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THREE DIMENSIONAL PPI DISPLAYS BY STEREOSCOPIC METHODS



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THREE DIMENSIONAL PPI DISPLAYS BY STEREOSCOPIC METHODS

by

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June 1947

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ABSTRACT

Presentation of radar data in the conventional two-dimensional display does not fully utilize all of the information available from the radar equipment. In order to simultaneously employ range, azimuth and altitude data, all of which are readily obtainable, an indicator having three-dimensional presentation has been developed.

A brief resume of background theory in stereoscopy, together with an analysis of the binocular problem involved, is also presented herewith.

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THREE DIMENSIONAL PPI DISPLAYS BY STEREOSCOPIC METHODS

INTRODUCTION

The usual visual presentation of radar data upon a cathode ray tube screen does not take full advantage of all of the information available from the radar. When the common PPI is used, only details of range and bearing are utilized; when an RHI scope is viewed, only the range and height picture is presented upon the face of the tube. Similarly, only two of the three possible dimensions concerning the position of a target in space are presented simultaneously upon any one indicator scope at one time with any of the present equipment.

Furthermore, target range as presented on the customary PPI with its two-dimensional display of data is true range only when the target is in the same X-Y plane as the antenna. At all other times ranges measured on the PPI are values of slant range and vary from ground range as a function of target altitude, i.e., $\text{Ground Range} = R \cos \phi$. Hence, several targets with equal slant range may appear on the common PPI as having equal horizontal, or ground range, when in reality they do not. (Figure 1.)

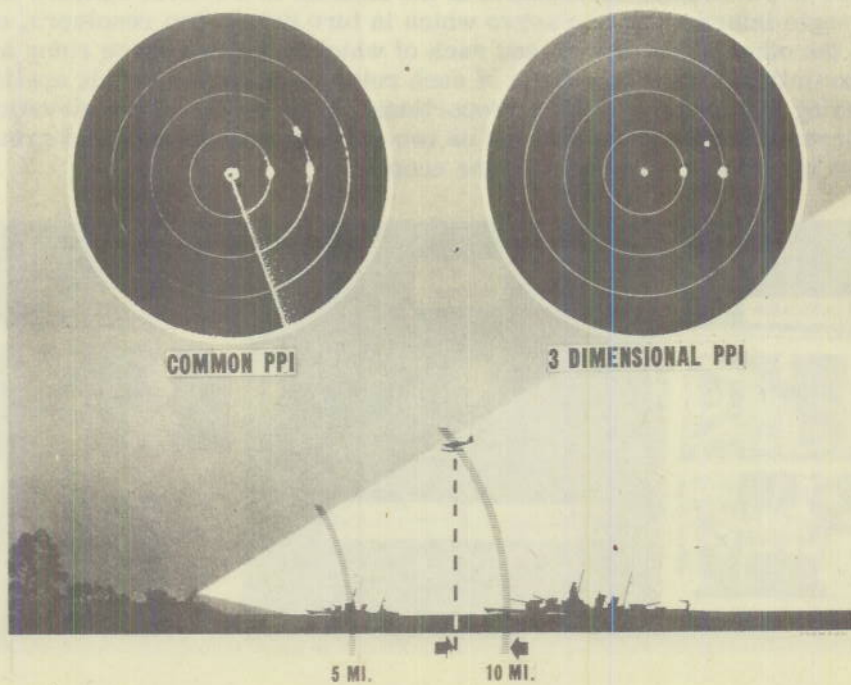


Fig. 1 - PPI and 3 Dimensional PPI Drawing

In order to take full advantage of all of the radar information, an indicator must be conceived which presents three-dimensional data locating a point in space, and does so in a simple and convenient manner. In short, we must give to the PPI a third sense-- that of depth perception, if we are to gain full utilization of our radar equipment. Herein we shall discuss a method of achieving this result, employing two PPI's in an adaptation of Wheatstone's theory of negative stereoscopic parallax, wherein the scope faces and patterns are viewed by reflection in two mirrors in which the two pictures are fused. (Figure 2 and Figure 3.)

PROCEDURE

For test purposes an SU radar set using a 30 inch dish with a 3° beam provides the radar information to operate the indicator. In addition to scanning at a variable speed up to six r.p.m., the antenna is made to nod upward from the horizontal axis at approximately 120 cycles per minute. Therefore, if the spinner is revolving at two r.p.m., it will simultaneously be nodding 60 times per revolution, or once per six degrees azimuth. From this solid figure being evolved by the radar beam, we obtain a three-dimensional view of the space swept by the beam. (Figure 4.)

The shape of the figure described by the radar beam is the complementary figure to an inverted cone whose base diameter is equal to the range of the beam and whose height is equal to the maximum altitude covered by the beam. The ideal scanner for this use, of course, would be of the Foster type where there is a high scanning rate per degree azimuth. However, the dish described above gives adequate coverage for the purposes of this test. (Figure 5 and 6.)

Geared to the nodding mechanism of the antenna is a 1-C.T. synchro which supplies elevation angle information to a servo which in turn drives two resolvers, one at one speed and the other at two speed, and each of which has a two phase rotor and two phase stator. Excitation of only one phase of each rotor is employed in this application. The stator winding furnishing a voltage proportional to the cosine of the elevation angle is used to correct the rate of the sweeps on two cathode ray tubes so that ground range is observed at all times on the faces of the scopes.

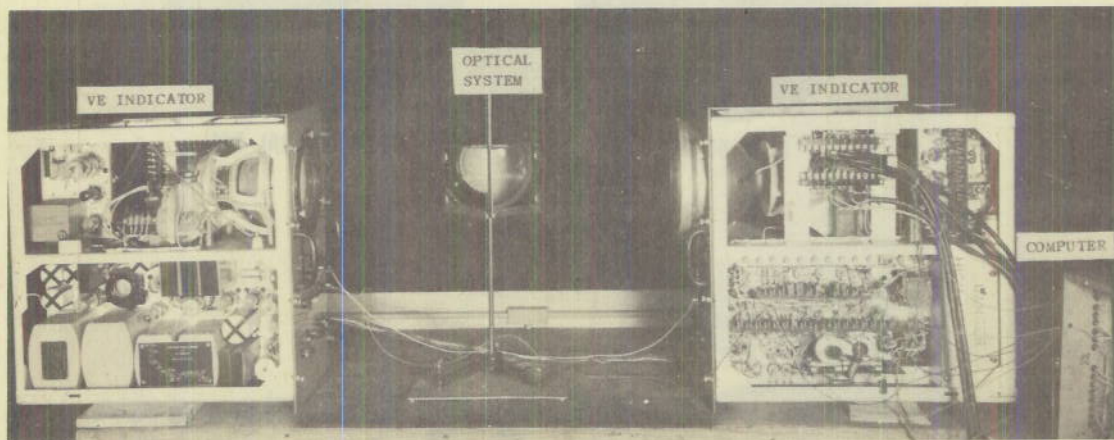


Fig. 2 - Indicators and Optical System

Fig. 3 - Schematic of Optical System

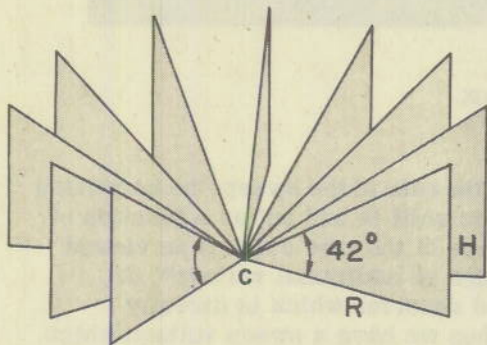
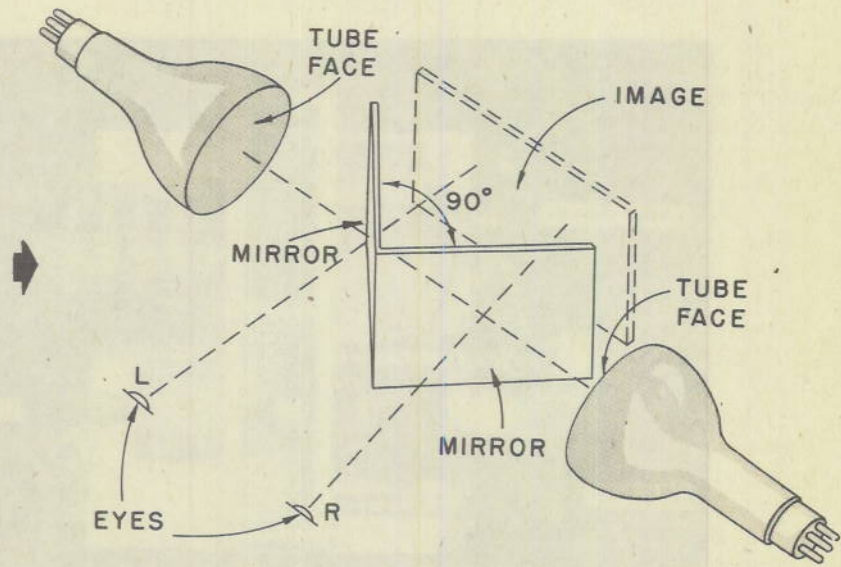
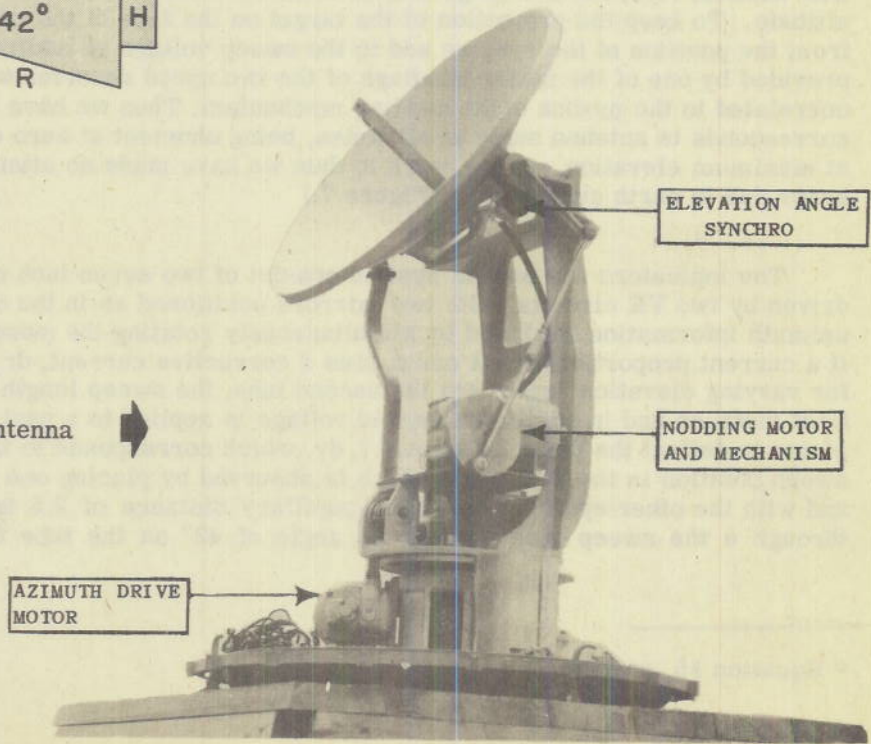


Fig. 4 - Antenna Pattern

Fig. 5 - Antenna



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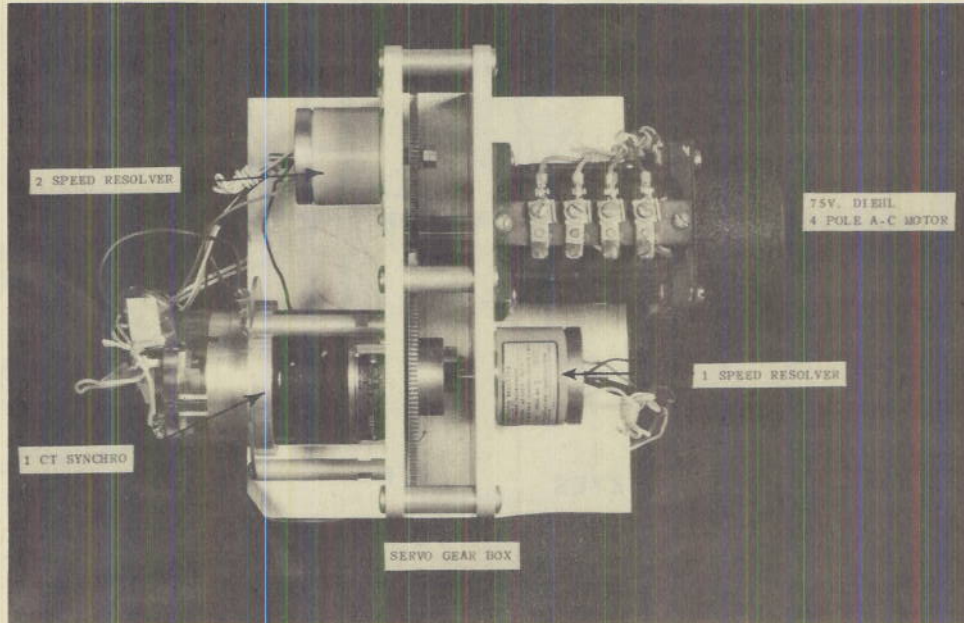


Fig 6 - Servo Gear Box

To present a stereoscopic picture, not only must the rate of the sweeps be corrected with antenna rise, but the range marks must be made to shift in and out as a function of altitude. To keep the projection of the target on the face of the tube correct as viewed from the position of the eye, we add to the sweep voltage an additional voltage*, dr , provided by one of the stator windings of the two speed resolver which is directly correlated to the motion of the nodding mechanism. Thus we have a sweep voltage which corresponds to antenna angle at all times, being shortest at zero elevation and longest at maximum elevation. In our work to date we have made no attempt to compensate for error due to earth curvature. (Figure 7.)

The indicators and optical system consist of two seven inch cathode ray tubes driven by two VE circuits, with two mirrors positioned as in the diagram. On one tube, azimuth information is placed by simultaneously rotating the sweep coil and applying to it a current proportional to $R \cos \phi$ plus a corrective current, dr , to correct the sweep for varying elevation angle. On the second tube, the sweep length is corrected as in the first instance and in addition a second voltage is applied to a part of horizontal deflection plates to deflect the beam an amount \dagger , dy , which corresponds to the apparent shift in sweep position in the X-Y plane which is observed by placing one eye e' , at the origin and with the other eye e at the inter-pupillary distance of 2.5 inches observing through e the sweep rise through an angle of 42° on the tube face. (Figure 8)

* Equation #5, page 6

† Equation #6

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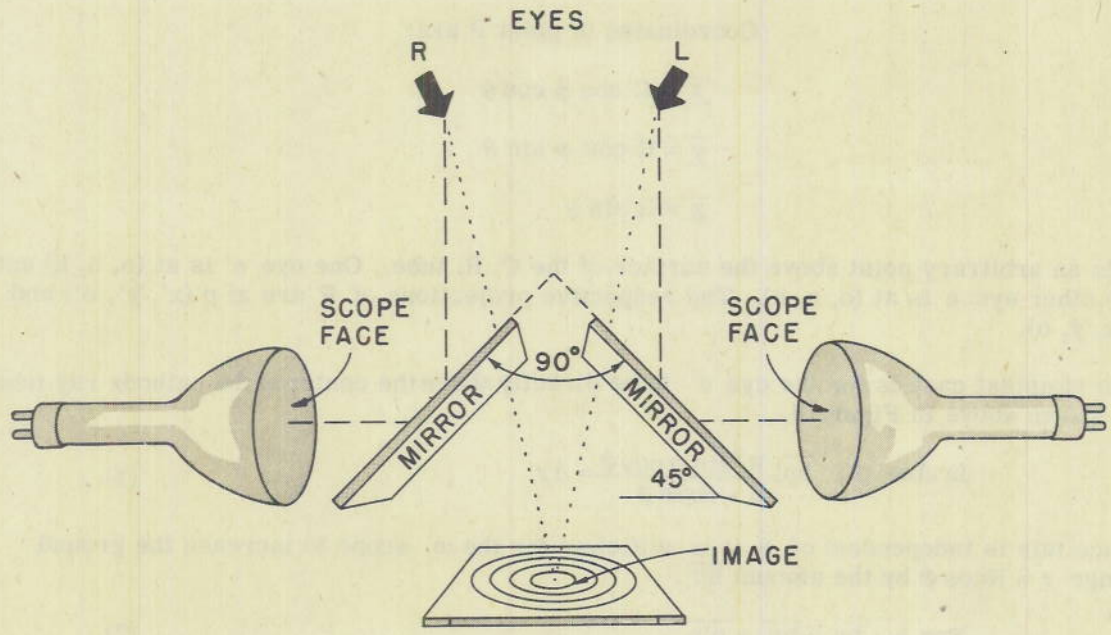


Fig. 7 - Sketch of Optical System

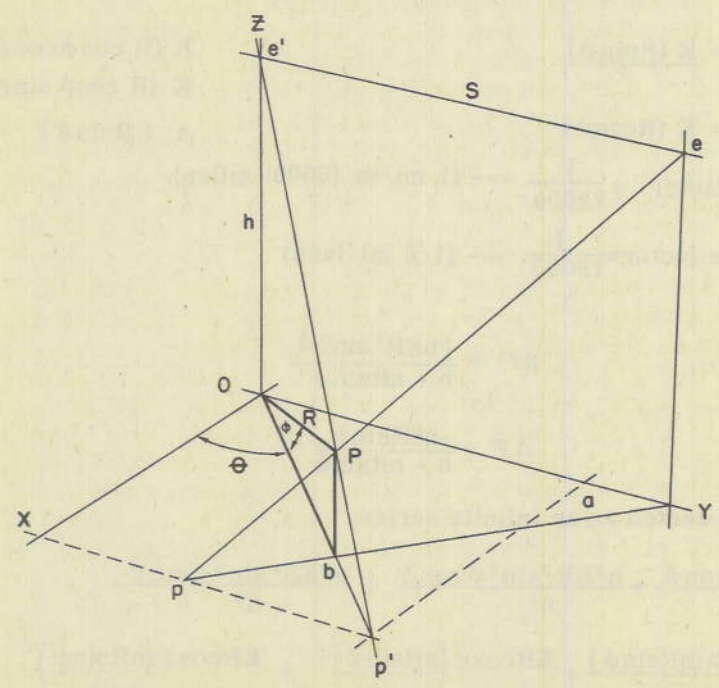


Fig. 8 - Illustration of Point P (3 dim)

Coordinates of point P are:

$$\bar{x} = R \cos \phi \cos \theta$$

$$\bar{y} = R \cos \phi \sin \theta$$

$$\bar{z} = R \sin \theta$$

P is an arbitrary point above the surface of the C. R. tube. One eye e' is at (o, o, h) and the other eye e is at (o, s, h) . The respective projections of P are at $p'(x', y', o')$ and $p(x, y, o)$.

The simplest case is for the eye e' to be directly above the center of the cathode ray tube as shown above in Figure 8.

$$\text{In } \Delta oe'p', \quad \bar{bp}' = \frac{R^2 \sin \phi \cos \phi}{h - R \sin \phi} = \Delta r' \quad (1)$$

Since this is independent of θ it is sufficient for the e' scope to increase the ground range $r = R \cos \phi$ by the amount \bar{bp}' .

$$\text{For } e \quad \bar{bp} = \bar{bp}' + \bar{p}'p \quad (2)$$

$$\text{as before } \bar{bp}' = \Delta r' = \left(\frac{\frac{1}{2} R^2 \sin 2\phi}{h - R \sin \phi} \right) \quad (3)$$

$$\bar{p}'p = \left(\frac{S R \sin \phi}{h - R \sin \phi} \right) = \Delta y' \quad (4)$$

$$\bar{pb} = K (R \sin \phi)$$

$$K (R \cos \phi \cos \theta) = \bar{x}$$

$$K (R \cos \phi \sin \theta) = \bar{y}$$

$$\bar{ob} = K (R \cos \phi)$$

$$n (R \sin \theta) = \bar{z}$$

$$K = \text{horiz. scale factor} = \frac{1}{72000} \quad \text{--- (1 mi/in (6000' miles))}$$

$$n = \text{vertical scale factor} = \frac{1}{18000} \quad \text{--- (1/4 mi/inch)}$$

$$\Delta r' = \frac{\frac{1}{2} n K R^2 \sin 2\phi}{h - n R \sin \phi} \quad (5)$$

$$\Delta y' = \frac{-n S R \sin \phi}{h - n R \sin \phi} \quad (6)$$

$\Delta r'$ may be represented as an infinite series.

$$\Delta r' = \frac{n K R^2 \sin \phi \cos \phi}{h} + \frac{n^2 K R^3 \sin^2 \phi \cos \phi}{h^2} + \frac{n^3 K R^4 \sin^3 \phi \cos \phi}{h^3}$$

$$\text{or } \Delta r' = \frac{K R \cos \phi (n R \sin \phi)}{h} + K R \cos \phi \left(\frac{n R \sin \phi}{h} \right)^2 + K R \cos \phi \left(\frac{n R \sin \phi}{h} \right)^3$$

$$\Delta r' = K R \cos \phi (x + x^2 + x^3 \dots) \quad \text{where } X = \frac{n R \sin \phi}{h}$$

$\Delta y'$ may also be represented by an infinite series.

$$\Delta y' = S(x + x^2 + x^3 - \dots) \text{ where } X = \frac{nR \sin \phi}{h}$$

$$\Delta y' \approx S(x + ax^2 - \dots) \text{ where } a = 1.25$$

$$\Delta y' \approx S(.97 + 1.4x^2) \quad 1.25 < \frac{b}{a} < 1.5 \text{ without appreciable error.}$$

The best ratio for $\frac{b}{a} = 1.287$ (by method of least squares)

Max. error = .58% at $nR \sin \phi = 4$

From the one-speed stator winding we get a voltage corresponding to $\sin \phi$ and from the two-speed stator $\sin 2\phi$ and $\cos 2\phi$. In order to lengthen the sweep in proportion to antenna angle we add to R an additional voltage proportional to $R^2 \sin \phi \cos \phi$ which is proportional to $R^2 \sin 2\phi$. dy is obtained by generating a sweep proportional to $aR \sin \phi + bR^2 \sin^2 \phi$, utilizing the fact that $1 - \cos 2\phi$ is proportional to $\sin^2 \phi$. (Figures 9 and 10)

On the first tube we have the sweep changing from R to $R + dr$ and back to R approximately 60 times per antenna revolution. While this is occurring on tube 1, the sweep on tube 2 is acting in synchronism with that on tube 1 and in addition it is being displaced from normal by an amount dy and back to normal in a manner such that the greatest displacement occurs when the physical length of the sweep is greatest, which is when the antenna angle is largest, and is least when the sweep is shortest, which is when the antenna is in the horizontal position.

In order to correct to ground range all elements of sweep must be multiplied by $\cos \phi$. As this value of $\cos \phi$ must be obtained independently of the resolver providing $\sin \phi$, a third resolver is added to furnish this voltage, and is driven in synchronism with the first one-speed resolver.

The overall effect is as though the observer were standing at the antenna with one eye centered on the vertical axis of rotation of the dish and the other $2.5/K$ inches away, where K is the horizontal scale factor. The stereoscopic combination of the two pictures on the cathode ray tubes will reproduce exactly the same picture as observed while standing in the above position. It has been found that it is not necessary to center the nose about the vertical axis, as might be expected, and calculations from such a standpoint (nose centered) are much more difficult. Further, it has been proven in the mathematical analysis that this is not a necessary assumption and that there is no significant difference in results between centering the eye and centering the nose.

Utilizing Wheatstone's experiments with negative stereoscopic parallax, the cathode ray tubes are mounted 34 inches apart and the mirrors located as in the diagram. (Figure 11).

Light coming from scope 1 is reflected by mirror AB into eye R as though it had come from ff. Likewise, light coming from scope 2 is reflected by mirror CD into eye L as though it too had come from ff. Thus both eyes apparently see the image at ff, and the observer gets the same impression by his sense of vision that he would by looking

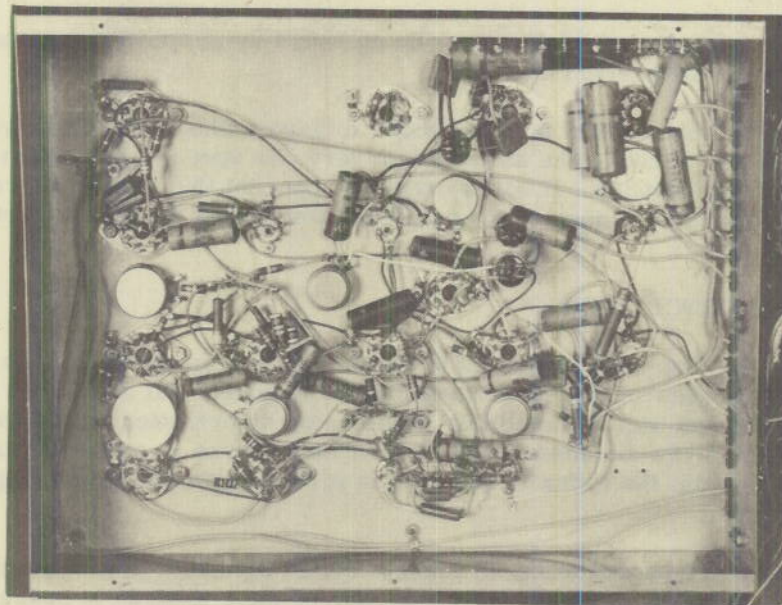


Fig. 10 - Photograph of Computer

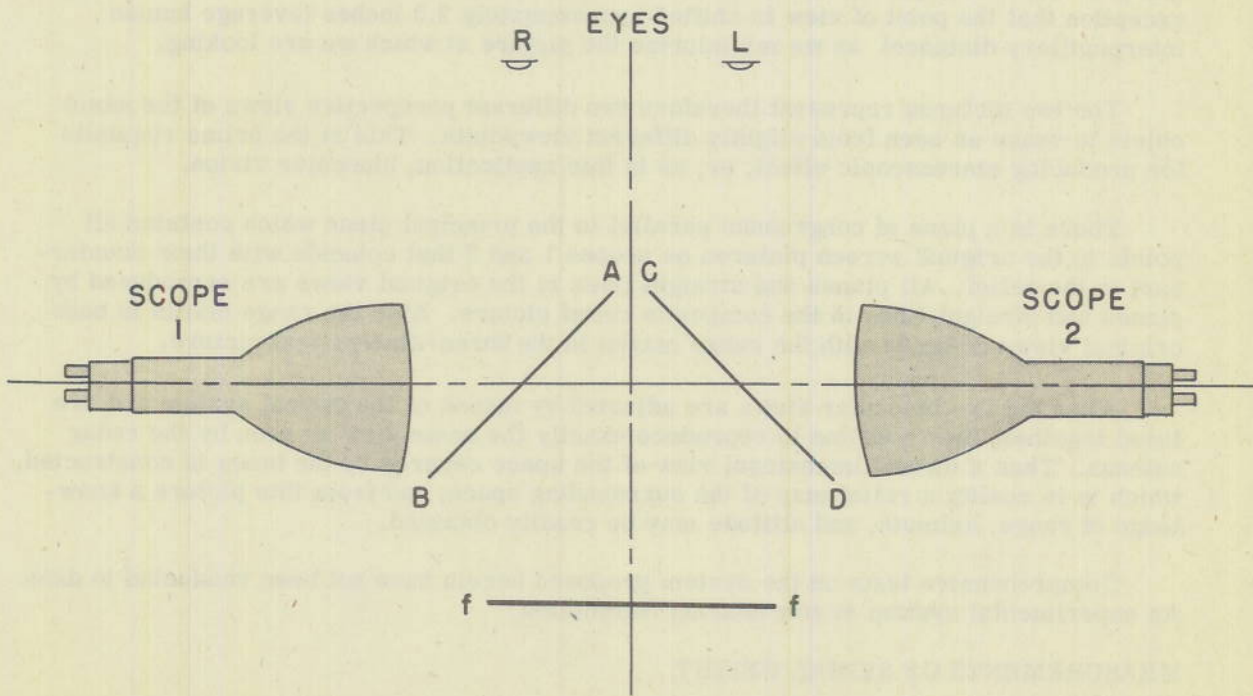


Fig. 11 - Sketch of Optical System

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at a three-dimensional view of the space being swept by the nodding antenna and not at one-plane patterns themselves at ff.

To obtain the best results it is desirable to have cathode ray tubes with highly persistent screens, to rotate the antenna at a very low speed, meanwhile nodding it at as high a frequency as possible. It will be readily apparent that these factors will have a heavy bearing upon the fullness of the patterns traced upon the tube screens.

SUMMARY AND CONCLUSIONS

Some question has been raised as to whether or not the fused picture was indeed a three-dimensional view of the space swept by the antenna; that necessary data for stereoscopic pictures was not available in the radar information as the usual visual presentation was in two dimensions only. It must be borne in mind that all of the information available from the radar is not used in presenting a one-plane map picture of the surrounding space.

In the usual PPI presentation all objects within the beam area appear as objects in a two-dimensional plane on the PPI irrespective of their altitude in space. However, the radar information actually contains data concerning three dimensions, namely R , θ , and ϕ , or range, azimuth, and elevation angle, which can be used to produce not only a flat map but also a location in space. By means of these three dimensions we can construct a picture from any point in space covered by the radar beam. Thus a composite picture is built up by looking from two points, since on one scope a picture using R , θ , and ϕ , is placed and on the second scope a second picture also utilizing R , θ , and ϕ . The latter picture is of the same volume in space as the former with the single exception that the point of view is shifted approximately 2.5 inches (average human interpupillary distance) as we miniaturize the picture at which we are looking.

The two pictures represent therefore two different perspective views of the same object in space as seen from slightly different viewpoints. This is the prime requisite for producing stereoscopic effect, or, as in this application, binocular vision.

There is a plane of congruence parallel to the principal plane which contains all points in the original screen pictures on scopes 1 and 2 that coincide with their counterpart in the relief. All planes and straight lines in the original views are reproduced by planes and straight lines in the composite relief picture. Also the range marks in both original views coincide with the range marks in the three-dimensional picture.

When the two binocular views are adjusted by means of the optical system and are fused together, they combine to reproduce exactly the same view as seen by the radar antenna. Thus a three-dimensional view of the space covered by the beam is constructed, which is in reality a relief map of the surrounding space, and from this picture a knowledge of range, azimuth, and altitude may be readily obtained.

Comprehensive tests on the system proposed herein have not been conducted to date. An experimental system is now nearing completion.

MEASUREMENTS OF SIGNAL HEIGHT

One possible method of measuring the apparent height of signals above the face of the PPI tube is shown below in Figure 12. With this arrangement, the virtual image of a

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movable point of light is made to coincide with the stereoscopic image of the target. A scale is provided to read target height. The length of the optical part from the point marked A to the movable point of light must be the same as the distance from A to the apparent image of the target if the target and the point of light are to coincide. (Figure 12).

With the apparatus as shown in Figure 12 a chart could be placed above the partially reflecting mirror so that it appears to coincide with the PPI surface. If the chart is made to the same scale as the PPI, the PPI and chart can be made to match and the apparatus could be employed in very precise off-shore navigation. By the addition of one more mirror between point A and the chart to eliminate the confusing mirror effect, the chart, or a piece of paper in its place, could be used for plotting target positions. There are several reports ‡ which discuss this method of plotting and navigating, also target designation and shore bombardment using similar equipment.

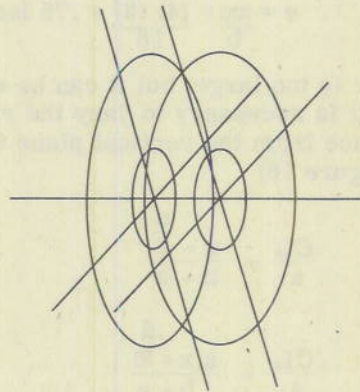
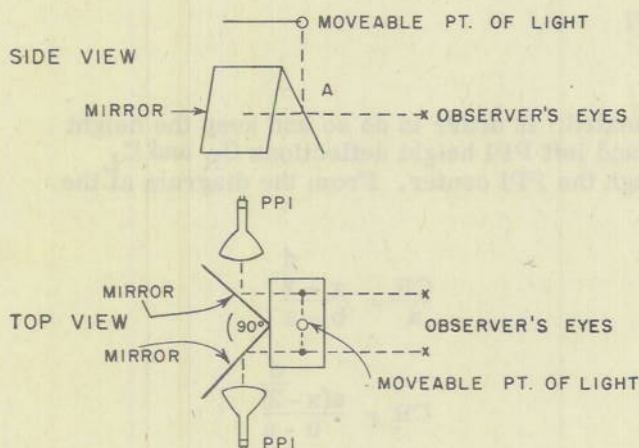


Fig. 12 - Measurement of Signal Height

Fig. 13 - Stereoscopic Range Circles

The partially reflecting surface could be a piece of plate glass, as prepared by Pittsburgh Plate Glass Company, with one side coated to make it non-reflecting so that there will be an image from the uncoated side only.

A second method of determining target height above the PPI face would be to compare it to stereoscopic range circles arranged in regular steps above the PPI face as shown below in Figure 13.

A third way would be to electronically generate a stereoscopic range spot which could be moved to coincide with the target.

SOME FURTHER CONSIDERATIONS

Let us consider the case of a signal being viewed off the axis. A diagram fitting such an occurrence is shown in Figure 14. If the signal is deflected by the same amount anywhere in the PPI for a given target height, then signals representing

‡ A Microfilm Chart Projector for Radar Navigation R.L. 658. Precise Navigation by Means of A Radar Map Superposed on the PPI R.L. 503.

the same height will all appear to lie in the same plane above the PPI face. However, signals will appear to lie closer to the vertical plane through the PPI center than they should.

To find the error e : $\frac{e}{a} = \frac{x}{b}$ or $e = \frac{ax}{b}$ where x is the perpendicular distance from

a vertical plane through the PPI center to the undistorted position of the signal. (Figure 15)

The worst error will exist when the signal is at the edge of the tube at maximum height, and the observer is as close as possible (about 16 inches) to the apparent position of the target.

suppose: $a = 4$ inches
 $b = 16$ inches
 $x = 3$ inches for a 7 inch PPI
 then: $e = \frac{ax}{b} = \frac{(4)(3)}{16} = .75$ inches

This error is too large, but it can be eliminated. In order to do so and keep the height constant, it is necessary to vary the right and left PPI height deflections C_R and C_L with distance from the vertical plane through the PPI center. From the diagram at the right: (Figure 16)

$$\frac{C_L}{a} = \frac{x+2}{b-a}$$

$$\frac{C_R}{a} = \frac{x-2}{b-a}$$

$$\frac{C_L}{a} = \frac{a(x+2)}{b-a}$$

$$\frac{C_R}{a} = \frac{a(x-2)}{b-a}$$

In order to avoid confusion in using these equations the proper signs must be chosen for x , C_L , and C_R .

LIMITS OF BINOCULAR PARALLAX §

The essence of stereoscopic sensation is the binocular parallax arising in the different perspectives presented to the two eyes: it cannot arise when both eyes view one and the same picture. According to J. W. French §, if we are fusing the binocular images of two points subtending an angle of one degree to one eye, then the limiting binocular parallax is about a half degree, i.e., if two points A and B subtend an angle of one degree to the left eye, and two other points C and D in the same horizontal line as A and B are viewed by the right eye, and if the images of A and C are fused, it will be impossible to fuse B and D into a stereoscopic image unless the angular separation of C and D to the right eye lies between 0.5 degrees and 1.5 degrees (French gives 0.6 and 1.6); for smaller or greater separations of C and D stereoscopic vision fails, and the images of C and D are not fused. The limiting separations depend on the angular separation of A and B as shown in Figures 17 and 18.

§ L. C. Martin, An Introduction to Applied Optics Vol II, (London: Pitman and Sons Ltd., 1930).

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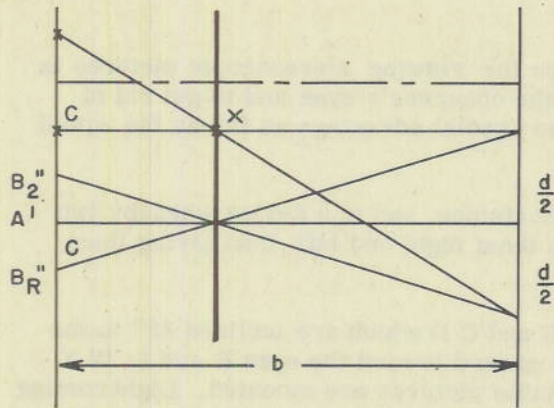


Fig. 14 - Signal Viewed off Axis

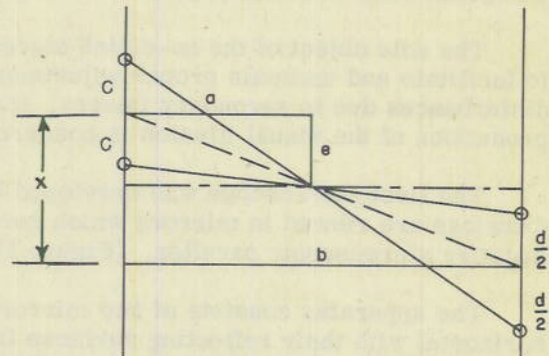


Fig. 15 - Error due to Signal Being Viewed off Axis.

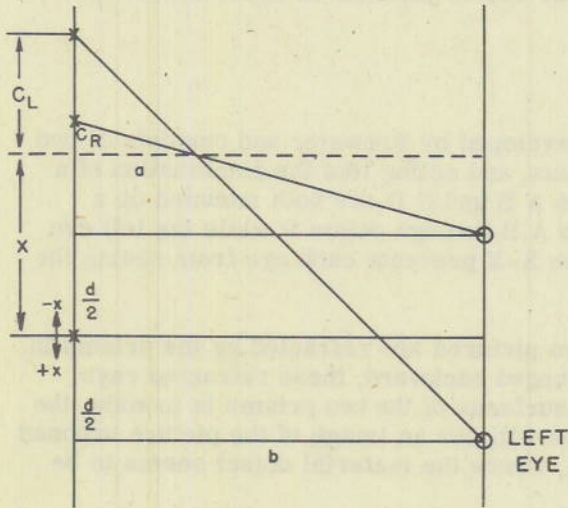


Fig. 16 - Elimination of Error

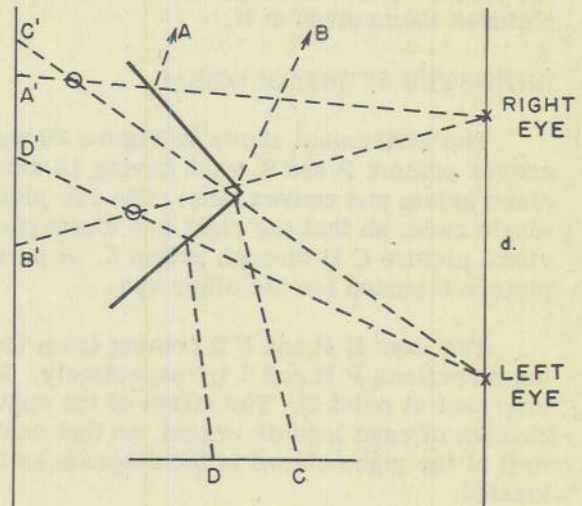


Fig. 17 - Limits of Binocular Parallax

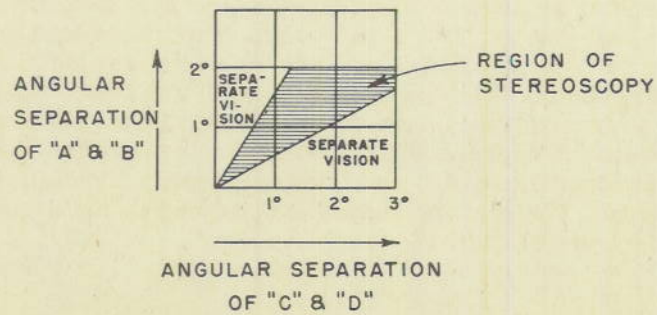


Fig. 18 Chart of Limiting Separations

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WHEATSTONE STEREOSCOPE**

The sole object of the so-called stereoscope for viewing stereoscopic pictures is to facilitate and maintain proper adjustment of the observer's eyes and to get rid of disturbances due to secondary images. It has no special advantage as far as the actual production of the visual illusion is concerned.

The first stereoscope was developed by Wheatstone, and is a device whereby two drawings are viewed in mirrors which perverts them right and left, thus giving them negative stereoscopic parallax. (Figure 19).

The apparatus consists of two mirrors A B and C D which are inclined 45° to the horizontal with their reflecting surfaces facing upward toward the eyes R and L. W X and Y Z are two vertical pieces upon which suitable pictures are mounted. Light coming from picture W X is reflected by mirror A B into the eye R as though it had come from the image ff. In a like manner light coming from Y Z is reflected from mirror C D into eye L as if it had also come from the image ff. Thus both eyes apparently see image ff, and if the images are different in detail the observer gets the same impression by his visual sense as though he were looking at the object extended in space and not at pictures themselves at ff.

BREWSTER STEREOSCOPE**

The instrument shown in Figure 20 was developed by Brewster and consists of two convex prisms P and S, each having 18 cm. focus, and acting like the combination of a plane prism and convex lens. The two pictures A B and C D are both mounted on a single card, so that the right eye views picture A B through prism P while the left eye views picture C D through prism S. A partition X-X prevents each eye from seeing the picture intended for the other eye.

The rays E P and F S coming from the two pictures are refracted by the prisms in the directions P R and S L respectively. Prolonged backward, these refracted rays intersect at point Q. The effect of the convex surfaces of the two prisms is to make the bundles of rays less divergent, so that each eye will see an image of the picture exposed to it at the place shown in the diagram as G-H, where the material object seems to be located.

** H.L.F. Von Helmholtz, Physiological Optics, (Optical Soc. of America, 1924-25).

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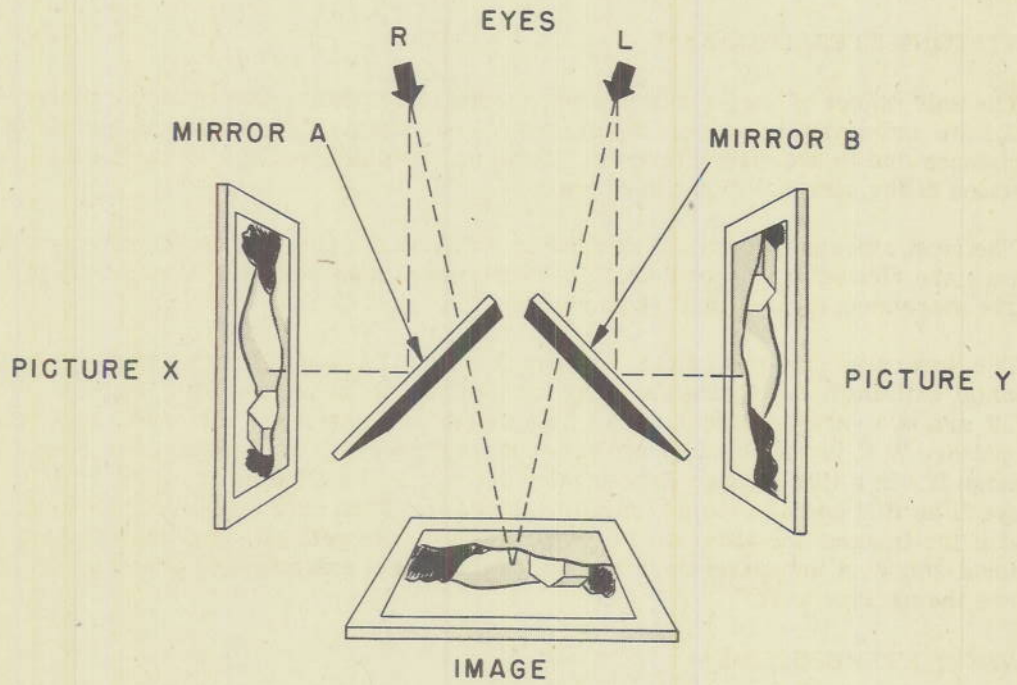


Fig. 19 - Wheatstone Stereoscope

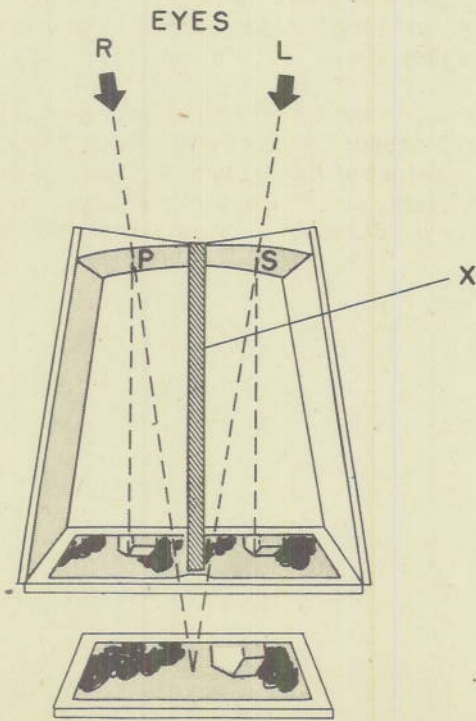


Fig. 20 - Brewster Stereoscope

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PLATE I

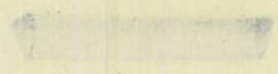


FIGURE 1



FIGURE 2

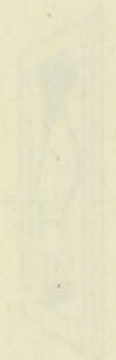


FIGURE 3



FIGURE 4



FIGURE 5

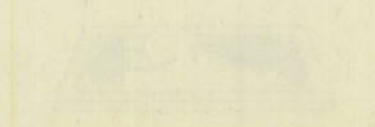


FIGURE 6

