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VIDEO MIXING
AND MINIMUM DETECTABLE SIGNAL
IN THE SPS-2 RADAR

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VIDEO MIXING AND MINIMUM DETECTABLE SIGNAL IN THE SPS-2 RADAR

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Problem No. 39R02-22 September 11, 1947

DEPARTMENT OF THE NAVY • OFFICE OF NAVAL RESEARCH

NAVAL RESEARCH LABORATORY
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ABSTRACT

The parameters of the proposed SPS-2 long range search radar were simulated, and the effect on the minimum detectable signal of mixing two video channels, representing adjacent elevation channels of the system, was determined for equal signals in the two channels and for signal in one channel only. In the former case the minimum detectable signal was 1.6 db lower and in the latter case 1.6 db higher than for a single unmixed channel. The minimum detectable receiver signal-to-noise ratio was found to be 7.4 db. The maximum free-space range of the system, with the stated parameters and for a target of one square meter cross-section, was calculated to be 318 nautical miles.

PROBLEM STATUS

This is an interim report on problem S1225 presenting results obtained in a study of one phase of this problem.

AUTHORIZATION

BuShips letter to NRL S-916-00413, 25 October 1945, Request for assignment of project to NRL to conduct research leading to the development of a long range search radar system (Confidential).

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VIDEO MIXING AND MINIMUM DETECTABLE SIGNAL IN THE SPS-2 RADAR

INTRODUCTION

1. The proposed SPS-2 long range search radar is a composite of eight separate radars with antennas directed as in Figure 1 to give adequate vertical coverage without vertical scanning. The elevation of a target may be estimated by noting in which of the eight channels the echo is received or more accurately by measuring the relative echo strength in two adjacent channels. A further possibility is to combine the (synchronized) video outputs of pairs of adjacent channels and to use these, either alone or in conjunction with one of the original channels, for obtaining the elevation angle. In order to evaluate this method it is necessary to know the effect on minimum detectable signal, and hence on the shape of the coverage diagram, of mixing signals from two similar video channels. This effect has been studied, and the results are discussed in this report.

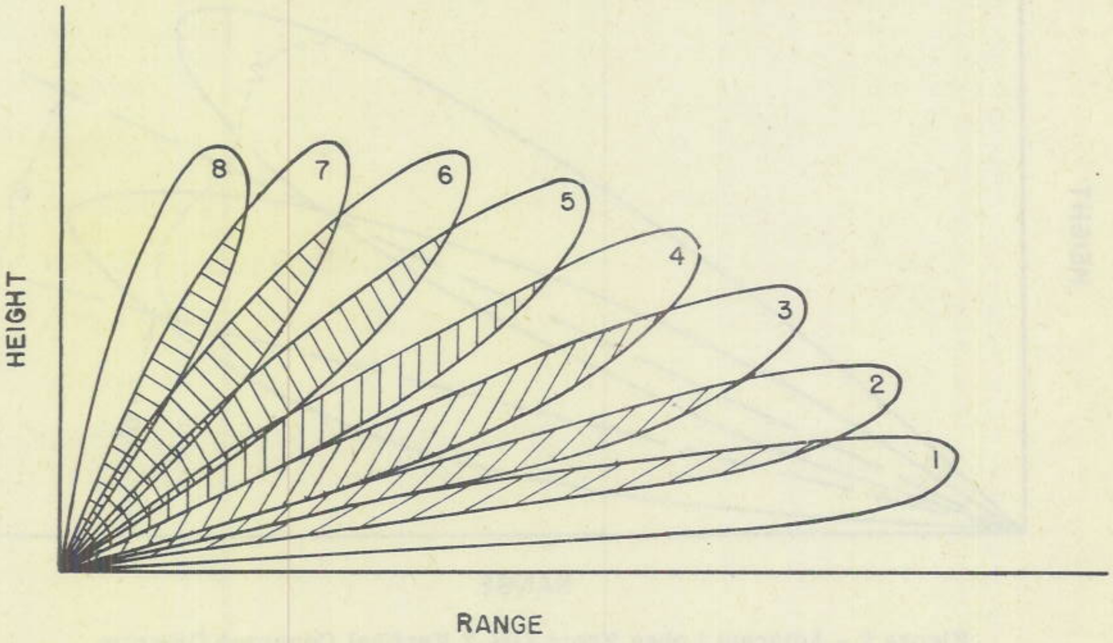

Figure 1 - Proposed SPS-2 Vertical Coverage Diagram

2. Some of the SPS-2 parameters lie outside the well-investigated range. Consequently in order to avoid extrapolation of data, they were simulated as closely as possible and the absolute value of the minimum detectable signal measured. After correction for the small deviations from these parameters, the receiver threshold power, P_{min}, was inserted into the radar range formula 2, along with the values of the other parameters which at present seem likely to be used, and the range to be expected from the system was calculated. In addition the data was carefully analyzed for information concerning the

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manner in which the observer scans the PPI in the six-position method of determining signal threshold.

THEORY

3. The video outputs of two receivers, whose input signals are received simultaneously are combined in a "linear" mixer in such a way that the resultant signal voltage is very nearly the sum of the individual signal voltages and the resultant noise power the sum of the individual noise powers. (This is based on reasoning and assumptions discussed in Appendix I.) For illustration, consider two adjacent antenna lobes as shown in Figure 2. The signal strength of the echoes from targets at an angle of elevation θ_1 is the same for both lobes. With identical receivers for the two lobes, the signal voltage in the combined video channel is twice that of either receiver taken separately, but the noise voltage is increased by a factor of only 1.41 (i.e., the noise power is doubled). Hence the video signal-to-noise ratio, as defined in Appendix I, is increased by 3 db. For a target at an angle of elevation θ_2 such that the echo received in channel 2 is negligible in comparison to that in channel 1, the combined video output consists of the same signal voltage as for channel 1 alone, but with a noise voltage multiplied by 1.41. Therefore, the video signal-to-noise ratio is decreased 3 db. As shown in Appendix I, however, these 3-db changes in video signal-to-noise ratio are equivalent to changes of only 1.6 db (approximately) in predetection signal-to-noise ratio. Thus the change in minimum detectable signal for these two cases of video mixing may be expected to be around 1.6 db. However, the combined noise appears to give a more nearly uniform background on the PPI, and, theoreti-

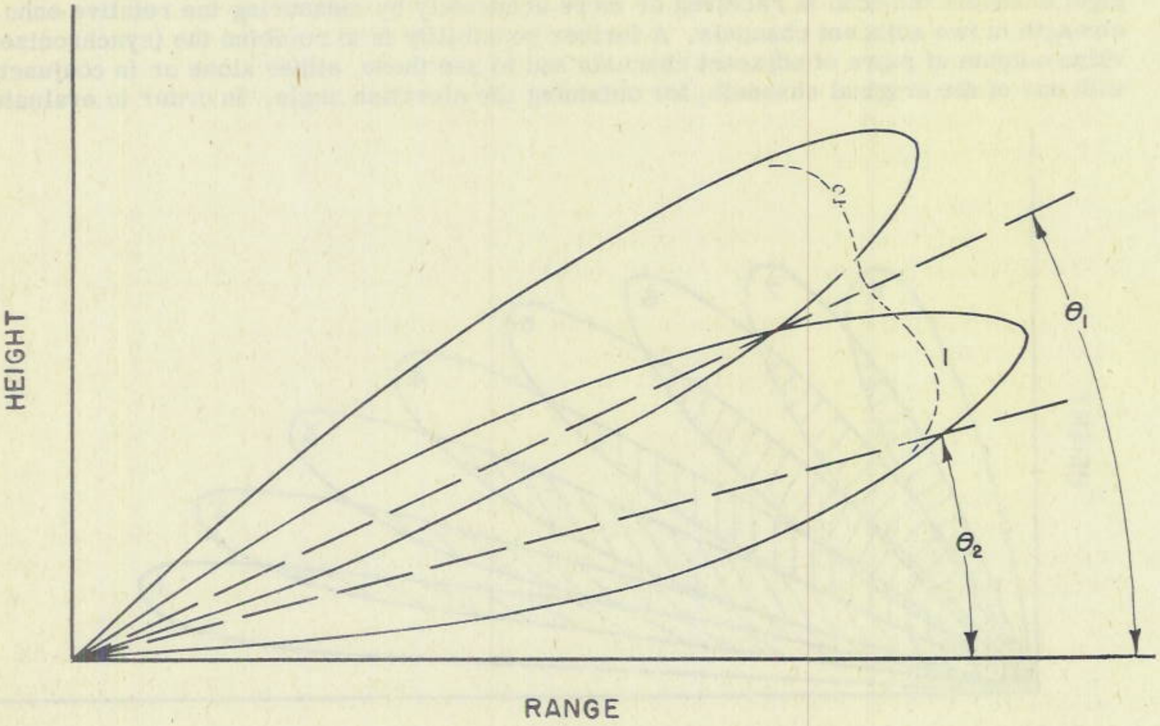

Figure 2 - Adjacent Lobes From SPS-2 Vertical Coverage Diagram

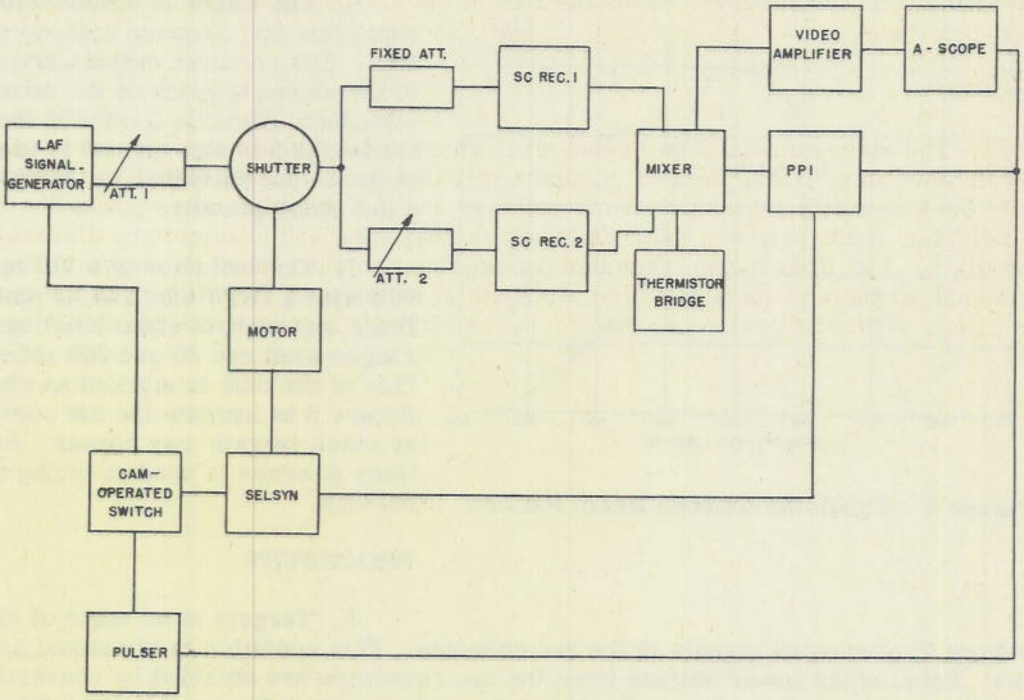

Figure 3 - Block Diagram of Equipment

cally, this may also affect the detectability of the signal. [1]

EQUIPMENT

4. Figure 3 is a block diagram of the equipment. Radar targets are simulated by a signal generator and a rotating disk shutter. The signals to the receivers are blocked by the disk except when a circular hole near its periphery sweeps across a fixed circular opening. As the aperture changes, the signal amplitude varies as shown in Figure 4, giving a beam pattern similar to that of a conventional radar. The shutter itself gives an 8.7° beam width, but it is rotated at six times the PPI speed of 10 rpm, giving an effective beam width of 1.4°. This technique would simulate targets at six different bearings, but five of them are eliminated by a cam which opens a switch in the synchronizing lead from the pulser to the signal generator.

5. Receivers 1 and 2 are early-model SC's modified to eliminate their characteristic logarithmic response and realigned to give about 0.7-Mc bandwidths. Similar receivers are used to minimize differences in the two channels. Two hundred Mc pulses of 5-μ seconds duration are supplied by a Model LAF signal generator to both channels to provide signal synchronization, and sufficient attenuation is used between the two receiver input circuits to prevent noise coherence. The range selector picks at random one of six ranges at which the target may appear and indicates to the operator, but not to the observer, the range selected. The repetition rate is 240 pps.

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[1] Payne-Scott, R.: "The Ultimate Visibility of Signals on a PPI Display," Commonwealth of Australia, Council for Scientific and Industrial Research, Division of Radiophysics, Report R. P. 252/1, 20 May 1945 (Confidential)

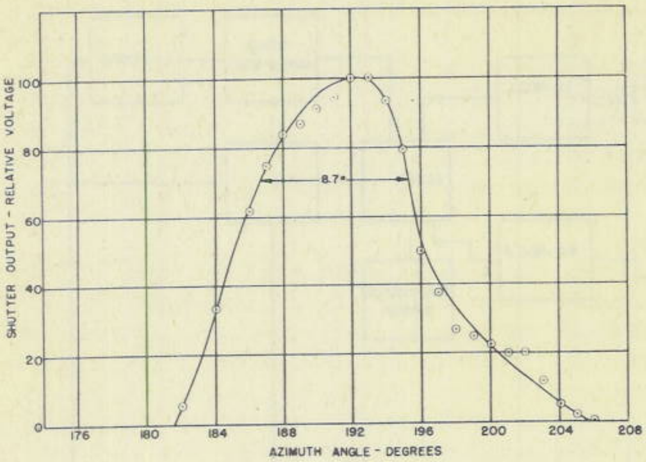

Figure 4 - Simulated Antenna Beam Pattern

6. The mixer is a 6SN7 with common plate and common cathode resistors. The receiver outputs are applied to the separate grids of the mixer, and a Western Electric D-164699 thermistor in a d-c bridge circuit is used to measure the individual or combined video noise powers.

7. The indicator is a VE remote PPI with a 7BP7 tube and an amber filter, but without video limiting. The ranges used are 80 and 200 miles. The face of the tube is marked as shown in Figure 5 to indicate the six positions at which targets may appear. An auxiliary **A**-scope is used in tuning the receivers.

PROCEDURE

8. Targets at an angle of elevation θ_1 (Figure 2) give equal signals in the two channels. This condition is simulated as follows: Equal noise power outputs from the two receivers are obtained by connecting them one at a time to the mixer, adjusting gain, and observing the thermistor bridge. Then the noise power of the combination is measured as a check against possible coherence in the two channels. With the gain settings unchanged, the input signals are adjusted to give for each receiver minimum detectable signals on the PPI, thus assuring also equal signal powers from the two channels. Then the receiver outputs are combined and the minimum detectable signal of the combination obtained by attenuating the signal generator output. Targets at angle θ_2 which return negligible signal to channel 2 are simulated in a similar way, and in this case there is a loss in minimum detectable signal.

9. Observations on the PPI are made in a darkened room which has just enough light to enable the observer to discern the outlines of objects in the room. The observer adjusts the CRT bias to near cut-off with no applied signal, and then turns up the video gain to give whatever screen brightness he prefers. After he has spent twenty minutes becoming "scope adapted" the operator adjusts the input signal to near minimum detectable and places it at one of the six range positions as chosen by the range selector. After the observer has watched the PPI for three revolutions, he calls the number of the position which seems most likely to be the signal position. Twenty such observations are made for each of four to six judiciously chosen values of the input signal, and the percentage correct of each group of readings is plotted against input signal strength as shown in Figure 6. Sufficient data are taken to insure knowing to within one-half db the attenuator setting for which the observer calls 90 percent of the

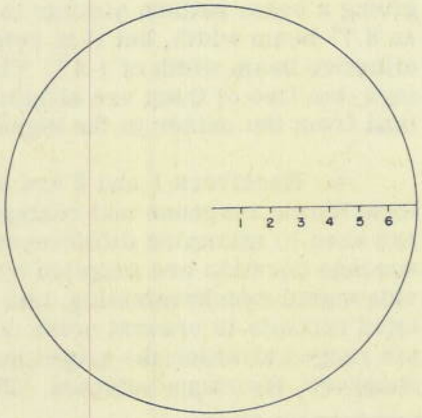

Figure 5 - Signal Positions as Marked on PPI Scope

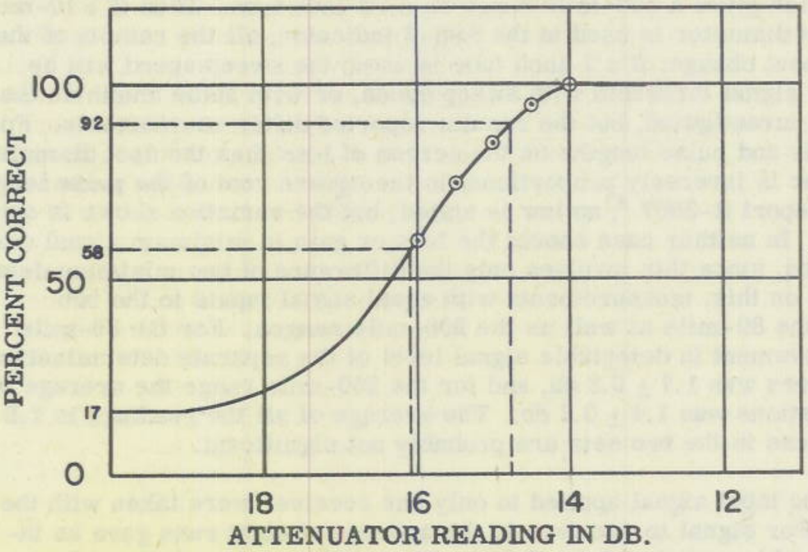

Figure 6 - Target Detectability vs. Signal Input

positions correctly. When this setting and the attenuation between signal generator and receiver is known, the minimum detectable signal at the receiver terminals is readily calculated.

10. This technique is followed in obtaining the minimum detectable signal for receiver 1, when it alone is connected to the mixer, by varying attenuator 1. With attenuator 1 set at this 90 percent point, and only receiver 2 is connected to the mixer, the minimum detectable signal for this receiver is found by varying attenuator 2. With attenuator 2 set at the 90 percent point and both receivers connected to the mixer, the minimum detectable signal for the combination is found by varying attenuator 1. The difference in the two settings of attenuator 1 gives the improvement in minimum detectable signal due to the combination when the signals in the two channels are equal. To obtain the loss in the minimum detectable signal of the combination when the signal appears only in channel 1, a similar procedure is used, but attenuator 2 is set to give negligible input signal to receiver 2.

RESULTS AND DISCUSSION

11. The six-position scheme is used because it seems capable of yielding more precise results than the other methods considered and because it allows comparison of some of the results with those of Report R-3007.[2] When the observer is permitted to watch the PPI for three revolutions rather than for only one, the resulting signal threshold is lower. However, it is felt that this corresponds reasonably well with operational procedure, since an operator will certainly want to verify a suspected (i.e., very weak) target by additional observations before taking action.

12. Although the maximum design range of the SPS-2 is 300 miles, the maximum range of the VE indicator is only 200 miles on a 7-inch tube. The sweep speed corresponds

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[2] Ashby, R. M., V. Josephson, S. Sydoriak, "Signal Threshold Studies," Report R-3007, 1 December 1946

almost exactly to that which gives a 300-mile range on a 10-inch tube. Thus if a 10-inch tube having the same spot diameter is used in the SPS-2 indicator, all the results of these measurements apply without change; if a 7-inch tube is used, the sweep speed will be slower. The variation of signal threshold with sweep speed, or with pulse length on the screen, has already been investigated, but the results reported differ considerably. For discontinuous backgrounds and pulse lengths on the screen of less than the spot diameter[3], the signal threshold power is inversely proportional to the square root of the pulse length on the screen. In NRL Report R-3007[4], no law is stated, but the variation shown is only about one-third as great. In neither case should the loss or gain in minimum signal due to video mixing be affected, since this involves only the difference of two minimum detectable signals. As a check on this, measurements with equal signal inputs to the two channels were made for the 80-mile as well as the 200-mile ranges. For the 80-mile range, the average improvement in detectable signal level of ten separate determinations by three different observers was 1.7 ± 0.3 db, and for the 200-mile range the average of seven separate determinations was 1.4 ± 0.3 db. The average of all the readings is 1.6 ± 0.3 db, and the differences in the two sets are probably not significant.

13. All data with the input signal applied to only one receiver were taken with the 200-mile-range sweep. For signal to receiver 1, the average of eight runs gave an increase in minimum detectable signal of 1.1 ± 0.2 db. This differed enough from the expected 1.6 db that the equipment was suspected. As a check the two receivers were interchanged so that the signal was applied to receiver 2 only and the 11 similar runs that followed showed an increase of 1.9 ± 0.3 db, or a weighted average for the two receivers of 1.6 ± 0.4 db.

14. The two receivers were then examined for differences which might explain these results, since the inherent accuracy of the method of measurement is known to be too good to permit experimental or "observational" errors of this order. The following differences noted are listed here without comment since their significance, if any, is not known. Upon close A-scope examination, the video noise output from receiver 2 appeared to have a finer structure with sharper peaks, as though higher-frequency components were present. This was probably due to the fact that the second-detector load resistors and by-pass condensers were not identical and the video bandwidths were consequently different. The detector characteristics, Figures 7 and 8, were measured by decreasing the gain until the noise disappeared, applying a c-w r-f signal to the antenna terminals, and measuring the d-c output voltage across the detector load resistor. The laws for the two detectors are very nearly the same, and the slopes of the curves at the operating points corresponding to the gain used in the threshold measurements are 1.6 for each detector. Figure 9 shows the output characteristics of linear, square-law, and intermediate-law detectors. Using the procedure described in Appendix I, and assuming a threshold signal-to-noise ratio (before detection) of 1 (see paragraph 21), it is seen that adding in the video of the intermediate-law receiver an equal noise power from another source causes an increase in the signal threshold of about 1.7 db. Very nearly the same result is obtained, however, if either the linear or square-law detector curves are used (1.6 db), so that the law of the detector is apparently of no appreciable consequence in this experiment.

15. Since the received signal varies inversely as the fourth power of the range, the 1.6-db decrease in the threshold signal (paragraph 12), is equivalent to a 10 percent

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[3] Payne-Scott, op. cit.

[4] Ashby, Josephson, and Sydoriak, op. cit.

[redacted]

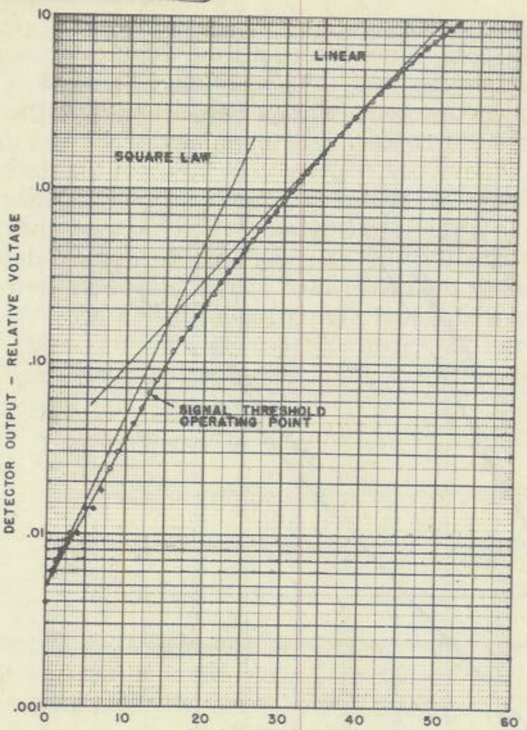

Figure 7 - Detector Characteristic for Receiver 1

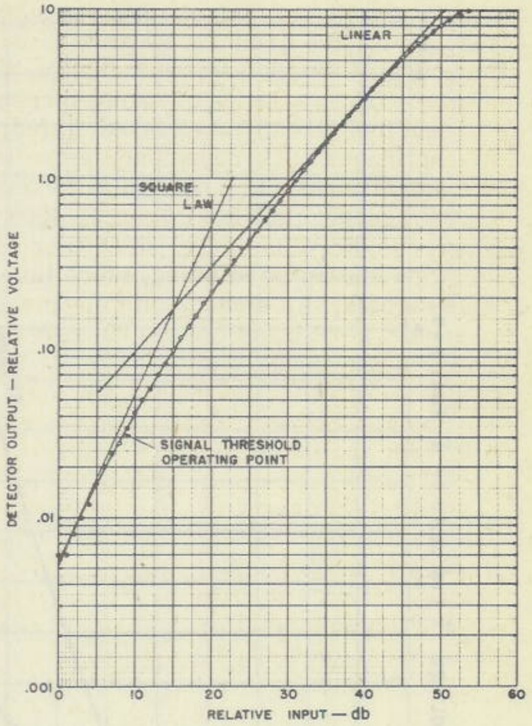

Figure 8 - Detector Characteristic for Receiver 2

increase in the maximum range of the system at angles of elevation corresponding to cross-overs of the antenna lobes, and the 1.6-db increase in the threshold signal (paragraph 13) corresponds to a 10 percent decrease in the maximum range at angles of elevation corresponding to the peaks of the original antenna lobes, provided negligible signal is being received from the other lobe. Thus the coverage diagram of the system is modified as indicated by the dotted line in Figure 2. The possible use of video mixing to obtain more accurate height finding will be discussed in a report now in preparation on the SPS-2 system. A more general discussion of video mixing may be found in NRL Report R-3008[5], which was received after these measurements were made.

16. The signal thresholds given here are easily converted to correspond to 50 percent probability of detection rather than to 90 percent. Following the reasoning given in the Payne-Scott Report[6], the conversion from the observer's scores (the fractional number guessed right) to the probability p, which excludes the observations called correctly by pure chance, is $p = (6/5\ s - 1/5) \times 100\%$. Then the values of s corresponding to the 50 percent and 90 percent values of p are 58 percent and 92 percent. The average of 64 curves exemplified in Figure 6 shows that for 50 percent probability of detection the minimum signal is 1.2 + 0.2 db below the 90 percent point.

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[5] Sydoriak, S. G.: "The Effects of Video Mixing Ratio and Limiting on Signal Threshold Power," Report R-3008, 1 February 1946

[6] op. cit.

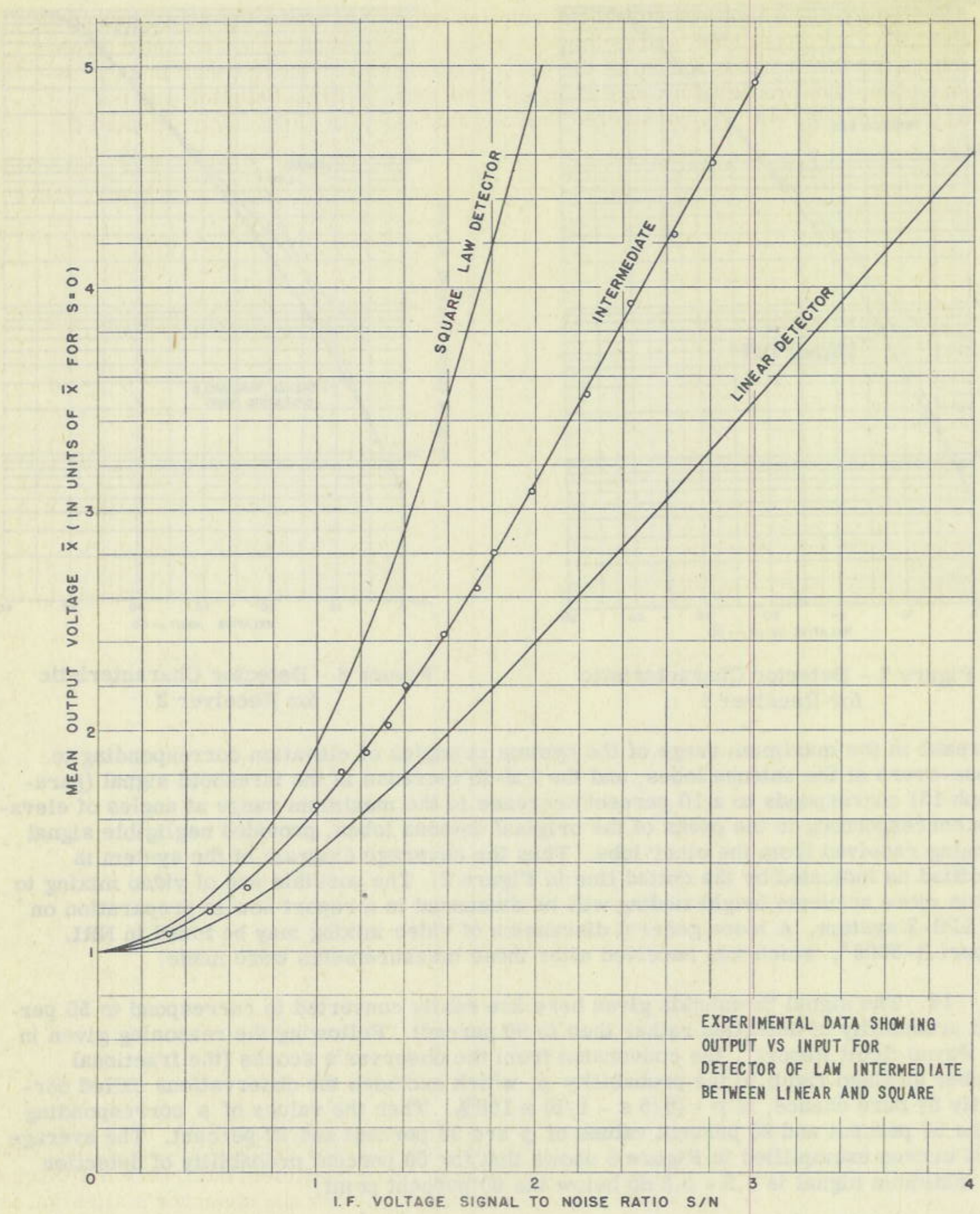

Figure 9 - Output Characteristics of Detectors

17. Figure 10, which is a plot of the relative number of times the signal appeared at each of the range positions, shows that the probability of occurence was not the same for each. Also the signal appeared at the same position two or more times in succession more often than predicted by pure chance. Hence the position selector was not a strictly random one, but it is considered unlikely that this has appreciably affected the results.

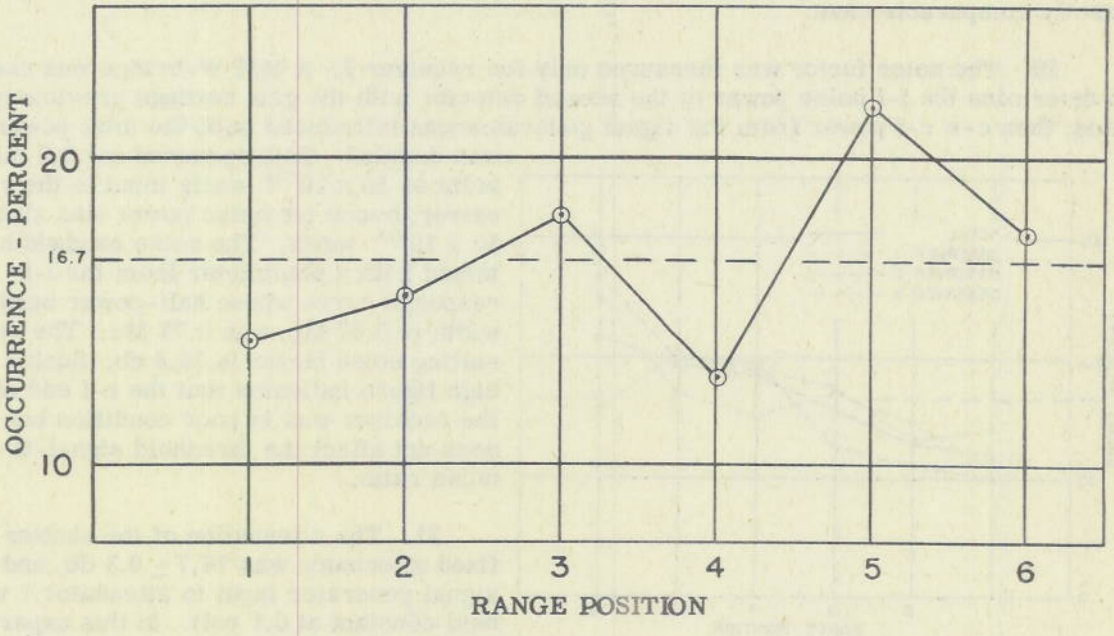

Figure 10 - Randomness of Range-Position Selector

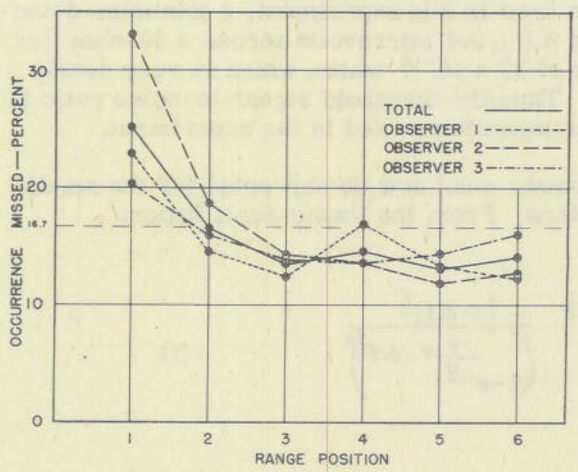

Figure 11 - Percent Missed for Each Range Position

18. Figure 11 shows the relative number of times each signal position was called incorrectly. Those near the center of the scope were missed somewhat more often than the others. However, this does not mean that the signal is inherently more difficult to detect near the center of the scope. Figure 12, giving the distribution among the signal positions of the incorrect calls, shows that when the signal was missed and the observer was guessing, he was more likely to call the positions near the edge of the scope. Thus his attention was concentrated more on the periphery than on the center of the screen. Sufficient data were not taken to determine whether the observer's attention is also concentrated on the periphery when he is required to scan the whole screen.

19. It was realized that the signal threshold for the six-position method is somewhat lower than that for the case where the

observer must scan the entire screen. Since at the time no factor was available for conversion between the two methods, provision was made whereby "random" range and bearing could be obtained manually. Three runs were made in which each observer determined in turn the thresholds for the two methods. The minimum detectable signal for the latter was found to be 3.0 ± 0.2 db higher. This figure is perhaps high, since the observers were not trained in the latter method, but it seems unlikely that further experience would reduce it by more than one db. NRL Report R-3007[7] gives 2.4 db for a similar but not exactly comparable case.

20. The noise factor was measured only for receiver 2. A MIT W-bridge was used to determine the i-f noise power at the second detector with the gain settings previously used; then c-w r-f power from the signal generator was introduced until the total power was doubled. This occurred for 3.9 microvolts or 30×10^{-14} watts input to the receiver; hence the noise power was also 30×10^{-14} watts. The noise bandwidth obtained with a planimeter from the i-f response curve whose half-power bandwidth is 0.67 Mc, was 0.77 Mc. The resulting noise factor is 19.8 db. Such a high figure indicates that the r-f end of the receiver was in poor condition but does not affect the threshold signal-to-noise ratio.

21. The attenuation of the shutter and fixed attenuator was 74.7 ± 0.3 db, and the signal generator input to attenuator 1 was held constant at 0.1 volt. In this experiment the two receivers were interchanged with respect to their positions shown in Figure 3. With this arrangement, when attenuator 1 was adjusted for minimum detectable signal to receiver 2, it read 13.3 ± 0.5 db. This gives, for the parameters used in this experiment, a minimum detectable signal of 88.0 ± 0.8 db below 0.1 volt, or 4.0 ± 0.4 microvolts across a 50-ohm line. This corresponds to a minimum signal power of 32×10^{-14} watts, which is very nearly equal to the noise power of 30×10^{-14} watts. Thus the threshold signal-to-noise ratio is 0.3 db for the six-position method and for the parameters used in the experiment.

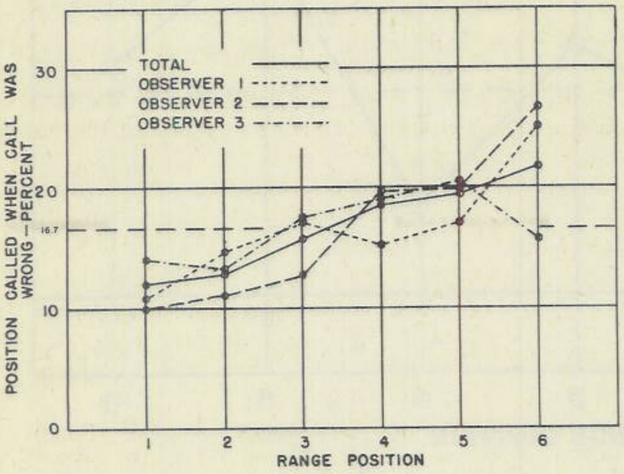

Figure 12 - Position Called When Call Was Wrong

22. This value of the threshold signal power must now be corrected for the small departures from the proposed SPS-2 parameters. From the Payne-Scott Report[8],

$$P_{min} \sim \theta_h^{-\frac{2}{3}} \cdot r^{-\frac{1}{2}} \frac{(\tau\cdot\Delta f)^{\frac{1}{2}}}{\left(1-e^{-\frac{\pi}{2}\cdot\tau\cdot\Delta f}\right)^{2}} \qquad (1)$$

---

[7] Ashby, Josephson, and Sydoriak, op. cit.

[8] op. cit.

where θ_h is the horizontal antenna beam width, r the pulse repetition frequency, τ the pulse length, Δf the i-f bandwidth, and $\Delta f'$ the noise bandwidth (about $1.1\,\Delta f$). The corrections tabulated from this formula agree well with the experimental data.[9] A conversion factor to the random-range random-bearing method of only 2 db has been allowed, since the observers were not so well trained in this as in the six-position method. In NRL Report R-3007[10] it is suggested that the threshold of the "good" radar operator is probably 3 db above that of the operators used in those experiments. This is somewhat questionable, since a signal 3 db above the threshold stands out very clearly. Nevertheless the 3 db has been included here as a safety factor. Also a duplexer and transmission line loss of 1 db has been added. These corrections are summarized in Table I. The resulting signal threshold power for the SPS-2, as nearly as can be determined with the available data, is 7.4 db above noise. Since the noise factor of the SPS-2 receiver itself is expected to be as low as 10 db or better, the threshold signal power will be about 10.1×10^{-14} watts.

APPLICATION TO MAXIMUM RANGE OF SPS-2 RADAR

23. Although the values of the parameters used here are not necessarily the final ones, it will be informative to use this value of the threshold power to calculate the range of the system from the radar range formula.

$$R_{max} = (4\pi)^{-\frac{3}{4}} \lambda^{\frac{1}{2}} \left(\frac{G^2 \ A \ P_t}{P_{min}} \right)^{\frac{1}{4}} \qquad (2)$$

Where:

R_{max}	is the maximum range, measured in the units used for λ and A,
λ	the operating wavelength,
G	the antenna power gain over an isotropic radiator,
A	the radar cross-section of the target,
P_t	the rms transmitter power during pulse, and
P_{min}	the minimum detectable power at receiver terminals, in same units as P_t.

The values of these parameters for the SPS-2 which at present seem most likely are $\lambda = 23$ cm, $G = 6600$, $A = 10^4$ cm^2 for smallest target which must be detected, and $P_t = 10^7$ watts.

Inserting these together with P_{min} into the above formula and converting to nautical miles gives $R_{max} = 318$ miles.

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[9] Haeff, A. V.: "Minimum Detectable Radar Signal and Its Dependence upon Parameters of Radar Systems," I.R.E. Proc. No. 34, pp. 857-861, 1946; Payne-Scott, op. cit.; Ashby, Josephson, and Sydoriak, op. cit.

[10] Ashby, Josephson and Sydoriak, op. cit.

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TABLE I

Parameter	Proposed Value for SPS-2	Value Used Here	Correction to P_{min}, db.
Antennae rotational speed	10 rpm	10 rpm	0
Pulse length	5μ sec 7 μsec	5μ sec	0 -1.5
Antennae horizontal beam width	1.6° — each way .7 x 1.6 = 1.12° effect	1.4°	~~-0.4~~ +0.4
Pulse repetition frequency	220	240	+0.2
I-F bandwidth	0.4 Mc	0.7 Mc	-1.0
Correction from six-position to random-range random-bearing method			+2.0
Good operator, above 90% point			+3.0
Duplexer and transmission line loss			+1.0
Noise factor	10 db 9	19.8	-9.8

$N_P = 1.$
$5\tau = \frac{2}{3}$
$\approx$

Net Correction to P_{min} ~~-5.0 db~~ -9.8

Threshold Signal Power for SPS-2 ~~10.1×10^{-14}~~ watts
3.3×10^{-14}
at recvr terminals

* * * * * * * *

Net Correction to Signal-to-Noise Ratio +7.1 db

Threshold Signal-to-Noise Ratio for SPS-2 7.4 db

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SUMMARY

24. The parameters of the SPS-2 radar were simulated as closely as possible with the available equipment, and measurements were made of the effect on the minimum detectable signal of combining the outputs of two video channels, first with equal signals in the two channels and second, with the signal in one channel only. It was found that in the first case the minimum detectable signal was 1.6 db lower and in the second case 1.6 db higher than for a single video channel.

25. The minimum detectable signal for a single channel was measured and the value obtained corrected for departures from the SPS-2 parameters. The minimum signal power for the SPS-2 was then 7.4 db above the receiver noise power. The maximum free space range of the system, using the parameters and allowing a 3 db safety factor, was calculated to be 318 miles.

26. Careful records were kept of all signal positions and the observer's guesses. It was found that positions near the center of the screen were missed more often than those near the edge and that when an observer guessed wrong he was more likely to guess a position near the edge of the screen, indicating that his attention was concentrated there.

SPS-2 visibility factor referred to receiver input (no TR loss, etc. included) is then

Factor	Correction DB
Ant. beam 1.65° one way = 1.17° two way $5 \log \frac{1.4}{1.17}$ =	+ 0.4
$S\tau = \frac{200}{300} \times \frac{7}{5}$ = negligible	
Rep. Freq – – – – – – – $245/240 \simeq 0$	+
I.F. τ/BW – – – – – – – – – –	− 1.0
6 pos. to random range & bearing	+ 2.0
100% betting/90% – – – –	+ .8
net	+ 2.2

Base ratio obtained here $\frac{32}{30} \frac{10^{-14}}{\cancel{\times 10^{-14}}}$ = + .3

Visibility factor = + 2.5 db

= P_{min}/KTB

aav 3/31/52

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APPENDIX I - A THEORETICAL ANALYSIS OF THE EFFECT OF VIDEO MIXING ON DETECTABILITY OF RADAR SIGNALS

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Changes in the "discernibility" of a radar signal are ordinarily brought about by variation of the pre-detection signal-to-noise ratio. A similar change in discernibility, however, may be brought about by "mixing" the video outputs of two receivers. If the output of one receiver contains both signal and noise, and the other noise only, the discernibility of the signal after mixing will be less. If both receiver outputs contain the same signal, but not the same noise (i.e., if the noise sources are independent of each other), then the signal discernibility in the mixed output will be improved. It is of interest to determine, from theoretical considerations, what will be the amount of this change in a given experiment. This change in discernibility, or detectability, is best expressed in terms of an <u>equivalent</u> change of pre-detection signal-to-noise ratio.

The problem is complicated by the fact that relationships between signal and noise are profoundly altered in the detection process -- at least in the case of signals of the same order of magnitude as the noise. And this is the case with which we are concerned.

The relationship between the mean value of <u>detector current</u> (or voltage across the detector load) and i-f signal-to-noise ratio is given in the curves of Figure 9.[11] This mean value is expressed in units of the no-signal value; that is, for $S = 0$, $\bar{x} = 1$ $(= \bar{x}_0)$.

Since the detector is a rectifier, this current is uni-directional, but fluctuating. It is important to recognize that the term "mean value" here means "the amplitude of the d-c component" of the detector current. Superimposed on this, of course, is a fluctuating (noise) current whose mean value is zero. The amplitude of the fluctuations may be described in terms of the <u>mean value of the deviation from the mean</u> (and also, of course, in terms of the r-m-s value of the deviation).

The value of the d-c component has already been designated $\bar{x}$ (and $\bar{x}_0$ for the no-signal value). Let the r-m-s value of the deviation from the mean (noise component) be called σ (the symbol used in statistical theory to designate the "standard deviation" of a variable having a probability distribution. Let the i-f signal-to-noise voltage ratio be called S.

The problem of discerning, or detecting, a signal in the presence of noise is one of observing a difference in the appearance of the indicator where there is a signal compared to its appearance where there is noise only. There are three characteristics of the detector output voltage when a signal is present which makes it different from the output for noise only:

(a) There is a difference in the mean value, or "d-c component," expressed mathematically by the difference: $\bar{x} - \bar{x}_0$.

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[11] Andrew, E. R.: "An Experimental Verification of Theoretical Relations between detector Current and Receiver Signal/Noise Ratios," Gt. Brit. R.R.D.E. Research Report No. 295, 23 August 1945, (Restricted)

(b) The r-m-s value of the fluctuating component is different when the signal is present. (See Appendix and Figure 10 of Andrew Report.)[1]

(c) The probability distribution of the noise amplitude, considered as a random variable with statistical properties, is different for different values of S. (See RCA Report PTR-6C of 6-25-43, by D. O. North). This difference is undoubtedly a factor in detectability on some types of display (e.g., A-scope).

While all three of these effects may contribute to discernibility, it is certainly true that the chief effect is the difference, $\bar{x} - \bar{x}_0$. The total range of variation of the other two effects is limited, i.e., for large S, the value of the noise component (σ) is asymptotic to a constant value, and the shape of the distribution curve tends to a normal distribution. The quantity $\bar{x} - \bar{x}_0$ is not so limited, however; as S increases indefinitely, so does the value of $\bar{x} - \bar{x}_0$. There may be only a small error, therefore, if for purposes of computation it is assumed that the quantity $\bar{x} - \bar{x}_0$ defines "the signal" and the symbol σ defines the noise.

(It may be of interest to observe that the video amplifiers following the detector amplify the quantities $\bar{x} - \bar{x}_0$ and σ, not the quantities $\bar{x}$ and $\bar{x}_0$ themselves, since the "d-c component" of the detector output is usually removed by condenser coupling.)

The ratio $(\bar{x} - \bar{x}_0)/\sigma_0$ may be considered analogous, therefore, to the i-f signal-to-noise ratio, and might in fact be called the video signal-to-noise ratio, although in using such a term it should be borne in mind that it is not a complete criterion of detectability since it ignores certain characteristics of "the signal". What has been assumed here, however, is that this ratio may be used as though it were a complete criterion of detectability, without excessive error in the result of computations based on this assumption. This is a possibly somewhat crude and certainly nonrigorous theoretical approach, but at the same time it is one which may provide a useful working relationship, good enough for many practical purposes.

The value of this ratio, which may be denoted by the symbol **V**, can be plotted as a function of S. The data for such a curve may be taken from the curves of Figure 9. The quantities $\bar{x}_0$ and σ_0 are for this purpose treated as constants.

It will be noted that the quantities $\bar{x}_0$ and σ_0 in the output of any given detector are directly related -- i.e., $\sigma_0 = k \cdot \bar{x}_0$. The question must therefore be answered why it is necessary to introduce the symbol σ_0 into the ratio denoted by **V**. The reason is that in the video mixing process to be discussed, the ratio between these two quantities does not remain constant.

Suppose now that the following video-mixing experiment is performed. Two receivers of equal bandwidth are adjusted to give equal noise output with no signal. There is assumed to be no coupling between the receiver input circuits, so that the two noise outputs may be considered as <u>independent</u> random variables. A signal, of amplitude S_1 relative to the noise, is fed into one receiver, but not into the other. The outputs of the two receivers are mixed, in a mixer which is so designed that there is direct linear addition of instantaneous amplitudes.

With this assumption, it can be shown that the resulting (mixed) value of no-signal r-m-s noise is $\sigma' = \sqrt{\sigma_0^2 + \sigma_0^2} = 1.41\,\sigma_0$, because the two mixed noise signals are independent random variables.

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[12] Andrew, op. cit.

What happens to the numerator of the expression for V is most conveniently analyzed by assuming that the d-c components of the two detector outputs are retained and applied to the mixer input along with the fluctuation components. Therefore, this assumption will be made here. Without this assumption, the following analysis would be slightly different but the result would be the same. (In actual practice, of course, the d-c components would usually be removed by condenser coupling.)

The quantities $\bar{x}$ and $\bar{x}_0$ (now considered as voltages), are mean values of "unidirectional" (i.e., non-negative) random variables. There is a theorem that the mean value of the sum of two or more variables is equal to the sum of their individual[13] mean values. Thus, after mixing, the new mean value of voltage at a "noise" position on the time scale is simply $\bar{x}_0' = \bar{x}_0 + \bar{x}_0 = 2\bar{x}_0$, and the new value of mean voltage at a "signal" position is $\bar{x}_1 + \bar{x}_0 = \bar{x}'$

Thus for the value of V after mixing, the result is

$$V_2 = \frac{\bar{x}' - \bar{x}_0'}{\sigma_0'} = \frac{(\bar{x}_1 + \bar{x}_0) - 2\bar{x}_0}{1.41\sigma_0}$$

$$= \frac{\bar{x}_1 - \bar{x}_0}{1.41\,\sigma_0} = \frac{\left(\dfrac{\bar{x}_1 + 0.41\,\bar{x}_0}{1.41}\right) - \bar{x}_0}{\sigma_0}$$

It is thus seen that the value $\bar{x}_2$ corresponding to V_2 is

$$\bar{x}_2 = \frac{\bar{x}_1 + 0.41\,\bar{x}_0}{1.41} = \frac{\bar{x}_1 + 0.41}{1.41}$$

since, on the curves of Figure 9, $\bar{x}_0 = 1$. Thus, $\bar{x}_1 = 1.41\,\bar{x}_2 - 0.41$

This relationship may now be used to compute directly, from the curves for $\bar{x}$ as a function of S (Figure 9), the value S_2 corresponding to V_2. Or, the problem may be inverted, by letting it be required to find the value of i-f signal-to-noise ratio (S_1) which must be employed in order for the mixed signal to have the same detectability as that of the single receiver when the i-f signal-to-noise ratio is S_2.

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For the unmixed case, let S = 1. This is S_2 in the foregoing expressions.

If a linear detector is assumed, the curve of Figure 9 gives for $\bar{x}_2$ the value of 1.44. Therefore $\bar{x}_1 = (1.41 \times 1.44) - 0.41 = 1.61$.

Corresponding to this value, S_1 is found to be 1.2. Thus the ratio $S_1/S_2 = 1.2$, or 1.6 db approximately.

If the intermediate-law detector curve is used instead, it is found that $\bar{x}_2 = 1.65$, so that $\bar{x}_1 = (1.41 \times 1.65) - 0.41 = 1.92$. It is then found that $S_1 = 1.22$, so that $S_1/S_2 = 1.22$, or approximately 1.7 db. (There is an uncertainity of about 0.1 db in reading the curve.)

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[13] See, for example, J. V. Uspensky, Introd. to Mathematical Probability, McGraw-Hill, 1937, p. 165

A similar calculation could be made for the mixing experiment in which both receivers have signals (synchronized) in their outputs, of equal amplitude. In this case, for V after mixing:

$$V_2 = \frac{2x_1 - 2x_O}{1.41\sigma_O}$$

so that $$x_2 = 1.41\,x_1 - 0.41$$

It is immediately observed that there is an inverse relationship between the values of x_1 and x_2 for this case and for the case where one receiver contained noise only. This means that the improvement in detectability for the second type of mixing is equal to the loss in detectability that occurred for the first type, as would be expected.

A close agreement is observed between these theoretically computed results and those found experimentally, although the experimentally obtained figure of 1.6 db, it should be noted, is an average of several experiments which gave individual results from 1.1 to 1.9 db.