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# ELECTRIC CABLE RESEARCH
# INVESTIGATION OF TRANSMISSION CHARACTERISTICS OF THREE-CONDUCTOR POWER CABLE IN USE BY THE U.S. NAVY

by

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# ABSTRACT

Measurements were made on three-conductor power cable for shipboard use to determine the effect of increased operating frequency (60-800 cps) on the operating characteristics of the cable, and to determine the extent of interference voltage induced in adjacent communication or control circuits by load currents in the power cable. Results of the investigation indicate the extent of reduction in current capacity and the increase in line voltage regulation as the frequency is increased. No interference voltage was observed in adjacent circuits under any condition of load in the power circuit.

# THE PROBLEM

The studies covered in this report were initiated in response to BuShips letter to NRL, S62-(10)(344c), dated 11 September 1946, Project SRD-677/47 on electric cable research.

This report concludes the work on this problem, and unless otherwise advised by the Bureau it will be closed one month from the mailing date of this report.

# ELECTRIC CABLE RESEARCH

## INTRODUCTION

The transmission characteristics of the power cable is one of the influential factors in determining the optimum operating frequency of the a-c power system on naval vessels. This study was conducted to determine the effect of frequency on the transmission characteristics of the power cables in use by the Navy Department. The particular characteristics investigated were:

(A) Effect of frequency on the current capacity and voltage regulation of the power cable.

(B) The extent of interference voltage induced in adjacent communication or control cable by a loaded power cable.

## ELECTRICAL CHARACTERISTICS OF CABLE

The so-called constants of a cable that determine its operating characteristics are resistance, inductance, and capacitance, per unit length. Because of the skin effect, the effective resistance of a conductor to alternating current is greater than that to direct current. At 60 cps the increase in resistance is generally not more than 2 or 3 percent; however, as the frequency increases the increase in effective resistance and the consequent reduction in current capacity becomes increasingly important.

The well-established theory of skin effect in isolated conductors of circular cross section permits the calculation of the effective resistance of such conductors at any usual operating frequency. The equations derived for these conditions, however, do not apply for stranded conductors forming a three-conductor cable since the theory for isolated conductors does not take into account the following factors:

(A) Proximity effect of adjacent conductors
(B) Stranding of conductors
(C) Spiraling of strands and conductors
(D) Iron losses in armor braid
(E) Coating of tin and other metals on strands
(F) Eddy currents in conductors and other adjacent metals.

Factors (B), (C), (D), (E), and (F) have a negligible effect compared to (A), the proximity effect of other conductors forming the cable. Moreover, this proximity effect increases with frequency and conductor size.

The complexity of the problem involved, has thus far prevented any successful mathematical analysis of the skin effect in a three-conductor stranded cable, therefore, an experimental determination was made of the skin effect in the power cables described in Table 1 and covered by Navy Standard Specification 15-C-1.

TABLE 1

DESCRIPTION OF CABLES

| Type | No. Conductors | Area Cir Mils Per Cond | Strands Per Conductor | Length Ft. | Finish | Purpose |
|---|---|---|---|---|---|---|
| THFA-300 | 3 | 296,400 | 91 | 50 | Armored | Power |
| THFA-150 | 3 | 157,600 | 61 | 50 | Armored | Power |
| THFA-50 | 3 | 49,080 | 19 | 50 | Armored | Power |
| TCOP-150 | 3 | 153,100 | 760 | 50 | Rubber-Covered | Power |
| TTHFA-10* | 10 Pair | 703 | 7 | 50 | Armored | Communication |
| TTHFA-1* | 1 Pair | 703 | 7 | 50 | Armored | Communication |
| MHFA-10* | 10 Pair | 2,828 | 7 | 50 | Armored | Control |
| DHFA-3* | 2 | 2,828 | 7 | 50 | Armored | Control |
| MCOS-6* | 6 | 1,005 | 10 | 50 | Shielded | Control |
| MHFF-2* | 2 | 2,613 | 26 | 50 | Rubber-Covered | Control |

* Cables tested for inductive interference

Because of the effect of eddy currents in the conductors and adjacent materials, the inductances of a three-conductor cable are also dependent upon frequency. For this reason, measurements of cable inductance at various frequencies were made.

## METHOD OF TEST

Since the power cables under study were three-conductor cables for use on three-phase circuits, the measurements were conducted with balanced three-phase load currents in the cable. The three conductors were shorted together at one end of the cable and a three-phase, low-voltage, variable frequency was applied at the other end. A diagram of this measuring circuit is shown in Figure 1.

To avoid errors caused by voltages induced in the measuring circuit by stray fields, shielded leads of twisted pairs to the cathode-ray oscilloscope were used. To reduce

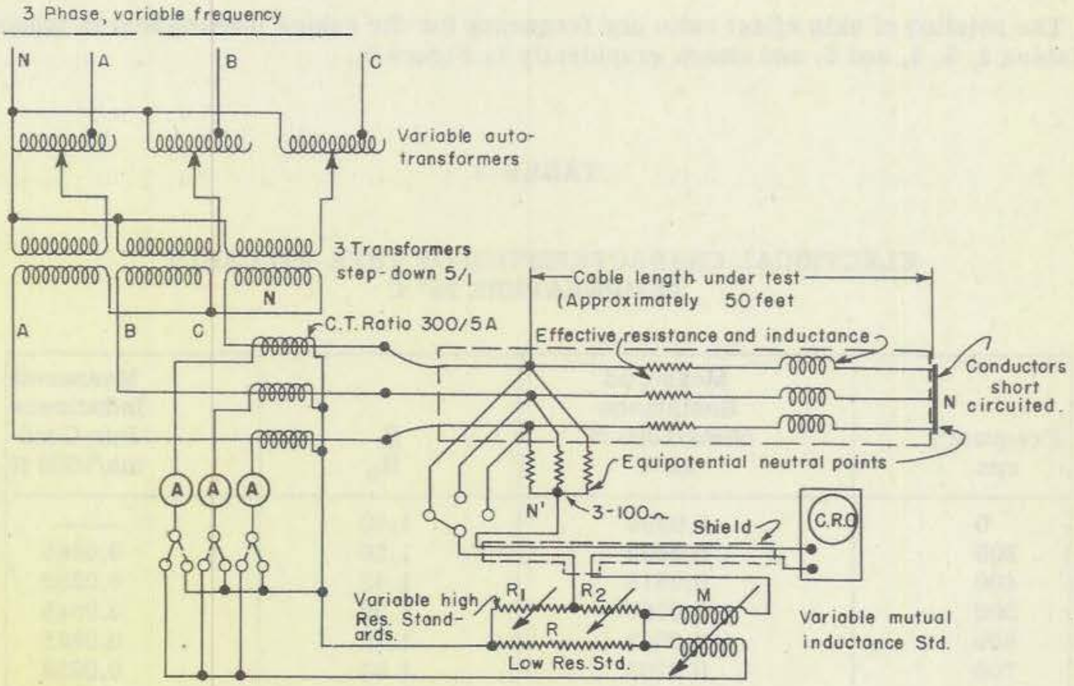

Fig. 1. Circuit Diagram of Test Equipment

further the effect of stray fields, the cathode-ray oscilloscope circuit was returned to an artificial neutral, N', formed by three 100-ohm Wye-connected resistors, instead of to the Neutral N formed at the end of the cable.

Phase voltages were adjusted to give equal conductor currents, thus forming a balanced Wye-connected load of the cable. Simultaneous measurement of the effective resistance and inductance of the cables at frequencies ranging from 300 to 800 cps were made by comparing the resistive and reactive voltage drops in each conductor of the cable with the voltage drops in a standard non-inductive resistance and a standard mutual inductance, respectively. The derivation of the equations used in calculating the effective resistance and inductance of the cable from these measurements is given in Appendix. By this method of test the error in the measured value of effective resistance is less than one percent.

## RESULT OF EFFECTIVE RESISTANCE MEASUREMENTS

In expressing the skin effect it is more convenient to use the ratio of effective or a-c resistance to the d-c or zero frequency resistance which in this study was measured by means of a Kelvin bridge. The skin effect ratio is herein designated by $\frac{R}{R_O}$, where R is the effective resistance at any frequency, and $R_O$ is the d-c resistance.

The relation of skin effect ratio and frequency for the cables investigated is tabulated in Tables 2, 3, 4, and 5, and shown graphically in Figure 2.

## TABLE 2

## ELECTRICAL CHARACTERISTICS OF THFA-300 CABLE TEMPERATURE 25° C

| Frequency cps | Measured Resistance ohms/1000 ft 25°C | $\frac{R}{R_o}$ | Measured Inductance Per Cond mh/1000 ft |
|---|---|---|---|
| 0 | 0.0365 | 1.00 | ----- |
| 300 | 0.0460 | 1.26 | 0.0865 |
| 400 | 0.0519 | 1.42 | 0.0855 |
| 500 | 0.0580 | 1.59 | 0.0845 |
| 600 | 0.0642 | 1.76 | 0.0832 |
| 700 | 0.0708 | 1.94 | 0.0818 |
| 800 | 0.0771 | 2.11 | 0.0808 |

## TABLE 3

## ELECTRICAL CHARACTERISTICS OF THFA-150 CABLE TEMPERATURE 25° C

| Frequency cps | Measured Resistance ohms/1000 ft 25°C | $\frac{R}{R_o}$ | Measured Inductance Per Cond mh/1000 ft |
|---|---|---|---|
| 0 | 0.0670 | 1.00 | ----- |
| 300 | 0.0728 | 1.08 | 0.0872 |
| 400 | 0.0763 | 1.14 | 0.0870 |
| 500 | 0.0804 | 1.20 | 0.0863 |
| 600 | 0.0850 | 1.27 | 0.0856 |
| 700 | 0.0894 | 1.33 | 0.0850 |
| 800 | 0.0950 | 1.43 | 0.0845 |

## TABLE 4

### ELECTRICAL CHARACTERISTICS OF THFA-50 CABLE TEMPERATURE 25° C

| Frequency cps | Measured Resistance ohms/1000 ft 25°C | $\frac{R}{R_o}$ | Measured Inductance Per Cond mh/1000 ft |
|---|---|---|---|
| 0 | 0.214 | 1.00 | ----- |
| 300 | 0.223 | 1.04 | 0.102 |
| 400 | 0.2255 | 1.05 | 0.102 |
| 500 | 0.2285 | 1.07 | 0.101 |
| 600 | 0.232 | 1.08 | 0.101 |
| 700 | 0.236 | 1.10 | 0.100 |
| 800 | 0.240 | 1.12 | 0.100 |

## TABLE 5

### ELECTRICAL CHARACTERISTICS OF TCOP-150 CABLE TEMPERATURE 25° C

| Frequency cps | Measured Resistance ohms/1000 ft 25°C | $\frac{R}{R_o}$ | Measured Inductance Per Cond mh/1000 ft |
|---|---|---|---|
| 0 | 0.0728 | 1.00 | ----- |
| 300 | 0.0755 | 1.04 | 0.0810 |
| 400 | 0.0785 | 1.08 | 0.0802 |
| 500 | 0.0817 | 1.12 | 0.0792 |
| 600 | 0.0850 | 1.17 | 0.0785 |
| 700 | 0.0890 | 1.22 | 0.0778 |
| 800 | 0.0932 | 1.28 | 0.0770 |

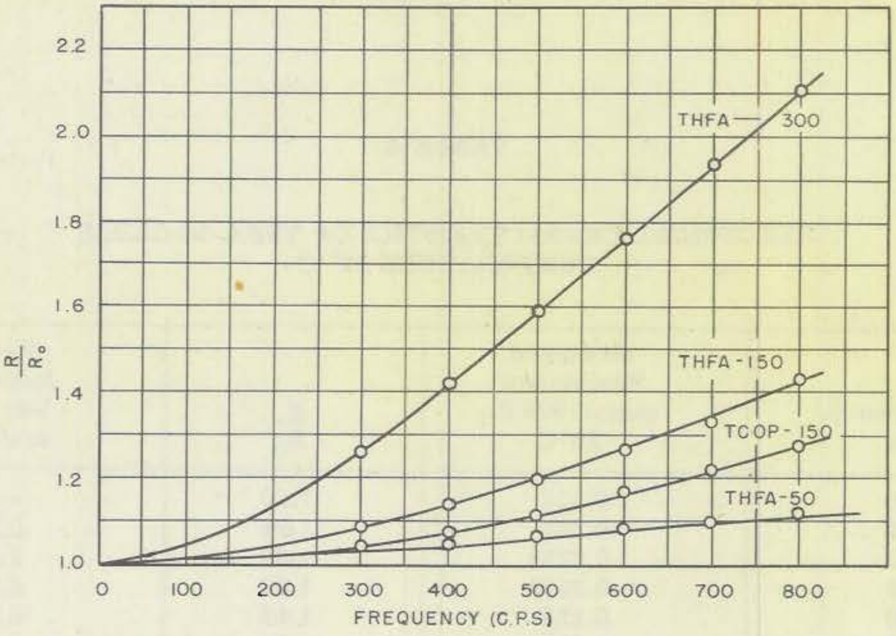

Fig. 2. Effect of Frequency on Skin Effect Ratio

In Figure 2 it will be noted that $\frac{R}{R_o}$ increases with frequency and that the rate of increase is greater for large size conductors. It will also be observed that the skin effect ratio at 60 cps is only slightly larger than unity. The difference between the skin effect ratio for the THFA-150 and the TCOP-150 cables (approximately the same cir mil area) is due principally to the difference in stranding and cable geometry.

## CURRENT CAPACITY OF CABLE

The increase in effective resistance with frequency causes the current capacity of the cable to be reduced as the frequency increases. Tables 6, 7, and 8 show the percent of zero frequency current capacity at frequencies ranging from 0 to 800 cps. The results are shown graphically in Figure 3.

The percent current capacity was calculated on the basis of equal $I^2R$ loss in the cable at all frequencies, which would produce the same maximum operating temperature for the cable. The zero frequency current, the maximum cable temperature, and the ambient temperature were obtained from Cable Comparison Guide (NavShips 250-660-23), "Data Pertaining to Electric Shipboard and Degaussing Cable." It should be noted that the current capacity decreases rapidly for large cables as the frequency increases.

## TABLE 6

### OPERATING CHARACTERISTICS OF THFA-300 CABLE 200-FT LENGTH OPERATING AT 440 V, 0.08 PF LAGGING

| Frequency cps | Maximum Load Current* Amperes | Reactance (ohms) | Resistance ohms 105° C | Percent Voltage Regulation | Percent of Zero Frequency Current Capacity |
|---|---|---|---|---|---|
| 0 | 348 | 0.0000 | 0.0095 | 1.38 | 100.0 |
| 300 | 310 | 0.0326 | 0.0120 | 3.74 | 89.0 |
| 400 | 292 | 0.0430 | 0.0136 | 4.41 | 83.9 |
| 500 | 276 | 0.0531 | 0.0153 | 5.12 | 79.4 |
| 600 | 262 | 0.0628 | 0.0168 | 5.50 | 75.3 |
| 700 | 249 | 0.0720 | 0.0184 | 5.91 | 71.6 |
| 800 | 239 | 0.0812 | 0.0202 | 6.30 | 68.8 |

## TABLE 7

### OPERATING CHARACTERISTICS OF THFA-150 CABLE 200-FT LENGTH OPERATING AT 440 V, 0.8 PF LAGGING

| Frequency cps | Maximum Load Current* Amperes | Reactance (ohms) | Resistance ohms 105° C | Percent Voltage Regulation | Percent of Zero Frequency Current Capacity |
|---|---|---|---|---|---|
| 0 | 235 | 0.0000 | 0.0175 | 1.57 | 100.0 |
| 300 | 226 | 0.0328 | 0.0190 | 3.15 | 96.0 |
| 400 | 220 | 0.0437 | 0.0200 | 3.74 | 93.5 |
| 500 | 214 | 0.0541 | 0.0210 | 4.23 | 91.2 |
| 600 | 208 | 0.0645 | 0.0222 | 4.72 | 88.6 |
| 700 | 203 | 0.0746 | 0.0234 | 5.12 | 86.5 |
| 800 | 196 | 0.0848 | 0.0248 | 5.50 | 83.5 |

## TABLE 8

### OPERATING CHARACTERISTICS OF THFA-50 CABLE 200-FT LENGTH OPERATING AT 440 V, 0.8 PF LAGGING

| Frequency cps | Maximum Load Current* Amperes | Reactance (ohms) | Resistance ohms 105° C | Percent Voltage Regulation | Percent of Zero Frequency Current Capacity |
|---|---|---|---|---|---|
| 0 | 110.0 | 0.0000 | 0.056 | 1.80 | 100.0 |
| 300 | 108.0 | 0.0384 | 0.0584 | 2.95 | 98.0 |
| 400 | 107.0 | 0.0513 | 0.0590 | 3.15 | 97.5 |
| 500 | 106.0 | 0.0630 | 0.0598 | 3.54 | 96.7 |
| 600 | 105.5 | 0.0755 | 0.0607 | 3.94 | 96.0 |
| 700 | 105.0 | 0.0880 | 0.0617 | 4.30 | 95.4 |
| 800 | 104.0 | 0.1010 | 0.0628 | 4.70 | 94.5 |

* Maximum allowable load current based on 105° G cable temperature with 40° C ambient.

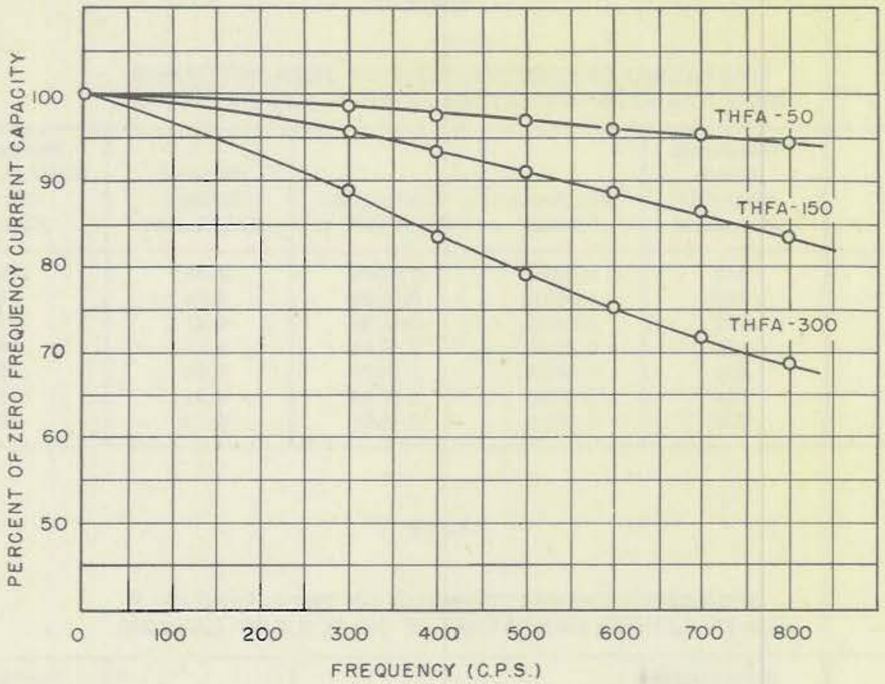

Fig. 3. Effect of Frequency on Current Capacity of Cable

## RESULTS OF EFFECTIVE INDUCTANCE MEASUREMENTS

Because of the close spacing of conductors, the inductance of a three-conductor cable is small and its effect on the operating characteristics of the cable may be neglected at the usual power frequencies. At higher frequencies, however, the effect of inductance on the voltage regulation of the cable should be considered.

The inductance per unit length of one conductor of a three-conductor cable depends on the geometry of the cable, i.e., stranding, size, and spacing of conductors. When these factors are known, the inductance can be calculated by means of the well-known theory of flux linkages per ampere. The calculated value is often incorrect, however, because of the difficulty of obtaining an accurate measurement of conductor spacing, and also because eddy currents, principally in the conductors and to a lesser degree in other adjacent metallic bodies, tend to reduce the inductance. This effect of eddy currents increases with frequency.

Measured values of inductance at frequencies ranging from 0 to 800 cps are given in Tables 2, 3, 4, and 5, and their relation to frequency is shown graphically in Figure 4. It will be observed that the inductance of all cables decreases as the frequency increases. The error in the measured inductance is less than 2 percent.

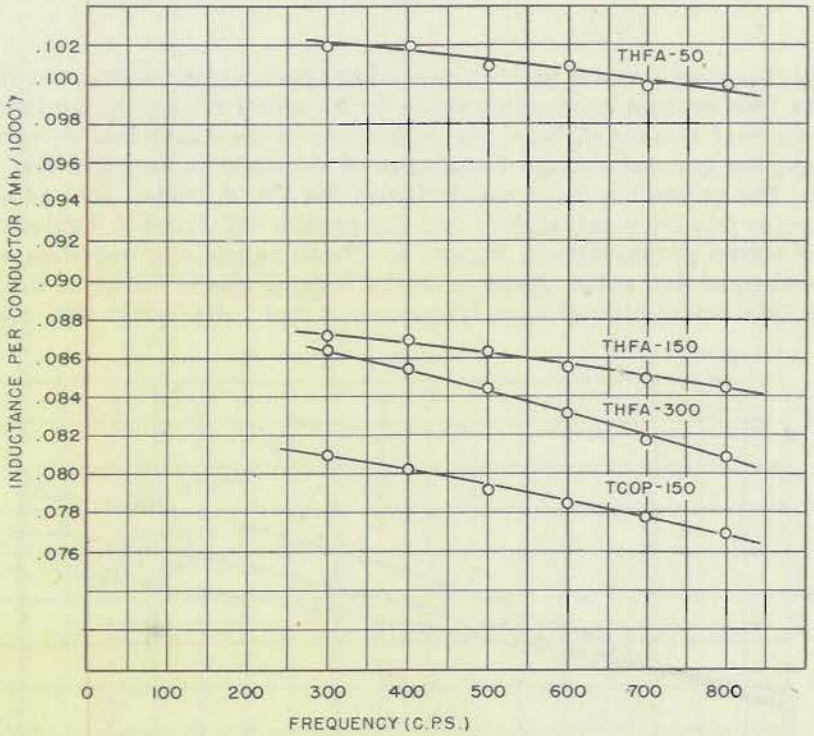

Fig. 4. Effect of Frequency on Inductance of Cable

## EFFECT OF ADJACENT MAGNETIC MATERIALS ON THE ELECTRICAL CHARACTERISTICS OF A THREE-CONDUCTOR POWER CABLE

To determine the effect of adjacent steel plates on the effective inductance and resistance, the cables were placed parallel to, and on the back of, a 4-inch steel channel, and the measurement of inductance and effective resistance were repeated. No change in these characteristics was observed over the frequency range from 60 to 800 cps.

## CAPACITANCE

Since the rated operating voltage of the cables under test is 440 volts and the usual length of cable per circuit does not exceed a few hundred feet, the capacitance charging current, even at 800 cps is negligible in comparison with the load current; therefore, no attempt was made to measure the capacitance of the cable or to calculate the charging current.

## VOLTAGE REGULATION

Efficient performance of a-c motors and other loads requires that the voltage at the terminals of the load be kept reasonably close to the rated voltage of the load device. Since the load current flowing through the impedance of the cable causes variation in the terminal voltage, the percent voltage regulation of the cable is an important operating characteristic. The percent voltage regulation of the THFA cables studied at various operating frequencies has been calculated, and the results tabulated in Tables 6, 7, and 8. The results are shown graphically in Figure 5. These values are based on a cable length of 200 feet operating at 440 volts, 3-phase, and a lagging power factor of 0.8. The load current used in this calculation at each frequency is that value which will result in the same $I^2R$ loss in the cable and, therefore, the same maximum operating temperature. The maximum operating temperature and the maximum ampere ratings of the cables are taken from pages 37 and 38 of the Cable Comparison Guide. This temperature is 105 degrees centigrade with 40 degrees centigrade ambient. For the purpose of simplicity, the ampere rating of the cable, given in the Cable Comparison Guide, is assumed to be at zero cps frequency instead of at 60 cps. The maximum load current at any higher frequency is calculated on this basis from information given in Figure 3. It will be noted that the maximum load current decreases as the frequency increases.

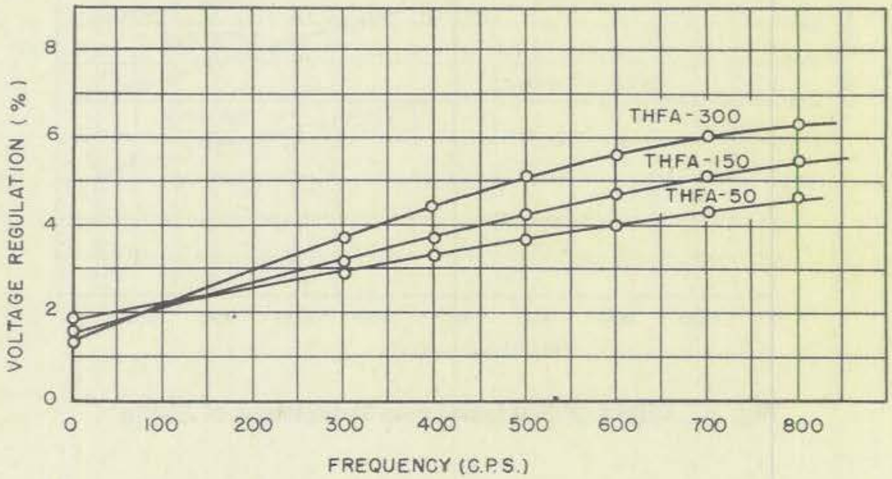

Fig. 5. Effect of Frequency on Voltage Regulation of Cable

The percent regulation is calculated at each frequency for the maximum load at that frequency by use of the equation:

$$\% \text{ Voltage Regulation} = \left[ \frac{\sqrt{(V_n \cos\theta + IR)^2 + (V_n \sin\theta + IX)^2} - V_n}{V_n} \right] 100$$

where

$V_n$ = rated voltage to neutral at load,

$\theta$ = power factor angle,

$I$ = maximum load current per conductor,

$R$ = effective or a-c resistance per conductor,

$X$ = effective inductive reactance per conductor.

Since the calculated percent voltage regulation at any frequency up to 800 cps does not greatly exceed the usual permissible value of 5 percent, the line voltage regulation does not appear to be a critical factor in the choice of operating frequency.

## INDUCTIVE INTERFERENCE

The inductive interference between a communication or control circuit and an adjacent loaded power cable was observed by placing the two cables parallel and in contact, and measuring the voltage induced in the communication circuit by means of a cathode-ray oscilloscope. Measurements of the induced voltage were made with the communication circuit terminated with various resistances ranging from 50 to 1000 ohms, as well as with a sound-powered telephone set. Measurements were made with balanced and unbalanced 800-cps power currents of 100 amperes, and with the neutral of the power cable grounded so that a third harmonic current of 20 amperes was flowing in the power cable. In addition to observing the induced voltage on the oscilloscope, the power frequency interference was observed audibly by means of a sound-powered telephone connected across one end of the communication cable under test, the other end being terminated in an equivalent impedance.

No measurable induced voltage was observed in any control or communication cable listed in Table 1 under the tests described in the preceding paragraph. The results of this interference study on twisted pairs is in agreement with the theory of inductive interference which indicates that negligible induced voltage exists when one or both circuits are transposed by twisting the conductors comprising the circuit.

## CONCLUSIONS

The results of the tests described in this report indicate the following:

(A) The effective resistance of three-conductor power cables is greater than that of the same conductors when isolated. This increase in skin effect ratio is dependent upon conductor size and is considerable for larger sizes of conductors.

(B) The maximum current capacity of power cables decreases as the frequency and conductor size increases. This reduction in capacity is considerable for cable above 150,000 cir mil area.

(C) No observable change in effective resistance or inductance of a three-conductor power cable occurs when it is placed parallel to, and in contact with, a steel plate.

(D) A slight decrease in inductance per conductor in a three-conductor power cable occurs as the operating frequency increases.

(E) The increased reactance and effective resistance of a power cable which results as the operating frequency is increased causes an increased line voltage drop under load. This drop is not excessive for frequencies below 400 cps on cable lengths of 200 feet or less.

(F) No appreciable interference voltage is induced in a communication or control circuit by a loaded adjacent three-conductor power cable, even at the closest possible spacing.

(G) The reduction in current capacity which occurs as the frequency increases appears to be the critical factor in the selection of operating frequency.

The results of this investigation indicate that shipboard power cable may be operated at frequencies higher than 60 cps, but with a reduction in current capacity and an increase in line voltage regulation. As this effect increases rapidly with conductor size, greater over-all economy in cable weight will be obtained with the use of smaller sizes of cables. No objectionable interference voltage is produced in communication or control circuits comprised of twisted pairs when adjacent to a loaded three-conductor power cable.

# APPENDIX

## DERIVATION OF EQUATIONS FOR USE IN CALCULATING EFFECTIVE RESISTANCE AND INDUCTANCE OF A THREE-CONDUCTOR CABLE

The circuit used in the measurement of the effective resistance and inductance of a three-conductor cable is shown in Figure 1 of this report. The essential portion of this diagram is also shown in Figure 6 to illustrate the derivation of the equations used in the calculation of the effective resistance and inductance of a three-conductor cable. In this circuit, balanced three-phase currents flow in the three conductors of the cable. Measurements are made on each of the conductors separately. The current $(I_c)$ in conductor A produces a voltage drop $(V_{an})$ between the point a and the neutral n. This voltage consists of a resistive component $(I_cR_c)$ in phase with the current and a reactive component $(I_cX_c)$ in quadrature with the current.

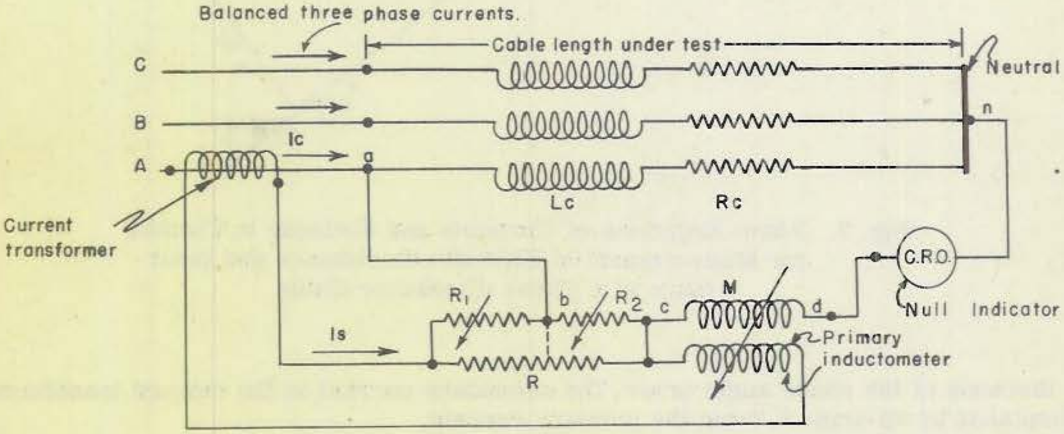

Fig. 6. Connection Diagram for Measurement of Effective Resistance and Inductance of a Three-Conductor Cable

The current $(I_s)$ in the secondary of the current transformer flows through the non-inductive resistance network consisting of the resistors R, $R_1$, and $R_2$, producing a resistive voltage drop $(V_{bc})$ between the points b and c. This secondary current flowing in the primary of the standard mutual inductometer induces a quadrature voltage $(V_{cd})$ in its secondary.

The magnitudes of $V_{bc}$ and $V_{cd}$ are independently adjustable by means of the variables $R_1$, $R_2$, and the mutual inductance M. The magnitude and phase of the total voltage $V_{bd}$, therefore, are adjustable and, with the proper connection of current transformer and inductometer, $V_{bd}$ can be made equal to and in phase opposition with $V_{an}$. This condition will result in zero voltage at the terminals of the cathode-ray oscilloscope which is used as a null indicator. The vector diagram for these voltages and currents at this condition of balance is shown in Figure 7.

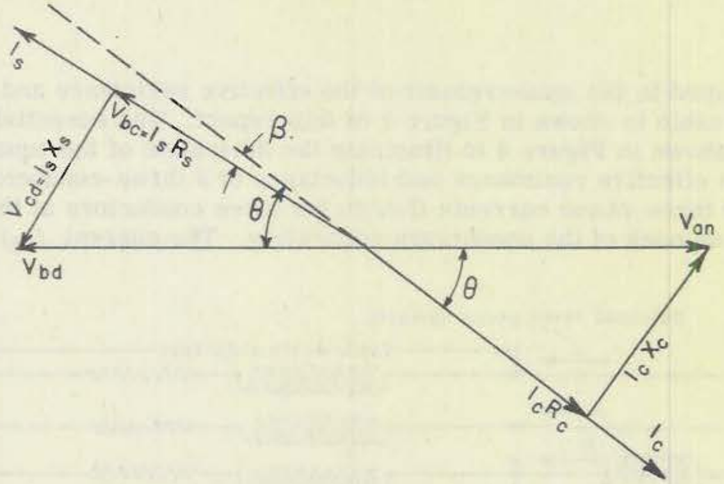

Fig. 7. Phase Relations of Currents and Voltages in Circuit for Measurement of Effective Resistance and Inductance of a Three-Conductor Cable

Because of the phase angle error, the secondary current in the current transformer is displaced by an angle $\beta$ from the primary current.

In Figure 7 the following relations may be seen:

$$V_{bd} = V_{an}, \tag{1}$$

$$I_cR_c = V_{an} \cos\theta, \tag{2}$$

$$I_sR_s = V_{bc} = V_{bd} \cos(\theta - \beta) = V_{an} \cos(\theta - \beta), \tag{3}$$

where $R_s$ is the equivalent resistance between points b and c of the resistance network; and

$$I_cX_c = V_{an} \sin\theta \tag{4}$$

$$I_sX_s = V_{cd} = V_{bd} \sin(\theta - \beta) = V_{an} \sin(\theta - \beta), \tag{5}$$

where $X_s = 2\pi fM$ and M = mutual inductance of the inductometer.

Dividing equation (3) by equation (2) gives

$$\frac{I_S R_S}{I_c R_c} = \frac{V_{bd}\cos(\theta-\beta)}{V_{an}\cos\theta} = \frac{\cos(\theta-\beta)}{\cos\theta} = \frac{\cos\theta\cos\beta + \sin\theta\sin\beta}{\cos\theta}.$$

Substituting $\frac{I_S}{I_c} = \frac{1}{r}$,

where r = true ratio of current transformer, gives

$$\frac{R_S}{rR_c} = \frac{\cos\theta\cos\beta + \sin\theta\sin\beta}{\cos\theta}.$$

Since $\beta$, the phase angle error of the current transformer, is less than 10 minutes, $\cos\beta$ is almost equal to unity. Then,

$$\frac{R_S}{rR_c} = 1 + \tan\theta\sin\beta. \tag{6}$$

Dividing equation (5) by equation (4) and substituting $\frac{I_S}{I_c} = \frac{1}{r}$, gives

$$\frac{X_S}{rX_c} = \frac{\sin(\theta-\beta)}{\sin\theta} = \frac{\sin\theta\cos\beta - \cos\theta\sin\beta}{\sin\theta}$$

$$\frac{X_S}{rX_c} = \cos\beta - \frac{\sin\beta}{\tan\theta} \approx 1 - \frac{\sin\beta}{\tan\theta} \tag{7}$$

Substituting $\frac{X_c}{R_c} = \tan\theta$ in equations (6) and (7) gives

$$\frac{R_S}{rR_c} = 1 + \frac{X_c}{R_c}\sin\beta \tag{6a}$$

$$\frac{X_S}{rX_c} = 1 - \frac{R_c}{X_c}\sin\beta \tag{7a}$$

When equations (6a) and (7a) are solved simultaneously for $R_c$ and $X_c$, the result is

$$R_c = \frac{\frac{R_S}{r} - \frac{X_S}{r}\sin\beta}{(1+\sin^2\beta)} \tag{8}$$

$$X_c = \frac{\frac{X_S}{r} + \frac{R_S}{r}\sin\beta}{(1+\sin^2\beta)} \tag{9}$$

Since $\beta$ is less than 10 minutes, $\sin^2\beta$ is negligible, and equations (8) and (9) reduce to

$$R_C = \frac{R_S - X_S \sin\beta}{r}$$

and

$$X_C = \frac{X_S + R_S \sin\beta}{r} .$$

A further approximation may be made with error of less than 0.1 percent by dropping the term containing $\sin\beta$. Then

$$R_C = \frac{R_S}{r} , \tag{8a}$$

and

$$X_C = \frac{X_S}{r} \tag{9a}$$

The equivalent resistance $R_S$ between points b and c on the resistance network is

$$R_S = \frac{R_2 R}{R_1 + R_2 + R} ,$$

and since R is negligibly small compared to $R_1 + R_2$,

$$R_S = \frac{R_2 R}{R_1 + R_2} . \tag{10}$$

Substituting this in equation (8a) gives

$$R_C = \frac{R_2 R}{r\,(R_1 + R_2)}$$

Since R, $R_1$, $R_2$, and r are known, $R_C$ may be readily calculated.

$$X_C = 2\pi f L_C$$

and

$$X_S = 2\pi f M.$$

Substituting these in equation (9a) gives

$$2\pi f L_C = \frac{2\pi f M}{r}$$

or

$$L_c = \frac{M}{r}.$$

Since M and r are known, $L_c$ may be readily calculated.

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Reference: "Resistance and Reactance of Three-Conductor Cables," by E. H. Salter, G. B. Shanklin, R. J. Wiseman, A.I.E.E. Transactions, Volume 53, 1934, Pages 1581-1589.

## SYMBOLS

The symbols used are defined in the report at the point where they are first introduced. They are repeated here for convenience in reference.

$f$ = Frequency (cycles per second)

$I_c$ = Current in one conductor of cable (amperes)

$I_s$ = Current in secondary of current transformer (amperes)

$L_c$ = Effective inductance of cable (henries per conductor)

$M$ = Mutual inductance of inductometer (henries)

$R_c$ = Effective resistance of cable (ohms per conductor)

$R_s$ = Equivalent resistance between points a and b of Figure 1a (ohms)

$R$ = Standard low resistance (ohms)

$R_1$ = Standard variable resistance (ohms)

$R_2$ = Standard variable resistance (ohms)

$r$ = True ratio of current transformer

$X_c$ = Reactance of cable (ohms per conductor)

$X_s$ = Mutual reactance of inductometer (ohms)

$\theta$ = Phase angle of cable current

$\beta$ = Phase angle error of current transformer

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