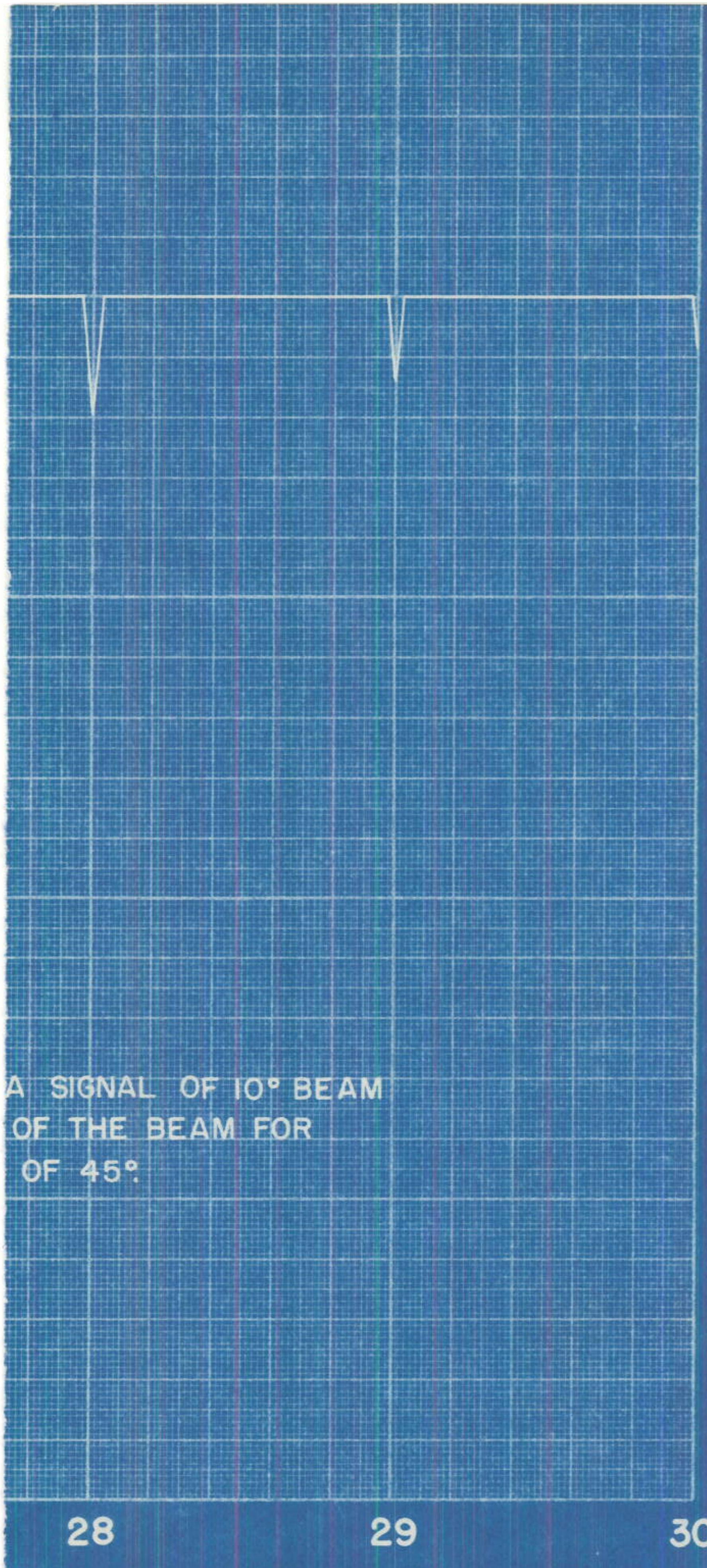

PROBABILITY
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
20
21
22
23


24
25
SPEED RATIO



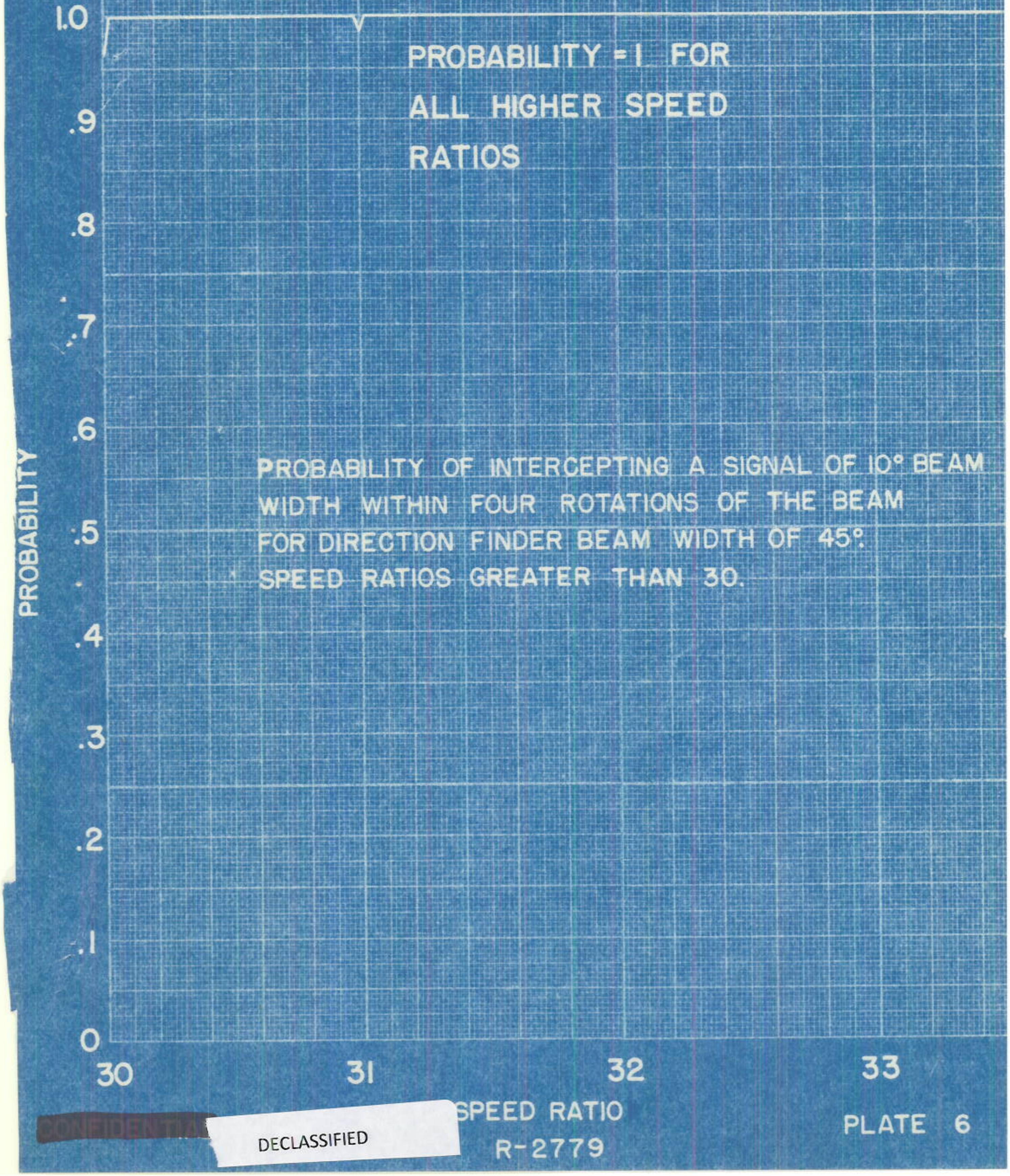
PROBABILITY OF INTERCEPTING
WIDTH WITHIN FOUR ROTATIONS
DIRECTION FINDER BEAM WIDTH
SPEED RATIOS 20-30.

R-2779



A SIGNAL OF 10° BEAM
OF THE BEAM FOR
OF 45°.

PLATE 5



PROBABILITY OF INTERCEPTING A SIGNAL OF 10° BEAM WIDTH WITHIN FOUR ROTATIONS OF THE BEAM FOR DIRECTION FINDER BEAM WIDTH OF 45°. SPEED RATIOS GREATER THAN 30.

~~CONFIDENTIAL~~ DECLASSIFIED

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PLATE 6