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GERMAN INTERFERENCE FILTERS

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ABSTRACT

The firm of Schott and Genossen manufactured "interference filters" which are essentially fixed separation, plane-parallel Fabry and Perot interference plates, the separation being about equal to the wave-length at which maximum transmission is desired and no greater than about 10 wave-lengths. When illuminated with white light, such a system shows a series of bright interference fringes when viewed with transmitted (or reflected) light. One of the transmitted fringes may be eliminated by incorporation of a colored glass cover opaque at that wave-length; a single bright fringe may then be seen in the visible spectrum, although usually a second bright fringe may be found in the near infra-red. The transmission may be about 30 percent at the maximum and about 0.15 percent at the minima on either side. The band width at half-intensity may be about 10 mμ.

The theory of the filters is discussed; experimental measurements on eleven available filters and properties of the filters computed from these measurements are presented; and suggestions for making filters for the ultra-violet and the infra-red are given.

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CHAPTER I

INTRODUCTION AND THEORY

Authorization. This investigation was requested by BuAer ltr. Aer-PH-1002-LMD of 19 Sept. 1945 to NRL.

The "interference filter" represents an ingenious application of the phenomenon of plane parallel plate interference to the production of practical filters of high transmission (about 30 percent) and quite narrow bandwidth (about 12 $m\mu$) useful in many applications. The filters have been described in (2) and (3). They were manufactured by Schott and Genossen, and it was stated in (2) that three series were available having maximum transmission coefficients of 10, 20 and 30 percent, respectively. Eleven filters were received at this Laboratory, each 5 cm in diameter and each of maximum transmission about 30 percent at wave-lengths within the range 445 to 690 $m\mu$. Spectral transmission curves of these filters are shown in Plates 3 to 8; these data will be discussed in more detail in the following chapter.

Description of an Interference Filter and its Action on Incident Light

An interference filter is made up by evaporating a thin, semitransparent film of silver or another chosen metal, depending on the part of the spectrum in which the filter is to be used, onto a glass plate (or quartz or fluorite, etc.), depositing on this a thin transparent film of nitrocellulose, or magnesium fluoride or other transparent material of a chosen thickness depending on the wave-length at which maximum transmission is desired, evaporating a second film of silver on top of the transparent separating film, and finally covering the whole assembly with a second glass plate. Either of the glass cover plates may be a conventional color filter for a purpose which will appear.

Reference (3), the German patent, states that collodion is used in the transparent film separating the two semitransparent silver films. The impression has been gained here that evaporated magnesium fluoride probably was used; the relative ease of making uniformly thick films by evaporation would appear to favor that method.

A sectional view of such a filter is shown in Plate 1A which is not drawn to scale. Plate 1B is a further simplification showing the transparent film mutually separating the plane-parallel, semitransparent silver films by the distance d . Such a plane-parallel plate interference producing system is perhaps most familiar in the form of the Fabry and Perot interferometer. The action of such a system on light incident upon it is well known and need be reviewed only for the sake of facilitating the discussion. For a more complete exposition of the theory see, for example, Schuster and Nicols (4) or Childs (5).

Light waves of wave-length λ and amplitude a_0 incident on the silver film at angle ϕ will be in part reflected and in part transmitted into the nitrocellulose as shown. Subsequently, at each interface the light is in part transmitted and in part reflected within the separating film so that eventually beams $a_1, a_2, a_3, \dots, a_n$ emerge as transmitted light, each in

succession being retarded in phase by an amount determined by the length of the path it has followed within the separating film. It can be shown that the difference in optical paths traversed by successive internally reflected rays is

$$D = 2\mu d \cos \theta \quad (1)$$

where μ = index of refraction of the separating film
 d = thickness of that film
 θ = angle of refraction of the incident light.

The phase retardation, β , is, hence,

$$\beta = \frac{2\pi}{\lambda} 2\mu d \cos \theta \quad (2)$$

where λ = wave-length
 and d and λ are expressed in the same units.

The transmitted light will be of maximum intensity whenever the phase retardation is an integral number of wave-lengths; i.e., and interference maximum occurs when

$$\begin{aligned} 2\mu d \cos \theta &= \lambda, 2\lambda, \dots m\lambda, \\ \text{or } \beta &= \frac{2\pi \mu d \cos \theta}{\lambda} = m \times 2\pi \end{aligned} \quad (3)$$

("Maximum of interference" denotes a bright fringe, "minimum of interference" a dark fringe).

An interference minimum occurs whenever

$$\begin{aligned} 2\mu d \cos \theta &= \lambda/2, 3\lambda/2, \dots (2m+1)\lambda/2 \\ \text{or } \beta &= (2m+1)\pi. \end{aligned} \quad (4)$$

Example. Let $\mu d = 1000$ millimicrons and let light fall on the filter at normal incidence so that $\theta = 0$, $\cos \theta = 1$. Maxima of interference (bright fringes) will be found at $\lambda \lambda$ 2000, 1000, 666, 500 — millimicrons, and intervening minima of interference $\lambda \lambda$ 4000, 1333, 800, 571, 445, — $m\mu$.

The alternate bright and dark fringes fall progressively closer together as shorter wave-lengths are approached; they are, however, equidistant in frequency provided only that the index of refraction remains constant over the frequency range under consideration. This may be seen from the following considerations. Let the thickness of the film be such that a bright fringe of order m occurs at λ_1 . Then the same film thickness will produce a bright fringe at some other wave-length λ_2 of order, say, $m+n$. Hence,

$$\begin{aligned} 2\mu d \cos \theta &= m\lambda_1 = (m+n)\lambda_2 \\ \text{Substituting } m &= \frac{n\lambda_2}{\lambda_1 - \lambda_2}, \\ 2\mu d \cos \theta &= \frac{n\lambda_1\lambda_2}{\lambda_1 - \lambda_2} \end{aligned} \quad (5)$$

In Eq. (5) put

$$\frac{1}{\lambda_{cm}} = \nu \text{ cm}^{-1}, \text{ the number of waves per cm. Then}$$

$$2\mu d \cos \theta = \frac{n}{\nu_2 - \nu_1} \quad (6)$$

or

$$\Delta \nu = \nu_2 - \nu_1 = \frac{n}{2\mu d \cos \theta} \quad (7)$$

where, as before, n is the number of fringes between wave numbers ν_1 and ν_2 . The frequency difference between successive bright fringes in the example above is

$$\Delta \nu = \frac{1}{2000 \times 10^{-7} \text{ cm}} = 5000 \text{ cm}^{-1}.$$

The use of these interference plates as filters requires that the unwanted interference maxima be eliminated leaving only the single maximum at the wave-length where transmission is desired. Thus, in the example given, if one of the glass covers were red, being opaque at 500 $m\mu$ and transmitting wave-lengths longer than 571 $m\mu$, a single visible transmission band would remain with its center at 666 $m\mu$. To be sure, the maxima at 1000 and 2000 $m\mu$ would also be present unless the glass cover were opaque at these wave-lengths, but their presence would not be a hindrance to the use of the filters in visual photometry or in any system in which the light measuring device was not sensitive to 2000 and 1000 $m\mu$ wave-lengths.

A second method of reducing the intensity at wave-lengths removed from the principal transmission band is described below under "Multiple Filters".

Complete Intensity Distribution

Relations (3) and (4) define the conditions under which bright and dark fringes occur. In order to obtain the complete expression for the intensity of the transmitted light it is necessary to find the vector sum of the amplitudes $a_1, a_2, a_3 \dots a_n$ represented in Plate 1. Let

$$a_0 = e^{i\omega t}.$$

Let T_1 and T_2 be factors by which the amplitude is reduced in passing through the upper and lower silver films, respectively, and let r_1 and r_2 be amplitude factors of reflection at the two silver surfaces. Then the amplitude of waves emerging along a_1 may be written

$$a_1 = T_1 T_2 e^{i(\omega t - \alpha)}$$

where α = the phase retardation resulting from a single traversal of the plate by the first ray.

Similarly,

$$a_2 = T_1 T_2 r_1 r_2 e^{i(\omega t - \alpha - \beta)}$$

$$a_3 = T_1 T_2 r_1^2 r_2^2 e^{i(wt - \alpha - 2\beta)}$$

$$a_4 = T_1 T_2 r_1^3 r_2^3 e^{i(wt - \alpha - 3\beta)}$$

$$\text{where } \beta = \frac{4\pi\mu d \cos \theta}{\lambda}, \text{ etc.}$$

The amplitude of the emerging light is the sum of these,

$$a = T_1 T_2 \left[1 + r_1 r_2 e^{-i\beta} + r_1^2 r_2^2 e^{-2i\beta} + \dots - r_1^n r_2^n e^{-ni\beta} \right] e^{i(wt - \alpha)} \quad (8)$$

The term in brackets is a geometric progression of the form $1 + x + x^2 + \dots + x^n$ where $x = r_1 r_2 e^{-i\beta}$, and it converges.

The sum has the value $\frac{1}{1 - r_1 r_2 e^{-i\beta}}$.

Hence,

$$a = \frac{T_1 T_2}{1 - r_1 r_2 e^{-i\beta}} e^{i(wt - \alpha)} \quad (9)$$

To find the intensity of the transmitted light, (9) must be multiplied by its conjugate complex, resulting in

$$\begin{aligned} I = a^2 &= \frac{T_1^2 T_2^2}{1 - r_1 r_2 (e^{-i\beta} + e^{i\beta}) + r_1^2 r_2^2} \\ &= \frac{T_1^2 T_2^2}{1 - 2r_1 r_2 \cos \beta + r_1^2 r_2^2} \\ &= \frac{T_1^2 T_2^2}{(1 - r_1 r_2)^2 + 4r_1 r_2 \sin^2 \beta/2}, \end{aligned} \quad (10)$$

by substituting $\cos \beta = 1 - \sin^2 \beta/2$.

Now, $T_1^2 = T_1$, $T_2^2 = T_2$ where T_1 and T_2 are the transmission coefficients giving the ratio of transmitted to incident intensities for the two silver films, and $r_1^2 = R_1$, $r_2^2 = R_2$ where R_1 and R_2 are the reflection coefficients of the two films. Thus, $T_1 T_2 = \sqrt{T_1 T_2} = T$, and $r_1 r_2 = \sqrt{R_1 R_2} = R$ where T and R are the geometric means of the transmission and reflectivity, respectively, of the two films. Making these substitutions in (10) the expression for the intensity becomes

$$I = \frac{T^2}{(1 - R)^2 + 4R \sin^2 \beta/2} \quad (11)$$

Eq. (11) is the familiar expression for the intensity in the interference fringes observed in the Fabry-Perot interferometer when the intensity of the incident light is unity. I represents the transmission of the complete plane-parallel plate interference system for any value of $\beta = \frac{4\pi\mu d \cos \theta}{\lambda}$.

It is convenient to consider normal incidence when $\cos \theta = 1$, and henceforth, unless specified, this condition will be assumed. By Eq. (11), maximum transmission will occur when $\sin^2 \beta/2 = 0$; i.e., when $\beta/2 = n\pi$ or $2\mu d = m\lambda$. Then

$$I_{\max} = \frac{T^2}{(1 - R)^2} \quad (12)$$

Similarly, minimum transmission occurs when $\sin^2 \beta/2 = 1$; i.e., when $\beta/2 = \frac{2m+1}{2} \pi$ or $2\mu d = \frac{2m+1}{2} \lambda$.

$$I_{\min} = \frac{T^2}{(1 - R)^2 + 4R} \quad (13)$$

(C.F. Eqs. (3) and (4)).

It is seen from Eq. (12) that $I_{\max} = 1.0$ if $T = 1 - R$, a condition not obtainable in metal films where absorption must occur; practically, a choice of metal and film thickness of the metal must be made for which R is high and $(R + T)$ approaches 1 as nearly as possible. For silver films the best value of R is accepted to be about 0.80.

Eq. 13 shows that the transmission is never zero, in the minima, but that it is lowest when R is large. The ratio of maximum to minimum transmission is obtained upon dividing Eq. (12) by Eq. (13),

$$\frac{I_{\max}}{I_{\min}} = \frac{(1 - R)^2 + 4R}{(1 - R)^2} = 1 + \frac{4R}{(1 - R)^2} \quad (14)$$

Examples:

(a). Let $R = 0.50$, $T = 0.50$.

$$I_{\max} = \frac{0.25}{0.25} = 1$$

$$I_{\min} = \frac{0.25}{2.25} = 0.11$$

$$\frac{I_{\max}}{I_{\min}} = 9$$

(b). Let $R = 0.90$, $T = 0.10$

$$I_{\max} = 1.0$$

$$I_{\min} = \frac{0.01}{0.01 + 3.6} = 0.00277$$

$$\frac{I_{\max}}{I_{\min}} = 361$$

The advantage of films of high reflectivity is obvious.

(c). More practically, let $R = 0.81$, $T = 0.10$, values characteristic of some of the German filters (see infra).

Then

$$I_{\max} = \frac{0.01}{0.036} = 0.278$$

$$I_{\min} = \frac{0.01}{3.27} = 0.003$$

$$\frac{I_{\max}}{I_{\min}} = 90.$$

Thus far only the positions of maxima and minima of interference and the values of transmission at the maxima and minima have been considered. Anticipating the experimental measurements, values of T , R and μd obtained for one of the filters have been substituted in Eq. (11) and the complete intensity curve over several fringes has been calculated by evaluating $\beta/2 = \frac{2\pi\mu d}{\lambda}$ at each wave-length. The assumption was made that R , T and the product μd remain constant over the wave-length range considered. The resulting intensity curve is shown in Plate 2. The "order of interference" or the number of internal reflections giving rise to each fringe (m of Eq. (3)) is shown for each fringe. These values of m were in fact determined by experimentally locating the three fringes shown as will be described below. While the computed maximum values of the intensity are equal for these fringes because of the assumption that R , T and μ remain constant throughout the spectrum, this would never be true with actual materials and the values of maximum transmission of the three fringes would differ.

The fringe chosen for use by the maker of this particular filter was that at $562 \text{ m}\mu$ for which $m = 2$; the third order fringe at $375 \text{ m}\mu$ was eliminated by the use of a green cover glass but could be located as a dark fringe by reflecting light from the clear-glass face of the filter; the fringe at $1124 \text{ m}\mu$ was found and measured by transmitted light.

The numerical calculations over one complete fringe are given in Table 3 which will be referred to later.

The Half Width

The term "half width" as applied to these filters means the full width of the transmission band at the ordinate equal to half the maximum intensity. This point has been marked on Plate 2. It has been seen that

$$I_{\max} = \frac{T^2}{(1 - R)^2}.$$

Furthermore,

$$I_{\text{half}} = 1/2 I_{\max} = \frac{T^2}{2(1 - R)^2} = \frac{T^2}{(1 - R)^2 + 4R \sin^2 \beta'/2} \quad (15)$$

where β' is the phase retardation at the wave-lengths of half intensity on either side of the maximum.

Simplifying,

$$\sin \beta/2 = \pm \frac{1-R}{2\sqrt{R}} = \sin \frac{2\mu d\pi}{\lambda}. \quad (16)$$

It has been seen that maximum transmission occurs when $\frac{2\mu d\pi}{\lambda} = m\pi$; hence, in a fringe of given order m the two positions of half-intensity will correspond to phase retardations $m\pi \pm \beta/2 = m\pi \pm \sin^{-1} \frac{(1-R)}{2\sqrt{R}}$. In the example of Plate 2, $R = 0.865$ and $\pm \sin^{-1} \frac{(1-R)}{2\sqrt{R}} = \pm 4.22^\circ = \pm 0.023\pi$ radians. μd was measured to be $562 \text{ m}\mu$ (see Chapter II); hence

$$\frac{2\pi\mu d}{\lambda} = \frac{1124\pi}{\lambda} = (m \pm 0.023)\pi,$$

and for the first order fringe half intensity occurred at

$$\lambda_1 = \frac{1124}{1.023} = 1098.7 \text{ m}\mu;$$

and at

$$\lambda_2 = \frac{1124}{0.977} = 1150.5 \text{ m}\mu.$$

The half-width was $\Delta\lambda = 52 \text{ m}\mu$. In frequency units the half width was

$$\Delta\nu = \frac{1}{\lambda_1} - \frac{1}{\lambda_2} = 41 \text{ cm}^{-1}.$$

For $m = 2$, $\lambda_{\text{max}} = 562 \text{ m}\mu$;

$$\lambda_1 = 555.6, \quad \lambda_2 = 568.5$$

$$\Delta\lambda = 12.9 \text{ m}\mu$$

and, again,

$$\Delta\nu = 41 \text{ cm}^{-1}.$$

For $m = 3$, $\lambda_{\text{max}} = 374.7 \text{ m}\mu$:

$$\lambda_1 = 371.8 \text{ m}\mu, \quad \lambda_2 = 377.6 \text{ m}\mu,$$

$$\Delta\lambda = 5.8 \text{ m}\mu$$

$$\Delta\nu = 41 \text{ cm}^{-1}.$$

But if the optical thickness were twice as great, $\mu d = 1124 \text{ m}\mu$, then the visible fringe at $562 \text{ m}\mu$ would result when $m = 4$, and half-intensity would occur for $\beta/2 = (4 \pm 0.023)\pi$, or at $\lambda_1 = 558.8$, $\lambda_2 = 565.2 \text{ m}\mu$; $\Delta\lambda = 6.4 \text{ m}\mu$, compared with $\Delta\lambda = 52 \text{ m}\mu$ for the fringe at this wave-length for $m = 2$. Thus, the fringes may be made narrower by increasing μd , but not indefinitely, for if μd becomes too great fringes do not appear in white light because the fringes of the many orders overlap and produce essentially uniform illumination throughout the spectrum. Reference (3) specifies that μd shall not be greater than 10λ .

The Fabry and Perot interferometer, it is true, employs plate separations of several millimeters or centimeters so that for approximately monochromatic divergent light falling on it, values of m of, say, 20000 may be used. The half width is then quite narrow and fringes due to wave-lengths very near together may be distinguished.

Multiple Layer Filters

If two interference filters are mounted on opposite sides of a glass plate so that they are mutually remote, the transmission of the assembly is obtained by multiplying the transmission factors of one filter by those of the other. Thus it can be seen that if, for one filter, $\mu_1 d_1 = 562 \text{ m}\mu$ giving rise to the fringe for $m = 2$ at $562 \text{ m}\mu$ (Plate 2), a second value $\mu_2 d_2$ could be chosen for which the $(m + n)$ th fringe would lie at $562 \text{ m}\mu$ while fringes of other orders would not coincide with those in the first plate. For example, if $m + n = 1$, the first order fringe would lie at $562 \text{ m}\mu$ when $\mu_2 d_2 = 281 \text{ m}\mu$, the second order at $281 \text{ m}\mu$, etc., so that only the second order fringe in $\mu_1 d_1$ and the first order fringe in $\mu_2 d_2$ would coincide. At other points in the spectrum the transmission would be somewhat lower than that of a single filter since it would be the product of small transmission factors except at points where the higher order fringes occur. The transmission factors at these points would be the products of the maximum of one filter and the low transmission factors of the second filter.

Reference (3) describes multiple layer filters in which two films of equal thickness and one film of somewhat greater thickness are deposited in succession, each being bounded by a silver reflecting film, of course. As in the example above, let the first two layers be of thickness $\mu_2 d_2 = 562 \text{ m}\mu$, and for the third, $\mu_3 d_3 = 281 \text{ m}\mu$. This filter differs from the preceding example only in that the two filters $\mu_1 d_1$ and $\mu_3 d_3$ are separated from each other by $\mu_2 d_2$ instead of by an intervening glass plate. Maxima will be produced at 562 millimicrons by $\mu_1 d_1$ and $\mu_3 d_3$. Now, it has been seen that the maximum transmission of a fringe would be 1.0 if there were no absorption in the metal films; i.e., if $T = 1 - R$ in Eq. (11). Hence, there is virtually no reflection of light by the metal films at the wave-length of maximum transmission and; hence, there can be no reflection of light in the intervening layer $\mu_2 d_2$. Therefore, $\mu_1 d_1$ and $\mu_3 d_3$ behave like two separated filters

If multiple filters exist among those examined at this Laboratory, the methods employed did not disclose them. All appeared to be single filters. However, reference (3) makes claims on the use of many films laid on each other to suppress all maxima but one by proper choice of the several films.

CHAPTER II

EXPERIMENTAL MEASUREMENTS

Spectral Transmission

The spectral transmission factors of eleven available filters were measured with a Beckman spectrophotometer, and the results are shown in Plates 3 to 8. Each filter is designated on the plates by the manufacturer's number together with the manufacturer's designation of wave-length of maximum transmission ($\lambda_{\max}$), transmission at maximum ($T_{\max}$) and half width (HwBr). The observed half-width and the half-width calculated from average values of R are also shown.

The beam of light emerging from the second slit of the Beckman spectrophotometer and falling on the sample was divergent, and the divergence was about 5 degrees. While the filters were placed in the spectrophotometer with their faces normal to the optical axis; the divergence of the light falling on them should broaden the observed fringes on the short wave-length side; that is,

$$\beta/2 = \frac{2\pi\mu d \cos \theta}{\lambda} = m\pi$$

is satisfied by a smaller value of λ when $\cos \theta < 1$. Pronounced broadening was not observed; but it should be remarked that the wave-length of maximum transmission may be considerably shifted to shorter wave-lengths when the filters are placed in an optical system askew so that $\theta \neq 90^\circ$ for all of the incident light.

The effective slit widths of the Beckman spectrophotometer, equal to the sum of the effective widths of each slit, are shown on each plate.

The Order of Interference

It has been shown that maxima of transmission occur when

$$2\mu d \cos \theta = m\lambda$$

where m is an integer. Let the visible interference fringe be the m^{th} fringe. Then the fringes in order at longer wave-lengths are the $(m-1)^{\text{th}}$, $(m-2)^{\text{nd}}$, etc. and at shorter wave-lengths, $(m+1)^{\text{th}}$, $(m+2)^{\text{nd}}$ etc. In most cases the $(m-1)^{\text{th}}$ fringe could be found by transmission measurements in the near infra-red. The wave-length scale of the Gaertner quartz monochromator with which these observations were made is not closely divided, and detailed transmission curves were not made, but only $\lambda_{\max}$ was determined as closely as possible by interpolation.

Since each filter incorporated a conventional glass color filter to eliminate fringes of shorter wave-length than the m^{th} fringe, it was necessary to determine the position of these fringes by reflection from the clear-glass side of the filter. This was done for normal incidence with a Bausch and Lomb constant deviation spectroscope as well as with the Gaertner monochromator. The fringes were dark by reflected light, of course.

Thus, for most of the filters the relation

$$2\mu d = m\lambda_m = (m-1)\lambda_{m-1} = (m+1)\lambda_{m+1}$$

could be solved for m and for μd . The computation assumes that μ remains constant over the wave-length interval λ_{m+1} to λ_{m-1} . The results are given in Table 1.

If the index of refraction of the transparent separating layer varies with wave-length, the effect is equivalent to a variation in thickness of the filter in proceeding through the spectrum. Since usually the index of refraction of transparent media increases in going from longer to shorter wave-lengths, the optical thickness increases toward shorter wave-lengths and the fringes become more closely spaced. The effect is more readily observable in thick films where there are many narrow fringes and, in fact, it affords a means of measuring the dispersion of the film material with accuracy if d is known and if μ is known at one wave-length.

Rand T

At the wave-length where the transmission of the filter is half the maximum value,

$$\frac{2\mu d\pi}{\lambda_{\text{half}}} = \sin^{-1} \left(\pm \frac{1-R}{2\sqrt{R}} \right).$$

By substituting the observed values of λ at half intensity into this expression, values of R were obtained. Upon substituting the computed value of R and the measured values of I_{max} into Eq. (12), values of T were obtained. These values together with the absorption $A = 1-(R+T)$ are given in Table 2 in two sets of numbers derived from the manufacturer's data and from the data obtained in the present measurements. The computed values depend on measurement of I_{max} , I_{half} and λ_{half} and incorporate errors in these measurements. Now, the transmission of a filter is the product of the transmission by interference and the transmission factors of the cover glasses, one of which in every case was colored. These factors were unknown. However in Eq. (15) these transmission factors would apply to both sides of the expression, provided the transmission of the colored glass filter was the same at λ_1 , λ_{max} and λ_2 in the notation used above, and, hence, Eq. (16) determining R is unaffected. However, when values of R are substituted into Eq. (12) to obtain T , the result is in error because I_{max} observed is somewhat lower than it would be if there were no cover glasses.

The results of Table 2 are intended to show the variations in the values of R and T to be expected in actual filters rather than to represent their precise values.

Minimum Transmission

Eqs. (11) shows that the minimum value of transmission, when $\sin^2 \phi/2 = 1$, is considerably less than the product of the transmissions of the two metal films, due to the redistribution of energy by interference. No attempt has been made to carry out the tedious procedures necessary for measuring the extremely small transmission coefficients at wave-lengths well

removed from the region of high transmission. But the fringes shown in Plate 2 were calculated from the data for filter 2433 from Table 2. These data for the fringe at $562\text{ m}\mu$ have been plotted as crosses on the experimental curve for that filter, Plate 5. They are also shown completely through the lateral minima on either side of this fringe in Table 3. Since the computation was based on data derived from the experimental curve, the deviation of computed from observed values probably is indicative of small errors in R, T and μd and of variations of these parameters with wave-length.

Table 3 shows that the minimum transmission factor of this filter was 0.0016; all factors on the short wave-length side were further reduced by the red cover glass, but this value would be approximately correct for the minimum at $749\text{ m}\mu$.

CHAPTER III

SUGGESTIONS

This Laboratory has had available time to make only one imperfect interference filter employing aluminum reflecting films and a thin film of nitrocellulose as the separating medium. It is expected that more extended efforts may be made as specific needs for the filters arise. Those obvious suggestions are made for certain specialized applications.

Ultra-Violet Aluminum reflecting films and evaporated quartz separating films (by evaporation of Santocell, Monsanto Chemical Corporation) on quartz cover plates.

Infra-Red Gold, copper or nickel reflecting films (see ref. (3), (5) on fluorite or rock salt plates with evaporated magnesium fluoride or quartz as the separating film. Measurements at this Laboratory indicate that thick evaporated quartz films are quite transparent except in the $9\ \mu$ region where a small decrease in transmission occurred due to selective reflection.

It is not inconceivable that, at wave-lengths of several microns, the filters might be made with thin foil spacers or very accurate screw adjustments for maintaining the correct separation. The filters then would become applicable to the measurement of the dispersion of liquids at those wave-lengths by measurement of the displacement of the fringes upon introduction of the liquid between the plates. Since organic liquids absorb strongly in the infra-red, the use of small plate separations would be advantageous.

There are many other applications of these filters which may be explored at this Laboratory.

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TABLE 1

Observed Positions of Fringes and Computed
Values of m and μd . λ in millimicrons

Filter	m	λ_m	μd ($= \frac{m\lambda_m}{2}$)	λ_{m-1}	μd ($= \frac{\lambda_m - \lambda_{m-1}}{2}$)	λ_{m+1}	μd ($= \frac{(m+1)\lambda_{m+1}}{2}$)
2092	2	444	444	890	442	—	—
2093	2	464	464	950	475	—	—
2561	2	522	522	1025	512	—	—
2445	2	546	546	1075	537	373	559
2569	2	554	554	1100	550	377	565
2433	2	562	562	1110	555	371	556
2434	2	586	586	1179	589	396	594
2554	2	605	605	1260	630	406	609
2552	2	648	648	—	—	440	660
2360	2	650	650	1270	635	—	—
2335	2	690	690	1370	685	467	700

TABLE 2

Computed Values of R, T and A

Filter	From Observed Values of $I_{\max}$ and λ_{half}			From German Values of $I_{\max}$ and λ_{half}		
	<u>R</u>	<u>T</u>	<u>A</u>	<u>R</u>	<u>T</u>	<u>A</u>
2092	84.7 %	9.9 %	5.4 %	84.3 %	9.1 %	6.6 %
2093	81.9	11.0	7.1	82.9	10.1	7.0
2561	88.8	6.1	5.1	87.6	6.5	5.9
2445	87.2	7.0	5.8	88.2	6.4	5.4
2569	89.3	5.9	4.8	89.9	5.7	4.4
2433	86.5	7.5	6.0	88.1	7.1	4.8
2434	90.5	4.8	4.7	90.6	5.1	4.3
2554	84.4	8.0	7.6	88.2	7.0	4.8
2552	80.7	10.1	9.2	80.3	10.3	9.4
2360	83.4	7.4	9.2	89.1	5.6	5.3
2335	91.0	4.7	4.3	88.2	6.4	5.4

Note: The values of T are too low and of A, too high due to neglect of the (unknown) transmission of the glass covers in making the calculations.

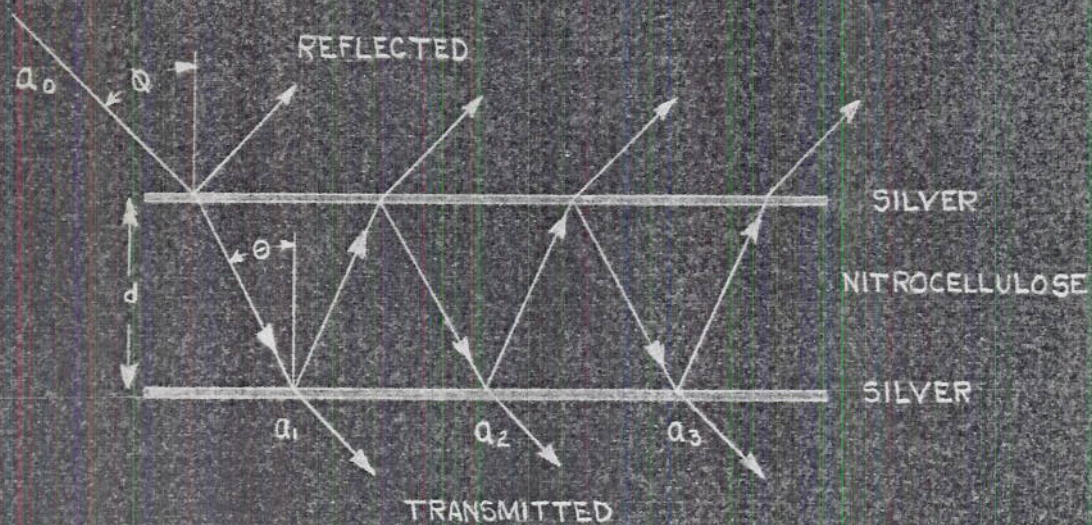
TABLE 3

Computed Intensity Distribution for Filter 2433 in the Visible Region;
 $R = 0.865$, $T = 0.075$, $\mu d = 562m\mu$.

$\lambda_{m\mu}$	Transmission	λ	Transmission
450	0.0016 min	600	0.010
460	0.0020	620	0.0052
480	0.0025	640	0.0034
500	0.0040	660	0.0025
520	0.0065	680	0.0021
540	0.0216	700	0.0018
545	0.0348	749	0.0016 min
550	0.0645	800	0.0017
555	0.114	850	0.0022
558	0.215		
560	0.255		
562	0.311 max		
564	0.295		
566	0.255		
570	0.140		
575	0.066		
580	0.039		
590	0.018		



A. CROSS SECTION OF INTERFERENCE FILTER



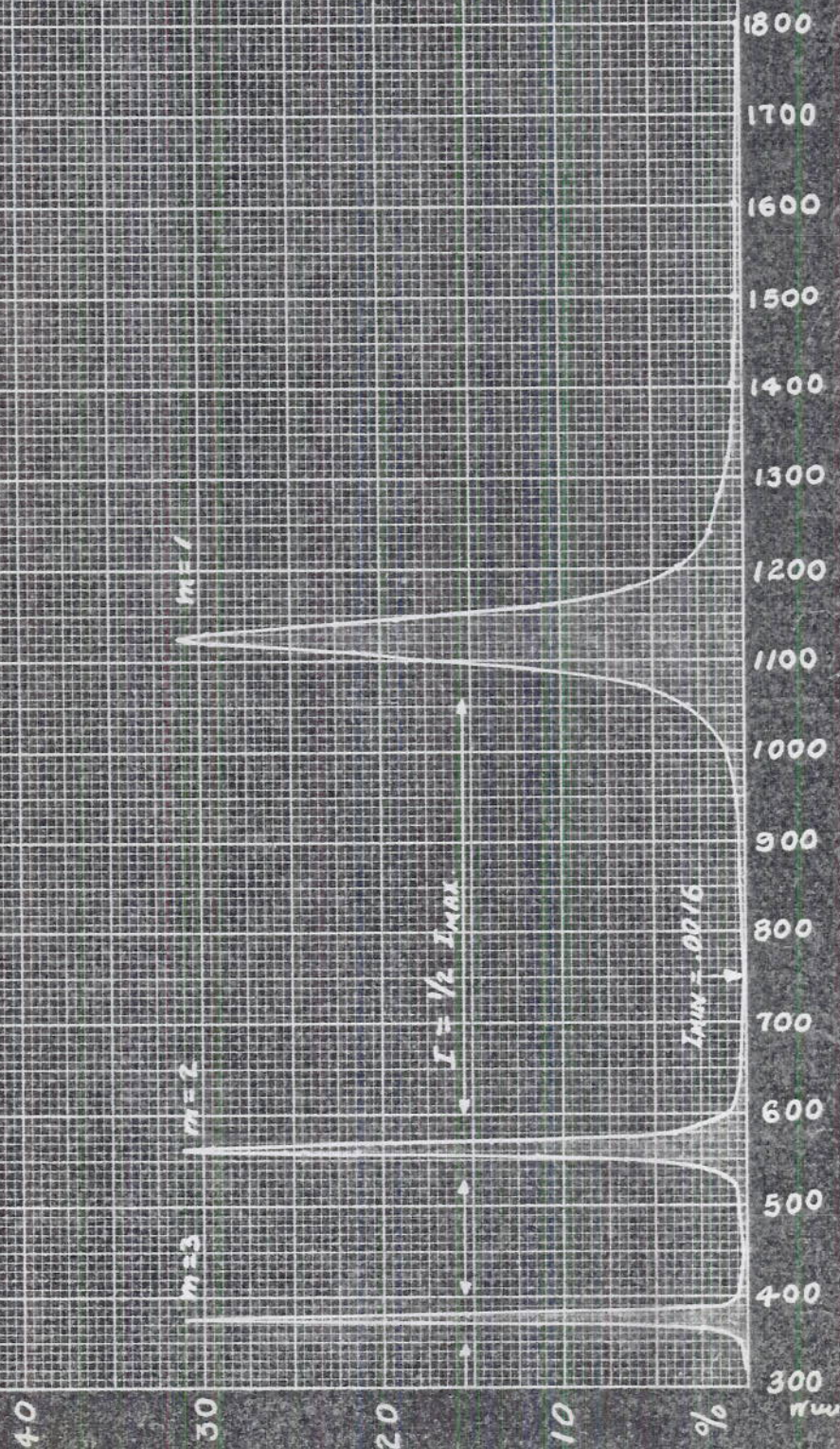
B. PATHS OF MULTIPLY REFLECTED RAYS BETWEEN THE SILVER FILMS.

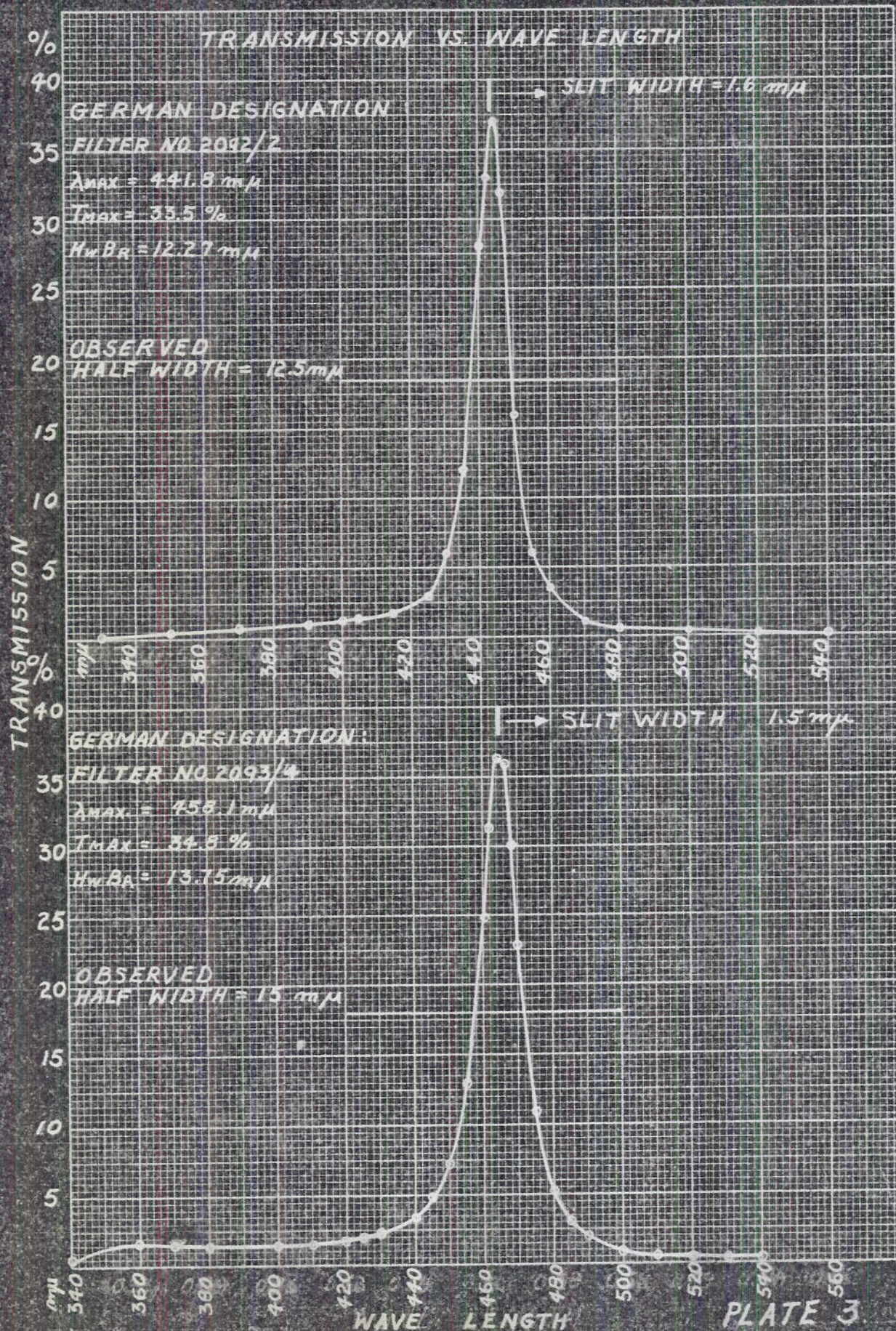
CALCULATED INTENSITY DISTRIBUTION

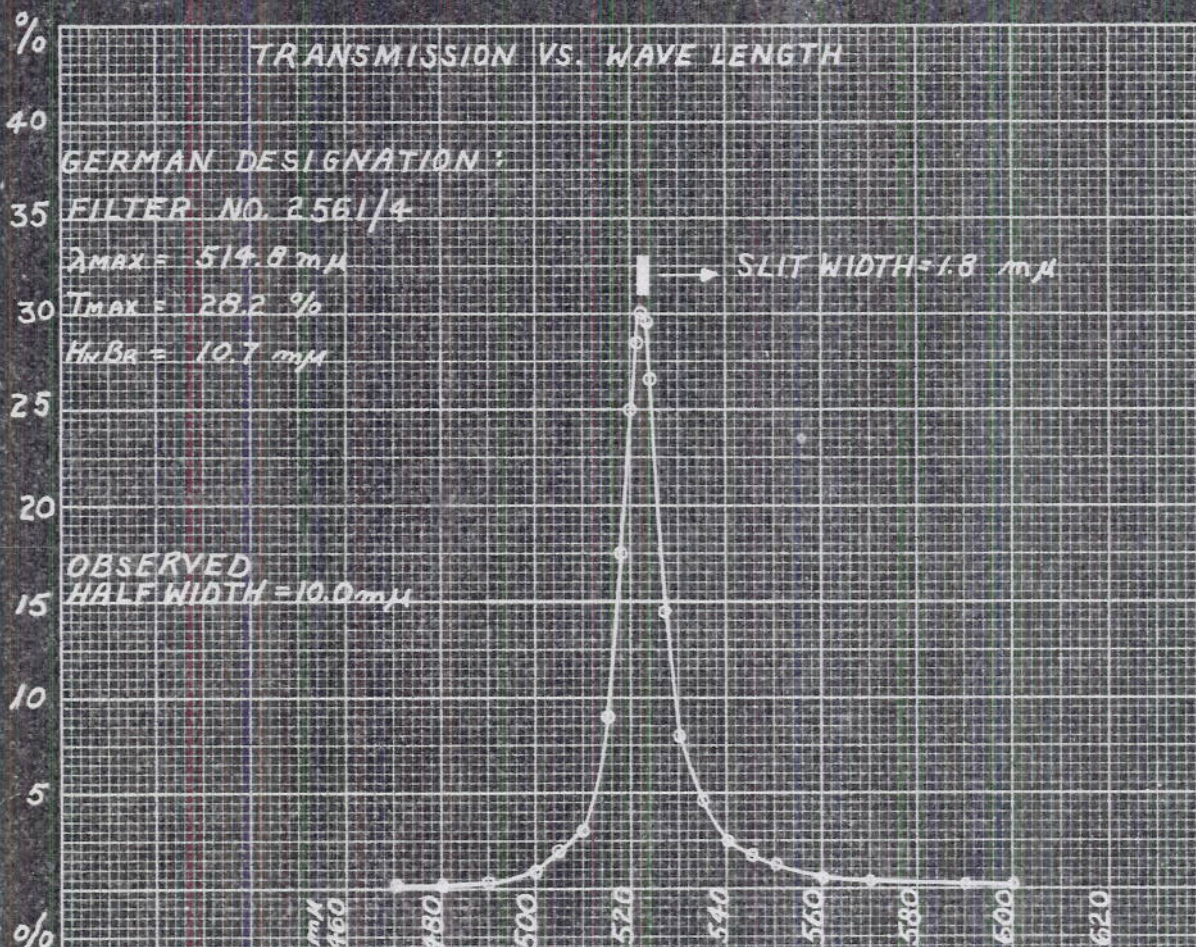
FILTER 2433: $T = 0.15$, $R = 0.65$, $\lambda_0 = 562 \text{ m}\mu$

THE INTENSITIES WERE COMPUTED FROM $I = \frac{T^2}{(1-R)^2 + 4R \sin^2 \theta/2}$

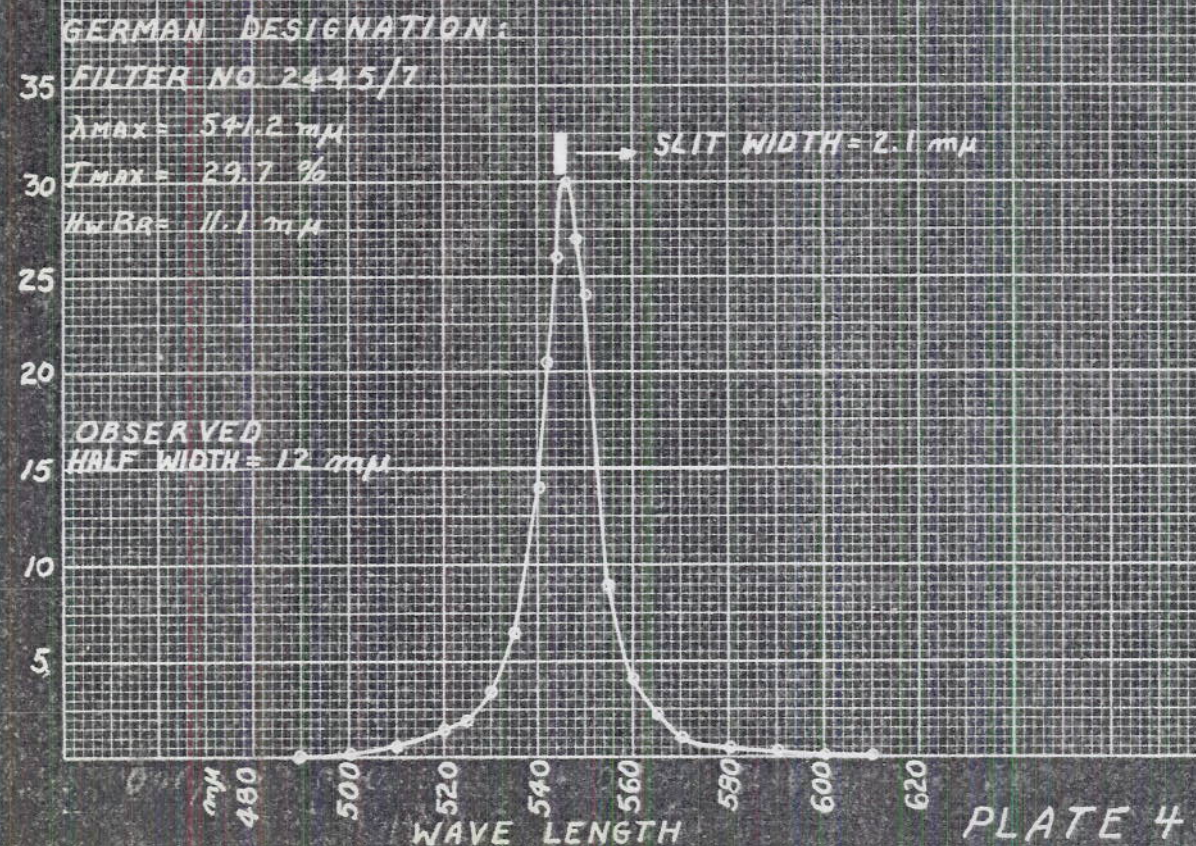
$$\theta/2 = \frac{2\pi d \sin \alpha}{\lambda}$$







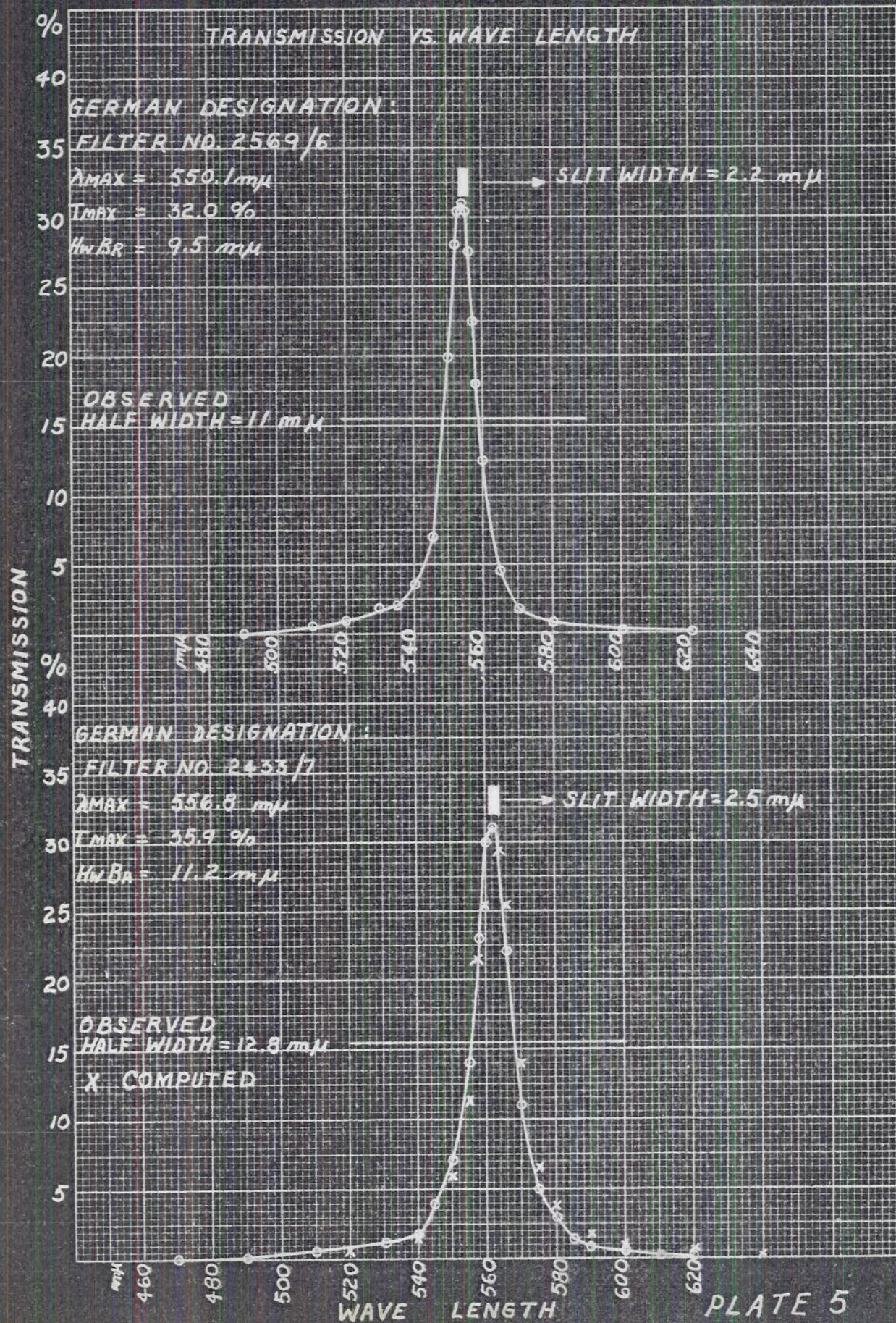
TRANSMISSION



WAVE LENGTH

PLATE 4

H-2731



%

TRANSMISSION VS. WAVE LENGTH

40

GERMAN DESIGNATION:

35

FILTER NO. 2434/6

$\lambda_{MAX} = 582.4 \text{ m}\mu$

30

$T_{MAX} = 29.1 \%$

$HwBr = 8.9 \text{ m}\mu$

25

OBSERVED
HALF WIDTH = $10.0 \text{ m}\mu$

20

15

10

5

TRANSMISSION

40

35

GERMAN DESIGNATION:

30

FILTER NO. 2554/2

$\lambda_{MAX} = 602.4 \text{ m}\mu$

25

$T_{MAX} = 34.9 \%$

$HwBr = 11.9 \text{ m}\mu$

20

15

OBSERVED
HALF WIDTH = $18 \text{ m}\mu$

10

5

0

WAVE LENGTH

600

620

640

660

680

700

720

740

760

780

800

820

840

860

880

900

920

940

960

980

1000

1020

1040

1060

1080

1100

1120

1140

1160

1180

1200

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1820

1840

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1880

1900

1920

1940

1960

1980

2000

2020

2040

2060

2080

2100

2120

2140

2160

2180

2200

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4000

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4100

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4140

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4220

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6000

6020

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6060

6080

6100

6120

6140

6160

6180

6200

6220

6240

6260

6280

6300

6320

6340

6360

6380

6400

6420

%

TRANSMISSION VS. WAVE LENGTH

40

GERMAN DESIGNATION:

35 FILTER NO. 2552/2

$\lambda_{MAX} = 641.1 \text{ m}\mu$

30 $T_{MAX} = 27.6 \%$

$HwBr = 11.3 \text{ m}\mu$

25

20

15

OBSERVED
HALF WIDTH = 11.5 mμ

10

5

%

40

GERMAN DESIGNATION

35 FILTER NO. 2360/1

$\lambda_{MAX} = 634.2 \text{ m}\mu$

30 $T_{MAX} = 28.2 \%$

$HwBr = 12.1 \text{ m}\mu$

25

20

15

10

OBSERVED
HALF WIDTH = 21.6 mμ

5

%

40

H-7131

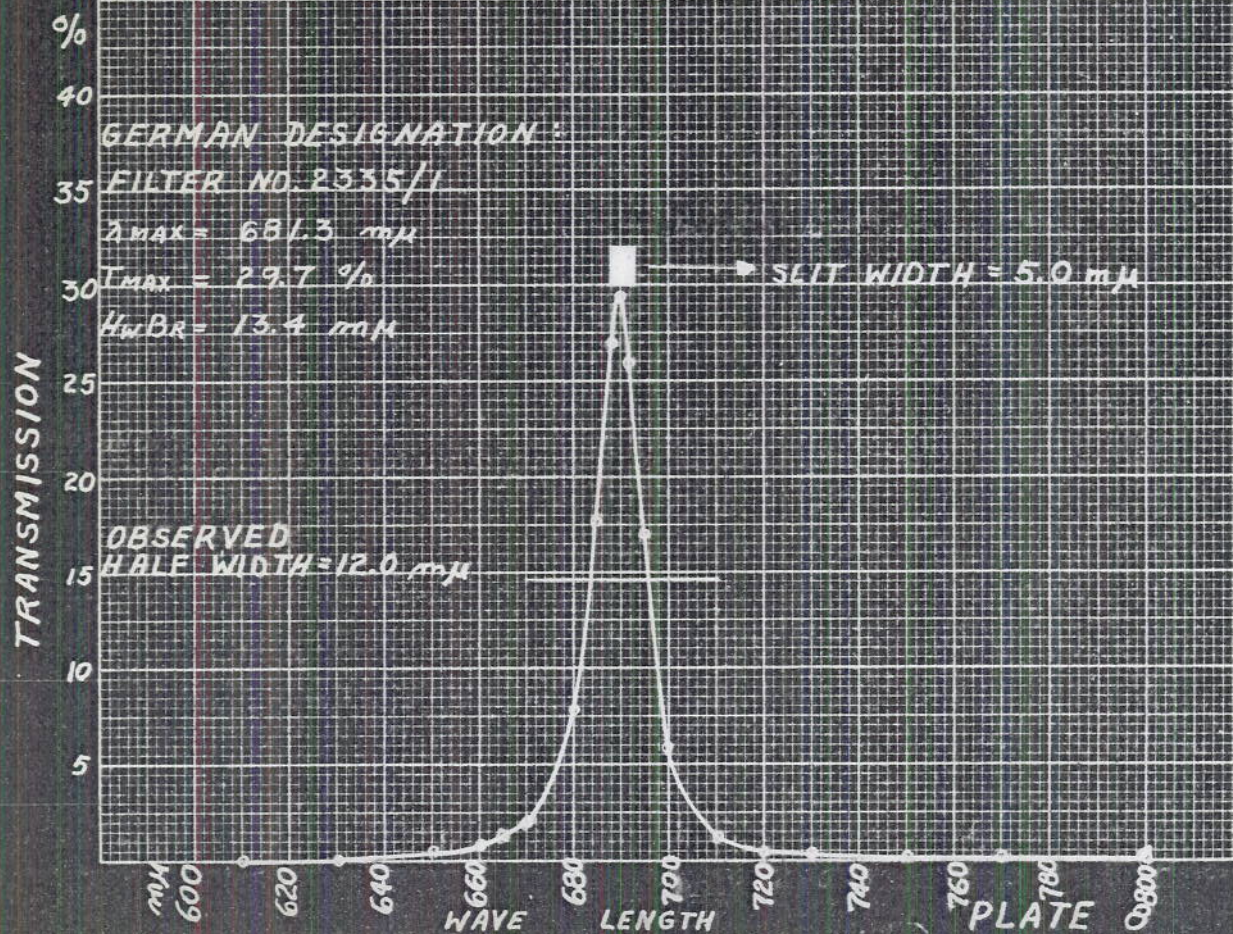
→ SLIT WIDTH = 5.8 mμ

SLIT WIDTH = 2.6 mμ

WAVE LENGTH

PLATE 7

TRANSMISSION VS. WAVE LENGTH



H-2731

Distribution:

BuAer (20)