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Final Report: Large scale dynamics and geometry in stochastic systems

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5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER
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611104

6. AUTHORS

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAMES AND ADDRESSES
University of Arizona
PO Box 210158, Rm 510

Tucson, AZ 85721 -0158

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS (ES)
U.S. Army Research Office
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Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES
The views, opinions and/or findings contained in this report are those of the author(s) and should not contrued as an official Department of the Army position, policy or decision, unless so designated by other documentation.

14. ABSTRACT

15. SUBJECT TERMS

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a. REPORT	b. ABSTRACT	c. THIS PAGE			Sunder Sethuraman
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					520-621-1774

RPPR Final Report
as of 09-Oct-2022

Agency Code: 21XD

Proposal Number: 70874NCH **Agreement Number: W911NF-18-1-0311**
INVESTIGATOR(S):

Name: Sunder Sethuraman
Email: sethuram@math.arizona.edu
Phone Number: 5206211774
Principal: Y

Organization: **University of Arizona**
Address: PO Box 210158, Rm 510, Tucson, AZ 857210158
Country: USA
DUNS Number: 806345617 EIN: 866004791
Report Date: 08-Oct-2022 Date Received: 07-Oct-2022
Final Report for Period Beginning 13-Jul-2018 and Ending 08-Jul-2022
Title: Large scale dynamics and geometry in stochastic systems
Begin Performance Period: 13-Jul-2018 **End Performance Period:** 08-Jul-2022
Report Term: 0-Other
Submitted By: Sunder Sethuraman Email: sethuram@math.arizona.edu
Phone: (520) 621-1774
Distribution Statement: 1-Approved for public release; distribution is unlimited.

STEM Degrees: 7 **STEM Participants:** 9

Major Goals: Major goals of the grant are to understand `scales' in stochastic systems and connections to geometry. Interacting particle systems as spatially discrete models of physical and other phenomena are studied. Also, stochastic networks, and flows of Markov chains are to be investigated. Problems include understanding large scale features as a consequence of types of microscopic interactions. Such an understanding allows to classify behaviors in complex systems at a `high' level, yielding a description in reduced variables, depending on the type of `street' level dynamics.

In this context, the main aim is to identify universal behaviors and provide a detailed understanding of the scaling limits in different settings which, although fundamental, have been difficult to analyze as they involve nonlinear, singular, or heterogeneous components incorporating geometry. To this end, a departure from previous methods and a common theme of the work is to develop new robust mathematical methods for `local' homogenizations which take account of both the stochastic particle interactions and the complex structure of the underlying graphs.

As such, we hope to address a host of previously intractable problems including capturing `interface flow', `flows through traps', and `non-Gaussian effects in fluctuations'. These limits are anticipated to connect integrally with notions in geometry, statistics, and analysis.

Accomplishments: Research:

5 papers appeared after 2018, which were started near when the original grant was submitted (J. Stat. Phys, Ann. Appl. Probab (2 papers), Elec J. Probab., Springer Proc. Math. Stat.).

In the period of review, July 2018 - July 2022, 8 papers appeared (Elec J. Probab, Sankhya, Ann. Appl Probab, Rocky Mountain J. Math, Commun Math Phys, AMS Contemp. Math, Ann. Inst. Henri Poincare, AISTATS 2022).

Also, in this period of review, 4 papers are accepted/waiting to be published (Ann. Appl Probab, SIAM J. Appl. Math., Tunisian J. Math, J. Stat. Phys.). In addition, 2 papers are in review (Memoirs AMS, Ann. Probab.). Finally, 5 papers are works-in-progress.

Dissemination: 13 invited lectures in conferences. 8 seminars/colloquia at outside institutions.

Conference Organization: Frontier Probability Days 2020, Dec 2020, Las Vegas. Another session at AIMS Dynamical systems and differential equations was postponed to 2023.

RPPR Final Report
as of 09-Oct-2022

Mentoring:

PhD students graduated: William Lippitt (Math, 2019), Derick Bishop (Stat, 2022).

PhD students continuing: Conner Hatton (Math), Alex Loomis (Math), Michael Phizicky (Math, 2019-2021).

MS students graduated: Conner Hatton (MS 2022), Owen MacDonald (Math 2022), Brady Gales (Applied Math, 2019).

Postdocs: Jianfei Xue (2018-2020), William Lippitt (2020), Michael Conroy (2021 - 2024).

Undergrads: Lexi Garrabrant (2019-2020), Tristin Solarzano (2019).

Highschool Outreach: 7 students in Summer 2020, and 5 students in Summer 2021.

Honors during the period:

1. Fellowship of the IMS (Institute of Mathematical Statistics)

2. Fellowship of the JSPS (Japan Society for the Promotion of Science)

3. Simons Foundation Fellow

Selected highlights in the articles

1. A derivation from general microscopic systems of mean-curvature and Huygens' interface flows. This work led to results in PDE with respect to Allen-Cahn equations with nonlinear diffusion terms, and also to discretizations.

2. How strong random environments (random traps) in microscopic settings, which do not average out, lead to singular SPDE continuum space time mass distributional limits.

3. New concept of Markovian stick-breaking process, which can be used in Bayesian network estimation, but also interestingly connects with the empirical limits in Metropolis MC's and also the stationary distributions of mRNA dynamic models.

4. Derivation of coupled singular SPDES, such as KPZ coupled systems, associated with non-Gaussian scalings, from types of `rigid' traffic flows.

5. A first derivation of how continuum boundary conditions arise as a result of microscopic defects in the flow, say `slow' locations, in a general class of models.

6. Determining the (large deviation) cost of atypical events of a distinguished particle in traffice, and also how these atypical events are typically achieved.

Highlights in the outreach: The two summer programs, supported by AEOP, running 6 weeks in 2020 and 2021, on the topics `Math of Neural Networks' and `Math of Opinions and Epidemics' were valuable to both students and mentors. Students learned online, and wrote computer programs to simulate, as well as analytical calculated. These programs helped fill a void especially during the Covid isolation phase.

RPPR Final Report
as of 09-Oct-2022

Training Opportunities: The Pi has been fortunate to mentor students and postdocs during the grant period.

PhD students:

1. William Lippitt (Math, 2019 defense) `Stick-breaking processes, clumping, and a time-inhomogeneous Markov chain' (won best student in Math award).

2. Derick Bishop (Stat, 2022 defense) `the solution to the d-string optimal stopping problem in certain Bernoulli sequences.'

3. Conner Hatton (Math, 2024 expected) Current.

4. Alex Loomis (Math, 2023 expected) `Convergence of Multi-interacting Worlds ground state and other sequences through Stein's method' Current.

5. Michael Phizicky (Math 2019-2021) `Limits of inhomogeneous Markov chains' (has taken leave of absence due to Covid).

MS Students:

1. Conner Hatton (Math, 2022) `From range of random walk to voter models'

2. Owen MacDonald (Math, 2022) `Preferential attachment adjacent weight models'

3. Brady Gales (Applied Math, 2021) `Reinforced random walk'

Postdocs

1. Jianfei Xue (Math, 2018-2020) `Hydrodynamic limit of Young diagrams',
`Hydrodynamics in Sinai-type random environments'.

2. William Lippitt (Math, 2019-2020) `On the use of Markovian stick-breaking processes as a prior'; `Stationarity and inference and multistate promoter models of stochastic gene expression via stick-breaking measures.'

3. Michael Conroy (Math, 2020-2022) `Gumbel laws in symmetric exclusion processes'.

Undergraduate students

1. Lexi Garrabrant (2019-2020) Poster on `Records: Assessing change in temperature and other data'; Helper in Highschool outreach 2020. BS (Math 2020)

2. Tristan Solarzano (2019-2020) Poster on `Records: Assessing change in temperature and other data' BS (Math 2020)

Highschool Outreach

1. 7 students mentored in a 6 week online program, `Math of Neural Networks' in Summer 2020. Funded by AEOP.

2.5 students mentored in a 6 week online program,`Math of Opinions and Epidemics' in Summer 2021. Funded by AEOP.

RPPR Final Report
as of 09-Oct-2022

Results Dissemination: The PI was fortunate to be invited to several conferences and universities to give talks on the research, some of them plenary talks.

13 Lectures in conferences.
8 seminars/colloquia at outside universities

Also, the PI helped organize Frontier Probability Days 2022, Dec 3-5, 2021 in Las Vegas, NE, a national conference in probability and applications.

In addition, the PI puts preprints on the arXiv for quick dissemination.

In conferences:

1. Approximating geodesics via random points. Nov 5- Nov 8, 2018, Kyoto-RIMS, Kyoto, Japan.

2. On hydrodynamics of Young diagrams. ICTS Program, Universality in random strucures, Bangalore, India, Jan. 28 - Feb. 5, 2019

3. On microscopic derivation of a mean curvature flow. Seminar in Stochastic Processes, Salt Lake City, UT, Mar 13-16, 2019

4. On microscopic derivation of a mean curvature flow. Dynamics, random media, and universality in complex phenomena, Munster, Aug 26-30, 2019.

5. Stickbreaking, clumping, and Markov chain occupation laws. AMS Western Sectional meeting, MM Rao Session, Riverside, CA, Nov 9, 2019

6. On microscopic derivation of a mean curvature flow. Particle Systems and Partial Differential Equations VIII, Lisbon, Dec 2-6, 2019.

7. On microscopic derivation of a mean curvature flow. Hydrodynamic limits and related topics--one day symposium. Waseda University, Tokyo, Japan, Dec. 20, 2019.

8. Approximating geodesics via random points. JMM Spatial Stochastic Models. Denver, Co., Jan 16, 2020.

9. SPDE hydrodynamic limit in Sinai-type random environments, Bernoulli-IMS One-World Symposium, August 24-31, 2020

10, On microscopic derivation of a mean-curvature flow. 10th International Conference on Stochastic Analysis and its Applications (ICSAA), Kyoto University, Japan, Sep 6-10, 2021

11. Boundary behaviors, condensation, and effects of slow sites in zero-range systems, Montreal Workshop Probability and PDE, March, 2022.

12. On atypical motions of a tagged particle in asymmetric simple exclusion. Probability Conference (in honor of Raghu Varadhan), Seoul National University, South Korea, Jun 13-17, 2022.

13. On atypical motions of a tagged particle in asymmetric simple exclusion. Probability Conference (in honor of Herbert Spohn), Muenster, Germany, Jun 20-25, 2022.

In seminars/colloquia:

1. Stick breaking processes, Clumping, and Occupation laws of Markov chains. Temple University, Nov. 26, 2018

2. Stick breaking, clumping, and occupation laws of Markov chain, IMPA, Jan 16, 2019.

3. On Hydrodynamic limits of Young Diagrams. Kansas University, February 21, 2019.

4. On microscopic derivation of a mean curvature flow, Brown University, Apr. 5, 2019.

5. On microscopic derivation of a mean curvature flow, New York University, April 12, 2019.

6. On microscopic derivation of a mean curvature flow, Columbia University, Apr. 19, 2019

7. Mean-curvature interface limit from Glauber+Zero-range microscopic dynamics, Northwestern-U. Minnesota, Oct 21, 2020

8. Boundary behaviors, condensation, and effects of slow sites in zero-range systems, IMPA, Rio de Janeiro, Feb 9, 2022

Honors and Awards: The PI was fortunate to be honored:

1. Fellowship of the IMS (Institute of Mathematical Statistics), 2018

2. Fellowship of the JSPS (Japan Society for the Promotion of Science), 2018

3. Simons Foundation Fellow, 2018

Protocol Activity Status:

Technology Transfer: The PI would like to interact with DoD laboratories. With PhD student Conner Hatton, there will be interaction with Nevada National Security Site, planned for 2022-2023.

PARTICIPANTS:

Participant Type: PD/PI
Participant: Sunder Sethuraman
Person Months Worked: 9.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: Postdoctoral (scholar, fellow or other postdoctoral position)
Participant: Jianfei Xue
Person Months Worked: 1.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: Postdoctoral (scholar, fellow or other postdoctoral position)
Participant: William Lippitt
Person Months Worked: 5.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: Graduate Student (research assistant)
Participant: William Lippitt
Person Months Worked: 3.00 **Funding Support:**
Project Contribution:
National Academy Member: N

RPPR Final Report
as of 09-Oct-2022

Participant Type: Postdoctoral (scholar, fellow or other postdoctoral position)
Participant: Michael Conroy
Person Months Worked: 1.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: Graduate Student (research assistant)
Participant: Alex Loomis
Person Months Worked: 3.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: Graduate Student (research assistant)
Participant: Conner Hatton
Person Months Worked: 1.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: High School Student
Participant: Mathew Colson
Person Months Worked: 1.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: Undergraduate Student
Participant: Lexi Garrabrant
Person Months Worked: 1.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: High School Student
Participant: Kathleen Ge
Person Months Worked: 1.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: High School Student
Participant: Marco Manzo
Person Months Worked: 1.00 **Funding Support:**
Project Contribution:
National Academy Member: N

RPPR Final Report
as of 09-Oct-2022

Participant Type: High School Student
Participant: Rossisela Wong
Person Months Worked: 1.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: High School Student
Participant: Ted Roberts
Person Months Worked: 1.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: High School Student
Participant: Yulia Savine
Person Months Worked: 1.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: High School Student
Participant: Isabella Ducey
Person Months Worked: 1.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: High School Student
Participant: Karl Ramus
Person Months Worked: 1.00 **Funding Support:**
Project Contribution:
National Academy Member: N

International Collaboration:

MEX
JPN
KOR
FRA
BRA

ARTICLES:

RPPR Final Report
as of 09-Oct-2022

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: Elec. J. Probab.
Publication Identifier Type: DOI Publication Identifier: 10.1214/18-EJP237
Volume: 23 Issue: 130 First Page #: 1
Date Submitted: 4/5/22 12:00AM Date Published: 10/17/18 5:24PM
Publication Location:
Article Title: Hydrodynamic limits for long-range asymmetric interacting particle systems
Authors: Sunder Sethuraman, Doron Shahar
Keywords: interacting particle system; long-range; asymmetric; hydrodynamic; Burgers; anoma- lous; misanthrope; zero-range; exclusion.
Abstract: We consider the hydrodynamic scaling behavior of the mass density with respect to a general class of mass conservative interacting particle systems on Zn, where the jump rates are asymmetric and long-range of order ?x?^{?(n+?)} for a particle displacement of order ?x?. Two types of evolution equations are identified depending on the strength of the long-range asymmetry. When 0 < ? < 1, we find a new integro-partial differential hydrodynamic equation, in an anomalous space-time scale. On the other hand, when ? ? 1, we derive a Burgers hydrodynamic equation, as in the finite-range setting, in Euler scale.
Distribution Statement: 1-Approved for public release; distribution is unlimited.
Acknowledged Federal Support: **Y**

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: Electronic Journal of Probability
Publication Identifier Type: ISSN Publication Identifier: 1083-6489
Volume: 25 Issue: 58 First Page #: 1
Date Submitted: 4/13/22 12:00AM Date Published: 4/12/20 11:10PM
Publication Location:
Article Title: On Hydrodynamic Limits of Young Diagrams
Authors: Ibrahim Fatkullin, Sunder Sethuraman, Jianfei Xue
Keywords: Young diagram; Gibbs measure; interacting particle system; zero-range; weakly; hydrodynamic; shape; dynamic.
Abstract: We consider a family of stochastic models of evolving two-dimensional Young diagrams, given in terms of certain energies, with Gibbs invariant measures. ‘Static’ scaling limits of the shape functions, under these Gibbs measures, have been shown in the literature. The purpose of this article is to study corresponding, but less understood, ‘dynamical’ limits. We show that the hydrodynamic scaling limits of the diagram shape functions may be described by different types of parabolic PDEs, depending on the energy structure.
Distribution Statement: 1-Approved for public release; distribution is unlimited.
Acknowledged Federal Support: **Y**

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: Sankhya A
Publication Identifier Type: DOI Publication Identifier: https://doi.org/10.1007/s13171-020-00236-x
Volume: Issue: First Page #:
Date Submitted: 4/13/22 12:00AM Date Published: 11/5/20 11:17PM
Publication Location:
Article Title: Stick-Breaking processes, Clumping, and Markov Chain Occupation Laws
Authors: Zach Dietz, William Lippitt, Sunder Sethuraman
Keywords: RAM, GEM, Dirichlet, Inhomogeneous, Markov, Stick-breaking, Empirical, Clumping
Abstract: We connect the empirical or ‘occupation’ laws of certain discrete space time- inhomogeneous Markov chains, related to simulated annealing, to a novel class of ‘stick-breaking’ processes, a ‘nonexchangeable’ generalization of the Dirichlet process used in nonparametric Bayesian statistics. To make this unexpected correspondence, we examine an intermediate ‘clumped’ struc- ture in both the time-inhomogeneous Markov chains and the stick-breaking processes, perhaps of its own interest, which records the sequence of differ- ent states visited and the scaled proportions of time spent on them. By matching the associated intermediate structures, we identify the limits of the empirical measures of the time-inhomogeneous Markov chains as types of stick-breaking processes
Distribution Statement: 1-Approved for public release; distribution is unlimited.
Acknowledged Federal Support: **Y**

RPPR Final Report
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Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: Annals of Applied Probability
Publication Identifier Type: DOI Publication Identifier: 10.1214/20-AAP1639
Volume: 31 Issue: 4 First Page #: 1966
Date Submitted: 4/13/22 12:00AM Date Published: 8/1/21 11:23PM
Publication Location:
Article Title: DERIVATION OF COUPLED KPZ-BURGERS EQUATION FROM MULTI-SPECIES ZERO-RANGE PROCESSES
Authors: Cedric Bernardin, Tadahisa Funaki, Sunder Sethuraman
Keywords: Interacting, particle system, zero-range, multi-species, weakly asymmetric, fluctua- tion, field, Burgers, coupled, KPZ, nonlinear fluctuating hydrodynamics.
Abstract: We consider the fluctuation fields of multi-species weakly-asymmetric zero-range interacting particle systems in one dimension, where the mass density of each species is conserved. Although such fields have been stud- ied in systems with a single species, the multi-species setting is much less understood. Among other results, we show that when the system starts from stationary states with a particular property, the scaling limits of the multi- species fluctuation fields, seen in a characteristic traveling frame, solve a coupled Burgers SPDE, which is a formal spatial gradient of a coupled KPZ equation.
Distribution Statement: 1-Approved for public release; distribution is unlimited.
Acknowledged Federal Support: **Y**

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: Rocky Mountain Journal of Mathematics
Publication Identifier Type: DOI Publication Identifier: 10.1216 / rmj.2021.51.1603
Volume: 51 Issue: 5 First Page #: 1603
Date Submitted: 4/13/22 12:00AM Date Published: 1/19/21 12:06AM
Publication Location:
Article Title: REMARKS ON THE RANGE AND MULTIPLE RANGE OF A RANDOM WALK UP TO THE TIME OF EXIT
Authors: Thomas Doehrman, Sunder Sethuraman, Shankar C. Venkataramani
Keywords: random walk, range, multiple, Brownian motion, exit, time, constrained, polyharmonic.
Abstract: We consider the scaling behavior of the range and p-multiple range, that is the number of points visited and the number of points visited exactly p ? 1 times, of a simple random walk on Zd , for dimensions d ? 2, up to time of exit from a domain DN of the form DN = N D, where D ? Rd , as N ? ?. Recent papers have discussed connections of the range and related statistics with the Gaussian free field, identifying in particular that the distributional scaling limit for the range, in the case D is a cube in d ? 3, is proportional to the exit time of Brownian motion. The purpose of this note is to give a concise, different argument that the scaled range and multiple range, in a general setting in d ? 2, both weakly converge to proportional exit times of Brownian motion from D, and that the corresponding limit moments are “polyharmonic”, solving a hierarchy of Poisson equations.
Distribution Statement: 1-Approved for public release; distribution is unlimited.
Acknowledged Federal Support: **Y**

RPPR Final Report
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Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: Contemporary Mathematics
Publication Identifier Type: DOI Publication Identifier: 10.1090/conm/774/15571
Volume: 774 Issue: First Page #: 153
Date Submitted: 4/15/22 12:00AM Date Published: 11/18/21 5:14PM
Publication Location:
Article Title: On the use of Markovian stick-breaking priors
Authors: William Lippitt, Sunder Sethuraman
Keywords: Markovian, stick-breaking, prior, Dirichlet, posterior, consistency, histogram, density, estimation, smoothing, geometric, contingency
Abstract: .Recently, a 'Markovian stick-breaking' process which generalizes the Dirichlet process (?, ?) with respect to a discrete base space X was introduced. In particular, a sample from from the 'Markovian stick-breaking' processs may be represented in stick-breaking form SUM_{i?1} P_i?(T_i) where {T_i} is a stationary, irreducible Markov chain on X with stationary distribution ?, instead of i.i.d. {T_i} each distributed as ? as in the Dirichlet case, and {P_i} is a GEM(?) residual allocation sequence. Although the previous motivation was to relate these Markovian stick-breaking processes to empirical distributional limits of types of simulated annealing chains, these processes may also be thought of as a class of priors in statistical problems. The aim of this work in this context is to identify the posterior distribution and to explore the role of the Markovian structure of {T_i} in some inference test cases.
Distribution Statement: 1-Approved for public release; distribution is unlimited.
Acknowledged Federal Support: **Y**

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: Annales de Li'Institute Henri Poincare Probabilites et Statistiques
Publication Identifier Type: DOI Publication Identifier: 10.1214/20-AIHP1129
Volume: 57 Issue: 3 First Page #: 1702
Date Submitted: 4/15/22 12:00AM Date Published: 11/13/20 5:21PM
Publication Location:
Article Title: Asymptotics of PDE in random environment by paracontrolled calculus
Authors: Tadahisa Funaki, Masato Hoshino, Sunder Sethuraman, Bin Xie
Keywords: Paracontrolled calculus; Quasilinear stochastic PDE; PDE in random environment
Abstract: We apply the paracontrolled calculus to study the asymptotic behavior of a certain quasilinear PDE with smeared mild noise, which originally appears as the space-time scaling limit of a particle system in random environment on one dimensional discrete lattice. We establish the convergence result and show a local in time well-posedness of the limit stochastic PDE with spatial white noise. It turns out that our limit stochastic PDE does not require any renormalization. We also show a comparison theorem for the limit equation.
Distribution Statement: 1-Approved for public release; distribution is unlimited.
Acknowledged Federal Support: **Y**

RPPR Final Report
as of 09-Oct-2022

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 2-Awaiting Publicat
Journal: Annals of Applied Probability
Publication Identifier Type: DOI Publication Identifier:
Volume: Issue: First Page #:
Date Submitted: 10/3/22 12:00AM Date Published:
Publication Location:
Article Title: ON A NONLINEAR SPDE DERIVED FROM A HYDRODYNAMIC LIMIT IN A SINAI-TYPE RANDOM ENVIRONMENT
Authors: Claudio Landim, Carlos G. Pacheco, Sunder Sethuraman, Jianfei Xue
Keywords: Sinai random environment, Brox diffusion, SPDE, interacting particle system, zerorange, hydrodynamic, quasilinear, inhomogeneous, annealed, quenched, regularization.
Abstract: We derive a continuum mean-curvature flow as a certain hydrodynamic scaling limit of a class of Glauber+Zero-range particle systems. The Zero-range part moves particles while preserving particle numbers, and the Glauber part governs the creation and annihilation of particles and is set to favor two levels of particle density. When the two parts are simultaneously seen in certain different time-scales, the Zero-range part being diffusively scaled while the Glauber part is speeded up at a lesser rate, a mean-curvature interface flow emerges, with a homogenized 'surface tension-mobility' parameter reflecting microscopic rates, between the two levels of particle density. We use relative entropy methods, along with a suitable 'Boltzmann-Gibbs' principle, to show that the random microscopic system may be approximated by a 'discretized' Allen-Cahn PDE with nonlinear diffusion. In turn, we show the behavior, especially generation and propagation of interface properties, of this 'discretized'
Distribution Statement: 2-Distribution Limited to U.S. Government agencies only; report contains proprietary info
Acknowledged Federal Support: **Y**

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: Communications in Mathematical Physics
Publication Identifier Type: Other Publication Identifier: https://arxiv.org/pdf/2004.05276.pdf
Volume: Issue: First Page #:
Date Submitted: 7/1/22 12:00AM Date Published:
Publication Location:
Article Title: MEAN CURVATURE INTERFACE LIMIT FROM GLAUBER+ZERO-RANGE INTERACTING PARTICLES
Authors: Perla El Kettani, Tadahisa Funaki, Danielle Hilhorst, Hyunjoon Park, Sunder Sethuraman
Keywords: interacting, particle system, zero-range, Glauber, relative entropy, motion by mean curvature, Allen-Cahn equation, nonlinear diffusion, surface tension.
Abstract: We derive a continuum mean-curvature flow as a certain hydrodynamic scaling limit of a class of Glauber+Zero-range particle systems. The Zero-range part moves particles while preserving particle numbers, and the Glauber part governs the creation and annihilation of particles and is set to favor two levels of particle density. When the two parts are simultaneously seen in certain different time-scales, the Zero-range part being diffusively scaled while the Glauber part is speeded up at a lesser rate, a mean-curvature interface flow emerges, with a homogenized 'surface tension-mobility' parameter reflecting microscopic rates, between the two levels of particle density. We use relative entropy methods, along with a suitable 'Boltzmann-Gibbs' principle, to show that the random microscopic system may be approximated by a 'discretized' Allen-Cahn PDE with nonlinear diffusion. In turn, we show the behavior, especially generation and propagation of interface properties, of this 'discretized'
Distribution Statement: 2-Distribution Limited to U.S. Government agencies only; report contains proprietary info
Acknowledged Federal Support: **Y**

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as of 09-Oct-2022

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 2-Awaiting Publicat
Journal: SIAM Journal on Applied Mathematics
Publication Identifier Type: Other Publication Identifier:
Volume: Issue: First Page #:
Date Submitted: 10/3/22 12:00AM Date Published:
Publication Location:
Article Title: STATIONARITY AND INFERENCE IN MULTISTATE PROMOTER MODELS OF STOCHASTIC GENE EXPRESSION VIA STICK-BREAKING MEASURES
Authors: William Lippitt, Sunder Sethuraman, Xueying Tang
Keywords: multistate, promoter, mRNA, protein, Bayesian, inference, model validation, stickbreaking, Dirichlet, Markovian, stationary distribution, constructive.
Abstract: In a general stochastic multistate promoter model of dynamic mRNA/protein interactions, we identify the stationary joint distribution of the promoter state, mRNA, and protein levels through an explicit `stick-breaking' construction perhaps of interest in itself. This derivation is a constructive advance over previous work where the stationary distribution is solved only in restricted cases. Moreover, the stick-breaking construction allows to sample directly from the stationary distribution, permitting inference procedures and model selection. In this context, we discuss numerical Bayesian experiments to illustrate the results.
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Acknowledged Federal Support: **Y**

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 2-Awaiting Publicat
Journal: Tunisian Journal of Mathematics
Publication Identifier Type: Other Publication Identifier: https://arxiv.org/pdf/2112.13081.pdf
Volume: Issue: First Page #:
Date Submitted: 10/3/22 12:00AM Date Published:
Publication Location:
Article Title: Singular limit of an Allen-Cahn equation with nonlinear diffusion
Authors: Perla El Kettani, Tadahisa Funaki, Danielle Hilhorst, Hyunjoon Park, Sunder Sethuraman
Keywords: Allen-Cahn equation, Mean curvature flow, Singular limit, Nonlinear diffusion, Interface, Surface tension
Abstract: We consider an Allen-Cahn equation with nonlinear diffusion, motivated by the study of the scaling limit of certain interacting particle systems. We investigate its singular limit and show the generation and propagation of an interface in the limit. The evolution of this limit interface is governed by mean curvature flow with a novel, homogenized speed in terms of a surface tension-mobility parameter emerging from the nonlinearity in our equation.
Distribution Statement: 2-Distribution Limited to U.S. Government agencies only; report contains proprietary info
Acknowledged Federal Support: **Y**

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 4-Under Review
Journal: Memoirs of the AMS
Publication Identifier Type: Other Publication Identifier: https://arxiv.org/pdf/2112.13973.pdf
Volume: Issue: First Page #:
Date Submitted: 4/15/22 12:00AM Date Published:
Publication Location:
Article Title: Schauder estimate for quasilinear discrete PDEs of parabolic type
Authors: Tadahisa Funaki, Sunder Sethuraman
Keywords: Schauder estimate, H ?older estimate, Nash estimate, quasilinear discrete PDE, Allen-Cahn equation, polylinear interpolation, fundamental solution.
Abstract: We investigate quasilinear discrete PDEs of reaction-diffusion type with nonlinear diffusion term defined on an n-dimensional unit torus discretized with mesh size 1/N, in terms of discrete Laplacians and gradients. We establish L-infinity bounds and space-time Holder estimates, both uniform in N, of the first and second spatial discrete derivatives of the solutions. We show how these estimates depend on constant prefactors. The motivation for this work stems originally from the study of hydrodynamic scaling limits of interacting particle systems.
Distribution Statement: 2-Distribution Limited to U.S. Government agencies only; report contains proprietary info
Acknowledged Federal Support: **Y**

RPPR Final Report
as of 09-Oct-2022

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 3-Accepted
Journal: Journal of Statistical Physics
Publication Identifier Type: Other Publication Identifier: https://arxiv.org/pdf/2202.13286.pdf
Volume: Issue: First Page #:
Date Submitted: 10/3/22 12:00AM Date Published:
Publication Location:
Article Title: Motion by mean curvature from Glauber-Kawasaki dynamics with speed change
Authors: Tadahisa Funaki, Patrick van Meurs, Sunder Sethuraman, Kenkichi Tsunoda
Keywords: Hydrodynamic limit, Motion by mean curvature, Glauber-Kawasaki dynamics, Sharp interface limit
Abstract: We derive a continuum mean-curvature flow as a certain hydrodynamic scaling limit of Glauber-Kawasaki dynamics with speed change. The Kawasaki part describes the movement of particles through particle interactions. It is speeded up in a diffusive space-time scaling. The Glauber part governs the creation and annihilation of particles. The Glauber part is set to favor two levels of particle density. It is also speeded up in time, but at a lesser rate than the Kawasaki part. Under this scaling, a mean-curvature interface flow emerges, with a homogenized ‘surface tension-mobility’ parameter reflecting microscopic rates. The interface separates the two levels of par- ticle density. These results extend previous results to beyond nearest neighbor interactions.
Distribution Statement: 2-Distribution Limited to U.S. Government agencies only; report contains proprietary info
Acknowledged Federal Support: **Y**

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 4-Under Review
Journal: Annals of Probability
Publication Identifier Type: Publication Identifier:
Volume: Issue: First Page #:
Date Submitted: 7/8/22 12:00AM Date Published:
Publication Location:
Article Title: Condensation, boundary conditions, and effects of slow sites in zero-range systems
Authors: Sunder Sethuraman, Jianfei Xue
Keywords: interacting particle system, zero-range, hydrodynamic, boundary condition, defect, inhomogeneity, condensation
Abstract: We consider the diffusive hydrodynamic mass scaling limit of a zero-range particle system on a 1D discrete torus with a finite number of defects. In such a model, a particle at x jumps equally likely to a neighbor with rate depending only on a function of k, the number of particles at x, say g(k)=k -\alpha. A defect, however, may be present at specific sites in that the jump rate is slowed down there to N -\beta g(k), in terms of a scaling parameter N. In three regimes, depending on \alpha and \beta, the scaling pde limits are different, with boundary conditions reflecting interaction with the slow site and condensation on it.
Distribution Statement: 2-Distribution Limited to U.S. Government agencies only; report contains proprietary info
Acknowledged Federal Support: **Y**

CONFERENCE PAPERS:

Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: AI STATS 2022
Date Received: 03-Oct-2022 Conference Date: 30-Mar-2022 Date Published:
Conference Location: Virtual
Paper Title: Norm agnostic linear bandits
Authors: Brady Gales, Sunder Sethuraman, Kwang-Jun Sung
Acknowledged Federal Support: **Y**

DISSERTATIONS:

RPPR Final Report
as of 09-Oct-2022

Publication Type: Thesis or Dissertation
Institution: University of Arizona
Date Received: 09-May-2022 Completion Date: 5/6/22 7:00AM
Title: Almost sure convergence of adjacent weights in a preferential attachment random graph
Authors: Owen, MacDonald
Acknowledged Federal Support: **Y**

Publication Type: Thesis or Dissertation
Institution: University of Arizona
Date Received: 08-Jun-2022 Completion Date: 6/7/22 9:18PM
Title: The Solution of the d-string Optimal Stopping Problem in Certain Bernoulli Sequences
Authors: Derick, Bishop
Acknowledged Federal Support: **Y**

Publication Type: Thesis or Dissertation
Institution: University of Arizona
Date Received: 08-Jun-2022 Completion Date: 6/7/22 10:14PM
Title: GAMBLER'S RUIN: THE LIMITING DISTRIBUTION OF POINTS VISITED EXACTLY ONCE OF A SIMPLE RANDOM WALK UP TO TIME OF EXIT AND THE INFLUENCER VOTER MODEL
Authors: Conner, Hatton
Acknowledged Federal Support: **Y**

WEBSITES:

URL: https://www.math.arizona.edu/~sethuram/
Date Received: 05-May-2022
Title: Website of PI: Sunder Sethuraman
Description: Website which contains articles, notes, and other information about the research of the PI.

Partners

,

I certify that the information in the report is complete and accurate:
Signature: Sunder Sethuraman
Signature Date: 10/7/22 1:51PM

Final Report from July 2018 - July 2022 for Grant ARO W911NF-18-1-0311

PI: Sunder Sethuraman, University of Arizona

October 7, 2022

The Army grant was awarded July 2, 2018 and ended July 8, 2022. The first update was from July 2018 to August 2019. The second update was from August 2019 to July 2020. The third update was from July 2020 and April 2022. These were sent as emails to Dr. Myers initially and then uploaded in March 2022.

Now, we summarize the overall activity, starting with lists of papers, students, and talks given. Then, we discuss the work from April 2022 to July 2022 in a section. We have also appended the previous reports in two other sections.

In the following, in authorship of papers, the standard convention in the field of the PI is to list authors alphabetically.

Contents

1 Summary of papers, students, and talks given

We begin with a list of papers, followed by a list of students/postdocs mentored and their projects, and then a list of the talks given.

1.1 Papers written from July 2018 - July 2022

We first list papers which appeared in this period, which acknowledge the previous grant ARO W911NF-14-1-0179. Then, we list the appeared/accepted papers under the current grant, and then submitted papers, followed by works-in-progress.

1.1.1 Papers appeared in this period which acknowledge previous grant support

1. E. Davis and S. Sethuraman Consistency of modularity clustering in random geometric graphs. *Ann. Appl. Probab.* **28** (2018) 2003–2062.

2. S. Sethuraman and S. Venkataramani (2018) On the asymptotic growth of a superlinear preferential attachment scheme. In *Probability and Analysis in Interacting Particle Systems: In Honor of SRS Varadhan, Berlin, August 2016* Springer Proceedings in Mathematics & Statistics **283** 243–265. Available at arXiv: 1704.05568

3. J. Lega, S. Sethuraman, A. Young On collision times of self-sorting interacting particles in one-dimension with random initial positions and velocities. *J. Stat. Phys.* **170** (2018) 1088–1122.

4. S. Sethuraman, D. Shahar, Hydrodynamics for asymmetric long-range interacting particle systems. 2018 *Elec J. Probab.* **23** (2018) Paper 130, 54 pgs.

5. E. Davis, S. Sethuraman, Approximating geodesics via random points. *Ann. Appl. Probab.* **29** (2019) 1446–1486.

1.1.2 Papers appeared/accepted which acknowledge the current grant ARO W911NF-18-1-0311

1. I. Fatkullin, S. Sethuraman, J. Xue On hydrodynamic limits of Young diagrams. *Elec. J. Probab.* **25** (2020) Paper 58, 44pgs.

2. Z. Dietz, W. Lippitt, S. Sethuraman, (2021) Stick-breaking processes, clumping, and Markov chain occupation laws. *Sankhya A*
https://doi.org/10.1007/s13171-020-00236-x

3. Bernardin, C., Funaki, T., Sethuraman, S. (2021) Derivation of coupled KPZ-Burgers equation from multi-species zero-range processes. *Ann. Appl. Probab.* **31** 1966–2017.

4. Doehrman, Sethuraman, S., Venkataramani, S.C. (2021) Remarks on the range and multi-range of a random walk up to the time of exit. *Rocky Mountain J. Math.* **51**, No. 5, 1603–1614.

5. Lippitt, W., Sethuraman, S. (2021) On the use of the Markovian stick-breaking priors. *AMS Contemporary Math. Volume in honor of M.M. Rao. Stochastic Processes and Functional Analysis: New Perspectives* **774** 153–174.

6. Funaki, T, Hoshino, M., Sethuraman, S., Xie, B. (2020) Asymptotics of PDE in random environment by paracontrolled calculus. *Ann. I.H.P. Probab. Stat.* **57** 1702–1735.

7. El Kettani, P., Funaki, T., Hilhorst, D., Park, H., Sethuraman, S. (2020) Mean curvature interface limit from Glauber + Zero-range interacting particles. Commun. Math. Phys. https://doi.org/10.1007/s00220-022-04424-8

8. Gales, S.B., Sethuraman, S., Jun, Kwang Sung. (2021) Norm-Agnostic linear bandits. AISTATS 2022 73–91; arXiv:2205.01257

9. Landim, C., Pacheco, C.G., Sethuraman, S., Xue, J. (2020) On hydrodynamic limits in Sinai-type random environments. To appear in *AAP*. https://www.math.arizona.edu/ sethuram/papers/lpsx10.pdf

10. Lippitt, W., Sethuraman, S., Tang, X. (2021) Stationarity and inference and multistate promoter models of stochastic gene expression via stick-breaking measures. To appear in SIAM J. Math. Analysis; arXiv:2108.10896

11. El Kettani, P., Funaki, T., Hilhorst, D., Park, H.J., Sethuraman, S. (2021) Singular limit of an Allen-Cahn equation with nonlinear diffusion. To appear in Tunisian J. Math.; arXiv:2112.13081

12. Funaki, T., van Meurs, P., Sethuraman, S., Tsunoda, K. (2022) Motion by mean curvature from Glauber-Kawasaki dynamics with speed change. *Accepted in J. Stat. Phys.* arXiv:2202.13286

1.1.3 Papers submitted in this period which acknowledge current support

1. Funaki, T., Sethuraman, S. (2021) Schauder estimate for quasilinear discrete PDEs of parabolic type. arXiv:2112.13973 (in review at *Memoirs AMS*).

2. Sethuraman, S., Xue, J. Condensation, boundary conditions, and effects of slow sites in zero-range systems. arXiv:2205.10252 (in review at *Ann. Probab.*).

1.1.4 Works-in-Progress

1. Funaki, T., van Meurs, P., Sethuraman, S., Tsunoda, K. Constant-speed interface flow from unbalanced Glauber-Kawasaki dynamics. To be submitted soon.

2. Conroy, M., Sethuraman, S. Gumbel limits in symmetric exclusion processes. To be submitted soon.

3. Sethuraman, S., Varadhan, S.R.S. How does the system typically organize to achieve atypical behaviors of a tagged particle in asymmetric simple exclusion?

4. Loomis, A., Sethuraman, S. MIW sequences and density approximation via Stein's method.

5. Hatton, C., Sethuraman, S. Multi-range of random walk on the interval.

1.2 Students/Postdocs mentored from July 2018 - July 2022

PhD students

1. William Lippitt, "Stick-breaking processes, clumping, and a time-inhomogeneous Markov chain" (Math) UA Math Bartlett Award 2019. Defended Summer 2019.

2. Derick Bishop, "The solution of the d-string optimal stopping problem in certain Bernoulli sequences." (Stat) Comprehensive Fall 2021. Defended Summer 2022.

3. Conner Hatton, Topics to be determined. (Math) Expected 2024.

4. Alex Loomis, 'Convergence of MIW ground state and other sequences through Stein's method.' (Math) Expected 2023.

5. Michael Phizicky, 'Limits of inhomogeneous Markov chains' (Math) 2019-2021.

MS students

1. Conner Hatton. 'From range of random walk to voter models.' (Math) Defended June, 2022.

2. Owen Macdonald, 'Preferential attachment adjacent weights.' (Math) Defended May, 2022.

3. Brady Gales, 'Reinforced random walks' (Applied Math) Received 2021.

Postdocs

1. Jianfei Xue (2017 - 2020). 'Hydrodynamic limit of Young diagrams', 'Hydrodynamics in Sinai-type random environments'.

2. William Lippitt (2019-2020). 'On the use of Markovian stick-breaking processes as a prior'; 'Stationarity and inference and multistate promoter models of stochastic gene expression via stick-breaking measures.'

3. Michael Conroy (2020-2023) 'Gumbel laws in symmetric exclusion processes.'

Undergrads

1. Lexi Garrabrant and Tristin Solarzano. Research project on records in climate data. Presented poster 'Records: Assessing change in temperature and other data' at UBRP Jan 20, 2020.

Highschool students and outreach

1. Highschool: Ran an online summer 2020 program 'Math of Neural Networks' with 7 Tucson area highschool junior students, June 10-July 20, 2020. Funding: HSAP Army Research Office. Lexi Garrabrant was a co-mentor. Students: Jean Benimana, Kathleen Ge, Marco Manzo, Mathew Colson, Mia Monteverde, Nicole, Lee, Rossisela Wong.

2. Highschool: Ran an online summer 2021 program 'Math Models of

Opinions and Epidemics' with 5 Tucson highschool students, June 9 - July 19, 2021. Funding: HSAP Army Research Office. Students: Lily Wood, Ted Roberts, Yulia Savine, Isabella Ducey, Karl Ramus.

1.3 Talks given from July 2018 - July 2022

We list conference talks and those at institutes.

Conference talks

1. Approximating geodesics via random points. Nov 5- Nov 8, 2018, Kyoto-RIMS, Kyoto, Japan.

2. On hydrodynamics of Young diagrams. ICTS Program, Universality in random structures, Bangalore, India, Jan. 28 - Feb. 5, 2019

3. On microscopic derivation of a mean curvature flow. Seminar in Stochastic Processes, Salt Lake City, UT, Mar 13-16, 2019

4. On microscopic derivation of a mean curvature flow, Dynamics, random media and universality of complex phenomena, Munster, Aug 26-30, 2019.

5. Stickbreaking, clumping, and Markov chain occupation laws. AMS Western Sectional Meeting, MMRao section, Riverside, CA, Nov. 9, 2019

6. On microscopic derivation of a mean curvature flow, Particle systems and PDE VIII, Lisbon, Portugal, Dec. 2-6, 2019

7. On microscopic derivation of a mean curvature flow, Hydrodynamic limit and related topics–one day symposium, Waseda University, Dec. 20, 2019

8. Approximating geodesics via random points. JMM, Spatial Stochastic Models an MRC event, Denver, CO, Jan 16, 2020

9. SPDE hydrodynamic limit in Sinai-type random environments, Bernoulli-IMS One-World Symposium, August 24-31, 2020

10. On microscopic derivation of a mean curvature flow, 10th International Conference on Stochastic Analysis and its Applications (ICSAA), Kyoto University, Japan, Sep 6-10, 2021

11. Boundary behaviors, condensation, and effects of slow sites in zero-range systems, Montreal Workshop Probability and PDE, March, 2022.

12. Atypical behaviors of a tagged particle in asymmetric simple exclusion, Symposium on Interacting Stochastic Systems (in honor of Raghu

Varadhan), Seoul National University, Jeju Island, South Korea, Jun 12-18, 2022.

13. Atypical behaviors of a tagged particle in asymmetric simple exclusion, Conference in honor of Herbert Spohn, Muenster, Germany, Jun 21 - 25, 2022.

Talks at institutes

1. Stick breaking processes, Clumping, and Occupation laws of Markov chains. Temple University, Nov. 26, 2018

2. Stick breaking, clumping, and occupation laws of Markov chain. IMPA, Jan 16, 2019.

3. On Hydrodynamic limits of Young Diagrams. Kansas University, February 21, 2019.

4. On microscopic derivation of a mean curvature flow, Brown University, Apr. 5, 2019.

5. On microscopic derivation of a mean curvature flow, New York University, April 12, 2019.

6. On microscopic derivation of a mean curvature flow, Columbia University, Apr. 19, 2019

7. Mean-curvature interface limit from Glauber+Zero-range microscopic dynamics, Northwestern-U. Minnesota, Oct 21, 2020

8. Boundary behaviors, condensation, and effects of slow sites in zero-range systems, IMPA online, Rio de Janeiro, Feb 9, 2022

2 Update from April 2022 - July 2022

The following summarizes work done between April 2022 - July 2022 on grant ARO W911NF-18-1-0311.

2.1 Personnel

People in the group: M. Conroy (postdoc), D. Bishop (Stat PhD student), C. Hatton, A. Loomis, O. MacDonald (PhD/MS Math students).

Personnel supported on the grant in this period:

1. Alex Loomis, Math PhD student, 1 month summer support.

2. Michael Conroy, Postdoc, 1 month summer support.

3. The PI was also supported for 1.5 summer months.

2.2 Research acknowledging ARO W911NF-18-1-0311

Largely, the research was on connecting micro (street level) dynamics/features with macro (zoomed out, bird's view) reduced description in terms of continuum structure, flows, pdes, spdes, etc. Specific problems considered how interface flow and fronts develop, how atypical events occur in traffic, the role of 'stick-breaking' measures in a variety of settings, including mRNA dynamic models, random network structures, etc.

Highlights include:

1. In work-in-progress (to be submitted in days), with Michael Conroy, solved a 30 year conjecture on front propagation in exclusion processes where particles may jump symmetrically but not on each other. Here, starting from a backed up road full of particles (cars), the lead particle in a superdiffusive scaling $\sqrt{t \log t}$ obeys a Gumbel law.

2. In a microscopic model, with movement of particles and also birth/death of them, we show that if the birth/death rates are balanced, then mean-curvature flow results in the continuum limit (Funaki-van Meurs-Sethuraman-Tsunoda). If the rates are not balanced, then Huygens' flow, where the interface propagates with constant speed in the normal direction, results. The Huygens' flow result is the first of its kind. The mean-curvature result shows the robustness of a technique that we developed earlier.

3. In mRNA dynamic models, the stationary distribution is not understood in closed form, preventing accurate statistical estimation of parameters of the model from data. We showed that the distribution however has stick-breaking form, and fits into the framework of Markovian stick-breaking measures previously developed during the grant period (Lippitt-Sethuraman-Tang). We also showed that identification is possible of parameters using Bayesian approaches and the stick-breaking form.

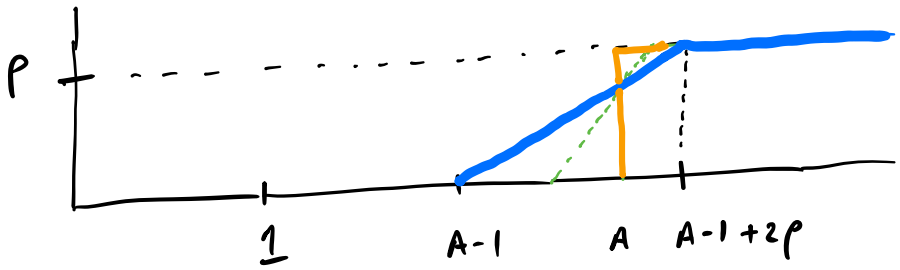


Figure refers to the 'typical' behavior of the traffic process subject to the 'atypical' event that a distinguished particle, initially at 0, is at a location A much larger than its typical position at macro time 1. Since particles cannot jump on to each other, the process arranges so that initially there are no particles in front of the distinguished particle (blue profile). The mass then flows. The distinguished particle must speed up its rate so that both it and the mass flow both arrive at A at time 1 (bottom of yellow profile). The cost of this arrangement is exactly the large deviation cost of the atypical event. See the next description in item 4.

4. The typical behavior of a distinguished particle in asymmetric exclusion, where particle jumps have a drift, is known to follow an ODE, which involves the hydrodynamic density of mass of the bulk particle motion. We find the large deviation costs of the atypical behavior of the particle, and also show how these events are achieved (Sethuraman-Varadhan).

2.2.1 Papers appeared April 2022 - July 2022, which acknowledge current support

1. Doehrman, Sethuraman, S., Venkataramani, S.C. (2022) Remarks on the range and multi-range of a random walk up to the time of exit. *Rocky Mountain J. Math.* **51**, No. 5, 1603–1614; arXiv:2003.07960

2. Gales, S.B., Sethuraman, S., Jun, Kwang Sung. (2021) Norm-Agnostic linear bandits. AISTATS 2022 73–91. arXiv:2205.01257

3. El Kettani, P., Funaki, T., Hilhorst, D., Park, H., Sethuraman, S. (2020) Mean curvature interface limit from Glauber + Zero-range interacting particles. Commun. Math. Phys. https://doi.org/10.1007/s00220-022-04424-8

2.2.2 Papers accepted April 2022 - July 2022, which acknowledge current support

1. Landim, C., Pacheco, C.G., Sethuraman, S., Xue, J. (2020) On hydrodynamic limits in Sinai-type random environments. To appear in *AAP*. arXiv:2006.00583; https://www.math.arizona.edu/ sethuram/papers/lpsx10.pdf

2. Lippitt, W., Sethuraman, S., Tang, X. (2021) Stationarity and inference and multistate promoter models of stochastic gene expression via stick-breaking measures. To appear in SIAM J. Math. Analysis; arXiv:2108.10896

3. El Kettani, P., Funaki, T., Hilhorst, D., Park, H.J., Sethuraman, S. (2021) Singular limit of an Allen-Cahn equation with nonlinear diffusion. To appear in Tunisian J. Math.; arXiv:2112.13081

4. Funaki, T., van Meurs, P., Sethuraman, S., Tsunoda, K. Motion by mean curvature from Glauber-Kawasaki dynamics with speed change. Accepted in *J. Stat. Phys.* arXiv:2202.13286

2.2.3 Papers submitted/in review April 2022 - July 2022, which acknowledge current support

1. Funaki, T., Sethuraman, S. (2021) Schauder estimate for quasilinear discrete PDEs of parabolic type. arXiv:2112.13973 Submitted to *Memoirs AMS*

2. Sethuraman, S., Xue, J. Condensation, boundary conditions, and effects of slow sites in zero-range systems. arXiv:2205.10252 Submitted to *Ann. Probab.*

2.2.4 Works-in-progress

1. Funaki, T., van Meurs, P., Sethuraman, S., Tsunoda, K. Constant-speed interface flow from unbalanced Glauber-Kawasaki dynamics. To be submitted in days.

2. Conroy, M., Sethuraman, S. Gumbel limits in symmetric exclusion processes. To be submitted in days.

3. Sethuraman, S., Varadhan, S.R.S. How does the system typically organize to achieve atypical behaviors of a tagged particle in asymmetric simple exclusion? All the work has been done.

4. Loomis, A., Sethuraman, S. MIW sequences and density approximation via Stein's method.

5. Hatton, C., Sethuraman, S. Multi-range of random walk on the interval.

2.3 Students and mentoring April 2022 - July 2022

1. Derick Bishop "The solution of the d-string optimal stopping problem in certain Bernoulli sequences." Comprehensive Fall 2021. Defended Summer 2022.

2. Conner Hatton 'From range of random walk to voter models.' Defended June, 2022. Expected 2024.

3. Alex Loomis 'Convergence of MIW ground state and other sequences through Stein's method.' Expected 2023.

4. Owen MacDonald 'Preferential attachment adjacent weights.' Defended May, 2022.

5. Michael Conroy (2020-2023) 'Gumbel laws in symmetric exclusion processes.'

2.4 Conferences organized

1. The postponed Frontier Probability Days 2020 finally took place Dec 3-5, in Las Vegas NE. The PI is an organizer of this regular national probability and applications conference.

2. A session at AIMS Dynamical systems and differential equations is now being planned for Summer 2023, after a postponement of 3 years.

2.5 Talks given April 2022 - July 2022

1. Atypical behaviors of a tagged particle in asymmetric simple exclusion, Symposium on Interacting Stochastic Systems (in honor of Raghu Varadhan), Seoul National University, Jeju Island, South Korea, Jun 12-18, 2022.

2. Atypical behaviors of a tagged particle in asymmetric simple exclusion, Conference in honor of Herbert Spohn, Muenster, Germany, Jun 21 - 25, 2022.

3 Update from July 2020 - April 2022

The following summarizes work done on the grant ARO W911NF-18-1-0311 between July 2020 - April 2022.

3.1 Personnel

People in the group: M. Conroy (new postdoc), W. Lippitt, J. Xue (postdocs), M. Phyzicky, D. Bishop (Stat PhD student), B. Gales (Applied Math PhD student), C. Hatton, A. Loomis, O. MacDonald (Math PhD students),

Personnel supported on the grant between July 2020 - April 2022:

1. Alex Loomis, Math PhD student: 1 semester full RA support.

2. Conner Hatton, Math PhD student: 3K summer support.

3. 4 Highschool students (Ted Roberts, Karl Ramus, Yulia Savine, Isabella Ducey) were also supported as part of a AEOP supplement at 720$ participant support each.

4. The PI was also supported in parts including two summers for 2.65 mo.'s.

3.2 Research acknowledging ARO W911NF-18-1-0311

The themes of research have been to understand quantitatively the approximation of complex micro systems of interacting agents by reduced description continuum laws. Such an approach has resonance with ARO aims to understand multiscale phenomena in relevant problems.

For example, interface flows–how segregation of types (safe v. troubled regions) occur and evolve. Also, how to handle disorder, even disorder at a location can sometimes have continuum effects–for example, in a domain with traps, how does the type of traps affect flow of agents or particles? We have also been interested in new Bayesian estimation procedures, which we have applied to assess mRNA levels in cells.

3.2.1 Papers appeared July 2020 - April 2022, which acknowledge current grant ARO W911NF-18-1-0311

1. C. Bernardin, T. Funaki, S. Sethuraman (2020) Derivation of coupled KPZBurgers equation from multi-species zero-range processes. in Ann. Appl. Probab.

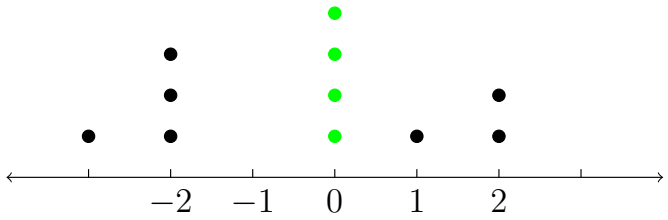


Figure 1: Here, the origin is a 'slow' site or 'trap' for traffic. In work-in-progress, we understand the scale of buildup on the slow site as a function of how 'slow' it is, and connect to continuum PDE/atom systems with boundary conditions.

2. T. Funaki, M. Hoshino, S. Sethuraman, B. Xie, Asymptotics of PDE in random environment by paracontrolled calculus. (2020) in Ann. IHP Prob Stat.

3. Z. Dietz, W. Lippitt, S. Sethuraman, Stick-breaking processes, clumping, and Markov chain occupation laws (2019) in Sankhya A.

4. W. Lippitt, S. Sethuraman, On the use of the Markovian stick-breaking priors. (2020) in AMS Proceedings Volume in honor of MM Rao;

3.2.2 Papers accepted July 2020 - April 2022, which acknowledge current grant ARO W911NF-18-1-0311

1. T. Doehrman, S. Sethuraman, S.C. Venkataramani, Remarks on the range and multiple range of random walk up to the time of exit. Accepted in Rocky Mountain J. Math.

2. C. Landim, C.G. Patheco, S. Sethuraman, J. Xue, On hydrodynamic limits in Sinai-type random environments. (2020) Accepted in Ann. Appl. Probab.

3. B. Gales, S. Sethuraman, K-S., Jun, Norm agnostic linear bandits. Accepted in AISTATS. (Here, the authors are listed as graduate student, and advisors).

3.2.3 Papers submitted/still-in-review July 2020 - April 2022, which acknowledge current grant ARO W911NF-18-1-0311

1. P. El Kettani, T. Funaki, D. Hilhorst, H.J. Park, S. Sethuraman, Mean curvature interface limit from Glauber + Zero-range interacting particles.

(2020) Still in review.

2. W. Lippitt, S. Sethuraman, X. Tang. (2021) Stationarity and inference and multistate promoter models of stochastic gene expression via stick-breaking measures. Available at arXiv:2108.10896; Revise at SIAP.

3. T. Funaki, S. Sethuraman. (2021) Schauder estimate for quasilinear discrete PDEs of parabolic type. arXiv:2112.13973 Submitted

4. P. El Kettani, T. Funaki, D. Hilhorst, H.J. Park. Sethuraman, S. (2021) Singular limit of an Allen-Cahn equation with nonlinear diffusion. arXiv:2112.13081 Submitted.

5. T. Funaki, P. van Meurs, S. Sethuraman, K. Tsunoda. Motion by mean curvature from Glauber-Kawasaki dynamics with speed change. Submitted.

3.2.4 Works-in-progress July 2020 - April 2022

1. S. Sethuraman, J. Xue. Condensation, boundary behaviors, and effects of slow sites in zero-range processes. About to submit in a few days.

2. M. Conroy, S. Sethuraman. Limits of a tagged particle in asymmetric exclusion starting from step initial profiles.

3. S. Sethuraman, S.R.S. Varadhan, Large deviations of a tagged particle in 1D asymmetric exclusion processes.

4. A. Loomis, S. Sethuraman, Convergence of MIW ground states and other sequences through Stein's method.

5. C. Hatton, S. Sethuraman, On the number of points visited once by random walk up to the time of exit from an interval.

6. D. Bishop, S. Sethuraman, Optimal stopping of consecutive records and other record structures.

3.3 Students, mentoring, outreach July 2020 - April 2022

1. Outreach: Ran a 6 week highschool summer program in Summer 2021 Math of Opinions and Epidemics, sponsored by the Army Apprentice Research office, where 5 highschool students learned about voter and reed-frost models both theoretically and in octave code.

2. PhD thesis work of D. Bishop: Continuing to work with this Statistics student on 'Results on optimal stopping in a sequence of Bernoulli trials'.

He will defend in May 2022.

3. Worked with A. Loomis on problems in understanding the multi-interacting worlds approximation to the Schroedinger's equation of quantum physics. He passed his comprehensive exam in Jan 2022, and now is ABD.

4. Worked with B. Gales on problems in optimal selection and bandit problems (e.g. how to optimize recommendations for users)

5. Worked with C. Hatton on problems in 'random walk' and 'voter models'. He will defend a MS thesis June 2022.

6. Worked with O. MacDonald on 'random graphs'. to get them started on PhD research. He will defend a MS thesis May 2022.

7. Working with postdoc M. Conroy on problems in 'interacting particle systems', and have a working paper in progress.

3.4 Conferences and seminars organized

The PI helped organize these conferences and seminars:

1. Frontier Probability Days 2020, Dec 3-5, 2021, Las Vegas. http://lechen.faculty.unlv.edu/FPI

2. Session on Stochastic Analysis and Large Scale Interacting Systems in AIMS on Dynamical systems and differential equations https://aimsconference.org/AIMS-Conference/ Postponed to Summer 2023.

3. Joint online seminar in probability/mathphysics between U. Utah and U. Arizona Fall 2020 - Spring 2021.

4. Joint online seminar in probability/statistical physics between IMPA (Rio de Janeiro), UBA (Buenos Aires), IST (Lisbon), UA (Tucson); https://spmes.impa.br/

3.5 Talks given July 2020 - April 2022

1. SPDE hydrodynamic limit in Sinai-type random environments, Bernoulli-IMS One-World Symposium, August 24-31, 2020

2. On microscopic derivation of a mean curvature flow, 10th International Conference on Stochastic Analysis and its Applications (ICSAA), Kyoto University, Japan, Sep 6-10, 2021

3. Boundary behaviors, condensation, and effects of slow sites in zero-range systems, IMPA, Rio de Janeiro, Feb 9, 2022

4. Boundary behaviors, condensation, and effects of slow sites in zero-range systems, Montreal Workshop in Probability and PDE, CIRM, March, 2022.

5. (upcoming) Probability Conference (in honor of Raghu Varadhan), Seoul National University, South Korea, Jun 12-17, 2022.

6. (upcoming) Conference in honor of Herbert Spohn, Muenster, Germany, Jun 18 - 23, 2022.

4 Update from July 2018 - August 2019

The following summarizes work on the grant ARO W911NF-18-1-0311 between July 2018 and August 2019.

4.1 Honors

The PI has been fortunate to be honored in 2018:

1. Fellowship of the IMS (Institute of Mathematical Statistics),
2. Fellowship of the JSPS (Japan Society for the Promotion of Science),
3. Simons Foundation Fellow.

4.2 Personnel

Personnel on the grant between July 2018 - August 2019:

1. William Lippitt (a Math PhD student) – 2 semesters .25 FTE (half RA for year)

2. Jianfei Xue (a Math postdoc) – .5 month in Summer 2019

3. the PI: 3.3 months from Summer 2018 to Summer 2019.

4.3 Previous research under ARO W911NF-14-1-0179, which has appeared after 2018

In 2018-2019, 5 papers appeared, which were started under and acknowledge the previous grant ARO W911NF-14-1-0179

A. Lega, Joceline; Sethuraman, Sunder; Young, Alexander L. On collisions times of 'self-sorting' interacting particles in one-dimension with ran-

dom initial positions and velocities. J. Stat. Phys. 170 (2018), no. 6, 1088–1122.

B. Davis, Erik; Sethuraman, Sunder Consistency of modularity clustering on random geometric graphs. Ann. Appl. Probab. 28 (2018), no. 4, 2003–2062.

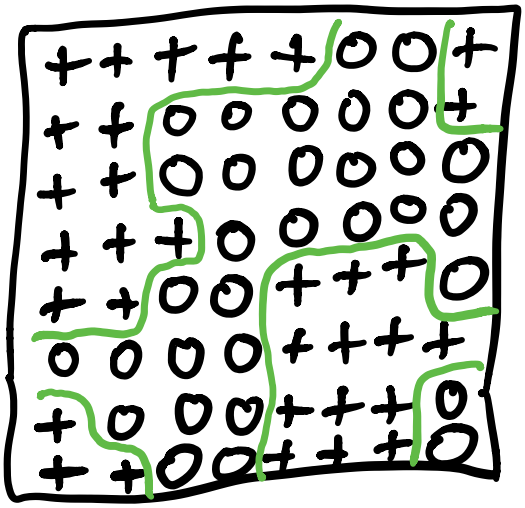
C. Sethuraman, Sunder; Shahar, Doron Hydrodynamic limits for long-range asymmetric interacting particle systems. Electron. J. Probab. 23 (2018), Paper 130, 54 pp.

D. Davis, Erik; Sethuraman, Sunder Approximating geodesics via random points. Ann. Appl. Probab. 29 (2019), no. 3, 1446–1486.

E. Sethuraman, Sunder; Venkataramani, Shankar C. On the growth of a superlinear preferential attachment scheme. Probability and analysis in interacting physical systems, 243–265, Springer Proc. Math. Stat., 283, Springer, Cham, 2019.

4.4 Current research acknowledging ARO W911NF-18-1-0311

The subject of the current grant is to understand 'scales' in stochastic systems and connections to geometry. Interacting particle systems as a spatially discrete model of physical phemonena are studied. Also, networks, and flows of Markov chains are to be investigated. Problems include understanding large scale features as a consequence of types of microscopic interactions. Such an understanding allows to classify behaviors in complex systems at a 'high' level, yielding a description in reduced variables, depending on the type of 'street' level dynamics.



The figure shows a discrete (cartoon) interface between two species. Part of work-in-progress to capture continuum limits and evolution of the interface.

4.4.1 Papers submitted July 2018 - August 2019, which acknowledge current grant ARO W911NF-18-1-0311

1. (submitted to Elec J Probab) I. Fatkullin, S. Sethuraman, J. Xue (2019) On hydrodynamic limits of Young diagrams. arXiv:1809.03592

2. (submitted to Sankhya A.) Z. Dietz, W. Lippitt, S. Sethuraman, Stick-breaking processes, clumping, and Markov chain occupation laws (2019) arXiv:1901.08135

Abstract: In a time-inhomogeneous Markov chain model of simulated annealing, where the state switches at time n with chance $O(1/n)$, the empirical measure (frequencies) of the chain is found. Interestingly, the limit measure is a generalization of the Dirichlet process measure, used ubiquitously in Bayesian statistics as a prior.

4.4.2 Works-in-progress July 2019 - August 2019

1. Will submit (likely to Ann. Appl. Probab.) with C. Bernardin and T. Funaki on deriving coupled KPZ-Burger stochastic partial differential equations.

Abstract: We consider the fluctuation fields of multi-species weakly-asymmetric zero- range interacting particle systems in one dimension, where the mass density of each species is conserved. Although such fields have been studied in systems with a single species, the multi-species setting is much less understood. Among other results, we show that, when the system starts from stationary states, with a particular property, the scaling limits of the multi-species fluctuation fields, seen in a characteristic traveling frame, solve a coupled Burgers SPDE, which is a formal spatial gradient of a coupled KPZ equation.

2. Finalizing a paper with T. Funaki, and D. Hilhorst and two of her students, on deriving mean curvature flow of an interface between small and large density mass from microscopic systems. I have talked several times on this work, and will be talking on in at an upcoming meeting in Muenster, Germany Aug 30, 2019.

Abstract: We derive the motion by mean curvature in the diffusive space-time scaling limit directly from the microscopic interacting particle system called the Glauber-Zero range process. The Glauber part which governs the creation and annihilation of particles is also speeded up, but slower compared with the diffusive time scale. The Zero range part produces a nonlinear diffusion operator. We apply the relative entropy method and the Boltzmann-Gibbs principle in the probabilistic part. Then we combine this with the asymptotic expansion up to the second order and the comparison method for the discrete nonlinear Allen-Cahn equation in the PDE part.

3. Finalizing a work on scaling limit of interacting random walks seen in a Sinai random trap setting with C. Landim and J. Xue.

Abstract: It is known that a scaling limit of random walk in certain Sinai trap settings in one dimension, that is when the walk at x moves left or right depending on the values of auxilliary iid random variables indexed by x, and these iid random variables are such that the walk is not transient (a condition is known), converges to a Brox diffusion, which keeps in its description the noise from the enviornment. The project here is to consider a large number of walks in such an enviornment and to understand their mass evolution in space and time. What we find is that the mass evolution equation relates to the Brox diffusion generator, also keeping in its description the noise from the environment.

4. Started work on large deviations of a tagged particle in asymmetric

exclusion processes in one dimension with SRS Varadhan.

Abstract: It is known that the rate function of X_t/t where X_t is the position of a distinguished particle interacting with others by nearest-neighbor asymmetric exclusion, near its zero, looks like the rate function of a Poisson process. This work explains the optimal ways the particle behaves to deviate from its mean.

5. In progress: Range of Random walk up to the time of exit from a domain, with T. Doehrman, S. Venkataramani.

Abstract: The scaling behavior of the range of fair random walk (number of distinct sites visited) up to time n is well known. Here, we examine the behavior of the range up to the time of exit from a domain of width N in R^d. We find the range, when scaled by $N^2/\log N$ in $d = 2$, and N^2 in $d \geq 3$, converges to the time of exit of a Brownian motion from the scaled Domain, multiplied by a constant. We will write this up in Fall and submit then.

6. Posterior distribution of the stick breaking processes described in (1), with W. Lippitt. We are trying to compute it.

7. Postscript: There are other things that I worked on during the sabbatical in 2017-2018, which will get written up hopefully in Fall or Spring this coming academic year 2019-2020.

4.5 Students and mentoring July 2018 - August 2019

1. PhD thesis work of W. Lippitt: Working on understanding flows in inhomogeneous Markov chains and connections to Bayesian statistics. See descriptions of the papers earlier. William defended in Summer 2019.

2. PhD thesis work of D. Bishop: Working with this Statistics student on 'Results on optimal stopping in a sequence of Bernoulli trials'.

Abstract: Consider a sequence of 'records' in data. It is known that the these records follow independent Bernoulli trials where the nth one has success probability of being a record with probability $1/n$. Although there will be an infinite number of 'records', there will be only a finite number of consecutive 'records'. The problem is to understand the optimal strategy to find the 'last' time that two consecutive records occur in a more general situation when the success probability is of form $a/(a + b + n - 1)$. This is an infinite horizon 'secretary' problem. The answer it turns out is to wait

for a time $t_{a,b}$ and then declare the next consecutive 'record' pair as the last pair. The success of this strategy is roughly 1/3, similar to the 'secretary' problem.

3. Postdoctoral work of Jianfei Xue: Working with Jianfei on a number of projects in the scaling limits of interacting particle systems, such as those modeling polymer dynamics (Young diagrams), and motion of traffic in trap environments. See descriptions of the papers earlier.

4.6 Talks given July 2018 - August 2019

Conference talks:

1. Approximating geodesics via random points. Nov 5- Nov 8, 2018, Kyoto- RIMS, Kyoto, Japan.

2.On hydrodynamics of Young diagrams. ICTS Program, Universality in random structures, Bangalore, India, Jan. 28 - Feb. 5, 2019

3. On microscopic derivation of a mean curvature ow. (plenary) Seminar in Stochastic Processes, Salt Lake City, UT, Mar 13-16, 2019

4. On microscopic derivation of a mean curvature ow, Dynamics, random media and universality of complex phenomena, Munster, Aug 26-30, 2019.

Talks at Universities:

5. Stick breaking processes, Clumping, and Occupation laws of Markov chains. Temple University, Nov. 26, 2018

6. On Hydrodynamic limits of Young Diagrams. Kansas University, February 21, 2019.

7. On microscopic derivation of a mean curvature ow, Brown University, Apr. 5, 2019.

8. On microscopic derivation of a mean curvature ow, New York University, April 12, 2019.

9. On microscopic derivation of a mean curvature ow, Columbia University, Apr. 19, 2019