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Agency Code: 21XD

Proposal Number: 69816ELYIP
Agreement Number: W911NF-17-1-0482

INVESTIGATOR(S):

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Report Date: 31-Dec-2021
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Title: Disorder and Interactions in Dirac Materials
Begin Performance Period: 01-Sep-2017
End Performance Period: 30-Sep-2021
Report Term: 0-Other
Submitted By: Ph.D Rahul Nandkishore
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STEM Degrees:
STEM Participants:

Major Goals: To investigate the new quantum states of matter that could arise as a result of the interplay of disorder and interactions in Dirac semimetals. This was further organized into two subthrusts (i) exploring new physics driven by disorder in existing Dirac semimetal systems, and (ii) exploring correlation driven phases in emerging Dirac systems, such as nodal line semimetals and Moire heterostructures.

Accomplishments: We made a great deal of progress, substantially accomplishing all the major goals. These have been reported in 18 peer reviewed published papers, and three preprints still under review.

The single most important `research theme' involved discovery of new kinds of localized phases arising from the interplay of disorder and interactions on the surface of conventional topological insulators, a setting where localization had been previously believed to be impossible. These results were reported in a sequence of papers, namely Phys. Rev. B 98, 054205 (2018), Phys. Rev. B 99, 045125 (2019), Phys. Rev. B 99, 165108 (2019) [Editor's suggestion], and Phys. Rev. B 103, 075120 (2021). This constituted very substantial progress on `thrust 1' of our major goals. Other significant developments under the same thrust included Proc. Natl. Acad. Sci. 115, 10570 (2018) (explaining magnetoresistance in topological semimetals from interplay with disorder), Phys. Rev. B 97, 125121 (2018) (exploring interplay of disorder and Coulomb interactions in three dimensional systems with quadratic band crossings), and Phys. Rev. B 97, 184205 (2018) (explaining how a Mott glass could arise from interplay of disorder and interactions in a one dimensional Dirac system).

Meanwhile, `thrust 2' was concerned with exploring new correlation driven phases in emerging Dirac fermion systems. We also made very substantial progress on this effort, which is best organized by the system under study. We studied the following classes of systems (i) nodal line systems (ii) twisted bilayer graphene and related Moire heterostructures (iii) Kagome metals, and (iv) one dimensional Dirac systems. On nodal line systems, our key results were reported in Phys. Rev. B 97, 134521 (2018), Phys. Rev. Research 2, 043209 (2020), Phys. Rev. B 103, 081103 (2021), and arXiv:2102.04470, all exploring the unusual superconducting states that may arise in this setting. On twisted bilayer graphene and related Moire heterotructures, our results were reported in Phys. Rev. B 98, 214521 (2018), Phys. Rev. B 100, 085136 (2019), Phys. Rev. B 100, 115128 (2019), Phys. Rev. B 102, 245122 (2020), and Phys. Rev. B 101, 235121 (2020). On Kagome metals, our results were reported in Phys. Rev. B 104, 045122 (2021), arXiv:2106.09717 and arXiv:2107.09050. And finally, for one dimensional Dirac systems our results were reported in Phys. Rev. D 100, 094504 (2019) and Phys. Rev. B 101, 075430 (2020).

Training Opportunities: This grant supported the training of one PhD student (Yu-Ping Lin), who is expected to graduate this year. It also contributed to the support and training of four postdoctoral scholars: Dr. Itamar Kimchi (now faculty at Georgia Tech), Dr. Yang-Zhi Chou (now postdoc at Maryland), Dr. Jason Iaconis (now research scientist at IonQ) and Dr. Michael Pretko (now at the Department of Defense).

Results Dissemination: Results were reported in eighteen published peer reviewed papers and three preprints currently under review. I also gave six invited talks at various conferences and seminars on published work supported by this grant.

Honors and Awards: Nothing to Report

Protocol Activity Status:

Technology Transfer: Nothing to Report

PARTICIPANTS:

Participant Type: Postdoctoral (scholar, fellow or other postdoctoral position)
Participant: Yang-Zhi Chou
Person Months Worked: 9.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: Postdoctoral (scholar, fellow or other postdoctoral position)
Participant: Itamar Kimchi
Person Months Worked: 1.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: Postdoctoral (scholar, fellow or other postdoctoral position)
Participant: Jason Iaconis
Person Months Worked: 7.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: Postdoctoral (scholar, fellow or other postdoctoral position)
Participant: Michael Pretko
Person Months Worked: 2.00 **Funding Support:**
Project Contribution:
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Participant Type: PD/PI
Participant: Rahul Nandkishore
Person Months Worked: 6.00 **Funding Support:**
Project Contribution:
National Academy Member: N

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Participant Type: Graduate Student (research assistant)
Participant: Yu-Ping Lin
Person Months Worked: 15.00 **Funding Support:**
Project Contribution:
National Academy Member: N

International Travel:

CAN 6 days

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International Collaboration:

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Journal: Physical Review B
Publication Identifier Type: Publication Identifier:
Volume: Issue: First Page #:
Date Submitted: 8/21/18 12:00AM Date Published:
Publication Location:
Article Title: Gapless insulating edges of dirty interacting topological insulators
Authors: Yang-Zhi Chou, Rahul Nandkishore, Leo Radzihovsky
Keywords: topological insulators, interactions, disorder, localization, symmetry breaking
Abstract: We demonstrate that a combination of disorder and interactions in a two-dimensional bulk topo- logical insulator can generically drive its helical edge insulating. We establish this within the frame- work of helical Luttinger liquid theory and exact Emery-Luther mapping. The gapless glassy edge state spontaneously breaks time-reversal symmetry in a `spin glass' fashion, and may be viewed as a localized state of solitons which carry half integer charge. Such a qualitatively distinct edge state provides a simple explanation for heretofore puzzling experimental observations. This phase exhibits a striking non-monotonicity, with the edge growing less localized in both the weak and strong disorder limits.
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Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
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Volume: 97 Issue: First Page #: 125121
Date Submitted: 3/21/18 12:00AM Date Published: 3/15/18 6:00AM
Publication Location:
Article Title: Interplay of Coulomb interactions and disorder in three dimensional quadratic band crossings without time-reversal symmetry and with unequal masses for conduction and valence bands
Authors: Ipsita Mandal, Rahul Nandkishore
Keywords: disorder, Coulomb interactions, quadratic band crossing, Luttinger semimetal
Abstract: In this work we analyze the interplay of Coulomb interactions and disorder in three dimensional quadratic band crossing materials. Our work differs from previous analyses in that we allow disorder to break time reversal symmetry, and allow the the electron and hole bands to have different band masses. We show that whereas asymmetry in the two masses is irrelevant in the clean system, it is relevant in the presence of disorder, such that the `e♂ffective masses' of the conduction and valence bands should become sharply distinct in the low-energy limit. We further show that the problem flows to strong disorder, but time-reversal-symmetry-breaking disorder grows asymptotically more slowly than time-reversal-symmetry-preserving disorder, which we therefore expect should dominate the strong-coupling phase.
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Publication Identifier Type: DOI Publication Identifier: 10.1103/PhysRevB.97.134521
Volume: 97 Issue: First Page #: 134521
Date Submitted: 4/27/18 12:00AM Date Published:
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Article Title: Exotic Superconductivity with Enhanced Energy Scales in Three Band Crossing Materials
Authors: Yu-Ping Lin and Rahul Nandkishore
Keywords: superconductivity, three band crossing, spin one
Abstract: Three band crossings can arise in three dimensional quantum materials with certain space group symmetries. The low energy Hamiltonian supports spin one fermions and a flat band. We study the pairing problem in this setting. We write down a minimal BCS Hamiltonian and decompose it into irreducible pairing channels. We then solve the resulting gap equations in channels with zero total angular momentum. We fi♀nd that in the spin singlet channel (and also in an unusual `spin quintet' channel), superconductivity is enormously enhanced, with the critical temperature linear in interaction strength. These states also support a nodal surface of Bogoliubov-de Gennes quasiparticles. Meanwhile, in the spin triplet channel, the weak coupling BCS problem has no nontrivial solutions. Three band crossings thus represent an exciting new platform for realizing exotic superconducting states with enhanced energy scales.
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Publication Location:
Article Title: Mott glass from localization and confinement
Authors: Yang-Zhi Chou, Rahul Nandkishore and Leo Radzihovsky
Keywords: localization, confinement, Mott glass, one dimension
Abstract: We study a system of Dirac fermions in one spatial dimension with linearly confining interactions and short-range disorder. We focus on the zero temperature properties of this system, which we characterize using bosonization and the Gaussian variational method. We compute the static compressibility and ac conductivity, and thereby demonstrate that the system is incompressible, but exhibits gapless optical conductivity. This corresponds to a “Mott-glass” state, distinct from an Anderson and a fully gapped Mott insulator, arising due to the interplay of disorder and charge confinement. We argue that this “Mott-glass” phenomenology should persist to non-zero temperatures.
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Volume: 99 Issue: First Page #: 045125
Date Submitted: 7/30/19 12:00AM Date Published: 1/14/19 7:00AM
Publication Location:
Article Title: Localization driven correlated states of two isolated interacting Helical edges
Authors: Yang-Zhi Chou
Keywords: Dirac, Luttinger liquid, localization, drag
Abstract: We study the localization-driven correlated states among two isolated dirty interacting helical edges as realized at the boundaries of two-dimensional topological insulators. We show that an interplay of time-reversal symmetric disorder and strong inter-edge interactions generically drives the entire system to a gapless localized state, preempting all other intra-edge instabilities. For weaker interactions, an anti-symmetric interlocked fluid, causing a negative perfect drag, can emerge from dirty edges with different densities. The corresponding experimental signatures are also discussed.
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Volume: Issue: First Page #:
Date Submitted: 10/13/18 12:00AM Date Published:
Publication Location:
Article Title: Non-saturating large magnetoresistance in semimetals
Authors: Ian A. Leahy, Yu-Ping Lin, Peter E. Siegfried, Andrew C. Treglia, Justin C. W. Song, Rahul M. Nandkish
Keywords: Magnetoresistance, topological semimetals, disorder
Abstract: Bloch band energy dispersion, microscopic trajectories, and the disorder scattering of particles are often intimately intertwined in determining magnetoresponse in real materials. We investigate the magnetic susceptibility (?), the tangent of the Hall angle (tan?H) and magnetoresistance in four different non-magnetic semimetals with high mobilities. We find that the temperature dependence of ? and the field dependence of the longitudinal and Hall resistivities allow us to deduce the origins of the non-saturating large magnetoresistance observed in these materials. In particular, the susceptibility and transport data for NbP and TaP are strongly indicative of these materials being uncompensated semimetals with linear dispersion, in which the non-saturating magnetoresistance arises due to guiding center motion. Meanwhile, the corresponding measurements on NbSb2 and TaSb2 suggest that these materials are compensated semimetals, with a magnetoresistance that emerges from compensation.
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Journal: Physical Review B
Publication Identifier Type: Publication Identifier: 10.1103/PhysRevB.98.214521
Volume: 98 Issue: First Page #: 214521
Date Submitted: 7/30/19 12:00AM Date Published: 12/28/18 7:00AM
Publication Location:
Article Title: Kohn-Luttinger superconductivity on two orbital honeycomb lattice
Authors: Yu-Ping Lin, Rahul M. Nandkishore
Keywords: bilayer graphene, superconductivity, Kohn-Luttinger
Abstract: Motivated by experiments on twisted bilayer graphene, we study the emergence of superconductivity from weak repulsive interactions in the Hubbard model on a honeycomb lattice, with both spin and orbital degeneracies, and with the filling treated as a tunable control parameter. The attraction is generated through the Kohn-Luttinger mechanism. We find, consistent with old studies of single layer graphene, that the leading instability is in a d-wave channel close to Van Hove filling, and is in an f-wave channel away from Van Hove filling. The d-wave instability further has a twelve-fold degeneracy while the f-wave instability has a ten-fold degeneracy. We investigate the symmetry breaking perturbations to this model. Combining this with a Ginzburg-Landau analysis, we conclude that close to Van Hove filling, a spin singlet d+id state should form (consistent with several other investigations of twisted bilayer graphene), whereas away from Van Hove filling we propose an unusual spin and orbi
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Volume: 99 Issue: 16 First Page #: 165108
Date Submitted: 7/30/19 12:00AM Date Published: 4/1/19 6:00AM
Publication Location:
Article Title: Localized surfaces of three-dimensional topological insulators
Authors: Yang-Zhi Chou, Rahul M. Nandkishore, Leo Radzihovsky
Keywords: Dirac, disorder, localization, interactions, three dimensions
Abstract: We study the surface of a three-dimensional spin chiral Z2 topological insulator (class CII), demonstrating the possibility of its localization. This arises through an interplay of interaction and statistically symmetric disorder that confines the gapless fermionic degrees of freedom to a network of one-dimensional helical domain walls that can be localized. We identify two distinct regimes of this gapless insulating phase, a "clogged" regime wherein the network localization is induced by its junctions between otherwise metallic helical domain walls, and a "fully localized" regime of localized domain walls. The experimental signatures of these regimes are also discussed.
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Volume: 100 Issue: First Page #: 085136
Date Submitted: 9/11/19 12:00AM Date Published: 8/26/19 6:00AM
Publication Location:
Article Title: A chiral twist on the high-Tc phase diagram in Moire heterostructures
Authors: Yu-Ping Lin, Rahul M. Nandkishore
Keywords: Moire, graphene, parquet, renormalization, superconductor
Abstract: We show that the large orbital degeneracy inherent in Moire heterostructures naturally gives rise to a 'high-Tc' like phase diagram with a chiral twist - wherein an exotic quantum anomalous Hall insulator phase is flanked by chiral d + id superconducting domes. Specifically, we analyze repulsively interacting fermions on hexagonal (triangular or honeycomb) lattices near Van Hove filling, with an SU(Nf) flavor degeneracy. This model is inspired by recent experiments on graphene Moire heterostructures. At this point, a nested Fermi surface and divergent density of states give rise to strong (ln^2) instabilities to correlated phases, the competition between which can be controllably addressed through a combination of weak coupling parquet renormalization group and Landau-Ginzburg analysis. We solve the resulting problem, exhibiting the rich phase diagram that arises as a result of orbital degeneracy.
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Date Submitted: 1/30/20 12:00AM Date Published: 9/13/19 12:00PM
Publication Location:
Article Title: Superconductor versus insulator in twisted bilayer graphene
Authors: Yang-Zhi Chou, Yu- Ping Lin, Sankar Das Sarma and Rahul M. Nandkishore
Keywords: Graphene, superconductor, insulator, commensuration, Luttinger
Abstract: We present a simple model that we believe captures the key aspects of the competition between superconducting and insulating states in twisted bilayer graphene. Within this model, the superconducting phase is primary, and arises at generic fillings, but is interrupted by the insulator at commensurate fillings. Importantly, the insulator forms because of electronelectron interactions, but the model is agnostic as to the superconducting pairing mechanism, which need not originate with electron-electron interactions. The model is composed of a collection of crossed one-dimensional quantum wires whose intersections form a superlattice. At each superlattice point, we place a locally superconducting puddle which can exchange Cooper pairs with the quantum wires. We analyze this model assuming weak wire-puddle and wire-wire couplings. We show that the system is superconducting at `generic' incommensurate fillings, and insulating at commensuration, consistent with the experiments.
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Date Submitted: 8/17/20 12:00AM Date Published: 6/1/20 6:00AM
Publication Location:
Article Title: Magic-angle semimetals with chiral symmetry
Authors: Yang-Zhi Chou, Yixing Fu, Justin H. Wilson, E. J. König, J. H. Pixley
Keywords: Graphene, Moire, quasi periodic
Abstract: We construct and solve a two-dimensional, chirally symmetric model of Dirac cones subjected to a quasiperiodic modulation. In real space, this is realized with a quasiperiodic hopping term. This hopping model, as we show, at the Dirac node energy has a rich phase diagram with a semimetal-to-metal phase transition at intermediate amplitude of the quasiperiodic modulation, and a transition to a phase with a diverging density of states (DOS) and subdiffusive transport when the quasiperiodic hopping is strongest. We further demonstrate that the semimetal-to-metal phase transition can be characterized by the multifractal structure of eigenstates in momentum space and can be considered as a unique “unfreezing” transition. This unfreezing transition in momentum space generates flat bands with a dramatically renormalized bandwidth in the metallic phase similar to the phenomena of the band structure of twisted bilayer graphene at the magic angle. We characterize the nature of this transition n
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Volume: 101 Issue: 7 First Page #:
Date Submitted: 8/17/20 12:00AM Date Published: 2/1/20 2:00PM
Publication Location:
Article Title: Nonmonotonic plasmon dispersion in strongly interacting Coulomb Luttinger liquids
Authors: Yang-Zhi Chou, Sankar Das Sarma
Keywords: Dirac, one dimension, plasmon, Coulomb interaction
Abstract: We construct and solve a two-dimensional, chirally symmetric model of Dirac cones subjected to a quasiperiodic modulation. In real space, this is realized with a quasiperiodic hopping term. This hopping model, as we show, at the Dirac node energy has a rich phase diagram with a semimetal-to-metal phase transition at intermediate amplitude of the quasiperiodic modulation, and a transition to a phase with a diverging density of states (DOS) and subdiffusive transport when the quasiperiodic hopping is strongest. We further demonstrate that the semimetal-to-metal phase transition can be characterized by the multifractal structure of eigenstates in momentum space and can be considered as a unique “unfreezing” transition. This unfreezing transition in momentum space generates flat bands with a dramatically renormalized bandwidth in the metallic phase similar to the phenomena of the band structure of twisted bilayer graphene at the magic angle. We characterize the nature of this transition
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Journal: Physical Review D
Publication Identifier Type: DOI Publication Identifier: 10.1103/PhysRevD.100.094504
Volume: 100 Issue: 9 First Page #:
Date Submitted: 8/17/20 12:00AM Date Published: 11/1/19 6:00AM
Publication Location:
Article Title: Phase structure of the (1+1) dimensional massive Thirring model from Matrix Product States
Authors: Mari Carmen Bañuls, Krzysztof Cichy, Ying-Jer Kao, C.-J. David Lin, Yu-Ping Lin, David T.-L. Tar
Keywords: Dirac, Thirring model, matrix product simulation
Abstract: We construct and solve a two-dimensional, chirally symmetric model of Dirac cones subjected to a quasiperiodic modulation. In real space, this is realized with a quasiperiodic hopping term. This hopping model, as we show, at the Dirac node energy has a rich phase diagram with a semimetal-to-metal phase transition at intermediate amplitude of the quasiperiodic modulation, and a transition to a phase with a diverging density of states (DOS) and subdiffusive transport when the quasiperiodic hopping is strongest. We further demonstrate that the semimetal-to-metal phase transition can be characterized by the multifractal structure of eigenstates in momentum space and can be considered as a unique “unfreezing” transition. This unfreezing transition in momentum space generates flat bands with a dramatically renormalized bandwidth in the metallic phase similar to the phenomena of the band structure of twisted bilayer graphene at the magic angle. We characterize the nature of this transition
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Date Submitted: 10/11/21 12:00AM Date Published: 11/9/20 7:00AM
Publication Location:
Article Title: Chiral flat band superconductivity from symmetry-protected three-band crossings
Authors: Yu-Ping Lin
Keywords: Dirac, superconductivity, flat band
Abstract: We construct and solve a two-dimensional, chirally symmetric model of Dirac cones subjected to a quasiperiodic modulation. In real space, this is realized with a quasiperiodic hopping term. This hopping model, as we show, at the Dirac node energy has a rich phase diagram with a semimetal-to-metal phase transition at intermediate amplitude of the quasiperiodic modulation, and a transition to a phase with a diverging density of states (DOS) and subdiffusive transport when the quasiperiodic hopping is strongest. We further demonstrate that the semimetal-to-metal phase transition can be characterized by the multifractal structure of eigenstates in momentum space and can be considered as a unique "unfreezing" transition. This unfreezing transition in momentum space generates flat bands with a dramatically renormalized bandwidth in the metallic phase similar to the phenomena of the band structure of twisted bilayer graphene at the magic angle. We characterize the nature of this transition
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Volume: 103 Issue: First Page #: L081103
Date Submitted: 10/11/21 12:00AM Date Published: 2/8/21 7:00AM
Publication Location:
Article Title: Dual Haldane sphere and quantized band geometry in chiral multifold fermions
Authors: Yu-Ping Lin and Wei-Han Hsiao
Keywords: Dirac, topological superconductor, flat band
Abstract: We show that the chiral multifold fermions present a dual Haldane sphere problem in momentum space. Owing to the Berry monopole at the degenerate point, a dual Landau level emerges in the trace of quantum metric, with which a quantized geometric invariant is defined through a surface integration. We further demonstrate potential manifestations in the measurable physical observables. With a lower bound derived for the finite spread of Wannier functions, anomalous phase coherence is identified accordingly for the flat band superconductivity. Potential experimental probes of the quantum metric are also discussed.
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Article Title: Parquet renormalization group analysis of weak-coupling instabilities with multiple high-order Van Hove points inside the Brillouin zone
Authors: Yu-Ping Lin and Rahul M. Nandkishore
Keywords: Twisted bilayer graphene, renormalization group, high order Van Hove
Abstract: We analyze the weak-coupling instabilities that may arise when multiple high-order Van Hove points are present inside the Brillouin zone. The model we consider is inspired by twisted bilayer graphene, although the analysis should be more generally applicable. We employ a parquet renormalization group analysis to identify the leading weak-coupling instabilities, supplemented with a Ginzburg-Landau treatment to resolve any degeneracies. Hence we identify the leading instabilities that can occur from weak repulsion with the power-law divergent density of states. Four correlated phases are uncovered along distinct fixed trajectories, including the s-wave ferromagnetism, p-wave chiral/helical superconductivity, f-wave valley-polarized order, and p-wave polar valley-polarized order. The phase diagram is stable against band deformations which preserve high-order Van Hove singularity.
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Volume: 103 Issue: 7 First Page #: 075120
Date Submitted: 10/11/21 12:00AM Date Published: 2/1/21 7:00AM
Publication Location:
Article Title: Marginally localized edges of time-reversal symmetric topological superconductors
Authors: Yang-Zhi Chou, Rahul M. Nandkishore
Keywords: Localization, majorana, topological superconductor
Abstract: We demonstrate that the one-dimensional helical Majorana edges of two-dimensional time-reversal symmetric topological superconductors (class DIII) can become gapless and insulating by a combination of random edge velocity and interaction. Such a gapless insulating edge breaks time-reversal symmetry inhomogeneously, and the local symmetry broken regions can be regarded as static mass potentials or dynamical Ising spins. In both limits, we find that such gapless insulating Majorana edges are generically exponentially localized and trap Majorana zero modes. Interestingly, for a statistically time-reversal symmetric edge (symmetry is broken locally, but the symmetry breaking order parameter is zero on average), the low-energy theory can be mapped to a Dyson model at zero energy, manifesting a diverging density of states and exhibiting marginal localization (i.e., a diverging localization length). Experimental signatures are also discussed.
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Authors: Yu-Ping Lin, Rahul M. Nandkishore
Keywords: kagome metals, chern insulator, charge density wave
Abstract: We investigate how the real and imaginary charge density waves interplay at the Van Hove singularity on the hexagonal lattices. A phenomenological analysis indicates the formation of 3Q complex orders at all three nesting momenta. Under a total phase condition, unequal phases at the three momenta break the rotation symmetry generally. The 3Q complex orders constitute a rich Haldane-model phase diagram. When effective time-reversal symmetries arise under 1-site translations, the Dirac semimetals are protected. The breakdown of these symmetries gaps the Dirac points and leads to the trivial and Chern insulator phases. These phases are deformations of purely real and imaginary orders, which exhibit trivial site and/or bond density and chiral flux orders, respectively. The exotic single-Dirac-point semimetals also appear along the gapless phase boundary. We comment on relation for recent experiments on Kagome metals.
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Authors: Yu-Ping Lin, Rahul M. Nandkishore
Keywords: Kagome superconductors, double dome
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Article Title: Band geometry from position-momentum duality at topological band crossings
Authors: Yu-Ping Lin, Wei-Han Hsiao
Keywords: band geometry, topological band crossings,
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Partners

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International collaborationsThe work [Phys. Rev. B 97, 125121 (2018)] involved a collaboration with Dr. Ipsita Manda

I certify that the information in the report is complete and accurate:
Signature: Rahul Nandkishore
Signature Date: 11/11/21 8:57PM

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1) Summary of accomplishments made during this grant period

We had a very successful grant and successfully accomplished all the major goals. Results were reported in eighteen published papers and three preprints under review. These are listed below.

(i) **Interplay of Coulomb interactions and disorder in three dimensional quadratic band crossings without time-reversal symmetry and with unequal masses for conduction and valence bands** Ipsita Mandal and Rahul M. Nandkishore, Phys. Rev. B 97, 125121 (2018)

Coulomb interactions famously drive three dimensional quadratic band crossing semimetals into a non-Fermi liquid phase of matter. In a previous work by the PI, Phys. Rev. B 95, 205106 (2017), the effect of disorder on this non-Fermi liquid phase was investigated, assuming that the bandstructure was isotropic, assuming that the conduction and valence bands had the same band mass, and assuming that the disorder preserved exact time-reversal symmetry and statistical isotropy. It was shown that the non-Fermi liquid fixed point is unstable to disorder, and that a runaway flow to strong disorder occurs. In this work, we extend that analysis by relaxing the assumption of time-reversal symmetry and allowing the electron and hole masses to differ (but continuing to assume isotropy of the low energy bandstructure). We first incorporate time-reversal symmetry breaking disorder, and demonstrate that there do not appear any new fixed points. Moreover, while the system continues to flow to strong disorder, time-reversal-symmetry-breaking disorder grows asymptotically more slowly than time-reversal-symmetry-preserving disorder, which we therefore expect should dominate the strong-coupling phase. We then allow for unequal electron and hole masses. We show that whereas asymmetry in the two masses is irrelevant in the clean system, it is relevant in the presence of disorder, such that the 'effective masses' of the conduction and valence bands should become sharply distinct in the low-energy limit. We calculate the RG flow equations for the disordered interacting system with unequal band masses, and demonstrate that the problem exhibits a runaway flow to strong disorder. Along the runaway flow, time-reversal-symmetry-preserving disorder grows asymptotically more rapidly than both time-reversal-symmetry-breaking disorder and the Coulomb interaction.

(ii) **Mott glass from localization and confinement** Yang-Zhi Chou, Rahul M. Nandkishore and Leo Radzihovsky, *Phys. Rev. B* 97, 184205 (2018)

A Mott glass is a conjectured phase of matter which is incompressible, but looks gapless in optical conductivity. No examples of a Mott glass phase are presently known. In this work, we showed that a Mott glass phase can unambiguously be realized in a one dimensional Dirac fermion system with confining interactions. Specifically, we studied a system of fermions in one spatial dimension with linearly confining interactions and short-range disorder. We focused on the zero temperature properties of this system, which we characterized using bosonization and the Gaussian variational method. We computed the static compressibility and ac conductivity, and thereby demonstrated that the system is incompressible, but exhibits gapless optical conductivity. This corresponds to a "Mott-glass" state, distinct from an Anderson and a fully gapped Mott insulator, arising due to the interplay of disorder and charge confinement. We argued that this "Mott-glass" phenomenology should persist to non-zero temperatures.

(iii) **Gapless insulating edges of dirty interacting topological insulators** Yang-Zhi Chou, Rahul M. Nandkishore and Leo Radzihovsky, Phys. Rev. B 98, 054205 (2018)

It is widely believed that at the boundary of a bulk topological insulator, there must be a metallic state that is protected against localization. In the presence of topological order or symmetry breaking, a gapped edge is also possible. Here we demonstrate the existence of a third possibility. Specifically, we demonstrate that a combination of disorder and interactions in a two-dimensional bulk topological insulator can generically lead to a localized (gapless and insulating) edge. We establish this within the framework of helical Luttinger liquid theory and exact Emery-Luther mapping. The gapless glassy edge state spontaneously breaks time-reversal symmetry in a `spin glass' fashion, and may be viewed as a localized state of solitons which carry half integer charge. Such a qualitatively distinct edge state provides a simple explanation for heretofore puzzling experimental observations. This phase exhibits a striking non-monotonicity, with the edge growing less localized in both the weak and strong disorder limits. It also breaks new ground in our understanding of what can happen at the boundary of a topological insulator.

(iv) **Localization driven correlated states of two isolated interacting Helical edges***,* Yang-Zhi Chou, Phys. Rev. B 99, 045125 (2019)

In this work (carried out by the PI's ARO funded postdoc), we examined the behavior of the surface states of two 2-dimensional topological insulator samples placed in close proximity. Building on our earlier work (i) above, we showed that an interplay of time-reversal symmetric disorder and strong inter-edge interactions generically drives the entire system to a gapless localized state, preempting all other intra-edge instabilities. The instability to localization sets in for weaker interactions than are needed for a single isolated edge. We also identify a parameter regime in which the `symmetric' mode is localized, but the antisymmetric mode is not. This regime corresponds to a system with a negative perfect drag between the two edges. The corresponding experimental signatures are also discussed.

(v) **Localized phases of three dimensional topological insulators,** Yang-Zhi Chou, Rahul Nandkishore and Leo Radzihovsky. Phys. Rev. B 99, 165108 (2019) [Editor's suggestion]

This work extends the notion of localization on the surface of disordered interacting topological insulators (introduced in (i) above) to topological insulators in three spatial dimensions.

Specifically, we study the surface of a three-dimensional spin chiral Z2 topological insulator (class CII), demonstrating the possibility of its localization. This arises through an interplay of interaction and statistically-symmetric disorder, that confines the gapless fermionic degrees of freedom to a network of one-dimensional helical domain-walls that can be localized. We identify two distinct regimes of this gapless insulating phase, a `clogged' regime wherein the network localization is induced by its junctions between otherwise metallic helical domain-walls, and a `fully localized' regime of localized domain-walls. The experimental signatures of these regimes are also discussed.

(vi) **Exotic Superconductivity with Enhanced Energy Scales in Materials with Three Band Crossings** Yu-Ping Lin and Rahul Nandkishore**,** *Phys. Rev. B* 97, 134521 (2018)

Three band crossings can arise in three-dimensional quantum materials with certain space group symmetries. The low energy Hamiltonian supports spin *one* fermions and a flat band. We study the pairing problem in this setting. We write down a minimal BCS Hamiltonian and decompose it into spin-orbit coupled irreducible pairing channels. We then solve the resulting gap equations in channels with zero total angular momentum. We find that in the *s*-wave spin singlet channel (and also in an unusual *d*-wave 'spin quintet' channel), superconductivity is enormously enhanced, with a possibility for the critical temperature to be *linear* in interaction strength. Meanwhile, in the *p*-wave spin triplet channel, the superconductivity exhibits features of conventional BCS theory due to the absence of flat band pairing. Three band crossings thus represent an exciting new platform for realizing exotic superconducting states with enhanced energy scales. We also discuss the effects of doping, nonzero temperature, and of retaining additional terms in the k▪p expansion of the Hamiltonian.

(vii) **Kohn Luttinger superconductivity on two orbital honeycomb lattice** Yu-Ping Lin and Rahul M. Nandkishore, Phys. Rev. B 98, 214521 (2018)

Motivated by experiments on twisted bilayer graphene, we study the emergence of superconductivity from weak repulsive interactions in the Hubbard model on a honeycomb lattice, with both spin and orbital degeneracies, and with the filling treated as a tuneable control parameter. The attraction is generated through the Kohn-Luttinger mechanism. We find, consistent with old studies of single layer graphene, that the leading instability is in a d-wave channel close to Van Hove filling, and is in an f-wave channel away from Van Hove filling. The d-wave instability further has a twelve-fold degeneracy while the f-wave instability has a ten-fold degeneracy. We investigate the symmetry breaking perturbations to this model. Combining this with a Ginzburg-Landau analysis, we conclude that close to Van Hove filling, a spin singlet d + id state should form (consistent with several other investigations of twisted bilayer graphene), whereas away from Van Hove filling we propose an unusual spin and orbital singlet f-wave state.

(viii) **Non-saturating large magnetoresistance in semimetals** Ian A. Leahy, Yu-Ping Lin, Peter E. Seigfried, Andrew C. Treglia, Justin C.W. Song, Rahul M. Nandkishore and Minhyea Lee, Proc. Natl. Acad. Sci. 115, 10570 (2018)

Bloch band energy dispersion, microscopic trajectories, and the disorder scattering of particles are often intimately intertwined in determining magnetoresponse in real materials. We investigate the magnetic susceptibility (χ), the tangent of the Hall angle (tanθH) and magnetoresistance in four different non-magnetic semimetals with high mobilities. We find that the temperature dependence of χ and the field dependence of the longitudinal and Hall resistivities allow us to deduce the origins of the non-saturating large magnetoresistance observed in these materials. In particular, the susceptibility and transport data for NbP and TaP are strongly indicative of these materials being uncompensated semimetals with linear dispersion, in which the non-saturating magnetoresistance arises due to guiding center motion. Meanwhile, the corresponding measurements on NbSb2 and TaSb2 suggest that these materials are compensated semimetals, with a magnetoresistance that emerges from charge compensation. Our results indicate how a combination of magnetotransport and susceptibility measurements may be used to categorize the increasingly ubiquitous non-saturating large magnetoresistance reported in topological semimetals.

(ix) **A chiral twist on the high-Tc phase diagram in Moire heterostructures**. Yu-Ping Lin and Rahul M. Nandkishore, *Phys. Rev. B*, 100, 085136 (2019)

We show that the large orbital degeneracy inherent in Moire heterostructures naturally gives rise to a 'high-Tc' like phase diagram with a chiral twist - wherein an exotic quantum anomalous Hall insulator phase is flanked by chiral d + id superconducting domes. Specifically, we analyze repulsively interacting fermions on hexagonal (triangular or honeycomb) lattices near Van Hove filling, with an SU(Nf) flavor degeneracy. This model is inspired by recent experiments on graphene Moire heterostructures. At this point, a nested Fermi surface and divergent density of states give rise to strong (ln^2) instabilities to correlated phases, the competition between which can be controllably addressed through a combination of weak coupling parquet renormalization group and Landau-Ginzburg analysis. For Nf = 2 (i.e. spin degeneracy only) it is known that chiral d + id superconductivity is the unambiguously leading weak coupling instability. Here we show that Nf ≥ 4 leads to a richer (but still unambiguous and fully controllable) behavior, wherein at weak coupling the leading instability is to a fully gapped and chiral Chern insulator, characterized by a spontaneous breaking of time reversal symmetry and a quantized Hall response. Upon doping this phase gives way to a chiral d+id superconductor. We further consider deforming this minimal model by introducing an orbital splitting of the Van Hove singularities, and discuss the resulting RG flow and phase diagram. Our analysis thus bridges the minimal model and the practical Moire band structures, thereby providing a transparent picture of how the correlated phases arise under various circumstances. Meanwhile, a similar analysis on the square lattice predicts a phase diagram where (for Nf > 2) a nodal staggered flux phase with 'loop current' order gives way upon doping to a nodal d-wave superconductor.

(x) **Superconductor versus insulator in twisted bilayer graphene.** Yang-Zhi Chou, Yu-Ping Lin, Sankar Das Sarma and Rahul M. Nandkishore, Phys. Rev. B **100**, 115128 (2019)

We present a simple model that we believe captures the key aspects of the competition between superconducting and insulating states in twisted bilayer graphene. Within this model, the superconducting phase is primary, and arises at generic fillings, but is interrupted by the insulator at commensurate fillings. Importantly, the insulator forms because of electron-electron interactions, but the model is agnostic as to the superconducting pairing mechanism, which need not originate with electron-electron interactions. The model is composed of a collection of crossed one-dimensional quantum wires whose intersections form a superlattice. At each superlattice point, we place a locally superconducting puddle which can exchange Cooper pairs with the quantum wires. We analyze this model assuming weak wire-puddle and wire-wire couplings. We show that for a range of repulsive intrawire interactions, the system is superconducting at `generic' incommensurate fillings, with the superconductivity being `interrupted' by an insulating phase at commensurate fillings. We further show that the gapped insulating states at commensurate fillings give way to gapless states upon application of external Zeeman fields. These features are consistent with experimental observations in magic-angle twisted bilayer graphenes despite the distinct microscopic details. We further study the full phase diagram of this model and discover that it contains several distinct correlated insulating states, which we characterize herein.

(xi) **Magic-angle semimetals with chiral symmetry.** Yang-Zhi Chou ,Yixing Fu , Justin H. Wilson, E. J. König, and J. H. Pixley. *Phys. Rev. B* 101, 235121 (2020)

We construct and solve a two-dimensional, chirally symmetric model of Dirac cones subjected to a quasiperiodic modulation. In real space, this is realized with a quasiperiodic hopping term. This hopping model, as we show, at the Dirac node energy has a rich phase diagram with a semimetal-to-metal phase transition at intermediate amplitude of the quasiperiodic modulation, and a transition to a phase with a diverging density of states (DOS) and subdiffusive transport when the quasiperiodic hopping is strongest. We further demonstrate that the semimetal-to-metal phase transition can be characterized by the multifractal structure of eigenstates in momentum space and can be considered as a unique “unfreezing” transition. This unfreezing transition in momentum space generates flat bands with a dramatically renormalized bandwidth in the metallic phase similar to the phenomena of the band structure of twisted bilayer graphene at the magic angle. We characterize the nature of this transition numerically as well as analytically in terms of the formation of a band of topological zero modes. For pure quasiperiodic hopping, we provide strong numerical evidence that the low-energy DOS develops a divergence and the eigenstates exhibit Chalker (quantum-critical) scaling despite the model not being random. At particular commensurate limits the model realizes higher-order topological insulating phases. We discuss how these systems can be realized in experiments on ultracold atoms and metamaterials.

(xii) **Nonmonotonic plasmon dispersion in strongly interacting Coulomb Luttinger liquids.** Yang-Zhi Chou and Sankar Das Sarma. Phys. Rev. B 101, 075430 (2020)

We demonstrate that the plasmon in one-dimensional Coulomb interacting electron fluids can develop a finite momentum maxon-roton-like nonmonotonic energy-momentum dispersion. Such an unusual nonmonotonicity arises from the strongly interacting 1/r Coulomb potential going beyond the conventional band linearization approximation used in the standard bosonization theories of Luttinger liquids. We provide details for the nonmonotonic plasmon dispersion using both bosonization and random-phase approximation theories. We also calculate the specific heat including the nonmonotonicity and discuss possibilities for observing the nonmonotonic plasmon dispersion in various physical systems, including semiconductor quantum wires, carbon nanotubes, and the twisted bilayer graphene at subdegree twist angles, which naturally realize one-dimensional domain-wall states. We provide results for several different models of long-range interaction showing that the nonomonotonic charge collective mode dispersion is a generic phenomenon in one-dimensional strongly interacting electron systems.

(xiii) **Phase structure of the (1 + 1)-dimensional massive Thirring model from matrix product states.** Mari Carmen Bañuls, Krzysztof Cichy, Ying-Jer Kao, C. -J. David Lin, Yu-Ping Lin, David T. -L. Tan, Phys. Rev. D 100, 094504 (2019)

Employing matrix product states as an ansatz, we study the nonthermal phase structure of the (1 + 1)-dimensional massive Thirring model in the sector of a vanishing total fermion number with staggered regularization. In this paper, details of the implementation for this project are described. To depict the phase diagram of the model, we examine the entanglement entropy, the fermion bilinear condensate, and two types of correlation functions. Our investigation shows the existence of two phases, with one of them being critical and the other gapped. An interesting feature of the phase structure is that the theory with the nonzero fermion mass can be conformal. We also find clear numerical evidence that these phases are separated by a transition of the Berezinskii-Kosterlitz-Thouless type. Results presented in this paper establish the possibility of using the matrix product states for probing this type of phase transition in quantum field theories. They can provide information for further exploration of scaling behavior, and they serve as an important ingredient for controlling the continuum extrapolation of the model.

(xiv) **Marginally localized edges of time reversal symmetric topological superconductors.** Yang-Zhi Chou and Rahul M. Nandkishore, *Phys. Rev. B*, **103**, 075120 (2021)

We demonstrate that the one-dimensional helical Majorana edges of two-dimensional time-reversal symmetric topological superconductors (class DIII) can become gapless and insulating by a combination of random edge velocity and interaction. Such a gapless insulating edge breaks time-reversal symmetry inhomogeneously, and the local symmetry broken regions can be regarded as static mass potentials or dynamical Ising spins. In both limits, we find that such gapless insulating Majorana edges are generically exponentially localized and trap Majorana zero modes. Interestingly, for a statistically time-reversal symmetric edge (symmetry is broken locally, but the symmetry breaking order parameter is zero on average), the low-energy theory can be mapped to a Dyson model at zero energy, manifesting a diverging density of states and exhibiting marginal localization (i.e., a diverging localization length). Although the ballistic edge state transport is absent, the localized Majorana zero modes reflect the nontrivial topology in the bulk. Experimental signatures are also discussed.

(xv) **Parquet renormalization group analysis of weak-coupling instabilities with multiple high-order Van Hove points inside the Brillouin zone**. Yu-Ping Lin and Rahul M. Nandkishore, *Phys. Rev. B* **102**, 245122 (2020)

We analyze the weak-coupling instabilities that may arise when multiple high-order Van Hove points are present inside the Brillouin zone. The model we consider is inspired by twisted bilayer graphene, although the analysis should be more generally applicable. We employ a parquet renormalization group analysis to identify the leading weak-coupling instabilities, supplemented with a Ginzburg-Landau treatment to resolve any degeneracies. Hence we identify the leading instabilities that can occur from weak repulsion with the power-law divergent density of states. Five correlated phases are uncovered along distinct stable fixed trajectories, including s-wave ferromagnetism, p-wave chiral/helical superconductivity, d-wave chiral superconductivity, f-wave valley-polarized order, and p-wave polar valley-polarized order. The phase diagram is stable against band deformations which preserve the high-order Van Hove singularity.

(xvi) **Dual Haldane sphere and quantized band geometry in chiral multifold fermions**. Yu-Ping Lin and Wei-Han Hsiao. *Phys. Rev. B* **103**, L081103 (2021)

We show that the chiral multifold fermions present a dual Haldane sphere problem in momentum space. Owing to the Berry monopole at the degenerate point, a dual Landau level emerges in the trace of quantum metric, with which a quantized geometric invariant is defined through a surface integration. We further demonstrate potential manifestations in the measurable, physical observables. With a lower bound derived for the finite spread of Wannier functions, anomalous phase coherence is identified accordingly for the flat band superconductivity. We briefly comment on the stability of these results under perturbations. Potential experimental probes of the quantum metric are also discussed.

(xvii) **Chiral flat band superconductivity from symmetry-protected three-band crossings**. Yu-Ping Lin. *Phys. Rev. Research* **2**, 043209 (2021)

We show that chiral (nearly) flat band superconductivity can develop and host novel Majorana fermions at a time-reversal pair of symmetry-protected three-band crossing points. Based on symmetry analysis, mean-field study, and superfluid stiffness calculation, we determine and analyze the irreducible pairing channels with flat band pairings in the low-energy spin-1 fermion theory. Flat band pairing can enhance superconductivity dramatically, where the critical temperature scales linearly in the interaction strength. While fully gapped flat band pairing states develop in the single-component pairing channels, we find chiral p±ip flat band superconductivity in the multicomponent pairing channels. Three-dimensional itinerant Majorana fermions arise at the bulk nodal points, whereas Majorana arcs appear on the surface.

(xviii) **Complex charge density waves at Van Hove singularity on hexagonal lattices: Haldane-model phase diagram and potential realization in the kagome metals AV3Sb5 (A = K, Rb, Cs)** *[Editor's suggestion].* Yu-Ping Lin and Rahul M. Nandkishore, *Phys. Rev. B* **104**, 045122 (2021)

We investigate how the real and imaginary charge density waves interplay at the Van Hove singularity on the hexagonal lattices. A phenomenological analysis indicates the formation of 3Q complex orders at all three nesting momenta. Under a total phase condition, unequal phases at the three momenta break the rotation symmetry generally. The 3Q complex orders constitute a rich Haldane-model phase diagram. When effective time-reversal symmetries arise under 1-site translations, the Dirac semimetals are protected. The breakdown of these symmetries gaps the Dirac points and leads to the trivial and Chern insulator phases. These phases are deformations of purely real and imaginary orders, which exhibit trivial site and/or bond density and chiral flux orders, respectively. The exotic single-Dirac-point semimetals also appear along the gapless phase boundary. We further show that the theoretical model offers transparent interpretations of experimental observations in the kagome metals AV3Sb5 with A=K,Rb,Cs. The topological charge density waves may be identified with the complex orders in the Chern insulator phase. Meanwhile, the lower-temperature symmetry-breaking phenomena may be interpreted as the secondary orders from the complex order ground states. Our work sheds light on the nature of the topological charge density waves in the kagome metals AV3Sb5 and may offer useful indications to the experimentally observed charge orders in the future experiments.

Preprints

(i) **Higher-order topological insulators from 3Q charge bond orders on hexagonal lattices: a hint to kagome metals.** Yu-Ping Lin, arXiv: 2106.09717

We show that unconventional boundary phenomena occur in the 3Q charge bond orders on the hexagonal lattices. At the Van Hove singularity with three nesting momenta, 3Q orders can trigger a C6-symmetric insulator under bond modulations. On the kagome lattice, in-gap corner states appear in the energy spectrum and carry fractional corner charge −2e/3. Such corner phenomena originate from the corner filling anomaly and indicate a higher-order topological insulator. The ingap corner states are also observed on the triangular lattice. The honeycomb lattice does not support fractional corner charges, while in-gap edge states are observed. We discuss possible indications to the experimentally uncovered charge bond orders in the kagome metals AV3Sb5 with A = K, Rb, Cs. With layer stacking along the out-of-plane direction, the corner states can constitute the hinge states with fractional charge densities.

(ii) **Band geometry from position-momentum duality at topological band crossings**. Yu-Ping Lin and Wei-Han Hsiao, arXiv: 2102.04470

We show that the position-momentum duality offers a transparent interpretation of the band geometry at the topological band crossings. Under this duality, the band geometry with Berry connection is dual to the free-electron motion under gauge field. This identifies the trace of quantum metric as the dual energy in momentum space. The band crossings with Berry defects thus induce the dual energy quantization in the trace of quantum metric. For a Z nodal-point or nodal-surface semimetal, a dual Landau level quantization occurs owing to the Berry charge. Meanwhile, a nodalloop semimetal exhibits a Berry vortex line, leading to the quantized dual rotational energy about the nodal loop. A Z2 monopole brings about another dual rotational energy, which originates from the link with an additional nodal line. Nontrivial band geometry generically induces finite spread in the Wannier functions. While the spread manifests a quantized lower bound in a Z nodal-point or nodal-surface semimetal, logarithmic divergence occurs in a nodal-loop semimetal. The band geometry at the band crossings may be probed experimentally by a periodic-drive measurement.

(iii) **Kagome superconductors from Pomeranchuk fluctuations in charge density wave metals.** Yu-Ping Lin and Rahul M. Nandkishore, arXiv: 2107.09050

Motivated by the recent experiments on the kagome metals AV3Sb5 with A = K, Rb, Cs, which see onset of charge density wave (CDW) order at ~ 100 K and superconductivity at ~ 1 K, we explore the onset of superconductivity, taking the perspective that it descends from a parent CDW state. In particular, we propose that the pairing comes from the Pomeranchuk fluctuations of the reconstructed Fermi surface in the CDW phase. This scenario naturally explains the large separation of energy scale from the parent CDW. Remarkably, the phase diagram hosts the doubledome superconductivity near two reconstructed Van Hove singularities. These singularities occur at the Lifshitz transition and the quantum critical point of the parent CDW. The first dome is occupied by the dxy-wave nematic spin-singlet superconductivity. Meanwhile, the (s+dx2−y2)-wave nematic spin-singlet superconductivity develops in the second dome. Our work sheds light on an unconventional pairing mechanism with strong evidences in the kagome metals AV3Sb5.

2) **Problems or delays** (If applicable)

None

3) Submissions or publications

Published articles

(i) **Interplay of Coulomb interactions and disorder in three dimensional quadratic band crossings without time-reversal symmetry and with unequal masses for conduction and valence bands** Ipsita Mandal and Rahul M. Nandkishore, Phys. Rev. B 97, 125121 (2018)

(ii) **Mott glass from localization and confinement** Yang-Zhi Chou, Rahul M. Nandkishore and Leo Radzihovsky, *Phys. Rev. B* 97, 184205 (2018)

(iii) **Exotic Superconductivity with Enhanced Energy Scales in Materials with Three Band Crossings** Yu-Ping Lin and Rahul Nandkishore**,** *Phys. Rev. B* 97, 134521 (2018)

(iv) **Gapless insulating edges of dirty interacting topological insulators** Yang-Zhi Chou, Rahul M. Nandkishore and Leo Radzihovsky, Phys. Rev. B 98, 054205 (2018)

(v) **Localization driven correlated states of two isolated interacting Helical edges***,* Yang-Zhi Chou, Phys. Rev. B 99, 045125 (2019)

(vi) **Localized phases of three dimensional topological insulators,** Yang-Zhi Chou, Rahul Nandkishore and Leo Radzihovsky. Phys. Rev. B 99, 165108 (2019) [Editor's suggestion]

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4) "Technology transfer" (any specific interactions or developments which would constitute technology transfer of the research results, if applicable [IE patents or inventions])

None