NRL REPORT NO. R-3071

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THE EFFECTS OF GROUND REFLECTION AND ANTENNA CHARACTERISTICS ON CONTROL LINES FOR BEAM RIDING MISSILES

NAVAL RESEARCH LABORATORY

Washington, D.C.

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by

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CONTENTS

ABSTRACT

A general formula is developed for the voltage which is effective in steering a beam-rider such as the Lark missile. This formula involves antenna pattern, squint angle, reflection coefficient of the ground, wave length, and the relative positions of the missile, antenna and the ground. A crossover zone is defined, determining the number of angular positions with respect to the antenna that a missile with an ideal steering mechanism may assume. Two methods are developed for determining the extent of this crossover zone by means of the voltage formula: an analytic method for use with low flying missiles, and a graphical method for use when radiation from the antenna side lobes reaches the missile. Numerical examples of each method are discussed in terms of the SP radar, used in the Lark project. Suggestions are made for changing the SP antenna system to reduce the size of the crossover zones.

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THE EFFECTS OF GROUND REFLECTION AND ANTENNA CHARACTERISTICS ON CONTROL LINES FOR BEAM RIDING MISSILES

GENERAL CONSIDERATIONS

This report considers the general question of determining the voltages effective in steering a beam-riding missile at various positions in the field of a circularly scanning radar antenna. The geometry of the situation that will be considered is given in Figure 1(A). Figure 1(B) shows the missile and its image or reflection by the ground, as seen from the antenna. All linear dimensions in Figure 1(B) are in angular units, where the appropriate angles are measured from the antenna. In scanning, the main lobe of the antenna pattern moves in a cone whose center, 0, is at the point and train angle of the antenna and whose half angle is the squint angle, α. It is convenient to refer to this cone as the scan circle whose radius, measured in angular units, is the squint angle, α. The scan phase, t (Figure 1(B)), is determined by the angular displacement of the lobe nose from the top of the scan circle.

An ideal beam-riding missile will be steered by the voltages in the radar field to a position on a straight line running out from the center of the antenna through the center of the scan circle, and thenceforth the missile will remain on this line. Qualitatively this ultimate course is determined as a locus in space, on any point of which the missile receives equal voltages from all parts of the radar scan cycle. Unfortunately the antenna pattern and the reflection of rays from the ground may produce several such equi-voltage loci in space besides the line through the geometric center of the scan circle. Any such locus, to which the ideal missile may be steered and on which it will remain, is termed a control line. This report discusses the influence of antenna pattern and ground on the number and location of these control lines.

Besides the geometric variables defined and tabulated in Figure 1, it is necessary to know the electrical constants of the system. R is the complex reflection coefficient of the ground at the angles considered, and λ, the radar's wavelength. Also some specifications of the antenna pattern must be given. In this report antenna patterns are considered symmetrical. Here symmetry means that equiphase and equipower surfaces of the field in front of the radar antenna (non-scanning) are surfaces of revolution, with the axis of revolution being the line through the antenna in the direction of the nose of the main lobe of the pattern.

The voltage in any symmetrical pattern, in which field intensity becomes negligible at sufficiently large angles from the center of the main lobe, can be represented by

$$E(q) = e^{-aq^2}\,[1 + bq^2 + cq^4 + \ldots] \text{ volts/meter} \qquad (1)$$

where q is defined in Figure 1, and a, b, c, are in general complex constants determined by the lobe shape. From the geometry

$$q^2 = (\epsilon + \alpha \sin t)^2 + (\phi + \alpha \cos t - \theta)^2$$

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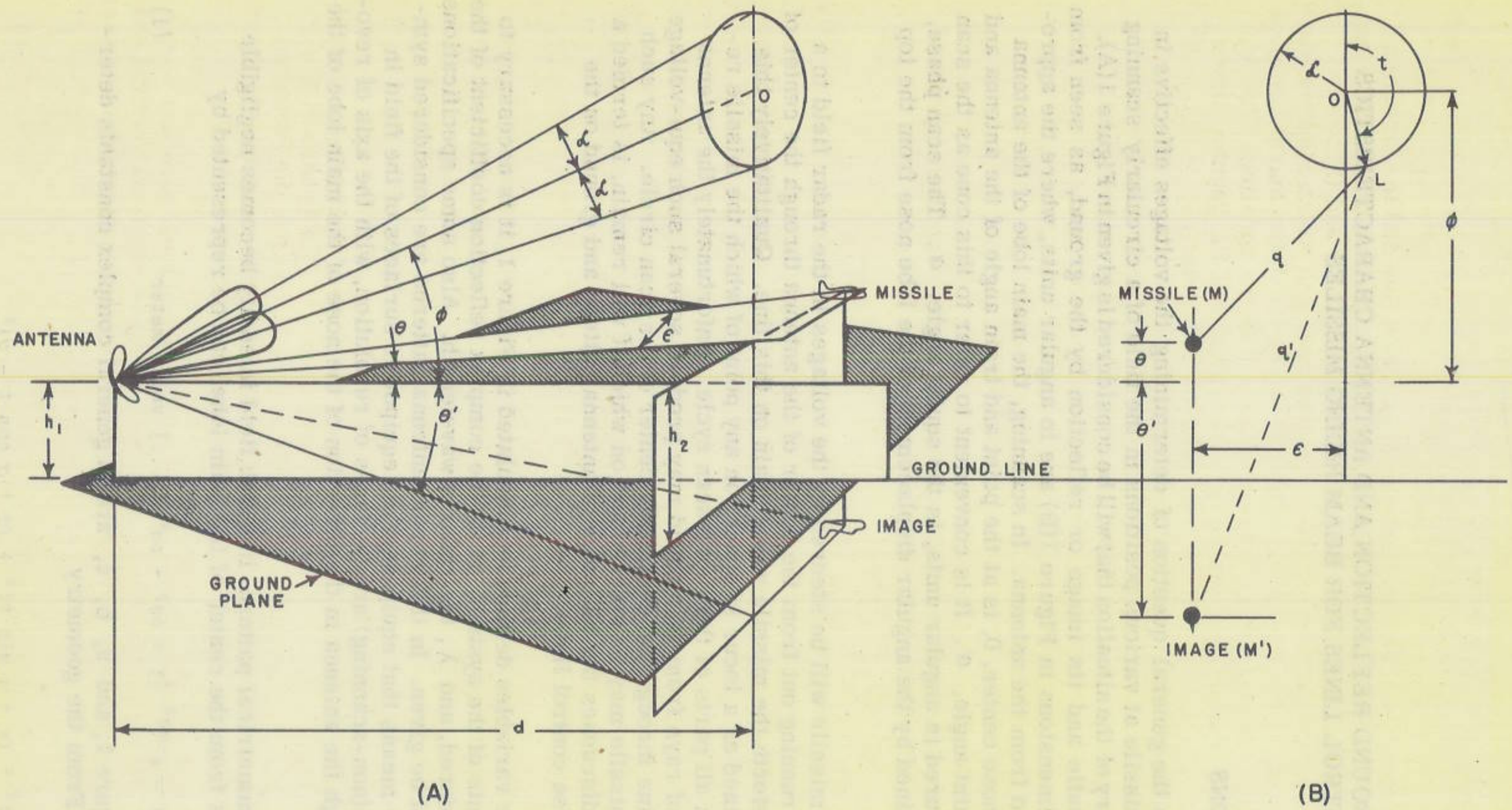

d	horizontal range of missile from antenna	α	squint angle (lobe max. to crossover)
h_1	height of radar antenna above ground	0	free-space crossover of antenna pattern
h_2	height of missile above ground	L	position of lobe nose
θ	elevation angle of missile from antenna	t	scan phase of the lobe nose
θ'	depression angle of missile image from antenna	q	angle at antenna between missile and lobe nose
ϕ	elevation angle of antenna	q'	angle at antenna between missile image and lobe nose
ϵ	bearing error of missile position		

Fig. 1 - Geometric Relations Between Missile, Antenna System, and the Reflecting Surface of the Ground

and

$$q'^2 = (\epsilon + \alpha \sin t)^2 + (\phi + \alpha \cos t + \theta')^2 \tag{2'}$$

(q in Figure 1 represents the angle between the antenna-missile line and the direction of the lobe maximum, and q' represents the angle between the direction of the lobe maximum and the ray reflected to the missile by the ground). Equations (2) and (3) are the plane geometry approximations to the correct spherical trigonometric relations between the variables. The error made in the approximation is of the order of the fourth power of the largest angular dimension, expressed in radians, appearing in the equations. This error is therefore negligible for low flying missiles as the largest of the angular dimensions q, q', θ, θ', α, ϕ, ϵ, will be of the order of a tenth radian.

In order to make the computations tractable the assumption is made that $\theta = \theta'$. This assumption is physically equivalent to the missile being at great ranges and at heights greater than the radar antenna height. From the geometry of Figure (1):

$$\theta = \tan^{-1} \frac{h_2 - h_1}{d} \quad \text{and} \quad \theta' = \tan^{-1} \frac{h_2 + h_1}{d}$$

Expanding the arc tangent in series and keeping only the first term (permitted in view of the smallness of h_1/d, h_2/d), there is obtained

$$\theta \approx \frac{h_2 - h_1}{d} \quad \text{and} \quad \theta' \approx \frac{h_2 + h_1}{d} \tag{3}$$

The assumption that $h_2 \gg h_1$ leads immediately to $\theta = h_2/d = \theta'$ (See Appendix for discussion).

Let $f/2\pi$ $[g/2\pi]$ be the path length in wavelengths traveled by the direct [reflected] ray. Then, from the geometry of Figure (1):

$$\frac{f}{2\pi} = \frac{1}{\lambda}\sqrt{d^2 + (h_2 - h_1)^2}, \quad \frac{g}{2\pi} = \frac{1}{\lambda}\sqrt{d^2 + (h_2 + h_1)^2} \tag{4}$$

Thus e^{if} $[e^{ig}]$ is the phase of the field at the missile due to the path traveled by the direct [reflected] ray, and therefore the total voltage, $E(t)$, for a given scan phase t is given by:

$$E(t) = E(q)e^{if} + RE(q')e^{ig} \text{ volts/meter.} \tag{5}$$

(Since $E(q)$ and $E(q')$ and R are in general all complex quantities, their phases must be accounted for as well as the phases of f and g. To this end let $\beta \equiv$ [phase of $E(q)$] - [phase of $RE(q')$].) The inverse distance law of field strength requires the reflected field to be weaker than the direct field even with $|R| = 1$; but this weakening is less than 2 percent for $\theta \leq 5°$, and may be neglected or considered as absorbed into R.

For vertical steering, the missile circuits weight the signal voltages received when the scan phase angle is t, by the factor $\cos(t)$, where t is measured from vertical scan position. Thus in one scan cycle the total error voltage E effective in steering the missile in a vertical plane is given by

$$E = \int_0^{2\pi} \cos t \, |E(t)| \, dt \tag{6}$$

The left-right error voltage is similarly given by

$$E = \int_0^{2\pi} \sin t \, |E(t)| \, dt$$

The absolute value of the voltage must be taken because the averaging circuit does not pay any attention to the phase of the signal.

With the insertion of expression (5) for E(t), (6) becomes

$$E = \int_0^{2\pi} \cos t \, \{|E(q)|^2 + |RE(q')|^2 + 2|RE(q)E(q')| \cos (f-g+\beta)\}^{\frac{1}{2}} \, dt \tag{7}$$

If the total error signal E is zero, the missile is undeflected and remains so as long as its elevation angle θ and the antenna elevation angle ϕ are such that $E = 0$. Under the assumption $\theta = \theta'$, if E is zero at a given range for a certain θ, E will be zero for this θ for all ranges; and such a value of θ is termed a crossover. A crossover zone is defined as the range, or ranges, of values of θ within which crossovers may occur, and outside of which none occur. Since f and g are fixed in the integration, being independent of t, it is possible to let $\cos (f-g+\beta) = K$, a constant, throughout the integration.

The variation of β will be assumed to be small throughout a scan cycle. β is very nearly constant when, throughout the scan cycle, the two rays emerge from the main lobe and/or from the maximum of a side lobe in a radar antenna having deep pattern minima. Equation (7) may now be written as:

$$E(\theta, K) = \int_0^{2\pi} \cos t \, \{|E(q)|^2 + |RE(q')|^2 + 2K|RE(q)E(q')|\}^{\frac{1}{2}} \, dt \tag{7a}$$

Crossovers occur when and only when θ and K are such that $E(\theta, K) = 0$ and $\cos (f-g+\beta) = K$ simultaneously. Define $\theta_+ \, [\theta_-]$ as the value of θ for which $E(\theta, +1) = 0 \; [E(\theta, -1) = 0]$. Now θ_+, for instance, is not a crossover necessarily because for this θ value, $\cos (f-g+\beta) \neq 1$ in general. But only between θ_+ and θ_- are crossovers found, for outside such a range it is required that $\cos (f-g+\beta) > 1$, which is impossible for real values of f and g. Thus the crossover zone is bounded by θ_+ and θ_-, and these bounds are the θ values determined by the + and - signs in

$$E(\theta, \pm 1) = 0 = \int_0^{2\pi} \cos t \, \big| |E(q)| \pm |RE(q')| \big| \, dt \tag{8}$$

The variables which appear in equation (8) are seen from (2) and (1) to be only the missile and antenna elevation angles, the squint angle, the appropriate absolute reflection coefficient of the ground, and the antenna pattern parameters. The scan phase angle is the variable of integration, which does not appear in the final expression. The wavelength, antenna height, and phase shift of the reflection coefficient have been eliminated from consideration in delimiting the crossover zone, but they are necessary in determining the locations of crossovers within the zone, since they determine the relative phase of the direct and reflected rays.

Within the crossover zone, a crossover occurs approximately twice every time (f-g) in equation (7) varies through 2π radians (if β remains essentially constant in this range). For in 2π radians, $\cos(f-g+\beta)$ takes on each value between -1 and +1 twice. Setting $E(\theta, K) = 0$ in (7a) determines for each θ in the crossover zone a value $K(\theta)$ of K between -1 and +1. The expectation, then, is that $\cos(f-g+\beta) = K(\theta)$ for two θ values in a range of θ such that (f-g) varies through 2π radians. But $\cos(f-g+\beta) = K(\theta)$ is the condition for a crossover, therefore two crossovers are expected in a change of 2π in (f-g), as stated.

To determine the θ range giving a 2π radian variation of (f-g), the radicals in (4) are expanded and higher powers of h_2/d are neglected as they are small:

$$\frac{f}{2\pi} = \frac{d}{\lambda}\left[1 + \frac{1}{2}\left(\frac{h_2 - h_1}{d}\right)^2 + \dots\right]$$

$$\frac{g}{2\pi} = \frac{d}{\lambda}\left[1 + \frac{1}{2}\left(\frac{h_2 + h_1}{d}\right)^2 + \dots\right]$$

$$\text{or } (f-g) = \frac{-2\pi d(2h_1 h_2)}{\lambda d^2} = \frac{-4\pi\ h_1 \theta}{\lambda}$$

Thus (f-g) increases by 2π radians for a variation $d\theta = (2\pi) \cdot \lambda/4\pi h_1$ radians or $d\theta = \lambda/2h_1$ radians. For an S-band radar with antenna 90 feet above level ground, $d\theta = 0.00185$ radians $= 0.106°$. Thus, in a crossover zone, a normally mounted shipborne radar will have between 5 and 10 crossovers per degree change in missile elevation angle.

Crossovers thus occur so frequently in the crossover zone for the SP system that there is no reason to specify exactly the location of individual crossovers. Besides, to do this would have required use of the phase of the reflection coefficient as well as its magnitude.

It should be pointed out that alternate crossovers are unstable in the sense that a missile finding itself slightly above an unstable crossover will receive greater effective voltage due to the lower part of the scan and therefore have a "go up" order instead of the correct "go down" order. Thus the missile is driven away from the unstable crossover instead of toward it. The situation is illustrated in Figure 2. The same considerations hold for the left-right orders to the missile. If there are more than one crossover in the left-right sense, alternate ones are likewise unstable. A control line is a line in space which is at once a stable crossover in the up-down and in the left-right senses.

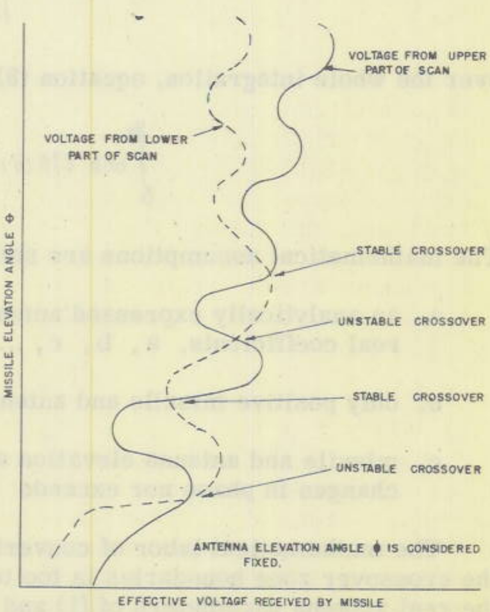

Fig. 2 - Nature of Crossovers

Since in application to the SP system the crossovers lie so closely together in the up-down sense, it will be more immediately informative to determine the extent of the crossover zone through equation (8) than it would be to find the position of individual crossovers. In the following sections two methods of determining the boundaries of crossover zones are developed, based on equation (8).

If the radar which is used to guide the missile is at the same time a search radar (and the squint angle is small—see Appendix) the antenna will seem to be "on target" whenever the sought target lies on a control line for the given antenna orientation. For when the target is on a control line, echo voltages as well as radar field voltages are roughly equal over all parts of the scan cycle. Therefore the problem of determining possible locations of the target is mathematically the same as that of determining possible control lines for the missile. The difficulty of the tactical problem is increased because of the uncertainty as to which control line is the correct one for the missile to follow out. Although the dynamics of the missile and of the antenna system are different, general principles of the beam-rider problem are the same as those in the problem of target location and acquisition with conically scanning search and fire-control radars.

CROSSOVERS AT VERY LOW MISSILE AND ANTENNA ELEVATION ANGLES

The integration of (8) may be performed either numerically or by using analytic functions for the integrand. In the latter case, analytic functions may be substituted only when the expression in the absolute value signs is well behaved over the whole range of the integration. Physically, this will be the case only when the missile is near the position indicated by the antenna's orientation or where the side lobes of the pattern do not enter the integration. Mathematically:

Whenever β is constant and

$$|E(q)| > |RE(q')| \qquad (9)$$

over the whole integration, equation (8) can be written as

$$\int_0^{2\pi} \cos t\,|E(q)|\,dt = \pm \int_0^{2\pi} \cos t\,|RE(q')|\,dt \qquad (10)$$

The mathematical assumptions are satisfied if:

a. an analytically expressed antenna pattern is used such as equation (1) with real coefficients, a, b, c, ...

b. only positive missile and antenna elevation angles are used

c. missile and antenna elevation angles are chosen so that RE(q') neither changes in phase nor exceeds |E(q)| in magnitude.

The mathematical labor of converting (10) into an explicit formula for determining the crossover zone boundaries is too tedious to reproduce here. With a, b, and c as the real antenna parameters of (1) and with the other variables as previously defined, (10) becomes the equality (11) = (11'), where (11) and (11') are as follows:

$$\left\{\begin{aligned} &\left[1 + \frac{a^2\alpha^2 x}{2} + \frac{a^2\alpha^4}{2}\right]\left[1 + xb + x^2 c\right] \\ &-\left[\frac{b}{a} + \frac{2cx}{a}\right]\left[1 + \frac{3a^2\alpha^2 x}{2} - \frac{3a^2\alpha^4}{2}\right] \\ &+ \frac{c}{a^2}\left[a^2\alpha^2 x - a^2\alpha^4\right] + \cdots \end{aligned}\right\} 2\pi\alpha a\,(\phi - \theta) \tag{11}$$

$$\pm \left\{\begin{aligned} &\left[1 + \frac{a^2\alpha^2 x'}{2} + \frac{a^2\alpha^4}{2}\right]\left[1 + x'b + (x')^2 c\right] \\ &-\left[\frac{b}{a} + \frac{2cx'}{a}\right]\left[1 + \frac{3a^2\alpha^2 x'}{2} - \frac{3a^2\alpha^4}{2}\right] \\ &+ \frac{c}{a^2}\left[a^2\alpha^2 x' - a^2\alpha^4\right] + \cdots \end{aligned}\right\} 2\pi\alpha a\,|R|\,(\phi + \theta)\,e^{-4a\theta\phi} \tag{11'}$$

where

$$x = \epsilon^2 + \alpha^2 + (\phi - \theta)^2$$

$$x' = \epsilon^2 + \alpha^2 + (\phi + \theta)^2$$

In the following paragraphs, the equation (11) = (11') is discussed as applied to the SP radar, which is projected for use with the Lark beam-riding missile. The formulas and conclusions will also hold for similar systems.

In the case of the present SP antenna, a sufficiently accurate representation of the main lobe of the antenna can be realized by setting b and c in (1) equal to zero. This means that side lobes are not taken into consideration, but at present this is impossible with analytic functions anyway, because conditions (9) will not hold. The proper choice of the parameter a is found by fitting a parabola to the antenna pattern in a plot of the antenna beam pattern in decibels versus angle from beam nose (Figure 3). A good fit to the SP pattern is a parabola which is down 16 decibels at an angle of 4 degrees off beam nose. According to the formula $\sqrt{\text{power}}$ = (const)exp [-a (angle in degrees)^2], a comes out to be 0.1141/(degree^2). The accuracy of this representation is discussed in the Appendix.

Since the problem is concerned with small values, the quantity $a^2\alpha^2 x$ is small with respect to 1 (see Appendix for discussion) and on neglecting this term and the even smaller $a^2\alpha^4$, expression (11) becomes $2\pi\alpha a(\phi-\theta)$. Similarly (11') becomes $\pm 2\pi\alpha a|R|(\phi+\theta)e^{-4a\phi\theta}$. Equating (11) to (11') and eliminating the common factor $2\pi\alpha a$, there follows:

$$(\phi - \theta) = \pm|R|\,(\phi + \theta)\,e^{-4a\phi\theta} \tag{12}$$

This is the equation which holds at the two boundaries of the crossover zone.

Figure 4 is a plot of the extent of the crossover zones for various antenna elevation angles ϕ. The boundaries of the crossover zones are plotted from (12). For fixed antenna elevation angle ϕ and reflection coefficient R the crossover zone is the range of missile elevation angles θ lying between the crossover zone boundaries. Units of ϕ and θ in the plot are taken as $1/\sqrt{2a}$. This unit can be calculated to be 1.201 times the half width

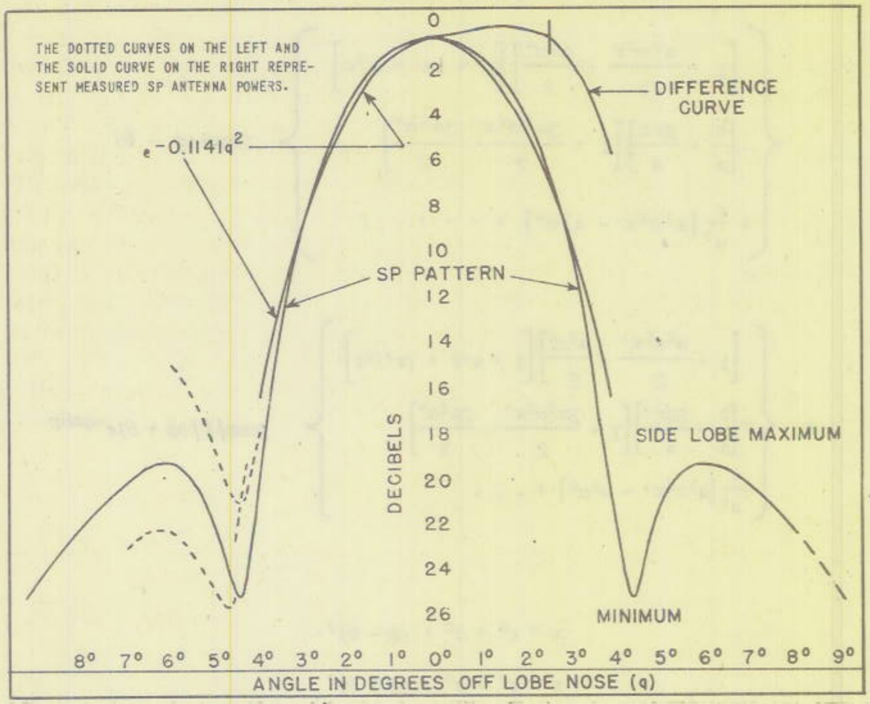

Fig. 3 - Antenna Pattern of the SP Radar, Made Symmetric

of the antenna pattern at half-power points. A most striking feature of the plot is the swiftness of the narrowing down of the crossover zone to the true position, $\theta = \phi$, once the elevation angles exceed $1/\sqrt{2a}$. This is due to the fact that above this elevation angle the reflected ray is very weak.

Figure 4 shows the influence of the absolute value of the reflection coefficient on the extent of the crossover zones for small elevation angles. Figures 5, 6, 7 and 8 show theoretical and experimental values of the reflection coefficient of relatively smooth sea water for varying reflection angles for vertically and horizontally polarized 10-cm and 3.2-cm radiation. These four plates are reproduced from Radiation Laboratory Report 568: "Further Measurements of 3- and 10-cm Reflection Coefficients of Sea Water at Small Grazing Angles," 17 May 1944. The dots are the measured points and the solid lines are the theoretical reflection coefficients. The ρ of these figures is identical with $|R|$ of this report and the abscissas "Grazing Angle, Degrees" are the same as θ' here.

The features in these curves of greatest interest for this investigation are the scatter of the experimental points and the relatively low reflection coefficient for vertical polarization. The scatter seems to be a feature of the reflection and not of the measuring technique, as the same general dispersion of experimental points was found in independent measurements taken on 700 megacycles per second by members of the Wave Propagation Section of NRL. The result of the scatter is to reduce the effective reflection coefficient to about 0.8 for horizontal polarization on 10- and 3-cm radiation whereas the theoretical curve is an adequate fit to the points for vertical polarization on these frequencies.

The effect of using the theoretical curve of Figure 5 to specify the reflection coefficient determining the boundaries of the crossover zone is illustrated by the dotted curves labeled "SP Radar" in Figure 4. For a given missile elevation angle θ, the "grazing angle" of the reflected ray is also θ, by virtue of the assumption $\theta = \theta'$. Therefore, for a given θ, the dotted curve on Figure 4 crosses the solid contour whose $|R|$ value is the absolute reflection coefficient for this θ value, as read from Figure 5. The crossover zone determined by the dotted curves, then, is the zone throughout which a low-flying missile may find control lines, if it is guided over sea water by the present SP system with vertical polarization. With horizontal polarization this zone is determined approximately by the $|R| = 0.8$ contours on Figure 4. Since the dotted curves narrow down to the line $\theta = \phi$ (presumed the correct position for the missile) faster than the $|R| = 0.8$ contours do, the conclusion is immediate that, with the SP radar, vertical polarization is preferable to horizontal for the purpose of keeping the crossover zones narrow.

A choppy sea surface will reduce the reflection coefficients on both vertical and horizontal polarizations, but quantitative data on this effect are lacking.

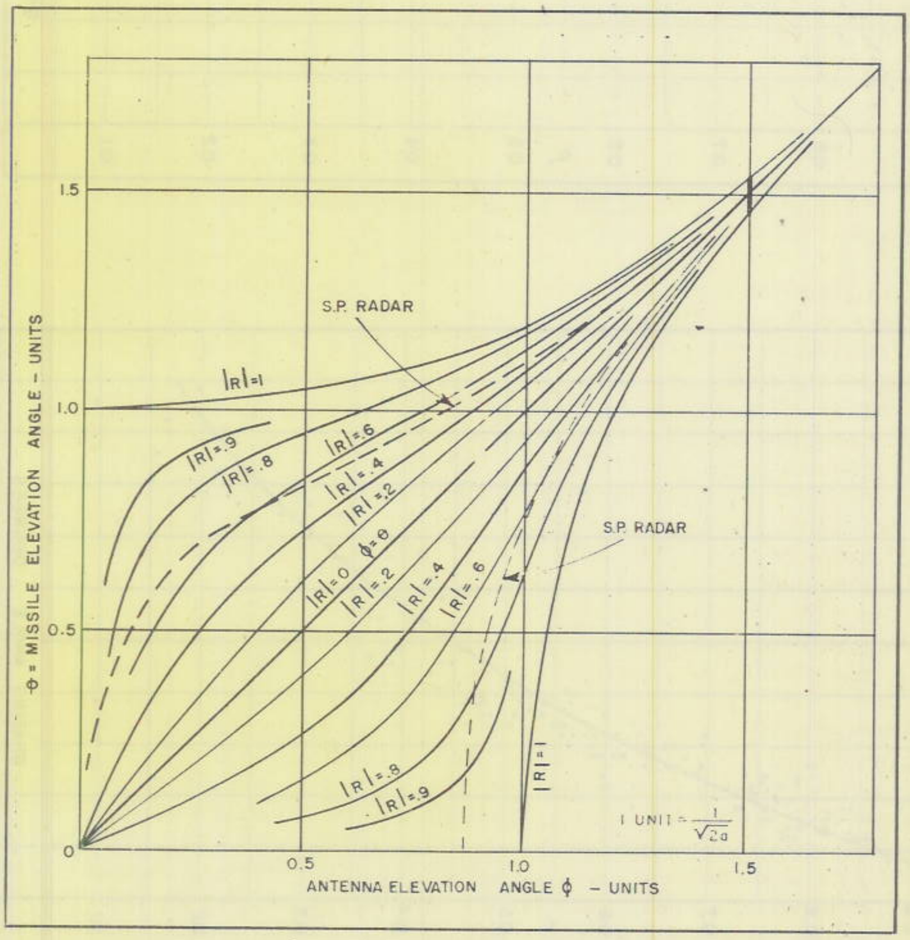

Fig. 4 - Crossover Zone Boundaries
as Function of Reflection Coefficient and Antenna Elevation Angle

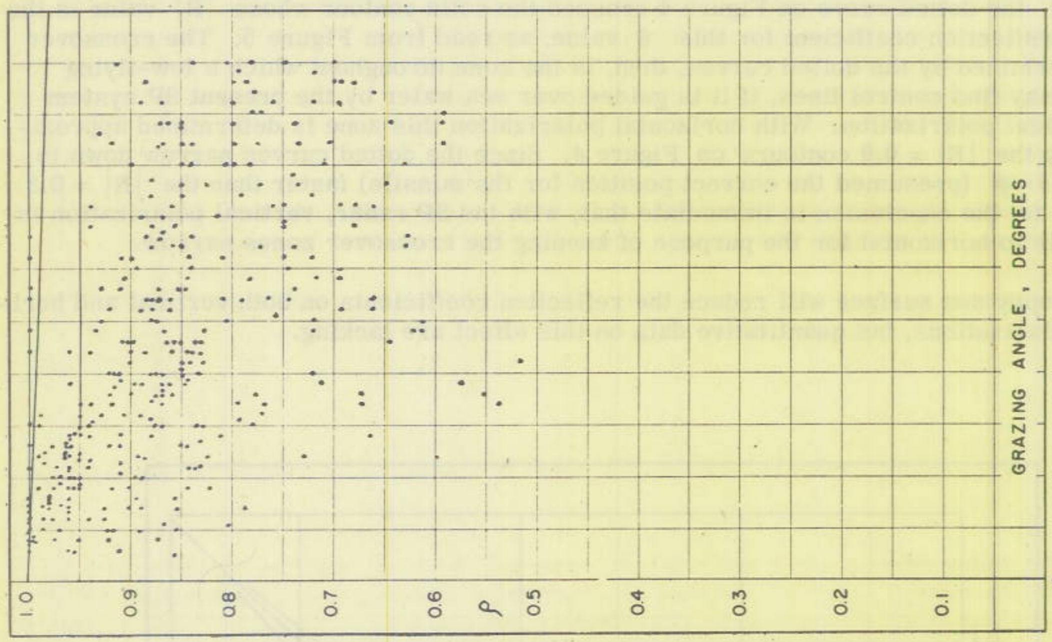

Fig. 6 - Sea Water Reflection Coefficient vs Grazing Angle at 10 cm - Horizontal Polarization

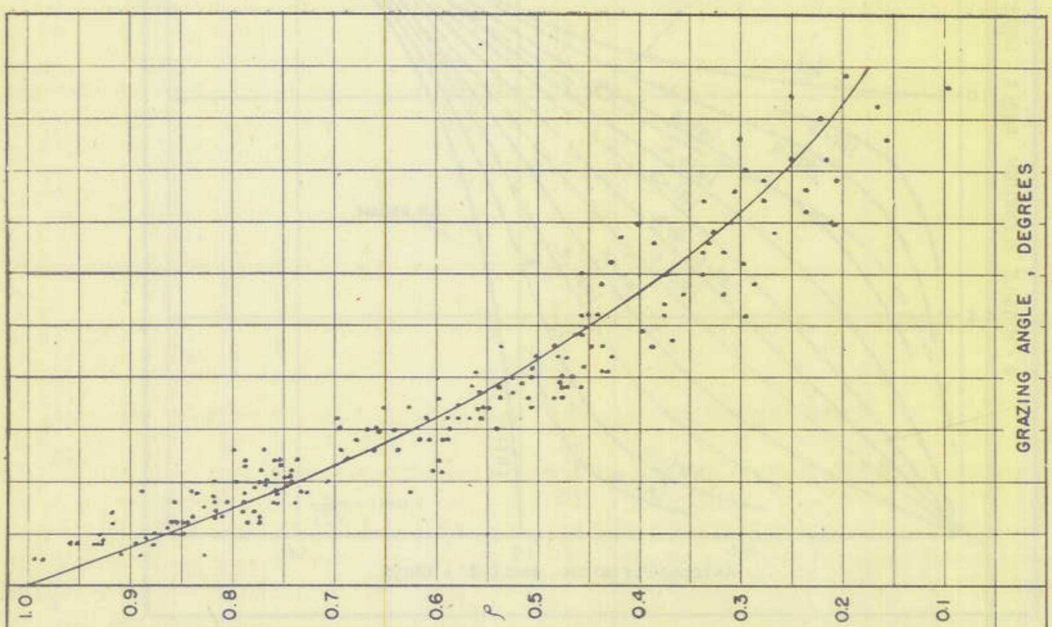

Fig. 5 - Sea Water Reflection Coefficient vs Grazing Angle at 10 cm -- Vertical Polarization

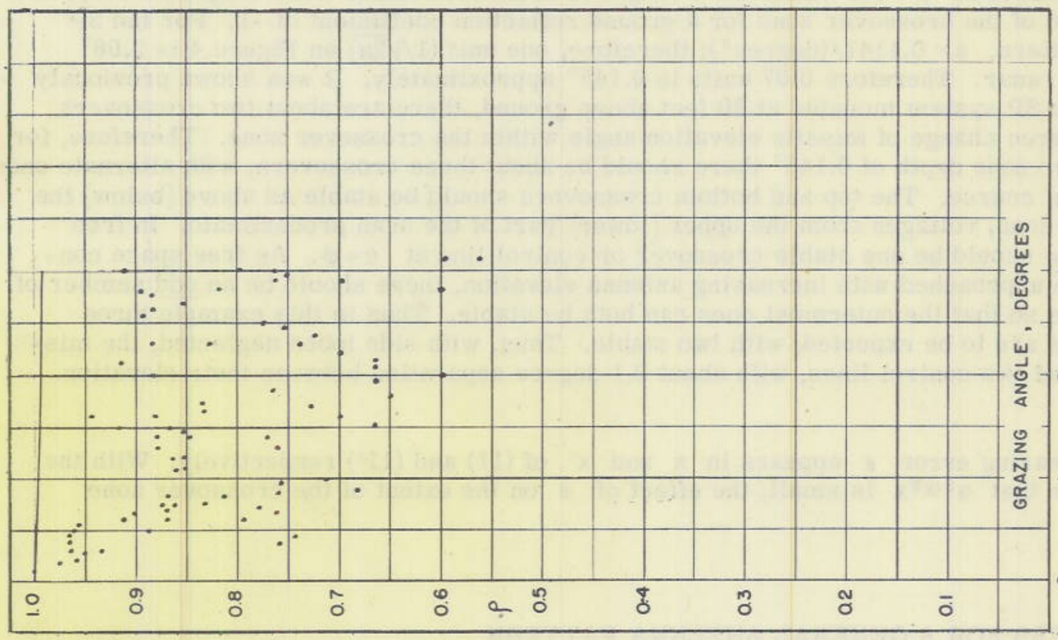

Fig. 8 - Sea Water Reflection Coefficient vs Grazing Angle at 3.2 cm -- Horizontal Polarization

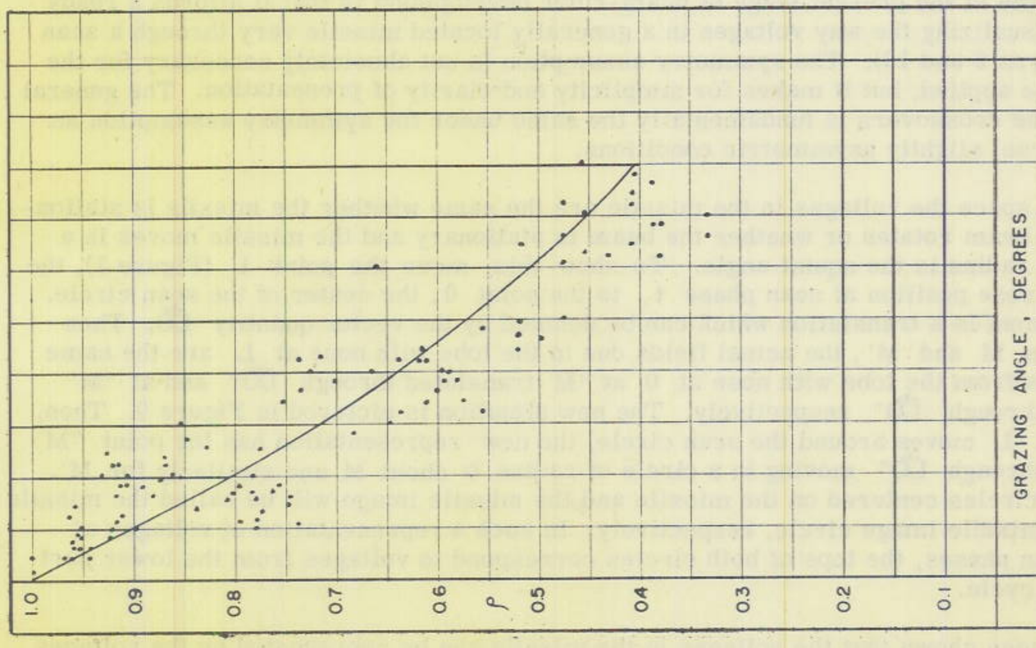

Fig. 7 - Sea Water Reflection Coefficient vs Grazing Angle at 3.2 cm -- Vertical Polarization

As an illustration, on Figure 4 the heavy vertical line joining the $|R| = 1$ crossover zone boundaries at $\phi = 1.5$ units has a length of about 0.07 units. This length corresponds to the depth of the crossover zone for a ground reflection coefficient of -1. For the SP antenna pattern, $a = 0.1141/(\text{degree}^2)$; therefore, one unit $(1/\sqrt{2a})$ on Figure 4 is 2.08° for the SP radar. Therefore 0.07 units is 0.145° approximately. It was shown previously that for the SP system mounted at 90 feet above ground, there are about two crossovers per 0.1 degree change of missile elevation angle within the crossover zone. Therefore, for a crossover zone depth of 0.145° there should be about three crossovers, with alternate ones unstable, of course. The top and bottom crossovers should be stable as above [below] the crossover zone, voltages from the upper [lower] part of the scan predominate. In free space there should be one stable crossover or control line at $\theta = \phi$. As free space conditions are approached with increasing antenna elevation, there should be an odd number of crossovers so that the outermost ones can both be stable. Thus in this example three crossovers are to be expected, with two stable. Thus, with side lobes neglected, the missile can find two control lines, with about 0.1 degree separation between their elevation angles.

The bearing error ϵ appears in x and x' of (11) and (11') respectively. With the assumption that $a^2\alpha^2x$ is small, the effect of ϵ on the extent of the crossover zone vanishes.

CROSSOVERS FOR A GENERAL ANTENNA PATTERN

In this section a method is developed for determining the extent of the crossover zone for a general geometric relation of missile, reflecting surface, and radar antenna, with the sole restriction that the antenna pattern be symmetric. Probably the principal utility of this scheme at the present stage of beam-rider development is that it affords a ready means of visualizing the way voltages in a generally located missile vary through a scan cycle (Figures 9 and 10). The symmetry assumption is not absolutely necessary for the method to be applied, but it makes for simplicity and clarity of presentation. The general pattern of the crossovers is fundamentally the same under the symmetry assumption as under the true, slightly asymmetric conditions.

In free space the voltages in the missile are the same whether the missile is stationary and the beam rotates or whether the beam is stationary and the missile moves in a cone whose radius is the squint angle. To show this, move the point L (Figure 1), the actual lobe nose position at scan phase t, to the point 0, the center of the scan circle. This movement is a translation which can be denoted by the vector quantity $\overrightarrow{LO}$. Then at the points M and M', the actual fields due to the lobe with nose at L are the same as the fields from the lobe with nose at 0 at "M translated through $\overrightarrow{LO}$" and at "M' translated through $\overrightarrow{LO}$" respectively. The new situation is pictured in Figure 9. Then, as the point L moves around the scan circle, the new representation has the point "M translated through $\overrightarrow{LO}$" moving in a circle of radius α about M and similarly for M'. These new circles centered on the missile and the missile image will be called the missile circle and missile image circle, respectively. In such a representation of voltages at various scan phases, the tops of both circles correspond to voltages from the lower part of the scan cycle.

It has been shown that the voltages in the missile can be represented by the voltages in a circle of radius, squint angle α, centered on the missile position. The latter voltages are due to a radar antenna pattern with main lobe nose fixed at the center of the scan circle (neglecting the reflected rays). The integration of (6) can be performed by a graphical method. Draw equivoltage contours of the antenna pattern (for fixed range) with nose of the main lobe at the center of the scan circle. These equivoltage contours will be circles centered on the scan circle center 0, because of the postulated symmetry of the lobe pattern. Then the net voltage effective in giving the missile an "up" or "down" order is determined by the contours intersected by the missile circle. Divide the missile circle into a number of equal arcs and perform the integration (6) approximately by adding an increment to the effective voltage for each arc. This increment is determined by multiplying the cosine of the angular displacement of the arc from the bottom of the circle by the approximate mean value of the absolute voltages intercepted by the arc. If the resulting sum bears a plus [minus] sign the resulting order to the missile is "go down" ["go up"] because the plus [minus] excess represents an excess of signal during the upper [lower] part of the scan cycle. If the sum comes out zero, the missile is on a crossover.

When both direct and reflected rays are important, the absolute values of the fields at the missile still enter the integrand in (6) so that the relative phase of the field at different angles from the beam nose must be considered, as well as the relative amplitudes. Hence when the reflected ray leaves the antenna on a side lobe, the phase of the field in the side lobe relative to that in the main lobe must be known. When the minima in the lobe pattern are deep, the fields in the first side lobe are approximately 180° out of phase with the fields in the main lobe (at a fixed range from the antenna). But a non-zero minimum means that, although the voltage in the nose of the side lobe is about 180° out of phase with the voltage in the main lobe, near the minimum of the field between the lobes, the phase varies continuously between 0° and 180°. The deeper the minimum is, the quicker the phase shift near the minimum will be as one passes from main lobe into side lobe.

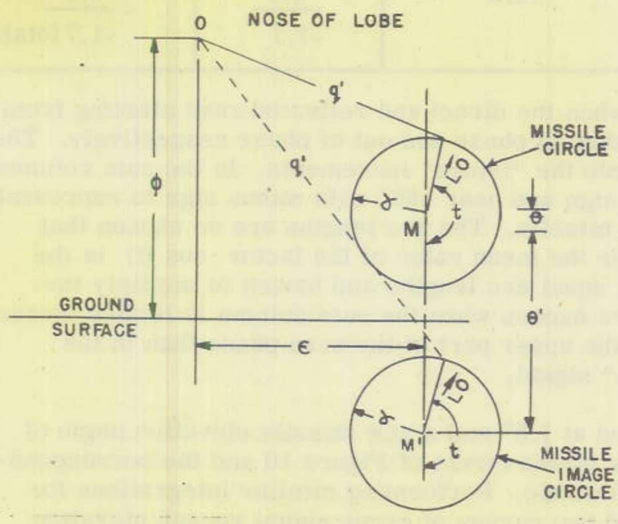

Fig. 9 - Diagram for Obtaining Voltages in Missile from Radar Pulse at Scan Phase t When Lobe Nose Is Considered Fixed Throughout Scan Cycle

The swiftness of this phase shift might be determined through the minimum between main and side lobes by proper choice of the complex parameters a, b, c, — of (1), so that the absolute value of the field as given by the representation fits that determined by actual antenna measurements. This would not simplify the integration of (6), however, as near the side lobes the integrand is far from analytic. Furthermore, the actual asymmetry of the antenna near the side lobes makes the symmetric representation (1) a poor fit there. The dotted curves on the left of Figure 3 show this asymmetry. Therefore in the following paragraphs the pattern will be treated as if the voltage underwent a 180° phase shift exactly at the minimum. However, the assumption of symmetrical pattern does not fundamentally change the behavior of the voltages at the missile; it only displaces slightly the calculated positions of crossovers.

As an example of the type of integration possible, the characteristics of the actual SP antenna for squint angle and for field contours will be used. In the example illustrated in Figure 10, the antenna elevation angle ϕ is 1.5°, and the solid missile circle is centered at $\theta = 2.5°$ and $\epsilon = 4.5°$. θ' is equal to θ here so that the center of the solid missile circle is at 2.5° + 1.5° = 4° below the center of the scan circle, 0. The radii of the missile circles are 0.63°, the SP squint angle. The contours of the lobe pattern given as circles about 0 are determined by the SP antenna pattern measurements presented in Figure 3. In the lower right corner of Figure 10 is part of a relative voltage vs angle-from-lobe-nose plot of the SP pattern. The minimum is shallow, so that the phase shift across the minimum is slow. This slowness is neglected in the illustration.

For the integration of (8) the following table of increments is formed:

Angle from bottom of circle	Increment from missile circle	Increment from missile image circle	Sum in phase	Sum out of phase
0° - 30°	-1.2	-1.9	3.1	0.7
30° - 90°	-1.5	-1.9	3.4	0.4
90° - 150°	-1.7	-1.95	-3.65	-0.25
150° - 180°	-1.5	-1.95	-3.45	-0.45
180° - 210°	-1.1	-1.9	-3.0	-0.8
210° - 270°	1.2	-1.9	-0.7	-3.1
270° - 330°	1.3	-2	0.7	3.3
330° - 360°	0.0	-1.9	1.9	1.9
			-1.7	+1.7 totals

The two sum columns represent the fields when the direct and reflected rays starting from the antenna in the main lobe are added together in phase and out of phase respectively. The reflection coefficient has been multiplied into the "image" increments. In the sum columns, increments near 0° are added in with plus sign and near 180° with minus sign to represent their relative effect as error signals to the missile. The arc lengths are so chosen that the length of the arc compensates exactly for the mean value of the factor cos (t) in the arc; this is an easier procedure than taking equal arc lengths and having to multiply the factors cos (t) in later. Thus a net positive excess when the sum column is totaled means that the missile received greater signal in the upper part of the scan phase than in the lower, and therefore it receives a "go down" signal.

If the antenna elevation angle ϕ is fixed at 1.5° and a new missile elevation angle of 4° is chosen, the missile circle is the upper dotted circle of Figure 10 and the corresponding missile image circle is the lower dotted circle. Performing similar integrations for various missile elevation angles θ will yield two curves of error signal versus elevation angle θ, for fixed ϵ and fixed antenna elevation angle ϕ. One curve is based on the assumption that the direct and reflected rays from the main lobe are totally in phase, and the other on the assumption that they are totally out of phase. The boundaries of the crossover zone will then be the zeros of these error-signal curves. The missile of the example lies in the crossover zone, as is apparent from the fact that the direction of its orders depends on the relative phase of direct and reflected fields at the missile. Thus, since the relative phase is a continuous function of θ, there will be a crossover (where the sum column totals zero) somewhere between the θ value giving a "go up" order and one giving a "go down" order. This relative phase is the same as that determined by the quantity $(f-g+\beta)$ of equation (7).

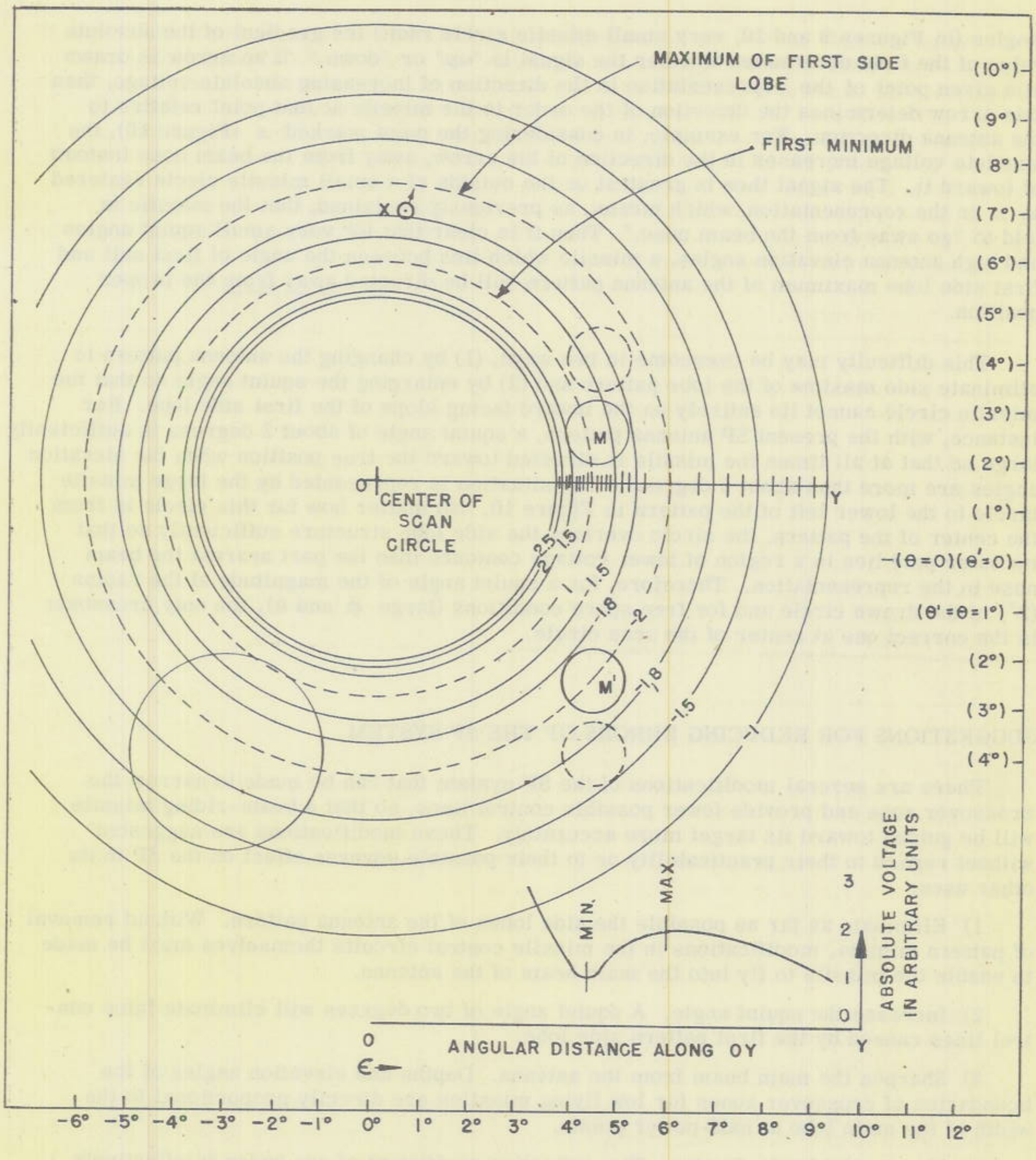

Fig. 10 - Sample of Graphical Integration of Error Voltages in Missile Riding SP Antenna Beam

There are a number of deductions which can be made from inspection of Figure 10. In the first place, when the elevation angles ϕ and θ are great, the missile image circle is so far away from the beam nose that voltages on this circle are negligibly small and free space conditions may be assumed. Secondly, in the case of very small squint angles (in Figures 9 and 10, very small missile circle radii) the gradient of the absolute value of the field determines whether the signal is "up" or "down." If an arrow is drawn at a given point of the representation in the direction of increasing absolute voltage, then this arrow determines the direction of the order to the missile at that point relative to the antenna direction. For example, in considering the point marked x (Figure 10), the absolute voltage increases in the direction of the arrow, away from the beam nose instead of toward it. The signal then is greatest on the outside of a small missile circle centered at x in the representation, which means, as previously explained, that the missile is told to "go away from the beam nose." Thus it is clear that for very small squint angles and high antenna elevation angles, a missile which lies between the angle of first null and first side lobe maximum of the antenna pattern will be directed away from the proper position.

This difficulty may be overcome in two ways, (1) by changing the antenna pattern to eliminate side maxima of the lobe pattern and (2) by enlarging the squint angle so that the missile circle cannot lie entirely on the inward facing slope of the first side lobe. For instance, with the present SP antenna pattern, a squint angle of about 2 degrees is sufficiently large so that at all times the missile is directed toward the true position when the elevation angles are more than about 5 degrees. The situation is represented by the large missile circle to the lower left of the pattern in Figure 10. No matter how far this circle is from the center of the pattern, the circle overlaps the side lobe structure sufficiently so that its outer part lies in a region of lower voltage contours than the part nearest the beam nose in the representation. Therefore, for a squint angle of the magnitude of the radius (2°) of the drawn circle and for free space conditions (large ϕ and θ), the only crossover is the correct one at center of the scan circle.

SUGGESTIONS FOR REDUCING ERRORS OF THE SP SYSTEM

There are several modifications of the SP system that can be made to narrow the crossover zone and provide fewer possible control lines, so that a beam-riding missile will be guided toward its target more accurately. These modifications are suggested without regard to their practicability or to their possible adverse effect on the SP in its other uses:

1) Eliminate as far as possible the side lobes of the antenna pattern. Without removal of pattern minima, modifications in the missile control circuits themselves must be made to enable the missile to fly into the main beam of the antenna.

2) Increase the squint angle. A squint angle of two degrees will eliminate false control lines caused by the first pattern side lobe.

3) Sharpen the main beam from the antenna. Depths and elevation angles of the boundaries of crossover zones for low flying missiles are directly proportional to the width of the main lobe at half-power points.

4) Use vertical polarization. The reflection coefficient of sea water is effectively less for vertical than horizontal polarization in the most important range of reflection angles, and reduction of field strength due to the reflected ray minimizes the adverse effect of sea reflections.

5) The use of different squint angles on successive scan cycles might make possible effective "fill in" of the minimum between main and first side lobe.

APPENDIX

1. ERROR INTRODUCED BY THE ASSUMPTION OF A FLAT EARTH

The error introduced by this assumption is negligible when the missile elevation angle is sufficiently high to prevent formation of multiple control lines. The flat earth assumption merely shifts effective antenna and missile heights slightly and reduces negligibly the reflection coefficient of the ground through "spreading" the reflected rays when they bounce off the earth's convex surface. Full discussion is found in Federal Communications Commission Report No. 39920.

2. ERROR INTRODUCED BY THE ASSUMPTION $\theta = \theta'$

With flat earth, $\theta = \tan^{-1}(h_2 - h_1)/d$ and $\theta' = \tan^{-1}(h_2 + h_1)/d$ exactly. Let $(h_2 + h_1)/d = 0.1$. Expanding $\tan^{-1}(0.1)$ in series we have $\theta' = 0.1 - (.001/3) + \ldots$ or $\theta' = 0.1$ radians with an error of 0.3 percent. The error is smaller with smaller θ'; thus for angles less than about 6°, the first term of the arc tangent series may be used with an accuracy within 0.3 percent. Therefore $\theta = (h_2 - h_1)/d$ and $\theta' = (h_2 + h_1)/d$. to within 0.3 percent. In the limit of great ranges with the ratio h_2/d fixed, $h_1/d = 0$ and $\theta = \theta'$ exactly in the limit. When h_1/d is finite, the error $(\theta' - \theta) = 2h_1/d$. For antenna height 90 feet and horizontal missile range $d = 6000$ yards, $2h_1/d = 0.01$ radian or 0.57°. This means that for this height and missile range the "missile image circle" should be shifted downward .57° before the numerical integration is begun. A shift this small does not essentially alter the position of crossovers. It merely means that a missile following a crossover out from range 6000 yards will change in elevation angle not more than .57°, but the numbers and general pattern of the crossovers are not changed.

3. THE EFFECT OF NEGLECTING $a^2\alpha^2x$ IN EQUATIONS (11) and (11')

With the SP antenna, $a = 0.1141/(\text{degree}^2) = 0.63$ degrees, and x must be held less than 2.5° or otherwise either the direct or reflected ray lies in a side lobe, which was forbidden in the derivation of (11') by the condition (9). The quantity $a^2\alpha^2x = .0324$ for $x = 2.5°$.

The quantity .0324 can be considered small with respect to 1 as the error introduced by this assumption is probably of smaller order of magnitude than that introduced by assuming a symmetrical main lobe.

The assumption $a^2\alpha^2x \ll 1$ is equivalent to assuming a very small squint angle. In fact equation (12) is an immediate consequence of this and taking the lobe pattern as ϵ^{-aq^2}.

The assumption of negligibly small squint angle is equivalent to the expression $|E(q) \pm RE(q')|$. of (8) being taken as closely representable by a linear function of ϵ and ϕ throughout the region of integration (which is a circle of radius α in the ϕ, ϵ plane). For a control line this expression will be constant, making the result of the integration zero. Echo voltages in the radar are proportional to the squares of field voltages, so if the field voltages are essentially constant throughout the scan cycle, so will be the echo voltages. Thus, as stated in the first section of this report, when the squint angle is small, the radar system will act as if it were "on target." There will be slight discrepancies between such "on target" positions and control lines when the squint angle is enlarged.

4. ERROR INTRODUCED BY THE ASSUMPTION $b = 0$ and $c = 0$ IN EQUATIONS (11) AND (11')

Compare in Figure 3 the parabola $\epsilon^{-0.1141q^2}$ to the actual shape of the main lobe. The difference between the two curves on the decibel versus angle plot is twice the logarithm of the factor $1 + bq^2 + cq^4$ of (1), which was assumed equal to 1 for the parabola. The difference curve is plotted at the top of the graph. The difference is zero at the apex of the parabola, $q = 0$, and at $q = 2.5°$. So $1 + bq^2 + cq^4 = 1$ at $q = 0$ and $q = 2.5°$. Thus $bq^2 + cq^4 = 0$ at $q = 2.5°$ or $b/c = -6.25$. Now $1 + bq^2 + cq^4$ is maximum at $2qb + 4q^3c = 0$ or at $q^2 = -b/2c$.

Examination of the difference curve shows that the factor makes a difference in voltage of about 0.2 db (or 1.046 times) at maximum; that is $[1 + b(b/2c) + c(b^2/4c^2)]^2 = 1.046 = (1.023)^2$. Using this and $b/c = -6.25$ to determine b and c, there follows: $b = .0064$ and $c = -.001$. In equation (1) a is 0.1141. Therefore $b/a < 0.1$, and neglecting b and c makes little difference in (11) and (11').

* * *

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