NRL REPORT NO. R-31

DIRECTION FINDER BANDWIDTH REQUIREMENTS

S. F. GEORGE

Approved by:

FR-3182

Mr. E. A. Speakman, Head, Radio Countermeasures Section
Mr. L. A. Gebbard, Supt. Radio Division II
Problem 39R06-14

1 October 1947

NAVAL RESEARCH LABORATORY
COMMODORE H. A. SCHADE, USN, DIRECTOR
WASHINGTON, D.C.

DISTRIBUTION

BuShips Attn: Code 925D	(10)
CNO Attn: Code Op-413-B2	(5)
ONR Attn: Code N482	(2)
CO, ONR, Boston	(1)
Dir., USNEL	(2)
Cdr., NATC, Patuxent	(1)
SNLO, USNELO, Ft. Monmouth	(1)
CO, Watson Lab., AMC, Ft. Monmouth Attn: WLEOE-1	(1)
CG, AMC, Wright Field Attn: TSELO	(1)
CO, Watson Lab., AMC, Cambridge Attn: WLCRD	(1)
OCSigO, Attn: Ch. Eng. & Tech. Div., SIGTM-S	(1)
CO, SCEL Attn: Dir. of Eng.	(2)
RDB Attn: Library Attn: Navy Secretary	 (2) (1)
Science and Technology Project Attn: Mr. J. H. Heald, Ch.	(2)

CONTENTS

	Page
Abstract	iv
Problem Status	iv
SYMBOLS	v
INTRODUCTION	1
THE SENSE CIRCUIT	1
PROCEDURE OF ANALYSIS	2
THE COLLECTOR SYSTEM	2
GONIOMETER ACTION ANALYSIS	4
1. An Electronic Goniometer	4
2. A Mechanical Goniometer	8
THE SIGNAL INTRODUCED TO THE RECEIVER	12
RECEIVER BANDWIDTH REQUIREMENTS	12
1. The Radio Frequency Section	12
The Causes of Bearing Error	14
The Causes of Blur	15
Determination of Bandwidth	16
2. Frequency Conversion	18
3. The Intermediate Frequency Section	19
4. The Audio Section	19
5. The Second Detector	19
6. Detector Output Amplification	20
7. Bandwidth Requirements Following the Second Detector	22
FUNDAMENTAL LIMITATIONS ON ROTATION OR MODULATION FREQUENCIES	30
SYSTEMS FOR REDUCING BANDWIDTH REQUIREMENTS	30
CONCLUSIONS	31
ACKNOWLEDGMENTS	33
APPENDIX I THE CONCEPTS OF BLUR AND PULL-IN	34
APPENDIX II HARMONIC ANALYSIS OF SECOND DETECTOR OUTPUT	36

ABSTRACT

An incoming signal, whose accurate bearing is being sought by a goniometer type direction finder, is traced by mathematical analysis from the collector, through the goniometer, through the receiver sections, and finally to the cathode ray indicator which provides a visual bearing indication. At each point in the analysis, the effects of the various components of the system on the signal are considered, with special emphasis on those effects which tend to cause inaccurate or obscured bearing presentation. The greatest emphasis is placed on the bandwidth requirements, in the various sections of the direction finding receiver, that are essential to the maintenance of accurate and sharply defined bearings. The analyzed receiver input consists of two side-bands symmetrically displaced about the carrier frequency of the incoming signal by the goniometer modulation or rotation frequency. It is found that, in the sections of the receiver prior to the second detector a change in the relative phase shift of the side-bands causes bearing error whereas a change in the relative amplitudes of the side-bands causes obscured indication. The radio frequency and intermediate frequency bandwidths needed to prevent inaccurate and obscured bearing indications are shown to be directly proportional to the ratio between the goniometer modulation frequency and signal carrier frequency. Carefully designed frequency conversion and linear detection systems do not contribute to bearing inaccuracy or obscurity. The total bandwidth required of the circuits following the second detector is found to be proportional to the goniometer modulation frequency alone. Mention is made of methods which can be used to reduce the rather severe bandwidth requirements explained in this work.

PROBLEM STATUS

The analysis which resulted in this report was undertaken in conjunction with Problem S1431, relating to the remote control of direction finder stations. However, this report is believed to be of interest to those concerned with many phases of direction finder research and development.

SYMBOLS

Symbol	Definition	Page First Use
E	Electric field intensity of incoming signal	2
E_m	Maximum instantaneous value of electric field intensity	2
ω_c	Angular velocity of carrier signal	2
f_c	Carrier frequency of incoming signal	2
ϕ_c	Arbitrary phase angle associated with incoming signal	2
e_N, e_S, e_E, e_W	Voltages induced in monopoles of Adcock array	3
h_e	Effective height of each monopole	3
d	Distance of each monopole from center of array	3
λ_c	Wavelength of carrier signal	3
α	Angle of arrival of incoming signal	3
V_1, V_2	Balance modulator pentode vacuum tubes	4
i_1, i_2	Plate currents in V_1 and V_2 respectively	4
Z	Combining impedance in electronic goniometer	4
$e_{AN}, e_{AS}, e_{AE}, e_{AW}$	Audio modulating voltages	5
A_m	Maximum instantaneous value of modulating signal	5
ω_m	Angular velocity of modulating signal	5
f_m	Audio frequency of modulating signal	5
m	Modulation factor	5
k	Gain constant associated with balanced modulator	5
ϕ_a	Phase angle associated with balanced modulator	5
ϕ_s	Total phase shift = $\phi_a + \phi_c$	5
$e_{NT}, e_{ST}, e_{ET}, e_{WT}$	Balanced modulator output voltages	5
e_T	Combined balanced modulator output	6
h	Physical height of each monopole	7
e_N', e_S', e_E', e_W'	Adcock voltages coupled to transmission lines	9
a, a'	Gains through coupling device and differential connection	9

Symbol	Definition	Page First Used
ψ_c, β_c	Phase shifts through coupling device and differential connection	9
ϕ_T	Total phase shift = $\phi_c + \psi_c + \beta_c$	9
$e_{NS}, e_{EW}, e_{NS}', e_{EW}'$	Differential voltages between monopoles	9
H_{NS}, H_{EW}	Magnetic fields from differential voltages	10
$\Phi_{NS}, \Phi_{EW}, \Phi_T$	Magnetic flux due to magnetic fields	10
θ	Angle of rotor clockwise from North	10
ω_r	Angular velocity of rotor frequency rotation	10
N	Number of turns in rotor field coil	11
e_r	Rotor output voltage	11
$A, A_{rf}, A_{fc}, A_{if}$	Side-band amplitude of receiver input, r-f, f-c, and i-f outputs	12
$e_{i(rf)}, e_{i(fc)}$	Voltage input to r-f and f-c sections	12
$e_{o(rf)}, e_{o(fc)}, e_{o(if)}, e_d$	Voltage output from r-f, f-c, i-f and detector sections	13
G	Gain factor of radio frequency section	13
$\gamma_r, \gamma_f, \gamma_i$	Side-band amplitude ratios of r-f, f-c, and i-f sections	13
$\phi_{rf}, \phi_{fc}, \phi_{if}$	Side-band symmetric phase shift in degrees	13
$\delta_r, \delta_f, \delta_i$	Side-band phase shift difference in degrees	13
E_{rf}	Envelope of side-bands	14
ϵ	Bearing error in degrees	15
B_{rf}	Decimal value of blur	15
A_d	Detector output factor	20
S_n	Sum to n terms of a particular series	27
A_o, A_n	Constants associated with Harmonic analysis	36

DIRECTION FINDER BANDWIDTH REQUIREMENTS

INTRODUCTION

The extent to which direction finder receiver characteristics can be responsible for inaccurate and obscured bearings has not been determined previously by a quantitative analysis. It has been discussed qualitatively and demonstrated experimentally that insufficient receiver band-widths can be responsible for both bearing error and blur,* but there has existed a serious need for a definite and rigorous formulation of the exact cause and effect. It is hoped that the mathematical analysis herein presented will satisfy this need and in addition afford a concrete foundation for the future design of receivers to be used in goniometer type direction finders. It is important to emphasize that the material in this work is applicable, at least in part if not in the large, to any direction finder system in which there is side-band input to the receiver. This side-band input can result from mechanical or electronic goniometers as such, or from a number of circuits involving carrier suppressed balanced modulators, multivibrators, etc. The results obtained from the analysis should find immediate application in the problem of remoting bearing information over both long and short distances.

THE SENSE CIRCUIT

Following this paragraph no mention will be made of sense indication or sense circuits. It was considered advisable to eliminate any discussion of sense indication for these reasons: (1) there are so many different methods of sense indication that the subject would require a separate and quite lengthy report in itself, (2) since sense is almost always introduced as a single frequency there are no side-bands involved, and finally (3) in most direction finders, bearing error and blur are not initially affected by the sense circuits. In those few direction finders where a sense voltage is an integral part of, and cannot be divorced from, the initial bearing indication, the material in this report must be considered with reservation.

---

* *See Appendix 1 for a discussion and quantitative definition of blur. Qualitatively, blur may be considered as any reduction in the definition of the bearing pattern, whether aural or visual, resulting from any diminution in the sharpness of the indication.*

PROCEDURE OF ANALYSIS

For the benefit of those readers who, in the past, have not fully appreciated the equivalence in outputs of the mechanical and electronic types of goniometers, the first subdivision of this report treats the incoming signal from the time it first impinges upon the collector until it is ready to be introduced into the receiver. This is divided into two parts: (1) a discussion of the signal prior to the goniometer and (2) an analysis of the goniometer action upon the signal. The second and major subdivision of the report concerns the action of the various parts of the receiver upon the side-band input. This is treated in six closely related sections:

(1) The r-f section
(2) The frequency conversions
(3) The i-f section
(4) The audio section
(5) The second detector
(6) Detector output amplification

In each of these sections in turn there is a discussion of the causes of bearing error and blur, how both may be determined, and how to design for their prevention. In the last section is an additional discussion of pull-in * which is often confused with blur.

THE COLLECTOR SYSTEM

Before undertaking an analysis of the action of the goniometer itself, it is desirable to examine very briefly the incoming signal and determine its behavior through the collector system. The exact type of antenna array used in this analysis is immaterial since it will be assumed to be error-free of itself. A short-base Adcock array will be used for convenience. Let the electric field intensity E, as measured for reference at the center of the array, be denoted by:

$$E = E_m \sin (\omega_c t + \phi_c) \qquad (1)$$

Where E_m is the maximum value of field intensity, $f_c = \frac{\omega_c}{2\pi}$ is the radio frequency of the carrier signal, and ϕ_c is an arbitrary phase constant. †

---

* *See Appendix 1 for a discussion of pull-in vs. blur. Pull-in may be qualitatively defined as the movement of the ends of the propeller-shaped pattern of an Automatic Bearing Indicator toward the center of the CRT resulting from blur.*

† *It would have been more general to have considered the case of a modulated incoming signal carrier. However, this would have resulted in needless complication of form; and, ordinary modulation has no deleterious effect on direction finder operation.*

Geometrical considerations (See Figure 1) enable the writing of the four voltages induced by the incoming signal into the four monopoles:

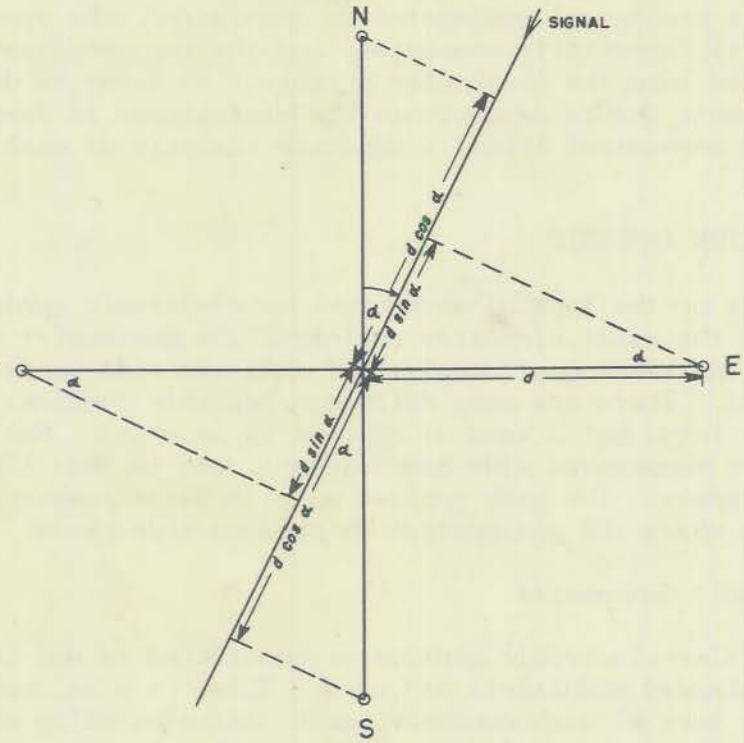

Figure 1. Geometrical Consideration of Adcock Antenna Array

$$\left.\begin{aligned} e_N &= E_m\, h_e \sin\left(\omega_c t + \phi_c - \frac{2\pi d}{\lambda_c}\cos\alpha\right) \\ e_S &= E_m\, h_e \sin\left(\omega_c t + \phi_c + \frac{2\pi d}{\lambda_c}\cos\alpha\right) \\ e_E &= E_m\, h_e \sin\left(\omega_c t + \phi_c - \frac{2\pi d}{\lambda_c}\sin\alpha\right) \\ e_W &= E_m\, h_e \sin\left(\omega_c t + \phi_c + \frac{2\pi d}{\lambda_c}\sin\alpha\right) \end{aligned}\right\} \quad \cdots\cdots\cdot (2)$$

where h_e is the effective height* of each monopole, d is the distance of each monopole from the center of the antenna array, λ_c is the wavelength of the incoming signal carrier, and α is the angle of arrival of the incoming signal as measured in a clockwise direction from North.

* *For consideration of* h_e *see Sec. 137 of "Phenomena in High - Frequency Systems" by August Hund, McGraw - Hill Book Company, 1936.*

At this point in the analysis the procedure depends upon the type of goniometer employed. In particular, if an electronic goniometer is in question, the four induced monopole voltages given by equations (2) are fed individually into separate balanced modulator units, one at the base of each monopole. If a mechanical goniometer is considered, the opposing pairs of monopoles are differentially connected, and the two resultant difference voltages are fed into the goniometer stators. In order to determine the goniometer outputs and to demonstrate the equivalence in form between the electronic and mechanical types, a separate analysis of each will be presented.

GONIOMETER ACTION ANALYSIS

Regardless of the type of mechanical or electronic goniometer used, and independent of the exact circuitry employed, the goniometer output under consideration in this report consists of only two side-bands with the carrier suppressed. There are many different possible combinations to produce this resultant form, but a word of caution is in order. Not all goniometers produce carrier suppressed side-band outputs, and to this limited group this work does not apply. The work applies only to those conventional and numerous equipments where the goniometers do produce side-bands.

1. An Electronic Goniometer

The particular electronic goniometer considered is one in which carrier-suppression balanced modulators are used. There is a balanced modulator unit located at the base of each monopole, each unit consisting of a pair of parallel-fed vacuum tube amplifiers having push-pull outputs. The four outputs are introduced to a combining impedance and the voltage across this common impedance becomes the receiver input.

A typical balanced modulator circuit is shown in Figure 2. A number of other circuits could have been employed, but this one is sufficient to indicate conventional operation. As shown, the incoming induced carrier signal is fed in parallel and an internally generated audio modulating signal is fed in push-pull to the grids of V_1 and V_2. The plate currents, denoted by i_1 and i_2, of the amplifier pentodes flow thru the common impedance Z in opposite directions as indicated. An audio oscillator is used to generate the audio modulation voltage required. This voltage is passed thru a 90-degree phase splitter yielding two outputs, each of which is further split into two balanced voltages. These four resulting audio voltages, which are applied to the four modulator units, are symbolized as follows:

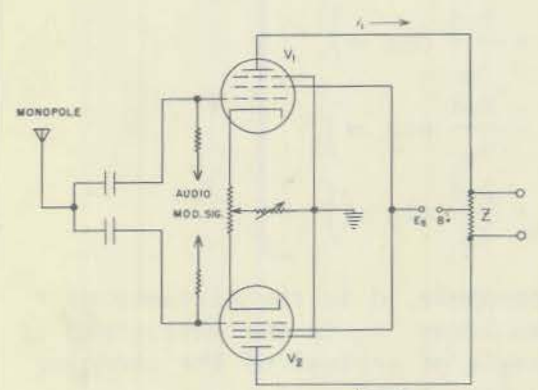

Figure 2. Typical Balanced Modulator Unit of An Electronic Goniometer.

$$\left.\begin{aligned} e_{AN} &= A_m \sin \omega_m t \\ e_{AS} &= A_m \sin (\omega_m t + \pi) = - A_m \sin \omega_m t \\ e_{AE} &= A_m \cos \omega_m t \\ e_{AW} &= A_m \cos (\omega_m t + \pi) = - A_m \cos \omega_m t \end{aligned}\right\} \; . \; . \; . \; . \; . \; . \; . (3)$$

where A_m is the maximum instantaneous value of the modulating signal and $f_m = \frac{\omega_m}{2\pi}$ is the frequency of audio modulation. Since this modulating voltage is to be used subsequently as a time or phase reference, it is convenient and does not limit generality to let e_{AN} be zero when $t = 0$.

The operation of only one monopole unit, say the North, will be analyzed in full and the others will follow readily from symmetry. Let e_{AN} from equations (3) and e_N from equations (2) be applied to the grids V_1 and V_2 as indicated in Figure 2. The outputs from the two tubes will be conventional amplitude modulation. It is of the utmost importance that tubes V_1 and V_2 are adjusted to have equal gain and phase shift. In fact this must be true of all eight tubes in the four modulator units. This will be assumed to hold here, since the discussion of goniometer errors is not a part of this work. Employing the conventional mathematical form, the output of V_1 will be:

$$i_1 = E_m h_e k (1 + m \sin \omega_m t) \sin \left(\omega_c t + \phi_a + \phi_c - \frac{2\pi d}{\lambda_c} \cos \alpha \right) \; . \; . \; . \; (4)$$

and for V_2:

$$i_2 = E_m h_e k (1 - m \sin \omega_m t) \sin \left(\omega_c t + \phi_a + \phi_c - \frac{2\pi d}{\lambda_c} \cos \alpha \right) \; . \; . \; . \; (5)$$

where m is the modulation factor dependent upon A_m and E_m, k is dependent upon the transconductance of the amplifier tubes and ϕ_a takes care of the phase shift through the amplifier tubes. The negative m in equation (5) arises from the fact that the audio modulation is introduced into V_1 and V_2 in push-pull. Combining the currents in equations (4) and (5) through the common impedance Z and letting $\phi_s = \phi_a + \phi_c$, the total voltage from the North monopole becomes:

$$e_{NT} = 2E_m h_e kZm \sin \omega_m t \sin \left(\omega_c t + \phi_s - \frac{2\pi d}{\lambda_c} \cos \alpha \right) \; . \; . \; . \; . \; . \; . \; . (6a)$$

By similar procedure the expressions for the voltage contribution from the other three monopoles can be derived; however, they may be rather easily written by inspection from symmetry:

$$e_{ST} = -\ 2E_m\ h_e\ k\ Z\ m\ \sin\omega_m t\ \sin\left(\omega_c t + \phi_s + \frac{2\pi d}{\lambda_c}\cos\alpha\right) \quad \text{(6b)}$$

$$e_{ET} = 2E_m\ h_e\ k\ Z\ m\ \cos\omega_m t\ \sin\left(\omega_c t + \phi_s - \frac{2\pi d}{\lambda_c}\sin\alpha\right) \quad \text{(6c)}$$

$$e_{WT} = -\ 2E_m\ h_e\ k\ Z\ m\ \cos\omega_m t\ \sin\left(\omega_c t + \phi_s + \frac{2\pi d}{\lambda_c}\sin\alpha\right) \quad \text{(6d)}$$

hese four equations contain several important parameters which call for onsideration. First, it is obvious that in order for any comparison system o work, the effective height, h_e, must be the same in all four monopoles. econdly, the fact that the same k and m are used in all four equations imlies that the tubes in the four modulation units are operating at the same oint and have identical dynamic characteristics over their operating range. s noted in Figure 2 there is provision made to adjust the operating points f the tubes and by selection the operating characteristics can be made the ame.

In order to simplify the writing of these equations during manipulation, et $k' = 2E_m\ h_e\ k\ Z\ m$. The resultant total output across Z, e_T, may now e written as the sum of the four components given by equations (6):

$$e_T = k'\ \sin\omega_m t\left\{\sin\left(\omega_c t + \phi_s - \frac{2\pi d}{\lambda_c}\cos\alpha\right) - \sin\left(\omega_c t + \phi_s + \frac{2\pi d}{\lambda_c}\cos\alpha\right)\right\}$$
$$+\ k'\ \cos\omega_m t\left\{\sin\left(\omega_c t + \phi_s - \frac{2\pi d}{\lambda_c}\sin\alpha\right) - \sin\left(\omega_c t + \phi_s + \frac{2\pi d}{\lambda_c}\sin\alpha\right)\right\} \quad (7)$$

Applying the trigonometric identity:

$$\sin a - \sin b \equiv 2\cos\frac{a+b}{2}\sin\frac{a-b}{2}$$

to the terms within the braces in equation (7) and simplifying the result gives:

$$e_T = -\ 2\ k'\left\{\sin\omega_m t\ \sin\left(\frac{2\pi d}{\lambda_c}\cos\alpha\right) + \cos\omega_m t\ \sin\left(\frac{2\pi d}{\lambda_c}\sin\alpha\right)\right\}\cos\left(\omega_c t + \phi_s\right) \quad (8)$$

Now equation (8) is an exact expression for the output of an electronic goniometer. It is complicated by the fact that terms of the form sin (c cos α) and sin (c sin α) occur. However, an approximation may be made which is justified in practice and which greatly simplifies the work.

That is, in conventional short-base Adcock systems the spacing between monopoles is small compared to the wavelength; i.e., in symbols $d \ll \lambda_c$. This means that $\frac{2\pi d}{\lambda_c}$ is small, and since sin α and cos α cannot exceed unity, then $\frac{2\pi d}{\lambda_c} \sin\alpha$ and $\frac{2\pi d}{\lambda_c} \cos\alpha$ are small. This justifies the approximation*:

$$\left.\begin{aligned} \sin\left(\frac{2\pi d}{\lambda_c}\cos\alpha\right) &\longrightarrow \frac{2\pi d}{\lambda_c}\cos\alpha \\ \sin\left(\frac{2\pi d}{\lambda_c}\sin\alpha\right) &\longrightarrow \frac{2\pi d}{\lambda_c}\sin\alpha \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (9)$$

The error of approximation is less than two percent for $d < \frac{\lambda_c}{20}$.

This approximation reduces expression (8) to:

$$e_T = -2k' \cdot \frac{2\pi d}{\lambda_c}\left\{\sin\omega_m t\cos\alpha + \cos\omega_m t\sin\alpha\right\} \cdot \cos(\omega_c t + \phi_s)$$

Combining the terms in braces:

$$e_T = -\frac{4\pi d\, k'}{\lambda_c}\sin(\omega_m t + \alpha)\cos(\omega_c t + \phi_s) \quad \ldots\ldots \quad (10)$$

To obtain the desired form it is now possible to apply a trigonometric identity to equation (10), yielding the sum of two sine terms †:

$$e_T = \frac{2\pi d k'}{\lambda_c}\left\{\sin\left[(\omega_c + \omega_m)t + \alpha + \phi_s\right] + \sin\left[(\omega_c - \omega_m)t + \pi - \alpha + \phi_s\right]\right\} \quad (11)$$

The effective height, which is one factor of k', is given by ‡ :

$$h_e = \frac{\lambda_c}{\pi \sin\frac{2\pi h}{\lambda_c}} \cdot \sin^2\left(\frac{\pi h}{\lambda_c}\right) \quad \ldots\ldots\ldots\ldots \quad (12)$$

---

* *There is a small octantal spacing error due to this approximation when using short-base Adcock systems. For the mathematical expression and discussion of this error see NRL report R-2707 by S. F. George and E. H. Flath on "Sensitivity and Accuracy Comparisons of DAU vs. DAJ Goniometers for DAJ - a Installations".*

† *The added π in the lower side-band has been introduced in order to avoid the negative frequency or amplitude otherwise obtaining.*

‡ *Bond, Donald S., "Radio Direction Finders", McGraw-Hill Book Co., New York, 1944, p 85.*

here $h \leqslant \frac{\lambda_c}{4}$ is the actual physical height of each monopole. If it can be ssumed that $h < \frac{\lambda_c}{4}$, the value of h_e is very closely h/2. It is now posible to combine all of the factors which make up the amplitude term of e_T n equation (11):

$$\frac{2\pi dk'}{\lambda_c} = \frac{2\pi d}{\lambda_c} \cdot 2F_m \frac{h}{2} \cdot kZm = k_T E_m \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (13)$$

here k_T is a parameter dependent upon the incoming signal frequency, the ollector system, and the balanced modulator circuits but is independent of ime and phase shift. Employing equation (13) in (11), the final form of he goniometer output, e_T, becomes:

$$e_T = k_T E_m \left\{ \sin\left[(\omega_c + \omega_m) t + \alpha + \phi_s\right] + \sin\left[(\omega_c - \omega_m) t + \pi - \alpha + \phi_s\right]\right\} \quad (14)$$

This final output voltage is seen to consist of two side-bands of qual amplitude whose frequencies are displaced symmetrically about the arrier by the amount of the modulation frequency. The actual carrier of he incoming signal has been suppressed. The equation can be thought of as epresenting two vectors of equal magnitude rotating in counterclockwise irections with angular velocities of $(\omega_c + \omega_m)$ and $(\omega_c - \omega_m)$ respectively. he vectors are initially, i.e., at time t=0, displaced from an arbitrary xis by phase shifts of α and $\pi - \alpha$ respectively.

2. A Mechanical Goniometer

The action of a mechanically rotated inductive goniometer will now be nalyzed. Unlike the electronic type goniometer, the monopoles are not eparately treated, but the opposite pairs are differentially connected irst and then the resultant outputs are applied to the two goniometer tators. It is not implied that the electronic goniometer does not operate n the differential principle, for it does; but the differential connection n the electronic goniometer occurs in the output across the common, comining impedance rather than in the input. Both types of goniometers operate on the same fundamental principle of employing the differential voltage of opposing monopoles.

As before, the incoming signal is given by equation (1). The signals induced in the four monopoles are likewise the same as before and are given by equations (2). From this point forward the analysis is different. Let it be assumed that the coupling device used between the monopole and connecting transmission line, whether it be of the cathode follower or transformer type, attenuates the induced voltage in the monopole by 1/a and shifts the phase by ψ_c. The resulting voltages are given by:

$$\left.\begin{aligned}
e_N' &= E_m h_e a \sin\left(\omega_c t + \phi_c + \psi_c - \frac{2\pi d}{\lambda_c}\cos\alpha\right)\\
e_S' &= E_m h_e a \sin\left(\omega_c t + \phi_c + \psi_c + \frac{2\pi d}{\lambda_c}\cos\alpha\right)\\
e_E' &= E_m h_e a \sin\left(\omega_c t + \phi_c + \psi_c - \frac{2\pi d}{\lambda_c}\sin\alpha\right)\\
e_W' &= E_m h_e a \sin\left(\omega_c t + \phi_c + \psi_c + \frac{2\pi d}{\lambda_c}\sin\alpha\right)
\end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; (15)$$

In these equations note that both a and ψ_c are functions of frequency and it is absolutely necessary that they be equivalent for each monopole. The opposite pairs of monopoles are now differentially connected in such a manner as to cause a further voltage attenuation of 1/a and phase shift β_c. The resulting differential voltages are:

$$e_{NS} = E_m h_e a a' \left\{ \sin\left(\omega_c t + \phi_c + \psi_c + \beta_c - \frac{2\pi d}{\lambda_c}\cos\alpha\right) - \sin\left(\omega_c t + \phi_c + \psi_c + \beta_c + \frac{2\pi d}{\lambda_c}\cos\alpha\right)\right\} \quad . \; . \; . \; . \; . \; (16)$$

$$e_{EW} = E_m h_e a a' \left\{ \sin\left(\omega_c t + \phi_c + \psi_c + \beta_c - \frac{2\pi d}{\lambda_c}\sin\alpha\right) - \sin\left(\omega_c t + \phi_c + \psi_c + \beta_c + \frac{2\pi d}{\lambda_c}\sin\alpha\right)\right\} \quad . \; . \; . \; . \; . \; (17)$$

Note here also that a' and β_c are functions of frequency and must be kept equal for both e_{NS} and e_{EW}. Simplification of the difference terms within braces yields:

$$\left.\begin{aligned}
e_{NS} &= -2E_m h_e a a' \sin\left(\frac{2\pi d}{\lambda_c}\cos\alpha\right)\cos\left(\omega_c t + \phi_c + \psi_c + \beta_c\right)\\
e_{EW} &= -2E_m h_e a a' \sin\left(\frac{2\pi d}{\lambda_c}\sin\alpha\right)\cos\left(\omega_c t + \phi_c + \psi_c + \beta_c\right)
\end{aligned}\right\} \quad . \; . \; . \; (18)$$

At this point it might be easier to combine the arbitrary phase constants into one, say $\phi_T = \phi_c + \psi_c + \beta_c$. It might also be advantageous to permit the same approximation as before, namely in equations (9):

$$\left.\begin{aligned}
e_{NS}' &= -2E_m h_e a a' \cdot \frac{2\pi d}{\lambda_c}\cos\alpha\cos\left(\omega_c t + \phi_T\right)\\
e_{EW}' &= -2E_m h_e a a' \cdot \frac{2\pi d}{\lambda_c}\sin\alpha\cos\left(\omega_c t + \phi_T\right)
\end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (19)$$

As a temporary expedient, let:

$$K_o = -\frac{4\pi d}{\lambda_c} E_m h_e aa'$$

so that:

$$\left.\begin{aligned} e_{NS}' &= K_o \cos\alpha \cos\left(\omega_c t + \phi_T\right) \\ e_{EW}' &= K_o \sin\alpha \cos\left(\omega_c t + \phi_T\right) \end{aligned}\right\} \quad (20)$$

These voltages are now introduced into the two quadrature stators of the goniometer. They create quadrature electromagnetic fields within the goniometer which are scanned by a rotating field coil or rotor. The pickup of this field coil constitutes the goniometer output and is the voltage with which this study is chiefly concerned. Let the stators with their fields and the rotor coil with displacement θ be illustrated diagramatically in Figure 3. In consideration of the figure, the magnetic fields may be represented by:

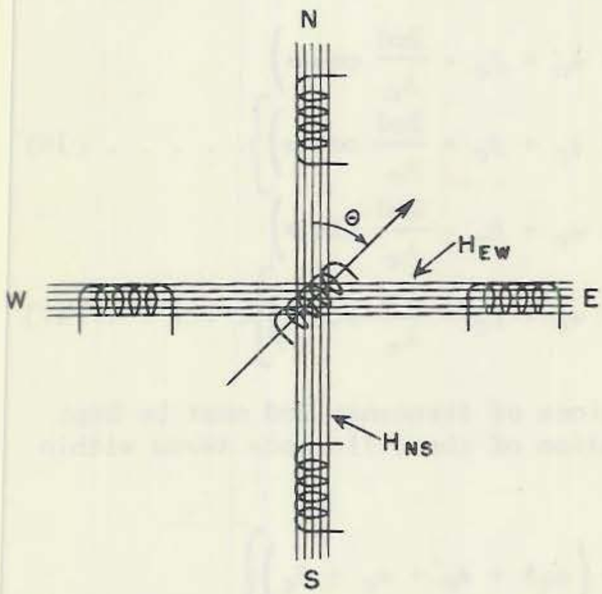

Figure 3. The Relationship of Magnetic Fields in An Inductive Mechanical Goniometer.

$$\left.\begin{aligned} H_{NS} &= K_0 K_1 \cos\alpha \sin\left(\omega_c t + \phi_T\right) \\ H_{EW} &= K_0 K_1 \sin\alpha \sin\left(\omega_c t + \phi_T\right) \end{aligned}\right\} (21)$$

where K_1 depends upon the impedance, area, and number of turns on the stator coils. (Note the change from the cosine to sine time function due to the fact that the current in the stators lags the applied voltage by 90°.) The total magnetic flux, generated by the current in the stators, passing through the rotor is given by the equations:

$$\left.\begin{aligned} \Phi_{NS} &= K_2 H_{NS} \sin\theta \\ \Phi_{EW} &= K_2 H_{EW} \cos\theta \end{aligned}\right\} \quad (22)$$

where K_2 depends upon the geometrical design of the transformer and the area of the rotor coil. The total flux cutting the rotor is the sum of the flux from the two stators and is:

$$\Phi_T = K_0 K_1 K_2 \left\{ \cos\alpha \sin\theta \sin\left(\omega_c t + \phi_T\right) + \sin\alpha \cos\theta \sin\left(\omega_c t + \phi_T\right) \right\} \quad (23)$$

Now, permitting the rotor to rotate continuously at angular velocity ω_r, there results

$$\theta = \omega_r t \quad (24)$$

where $\theta = 0$ at $t = 0$ establishes a reference for later synchronization with the bearing indicator. Therefore,

$$\Phi_T = K_0 K_1 K_2 \{\cos \alpha \sin \omega_r t + \sin \alpha \cos \omega_r t\} \sin (\omega_c t + \phi_T) \quad \ldots \ldots \ldots (25)$$

The voltage induced in the rotor field coil by this flux is given by:

$$e_r = -N \frac{\partial \Phi_T}{\partial t} \quad \ldots \ldots \ldots \ldots (26)$$

where N is the number of turns in the rotor coil. Differentiation and simplification of equation (25) in (26) yields:

$$e_r = -NK_0K_1K_2 [\omega_c \sin (\omega_r t + \alpha) \cos (\omega_c t + \phi_T) + \omega_r \cos (\omega_r t + \alpha) \sin (\omega_c t + \phi_T)]$$

$$e_r = -\frac{1}{2} NK_0K_1K_2 \{\omega_c \sin [(\omega_c + \omega_r) t + \alpha + \phi_T] + \omega_c \sin [(\omega_r - \omega_c) t + \alpha - \phi_T]$$
$$+ \omega_r \sin [(\omega_c + \omega_r) t + \alpha + \phi_T] + \omega_r \sin [(\omega_c - \omega_r) t - \alpha + \phi_T]\}$$

Finally,

$$e_r = -\frac{1}{2} NK_0K_1K_2 (\omega_c + \omega_r) \sin [(\omega_c + \omega_r) t + \alpha + \phi_T]$$
$$- \frac{1}{2} NK_0K_1K_2 (\omega_c - \omega_r) \sin [-(\omega_c - \omega_r) t + \alpha - \phi_T] \quad \ldots \ldots \ldots (27)$$

It is noted in equation (27) that the carrier and rotation angular velocitie are summed and differenced as multiplier coefficients. Now in practicable cases, since ω_r represents a physically rotating coil, $\omega_r << \omega_c$ always. The ω_r in conventional direction finders is always less than 10^3 whereas for the communication band ω_c is generally greater than 10^6, so the two angular velocities are at least three orders apart. This justifies the substitution of ω_c for both $(\omega_c + \omega_r)$ and $(\omega_c - \omega_r)$ in the coefficients:

$$e_r = -\frac{1}{2} NK_0K_1K_2 \, \omega_c \{\sin[(\omega_c + \omega_r) t + \alpha + \phi_T] + \sin [-(\omega_c - \omega_r) t + \alpha - \phi_T]\} \quad \ldots \ldots (28)$$

The same substitution cannot be made for the time dependent terms since the ω_c may be lowered subsequently in conversion. Replace the constant K_0 in equation (27):

$$-\frac{1}{2} NK_0K_1K_2 \, \omega_c = \frac{2\pi d}{\lambda_c} N \, aa' K_1 K_2 E_m h_e.$$

The value of h_e for a short-base Adcock using a differential connection is given as:*

$$h_e = \frac{4\pi dh}{\lambda_c}$$

Bond, Donald S, "Radio Direction Finders", McGraw-Hill Book Co. 1944, pp 100, 101.

since the horizontal distance between two monopoles is 2d. The coefficient of equation (28) can thus be written:

$$\frac{1}{2} NK_0K_1K_2 \ \omega_c = K_c \ E_m$$

where K_c depends upon system constants and parameters but does not have a time or phase shift factor.

The final form of the mechanical goniometer output then becomes:*

$$e_r = K_cE_m\{\sin[(\omega_c + \omega_r) \ t + \alpha + \phi_T] + \sin[(\omega_c - \omega_r) \ t + \pi - \alpha + \phi_T]\}. \quad (29)$$

THE SIGNAL INTRODUCED TO THE RECEIVER

A comparison of equations (14) and (29) shows them to be identical in form. A detailed comparison of the amplitudes is of no significance here since these do not alter the phase of the side-bands or the bandwidth requirements. The two important factors are: (1) that the upper and lower side-bands have equal amplitudes, and (2) that they have symmetrical phase shifts about some arbitrary constant. It is quite readily acceptable, then, to denote the output from either goniometer, and hence, the receiver input by the general expression:

$$e_{i(rf)} = A\{\sin[(\omega_c + \omega_m) \ t + \alpha + \phi_T] + \sin[(\omega_c - \omega_m) \ t + \pi - \alpha + \phi_T]\} \quad (30)$$

RECEIVER BANDWIDTH REQUIREMENTS

As stated previously in the Procedure of Analysis, this major subdivision is first expanded into six sections, corresponding to the various parts of the receiver. Although the sections are definitely interrelated, it is quite fortunate for this analysis that each section may be treated separately with respect to its effect on bearing error and blur. This means that in the final analysis the contribution of each section to both bearing error and blur may be determined. This is very important to the design engineer who is interested in the proper performance of each section of the receiver as well as the receiver on the whole.

1. The Radio Frequency Section

The form of the voltage input to the receiver is given by equation (30). It is important to note: (1) that both side-bands have the same amplitude, (2) that the carrier frequency itself is missing, (3) that the side-bands are separated by twice the modulating frequency, and (4) that the phase delay of the side-bands is symmetrically displaced from the bearing angle by an arbitrary phase constant. It is also important to consider the method of tuning in a signal of this type. Although the carrier frequency is not

---

* *The factor π has been added in the lower side-band term to avoid a physically meaningless negative frequency.*

present, the receiver is usually tuned very nearly to carrier frequency, especially when the modulating frequency is small. This is true because the overall selectivity of most receivers is not such as to distinguish between the two side-band frequencies. In the arguments which follow it will be assumed that the receiver is properly tuned when it is tuned to the carrier frequency.* This does not preclude examination of error and blur due to mistuning which occurs whenever the receiver is not tuned to the carrier frequency.

In passing through the radio frequency section of the receiver only two changes take place in the input signal: (1) the amplitudes of the side-bands change and (2) the phase shifts of the side-bands change. In each stage of radio frequency amplification there is an amplitude and phase change. The following analysis applies equally well to the stages one at a time or to the combined effect of all the stages. For simplicity, the effect of all of the stages working together will be considered here. Let the total amplitude change of the lower side-band be G, which is the product of the absolute gains of all the stages and let the total phase change be ϕ_r, which is the sum of the phase changes in the individual stages. For the upper side-band, let the new amplitude be $G\gamma_r$ where $0 \leq \gamma_r \leq 1$, and phase shift be $\phi_r + \delta_r$. Thus, the amplitude ratio between the two side-bands is now γ_r and the phase change difference is now δ_r. A word of explanation may be in order as to the generality of these assumed amplitude and phase changes. It will become obvious later in the analysis that it is immaterial which side-band is assumed to be of different amplitude and/or phase shift. The only important criterion is the ratio of amplitudes and the difference in phase shifts between the side-bands. Thus, the absolute amplitude and phase shifts in the side-bands is immaterial with regard to both error and blur; only their relative changes are significant. The output from the radio frequency section of the receiver is thus given by:

$$e_{o(rf)} = AG\{\gamma_r \sin[(\omega_c + \omega_m)t + \alpha + \phi_{rf} + \delta_r] + \sin[(\omega_c - \omega_m)t + \pi - \alpha + \phi_{rf}]\} \quad . \; . \; . \; . \; . \; (31)$$

where $\phi_{rf} = \phi_r + \phi_T$.

With the output in this form it is not readily applicable to both error and blur analysis. In order to handle this output, first the envelope will be obtained, since this contains the direction finder intelligence. The envelope is given by:† (See following page)

---

* *It is considered that, should the selectivity permit distinction between side-bands, the average operator would attempt to tune as closely as possible between them, thus coming very close to the carrier frequency.*

† *Terman, F. E., "Radio Engineers Handbook", McGraw Hill Book Co., 1943, pp. 568. For the superimposed waves* $e = E_o \sin \omega_o t + E_1 \sin(\omega_1 t + \phi_1) + E_2 \sin(\omega_2 t + \phi_2) + \ldots$ *the envelope is given as* $E = \{E_o^2 + E_1^2 + E_2^2 + \ldots + 2E_oE_1 \cos[(\omega_1 - \omega_o)t + \phi_1] + 2E_oE_2\cos[(\omega_2 - \omega_o)t + \phi_2] + \ldots + 2E_1E_2 \cos[(\omega_1 - \omega_2)t + (\phi_1 - \phi_2)] + \ldots\}^{1/2}$.

$$E_{rf} = \left\{A^2G^2\gamma_r^{\,2} + A^2G^2 + 2AG\gamma_r . AG \cos\left[\left(\overline{\omega_c + \omega_m} - \overline{\omega_c - \omega_m}\right)t + \left[\left(\alpha + \phi_{rf} + \delta_r - \overline{\pi - \alpha + \phi_{rf}}\right)\right]\right]\right\}^{1/2} = AG\left\{1 + \gamma_r^{\,2} - 2\gamma_r \cos 2\left[\omega_m t + \alpha + \frac{\delta_r}{2}\right]\right\}^{1/2} \quad . \; . \; . \; . \; (32)$$

For convenience in the remainder of this analysis, the square of this envelope will be employed:

$$E^2_{rf} = A^2G^2\left[1 + \gamma_r^{\,2} - 2\gamma_r \cos 2\left(\omega_m t + \alpha + \frac{\delta_r}{2}\right)\right]. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; (33)$$

This envelope will now be studied to see what error and blur, if any, exist due to unequal amplitude and phase changes in the side-bands.

The Causes of Bearing Error. -The first logical step in determining the cause of bearing error is to determine the bearing. To establish the null point, let E^2_{rf} of equation (33) equal zero and determine θ, where $\theta = \omega_m t$.

Assume:

$$E^2_{rf} = A^2G^2\left[1 + \gamma_r^{\,2} - 2\gamma_r^{\,2} \cos 2\left(\theta + \alpha + \frac{\delta_r}{2}\right)\right] = 0$$

where $A \neq 0$, $G \neq 0$, and $0 \leq \gamma_r \leq 1$.

This implies that

$$\cos 2\left(\theta + \alpha + \frac{\delta_r}{2}\right) = \frac{1 + \gamma_r^{\,2}}{2\gamma_r}$$

For $\gamma_r = 1$, this expression can hold and a null does exist at

$$2\left(\theta + \alpha + \frac{\delta_r}{2}\right) = \cos^{-1} 1.$$

For $0 \leq \gamma_r < 1$, this expression reduces to

$$\frac{1 + \gamma_r^{\,2}}{2\gamma_r} > 1 \text{ for } 1 + \gamma_r^{\,2} > 2\gamma_r \text{ since } (1 - \gamma_r)^2 > 1.$$

This cannot be possible, since the cosine cannot be greater than unity for real arguments. Hence, a null cannot exist for γ_r other than unity. For $\gamma_r \neq 1$, the bearing indication will be given by the minimum value of E_{rf}.

To obtain the minimum, and coincidently the maximum, value of the envelope it is necessary to obtain the first derivative of E_{rf} and then let it become zero. Differentiating equation (33) with respect to t:

$$2E_{rf}\frac{dE_{rf}}{dt} = A^2G^2\left[4\gamma_r\omega_m \sin 2\left(\theta + \alpha + \frac{\delta_r}{2}\right)\right]$$

Since $E_{rf} \neq 0$, then for $\frac{dE_{rf}}{dt} = 0$

$$\gamma_r \sin 2\left(\theta + \alpha + \frac{\delta_r}{2}\right) = 0$$

This can be true for $\gamma_r = 0$ or for $\sin 2\left(\theta + \alpha + \frac{\delta_r}{2}\right) = 0$. For the case of $\gamma_r = 0$, the bearing is indeterminate and, as will be seen later, the blur will be 100%. For $\gamma_r \neq 0$, a minimum value of the envelope will exist and $2\left(\theta + \alpha + \frac{\delta_r}{2}\right) = \sin^{-1}0$. Thus, so long as $\gamma_r \neq 0$, the minimum and maximum values of the envelope will be given alternately by:

$$2\left(\theta + \alpha + \frac{\delta_r}{2}\right) = 0,\ \pi,\ 2\pi,\ 3\pi,\ \ldots.$$

Inspection of equation (32) shows at once that minimum values of E_{rf} occur at $\left(\theta + \alpha + \frac{\delta_r}{2}\right) = 0,\ \pi,\ 2\pi,\ \ldots,\ n\pi\ldots$ and maximum values occur at $\left(\theta + \alpha + \frac{\delta_r}{2}\right) = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots, (n-\tfrac{1}{2})\pi, \ldots$.

From the above discussion, it can be seen that the bearing occurs at $\theta = -\alpha - \frac{\delta_r}{2}$. For $\delta_r = 0$, the correct bearing is $\theta = -\alpha$ and hence for $\delta_r \neq 0$ the bearing error is given in magnitude by:

$$\epsilon = \frac{\delta_r}{2} \quad \ldots \ldots \quad (34)$$

Equation (34) shows mathematically that the bearing error is a function of unequal side-band phase shift *only* and is independent of amplitude so long as both side-bands exist. Theoretically this is true; however, in the next section it will be found that unequal side-band amplitude causes blur or bearing obscurity leading to inaccuracy or error. Since this type of error results from the inability of the operator to determine the correct minimum rather than a fundamental error in the minimum, it will subsequently be considered one of the disadvantages of blur and will not be termed error. Thus, in this analysis, bearing error is caused solely by a phase shift inequality in the side-bands.

The Causes of Blur. -A quantitative measure for blur occuring prior to second detection has been defined in Appendix 1 as the ratio of the minimum to maximum value of the envelope resulting from a combination of the two side-bands. As determined in the preceeding section (The Causes of Bearing Error), a minimum value of the envelope occurs at $\left(\theta + \alpha + \frac{\delta_r}{2}\right) = n\pi$ and a maximum at $(n - \tfrac{1}{2})\pi$. From equation (32) the blur is readily seen to be:

$$\text{Blur} = B_{rf} = \frac{E_{rf(\text{min.})}}{E_{rf(\text{max.})}} = \frac{\sqrt{1 + \gamma_r^2 - 2\gamma_r \cos 2n\pi}}{\sqrt{1 + \gamma_r^2 - 2\gamma_r \cos (2n-1)\pi}}$$

$$= \sqrt{\frac{(1 - \gamma_r)^2}{(1 + \gamma_r)^2}} = \frac{1 - \gamma_r}{1 + \gamma_r}.$$

$$\therefore \%\ \text{Blur} = 100\,\frac{1 - \gamma_r}{1 + \gamma_r} \quad \ldots \ldots \quad (35)$$

Equation (35) reveals the illuminating fact that blur caused prior to second detection is a function of unequal side-band amplitude *only* and is entirely independent of phase angle so long as a minimum exists. For $\gamma_r = 1$, the side-bands are equal and no blur exists whereas for $\gamma_r = 0$, one side-band vanishes, the blur is 100%, and no bearing is possible. A plot of the percentage of blur against γ_r is given in Figure 4.

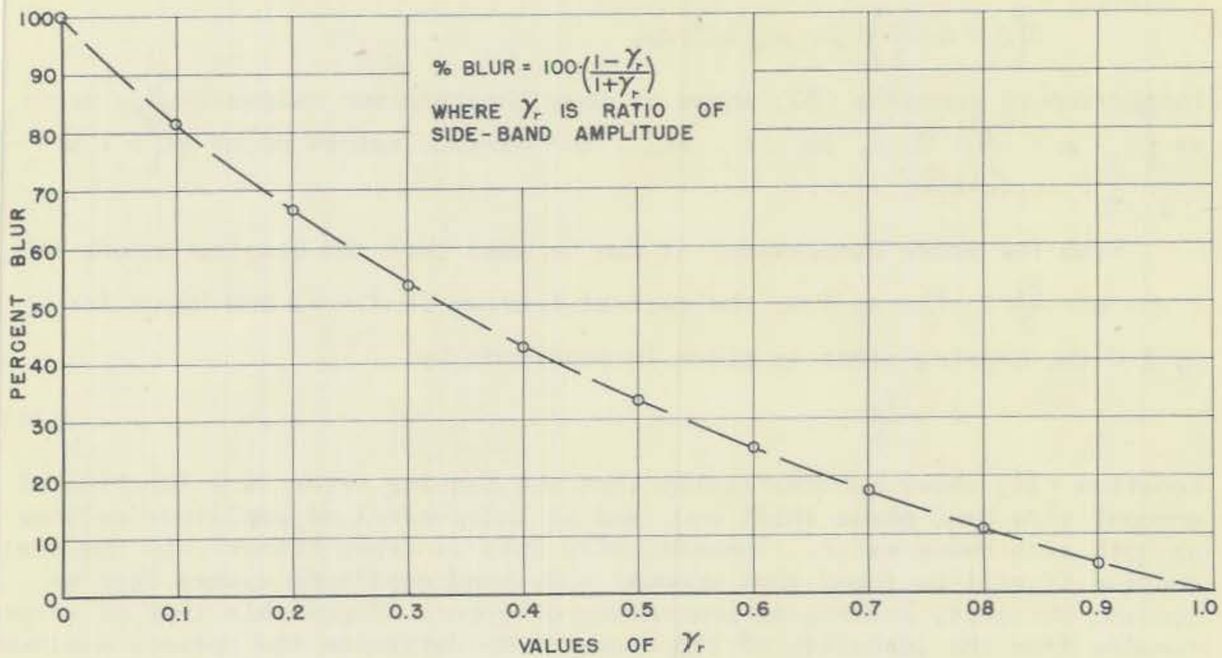

Figure 4. Percentage of Blur as a Function of Side-Band Amplitude

Determination of Bandwidth. -Once the factors contributing to bearing error and blur have been determined, the limits or tolerances allowable in the design of the circuits can be established. This is done in consideration of the specifications and in accordance with the desired over-all performance of the receiver for direction finder purposes. In view of the difficulty encountered in maintaining accurate and sharp bearings through the collector system, the transmission lines, and the goniometer, every effort is being made in present design to keep the receiver as free as possible from causing error and blur.

It has been shown that bearing error results from an unequal phase shift imparted to the two side-bands by the receiver, being in magnitude equal to half the absolute phase-shift difference. For example, should the lower side-band in passing through an amplifier be shifted in phase -181° and the upper side-band -179°, a bearing error of 1.0° would result. It is readily appreciated that the phase characteristic of the conventional amplifier is far from linear with respect to frequency, and hence it is possible for this error to become significant. This phase shift error always exists, the magnitude being directly proportional to the modulation frequency of the goniometer and inversely proportional to the bandwidth of the circuit under consideration. Unfortunately the error is generally the largest when the signal is properly tuned since in most amplifiers the phase characteristic has its greatest slope at the resonant frequency.

There are two ways in which this phase shift error can be treated: (1) a certain maximum error can be considered as tolerable and this can be calibrated out as part of the overall system error, and (2) the design can call for a sufficiently large bandwidth as to preclude any significant phase shift. If the first method is used, the actual bearing error caused by a phase shift difference is relatively unimportant, and hence the bandwidth criterion becomes one of maintaining a constant slope to the phase characteristic over the frequency range of the receiver. This means that, for each frequency to which the receiver can be tuned, the difference in phase shift between the side-bands must be constant but not necessarily negligible. If the second method is used, the actual phase difference between the side-bands must be made negligible over the entire frequency range of the receiver. Ultimately this is by far the most desirable method, but it is also the most severe on bandwidth requirements.

Blur was shown to result from an inequality in the amplitude of the two side-bands. Although as small a difference between side-bands as 1 db can cause as much as 5% blur, the question of blur prior to second detection is more or less academic. This is true for two reasons: (1) the selectivity curve of r-f amplifiers is generally quite symmetrical about the resonant frequency, and (2) the slope of the selectivity curve can readily be made very small around the resonant frequency. The first of these precludes blur for the case of a correctly tuned signal and the second of these reduces the tendency of appreciable blur even in the case of mistuning. However, as the ratio between the modulation and carrier frequency increases the danger of blur due to mistuning increases.

After the bandwidth demands have been established as functions of bearing error and blur, the actual circuit design becomes a problem in theoretical analysis. The theoretical approach is generally rather laborious, but there appears to be no alternative for design work. It is not the purpose of this work to discuss design methods, but the two references listed below might be of some aid, at least in the method of attack.*

Since the theoretical approach frequently becomes so complicated, experimental aid is suggested. However, the problem of accurately measuring small phase shifts is not too simple. A number of methods have been suggested in the literature and it would be helpful for an engineer desiring to measure phase to study them carefully. The accurate measurement of selectivity or the amplitude characteristic of a circuit is not new and can be easily achieved.

---

* *1. Markus, John & Zeluff, Vin, "Electronics for Engineers," McGraw-Hill Book Co., New York, 1945; Article by J. E. Maynard on "Universal Performance Curves for Tuned Transformers", p. 264 - 267 from the Electronics Magazine of February 1937.*

2. Terman, loc. cit. p. 135 - 172 Section 3 on Circuit Theory.

One way of determining the bearing error and blur due to insufficient r-f and i-f bandwidths is to use the automatic bearing indicator itself and simulate the carrier suppressed balanced modulator input. The important factor here is the knowledge of error and blur contributed from the circuits other than those being measured. It will be shown in Section 7 that this is relatively easy for circuits following second detection. Work along these lines has been done at Cruft Laboratory but references are not available.

2. Frequency Conversion

In discussing the process of frequency conversion, it will be assumed that no deleterious effects were produced in the radio frequency stages of the receiver. The form of the input to the converter will then be:

$$e_{i(fc)} = A_{rf}\{\sin[(\omega_c + \omega_m)t + \alpha + \phi_{rf}] + \sin[(\omega_c - \omega_m)t + \pi - \alpha + \phi_{rf}]\} \quad . (36)$$

The mathematical analysis of the process will not be reproduced here, but an outline of the steps needed will be given. First, a local oscillator voltage must be supplied whose frequency is either higher or lower than the carrier frequency of equation (36) by the desired intermediate frequency. Secondly, this local oscillator voltage-modulates the voltage given in equation (36) in the standard manner. There will be a number of frequencies in the resultant output, but only those near the intermediate frequency are wanted. The output from the converter is consequently passed through some type of frequency selective network in order to eliminate undesired frequencies. The process of frequency conversion itself does not cause bearing error or blur; however, reasonable care should be taken in the circuit design to insure maximum sensitivity. However, the frequency selective network following the converter, which is still a part of the conversion as a whole, may cause either error or blur or both. This network will change both the amplitude and phase of the remaining side-bands. Let the new amplitude of the lower side-band be A_{fc} and phase shift be $\phi_{fc} = \phi_{rf} + \phi_f$ where ϕ_f is due to the conversion stage alone. Let the new amplitude of the upper side-band be $A_{fc}\gamma f$ and phase shift be $\phi_{fc} + \delta_f$. Then the output of the frequency conversion section is given by:

$$e_{o(fc)} = A_{fc}\{\gamma_f \sin[(\omega_{if} + \omega_m)t + \alpha + \phi_{fc} + \delta_f]$$

$$+ \sin[(\omega_{if} - \omega_m)t + \pi - \alpha + \phi_{fc}]\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (37)$$

where ω_{if} is the new intermediate frequency and is a variable dependent upon the fixed carrier frequency of the incoming signal and the oscillator frequency which varies with tuning.*

---

* *It is readily acknowledged that ω_{if} is a variable dependent upon receiver tuning, however, the variation is not extensive because the receiver is generally tuned very close to the carrier frequency. For generality in this report ω_{if} must be considered a variable.*

Equation (37) has exactly the same form as equation (31) and hence the same arguments on bearing error and blur hold here as in the previous section.

3. The Intermediate Frequency Section

The analysis of the effects of intermediate frequency amplification in creating bearing error and blur can be handled in exactly the same manner as the radio frequency amplification. Each stage may be handled separately or all may be taken collectively to represent the whole section. The input given by equation (37) is transformed into:

$$e_{o(if)} = A_{if}\{\gamma_i \sin[(\omega_{if} + \omega_m)t + \alpha + \phi_{if} + \delta_i] + \sin[(\omega_{if} - \omega_m)t + \pi - \alpha + \phi_{if}]\} \quad (38)$$

where γ_i represents the ratio of side-band amplitudes and δ_i represents the phase shift difference. The causes of bearing error and blur are exactly the same as under Section 1, paragraphs entitled "The Causes of Bearing Error" and "The Causes of Blur". The bandwidth analysis under Section 1, paragraph entitled "Determination of Bandwidth", is the same except for specific values used. It must be remembered that $2f_m$ is a much larger percentage of f_{if} than f_c and therefore a large percentage bandwidth is required.

4. The Audio Section

Although the audio section of the receiver plays no role in obtaining a bearing when using ABI operation, it is useful for monitoring and aural null operation. In such cases the bandwidth requirements are the same as for any monitoring receiver. Although there is some hand rotation of a goniometer in aural null operation, the motion is very irregular and slight in the vicinity of the null, and any assumption regarding carrier suppression and side-band generation would be questionable. In the communications band and higher, any bandwidth fulfilling tuning and d-f requirements will also be adequate for a-f needs. Since these audio circuits are the least important in the application being considered, no further mention about them will be made.

5. The Second Detector

For direction finder purposes it is essential that linear or large signal detection take place. The purpose of the second detector is to reproduce as accurately as possible either the positive or negative envelope of the signal coming out of the intermediate frequency section of the receiver. For this reason both harmonic distortion and phase distortion must be kept to an absolute minimum. Detector design techniques will not be discussed in this report, but the importance of linear detection and careful design cannot be overemphasized. With existing diodes, such as the 6H6, calling for a large resistive load, it is possible to design detector circuits which produce no phase distortion and less than 1% harmonic distortion.

The envelope desired is derived from the following output of the intermediate frequency section, where it has been assumed that up to the second detector there has been no bearing error or blur:

$$e_{o(if)} = A_{if}\{\sin[(\omega_{if} + \omega_m)t + \alpha + \phi_{if}] + \sin[(\omega_{if} - \omega_m)t + \pi - \alpha + \phi_{if}]\} \quad (39)$$

The mathematics involved in obtaining the envelope of a form such as is given in equation (39) has already been illustrated on pages 13 and 14 under Section 1. The result given by equation (32) is immediately applicable upon the proper substitutions. In the case at hand then, the ideal detector output will be given by:*

$$e_d = -A_d[2 - 2\cos 2(\omega_m t + \alpha)]^{1/2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (40)$$

Here, the γ_r of equation (32) is unity and $\delta_r = 0$ since neither bearing error nor blur has been assumed. From the identity $\cos 2x \equiv 1 - 2\sin^2 x$ equation (40) may be rewritten as:

$$e_d = -2A_d\left|\sin(\omega_m t + \alpha)\right| \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (41)$$

An harmonic analysis for equation (41) has been performed in Appendix 2 and the result is given as:

$$e_d = -\frac{4A_d}{\pi} + \frac{8A_d}{\pi}\sum_{n=1}^{\infty}\frac{\cos 2n(\omega_m t + \alpha)}{4n^2 - 1} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (42)$$

An examination of equation (42) reveals the following facts about the output of the second detector: (1) there is a negative zero frequency or d-c component equal to 0.636 of the maximum envelope amplitude, (2) added to this are even harmonics of the modulation frequency whose amplitudes decrease as $\frac{1}{4n^2-1}$, and (3) the phases of these harmonics determine the bearing angle α.

A discussion of the bandwidth requirements for all of those circuits following the detector (see Section 7) is postponed until the d-c amplifier has been considered. This has been done so that the effects could be observed on the indicator directly.

6. Detector Output Amplification

From an inspection of equation (42) it is apparent that all of the circuits following the second detector must pass direct current as well as the the even harmonics of the modulation frequency. Between the detector and

* *The detection is performed in such a manner as to produce negative pulses because it is desired that negative voltages be applied to the grid of the d-c amplifier. The next section on the operation of the d-c amplifier will make this clear.*

the indicator there is located a d-c amplifier section consisting of one o two stages depending upon the type of indicator employed. As was the case with the goniometers, there are two possible types of indicators; electron and electromagnetic. Although either type of indicator may be associated with either goniometer, simplicity and convention usually lead to the association of the electronic indicator with the electronic goniometer and tl electromagnetic indicator with the mechanical goniometer.

If an electronic indicator is used, the same audio synchronizing voltage used for goniometer modulation must be introduced into one of the d-c amplifier stages. Here this voltage is phase split, balanced, modulated by the detector output and amplified. The resulting voltages are fed to the four plates of the CRT indicator producing the conventional propeller-shape pattern. If an electromagnetic type indicator is used, the detector output after amplification is introduced into a rotating field coil or magnetic yoke which is synchronized with the rotation of the mechanical goniometer.

Regardless of the type of indicator employed, the d-c amplifier acts i such a manner as to sharpen the bearing indication pattern considerably. This results from the inherent nonlinearity of the d-c amplifier. The degree of sharpening depends upon this nonlinearity as well as the "cut-off" voltage which is the smallest negative grid voltage needed to cut off the d-c amplifier and hence drive the spot to the center of the CRT. Obviously the characteristics of particular vacuum tubes used determine the d-c amplifier voltage linearity and hence (to a certain extent) the pattern shape an sharpness. Figure 5 shows a plot of a d-c amplifier voltage characteristic for a typical direction finder equipment.

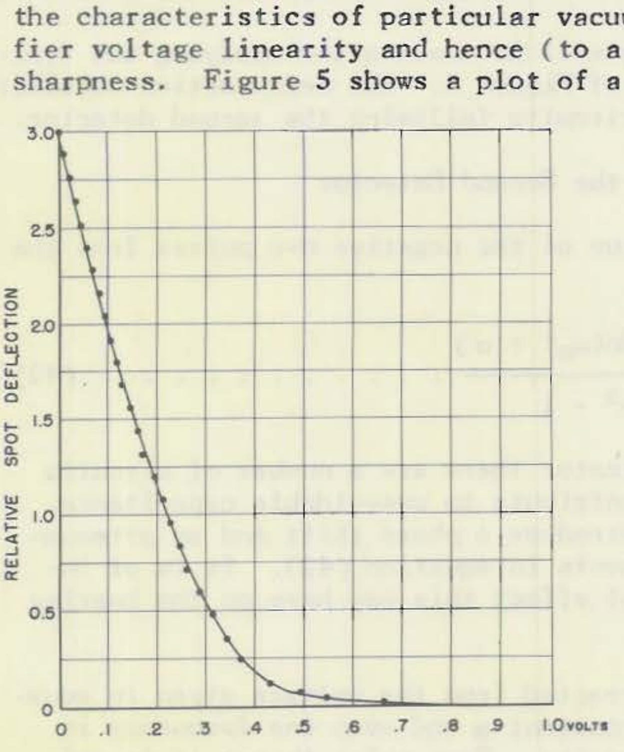

Figure 5. Characteristic of a Typical D-C Amplifier.

Before illustrating the effect of the non-linearity of the d-c amplifier, it is well to mention the manner of obtaining an indicator pattern on the CRT. First, of course, a signal must be located and tuned in. Then, the gain of the receiver is adjusted to produce a propeller-shaped pattern in which the points in the vicinity of the center of the CRT just touch. This means that the maximum negative voltage being fed to the grid of the d-c amplifier is just sufficient to cut off the tube. The resulting ideal pattern, assuming a perfectly linear d-c amplifier response, is shown in Figure 6.

Applying the typical non-linear characteristic of Figure 5 to this pattern, the actual scope presentation would resemble very closely that

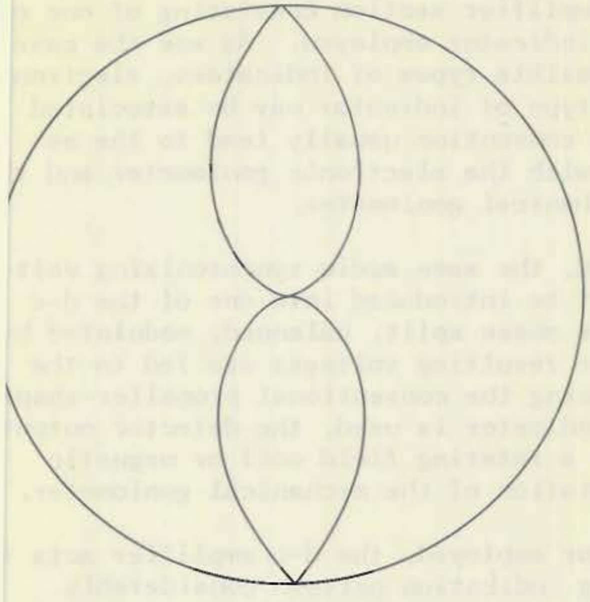

Figure 6. Ideal Pattern for Linear D-C Amplifier

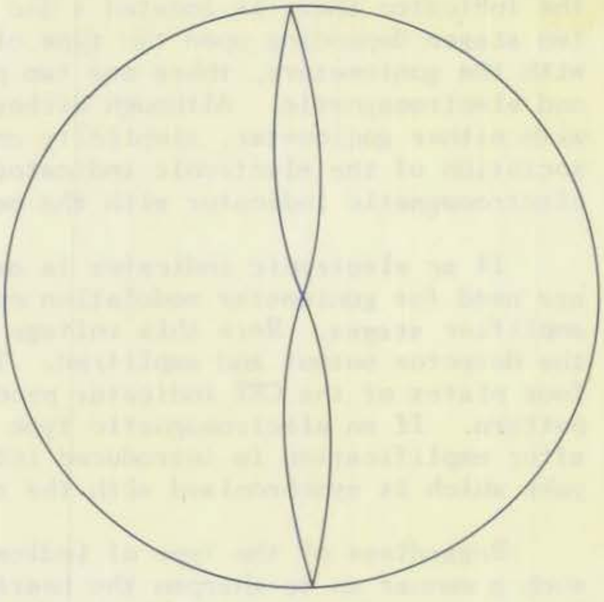

Figure 7. Ideal Pattern for Non-linear D-C Amplifier

given in Figure 7. The bearing quite arbitrarily has been assumed at $\alpha = 0$.

The remaining indicator patterns will be shown after applying the typical nonlinear d-c amplifier response of Figure 5. The next section considers the bandwidth requirements of those circuits following the second detector.

7. Bandwidth Requirements Following the Second Detector

In Section 5 the frequency spectrum of the negative d-c pulses from the second detector was found to be:

$$e_d = \frac{4A_d}{\pi} + \frac{8A_d}{\pi} \sum_{n=1}^{\infty} \frac{\cos 2n(\omega_m t + \alpha)}{4n^2 - 1} \quad \text{. . . . . (42)}$$

Now between the detector and the indicator there are a number of circuits and one or more vacuum tubes which contribute to unavoidable capacitance. This capacitance has a tendency to introduce a phase shift and an attenuation in the higher frequency components in equation (42). It is of importance, then, to determine just what effect this may have on the bearing indication.

The bearing information to be extracted from the voltage given in equation (42) is contained in the phase constant α and even one frequency is sufficient to indicate the correct bearing. From pulse theory it is evident that the addition of the higher harmonics contribute to the sharpness of the pattern. Theoretically, only when the series is infinite does a perfect null exist; a situation which, of course, is not practicable. And only when each component has the phase constant α is the correct bearing possible.

Again, however, this is neither possible nor strictly necessary.

Any change in the phase of the harmonics of the detector output has sim taneously two deleterious effects: (1) it causes a bearing error and (2) produces blur. These two effects can be explained logically by a brief con sideration of the mathematical equivalence of an extraneous phase shift in any particular frequency harmonic. The harmonic may be thought of as consisting of two components, one *in phase* with the bearing phase angle and th other in *phase quadrature* to the bearing phase angle. The in-phase component will not contribute to bearing error but will by virtue of its reduced magnitude, cause a slight amount of blur. The quadrature component will tend to produce a bearing error, but by virtue of its reduced amplitude will not be as serious as might be expected so long as the phase error is not large. In summary, then, a small phase change in the higher harmonics is not considered too serious in creating bearing error.

Attenuation of the harmonics, unaccompanied by a phase change, produces blur but no actual bearing error (only difficulty in determining the exact minimum). This is by far the most serious effect encountered in practice. In order to observe this effect, the patterns in Figures 8-13 are offered.

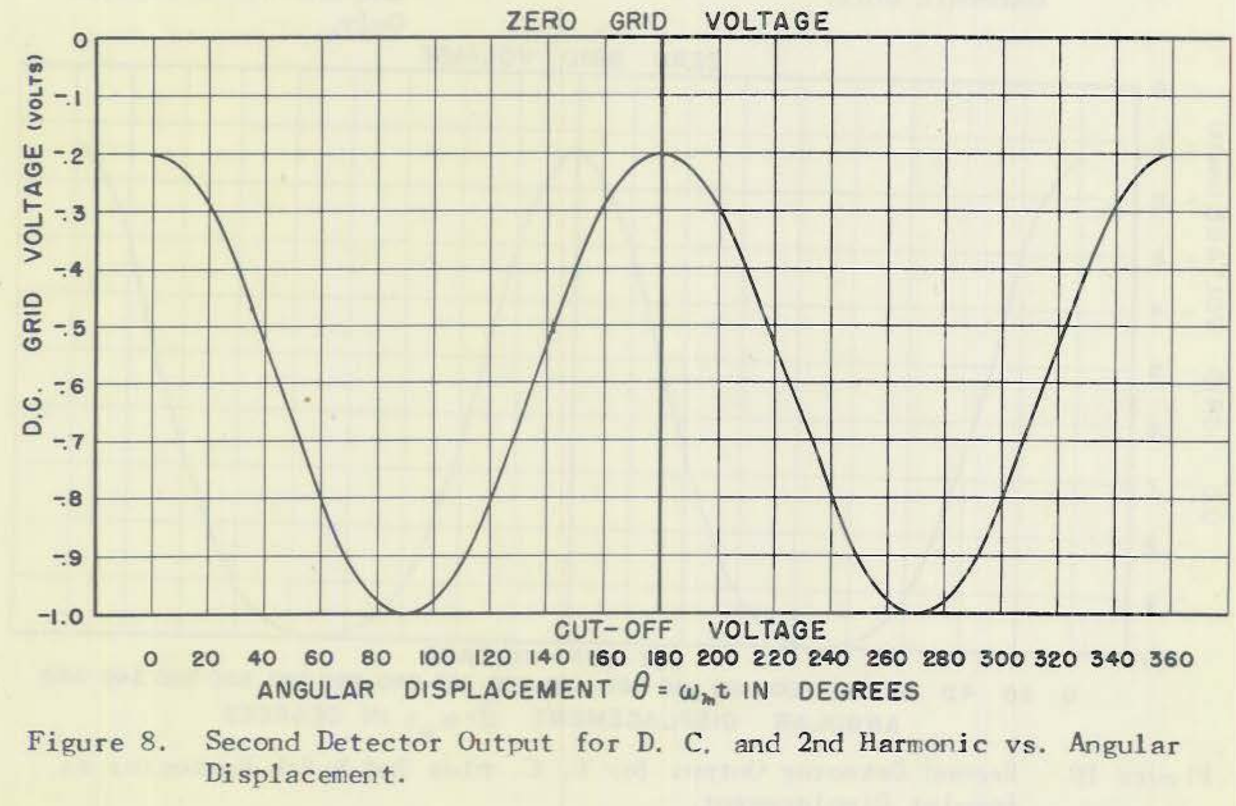

Figure 8. Second Detector Output for D. C. and 2nd Harmonic vs. Angular Displacement.

The first figure in each set shows the dynamic pulse shape and the second figure shows the resulting indicator pattern. A d-c component by itself would give a circular pattern on the CRT. No generality is lost by assuming $\alpha = 0$ in these figures.

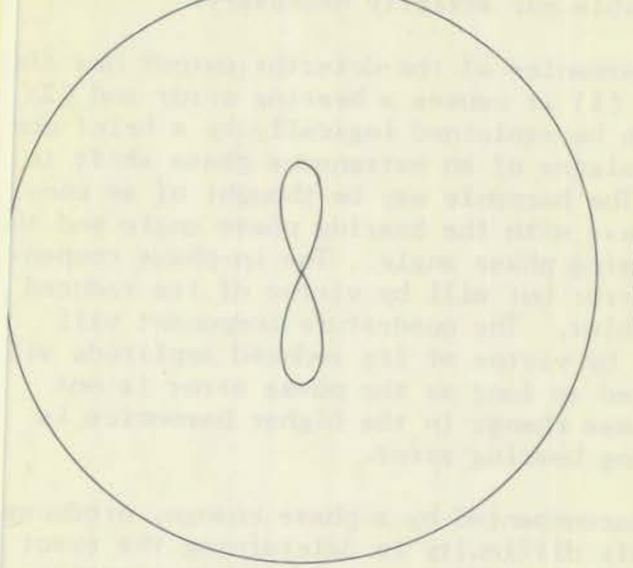

Figure 9. Pattern for D. C. and 2nd Harmonic Only.

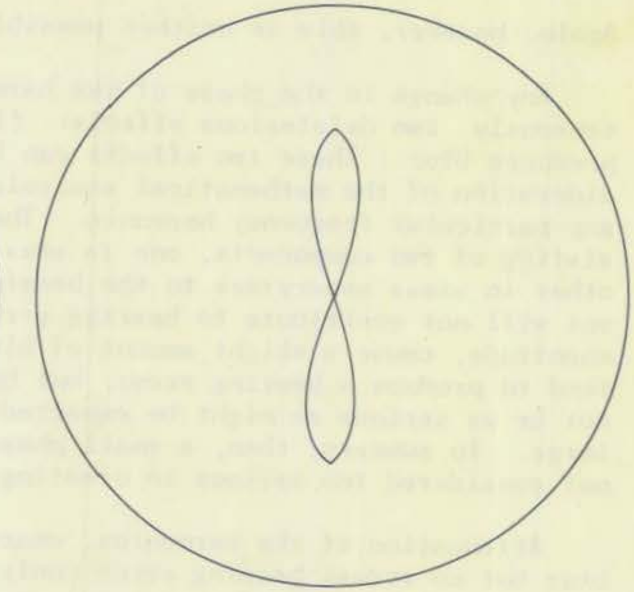

Figure 11. Pattern for D. C. plus 2nd and 4th Harmonics Only.

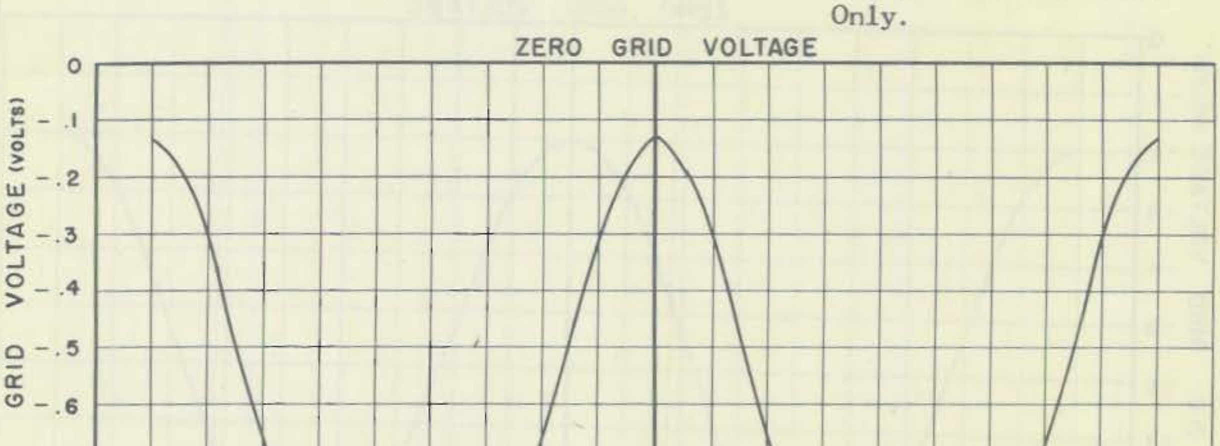

Figure 10. Second Detector Output for D. C. plus 2nd & 4th Harmonics vs. Angular Displacement.

The amount of blur created by harmonic attenuation can be measured from the curves in Figures 8, 10, and 12 and the amount of pull-in on a CRT may be measured from patterns in Figures 9, 11, and 13. For example, in the case where only harmonics up to and including the sixth are present, the

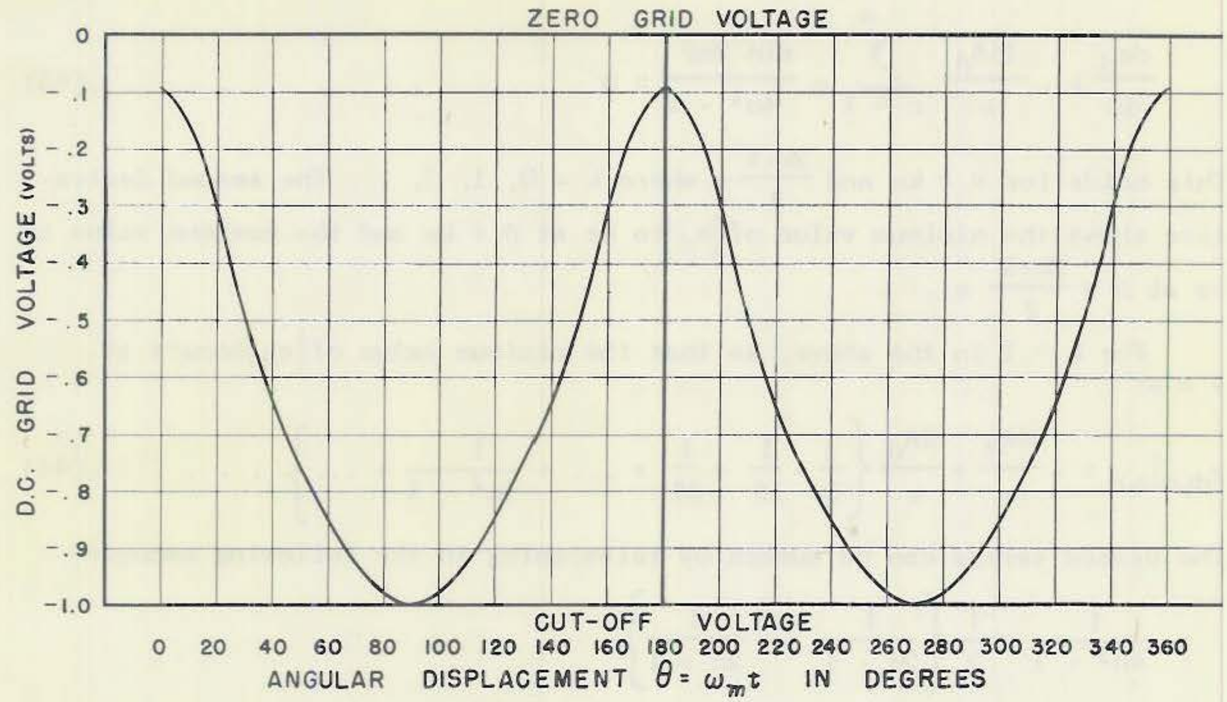

Figure 12. Second Detector Output for D. C. plus 2nd, 4th, & 6th Harmonics vs. Angular Displacement.

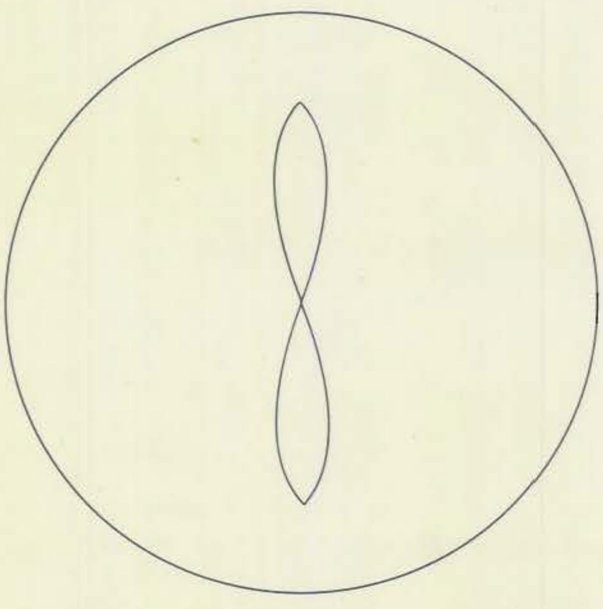

Figure 13. Pattern for D. C. plus 2nd, 4th, & 6th Harmonics Only.

blur is measured from Figure 12 as 9% and the pull-in from Figure 13 as roughly 30%. In this case, then, the nonlinearity of the d-c amplifier increases the deleterious effect of the blur over three-fold.

The bandwidth requirement upon the circuits following the second detector is established by a consideration of the maximum tolerable blur or pull-in. Just a word in passing about phase shift. In order to insure against error, the phase changes in the circuits following detection should be kept relatively small over the pass-band; i.e., until attenuation begins to take over. The relationship between blur and harmonic attenuation will now be obtained. To formulate blur mathematically, it is necessary once more to obtain the maximum and minimum dynamic values of e_d from equation (42). Letting $\theta = \omega_m t$ and $\alpha = 0$:

$$\frac{de_d}{d\theta} = -\frac{16A_d}{\pi}\sum_{n=1}^{\infty} n\,\frac{\sin 2n\theta}{4n^2-1} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (43)$$

his holds for $\theta = k\pi$ and $\frac{2k-1}{2}\pi$ where k = 0, 1, 2, ... The second deriva-
ive shows the minimum value of $|e_d|$ to be at $\theta = k\pi$ and the maximum value to
e at $\theta = \frac{2k-1}{2}\pi$.

For k = 1 in the above, so that the minimum value of $|e_d|$ occurs at
$= \pi$:

$$'d(min) = -\frac{4A_d}{\pi} + \frac{8A_d}{\pi}\left\{\frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \ldots + \frac{1}{4n^2-1} + \ldots\right\} \quad \ldots\ldots \quad (44)$$

'he braced series can be summed by telescoping in the following manner:

$$\frac{1}{4n^2-1} = \frac{1}{2}\left\{\frac{1}{2n-1} - \frac{1}{2n+1}\right\}$$

i.e.,

$$\frac{1}{3} = \frac{1}{2}\left\{1 - \frac{1}{3}\right\}, \text{ etc.}$$

Therefore:

Series
to n terms

$$= \frac{1}{2}\left\{1 - \frac{1}{3}\right\}$$
$$+ \frac{1}{2}\left\{\frac{1}{3} - \frac{1}{5}\right\}$$
$$+ \frac{1}{2}\left\{\frac{1}{5} - \frac{1}{7}\right\}$$

$$+ \frac{1}{2}\left\{\frac{1}{2n-5} - \frac{1}{2n-3}\right\}$$
$$+ \frac{1}{2}\left\{\frac{1}{2n-3} - \frac{1}{2n-1}\right\}$$
$$+ \frac{1}{2}\left\{\frac{1}{2n-1} - \frac{1}{2n+1}\right\}$$

$$\text{Sum} = \frac{1}{2}\left\{1 - \frac{1}{2n+1}\right\} = \frac{2n}{2(2n+1)}$$

Therefore,

$$\sum_{n=1}^{n} \frac{1}{4n^2 - 1} = \frac{n}{2n + 1}.$$

Therefore,

$$\left[e_{d(min)}\right]_o^n = -\frac{4A_d}{\pi} + \frac{8A_d}{\pi} \cdot \frac{n}{2n + 1}$$

$$= -\frac{4A_d}{\pi(2n + 1)}$$

$$\therefore \left[e_{d(min)}\right]_o^n = -\frac{4A_d}{\pi(2n + 1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (45)$$

For maximum value of e_d at $k = 1$ so that $\theta = \pi/2$:

$$e_{d(max.)} = -\frac{4A_d}{\pi} + \frac{8A_d}{\pi}\left\{-\frac{1}{3} + \frac{1}{5} - \frac{1}{35} + \ldots + \frac{(-1)^n}{4n^2 - 1} \pm \ldots\right\} \quad \ldots\ldots (46)$$

Unfortunately it is not possible to obtain the summation of this series to n in terms in simple closed form; however, a numerical term-by-term evaluation is practicable since the series converges quite rapidly. For convenience let the sum of n terms be denoted by S_n so that:

$$\left[e_{d(max.)}\right]_o^n = -\frac{4A_d}{\pi}(1 - 2\ S_n) \quad \ldots\ldots\ldots\ldots\ldots\ldots (47)$$

It can be shown that the average of S_{19} and S_{20} gives a numerical value whi agrees with S_∞ to four significant figures. Thus, only 20 values of S_n nee be computed and the subsequent values can be taken as S_∞. It is not difficult to show that $S_\infty = \frac{1}{2} - \frac{\pi}{4} = -0.2854$ to four decimal places and this figure will be used for S_{21} and all higher values.

Using equations (45) and (47) the blur can be written as:

$$\text{Blur} = \frac{1}{(2n + 1)(1 - 2\ S_n)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (48)$$

for harmonics of f_m up to and including the $2n^{th}$.

The percentage of blur is given by multiplying this value by 100. A plot of the percentage of blur as a function of the highest even harmonic passed is given in Figure 14. Although this blur has the same deleterious effect as that previously discussed, the equation (48) holds after detection where as equation (34) holds before detection. In Figure 15 is given the relationship between blur and pull-in for the d-c amplifier characteristic of

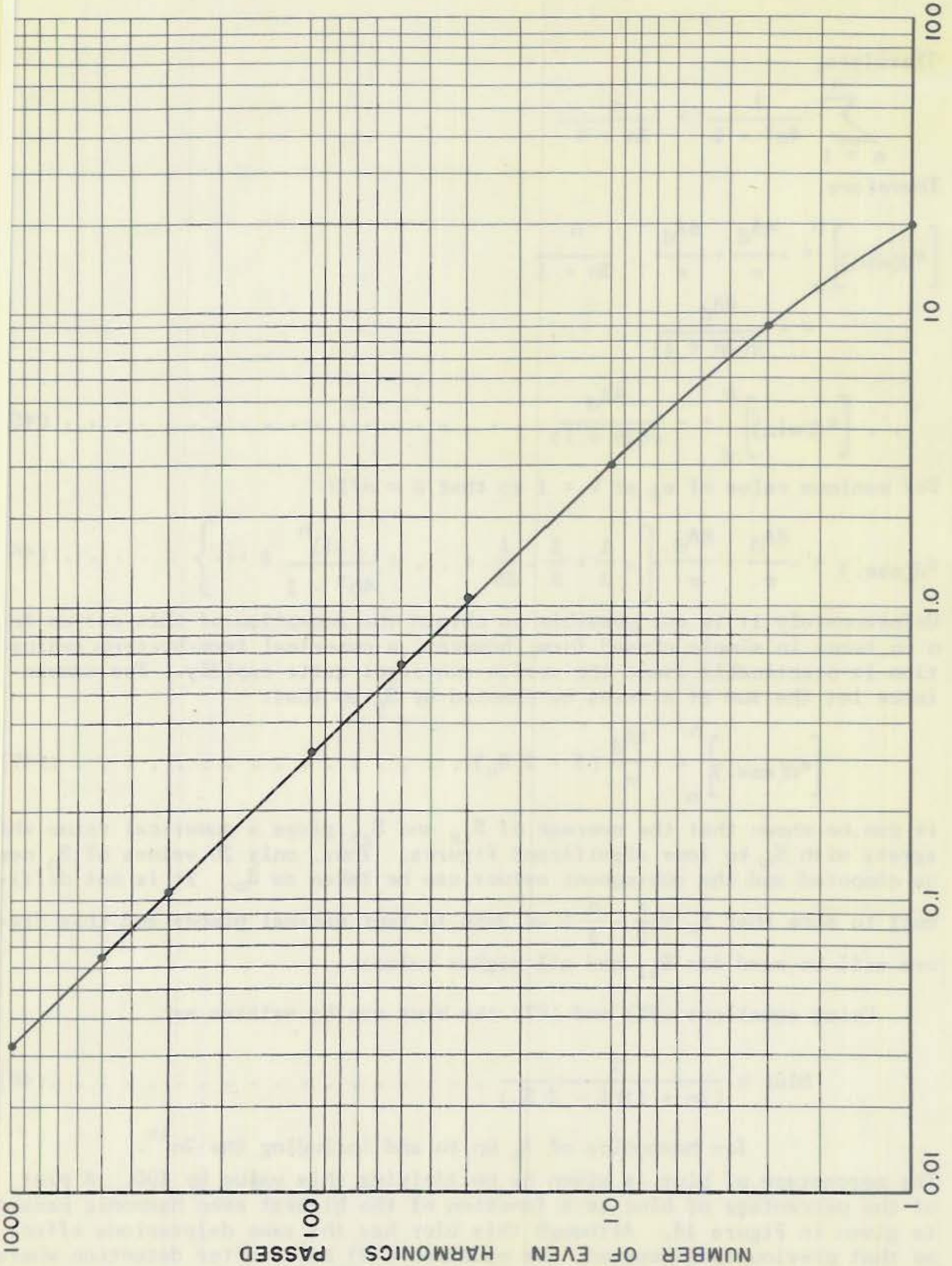

Figure 14. Percentage of Blur as a Function of Number of Even Harmonics of Modulation Frequency Passed Through D-C Amplifier.

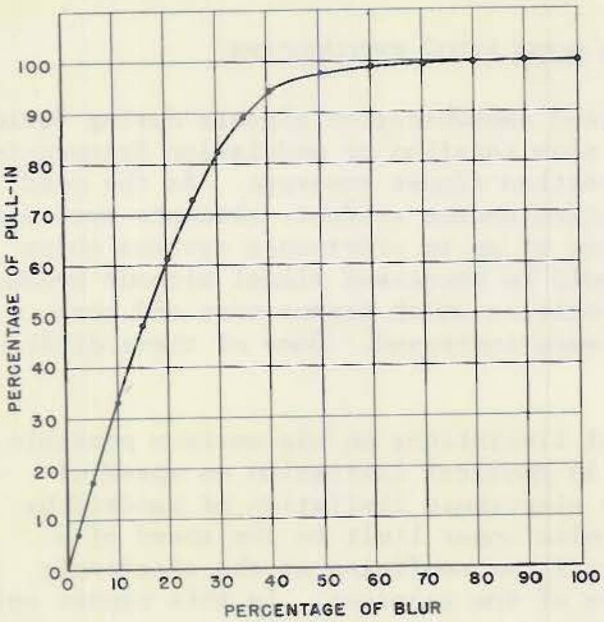

Figure 15. Relationship Between Blur and Pull-in for a Typical Equipment.

Figure 5. Here the gain of the receiver was so varied as to keep the pattern just closed at the center of the indicator CRT; i.e., maximum envelope just sufficient when rectified to cut off the d-c amplifier tube.

A few examples will illustrate the significance of Figures 14 and 15.

Example 1.

In order to obtain a pattern wit less than 0.1 percent blur, the circuits following detection must pass at least 320 even harmonics of the goniometer modulation or rotation frequency (see Figure 14). Assuming a 1200 r.p.m. rotational speed, i.e. f_m = 20 c.p.s., the bandwidth would need to be at least 12.8 kc. (320 x 2 x 20 cycles = 12.8 kc). If 1 percent blur is permitted the bandwidth would need to be only 1.3 kc. Since pull-in of the pattern on the CRT is perhaps easier to measure than blur, with the aid of the figures it can be calculated in this example a pull-in of less than 5% requires a bandwidth of at least 1 kc.

Example 2.

Suppose that in an electronic goniometer it is found especially convenient to use 60 cycle modulation in the balanced modulators. Then, for less than 0.1 percent blur, the bandwidth requirement would be at least 38.4 kc. If 1 percent blur were tolerated, the bandwidth could be reduced to 3.8 kc. For less than 5 percent pull-in, at least a 3 kc. bandwidth woulc be required.

Example 3.

The maximum modulation frequency known to be in use today in electronic goniometers is 150 c.p.s. Assuming this is used, then to keep the blur below 0.1 percent, a 96 kc bandwidth would be necessary. For 1 percent blur the bandwidth could be reduced to 9.6 kc. For 5 percent pull-in only a 7.5 kc. bandwidth would be required. These bandwidths can all be calculated with the aid of Figures 14 and 15.

The question arises as to just how much blur or pull-in should be tolerated. It is true that blur of 0.1 or even 1 percent means a very good direction finder pattern, but it must be remembered that this is only considering one of the numerous causes of blur. The final decision must rest in the ultimate performance specifications, but the modern trend is to desig for as little receiver blur as possible.

FUNDAMENTAL LIMITATIONS ON ROTATION OR MODULATION FREQUENCIES

The use of short duration and pulsed communication signals during World War II brought to light the fact that slow rotation or modulation frequencies were inadequate to insure complete direction finder coverage. As the need for higher and higher scanning frequencies became evident, attempts were made to increase the goniometer rotation rate or go to electronic systems where presumably the modulation frequency could be increased almost without bound. However, there arose a number of difficulties which became more and more serious as the modulation frequencies were increased. Some of these difficulties will be discussed here.

There are two types of fundamental limitations on the maximum possible rotation or modulation frequencies: (1) physical limitation on speed of mechanical goniometer rotation and (2) electronic limitation of bandwidths permissible. Although there is a definite upper limit to the speed of a physical rotor, such a limit is not nearly so confining as the electronic limit set by the bandwidth requirements of the receiver. In this report only the electronic limitation will be discussed.

The material in sections 1 and 7 of the text is background for a consideration of modulation frequency limitations. There are two problems here also: (1) the phase shift linearity of the radio and intermediate frequency sections of the receiver and (2) the bandwidth of the circuits following the second detector. It has been seen that both of these problems become more important as the modulation frequency increases. It would be hazardous to set an upper limit on the modulation frequency, since it is impossible to predict ultimate bandwidth capabilities. It must be remembered that as the intermediate frequency bandwidth is increased, the over-all receiver selectivity is decreased. This means not only decreased sensitivity but also an increased tendency to notice cross talk and adjacent channel interference. These are both very serious disadvantages and cannot be overlooked. It is these factors which ultimately determine the upper limit to the goniometer modulation frequency. The factors limiting the bandwidth following detection are chiefly wiring capacitance and interelectrode tube capacitance. By very careful design and selection of vacuum tube types these stray capacities can be reduced substantially.

In the next section several methods will be mentioned which have been used or suggested to eliminate the need for such large bandwidths both before and after the second detector.

SYSTEMS FOR REDUCING BANDWIDTH REQUIREMENTS

A number of systems are in use and many others have been suggested for reducing the bandwidth requirements in those direction finders employing the goniometer principle. Only a few representative methods will be mentioned here. They can be placed in two classes: (1) those reducing the bandwidths up to the second detector and (2) those reducing the bandwidth following the second detector.

The possibility of using a relatively high modulation frequency and then employing two separate receiver channels for the amplification of the side-bands, one for each side-band, was one of the first to be considered. The bearing still would be, in this case, a function of the relative phase shift of the side-bands and blur would be a function of side-band amplitude prior to detection. This means that the receiver channels would need to be identical throughout in both amplitude and phase shift - a situation which has been seen in the past to be very difficult to obtain. However, a modification of this system may be considered, that of using two different modulation frequencies to tag the antenna pick-ups. In reality, this turns out to be more than a slight modification, for the principle of indication is completely different. The amplitudes, rather than the phases of the two different frequencies, become the bearing criterion. However, the relative phases of the modulation frequencies must be preserved and must be present at the second detector along with the tagged signals. Differential detection takes place in which the signals are brought once again into synchronization to produce bearing indication. The merits of this system will not be considered here, although it can be said that the bandwidth requirements in the receiver are less severe than in the system previously discussed.

The logical way to reduce the bandwidth requirements of the circuits following detection seems to be to alter the shape or wave-form of the final output so as to reduce the harmonic content. Attempts along this line have resulted in simplification down to a single frequency sine wave. This could be some multiple of the goniometer modulation frequency and hence, being quite stable, would require only a relatively narrow bandwidth. This work is not the proper place to go into detail on specific systems, but any new system which simplifies the wave-form of the second detector output will reduce the bandwidth requirements accordingly. A word of caution may be inserted here to keep in mind that a system for reducing the bandwidth in one part of the receiver might demand an increase in the bandwidth of another part.

CONCLUSIONS

1. The output from a goniometer (of the type considered in this report), whether it be electronic or mechanical, is equivalent in form to the output of a carrier suppression balanced modulator; i.e., the output consists only of two side-bands whose frequencies are symmetrically displaced about the incoming signal carrier by the goniometer modulation or rotation frequency. The side-bands have equal amplitudes and their relative phase shifts contain the bearing information.

2. In passing through both the radio and intermediate frequency sections of the receiver, the two side-bands undergo a change in amplitude and a change in phase shift, with the possibility that these changes may not be the same for both side-bands. An unequal change in the phase-shift of the two side-bands causes bearing error. An unequal change in the amplitude of the side-bands causes reduced pattern sharpness or blur.

3. The amount of bearing error created by an unequal phase shift imparted to the two side-bands up to detection is numerically equal to one half the

difference between the phase shifts. This error can be calibrated out of the system provided the phase characteristics of the circuits have constant slope with respect to frequency over the effective tuning range of the receiver.

4. The amount of blur created by unequal amplitudes of the two side-bands is given by $\frac{1-\gamma}{1+\gamma}$ where γ is the ratio of the side-band amplitudes. This function is plotted in Figure 4. Blur, which is responsible for obscured bearings, is defined quantitatively as the ratio between the minimum value and the maximum value of the voltage envelope resulting from the combination of the side-bands.

5. The effect of blur is made manifest at the cathode ray indicator in a form called pull-in. This is observed as a retraction or pulling-in of the tips of the propeller-shaped pattern and is defined quantitatively as the distance from one tip of the pattern on the CRT to the nearest periphery divided by the effective radius of the CRT. Pull-in is a function of blur, receiver gain, and d-c amplifier linearity. In practice it is generally independent of receiver gain which is set so that maximum signal is just sufficient to cut off the d-c amplifier tube. The relationship between pull-in and blur, for the particular d-c response curve shown in Figure 5, is given in Figure 15.

6. Carefully designed frequency conversion and carefully designed linear detection should insure against bearing error and blur in these sections of the receiver.

7. The output from the second detector consists of pulsating direct current of repetition rate equal to twice the goniometer modulation frequency following the same form as a full rectified sine wave. Harmonic analysis reveals a steady state component and alternating current components of decreasing amplitude, which are even harmonics of the modulation frequency.

8. The ultimate pattern sharpness is determined by the bandwidth of the circuits following the second detector. Figure 14 gives the percentage blur following detection as a function of the number of even harmonics passed. It is found that the bandwidth required for a specified pattern sharpness is proportional to the goniometer modulation frequency.

9. The upper limit to the rate of goniometer rotation or electronic modulation is dictated by two bandwidth requirements: (1) the intermediate frequency bandwidth needed to maintain linear side-band phase relations, and (2) the bandwidth of those circuits following the second detector required to insure pattern sharpness. The first of these is in conflict with the selectivity required to prevent adjacent channel interference and works against high sensitivity. The second of these is limited by inter-electrode tube capacitance and unavoidable wiring capacitance.

10. Some fundamental methods are mentioned which reduce the large bandwidth requirements demanded by increased modulation frequencies. Two sys-

tems are required: (1) for reducing the needed intermediate frequency bandwidth and (2) for reducing the excessive bandwidth needed following detectio

ACKNOWLEDGMENTS

An acknowledgment is made to Dr. D. C. Harkin, Mathematics Consultant of Radio Division III of the Naval Research Laboratory, who rendered valuable assistance in the preparation of this report.

A further acknowledgment is made to the Messrs. Rupert L. Haskins, A. G. Loveberg, John R. Gruber, and James H. Trexler, all of Radio Division II of the Naval Research Laboratory who supplied much of the needed data and who reviewed the rough draft of the manuscript and made numerous valuable suggestions.

Much of the credit for the conception of this work is due to Mr. Robert W. Annis, also associated with Radio Division II of the Naval Research Laboratory, who did pioneering work on side-band characteristics and who brough to light the need for a mathematical analysis such as the one presented here

APPENDIX I

THE CONCEPTS OF BLUR AND PULL-IN

In any direction finder which operates on the null principle, one of the most significant factors in reading accurate bearings is the sharpness of bearing indication. Throughout the years of development of the art, this quality of "sharpness of indication" was known under a variety of names; but, with the advent of the automatic bearing indicator, with its propeller-shaped CRT pattern, the term "blur" appeared to be the most descriptive and gradually came into common use. Blur may be qualitatively defined as any reduction in the definition of the bearing pattern, whether aural or visual, resulting from any diminution in the sharpness of the indication. It may be caused by a multitude of factors some of the more serious of which are odd-phase re-radiation of the incoming signal, unbalance in the antennas, unbalance in the transmission lines, goniometer leakage, and insufficient receiver bandwidths. In this work only that blur caused by insufficient receiver bandwidths is considered.

A general quantitative definition of blur can be given which holds regardless of the cause. Blur is the voltage ratio of the minimum to maximum signal envelope under dynamic conditions. The numerical value will vary with respect to the point in the system at which it is measured and the conditions of measurement, such as signal level, etc., so that the expression is not complete unless all conditions of measurement are included. In this report only two causes of blur are discussed: (1) blur caused prior to the second detector by the unequal amplitude of the two side-bands, and (2) blur caused by a phase shift or attenuation in harmonics of the goniometer modulation frequency. The manner by which both of these change the ratio of minimum to maximum signal is discussed in Sections 1 and 7 of the text.

Although the measurement of blur from all causes may be achieved by direct measurement of pull-in on the CRT, it would be difficult if not impossible to isolate by this method the contributions to over-all blur from each separate part of the receiver. It would be possible to obtain the blur at any point in the receiver by applying the signal to an external oscilloscope and actually measuring the minimum and maximum envelope voltages. However, this would have to be done under dynamic conditions and would not be too accurate. Perhaps the simplest way to determine the blur prior to detection is to obtain a selectivity curve and apply equation (35). The easiest way to determine the blur after detection is to obtain an audio response curve and apply the graph of Figure 14.

Pull-in is a phenomenon unique to those direction finders employing automatic bearing indicators which have propeller-shaped patterns observed on a CRT. It may be qualitatively defined as the movement of the ends of the pattern toward the center of the CRT resulting from blur. Under ideal conditions, i.e., with no pull-in, the ends of the pattern would be sharp and extend to the periphery of the scope. As either the signal strength or gain of the receiver was increased, the pattern would get thinner and eventually become a straight line, but the ends would still remain at the outer edge of the scope. However, when blur exists, this does not happen. Instead, depending upon the amount of blur present, the ends of the pattern begin to round off and move toward the center of the scope. As the gain is increased the tendency becomes more prominent until finally a mere circle or dot remains at the center of the scope. This phenomenon has been named pull-in.

Pull-in is measured as the ratio of the magnitude of the movement of the pattern ends toward the center to the radius of the (no signal) circle, with the receiver gain so adjusted that the center of the pattern just closes. This means that the maximum voltage being fed the d-c amplifier is just sufficient to cut it off.

In order to conform to the definitions previously established by the Laboratory and in order to avoid confusion whenever blur is caused by factors external to the receiver, blur and pull-in are not considered synonymous. Pull-in may be considered the visual manifestation of blur and under any given set of conditions a definite relationship may exist between them. For example, in Figure 15 the relationship between the overall blur existing prior to d-c amplification and pull-in is given for the characteristic shown in Figure 5. A similar curve relating blur and pull-in may be obtained for any equipment provided the characteristic of the d-c amplifier can be determined. Pull-in, therefore, is not only a function of blur but also of the d-c amplifier characteristic.

APPENDIX II

HARMONIC ANALYSIS OF SECOND DETECTOR OUTPUT

In this appendix the harmonic content of the second detector output will be obtained. The wave-form to be analyzed previously has been given in section 5 equation (41) as follows:

$$e_d = -2A_d\left|\sin(\omega_m t + \alpha)\right|$$

The absolute value signs in this equation prevent the immediate determination of the frequency components present, and hence a Fourier analysis is suggested. The wave-form of the function to be analyzed is sketched in Figure 16. The analysis can be made simpler upon applying the transformation $x = 2(\omega_m t + \alpha)$ and then $e_d = -2A_d\left|\sin\frac{x}{2}\right| = f(x)$. Since $\sin x/2$ does not change sign in the interval $0 \leq x \leq 2\pi$, the complete analysis can be carried out within this interval on $f(x) = -2A_d \sin\frac{x}{2}$. As can be seen from the boundary conditions in Figure 16, this transformation greatly simplifies the work.

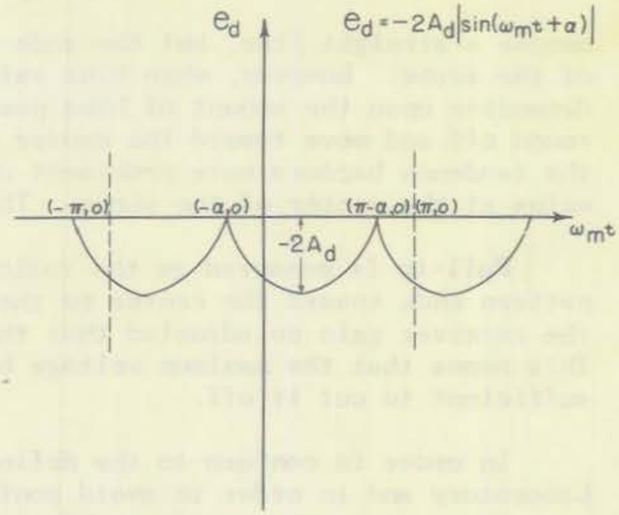

Figure 16. Wave-form of Second Detector Output

It is then possible to determine a function such that $f(x) = -2A_d \sin\frac{x}{2}$ in $0 \leq x \leq 2\pi$ and such that $f(x) = f(-x)$. Since such a function will be an *even* function, let

$$f(x) = \frac{A_o}{2} + \sum_{n=1}^{\infty} A_n \cos nx \quad \ldots \ldots \ldots \ldots \ldots \ldots (49)$$

where $A_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$

The evaluation of A_n follows:

$$A_n = \frac{1}{\pi}\int_0^{2\pi} -2A_d \sin\frac{x}{2}\cos nx\,dx \quad \ldots \ldots \ldots \ldots \ldots \ldots (50)$$

Let $u = \cos nx$ $\qquad dv = \sin\frac{x}{2}\,dx$

$du = -n \sin nx\,dx$ $\qquad v = -2\cos x/2$

$$\therefore\ A_n = -\frac{2A_d}{\pi}\left[-2\cos nx \cos\frac{x}{2}\right]_{x=o}^{x=2\pi} + \frac{2A_d}{\pi}\int_0^{2\pi} 2n \cos\frac{x}{2}\sin nx dx \quad . \ . \ . \ . \ . \ (51)$$

To evaluate $\int_0^{2\pi} \cos\frac{x}{2}\sin nxdx$, let:

$$p = \sin nx \qquad dq = \cos\frac{x}{2}dx$$

$$dp = n\cos nxdx \qquad q = 2\sin\frac{x}{2}$$

$$\therefore \int_0^{2\pi}\cos\frac{x}{2}\sin nxdx = \left[2\sin nx\sin\frac{x}{2}\right]_{x=o}^{x=2\pi} - \int_0^{2\pi} 2n\sin\frac{x}{2}\cos nxdx \quad . \ . \ . \ . \ (52)$$

Employing equation (50) in (52) and evaluating the limits, the last integral becomes:

$$\int_0^{2\pi}\cos\frac{x}{2}\sin nxdx = \frac{\pi n A_n}{A_d}$$

Substituting this last expression in equation (51) gives:

$$A_n = -\frac{2A_d}{\pi}\left[-2\cos nx\cos\frac{x}{2}\right]_{x=o}^{x=2\pi} + \frac{4nA_d}{\pi}\cdot\frac{\pi n A_n}{A_d}$$

$$= -\frac{2A_d}{\pi}(2 + 2\cos 2\pi n) + 4n^2A_n$$

Therefore:

$$A_n(4n^2 - 1) = \frac{8A_d}{\pi}$$

$$\left.\begin{aligned} \therefore\ A_n &= \frac{8A_d}{\pi(4n^2 - 1)} \\ \therefore\ \frac{A_o}{2} &= -\frac{4A_d}{\pi} \end{aligned}\right\} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (53)$$

The final result in equation (49) then becomes:

$$f(x) = -\frac{4A_d}{\pi} + \sum_{n=1}^{\infty} \frac{8A_d \cos nx}{\pi(4n^2 - 1)}$$

Reversing the transformation, $x = 2(\omega_m t + \alpha)$, the final form is:

$$e_d = -\frac{4A_d}{\pi} + \frac{8A_d}{\pi} \sum_{n=1}^{\infty} \frac{\cos 2n(\omega_m t + \alpha)}{4n^2 - 1}$$

This is the form of equation (42) discussed in Sections 5 and 7 of the text.