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RPPR Final Report

as of 15-Dec-2021

Agency Code: 21XD

Proposal Number: 66544MA

Agreement Number: W911NF-17-1-0020

INVESTIGATOR(S):

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Report Date: 02-Aug-2021

Date Received: 14-Dec-2021

Final Report for Period Beginning 03-Dec-2016 and Ending 02-May-2021

Title: Compatible Parameterizations based Isogeometric Analysis of Fluid-Structure Interaction with Applications in Blast Mitigation

Begin Performance Period: 03-Dec-2016

End Performance Period: 02-May-2021

Report Term: 0-Other

Submitted By: Xiaoping Qian

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Distribution Statement: 1-Approved for public release; distribution is unlimited.

STEM Degrees: 3

STEM Participants: 5

Major Goals: Our first goal is to understand how compatible parameterizations (tensor-product splines and triangular splines) in structure and fluid domains enable new spatial and temporal strategies for coupling isogeometric structure solver and isogeometric fluid solver to obtain more accurate and efficient coupled multi-physics analysis.

Our second goal is to further develop compatible parameterizations based analysis and optimization algorithms and apply them in optimal design of shape and material distribution in 3D deformable plate structures under blast load.

Accomplishments: Please see the uploaded document on research accomplishments.

Training Opportunities: The project provides the training for the following Ph.D. students:

1. Cunfu Wang, Ph.D. student, now an assistant professor at Xiamen University
2. Xilu Wang, Ph.D. student, now a senior software engineer at Gamma Technologies
3. Songtao Xia, Ph.D. student, now a research scientist at Siemens Corporate Technology
4. Ruochun Zhang, currently a Ph.D. student
5. Mehrdad Zareh, currently a Ph.D. student

Results Dissemination: Research results have been primarily disseminated through peer-reviewed journal publications.

Research presentations have also been made to ARL. Research visits to ARL were planned, but cancelled due to COVID.

Research results are being used by other research groups. A joint publication is currently pending.

Honors and Awards: The PI has been promoted to Elmer and Janet Kaiser Professor during this award period.

Protocol Activity Status:

RPPR Final Report as of 15-Dec-2021

Technology Transfer: Research presentations have also been made to ARL. Research visits to ARL were planned, but cancelled due to COVID.

Discussion were made with Ansys and with a small software company about commercializing the technology. These discussions did not lead to commercial implementation of the developed technology, but substantial interests still remain.

PARTICIPANTS:

Participant Type: PD/PI

Participant: Xiaoping Qian

Person Months Worked: 3.00

Project Contribution:

National Academy Member: N

Funding Support:

Participant Type: Graduate Student (research assistant)

Participant: Cunfu Wang

Person Months Worked: 12.00

Project Contribution:

National Academy Member: N

Funding Support:

Participant Type: Graduate Student (research assistant)

Participant: Songtao Xia

Person Months Worked: 12.00

Project Contribution:

National Academy Member: N

Funding Support:

Participant Type: Graduate Student (research assistant)

Participant: Xilu Wang

Person Months Worked: 6.00

Project Contribution:

National Academy Member: N

Funding Support:

Participant Type: Graduate Student (research assistant)

Participant: Mehrdad Zareh

Person Months Worked: 12.00

Project Contribution:

National Academy Member: N

Funding Support:

Participant Type: Graduate Student (research assistant)

Participant: Ruochun Zhang

Person Months Worked: 12.00

Project Contribution:

National Academy Member: N

Funding Support:

RPPR Final Report as of 15-Dec-2021

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First Page #:

Date Submitted: 12/13/21 12:00AM

Date Published: 12/17/17 10:06PM

Publication Location:

Article Title: Isogeometric shape optimization on triangulations

Authors: Cunfu Wang, Songtao Xia, Xilu Wang, Xiaoping Qian

Keywords: Isogeometric Analysis, Shape optimization, Bezier triangles

Abstract: The paper presents a Bézier triangle based isogeometric shape optimization method. Bézier triangles are used to represent both the geometry and physical fields. For a given physical domain defined by B-spline boundary, a coarse Bézier triangular parameterization is automatically generated. This coarse mesh is used to maintain parameterization quality and move mesh by solving a pseudo linear elasticity problem. Then a fine mesh for isogeometric analysis is generated from the coarse mesh through degree elevation and refinement. As the fine mesh retains the same geometric map as the coarse mesh, we can guarantee mesh validity with the coarse mesh only. This bi-level mesh allows us to achieve high numerical accuracy of isogeometric analysis and lower computational cost on mesh validity control and mesh movement. Due to the use of B-spline boundary, the optimized shape can be compactly represented with a relatively small number of optimization variables.

Distribution Statement: 2-Distribution Limited to U.S. Government agencies only; report contains proprietary info
Acknowledged Federal Support: Y

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Date Submitted: 12/13/21 12:00AM

Date Published: 4/20/18 2:10AM

Publication Location:

Article Title: Generating high-quality high-order parameterization for isogeometric analysis on triangulations

Authors: Songtao Xia, Xiaoping Qian

Keywords: Isogeometric analysis, Domain parameterization, Triangular Bézier splines, High-order elements, C^r smoothness

Abstract: This paper presents an approach for automatically generating high-quality high-order parameterizations for isogeometric analysis on triangulations. A B-spline represented boundary geometry is parameterized into a collection of high-order Bézier triangles or tetrahedra in 2D and 3D spaces, respectively. Triangular Bézier splines are used to represent both the geometry and physical fields over the triangulation. By imposing continuity constraints on the Bézier ordinates of the elements, a set of global smooth basis is constructed and used as the basis for analysis. To ensure high quality of the parameterization, both the parametric and physical meshes are optimized to reduce the shape distortion of the high-order elements relative to well-defined reference elements. The shape distortion is defined based on the Jacobian of the triangular Bézier splines, and its sensitivity with respect to the location of control points is derived analytically and evaluated efficiently.

Distribution Statement: 2-Distribution Limited to U.S. Government agencies only; report contains proprietary info
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CONFERENCE PAPERS:

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Paper Title: A NEW PLATE FORMULATION BASED ON TRIANGULAR ISOGEOMETRIC ANALYSIS

Authors: Mehrdad Zareh, Xiaoping Qian

Acknowledged Federal Support: Y

RPPR Final Report
as of 15-Dec-2021

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Date Received:

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Title: Isogeometric Analysis on Triangulations

Authors: Songtao, Xia

Acknowledged Federal Support: **N**

WEBSITES:

URL: <https://cdm.me.wisc.edu/tiga.htm>

Date Received: 13-Dec-2021

Title: Isogeometric Analysis on Triangulations

Description: We have developed a method for isogeometric analysis on the triangulation of a domain bounded by NURBS curves (or surfaces). In this method, both the geometry and the physical field are represented by bivariate (or trivariate) splines in Bernstein–Bézier form over the triangulation. We have developed a set of procedures to construct a parametric domain and its triangulation from a given physical domain, construct C_r -smooth basis functions over the domain, and establish a rational Triangular Bézier Spline (rTBS) based geometric mapping that C_r -smoothly maps the parametric domain to the physical domain and exactly recovers the NURBS boundaries at the domain boundary. As a result, this approach can achieve automated meshing of objects with complex topologies and allow highly localized refinement.

Partners

I certify that the information in the report is complete and accurate:

Signature: Xiaoping Qian

Signature Date: 12/14/21 3:28PM

Final Research Report (2021)

Project Report

(Award W911NF-17-1-0020)

Xiaoping Qian
University of Wisconsin-Madison

Research contributions in this project so far include

- Developed an approach for constructing C^r -smooth basis through triangulations over complex domain;
- Applied this approach in the solution of a set of higher-order PDEs including Cahn-Hilliard phase field models and Kirchhoff-Love shell formulation;
- Applied this approach in the solution of coupled fourth-order PDEs, von Karman equations which demonstrated high accuracy and optimal convergence rates;
- Developed an approach for automatically generating high-quality high-order parameterization for isogeometric analysis on triangulations;
- Developed a Bézier triangle based isogeometric shape optimization method.

The research results have been applied in two application contexts with publications currently under preparation. One is on fourth-order PDE based meta-material design and the other is on Toupin's gradient elasticity theory at finite strains formulation.

Technical details of the above contributions are presented below.

C^1 Triangular Isogeometric Analysis of the von Karman Equations

In this paper [1], we report the use of rational Triangular Bézier Splines (rTBS) to numerically solve the von Karman equations, a system of fourth order PDEs. C^1 smoothness of the mesh, generated by triangular Bézier elements, enables us to directly solve the von Karman systems of equations without using mixed formulation. Numerical results of benchmark problems show high accuracy and optimal convergence rate in L^1 , H^1 and H^2 norm for quadratic and cubic triangular Bézier elements. Results of this study show that triangular isogeometric analysis can efficiently and accurately solve systems of high order PDEs.

Triangulation-based isogeometric analysis of the Cahn–Hilliard phase-field model

This paper [2, 3] presents triangulation-based Isogeometric Analysis of the Cahn–Hilliard phase-field model. The Cahn–Hilliard phase-field model is governed by a time-dependent fourth-order partial

differential equation. The corresponding primal variational form involves second-order operators, making it difficult to be directly analyzed with traditional C^0 finite element analysis. In this paper, we construct C^1 Bernstein–Bézier simplicial elements through macro-element techniques, including various triangle-split based macro-elements in both 2D and 3D space. We extend triangulation-based isogeometric analysis to solving the primal variational form of the Cahn–Hilliard equation. We validate our method by convergence analysis, showing the nodal and degree-of-freedom advantages over C^0 Finite Element Analysis. We then demonstrate detailed system evolution from randomly perturbed initial conditions in periodic two-dimensional squares and three-dimensional cubes. We incorporate an adaptive time-stepping scheme in these numerical experiments. Our numerical study demonstrates that triangulation-based isogeometric analysis offers optimal convergence and time step stability, is applicable to complex geometry and allows local refinement.

Kirchhoff–Love shell formulation based on triangular isogeometric analysis

This article [4, 5] presents application of rational triangular Bézier splines (rTBS) for developing Kirchhoff–Love shell elements in the context of isogeometric analysis. Kirchhoff–Love shell formulation requires high continuity between elements because of higher order PDEs in the description of the problem. Non-uniform rational B-spline (NURBS)-based IGA has been extensively used for developing Kirchhoff–Love shell elements, as NURBS-based IGA can provide high continuity between and within elements. However, NURBS-based IGA has some limitations; such as, analysis of a complex geometry might need multiple NURBS patches and imposing higher continuity constraints over interfaces of patches is a challenging issue. Addressing these limitations, isogeometric analysis based on rTBS can provide C^1 continuity over the mesh including element interfaces, a necessary condition in finite elements formulation of Kirchhoff–Love shell theory. Based on this technology, we use C^r smooth rational triangular Bézier spline as the basis functions for representing both geometry and solution field. In addition to providing higher continuity for Kirchhoff–Love formulation, using rTBS elements we can achieve three significant challenging goals: optimal convergence rate, efficient local mesh refinement and analysis of geometric models of complex topology. The proposed method is applied on several examples; first, this technique is verified against multiple plate and shell benchmark problems; investigating the convergence rate on the benchmark problems demonstrates that the optimal convergence rate can be obtained by the proposed technique. We also apply our method on geometric models of complex topology or geometric models in which efficient local refinement is required. Moreover, a car hood is modeled with rTBS and structurally analyzed by using the proposed framework.

Generating high-quality high-order parameterization for isogeometric analysis on triangulations

This paper [6] presents an approach for automatically generating high-quality high-order parameterizations for isogeometric analysis on triangulations. A B-spline represented boundary geometry is parameterized into a collection of high-order Bézier triangles or tetrahedra in 2D and 3D spaces, respectively. Triangular Bézier splines are used to represent both the geometry and physical fields over the triangulation. By imposing continuity constraints on the Bézier ordinates of the elements, a set of global C^r smooth basis is constructed and used as the basis for analysis. To ensure high quality of the parameterization, both the parametric and physical meshes are optimized to reduce the shape distortion of the high-order elements relative to well-defined reference elements. The shape

distortion is defined based on the Jacobian of the triangular Bézier splines, and its sensitivity with respect to the location of control points is derived analytically and evaluated efficiently. Moreover, a sufficient condition is derived to guarantee the generated mesh is free of local self-intersection, thanks to the convex hull property of triangular Bézier splines. By using a Heaviside projection function, the non-negative Jacobian determinant constraints are formulated efficiently as a single optimization constraint. Several 2D and 3D numerical examples are presented to demonstrate that high-quality high-order elements are generated using our approach.

Isogeometric shape optimization on triangulations

The paper [7] presents a Bézier triangle based isogeometric shape optimization method. Bézier triangles are used to represent both the geometry and physical fields. For a given physical domain defined by B-spline boundary, a coarse Bézier triangular parameterization is automatically generated. This coarse mesh is used to maintain parameterization quality and move mesh by solving a pseudo linear elasticity problem. Then a fine mesh for isogeometric analysis is generated from the coarse mesh through degree elevation and refinement. As the fine mesh retains the same geometric map as the coarse mesh, we can guarantee mesh validity with the coarse mesh only. This bi-level mesh allows us to achieve high numerical accuracy of isogeometric analysis and lower computational cost on mesh validity control and mesh movement. Due to the use of B-spline boundary, the optimized shape can be compactly represented with a relatively small number of optimization variables. Due to the use Bézier triangles, this shape optimization method is applicable to structures of complex topology and allows for local refinement for analysis. By representing the squared distance between two Bézier curves as a Bézier form, a distance check scheme is also introduced to prevent intersections of design boundaries and control the thickness of structural connections. Numerical examples on minimal compliance design and design of negative Poisson ratios are presented to demonstrate the efficacy of the proposed method.

References

- [1] *Geometric Challenges in Isogeometric Analysis*, chapter C1 Triangular Isogeometric Analysis of the von Karman Equations. Springer International Publishing, 2022.
- [2] Ruochun Zhang and Xiaoping Qian. Triangulation based isogeometric analysis of the cahn-hilliard phase-field model. In *Technical Conferences and Computers and Information in Engineering Conference IDETC/CIE 2018 August 26-29, 2018, Quebec City, Quebec, Canada*, 2018.
- [3] Ruochun Zhang and Xiaoping Qian. Triangulation-based isogeometric analysis of the cahn-hilliard phase-field model. *Computer Methods in Applied Mechanics and Engineering*, 357:112569, 2019.
- [4] Mehrdad Zareh and Xiaoping Qian. A new plate formulation based on triangular isogeometric analysis. In *Technical Conferences and Computers and Information in Engineering Conference IDETC/CIE 2018 August 26-29, 2018, Quebec City, Quebec, Canada*, 2018.

- [5] Mehrdad Zareh and Xiaoping Qian. Kirchhoff–love shell formulation based on triangular isogeometric analysis. *Computer Methods in Applied Mechanics and Engineering*, 347:853–873, 2019.
- [6] Songtao Xia and Xiaoping Qian. Generating high-quality high-order parameterization for isogeometric analysis on triangulations. *Computer Methods in Applied Mechanics and Engineering*, 338:1–26, 2018.
- [7] Cunfu Wang, Songtao Xia, Xilu Wang, and Xiaoping Qian. Isogeometric shape optimization on triangulations. *Computer Methods in Applied Mechanics and Engineering*, 331:585–622, 2018.