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LINE AVERAGING CORRECTION OF STRUCTURE FUNCTION CALCULATED FROM SONIC ANEMOMETER DATA

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When using sonic anemometry to determine the temperature structure function parameter, there are multiple times in the data collection process where averaging occurs. In particular, this work considers the impact on the temperature structure function from spatial averaging of temperature between the piezoelectric transducer heads. It is found that the impact is nearly constant for separations greater than twice the transducer spacing. is found that the impact is

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Line Averaging Correction of Structure Function Calculated from Sonic Anemometer Data

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Abstract: When using sonic anemometry to determine the temperature structure function parameter, there are multiple times in the data collection process where averaging occurs. In particular, this work considers the impact on the temperature structure function from spatial averaging of temperature between the piezoelectric transducer heads. It is found that the impact is nearly constant for separations greater than twice the transducer spacing.

Sonic anemometers have been used for many years to measure virtual temperature based on the speed of flight of an ultrasonic pulse between pairs of piezoelectric transducers. However, while one would ideally measure temperature at a point in space, in reality temperature measurements are averaged over the distance between the transducer pairs (line averaging). Furthermore, many 3-dimensional anemometers have non-orthogonal transducer heads which, based upon the method used to calculate sonic temperature, could result in further averaging. Many anemometers block average data reporting it at a much lower rate than collected – yet another way temperature is averaged.

Generally, C_T^2 (and by extension, C_n^2) is calculated from sonic anemometer temperature measurements using a power spectrum method. In this case, block averaging may be utilized to reduce aliasing effects. When using the power spectrum method, line averaging is corrected by dividing the power spectrum by a transfer function that accounts for the effect [1]. We have recently been analyzing turbulence using the temperature structure function because it is inherently smooth making it easier to determine turbulence ranges (e.g., inertial) and structure function slopes. However, averaging of the data makes it challenging to determine turbulence parameters such as outer scale and C_T^2 with confidence. McCrae, *et al* [2] demonstrated that block averaging of measured data that often occurs within anemometer firmware would appear as a nearly constant offset being subtracted from the ideal Kolmogorov structure function. In this work, we extend this concept to show that this is the case not only for block averaging of data but also for the line averaging that occurs between the transducers for both the Kolmogorov and von Kármán based structure functions with zero inner scale. By extension, we believe that similar behavior will occur for whatever spectral model the data actually follows.

Based on Taylor's frozen flow hypothesis, we calculate the structure function as a function of radial separation, R, using

$$D_T\left(R\right)=D_T\left(\bar{u}\,\Delta t\right)=\left\langle\left[T\left(R_1\right)-T\left(R_1+R\right)\right]^2\right\rangle \tag{1}$$

where $\bar{u}$ is the mean wind speed, Δt is the time separation between the points and $\langle - \rangle$ indicate ensemble average.

It can be shown that this ideal structure function is reduced due to the averaging between transducers separated by the distance p and is given by

$$\tilde{D}_T\left(R\right)=\frac{1}{p^2}\int_{-p/2}^{p/2}\int_{-p/2}^{p/2}\left\{D_T\left(s+R-s'\right)-D_T\left(s-s'\right)\right\}ds\,ds' \tag{2}$$

The tilde notation is used to differentiate the line averaged structure function from its ideal counterpart.

For this work, we consider the structure function form of the Kolmogorov and von Kármán models. Eqs. (3) and (4) show both the more commonly used 3-D power spectrum model as well as its corresponding structure function. Note that for simplicity we are assuming an inner scale of zero in this analysis. Similar results are obtained for nonzero inner scale as well as other spectral models.

$$\Phi_{Kol}\left(\kappa\right)=0.033C_T^2\kappa^{-11/3}\Leftrightarrow D_{Kol}\left(R\right)=C_T^2R^{2/3} \tag{3}$$

$$\Phi_{vK}\left(\kappa\right)=0.033C_T^2(\kappa^2+\kappa_0^2)^{-11/6}\Leftrightarrow D_{vK}\left(R\right)==1.05C_T^2\left[1-0.59R^{1/3}\kappa_0^{1/3}K_{-1/3}\left(\kappa_0 R\right)\right] \qquad (4)$$

To examine the impact of line averaging, we numerically perform the integration in Eq. (2) for the Kolmogorov (Fig. 1) and von Kármán (Fig. 2) structure functions. In both figures, we show in (A) the theoretical and line averaged structure functions followed and (B) the difference between them as a function of radial separation. For these examples, a transducer separation of 15 cm was assumed. An outer scale spatial wavenumber of $\kappa_0 = 0.2m^{-1}$ was used in the von Kármán model. The magnitude of the offset between theoretical and line averaged results is dependent upon the inner and outer scale used with the von Kármán model. As emphasized by part (B) in each of these figures, the difference between the theoretical and line averaged structure functions rapidly approaches a constant whose magnitude is dependent upon the model representing the data. Thus, we propose that the data can be corrected for line averaging by adding an appropriate constant and fitting for outer scale and C_T^2. Thus

$$\tilde{D}_T\left(R\right)=D_T\left(R\right)+K, \qquad R>2p \qquad (5)$$

In which case, the structure function can be corrected for line averaging, as well as temporal averaging and background noise, by the simple addition of a constant. Once the structure function is corrected, it can be used to find C_T^2 and outer scale. Examples of this method of correcting the structure function are shown in Figs 3-4 for two cases of temperature data collected at 5 m height at 200 Hz sample rate and 5 minute temporal averages. (A) compares the fit of the measured structure function to the theoretical Kolmogorov and von Kármán structure functions (solid) as well as fitting to Eq. (5) by adding an offset (dashed). (B) shows the original measured structure function (solid) in addition to the corrected structure function (dashed). The Kolmogorov model was fit to $2p < R < \mathit{Inertial\ Range}$ with a maximum of $R = 1m$. The von Kármán was fit within the range $2p < R < 5m$ to include the transition in the structure function beyond the inertial range.

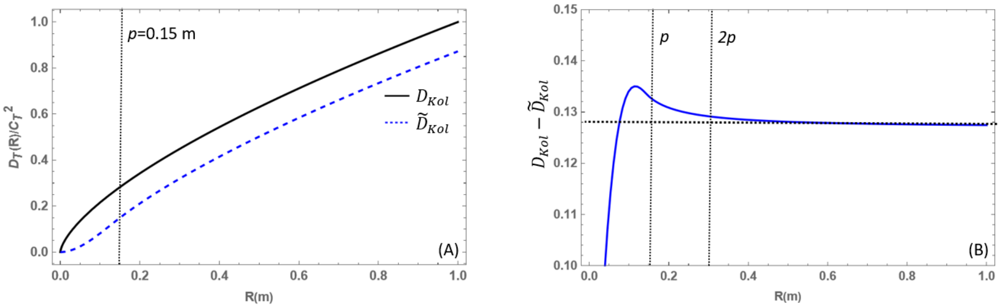

Fig. 1. Kolmogorov structure function. (A) Theoretical; (B) line averaged from Eq. (2).

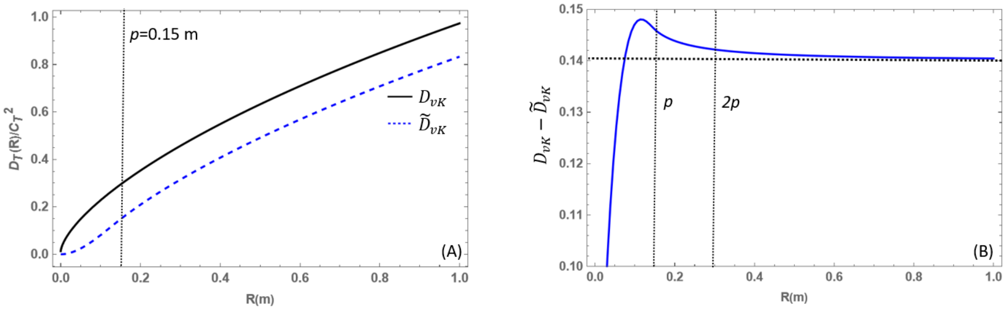

Fig. 2. von Kármán structure function. (A) Theoretical; (B) line averaged from Eq. (2).

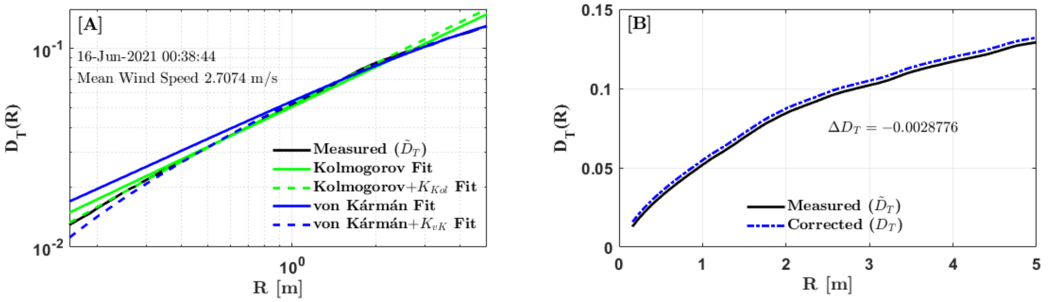

Fig. 3. First sample data set with (A) example of measured structure function fit with and without offset; (B) linear plot showing measured and corrected (addition of fit offset K_{Kol}) structure functions. Time shown in UTC.

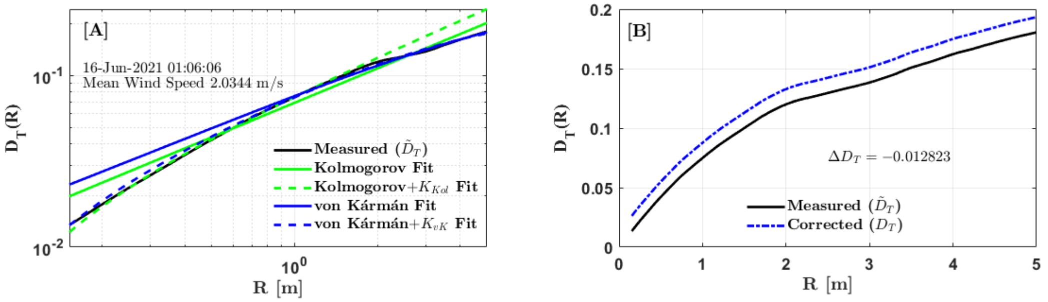

Fig. 4. Second sample data set with (A) example of measured structure function fit with and without offset; (B) linear plot showing measured and corrected (addition of fit offset K_{Kol}) structure functions. Time shown in UTC.

We have successfully shown that the inclusion of an offset term in the structure function improves the fit of measured data to theory. As evident in the shown examples, the magnitude of the fit term can vary depending upon spectral model and as the inner and outer scale of the atmosphere vary.

[1] C. J. Moore, "Frequency Response Corrections for Eddy Correlation Systems," *Boundary Layer Meteorology* **37**, 17-35 (1986).

[2] J. McCrae, Jr., S. Bose-Pillai, B. Wilson and S. Fiorino, "Investigating the Outer Scale of Turbulence with Time Domain Processing of Anemometer Data," *Proc. SPIE* **11386**, 1138604 (2021).